Determination of Polarization properties of Spatial Light Modulator (SLM) using Stokes Vector Polarimetry

Aim:
Determination of linear retardance and Optical rotation of a Spatial Light Modulator (SLM) measuring various Stokes vector elements.

1. INTRODUCTION

Optical polarimetry, ever since its discovery, has played important roles in gaining fundamental understanding of the nature of electromagnetic waves and answering some of the key questions related to the physics of light. Traditional polarimetry has also long being pursued for numerous practical applications in various branches of science and technology. Quantification of protein properties in solutions, testing of chiral (asymmetric) purity of pharmaceutical drugs, remote sensing in meteorology and astronomy, optical stress analysis of structures, and crystallography of biochemical complexes are just a smattering of its diversified uses. This particular polarimetry experiment involves determination of two important polarization characteristics of medium, namely, the linear retardance and optical rotation (defined subsequently), which has many practical applications.

2. THEORETICAL BACKGROUND

Stokes Mueller formalism

Polarization is a property which arises out of the transverse nature of the electromagnetic (EM) radiation, and is related to the orientation of the plane of vibration of its electric field. Mathematically, the propagation of the transverse EM wave (light) in free space can be represented by the transverse components of the associated electric fields as

$$E_x\left(z,t\right)=E_{ox}e^{i\left(\omega t\text{-}kz+\delta_x\right)}$$
$$E_y\left(z,t\right)=E_{oy}e^{i\left(\omega t\text{-}kz+\delta_y\right)} \tag{1}$$

Where E_x and E_y represent the electric fields in the x and y orthogonal directions respectively (z is the propagation direction of the wave), E_{0x} and E_{0y} represent the magnitude of the electric fields, δ_x and δ_y are phases associated with the two transverse electric field components, ω is the frequency of the light, k = 2×π/λ, the wave vector (λ is the wavelength of light in free space). The resultant electric field represented as the vector sum of E_x and E_y, describes the polarization state of light. Depending on the amplitudes and phases of the electric fields in Eq. (1), the polarization of light can be described as either linear, circular or elliptical polarizations. A linear polarization state can either be $\mathrm{E}_x(z,t)$, $\mathrm{E}_y(z,t)$ or a vector sum of these with $\delta_x = \delta_y$. For general elliptical polarization, $E_{0x} \neq E_{0y}$ and (δ_x - δ_y = any arbitrary phase angle). The circular polarization state is a special case, where $E_{0x} = E_{0y}$ and (δ_x - δ_y = π/2).

In fact several mathematical formulations have been developed to deal with the propagation of such polarized light and its interaction with any optical system. Among these, the Stokes-Mueller formalism is the most widely used, partly due to the fact that both the Stokes vectors and Mueller matrix can be measured with relative ease using any intensity-measuring instruments, including most polarimeters, radiometers and spectrometers. In this formalism, the state of polarization of a beam of light can be represented by four measurable quantities (intensities), when grouped in a 4 × 1 vector, are known as the Stokes vector, introduced by G. G. Stokes in 1852. The four Stokes parameters are defined relative to the following six intensity measurements (I) performed with ideal polarizers: I_H, horizontal linear polarizer (at 0^0 angle with respect to the horizontal axis of the laboratory frame) I_V, vertical linear polarizer (90^0); I_P, +45^0 linear polarizer; I_M, 135^0 (- 45^0) linear polarizer; I_R, right circular polarizer, and I_L, left circular polarizer. The Stokes vector (**S**) is defined as

$$\mathbf{S} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} I_H + I_V \\ I_H - I_V \\ I_P - I_M \\ I_R - I_L \end{bmatrix}. \tag{2}$$

where *I, Q, U* and *V* are Stokes vector elements. *I* is the total detected light intensity which corresponds to addition of the two orthogonal component intensities, *Q* is the difference in intensity between horizontal and vertical polarization states, *U* is the portion of the intensity that corresponds to the difference between intensities of linear + 45^0 and - 45^0 (135^0) polarization states, and *V* is the difference between intensities of right circular and left circular polarization states.

