

Electrical Transport Experiments

OHM'S LAW: Empirical fact, observed in all conductors (unless the current is very "large").

Voltage \propto Current

$$V = R \cdot I ; R: \text{Resistance, constant of proportionality}$$

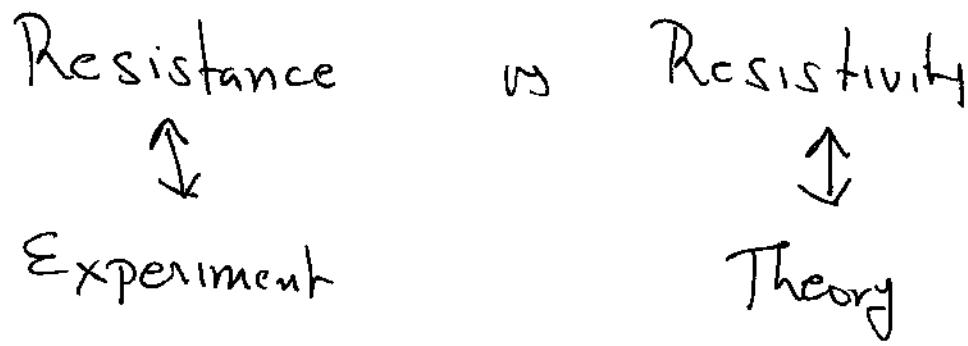
~~constant~~

Q. Why do we have a linear relationship? It is not at all obvious that under the action of an external electric field, the electrons should not accelerate but flow at a constant rate & that their flow rate should be proportional to the Voltage.

Q. What determines the value of resistance?
 & how & why it changes from material to material, & on changing the external conditions - temperature, magnetic field, pressure, etc.

It is a task of old-fashioned Solid State Physics to answer these questions (among many other related ones). In some of the experiments, we will try to figure out a way of connecting theory & ~~measurements~~ measurements.

(2)



We want to have a description of the conduction process. Since it involves electrons (& many many of them), we will need to use quantum statistical mechanics, [In this course, we will try to get away with as little quantum stat. mech. as possible], it is clear that we will need to change the variables used to describe the problem.

What is measured
[Experiment]

Voltage [Volts]

Current, I [Amperes]

Resistance [Ohm]

Conductance, G [Mho, Siemens]

What is calculated

Electric field, \vec{E}
[Volts per metre]

Current density, \vec{j}
[Amperes-metre $^{-2}$]

Resistivity, $\bar{\rho}$
 $\Omega\text{-m}$ [3D]; Ω [2D]
 $\Omega\text{-m}^{-1}$ [1D]

Conductivity, $\bar{\sigma}$
 $\bar{\sigma} = [\Omega]^{-1}$

(3)

Ohm's Law

$$V = RI$$

$$I = \left(\frac{1}{R}\right)V = GV$$

[Circuit theory]

Ohm's law

$$\vec{E} = \bar{\rho} \vec{j}$$

$$\vec{j} = \bar{\sigma} \vec{E}$$

[Physics]

→ Note: The quantities in the right column are vectors [\vec{E}, \vec{j}] or tensors/scalars: $\bar{\sigma}, \bar{\rho}$

It does not make much sense to give direction to macroscopic circuit quantities like current I , to give a g direction. This is sometimes done but you have to be cautious.

Q. Why do we need to talk in terms of electric fields, current densities & conductivity?

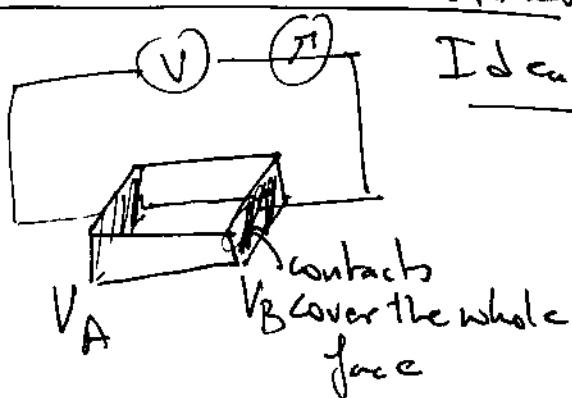
These are microscopic quantities & a description in terms of $\vec{E}, \vec{j}, \rho, \sigma$ instead of V, I, R allows us to bypass the details of the measurement & sample geometry. Currents flowing within a sample depend critically on the boundary conditions. It is only in the most trivial cases that the correspondence between resistance (R) & resistivity (ρ) is as trivial as $R = \frac{\rho l}{A}$

(4)

$R = \frac{\rho l}{A}$ will only work when the sample has a regular shape (cuboid or cylinder, e.g.)

& the current density within the sample is spatially uniform; $j(x, y, z) = j = \text{constant}$.

Ideal vs Real Geometries



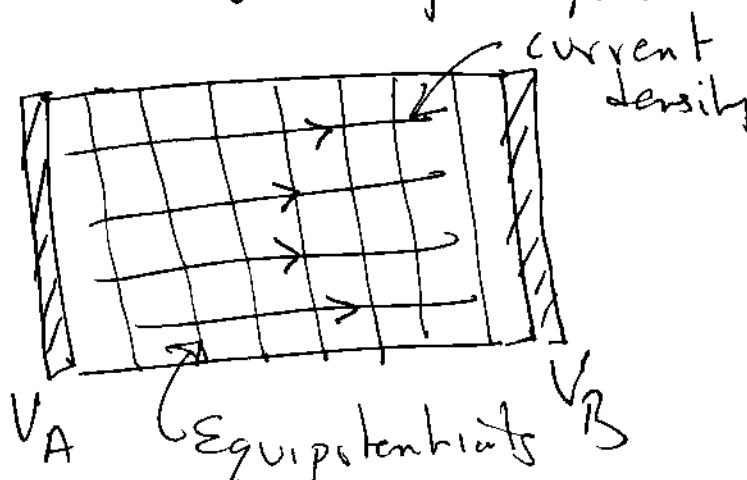
Ideal Geometry

- Contacts on the two sides cover the whole faces
⇒ each face is an equipotential surface

- Sample is of a perfectly regular shape.

