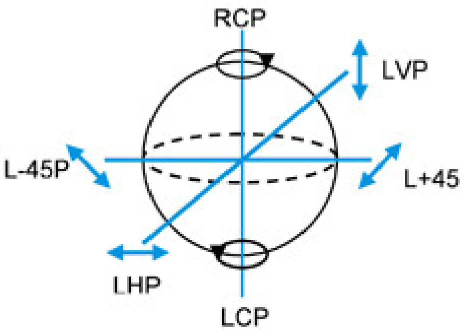
Experiment No. 3

Observation of Pancharatnam phase of light using a laser interferometer

Aim: Studying the Pancharatnam phase for polarized light by a simple arrangement with Michelson interferometer. Using linear polarizer and quarter waveplates, one will be able to observe the shift in fringe pattern and hence observe the Pancharatnam's geometric Phase.

1. Introduction

The Pancharatnam phase is a geometric phase associated with the polarization of light. When the polarization of a beam traverses a closed loop on the Poincare sphere, the final state differs from the initial state by a phase factor equal to half of the solid angle subtended by the closed loop at the centre of the sphere. The Poincare sphere is a space used to uniquely represent each state of polarization, as shown in Fig. 1. The axes of the Poincare sphere are the Stokes vectors: S_1= [±1 0 0], S_2= [0 ±1 0] and S_3= [0 0 ±1]. By this convention, the north and south poles are taken to be the states of circular polarization, and points on the equator correspond to linear polarization of different orientations with diametrically opposite points being orthogonal to each other. Points on the northern and southern hemispheres correspond to elliptically polarized light of different handedness.



$$
\begin{aligned}
S_0 &= I \\
S_1 &= Ip\cos 2\psi \cos 2\chi \\
S_2 &= Ip\sin 2\psi \cos 2\chi \\
S_3 &= Ip\sin 2\chi
\end{aligned}
$$

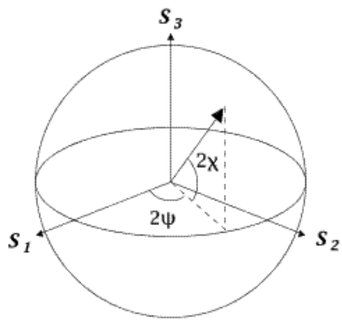


Figure 1: Poincare sphere representing various polarization states

By transforming the polarization of an input beam following a closed path on the Poincare sphere the wave acquires a geometric phase. It is important to note that we can vary the phase by changing the state of polarization without changing the optical path length. Changing the optical path length introduces dynamical phases.

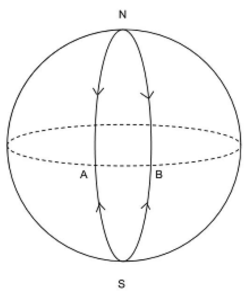
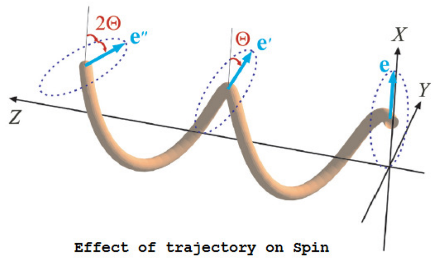


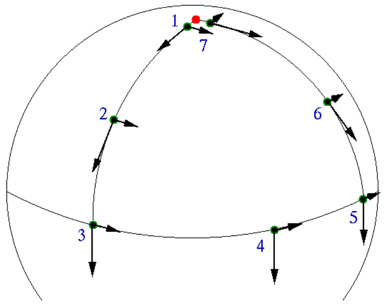
Figure 2: Closed path on Poincare Sphere acquires Geometric Phase.

2. Theoretical background

The Geometric Phase of light can be broadly classified into two types.