The Stokes vectors corresponding to the different polarization states are represented as

Unpolarized state	Horizontal polarization (0^0)	Vertical polarization (90^0)	+45^0 polarization	135^0 (-45^0) polarization	Left circular polarization (LCP)	Right circular polarization (RCP)
$\begin{pmatrix}1\\0\\0\\0\end{pmatrix}$	$\begin{pmatrix}1\\1\\0\\0\end{pmatrix}$	$\begin{pmatrix}1\\-1\\0\\0\end{pmatrix}$	$\begin{pmatrix}1\\0\\1\\0\end{pmatrix}$	$\begin{pmatrix}1\\0\\-1\\0\end{pmatrix}$	$\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}$	$\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$

For the above description of Stokes vector, each element has been normalized to the total detected intensity (by the first element in each case).

In Stokes formalism, the following polarization parameters of any light beam are defined:
Net degree of polarization

$$DOP = \frac{\sqrt{Q^2 + U^2 + V^2}}{I}, \tag{3}$$

Degree of linear polarization

$$DOLP = \frac{\sqrt{Q^2 + U^2}}{I}, \tag{4}$$

and Degree of circular polarization

$$DOCP = \frac{V}{I}. \tag{5}$$

Note that the degree of polarization of light must always be less than or equal to unity. Thus for physical realizability, the Stokes vector element should satisfy the following equation

$$\frac{\sqrt{Q^2+U^2+V^2}}{I} \le 1 \tag{6}$$

Thus far the polarization properties of a light beam have been discussed. However, any optical element (or medium through which light propagates) may also possess intrinsic polarization characteristics. Thus the polarization properties of light may get altered (or destroyed even) upon interacting with any optical element. The interaction of polarized light with optical element is mathematically described by a 4 × 4 matrix **M,** known as the Mueller matrix (named after its inventor Hans Mueller). The Mueller matrix describes the transfer function of any medium in its interaction with polarized light. The two descriptors are linked in the so-called Stokes-Mueller calculus – given a beam with a Stokes vector $\mathbf{S}_i$ impinging on an optical element with a Mueller matrix **M**, the light output from that element will have its polarization described by a Stokes vector as

$$\mathbf{S_o} = \mathbf{M}\,\mathbf{S_i}, \tag{7}$$

$$\begin{bmatrix} I_o \\ Q_o \\ U_o \\ V_o \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \begin{bmatrix} I_i \\ Q_i \\ U_i \\ V_i \end{bmatrix} = \begin{bmatrix} m_{11}I_i + m_{12}Q_i + m_{13}U_i + m_{14}V_i \\ m_{21}I_i + m_{22}Q_i + m_{23}U_i + m_{24}V \\ m_{31}I_i + m_{32}Q_i + m_{33}U_i + m_{34}V \\ m_{41}I_i + m_{42}Q_i + m_{43}U_i + m_{44}V \end{bmatrix}, \tag{8}$$

Note that while the Stokes vectors represent the polarization state of light, Mueller matrix represents the medium polarization characteristics.

Medium polarization characteristics

The different polarization properties of a medium are coded in the various elements of the Mueller matrix **M**. The three basic polarization properties are diattenuation (differential attenuation of orthogonal polarization), retardance (de-phasing of orthogonal polarization) and depolarization, the functional forms of **M** for these are well known. Here, we shall define the polarization property, retardance through the Mueller matrix formalism (this is of direct relevance to the measurements performed in this experiment).

