Quincke's Method for susceptibility measurement of a paramagnetic solution

Paramagnets are attracted to a magnet. This happens because of ~~the~~ their microscopic dipole moments $\vec{m}$, which get aligned in the presence of external magnetic field $\vec{H}_0$. On the other hand they align randomly in the absence of $\vec{H}$. The energy gain by such alignment is

$$U = -\vec{m} \cdot \vec{B}_0$$

Here; $\vec{H}$: Magnetic field
$\vec{B}$: Magnetic flux density.

$$\underbrace{\vec{H} = \frac{\vec{B}_0}{\mu_0} = \frac{\vec{B}}{\mu}}$$

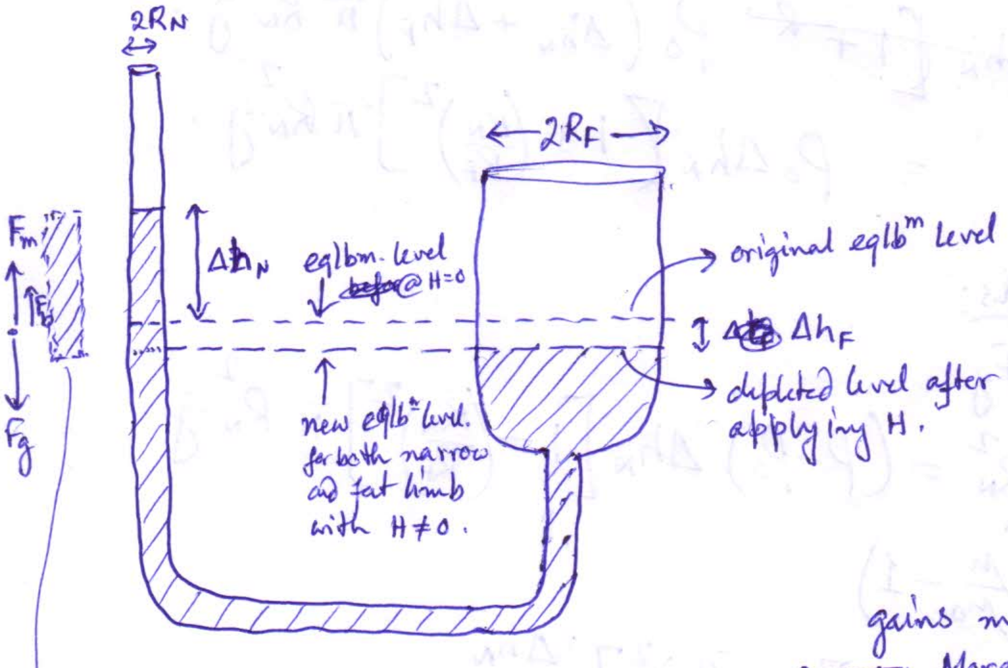
($\vec{H}$ is same in liquid or outside)

here $\vec{B}$ is the mag. flux density of the medium
$\vec{B}_0$ is " " " " of air/vacuum.
μ is the permeability of the medium.
μ_0 is the " " air/vacuum.

Writing ~~the~~ $\vec{m}$ in terms of ~~the~~ magnetic field (applied), the energy density when a magnetic substance is present in a field is:

$$u = \frac{1}{2}\mu H^2$$

Same energy in vacuum $u = \frac{1}{2}\mu_0 H^2$



→ Total volm. of liq. on which excess gravity force will act compared to the new eqlb^m fluid level.

Different Forces @ work:

1. Gravity force F_g acting on the excess liq. on the narrow limb for the volume of liq. above the new level (after switching on H) of liq. in the fat limb.

2. Magnetic force F_m will be upward. This is because the liq. between the pole pieces (i.e. the portion in mag. fld.) gains magnetic energy. ~~This gain in energy~~ More & more liquid trys to get into the region of mag. field (i.e. between pole pieces) so that system's energy is further minimized. This obviously cannot go on forever, and is compensated by downward gravity force.

(3) Buoyancy force: F_b Acts upward, and arises because the air column of certain height in narrow limb is displaced by fluid due to upward motion of fluid in it.

Calculation of F_m:

$$\Delta U = \frac{1}{2}(\mu-\mu_0)H^2 \cdot (\pi R_N^2 \Delta h_N)$$

Thus $F_m = -\frac{\Delta U}{\Delta h_N} = -\frac{1}{2}(\mu-\mu_0)H^2 \pi R_N^2$

Calculation of F_g:

$$F_g = mg. = \rho(\Delta h_N + \Delta h_F)\pi R_N^2 g$$

ρ → mass density of liquid.

Note $\pi R_N^2 \Delta h_N = \pi R_F^2 \Delta h_F \Rightarrow \Delta h_F = (\Delta h_N) \cdot \left(\frac{R_N}{R_F}\right)^2$

$$\therefore \quad F_g = \rho \Delta h_N \left[1 + \left(\frac{R_N}{R_F}\right)^2\right] \pi R_N^2 g.$$

Calculation of F_b:

$$F_b = \cancel{\rho_0 \Delta h_N [1 + R} \rho_0(\Delta h_N + \Delta h_F)\pi R_N^2 g.$$

ρ_0 → density of air.

$$= \rho_0 \Delta h_N \left[1 + \left(\frac{R_N}{R_F}\right)^2\right] \pi R_N^2 g.$$

Force balance demands:

$$F_m + F_b = F_g$$

$$\Rightarrow \frac{1}{2}(\mu-\mu_0)H^2 \pi R_N^2 = (\rho-\rho_0)\Delta h_N \left[1 + \left(\frac{R_N}{R_F}\right)^2\right] \pi R_N^2 g$$

Noting $\chi_m = \left(\frac{\mu}{\mu_0} - 1\right)$

$$\Rightarrow \chi_m = 2g\mu_0(\rho-\rho_0)\left[1 + \left(\frac{R_N}{R_F}\right)^2\right]\frac{\Delta h_N}{B_0^2}$$

if we can neglect ρ_0 w.r.t. ρ, and $\left(\frac{R_N}{R_F}\right)^2$ w.r.t. 1, $\chi_m = 2g\mu_0 \rho \frac{\Delta h_N}{B_0^2}$