Only then is $R = \frac{\rho l}{A}$

To understand this, let us look at a 2D projection of the sample (viewing it from top)

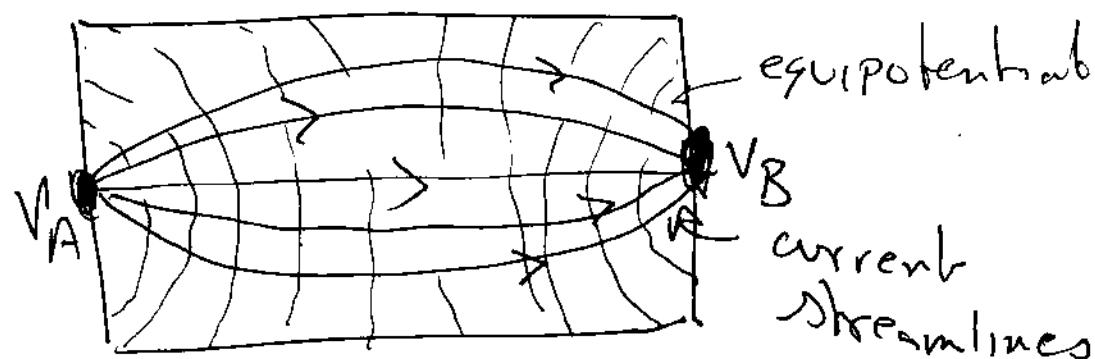


$$\begin{aligned} V_{AB} &= \int_A^B \vec{E} \cdot d\vec{l} \\ &= \int \rho \vec{j} \cdot d\vec{l} \end{aligned}$$

$$\begin{aligned} &\& I = \iint \vec{j} \cdot d\vec{s} = (\vec{j} * \text{Area}) \quad = f[\vec{j}] \cdot l \\ &\Rightarrow \boxed{V_{AB} = \frac{\rho l}{A}} \quad \left[\begin{array}{l} \text{if } j \text{ is const} \\ \text{if } j = \text{const} \\ f = \text{const} \end{array} \right] \end{aligned}$$

So you see, we only recover $R = \frac{P}{A}$ for the condition that J is spatially constant. (5)

Even for a regularly shaped sample, if we shrink the size of contacts



$$j = j(x, y, z) \neq \text{const.}$$

$$V'_{AB} = \int_A^B \vec{E} \cdot d\vec{l} = \int_A^B \vec{P} \vec{j} \cdot d\vec{l} = \iint_A^B \vec{j}(x, y, z) \cdot d\vec{l}$$

~~∅~~ — Depends on the
size of contacts!

— Would depend on the position of the
contacts.

[Of course the dimensional relationship between P & R is always valid (units of P & units of R are fixed). Hence ~~∅~~ the difference between V_{AB} & V'_{AB} ~~can~~ just be accommodated by a "geometric" conversion factor for a given expt;

(6)

To summarize:

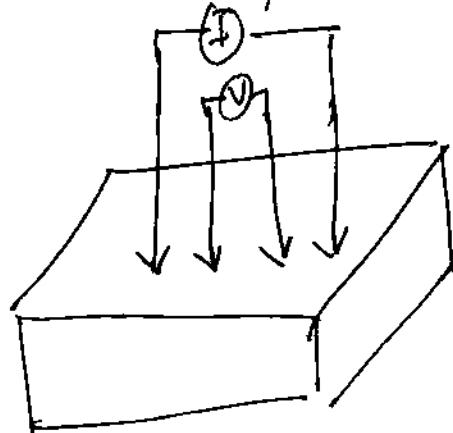
Since the current density $\vec{j}(x, y)$ is not the same in the two examples [even though f is the same & sample dimensions are the same], we will measure different resistances in the two examples.

Hence

- ① We shift the theoretical description to $(f, \vec{E}, \vec{j}, \sigma)$
- ② Of course in actual experiment, it is (V, I, R) that we measure. So we will have to deal with the painful (interesting, if you like) task of converting R to f by keeping in mind the boundary conditions.
- ③ Problem only becomes more messy (interesting) when we apply an external magnetic field.

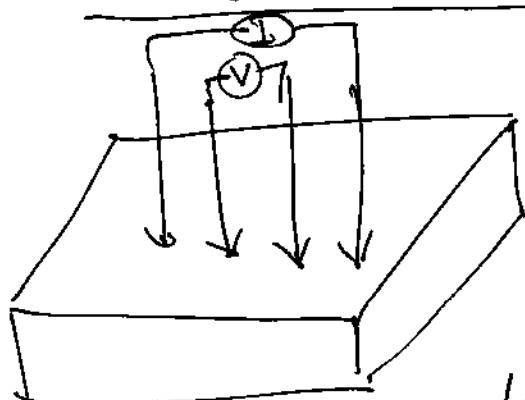
At this point, it is interesting to look at
the sample geometry in the three experiments. (7)

① Four-probe resistivity



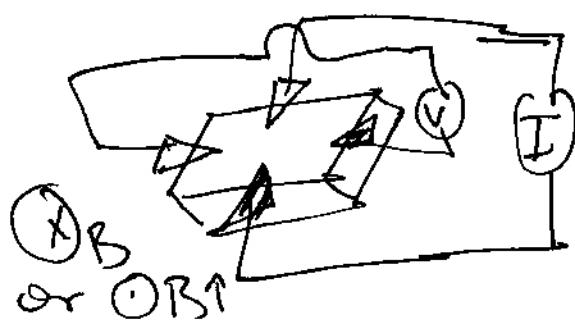
- Equidistant contacts, somewhere in the middle
- 4 contacts
- touch the top of the sample.

② MAGNETO RESISTANCE



- Same as the 4-probe set-up
- but with magnetic field \perp to current flow (more or less).

③ HALL EFFECT



- Contacts on the periphery [soft - of]
- magnetic field \perp to current flow
- [Van der Pauw geometry]

Why do we need 4 wires instead of 2 to measure resistance accurately?

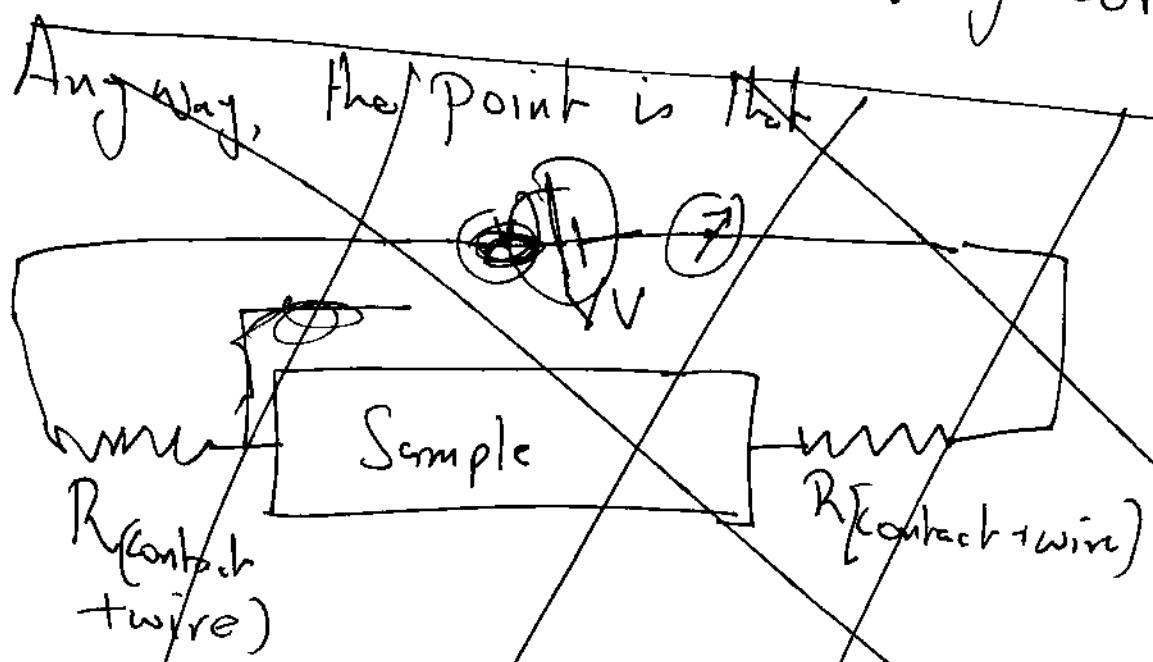
There are two reasons why a 4-wire geometry is important.

