Magnetic hysteresis

Maxwell equations in presence of matter:

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

← This is always true

$\vec{B}$ is the actual magnetic field that exists

$\vec{J}$ is the total current that acts as source of B.

When looking at the magnetic response of a material, it is convenient to think of the source term $\vec{J}$ to be comprising of two parts.

$$\vec{J} = \vec{J}_{free} + \vec{J}_{bound}$$

& consequently define two more fields, $\vec{H}$ & $\vec{M}$ such that

$$\vec{\nabla} \times \vec{H} = \vec{J}_{free}$$

$$\vec{\nabla} \times \vec{M} = \vec{J}_{bound}$$

$\vec{H}$ is thus the field generated by an electromagnet that I can control by turning a knob controlling the current.

$\vec{J}_{free}$ is the current that goes through wires (or in free space), in principle under our control.

$\vec{M}$: Magnetization of the material whose source is actually in the magnetic moment associated with electrons orbitals & spin. To be able to include it in Maxwell equations (classical), we just treat this as an additional current, $\vec{J}_{bound}$

$$\Rightarrow \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

$$\Leftrightarrow \vec{\nabla} \times (\vec{H} + \vec{M}) = \vec{J}_{free} + \vec{J}_{bound}$$

with

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

or alternatively, we could trivially define another quantity

$$\vec{J} = \mu_0 \vec{M}$$ [Sorry about having similar symbols for two very different quantities]

Note that the above relationship is always valid,
We also usually encounter an additional relationship, that of linear response between the magnetization & the applied field. This is especially true for paramagnetic & diamagnetic materials.

Linear response [not always valid]

$$\vec{M} = \chi \vec{H}$$

$$\Rightarrow \vec{B} = \mu_0 (\vec{H} + \chi \vec{H}) = \mu_0 (1 + \chi) \vec{H}$$

$$\vec{B} = \mu_0 \mu_r \vec{H}$$

χ — +ve & small for paramagnetic materials

χ — −ve & very small & diamagnetic materials

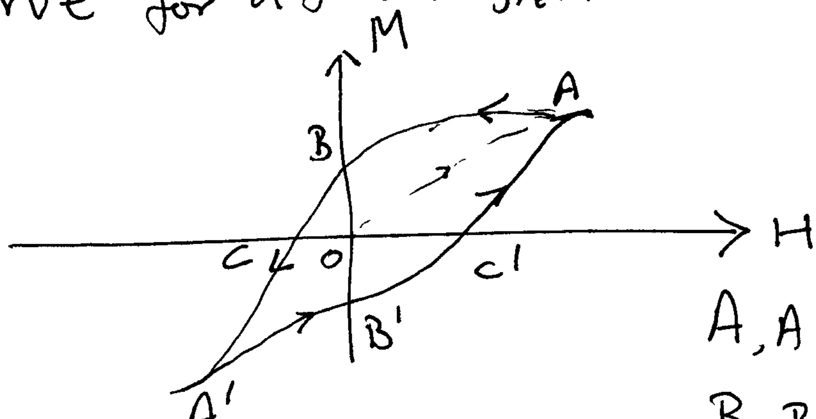
In the statistical mechanics course, we had calculated χ for simple systems.

Paramagnetism — Langevin, Brillouin, Pauli

Diamagnetism — Landau diamagnetism (free electron)
orbital diamagnetism
(for zero angular momentum states).

In this experiment, we will be focussing on ferromagnetic materials. These have a spontaneous magnetization below the Curie temperature. Ising model very crudely tries to approximate this phenomenon (not very successfully).
[Spin is not a classical variable, components of spin don't commute & obviously spin is not just ± 1).

→ Aim of the experiment: Measure the ~~H~~ M(H) curve for a ferromagnet.



A, A': saturation

B, B': Remanence

C, C': Coercivity

Hysteresis: History dependent magnetization.