(a) Spin Redirection Berry Phase



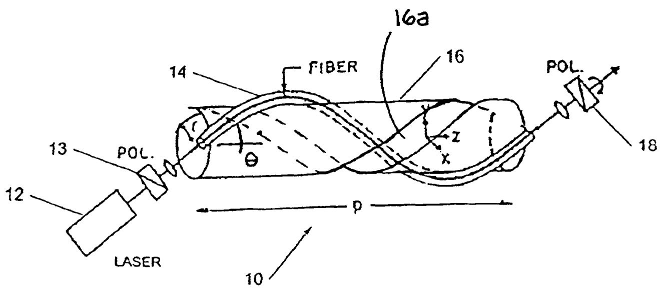


(a) **(b)**

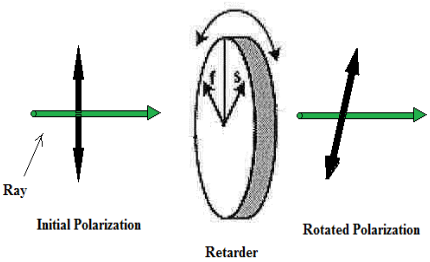
In this case the curved trajectory affects the evolution of spin (polarization) of light. The wave-propagation direction "k" changes its direction at all points whereas the polarization doesn't changes in its local reference frame. But to the observer the light changes its polarization. This can be explained using the above figures. In (a) the initial and final polarization of light remains same in its local reference frame but it acquires a phase difference of 2Θ in the observer's reference frame. Similar case has been denoted in the (b). We start with a vector at the position (1) as denoted by the red dot. Then we parallel transport this vector through the following path,

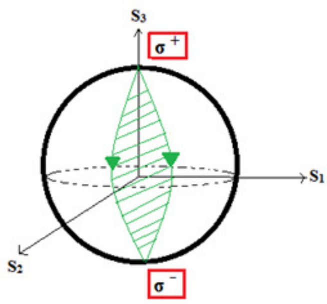
1 -> 2 -> 3-> 4 -> 5 -> 6 -> 7

Though as obvious from the figure the path 1 to 7 is a cyclic process, the final and initial orientation of the vector doesn't overlap. Although we had taken extreme care to transport the vector as parallel as possible to the previous orientation (at each point). We can see that the vector occupies a phase due to geometry i.e. a Geometric Phase. Another example where Berry's phase can be seen is a polarized light passing through a coiled fiber. As can be seen in the figure below, the polarization of light changes after it passes through a coiled fiber via total internal reflection.