- ① We do not want to ~~not~~ include the ~~resistor~~ resistance of the wire in our resistance measurement. This is usually a very small effect unless you have very long & thin wires & the sample resistance is less than say 100Ω .
- ② The fact that you are connecting two different materials to make contacts (e.g. copper & germanium) in ~~our~~ example can imply that the ~~junction~~ junction can itself have some resistance. This is called the contact resistance. One simple way to understand this is that any two materials will have different electron densities. When I join them, the chemical potential between the two should be the same & this can imply a transfer of electrons from one material to the other.

→ One material will have a slight net positive charge & the other a slight net negative charge.

(9)

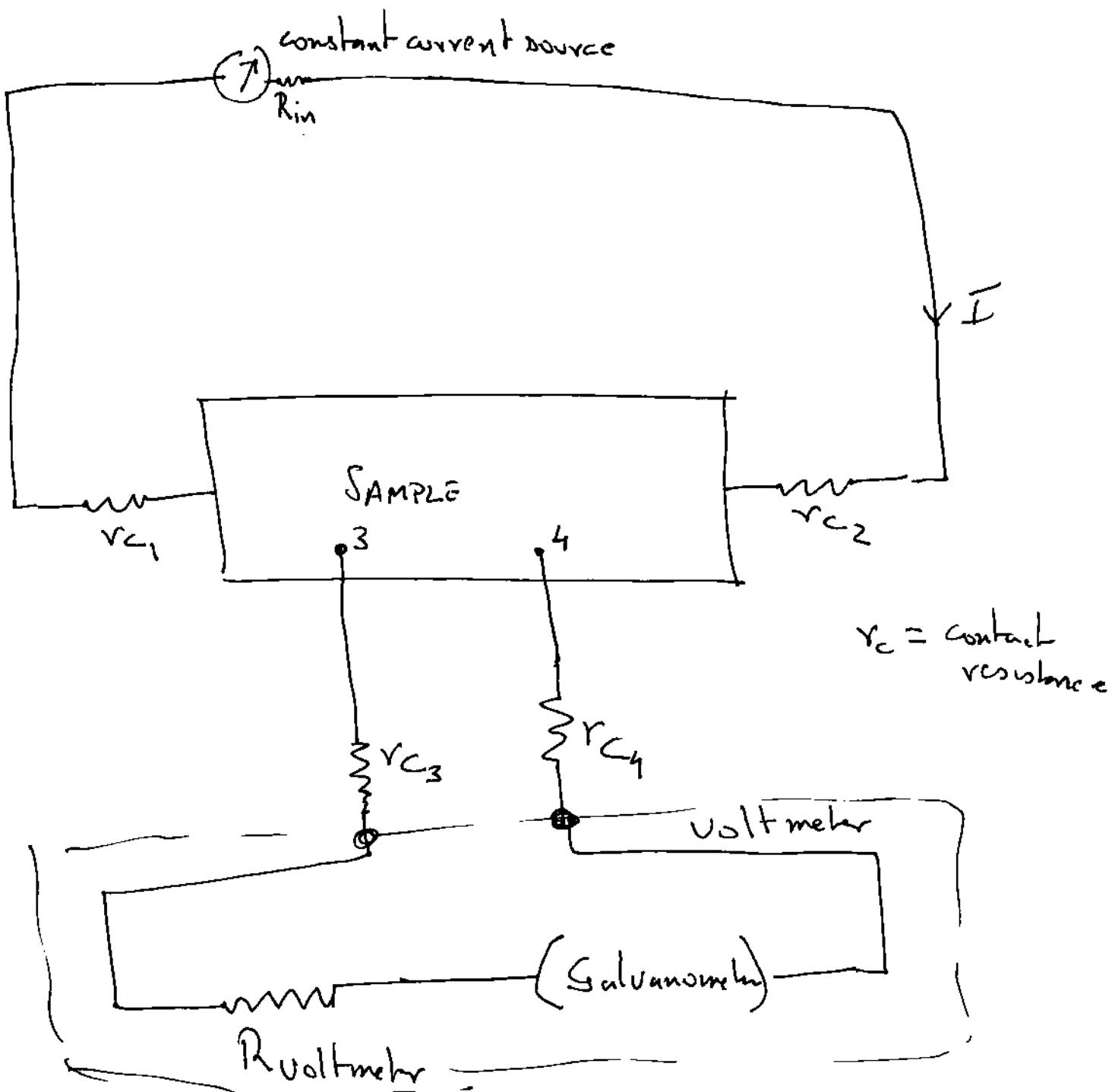
close to the junction region. This implies that electrons will not be able to freely move across the junction. [This situation described results in formation of a diode, called the Schottky barrier]. One can also visualize other types of barriers. E.g. if there is a thin ~~oxide surface layer~~ non-conducting oxide layer or grease on the surface, the electrons may need to tunnel across it or need some energy to cross the potential barrier they see. In this case, the barrier may simply be modelled as a resistance which is not dependent on the direction of current.



~~Anyways what I have drawn above is wrong. You cannot have a voltmeter &~~

(10)

In the 4-probe [4-wire] resistance measurement method, the current supply & the voltmeter are connected to different parts of the sample.



- Constant current source, by definition passes a constant current regardless of the resistance. Hence r_C does not matter
- As long as the voltmeter internal resistance, $R_{\text{voltmeter}}$ is much much larger than r_C , again r_C does not determine the measured voltage. Hence a true potential drop (more or less) can be measured by the voltmeter between points 3 & 4.

(a)

DRUDE THEORY

- Simplest zeroth order calculation of conductivity

└ Explains → Ohm's law & Hall effect

→ Geometric magnetoresistance

→ Cannot explain physical magnetoresistance.

[Please try to clearly understand the difference between geometric & physical magnetoresistance. Our experiment deals with geometric magnetoresistance]

~~Don't~~

B) Introduction: We consider the ^{average} behaviour of an ~~hypothetical~~ electron in a conducting solid.

[Electrically conducting solid = free electrons in a background of neutralizing +ve charge]

As we saw, Ohm's law $\Rightarrow \vec{G} = \rho \vec{J}$

$$\text{or } \vec{J} = \sigma \vec{E}$$

The first task is thus to make a theory for this observed linear relationship. Naturally, if we

simply had free electrons in an electric field,

$$\frac{d\vec{p}}{dt} = \vec{F} = q\vec{E} \Rightarrow \vec{p}(t) = qEt + \vec{p}_0 \Rightarrow \text{accelerated motion.}$$

To recover ~~and~~ the possibility of steady-state solution where the electrons don't constantly accelerate but reach a steady state drift velocity, we introduce an additional ingredient, collisions. (b)

Assumptions

(a) Crystal imperfections cause scattering of electrons which gives rise to finite conductivity & steady state behaviour.

