Fabry-Pérot Interferometry
Experiment 03

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1 Aim

- To measure the distance between the plates of a Fabry-Pérot etalon
- To measure wavelength of unknown laser source using the distance between plates
- To estimate the finesse of a fringe pattern.

2 Apparatus

The required apparatus for performing the experiment are:

1. Laser source
2. Fabry-Perot etalon
3. Power meter
4. Focusing lens
5. Screen
6. Optical table

3 Theory

The experiment is based on the phenomenon of multi-beam from a parallel glass slab. Consider a beam of light falling on a glass slab with refractive index n. This beam will undergo multiple refractions and internal reflections inside the slab, as shown below.

The higher order reflections become important if the reflection coefficient r is not very small. t denotes the fraction of amplitude transmitted on entering the slab and t' denotes the fraction transmitted when leaving the slab.

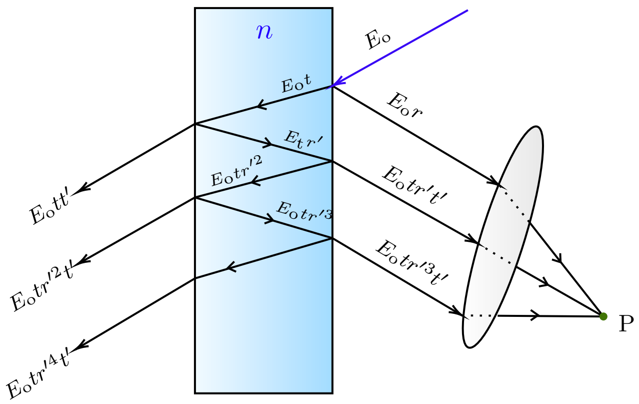


Figure 1: Multibeam Interference from a parallel slab

Consider the parallel reflected rays, each of which bears a fixed phase relation with the other. These rays are mutually coherent and will interfere when brought to focus at P. Suppose that the incident wave is given by $E_0 e^{i\omega t}$, then the optical field at P is given by:

$$\begin{aligned}
\widetilde{E}_{1\mathrm{r}} &= E_0 r e^{i\omega t} \\
\widetilde{E}_{1\mathrm{r}} &= E_0 r' t t' e^{i(\omega t - \delta)} \\
\widetilde{E}_{1\mathrm{r}} &= E_0 r'^3 t t' e^{i(\omega t - 2\delta)} \\
&\vdots \\
\widetilde{E}_{1\mathrm{r}} &= E_0 r'^{(2N-3)} t t' e^{i(\omega t - (N-1)\delta)}
\end{aligned} \tag{1}$$

The total reflected field is equal to the sum of these components, which gives us

$$\begin{aligned}
\widetilde{E}_{\mathrm{r}} &= \widetilde{E}_{1\mathrm{r}} + \widetilde{E}_{2\mathrm{r}} + \ldots + \widetilde{E}_{\mathrm{Nr}} \\
&= E_0 e^{-i\omega t}\left[r + r' t t' e^{-i\delta}\left(1 + r'^2 e^{-i\delta} + \ldots + \left(r'^2 e^{-i\delta}\right)^{N-2}\right)\right]
\end{aligned} \tag{2}$$

For $|r'^2| < 1$ and $N \to \infty$, the above sum becomes an infinite GP and we obtain,

$$\widetilde{E}_{\mathrm{r}} = E_0 e^{i\omega t}\left[r + \frac{r' t t' e^{-i\delta}}{1 - r'^2 e^{-i\delta}}\right] \tag{3}$$

We know that $r' = -r$ and $tt' + r^2 = 1$, from which we get

$$\widetilde{E}_{\mathrm{r}} = \left[\frac{1 - e^{-i\delta}}{1 - r^2 e^{-i\delta}}\right] E_0 e^{i\omega t} r \tag{4}$$

