Observing Pancharatnam-Berry Phase Using Interferometry
Experiment 02

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1 Aim

Using Michelson interferometer and simple optical elements viz. polarisers and waveplates to observe the manifestation of Pancharatnam-Berry phase through the shifting of fringe patterns.

2 Apparatus

The required apparatus for performing the experiment are:

1. Laser source
2. Two Quarter-Wave Plates
3. Two Mirrors
4. Linear Polariser
5. 50:50 Beam Splitter
6. Magnifier lens
7. Screen
8. Optical table

3 Theory

The experiment is based on the measurement of a geometric phase which is independent of the optical path length and is solely based on the geometry of the evolution of the electromagnetic wave. We will discuss about *Pancharatnam-Berry phase*, wherein the wave propagates in a fixed direction but undergoes a continuous change in the state of polarisation, which is represented by a closed loop in the Poincaré sphere.
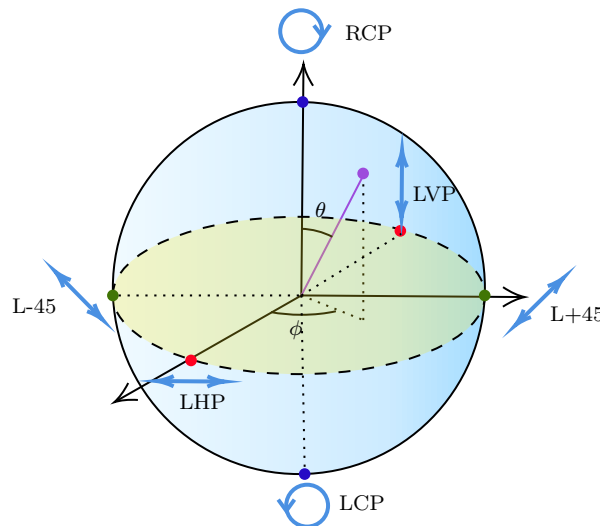

Figure 1: Poincaré sphere representation of polarisation states

The Poincaré sphere is a convenient representation of polarisation states of light where the Stokes vector for completely polarised light lie on the surface of the sphere and for partially polaised light lies in the interior. Change of polarisation can be depicted by rotation in the Poincaré sphere. The special points are shown in Fig. 1; the north and south pole represent the right and left circular polarised light. Along

the $+x$ and $-x$ direction, we have horizontal and vertical linear polarisation. Along $+y$ and $-y$ we have the diagonal linear polarisation states.
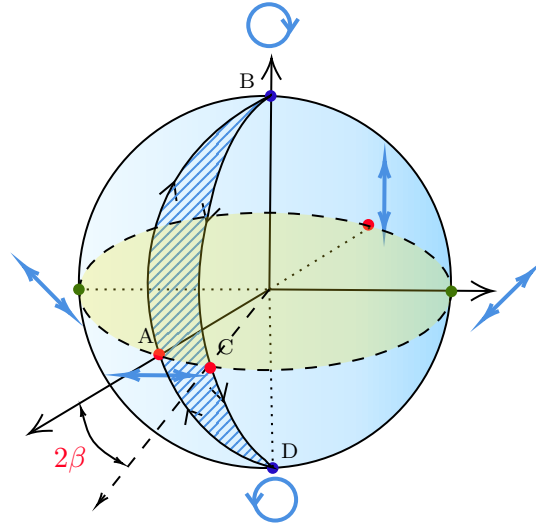

Figure 2: The evolution of polarisation along the closed loop, whose area gives us the geometric phase.

When a light beam's polarisation state is transformed along a closed loop as shown in Fig. 2 on the surface of the Poincaré sphere, it acquires a geometric phase shift Φ, related to the solid angle Ω subtended by the loop at the centre of the sphere:

$$\Phi = -\frac{1}{2}\Omega$$

3.1 Experimental Setup

We measure this geometric phase through a modified Michelson interferometer setup as shown below.
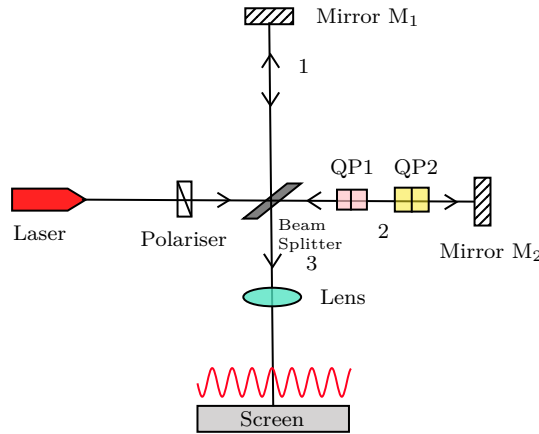

Figure 3: Schematic diagram of the experimental setup

The laser source is passed through a polariser to obtain a pure linearly polarised light which passes through a beam-splitter, creating a coherent superposition. The ray in arm 1 goes and reflects back from mirror M_1. The ray in arm 2 passes through two quarter waveplates.

On passing through QP_1, the linearly polarised light becomes right circularly polarised (path AB in Fig. 2). The second waveplate QP_2 is rotated by an angle β with the original polarization, which changes the right circularly polarised light backe to linear polarisation but introduces a phase (path BC in Fig. 2). On reflecting from mirror M_2, the evolution is similar but in an opposite sense (path CD and DA in Fig. 2). Thus, a closed loop is traversed in the Poincaré sphere.

The ray comes back to the beam-splitter, with linear polarisation but with an additional phase. The two

rays recombine and we obtain an interference pattern on the screen.

Throughout the experiment, we keep the mirrors fixed, hence the *dynamical phase* is kept constant. Hence, the fringe patterns due to interference is solely controlled by the *geometric phase*. Below, we show the mathematical calculation for the emergence of the geometric phase.