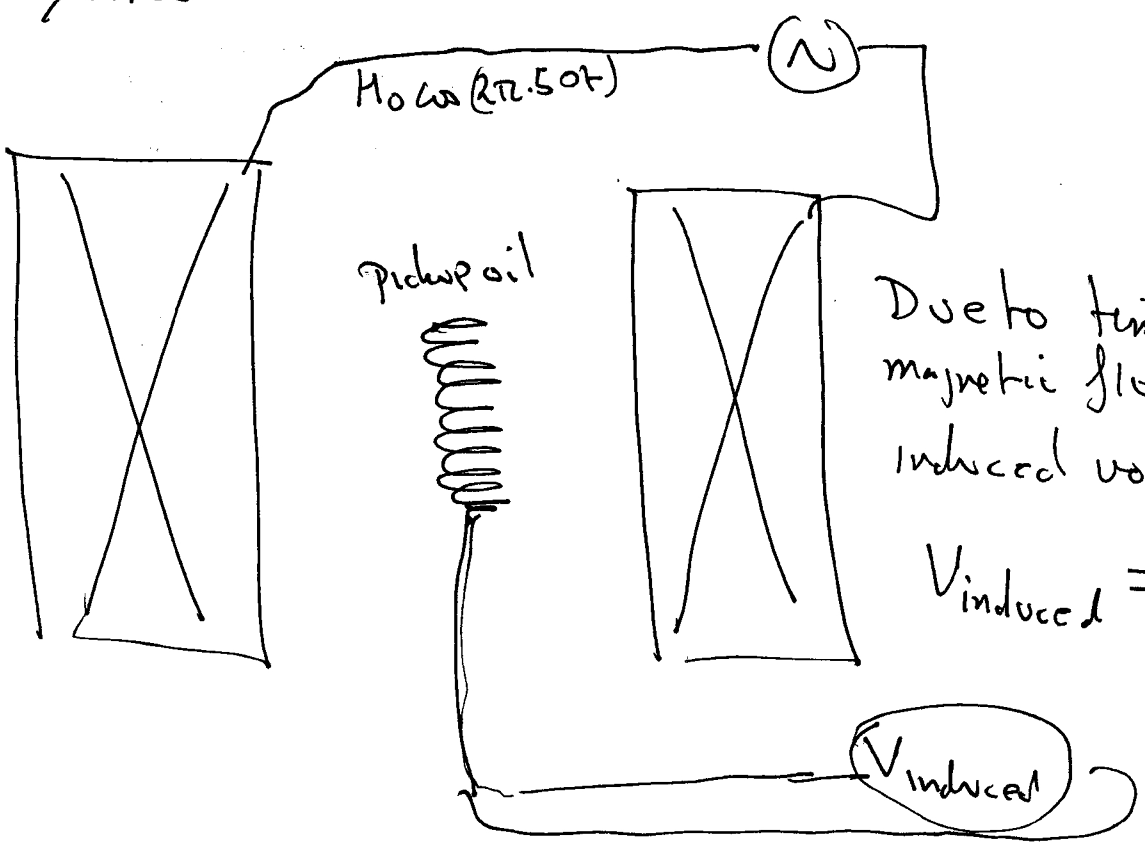
Basic Idea: Using a long solenoid whose current we can control to generate a homogeneous magnetic field that is time varying.

Applied magnetic field,

$$H_a(t) = H_a \cos(2\pi \cdot 50 t)$$

A 50 hz a.c magnetic field is generated by passing a 50 hz ac current.

→ Next we have a pickup coil, essentially another very small solenoid such that when inserted within the larger solenoid, the magnetic field it experiences is more or less uniform to the precision required



Due to time varying magnetic flux, there is an induced voltage (Lenz's law)

$$V_{induced} = -\frac{d\Phi_B}{dt}$$

Now with $\Phi_B = \text{Area} * B$
$$= \text{Area} * \left[\mu_0 (H + M)\right]$$

$\Rightarrow$ By measuring the induced voltage within the pick up coil with a ferromagnetic sample inside the pick up, we can estimate the value of magnetization, M,

$$H_a(t) = H_0 \cos(2\pi \cdot 50\, t)$$

$$V_{ind} = -\text{Area} * \frac{d}{dt}\left(\mu_0 [\vec{H}_a(t) + M(t)]\right)$$