The reflected flux density at P can be defined as,

$$\mathcal{I}_{\mathrm{r}}^{\mathrm{P}} = \frac{1}{2}\widetilde{E}_{\mathrm{r}}\widetilde{E}_{\mathrm{r}}^{*} = \frac{1}{2}E_0^2 \frac{2r^2(1 - \cos\delta)}{(1 + r^4 - 2r^2\cos\delta)} = \frac{\mathcal{F}\sin^2(\delta/2)}{1 + \mathcal{F}\sin^2(\delta/2)}\mathcal{I}_i \tag{5}$$

where $\mathcal{I}_i$ is the incident flux and $\mathcal{F} = \left(\frac{2r}{1-r^2}\right)^2$ is defined as the *coefficient of finesse*. Similarly for transmitted waves we have,

$$\widetilde{E}_{\mathrm{t}} = E_0 e^{i\omega t}\frac{tt'}{1 - r^2 e^{-i\delta}} \implies \mathcal{I}_{\mathrm{t}}^{\mathrm{P}} = \frac{1}{1 + \mathcal{F}\sin^2(\delta/2)}\mathcal{I}_i \tag{6}$$

From this we can observe that $\mathcal{I}_{\mathrm{i}} = \mathcal{I}_{\mathrm{r}} + \mathcal{I}_{\mathrm{t}}$. Maximum transmission intensity occurs when $\sin(\delta/2) = 0 \implies \delta_{\mathrm{M}} = 2\pi M$ for integral values of M while minimum intensity occurs when $\sin(\delta/2) = 1 \implies \delta_{\mathrm{m}} = (2m+1)\pi$. The maximum and minimum values of intensity are respectively,

$$\mathcal{I}_{\mathrm{t}}^{\max} = \mathcal{I}_{\mathrm{i}} \qquad\qquad \mathcal{I}_{\mathrm{t}}^{\min} = \left(\frac{1 - r^2}{1 + r^2}\right)^2 \mathcal{I}_{\mathrm{i}} \tag{7}$$

3.1 Fabry-Pérot Interferometer

Having seen the basics of multi-beam interference, we introduce the principle behaind our experimental setup. The Fabry-Pérot setup constitutes two plane parallel reflecting surfaces within which light rays are multiply reflected and then the transmitted rays are focussed on a screen where they interfere. If we keep the distance between the surfaces fixed, the setup is called an *etalon*.

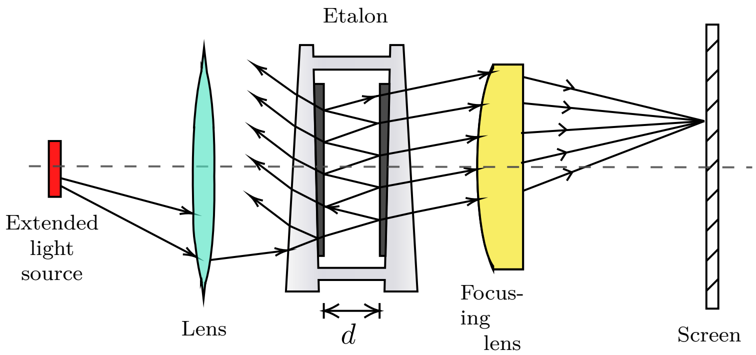


Figure 2: Schematic diagram of the Fabry-Perot experimental set-up

Sometimes transparent metal films are used to increase the reflectance $R = r^2$ but introducing this also incurs an absorptance A. Along with the transmittance T, the conservation equation becomes

$$R + T + A = 1 \tag{8}$$

Another complication due to the metallic film is the introduction of an additional phase ϕ which depends on θ_{i} (the angle of the incident rays). The phase difference between the two successively transmitted rays is,

$$\delta = \frac{4\pi n}{\lambda_0} d \cos\theta_{\mathrm{t}} + 2\phi \tag{9}$$

where n is the refractive index of the plates, λ_0 is the wavelength of the light source and θ_{t} is the angle of the transmitted ray. Considering small d and large λ_0, we can safely neglect ϕ. From this, the transmission ratio is given as,

$$\begin{aligned} \frac{\mathcal{I}_{\mathrm{t}}}{\mathcal{I}_{\mathrm{i}}} = \left(\frac{T}{1-R}\right)^2 \times \left(1 + \frac{4R}{(1-R)^2}\sin^2(\delta/2)\right)^{-1} &= \left(\frac{1-R-A}{1-R}\right)^2 \frac{1}{1+\mathcal{F}\sin^2(\delta/2)} \\ &= \left[1 - \frac{A}{1-R}\right]\left(1+\mathcal{F}\sin^2(\delta/2)\right)^{-1} \end{aligned} \tag{10}$$