3.2 Analysis using Jones matrices

The incident light from the laser, after passing through the polariser, can be denoted by the Jones vector

$$\mathbf{E}_{\text{in}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{1}$$

This light will pass through the waveplates. The Jones matrix for a quarter waveplate with its axis at an angle θ is given as,

$$\mathbf{J}_{\text{Q}}(\theta) = \begin{pmatrix} \cos^2\theta + i\sin^2\theta & (1-i)\sin\theta\cos\theta \\ (1-i)\sin\theta\cos\theta & \sin^2\theta + i\cos^2\theta \end{pmatrix} \tag{2}$$

Thus, on passing through QP_1 the state becomes,

$$\mathbf{E}_1 = \mathbf{J}_{\text{Q}}(45°)\begin{pmatrix} 1 \\ 0 \end{pmatrix} \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix} \tag{3}$$

On passing through QP_2 the state becomes,

$$\mathbf{E}_2 = \mathbf{J}_{\text{Q}}(+\beta)\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix} = e^{i\beta}\frac{1}{\sqrt{2}}\begin{pmatrix} \cos\beta + \sin\beta \\ \cos\beta - \sin\beta \end{pmatrix} \tag{4}$$

Now, on reflecting from the mirror M_2 the state becomes,

$$\mathbf{E}_3 = e^{i\beta}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} \cos\beta + \sin\beta \\ \cos\beta - \sin\beta \end{pmatrix} = e^{i\beta}\frac{1}{\sqrt{2}}\begin{pmatrix} \cos\beta + \sin\beta \\ \sin\beta - \cos\beta \end{pmatrix} \tag{5}$$

Again, after passing through QP_2 we get,

$$\mathbf{E}_4 = \mathbf{J}_{\text{Q}}(-\beta)e^{i\beta}\frac{1}{\sqrt{2}}\begin{pmatrix} \cos\beta + \sin\beta \\ \sin\beta - \cos\beta \end{pmatrix} = e^{2i\beta}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix} \tag{6}$$

Finally, on passing through QP_1 we have,

$$\mathbf{E}_5 = \mathbf{J}_{\text{Q}}(-45°)e^{2i\beta}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix} = e^{2i\beta}\frac{1}{2}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -i \end{pmatrix} = e^{2i\beta}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{7}$$

We see that we obtain the initial polarisation state but with an additional phase of $e^{2i\beta}$,

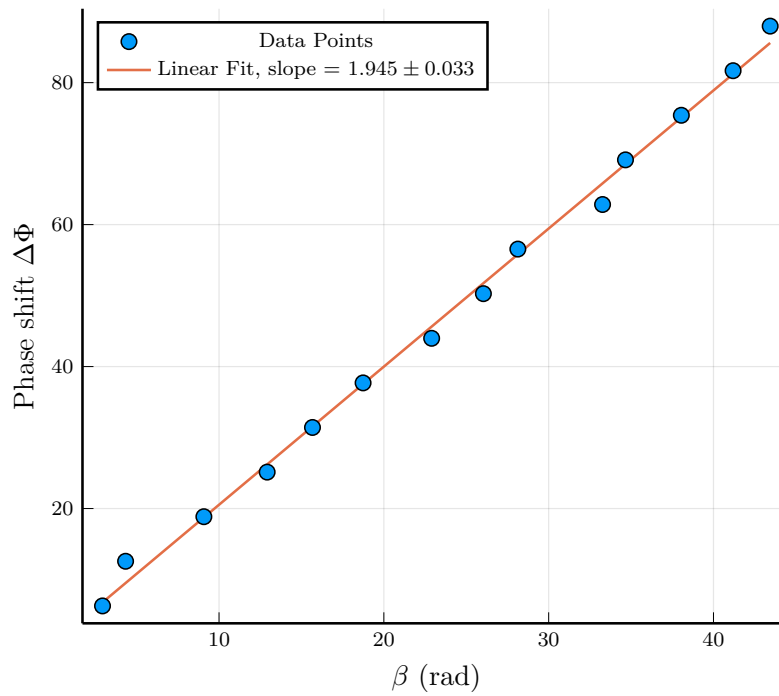$$\mathbf{E}_{\text{out}} = e^{-2i\beta}\mathbf{E}_{\text{in}}$$

4 Data and Calculations

After alignment of the Michelson interferometer, we varied the angle β of the second quarter waveplate and cumulatively counted the number of fringe shift occuring with the varying angle. The data is presented in the table below:

Table 1: Measured Phase Shift vs. Rotation Angle β

Angle β (°)	Angle β (rad)	Fringe Shift (n)	Phase $\Delta\Phi = 2\pi n$ (rad)
168	2.93	1	6.28
248	4.33	2	12.57
520	9.08	3	18.85
740	12.92	4	25.13
898	15.67	5	31.42
1074	18.74	6	37.70
1312	22.90	7	43.98
1492	26.04	8	50.27
1612	28.13	9	56.55
1906	33.27	10	62.83
1986	34.66	11	69.12
2180	38.05	12	75.40
2360	41.19	13	81.68
2489	43.44	14	87.96

The plot between the rotation angle β and the phase shift of the fringe pattern $\Delta\Phi = 2\pi n$ is shown below.

From the linear fit of the data points, we obtained a slope of $m = 1.945 \pm 0.033$ which is close to the theoretically expected value of $m_{\text{th}} = 2.0$ (since the geometric phase is 2β, $\Delta\Phi = 2\beta$)

5 Error Analysis

The percentage error from the theoretical value is,

$$\text{Error.}(\%) = \frac{2 - 1.945}{2} \times 100\% = 2.7\%$$

6 Discussion and Conclusion