Substitute e_1 & e_3

$\Rightarrow$

$$e_y = -g_y C_1 H_a + g_y e_3$$

$$= -g_y C_1 H_a + g_y \frac{g_1 A_c}{\omega} \mu_0 H + g_y \frac{g_1 A_s}{\omega} J$$

~~Now convert H_a to H using (1)~~

$$\cancel{\Rightarrow -g_y C_1 H - g_y C_1 N M + g_y \frac{g_1 A_c \mu_0 H}{\omega} + g_y \frac{g_1 A_s}{\omega} \mu_0 M}$$

Convert H to H_a ; $H = H_a - NM$

$\Rightarrow$

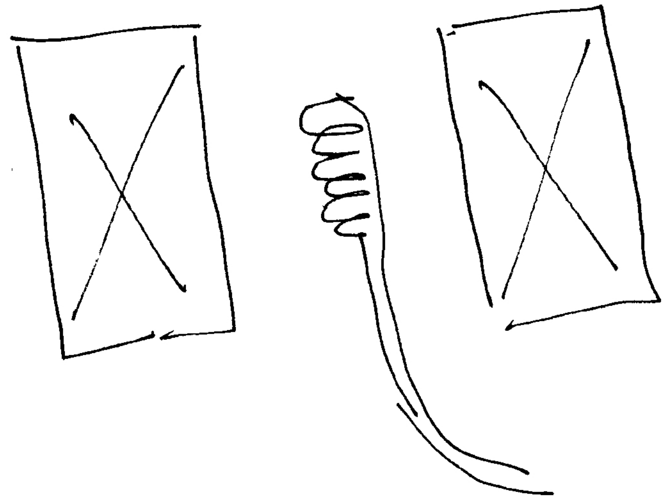
$$e_y = \left[-g_y C_1 H_a + \frac{g_y g_1 A_c H_a \mu_0}{\omega} \right]$$

$$\left[-\frac{N M \mu_0 g_y g_1 A_c}{\omega} + g_y g_1 A_s \mu_0 M \right]$$

First two terms have H_a & the other two terms have M

$\rightarrow$ Choose C_1 such that the first two terms cancel.

Basic Idea



→ A long solenoid generates a time dependent magnetic field, $H(t) = H_0 \cos \omega t$

$\omega = (2\pi * 50) \frac{rad}{sec}$

→ A small pick up coil measures the induced voltage. The sample is placed inside the pick up coil.

Aim: We want to plot M vs H for the sample.

→ What is measured is H vs t ← channel 1

M vs t ← channel 2

→ Go to "x y mode" to eliminate time & directly plot M vs H.

Magnetization is measured using Lenz's law:

Induced voltage (e_2)

$$e_2 = -\frac{\partial \Phi_B}{\partial t} = -\frac{\partial}{\partial t}\left[A_c \mu_0 H(t) + A_s \mu_0 M(t)\right]$$

Signal on the y-axis (Channel 2)

→ This should be the time dependent magnetization of the sample, following the applied magnetic field.

Signal on the x-axis (Channel 1)

→ The magnetic field seen by the sample
= (magnetic field in the solenoid) − demagnetization

→ Channel 2

Pick up voltage − the signal from empty coil.

(a) we need to modify e_2 such that the $H(t)$ part of the e_2 signal is completely eliminated & we only have a signal proportional to $M(t)$

So (1) Integrate $e_2 \longrightarrow e_3 = -\int e_2 \, dt$

(2) phase shift e_3 such that the signal is in phase with the magnetic field.

[Since $H = H_0 \cos \omega t$

$$\frac{\partial H}{\partial t} \propto \frac{H_0}{\omega} \cos(\omega t + \phi);$$

$\rightarrow$ The phase knob brings ϕ to zero

(3) ~~Add some~~ Subtract a fraction of the field signal, ~~$H_0 \cos$~~ so that only the magnetization part survives ~~in the~~ in e_3.

(4)

$$\boxed{H = H_a - NM}$$; N: demagnetization factor

H: the field the sample sees is less than what is applied by a constant factor N, called the demagnetization factor.