3.2 Sharpness and Finess

To analyse the sharpness of the peak, we find the half-width γ, which tells us the characteristic width of the peak. We obtain it by solving for δ where $\mathcal{I}_{\mathrm{t}} = \frac{1}{2}(\mathcal{I}_{\mathrm{t}})_{\max}$ from which we obtain the condition,

$$\frac{1}{1+\mathcal{F}\sin^2(\delta/2)} = \frac{1}{2} \implies \sin\left(\frac{\delta_{1/2}}{2}\right) = \frac{1}{\sqrt{\mathcal{F}}} \tag{11}$$

For large $\mathcal{F}$, the sin term is very small and hence can be approximated as simply $\frac{\delta_{1/2}}{2}$ which gives us $\gamma \equiv 2\delta_{1/2} = \frac{4}{\sqrt{\mathcal{F}}}$. The ratio of separation of adjacent maxima divided by the half-width is defined as the finesse, $\mathscr{F}$. The difference between maximas is $2\pi(m+1) - 2\pi m = 2\pi$ and hence,

$$\mathscr{F} = \frac{2\pi}{\gamma} = \frac{\pi\sqrt{\mathcal{F}}}{2} \tag{12}$$

4 Working Formula

The intensity of transmitted light will follow the relation,

$$\mathcal{T} = \frac{\tau_0}{1+\mathcal{F}\sin^2(2\pi L/\lambda)} \tag{13}$$

where τ_0 is the maximum possible intensity and L is the 'optical spacing' between the mirrors that depend on the refractive index, transmittance angle and the spacing d between the mirrors.

$$L = 2\pi n d \cos\theta_{\mathrm{t}} \tag{14}$$

The finesse $\mathscr{F}$ and the contrast $\mathcal{C}$ are defined as,

$$\mathscr{F} = \frac{\pi\sqrt{\mathcal{F}}}{2} = \frac{\pi r}{1-r^2} \qquad \mathcal{C} = \mathscr{F} + 1 \tag{15}$$

For circular fringes, the fringe radius χ_{m} is related to the m-th ring using,

$$\chi_{\mathrm{m}}^2 = \frac{D^2\lambda}{d} m \tag{16}$$

where D is the distance between the etalon and the screen and d is the distance between the mirrors.

5 Data and Calculations

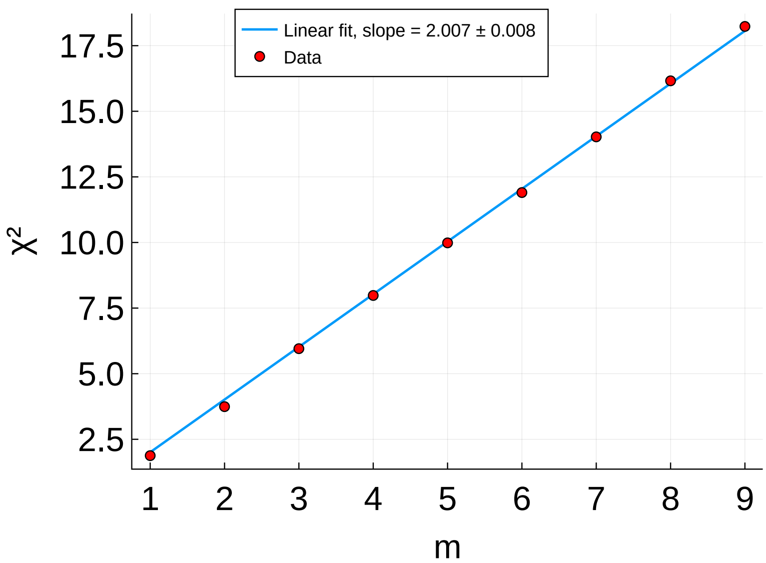
We perform three parts in the experiment. In the first part, we obtain the distance between the two mirrors of the etalon using a known laser source. In the second part, we use this distance between the mirrors to find the wavelength of an unknown laser source. In the third part, we obtain the finesse of the fringes formed due to interference.