(b) Pancharatnam Berry Phase

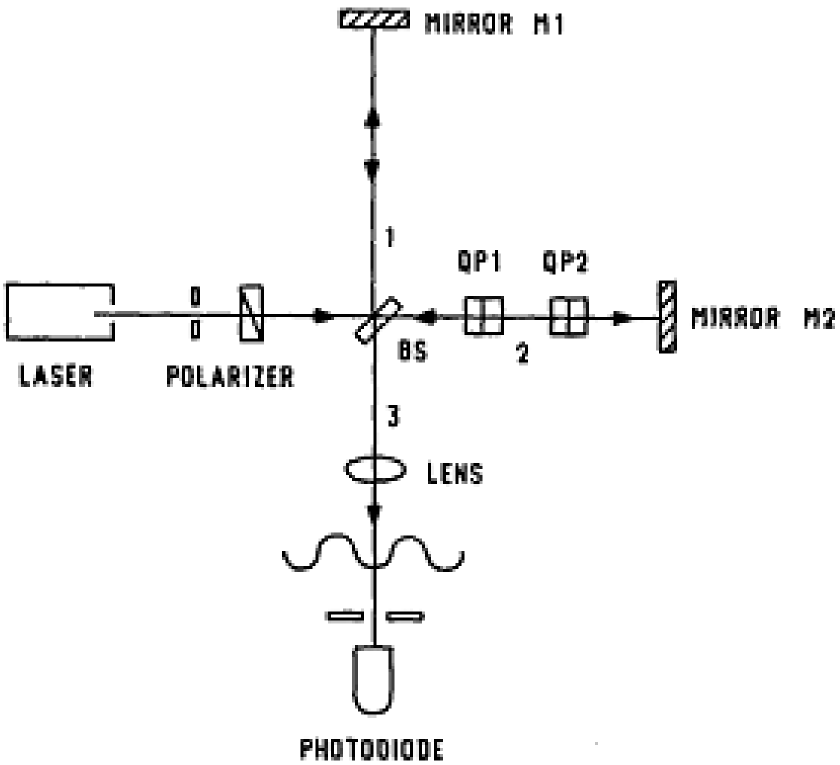


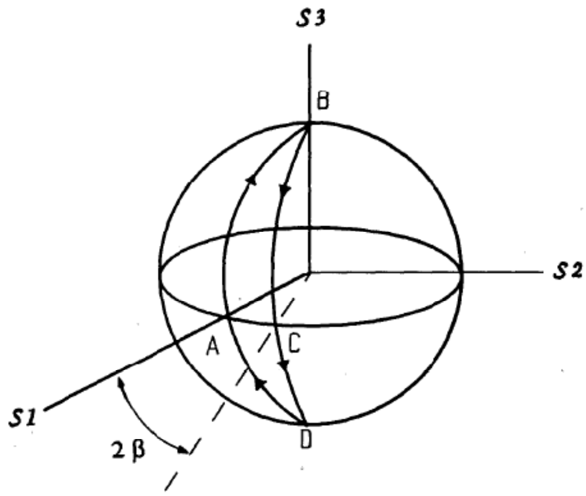


The Pancharatnam Berry phase involves change in polarization state of the light (ray) when it passes through optical elements like retarder. The difference between the Spin redirection Berry phase and Pancharatnam Berry phase is that the “k” –vector (propagation direction) doesn’t changes its direction. The direction propagation remains constant but the spin changes. For different combination of retarders we can generate different polarizations. The right hand side figure is the Poincare sphere representation of this case. Here σ^+ and σ^- states denote the RCP and LCP polarized light, respectively. We can reach to LCP from RCP using different paths on the sphere. Above shown by arrows are two different paths. And the shaded region gives us the Geometric Phase between them.

3. Experiment

Linearly polarized light from a He-Ne laser is divided into two equal beams by a 45°, 50:50 beam splitter. Beam 1 travels to a perpendicular mirror M_1 and is reflected back. Beam 2 traverses a quartz crystal multiple-order quarter-wave plate, QP1, whose optical axis is fixed at 45° relative to the incident polarization. This converts the linearly polarized light into a right circularly polarized beam that falls on a second quarter-wave plate, QP2, whose axis makes an angle β with the original polarization, which is free to rotate about the beam direction. The light emerging from QP2 strikes perpendicular mirror M2, from which it is reflected and made to retrace its path. The beam splitter BS combines portions of the returning light from arms 1 and 2 and gives rise to an interference pattern in arm 3 that allows the phase difference between the two return beams to be determined. By adjustment of mirrors M_1 and M_2 the interference fringes can be made vertical, and they are imaged by a lens on the screen. The shift of the interference phase can be recorded for various settings of the rotation angle β of QP2.





Outline of the closed path followed by the polarization state of the light on the Poincare sphere

In your experiment, light leaving the beam splitter in arm 2 is linearly polarized and may be represented by point A in the upper figure. After passing QP1 the light becomes right circularly polarized and is represented by point B. After passing QP2, whose axis is rotated through β, it is again linearly polarized and is represented by point C. The light is then reflected from mirror M2 and retraces its path through the quarter-wave plates, such that the representative point in the figure referred to a fixed reference frame moves to point D and then back to point A. At this stage the polarization state has made a closed circuit in parameter space and is back where it started. It may be shown that in completing the loop the light has suffered a phase shift equal to half the solid angle subtended by *ABCDA* at the center of the sphere, which is given by 2β. This is the Pancharatnam phase, which is closely related to the topological Berry phase and the Aharonov and Bohm phaseof a quantum particle. Its dependence on β can be determined by measurement of the interference pattern.

4. Experimental Procedure

[1] First align the Michelson interferometer and get interference fringes.

[2] Place the polarizer before the BS, the intensity of the fringes will be reduced, get the maximum intensity by rotating the polarizer.

[3] Place QWP1 and align the fast axis of it parallel to the input polarizer by getting the maximum intensity and then rotate it by 45° to generate the circularly polarized light. The fringes will disappear now, as the two beams of two arms of the interferometer are now orthogonally polarized and hence will not interfere.

[4] Place the second QWP(2) and get the fringes back.

[5] By rotating the QWP2 (changing β), observe the shift in the fringes.

[6] Record the shift of fringes (centre of a dark fringe to the next dark fringe) as a function of β.

[7] Record this for n number of consecutive fringe shifts (leading to phase change of $2n\pi$) and plot the accumulated phase shift this as a function of β. Using a linear fit of the obtained data prove that the Pancharatnam phase is = 2β.

Comment on the sources of errors in the measurements.

Suggested reading

M. V. Berry, "Quantal phase factors accompanying adiabatic changes", *Proc. R. Soc. London Ser. A,* vol. 392, pp. 45-57, 1984.

S. Pancharatnam, "Generalized theory of interference and its applications", *Proceedings of the Indian Academy of Science A,* vol. 44, pp. 247-262, 1956.