(a) The electrons will scatter/collide with an imperfection an average ever τ seconds. (τ = scattering time \approx picoseconds).

(b) These collision events cause electrons to completely randomize their momenta & lose all memory of the momentum value they had prior to collision.

(c) Under the influence of an electric field, the electrons very quickly reach steady state, due to collisions.

So, we can or rather let us write the equation of motion for a typical electron under the influence of external fields, with this additional collision term (c)

$$\frac{d\vec{P}}{dt} + \frac{\vec{P}}{\tau} = q(\vec{E} + \vec{v} \times \vec{B})$$

τ : Relaxation time

Pl. see Ashcroft & Mermin for a slightly better "justification" for this relation.

In steady state,

$$\frac{\vec{P}_{ss}}{\tau} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\text{or } \vec{v}_d = \left(\frac{q\tau}{m} \right) [\vec{E} + (\vec{v} \times \vec{B})]$$

So we get an expression for the average steady state drift velocity, \vec{v}_d

Let us first consider the case when $\vec{B} = 0$.

$$\Rightarrow \vec{v}_d = \left(\frac{q\tau}{m} \right) \vec{E}$$

$$\frac{q\tau}{m} = \mu = \text{Mobility}$$

= constant of proportionality between drift velocity & electric field.

μ : how fast does an ~~an~~ average electron move under the influence of electric field.

(d)

$$\text{Current density } \vec{J} = n q \vec{v}_d$$

n : # of electrons per unit volume

$$\vec{J} = \frac{\text{Coulomb}}{m^2 - s}$$

$$\Rightarrow \vec{J} = n q \mu \vec{E} = \frac{n q^2 \sigma}{m} \vec{E}$$

$$= \sigma \vec{E}$$

$$\sigma = \frac{n q^2 \epsilon}{m} = n q \mu : \text{conductivity}$$

\Rightarrow We recover microscopic Ohm's law very easily.
 [current density is proportional to electric field].

Note:

$$\vec{J} = \sigma \vec{E}$$

$$[\text{Vector}] = [\text{scalar or tensor}] \cdot [\text{Vector}]$$

σ : Determines the proportionality between
 \vec{J} & \vec{E} [Cause & effect]

: Can also determine the directions.

$$\text{resistivity } \rho \equiv [\sigma]^{-1}; \vec{E} = \rho \vec{J}$$

(e)

Next, let us look at the full Lorentz force acting on the electrons, i.e. what if there is also an external magnetic field, say in the z-direction, $\vec{B} = B \hat{z}$

Going back to Drude's Theory, the steady state solution now is

$$\frac{\vec{p}}{z} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\text{or } \vec{v}_d = \left(\frac{qz}{m} \right) (\vec{E} + \vec{v}_d \times \vec{B}) \quad \left[\begin{array}{l} \text{let us drop the} \\ \text{subscript } d \\ \vec{v}_d = v \end{array} \right]$$

$$\Rightarrow v_x = \mu E_x + \mu v_y B_z \quad \text{--- (1)}$$

$$v_y = \mu E_y - \mu v_x B_z \quad \text{--- (2)} \quad \mu = \frac{e z}{m}$$

$$v_z = \mu E_z \quad \text{--- (3)} \quad = \text{mobility}$$

Obviously the motion along the direction of the magnetic field is unaffected.

from (1) & (2), eliminating v_y

$$v_x = \mu E_x + \mu B_z [\mu E_y - \mu v_x B_z]$$

$$\text{or } v_x = \left[\frac{\mu}{1 + \mu^2 B_z^2} \right] (E_x + \mu B_z E_y) \quad \text{--- (4)}$$

& similarly eliminating v_x between (1) & (2)

\Rightarrow

$$v_y = \left(\frac{\mu}{1 + \mu^2 B_z^2} \right) [E_y - \mu B_z E_x] \quad \text{--- (5)}$$

From ④, ⑤ & ③ we can trivially write the expressions for the current densities j_x, j_y & j_z with

$$\vec{j} = nq \vec{v}$$

~~Note~~ — Note that now Ohm's law has become more interesting. Specifically an electric field in x direction also implies an electric field in y direction & vice versa. The equations for different components of current density (or the drift velocity) can still be compactly expressed as $\vec{j} = \bar{\sigma} \vec{E}$ but now $\bar{\sigma}$ becomes a matrix. Specifically, the Ohm's Law reads

$$\begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix} = \left[\frac{nq\mu}{1 + \mu^2 B_2^2} \right] \begin{pmatrix} 1 & \mu B_2 & 0 \\ -\mu B_2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

→ The linear relationship between current & voltage is still valid, only

$$\sigma \text{ (scalar)} \Rightarrow \bar{\sigma} \text{ (2nd rank tensor)}$$

$$\bar{\sigma} = \left[\frac{\sigma_0}{1 + \mu^2 B_2^2} \right] \begin{pmatrix} 1 & \mu B_2 & 0 \\ -\mu B_2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \sigma_0 = \frac{nq^2 e}{m} = n_a \mu$$

(9)

Conductivity Tensor

We have effectively expressed

$$j_x = \sigma_{xx} E_x + \sigma_{xy} E_y + \sigma_{xz} E_z$$

$$j_y = \sigma_{yx} E_x + \sigma_{yy} E_y + \sigma_{yz} E_z$$

$$j_z = \sigma_{zx} E_x + \sigma_{zy} E_y + \sigma_{zz} E_z$$

In the above example

$$\sigma_{xz} = \sigma_{yz} = \sigma_{zx} = \sigma_{zy} = 0$$

$$\& \quad \sigma_{xy} = -\sigma_{yx} = \frac{\sigma_0}{1+\mu^2 B_2^2} \mu B_2$$

$$\sigma_{xx} = \sigma_{yy} = \frac{\sigma_0}{1+\mu^2 B_2^2}$$

The symmetry relations $\sigma_{xx} = \sigma_{yy}$ & $\sigma_{xy} = -\sigma_{yx}$ are called Onsager's relation.

→ Note that μB_2 is a dimensionless quantity.

~~We see~~ There are two physical interpretations of this:

(a) $\mu B_2 = \tan \theta = E_y/E_x$

(b) $\mu B_2 = \frac{q e}{m} B_2 = (q B_2) c = \omega_c c$.

ω_c : cyclotron freq.

How many cyclotron orbits an electron completes before it suffers a collision

Unfortunately in the experiments we are doing $\mu B \ll 1$ (h)
 ~~ω_c~~ & hence Landau quantization effects are not at all important. A lot of interesting things happen when $\mu B \gg 1$. [Quantum Hall effects, Shubnikov de Haas effect, de Haas-Van Alphen effect, magnetophonon effects] ← Sorry I am dropping names in the hope that you may look these up sometime.