5.1 Distance between Plates

We set up the apparatus and obtained the interference fringes from the rays on a screen. We measured the outer radius of the fringes formed and then plotted it against the order of the fringe. For our setup, the distance between the etalon and the screen was $D = 66.2cm$ and a red laser source was used, with $\lambda = 655$ nm

n	Left Position (cm)	Right Position (cm)	Radius R (cm)	R^2 (cm^2)
1	2.46	11.00	4.27	18.23
2	2.73	10.77	4.02	16.16
3	3.02	10.51	3.75	14.03
4	3.30	10.20	3.45	11.90
5	3.63	9.95	3.16	9.99
6	3.93	9.58	2.83	7.98
7	4.36	9.24	2.44	5.95
8	4.84	8.71	1.94	3.74
9	5.41	8.15	1.37	1.88

Table 1: Radius of fringe for distance determination



We performed a linear fit of this data with $y = mx$ and obtained the slope as,

$$m = 2.007 \pm 0.008$$

Using Eq. (16), we can find the distance between the plates as,

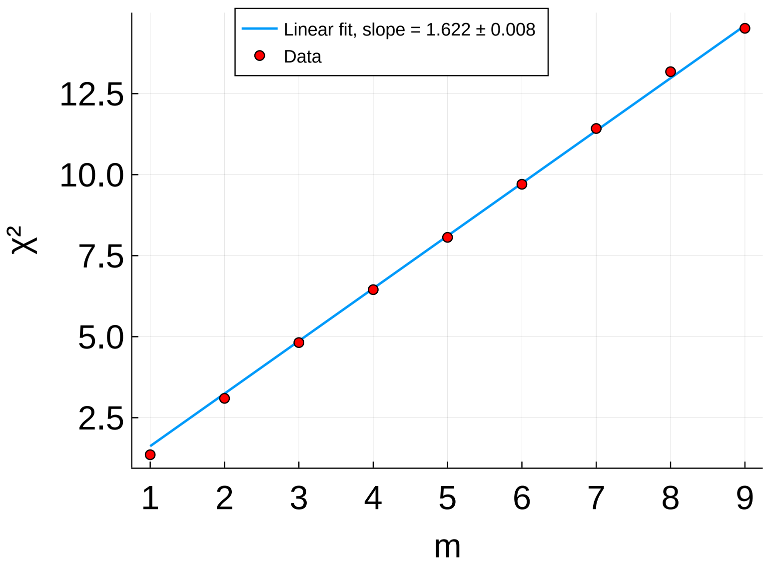
$$d = \frac{D^2\lambda}{\text{slope}} = \frac{(66.2 \text{ cm})^2 \times 655 \times 10^{-7} \text{ cm}}{2.007 \text{ cm}^2} = 0.143 \text{ cm}$$

5.2 Wavelength of Unknown Laser

In this part, we used a green laser source of undetermined wavelength. Similar as before, we calculated the radius of the fringe and plotted it with the fringe order.

n	Right Position (cm)	Left Position (cm)	Radius R (cm)	R^2 (cm^2)
1	7.97	5.64	1.17	1.36
2	8.58	5.06	1.76	3.10
3	8.99	4.60	2.20	4.82
4	9.36	4.28	2.54	6.45
5	9.65	3.97	2.84	8.07
6	9.90	3.67	3.12	9.70
7	10.17	3.41	3.38	11.42
8	10.41	3.15	3.63	13.18
9	10.58	2.96	3.81	14.52

Table 2: Radius of fringe for wavelength determination



We performed a linear fit of this data with $y = mx$ and obtained the slope as,

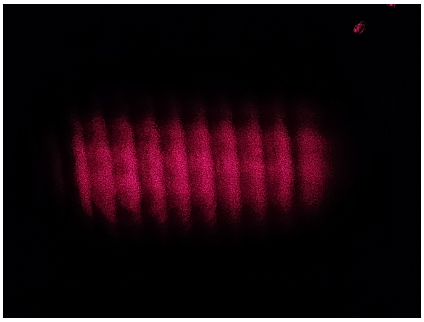
$$m = 1.622 \pm 0.008$$

Using the distance d previously obtained, we can estimate the wavelength of the laser source as,