R. Bhandari and J. Samuel, "Observation of Topological Phase by Use of a Laser Interferometer", *Phys. Rev. Lett.*, vol. 60, pp. 1211-1213, 1988.

J. Samuel and R. Bhandari, "General Setting for Berry's Phase", *Phys. Rev. Lett.*, vol. 60, pp. 2339-2342, 1988.

A. Tomita and R. Y. Chiao, "Observation of Berry's Topological Phase by Use of an Optical Fiber", *Phys. Rev. Lett.*, vol. 57 pp. 937-940, 1986.

Y. Aharonov et al., "Phase change during a cyclic quantum evolution", *Phys. Rev. Lett.*, vol. 58, pp. 1593-1596, 1987.

Additional reading: Read Stokes vector manual for background on Polarization.

~~**Please read the theoretical background portion of the manual for experiment no. 5 (3 by 3 Mueller matrix) for the definition of polarization state and Stokes Vector.**~~

The Poincaré sphere. A very convenient geometrical representation of all possible polarization states involves the intensiy normalized coordinates S_1, S_2 and S_3 defined above, as illustrated in the following figure. In this space, the DOP is nothing else but the distance of the representative point from origin. Thus the physical realizability condition implies that all acceptable Stokes vectors are represented by points located within the unit radius sphere, also called the *Poincaré sphere*. Totally polarized states are found at the surface of the sphere (point A) while partially polarized states are inside (point B). The other spherical coordinates, the point "latitude" and "longitude" are nothing else but twice the azimuth α and ellipticity ε, as shown by the last column of Table 2 for totally polarized states. This geometrical representation provides simple and intuitive descriptions of many aspects of the interaction between polarized light and samples and/or instruments.

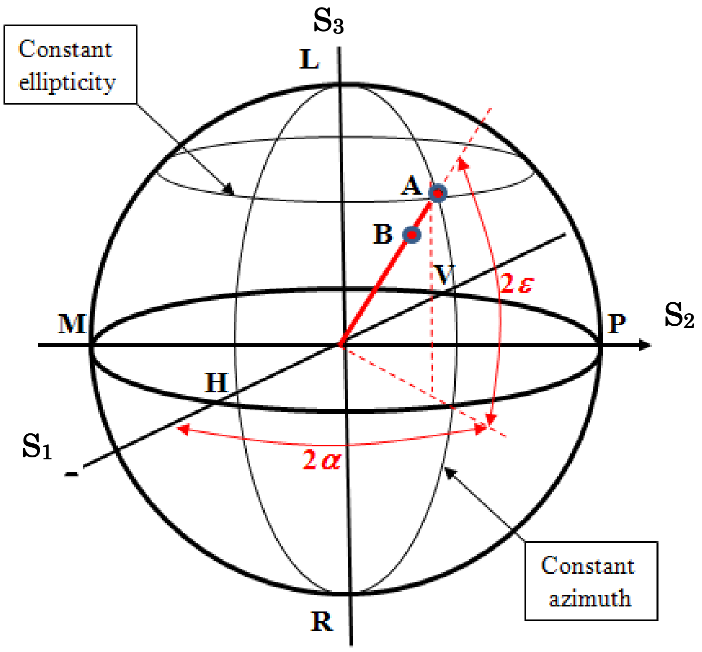


Figure: Geometrical representation of Stokes vectors within the Poincaré sphere. Any given polarization state is represented by a point whose Cartesian coordinates are the intensity normalized coordinates. The radial coordinate is the DOP, and the "longitude" and "latitude" are respectively 2α and 2ε.Totally polarized states are found at the surface of the unit radius sphere, while partially polarized states are inside (e.g., points A and B, respectively). Linearly polarizd states, among which H (0 deg.), V (90 deg.), P (+45 deg.) and M (-45 deg.) states, are on the "equator" while the L and R circular states are found at the "poles".