Anyway, coming back to the conductivity tensor, the relationship, $\vec{J} = \bar{\sigma} \vec{E}$

can of course be inverted to make current the independent variable.

$$\vec{E} = [\bar{\sigma}]^{-1} \vec{J} = \bar{f} \vec{J}; \quad \bar{f} = [\bar{\sigma}]^{-1}$$

Now the resistivity tensor is also a matrix whose components must be evaluated by inverting the matrix $\bar{\sigma}$.

$$\bar{f} = \begin{pmatrix} \frac{\sigma_{yy}}{\sigma_{xx}\sigma_{yy} + \sigma_{xy}^2} & -\frac{\sigma_{xy}}{\sigma_{xx}\sigma_{xy} + \sigma_{xy}^2} & 0 \\ \frac{\sigma_{xy}}{\sigma_{xx}\sigma_{xy} + \sigma_{xy}^2} & \frac{\sigma_{xx}}{\sigma_{xx}\sigma_{xy} + \sigma_{xy}^2} & 0 \\ 0 & 0 & \gamma_{\sigma_{zz}} \end{pmatrix}$$

Plugging in the values of σ_{xx} & σ_{xy} , (i)

$$\rho_{xx} = \frac{\sigma_{yy}}{\sigma_{xx}^2 + \sigma_{xy}^2} = \left(\frac{\sigma_0}{1 + \mu^2 B^2} \right) \left(\frac{1}{\frac{\sigma_0}{1 + \mu^2 B^2}} \right)^2 \left(1 + \mu^2 B^2 \right)$$

$\Rightarrow \boxed{\rho_{xx} = \frac{1}{\sigma_0}}$

Magnetic field
does not change
the resistivity
 ρ_{xx} !

$$\rho_{xy} = \frac{\mu B_z}{\sigma_0} = \frac{\mu B_z}{n g \mu} = \frac{B_z}{n g}$$

Note that some of the properties of ρ_{xy} are counterintuitive,
in particular both ρ_{xy} & σ_{xy} go to zero as $B \rightarrow 0$,
even though $\bar{\rho} = \bar{\sigma}^{-1}$

I don't want to confuse you further so let us move on.
[You don't need to worry too much about the specific
forms of $\bar{\rho}_{xx}$ & $\bar{\sigma}$. Just understand that these are
tensors.]

(J)

Let us finally come back to the measurements.

As I have been emphasizing, experimentally we measure currents & not current densities; voltages & not electric fields (usually).

$$V_{ab} = \int_a^b \vec{E} \cdot d\vec{l}; \quad I = \iint \vec{J} \cdot d\vec{a}$$

In ~~the~~ actual measurements the ratio of the voltage to current we measure ~~is~~ is related to ~~the~~ some component or a combination of components of the resistivity tensor. What we measure depends on the boundary conditions imposed on the electric fields, ~~&~~ E_y & E_x & currents j_x, j_y (for $B=B_2$) flowing through the sample.

— Let us consider two extreme examples:

CASE I:

→ HALL GEOMETRY: Ideally a thin long sample. In presence of B_2 , charges accumulate on the two edges of the sample such that the Lorentz force is balanced by the electric field of these accumulated charges.

⇒ transverse voltage (Hall voltage).

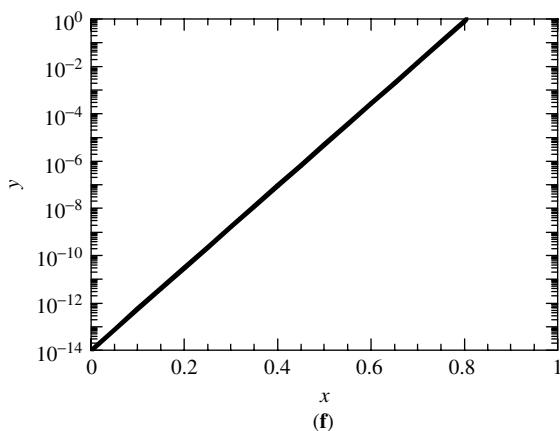


Fig. E1.1 (continued)

The slope m is

$$m = \frac{d[\log(y)]}{dx} = \frac{\beta}{\ln(10)} = \frac{\beta}{2.3036} = \frac{14}{2.3036 \times 0.8}$$

and the intercept at $x = 0$ is $y_o = 10^{-14}$.

To derive the four-point probe resistivity expression, we start with the sample geometry in Fig. 1.4(a). The electric field \mathcal{E} is related to the current density J , the resistivity ρ , and the voltage V through the relationship²

$$\mathcal{E} = J\rho = -\frac{dV}{dr}; \quad J = \frac{I}{2\pi r^2} \quad (1.3)$$

The voltage at point P at a distance r from the probe, is then

$$\int_0^V dV = -\frac{I\rho}{2\pi} \int_0^r \frac{dr}{r^2} \Rightarrow V = \frac{I\rho}{2\pi r} \quad (1.4)$$

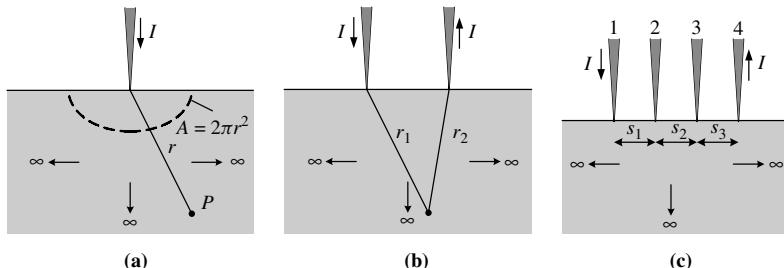


Fig. 1.4 (a) one-point probe, (b) two-point, and (c) collinear four-point probe showing current flow and voltage measurement.

For the configuration in Fig. 1.4(b), the voltage is

$$V = \frac{I\rho}{2\pi r_1} - \frac{I\rho}{2\pi r_2} = \frac{I\rho}{2\pi} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \quad (1.5)$$

where r_1 and r_2 are the distances from probes 1 and 2, respectively. The minus sign accounts for current leaving through probe 2. For probe spacings s_1 , s_2 , and s_3 , as in Fig. 1.4(c), the voltage at probe 2 is

$$V_2 = \frac{I\rho}{2\pi} \left(\frac{1}{s_1} - \frac{1}{s_2 + s_3} \right) \quad (1.6)$$

and at probe 3 it is

$$V_3 = \frac{I\rho}{2\pi} \left(\frac{1}{s_1 + s_2} - \frac{1}{s_3} \right) \quad (1.7)$$

The total measured voltage $V = V_{23} = V_2 - V_3$ becomes

$$V = \frac{I\rho}{2\pi} \left(\frac{1}{s_1} - \frac{1}{s_2 + s_3} - \frac{1}{s_1 + s_2} + \frac{1}{s_3} \right) \quad (1.8)$$