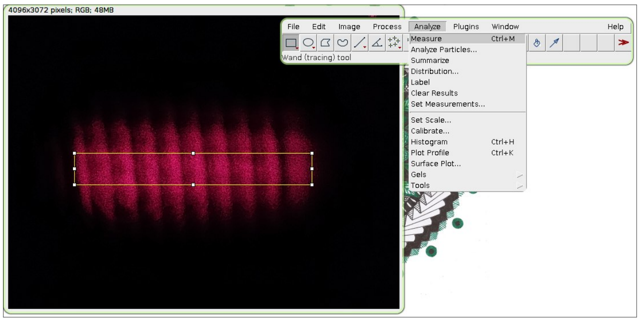
$$\lambda = \frac{\text{slope} \times d}{D^2} = \frac{1.622 \text{ cm}^2 \times 0.143 \text{ cm}}{(66.2 \text{ cm})^2} = 5.293 \times 10^{-5} \text{ cm} = 529.3 \text{ nm}$$

5.3 Finesse Calculation

For the finesse calculation, we need to analyse the fringes using the *ImageJ* software. One such picture of the fringes was taken using our phone camera and then using ImageJ, we found the distribution profule of the pixel.



(a) Actual image used for finesse analysis



(b) Image analysis using ImageJ software

Figure 3: Comparison of the two images

The distribution of the pixel is shown below. Through calibration, we set the pixel scale to the actual distance scale by measuring the distance between which the fringes extended (5.54 cm) and setting that scale in the ImageJ software, equating with the range of the pixels. The final plot is shown below.

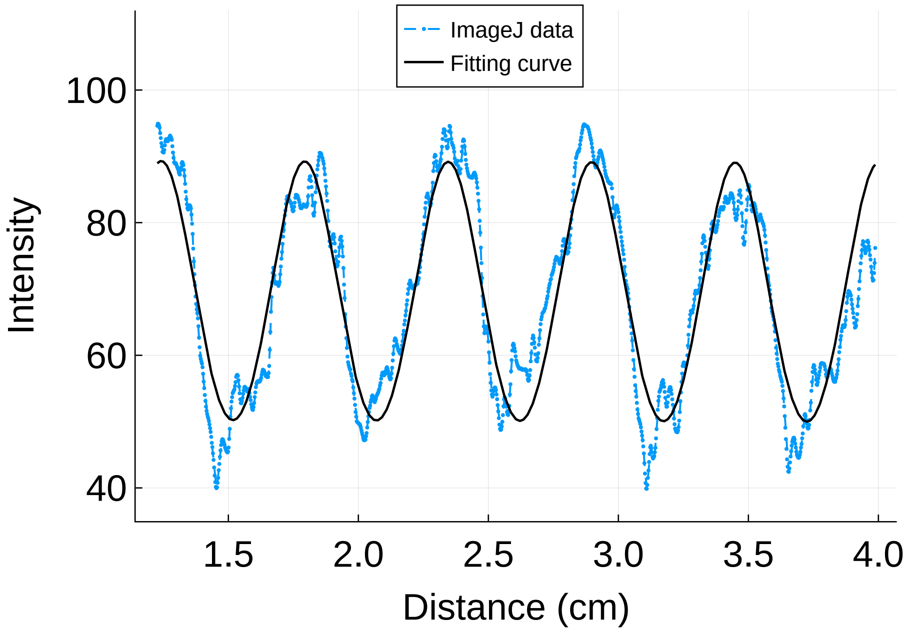


Figure 4: Fitting of the intensity profile

In the distribution data, we used a fitting function,

$$y = \frac{\tau_0}{1 + \mathcal{F}\left[\sin\left(\frac{x-a}{l}\right)\right]^2} + A\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] + c \tag{17}$$

The first part is the expected distribution for the intensity of transmitted light from Eq. (13). The second part takes care of the Gaussian background of the intensity and c is just to manage some offset.

The fitting is very sensitive to initial conditions and care needs to be taken so that the initial conditions roughly follow a trend with the original data. The data along with the fit curve is shown in Fig. 4.

After curve fitting, the coefficient of finesse was found out to be, $\mathcal{F} = 0.169$ from which we obtain the finesse as,

$$\mathscr{F} = \frac{\pi\sqrt{\mathcal{F}}}{2} = 0.645$$

6 Error Analysis

7 Discussion and Conclusion