Polarization states and ellipticity

For a purely monochromatic optical wave of frequency ω, propagating along the z axis, its electric field **E** vibrates in the xy plane according to:

$$\mathbf{E}(z,t)=\begin{bmatrix} E_{0x}\cos\left(\omega t-\beta z+\phi_x\right) \\ E_{0y}\cos\left(\omega t-\beta z+\phi_y\right) \end{bmatrix}=\mathrm{Re}\left[\exp\left(i\omega-i\beta z\right)\mathbf{J}\right] \tag{1}$$

with

$$\beta=\left|\boldsymbol{\beta}\right|=(n-ik)\frac{\omega}{c} \tag{2}$$

being the modulus of the propagation vector$\boldsymbol{\beta}$, c is the speed of light in vacuum and$n$and k are the real and imaginary part of the refractive index, which determine respectively the speed of light and the absorption in the medium. The amplitudes $E_{0\mathrm{i}}$ and phasesϕ_i are constants, and define the Jones vector **J**as [5-7]

$$\mathbf{J}=\begin{bmatrix} E_{0x}\exp\left(i\phi_x\right) \\ E_{0y}\exp\left(i\phi_y\right) \end{bmatrix} \tag{3}$$

The *polarization*of the wave -- the shape of the trajectory described by **E** in the xy plane -- depends only on the ratio of the amplitudes tan$\alpha$and the phase difference φ defined as

$$\tan\upsilon=\frac{E_{0y}}{E_{0x}}, \qquad \phi=\phi_y-\phi_x \tag{4}$$

This trajectory is in general elliptical and is represented in Figure below. Besides the parameters defined in Eq (4), the ellipse can also be described by the orientation (azimuth) $\alpha$of its major axis and its ellipticityε, which is positive (resp. negative) for left (resp. right) handedness. The ellipticityε varies between the two limits of zero (linearly polarized light) and +-45° (circularly polarized light), which thus represent the two limits of generally elliptical polarization. Table below lists the Jones vectors of usual polarization states (with H,V, P and M, for linear polarizations along the horizontal, vertical, + and -45° directions, and L and R for left and right circular polarizations).

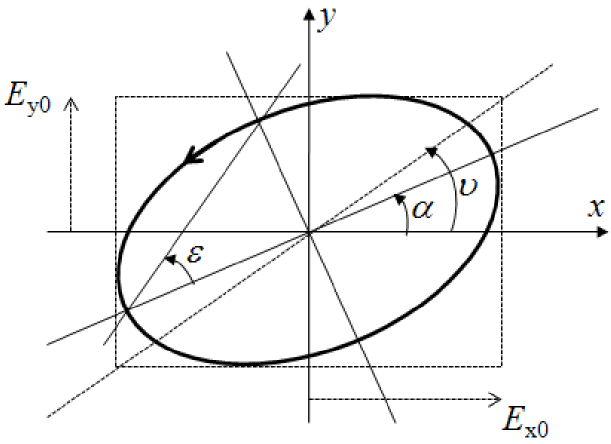


Figure: The polarization ellipse of a wave propagating in the z direction (towards the observer). E_{0x} and E_{0y} are the amplitudes of the field oscillations along the x and y directions; their ratio is equal to $\tan\upsilon$. α is the azimuth of the major axis of the ellipse and ε is its ellipticity. Ellipticity is positive or negative for left- or right-handed states.

Table1: Usual polarization states: Jones vectors, azimuths, ellipticities and shapes of the ellipses

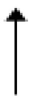
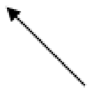
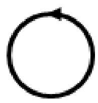
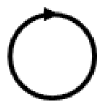
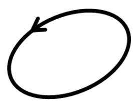
State	H	V	P	M	L	R	Elliptical
J	$\begin{bmatrix}1\\0\end{bmatrix}$	$\begin{bmatrix}0\\1\end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}$	$\begin{bmatrix}\cos\alpha\cos\varepsilon - i\sin\alpha\sin\varepsilon\\ \sin\alpha\cos\varepsilon + i\cos\alpha\sin\varepsilon\end{bmatrix}$
α	0	90°	45°	-45°	Undefined	Undefined	α
ε	0	0	0	0	45°	-45°	ε
Shape of the ellipse							

Table 2: Normalized Stokes vectors for usual totally polarized states (cf Table 1)

State	H	V	P	M	L	R	Elliptical
S	$\begin{bmatrix}1\\1\\0\\0\end{bmatrix}$	$\begin{bmatrix}1\\-1\\0\\0\end{bmatrix}$	$\begin{bmatrix}1\\0\\1\\0\end{bmatrix}$	$\begin{bmatrix}1\\0\\-1\\0\end{bmatrix}$	$\begin{bmatrix}1\\0\\0\\1\end{bmatrix}$	$\begin{bmatrix}1\\0\\0\\-1\end{bmatrix}$	$\begin{bmatrix}1\\ \cos 2\alpha\cos 2\varepsilon\\ \sin 2\alpha\cos 2\varepsilon\\ \sin 2\varepsilon\end{bmatrix}$