The resistivity ρ is given by

$$\rho = \frac{2\pi}{(1/s_1 - 1/(s_1 + s_2) - 1/(s_1 + s_2) + 1/s_3)} \frac{V}{I} \quad (1.9)$$

usually expressed in units of ohm · cm, with V measured in volts, I in amperes, and s in cm. The current is usually such that the resulting voltage is approximately 10 mV. For most four-point probes the probe spacings are equal. With $s = s_1 = s_2 = s_3$, Eq. (1.9) reduces to

$$\rho = 2\pi s \frac{V}{I} \quad (1.10)$$

Typical probe radii are 30 to 500 μm and probe spacings range from 0.5 to 1.5 mm. The spacings vary for different sample diameter and thickness.⁴ For $s = 0.1588$ cm, $2\pi s$ is unity, and ρ becomes simply $\rho = V/I$. Smaller probe spacings allow measurements closer to wafer edges, an important consideration during wafer mapping. Probes to measure metal films should not be mixed with probes to measure semiconductors. For some applications, *e.g.* magnetic tunnel junctions, polymer films, and semiconductor defects, microscopic four-point probes with probe spacings of 1.5 μm have been used.⁵

Semiconductor wafers are not semi-infinite in extent in either the lateral or the vertical dimension and Eq. (1.10) must be corrected for finite geometries. For an arbitrarily shaped sample the resistivity is given by

$$\rho = 2\pi s F \frac{V}{I} \quad (1.11)$$

where F corrects for probe location near sample edges, for sample thickness, sample diameter, probe placement, and sample temperature. It is usually a product of several

independent correction factors. For samples thicker than the probe spacing, the simple, independent correction factors contained in F of Eq. (1.11) are no longer adequate due to interactions between thickness and edge effects. Fortunately the samples are generally thinner than the probe spacings, and the correction factors can be independently calculated.

1.2.1 Correction Factors

Four-point probe correction factors have been calculated by the method of images,^{2,6} complex variable theory,⁷ the method of Corbino sources,⁸ Poisson's equation,⁹ Green's functions,¹⁰ and conformal mapping.^{11–12} We will give the most appropriate factors here and refer the reader to others where appropriate.

The following correction factors are for *collinear* or *in-line probes* with equal probe spacing, s . We write F as a product of three separate correction factors

$$F = F_1 F_2 F_3 \quad (1.12)$$

Each of these factors can be further subdivided. F_1 corrects for sample thickness, F_2 for lateral sample dimensions, and F_3 for placement of the probes relative to the sample edges. Other correction factors are discussed later in the chapter.

Sample thickness must be corrected for most measurements since semiconductor wafers are not infinitely thick. A detailed derivation of thickness correction factors is given by Weller.¹³ Sample thicknesses are usually on the order of the probe spacing or less introducing the correction factor¹⁴

$$F_{11} = \frac{t/s}{2 \ln\{[\sinh(t/s)]/[\sinh(t/2s)]\}} \quad (1.13)$$

for a *non-conducting* bottom wafer surface boundary, where t is the wafer or layer thickness. If the sample consists of a semiconducting layer on a semiconductor substrate, it is important that the layer be electrically isolated from the substrate. The simplest way to do this is for the two regions to be of opposite conductivity, *i.e.*, n -layer on a p -substrate or p -layer on an n -substrate. The space-charge region is usually sufficiently insulating to confine the current to the layer.

For a *conducting* bottom surface the correction factor becomes

$$F_{12} = \frac{t/s}{2 \ln\{[\cosh(t/s)]/[\cosh(t/2s)]\}} \quad (1.14)$$

F_{11} and F_{12} are plotted in Fig. 1.5. Conducting bottom boundaries are difficult to achieve. Even a metal deposited on the wafer back surface does not ensure a conducting contact. There is always a contact resistance. Most four-point probe measurements are made with insulating bottom boundaries.

For thin samples Eq. (1.13) reduces to

$$F_{11} = \frac{t/s}{2 \ln(2)} \quad (1.15)$$

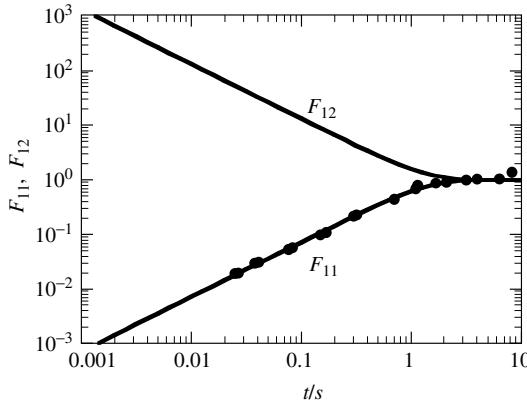


Fig. 1.5 Wafer thickness correction factors versus normalized wafer thickness; t is the wafer thickness, s the probe spacing. The data points are taken from ref. 15.

using the approximation $\sinh(x) \approx x$ for $x \ll 1$. Eq. (1.15) is valid for $t \leq s/2$. For very thin samples that satisfy the conditions for F_2 and F_3 to be approximately unity, we find from Eqs. (1.11), (1.12), and (1.15)

$$\boxed{\rho = \frac{\pi}{\ln(2)} t \frac{V}{I} = 4.532 t \frac{V}{I}} \quad (1.16)$$

Thin layers are often characterized by their *sheet resistance* R_{sh} expressed in units of ohms per square. The sheet resistance of uniformly doped samples is given by

$$\boxed{R_{sh} = \frac{\rho}{t} = \frac{\pi}{\ln(2)} \frac{V}{I} = 4.532 \frac{V}{I}} \quad (1.17)$$

subject to the constraint $t \leq s/2$. The sheet resistance characterizes thin semiconductor sheets or layers, such as diffused or ion-implanted layers, epitaxial films, polycrystalline layers, and metallic conductors.

The sheet resistance is a measure of the resistivity averaged over the sample thickness. The sheet resistance is the inverse of the sheet conductance G_{sh} . For *uniformly-doped* samples we find

$$R_{sh} = \frac{1}{G_{sh}} = \frac{1}{\sigma t} \quad (1.18)$$

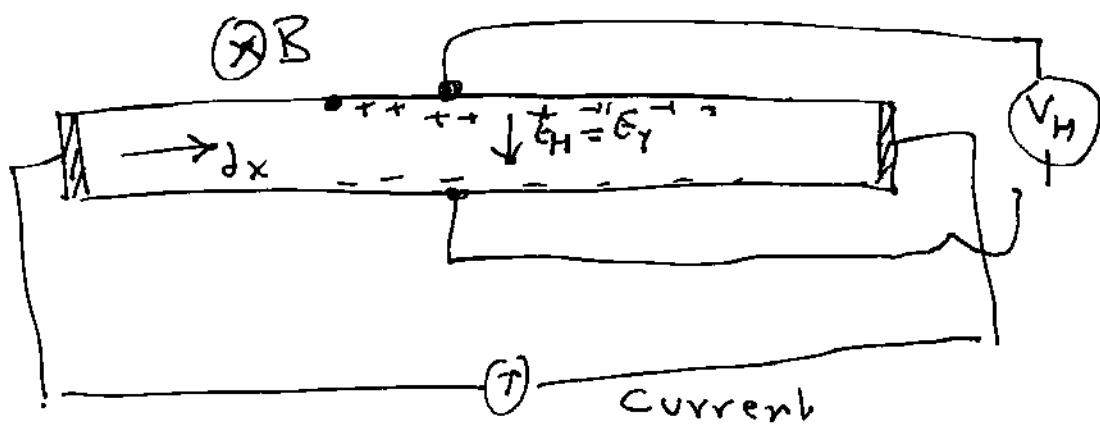
where σ is the conductivity and t the sample thickness. For *non uniformly-doped* samples

$$R_{sh} = \frac{1}{\int_0^t [1/\rho(x)] dx} = \frac{1}{\int_0^t \sigma(x) dx} = \frac{1}{q \int_0^t [n(x)\mu_n(x) + p(x)\mu_p(x)] dx} \quad (1.19)$$