Retardance

Retardance is the phase shift between two orthogonal polarizations of the light. Linear retardance (δ) arises due to difference in phase between orthogonal linear polarization states (between vertical and horizontal or between 45^0 and -45^0). Circular retardance (δ_C) arises due to difference in phase between RCP and LCP. In fact, the origin of retardance lies in the anisotropy of refractive indices (known as birefringence). Linear birefringence is the difference in refractive indices for two orthogonal linear polarizations (refractive indices $n_x \neq n_y$) and accordingly circular birefringence is the difference in refractive indices for two different circular polarizations ($n_L \neq n_R$). Due to this difference in refractive indices, the two orthogonal linear (circular) polarization states accumulate different phases while propagating through the medium (having path length L), manifesting as linear (circular) retardance

$$\delta = \frac{2\pi}{\lambda}(n_x - n_y)L$$

$$\delta_C = \frac{2\pi}{\lambda}(n_L - n_R)L \tag{9}$$

Many optical elements like calcite, quartz etc. posses linearly anisotropic refractive indices and accordingly exhibit linear retardance effects. Similarly, molecules like glucose, proteins, lipids possess asymmetric chiral structure and accordingly exhibit circular retardance. Note that circular retardance effect manifests as rotation of the plane of linearly polarized light about the axis of propagation (known as optical rotation, rotation angle $\psi = \frac{\delta_C}{2}$).

The general form of a Mueller matrix of a linear retarder with magnitude of retardance δ and axis of birefringence oriented at an angle θ with respect to the horizontal is

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\theta + \sin^2 2\theta\cos\delta & \sin 2\theta\cos 2\theta(1-\cos\delta) & -\sin 2\theta\sin\delta \\ 0 & \sin 2\theta\cos 2\theta(1-\cos\delta) & \sin^2 2\theta + \cos^2 2\theta\cos\delta & \cos 2\theta\sin\delta \\ 0 & \sin 2\theta\sin\delta & -\cos 2\theta\sin\delta & \cos\delta \end{pmatrix}. \tag{10}$$

Similarly, Mueller matrix for a circular retarder with retardance δ_C or optical rotation ψ is

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\psi & -\sin 2\psi & 0 \\ 0 & \sin 2\psi & \cos 2\psi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{11}$$

3. WORKING FORMULA

The experiment involves determination of linear retardance (δ) and optical rotation (ψ) of SLM by performing Stokes vector measurements. The underlying principle is outlined below.

Determination of linear retardance δ of a pure linear retarder (not applicable for the SLM)

Using Eq. (8) and Eq. (10), for incident horizontally (0^0) linearly polarized light (input Stokes vector $[1\ 1\ 0\ 0]^T$), the Stokes vector ($\mathbf{S_H}$) of light emerging from a pure linear retarder is

$$\mathbf{S_H} = \begin{pmatrix} 1 \\ \cos^2 2\theta + \sin^2 2\theta\cos\delta \\ \sin 2\theta\cos 2\theta(1-\cos\delta) \\ \sin 2\theta\sin\delta \end{pmatrix} \tag{12}$$

Using the same Eqs for incident $+45^0$ linearly polarized light (input Stokes vector $[1\ 0\ 1\ 0]^T$), the Stokes vector ($\mathbf{S_P}$) of emerging light emerging from a pure linear retarder is

$$\mathbf{S_P} = \begin{pmatrix} 1 \\ \sin 2\theta \cos 2\theta (1 - \cos\delta) \\ \sin^2 2\theta + \cos^2 2\theta \cos\delta \\ -\cos 2\theta \sin\delta \end{pmatrix} \tag{13}$$

Thus the second element of output Stokes vector [$\mathbf{S_H}$ (2)] in Eq. (12) and the third element [$\mathbf{S_p}$ (3)] of the output Stokes vector in Eq. (13) can be combined to yield the value for linear retardance δ of the sample as

$$\boldsymbol{\delta = \cos^{-1} ([S_H (2)]+ [S_P (3)]-1)} \tag{14}$$

Note, once the value for δ is determined the orientation of the axis of retardance θ can also be determined from Eq. (12) and (13).