$H = H_0 \cos \omega t$; $\omega = (2\pi * 50)$ rad/sec

There is a circuit that generates a voltage e_1 proportional to the applied field H_a.

$$\boxed{e_1 = C_1 H_a}$$

$$B = \mu_0 H + J$$

$$\Phi_B = A_c \mu_0 H + A_s J$$

The induced voltage

$$e_2 = -\frac{\partial \Phi_B}{\partial t}$$

This signal is integrated & phase shifted with an operational amplifier of gain g_1

$$e_3 = -g_1 \int e_2 \, dt$$

$$e_3 = \frac{g_1 A_c H \mu_0}{\omega} + \frac{g_1 A_s J}{\omega}$$ [Also a phase shift: H & J are periodic signals

Now, there is a differential amplifier with gain g_y

$$e_y = -g_y(e_1 - e_3)$$

$$= -g_y C_1 H_a + e_3 g_y$$

$$e_y = -g_y C_1 H_a + g_y \frac{g_1 A_c H}{\omega} \mu_0 + g_y \frac{g_1 A_s}{\omega} J$$

Write H as H_a; $H = H_a - NM$

~~too~~ in the second term.

$\Rightarrow$

$$e_y = -g_y C_1 H_a + \frac{g_y g_1 A_c}{\omega} \mu_0 H_a - \frac{g_y g_1 A_c}{\omega} \mu_0 NM + g_y \frac{g_1 A_s}{\omega} J$$

$\mu_0 M = J$

Now adjust C_1 such that the pick up signal cancels. [Note that this will change the x-axis scale also].

i.e. choose C_1 such that

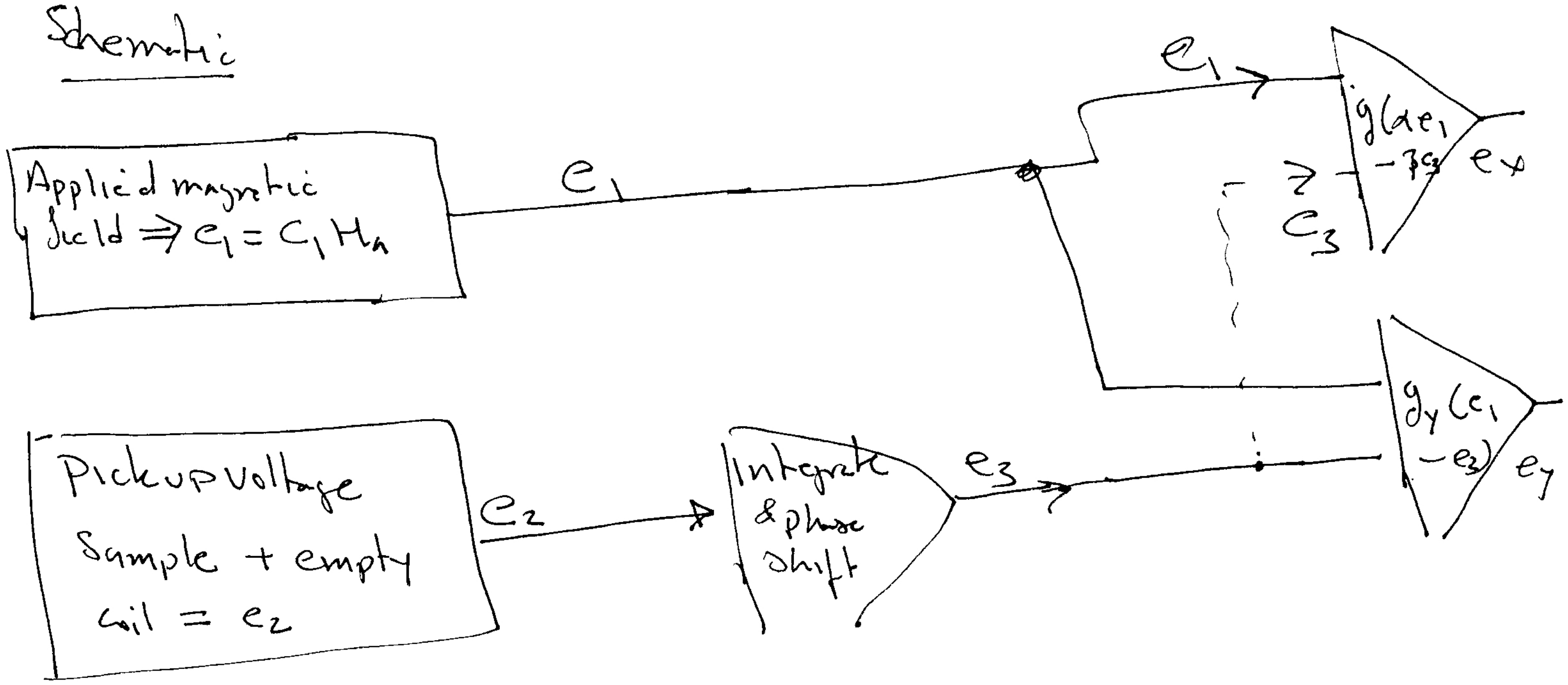
$$g_y C_1 = g_y \frac{g_1 A_c}{\omega} \mu_0 \Rightarrow \boxed{C_1 = \frac{g_1 A_c \mu_0}{\omega}}$$