HALL EFFECT

(k)

Ideal sample:



→ ~~Cross~~ The hall voltage (transverse to the magnetic field & ~~current~~ the applied electric field) is generated such that the current streamlines are unaffected by the applied ~~electric~~ magnetic field, i.e. $J_y = 0$

[I reemphasize that the charges are accumulating on the boundary of the sample. ~~deliberately~~ If the sample had no boundary or if the boundary was not easily accessible to the flowing charges, there will be no transverse Hall field.

So the boundary condition is

$$E_y \neq 0, J_y \approx 0 \Rightarrow V_y \approx 0 \quad [\text{everywhere in the sample}]$$

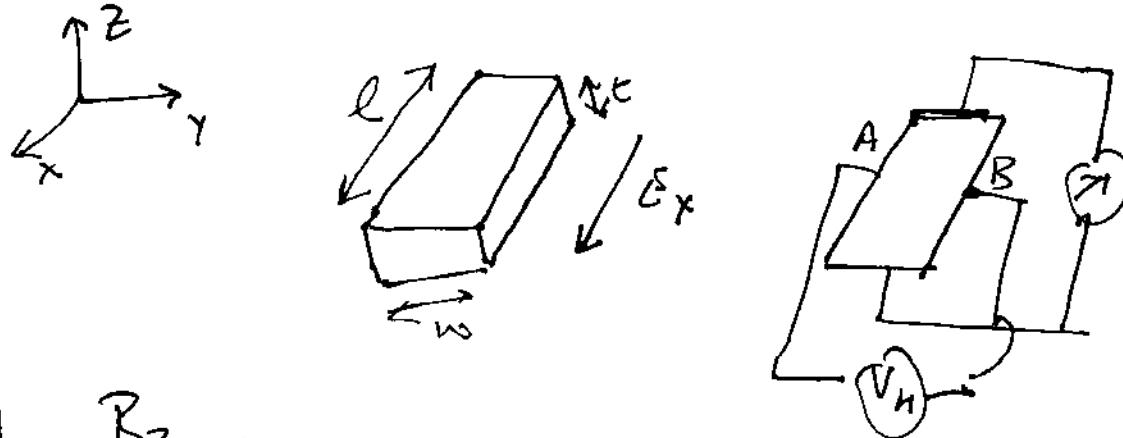
⇒ We have

$$V_y = \mu E_y - \mu v_x B_2$$

$$\text{with } V_y = 0 \Rightarrow E_y = \mu B_2 v_x$$

$$\text{or } nq E_y = \mu B_2 J_x \Rightarrow \frac{E_y}{J_x} = \frac{\mu B_2}{nq}$$

(e)



$$\frac{E_y}{J_x} = \frac{B_z}{nq} ; \text{ Since the sample is uniform & we are assuming that the current density & electric fields are also uniform within the sample}$$

$$V_H = \int_A^B \vec{E} \cdot d\vec{l} = E \cdot w ; I_x = \iint \vec{j}_x \cdot d\vec{a} = j \cdot w \cdot t$$

$$\Rightarrow R_H = \frac{V_H}{I} = \frac{E_y \cdot w}{J_x \cdot t \cdot w} = \frac{B_z}{nq \cdot t}$$

t: thickness of the sample

nt : effective carrier density per unit area
perpendicular to the magnetic field (cm^{-2})

\Rightarrow By measuring the Hall resistance, you can estimate

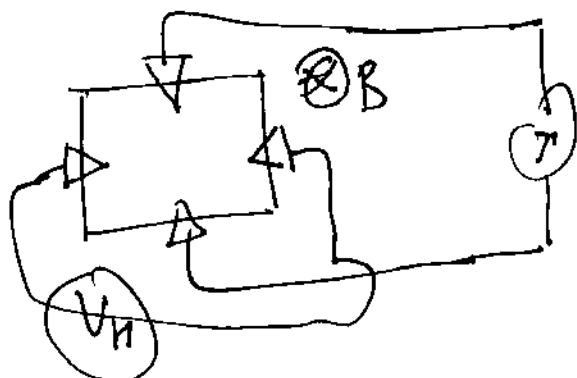
- ① The carrier density
- ② Sign of charge carriers

\rightarrow HALL Voltage is linear in B_z

\rightarrow If the line joining AB is ~~not~~ \perp to the current streamlines, you will also measure a millivoltage voltage.

m

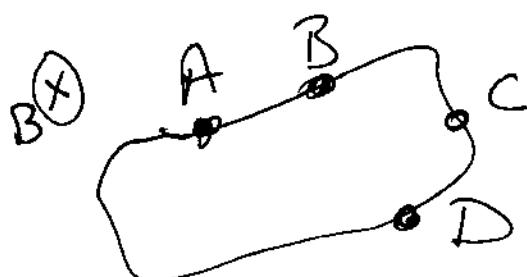
Van der Pauw Geometry: In the Hall effect experiment, instead of a long thin bar, we have something like



This is called the Van der Pauw geometry (or an approximation to it). The idea follows from a beautiful & non-trivial theorem & allows us to measure the resistivity, Hall resistance & magnetoresistance of an arbitrarily shaped sample. ~~just has to be of uniform thickness.~~

The only constraints on the sample geometry are

- ① The sample should not have any holes (simply connected)
- ② The sample should ~~of~~ be of uniform thickness but can be of any shape
- ③ Electrical contacts should be on the exact periphery
- ④ Electrical contacts should be points.

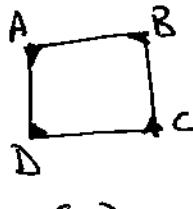


We can have any of these geometries,

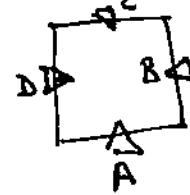
(n)



(i)



(ii)



(iii)



(iv) "Clover shaped sample"

To measure resistivity
(i), (iii) & (iv) are approximations to (ii) [contacts not on exact periphery
[contacts are not points]]

① Pass current through AB, I_{AB} ; measure voltage across CD, $V_{CD} \Rightarrow R_{AB, CD}$

② Pass current through BC, I_{BC} & measure $V_{DA} \Rightarrow R_{BC, DA}$

$$\Rightarrow \rho = f \left(\frac{R_{BC, DA}}{R_{AB, CD}} \right) \left(\frac{R_{BC, DA} + R_{AB, CD}}{2} \right) \times \frac{\pi \cdot \text{thickness}}{\text{thickness}^2}$$

~~16/17~~ f: correction factor, can be ~~approx.~~ calculated

Measuring the above ρ in presence of magnetic field directly yields $\rho_{xx}(B)$ - [Physical Magnetoresistance]

[Note that in our Drude theory $\rho_{xx}(B)$ is predicted to be $\rho_{xx}(B) = \rho_{xx}(B=0)$, independent of B . But this is in general not true.]