Determination of optical rotation ψ of a pure chiral sample (not applicable for the SLM)

Using Eq. (8) and Eq. (11), for incident horizontally (0^0) linearly polarized light (input Stokes vector $[1\ 1\ 0\ 0]^T$), the Stokes vector ($\mathbf{S_H}$) of light emerging from a pure chiral (circular retarder) sample is

$$\mathbf{S_H} = \begin{pmatrix} 1 \\ \cos 2\psi \\ \sin 2\psi \\ 0 \end{pmatrix} \tag{15}$$

Thus optical rotation ψ can be determined from this measurement alone as

$$\psi = \mathbf{0.5 \times \tan^{-1}} \left[\frac{S_H(3)}{S_H(2)}\right] \tag{16}$$

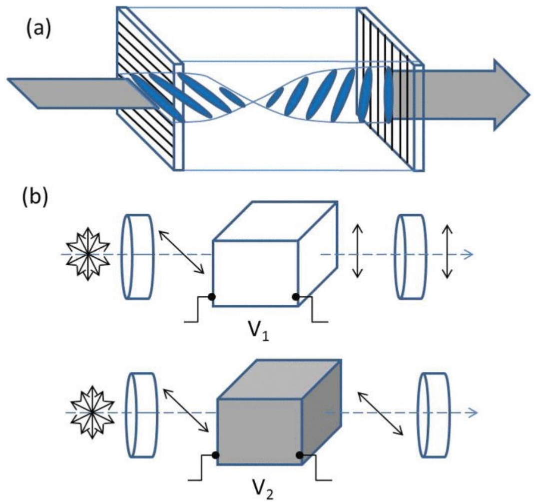
For further confirmation, ψ can also be determined from the other two incident linear polarization states (vertical 90^0 and $+45^0$) as

$$\psi = \mathbf{0.5 \times \tan^{-1}} \left[\frac{S_V(3)}{S_V(2)}\right] \tag{17}$$

$$\text{and } \psi = \mathbf{- 0.5 \times \tan^{-1}} \left[\frac{S_P(2)}{S_P(3)}\right] \tag{18}$$

4. SPATIAL LIGHT MODULATOR (SLM)

Spatial light modulators (SLMs) are opto-electronic devices that modulate the amplitude or phase of a beam of light, and are used in a number of other applications where one desires to manipulate the spatial profile of a light beam. The most common type of SLM is the liquid crystal display (LCD), which incorporates a pixelated layer of polarization-altering liquid crystals between two polarizers in order to modulate the intensity of the transmitted light. A simple sketch of an LCD-SLM pixel is shown in Fig. below.



(a) Operation of an LCD pixel showing the polarizers that act as the director and analyzer. Rotation of the polarization depends on the voltage across applied to the LCD.
(b) The transmissive and opaque states of an LCD pixel.
Ref: Am. J. Phys. **80**, 211 (2012)

The input beam is polarized horizontally before entering the liquid crystal assembly. The polarization can then be rotated depending on the voltage applied across the pixel. The analyzer on the output side transmits only the vertical component of light. The voltage applied to each pixel thus controls the intensity of transmitted light. Because the liquid crystal SLM matrix can be easily controlled by a computer, this scheme allows real-time manipulation of the wavefront of a light beam by creating amplitude and phase masks on-demand.

Since the SLM is made of liquid crystals which rotate and align according to the voltage applied to them, the general form of the Mueller matrix of an SLM can be written as a retarder matrix multiplied with a rotation matrix (***see additional study materials***). One can write this matrix using Eqs. 10 and 11 as

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\theta + \sin^2 2\theta\cos\delta & \sin 2\theta\cos 2\theta(1-\cos\delta) & -\sin 2\theta\sin\delta \\ 0 & \sin 2\theta\cos 2\theta(1-\cos\delta) & \sin^2 2\theta + \cos^2 2\theta\cos\delta & \cos 2\theta\sin\delta \\ 0 & \sin 2\theta\sin\delta & -\cos 2\theta\sin\delta & \cos\delta \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\psi & -\sin 2\psi & 0 \\ 0 & \sin 2\psi & \cos 2\psi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