After appropriately choosing C_1,

$$e_y = -\frac{N M \mu_0 g_y g_1 A_c}{\omega} + \frac{g_y g_1}{\omega} A_s \mu_0 M$$

$$e_y = \frac{g_y g_1}{\omega} \mu_0 M \left(A_s - A_c N\right)$$

Schematic



Ch1 ⇒ e_x : measures the magnetic field applied by the big solenoid. This field has to be corrected for the demagnetization ⇒ subtract part of the magnetic response from applied field.

$$H = H_a - NM$$

Ch2 ⇒ e_y : Measure sample magnetization via a pick up coil. The ~~sample~~ signal has to be corrected for the pick response of empty coil

The x-axis [channel 1 signal].

We already had

$$e_1 = C_1 H_a$$

with C_1 already chosen ~~to be~~

$$\boxed{C_1 = \frac{g_1 A_c \mu_0}{w}}$$

This value ensured that the channel 2 signal is only from the sample & the empty coil contribution is subtracted out.

To depict the hysteresis loop in real time, the x axis should be the field seen by the sample & not the applied field, that is we need to correct for the demagnetization factor.

$$H = H_a - NM \; ; \quad \text{we want } M(H) \text{ \& not } M(H_a)$$

N: demagnetization factor.

$\Rightarrow$ take part of the magnetization signal & ~~feed it to the~~ ~~add~~ subtract it ~~to the~~ from the signal e_1.

$\Rightarrow$

$$e_x = g_x(\alpha e_1 - \beta e_3)$$

$\Leftrightarrow$

$$e_x = g_x(\alpha e_1 - \beta e_3)$$

$$= g_x \alpha \frac{g_1 A_c}{\omega} H_a \mu_0 - g_x \beta\left(\frac{A_c \mu_0 H g_1}{\omega} + \frac{A_s \mu_0 M g_1}{\omega}\right)$$

$$e_x = \frac{g_x g_1}{\omega} \mu_0 A_c \left[\alpha H_a - \beta H - \beta M \frac{A_s}{A_c}\right]$$

now $H = H_a - NM$, eliminate H_a

$$e_x = K\left[\alpha H + \alpha NM - \beta H - \frac{\beta A_s}{A_c} M\right]$$

with $K = \frac{g_x g_1}{\omega} \mu_0 A_c$

or

$$e_x = K\left[(\alpha - \beta) H + \left(\alpha N - \frac{\beta A_s}{A_c}\right) M\right]$$

Now if we tweak α & β such that

$\alpha = \frac{A_s}{A_c}$ & $\beta = N$, the second term becomes zero

&

$$e_x = K\left[(\alpha - \beta) H\right]$$

or

$$H = \frac{e_x}{K(\alpha - \beta)} = \frac{e_x}{K\left(\frac{A_s}{A_c} - N\right)} =$$