Samples can have a real physical magnetoresistance.

We will not be worrying about physical magnetoresistance in this course.

* Having a symmetric sample helps reduce the effects of non-idealities. That is why the geometries (ii) & (iii) are most common?

(C)

S & with

$$I_{AC}, V_{BD} \quad \& \quad I_{BD}, V_{AC}$$

$$\Downarrow \\ R_H^{AC, BD}$$

$$R_H^{BD, AC}$$

as a function of magnetic field, we can determine the Hall resistance.

In principle only one set of measurement

[current through AC & voltage across BD, e.g. should suffice but it is good practice to measure both $R_{AC, BD}$ & $R_{BD, AC}$ & average the two.]

→ Also note that from the properties of the resistivity tensors, we have

~~$$R_{HALL}(B) = -R_{HALL}(-B)$$~~

~~$$& R_{magnetoresist.}(B) = R_{magnetoresist.}(-B)$$~~

Then again using the van der Pauw formula

$$f_{HALL} = f_{xy} = \frac{B}{nq} = \text{thickness} \left(\frac{R_{AC, BD}}{2} + R_{BD, AC} \right) + \text{thickness}$$

Working formula

$$\frac{B}{nq} = \text{thickness} \times \left[\frac{R_{AC, BD} + R_{BD, AC}}{2} \right]$$

What to do in the Hall effect experiment

(01)

- ① By plotting the voltage (Hall voltage) vs magnetic field for B in both directions,
→ determine the carrier density.
- ② From the sign of the Hall voltage, determine whether it is electrons or holes which are conducting
- ③ Try to change the configuration to magnetoresistance & see if you can measure any signal with magnetic field
- ④ Determine the resistivity of the sample ($B=0$) using the Van der Pauw formula. Ask us for the correction factor.
- ⑤ Check if $V_H(B) = -V_H(-B)$

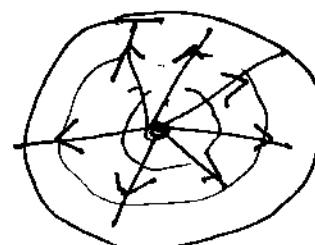
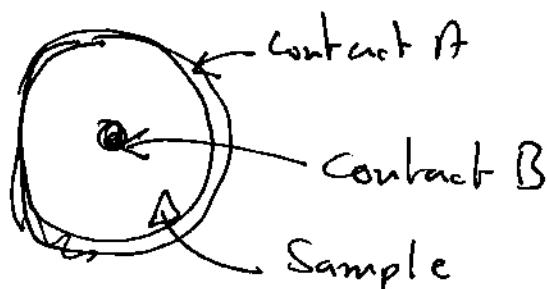
P

GEO METRIC MAGNETORESISTANCE :

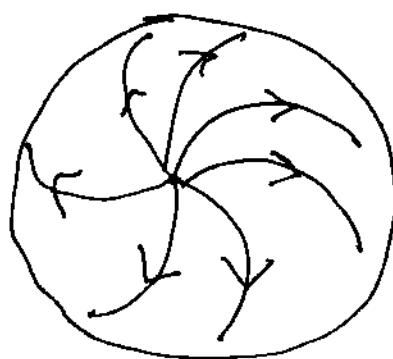
We saw that for the Hall field to develop, the sample must have a tangible boundary where the charges can accumulate & cancel the Lorentz force [$j_y \approx 0, E_y = 0$]

Now consider the opposite situation, ~~with~~ a sample without a boundary!

Corbing disc



$B = 0$
Current streamlines
& equipotentials



$B \neq 0$, current streamlines

\Rightarrow the current path becomes longer
 \Rightarrow increase in resistance when magnetic field is applied.

Physically it corresponds to

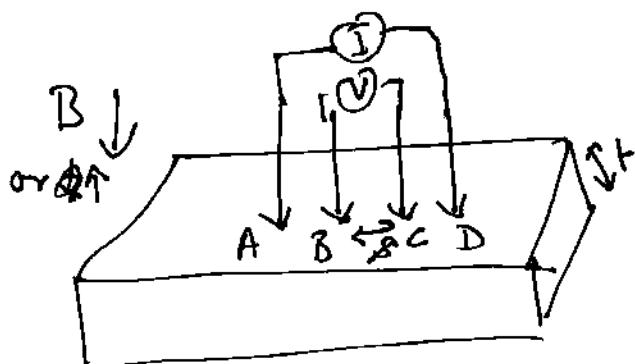
$$E_y \approx 0, j_y \neq 0$$

$$\Rightarrow j_x = \frac{\sigma_0}{1+\mu^2 B^2} E_x + \frac{\sigma_0 \mu B}{1+\mu^2 B^2} E_y$$

$$\left| \frac{E_x}{j_x} \right|_{\text{const}} = \frac{1+\mu^2 B^2}{\mu} \neq \rho(B)$$

Note that $\rho(B)$ is not a component of the resistivity tensor.

The geometry of our magnetoresistance experiment also seems to effectively imply an "infinite" sample



But the current streamlines in this case seem to be fairly complicated when the magnetic field is applied. Right now I am not able to derive the change in voltage $\frac{V(B)}{V(0)}$ with magnetic field (for a fixed current) from first principles but I think operationally the following should work.

① Measure the resistivity $\rho_0 (B=0)$ as you would do for the conventional 4-probe experiment without magnetic field

② Measure $R(B) - R(B=0)$ for both +ve & -ve directions of magnetic field

$$\frac{\Delta R(B)}{R(B=0)} = \frac{R(B) - R(B=0)}{R(B=0)} = \frac{\Delta \rho(B)}{\rho(B=0)} = \mu^2 B^2$$

(r)

- ③ \Rightarrow We should measure a quadratic dependence
 ↗ on B for the change in resistance.

From the coefficient \Rightarrow mobility μ .

- ④ Note that we have $P_0 = \frac{1}{nq\mu}$.

& we have μ

\Rightarrow Density of charge carriers (n)

\Rightarrow [Compare with what the people doing Hall effect experiment get]

- ⑤ From the mobility $\mu = \frac{qC}{m^{*e}}$; try to
 determine the value of the scattering time

in seconds. [m is not the free electron mass
 but the effective mass]. $m_{se}^{*} = 0.12 \times m_0$

m_{se}^{*e} = electron effective mass

m_0 = free electron mass = $9.1 \times 10^{-31} \text{ kg}$

μ should be of the order of $0.1 - 1 \frac{\text{m}^2}{\text{V}\cdot\text{s}}$

- ⑥ Verify ~~if~~ $R(B) = R(-B)$