As explained earlier, the Stokes vector for incident horizontally polarized light (S_H) and the Stokes vector for incident +45° linearly polarized light (S_P) can be written as,

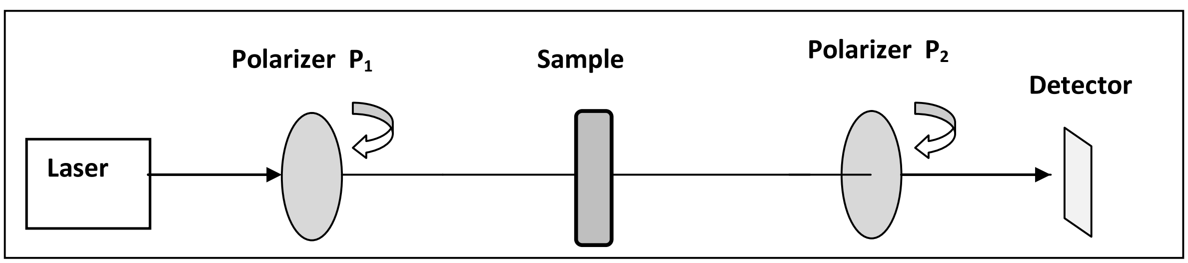
$$S_H = \begin{pmatrix} 1 \\ \cos 2\theta\cos(2\theta-2\psi) + \sin 2\theta\cos\delta\sin(2\theta-2\psi) \\ \sin 2\theta\cos(2\theta-2\psi) - \cos 2\theta\cos\delta\sin(2\theta-2\psi) \\ \sin(2\theta-2\psi)\sin\delta \end{pmatrix}, S_P = \begin{pmatrix} 1 \\ -\cos\delta\sin 2\theta\cos(2\theta-2\psi) + \cos 2\theta\sin(2\theta-2\psi) \\ \cos\delta\cos 2\theta\cos(2\theta-2\psi) + \sin 2\theta\sin(2\theta-2\psi) \\ -\cos(2\theta-2\psi)\sin\delta \end{pmatrix}$$

Thus linear retardance (δ) and optical rotation (ψ) of the SLM device can be calculated using the following relations

$$S_H(2)+S_P(3)=(1+\cos\delta)\cos 2\psi = X_1$$
$$S_H(3)-S_P(2)=(1+\cos\delta)\sin 2\psi = Y_1$$
$$\Rightarrow \delta=\cos^{-1}\left[\sqrt{X_1^{\,2}+Y_1^2}-1\right]$$
$$\psi=\frac{1}{2}\tan^{-1}\left(\frac{Y_1}{X_1}\right) \qquad \textbf{(19)}$$

4. EXPERIMENTAL PROCEDURE

A schematic of the experimental set-up is shown below. In the set-up; P1& P2 are linear polarizers mounted on rotational stage. The 650 nm line of a diode laser is used as the source. The light transmitted through the sample; passing through polarizer P2 (analyzer) is detected by a photodetector. Note that from Eq. (12)- (18), determination of linear retardance δ and optical rotation ψ for pure linear and circularly birefringent (respectively) samples do not involve circular polarization state measurements. In this experiment, these are therefore determined by performing Stokes vector measurements involving linear polarization measurements alone (the first three elements of the Stokes vector are used, the fourth element involving circular polarization is excluded).



Following is the details of the measurement procedure

The input polarizer P1 is pre-aligned. The marked angular positions in the rotation stage represent the following incident linear polarization states

0^0: horizontal polarization (H); 90^0: Vertical polarization (V); 45^0: + 45^0 polarization (P)

Align the analyzer (P2) to note down its angular positions (in the rotational stage) corresponding to the four required linear polarizations
0^0 horizontal (H); 90^0 Vertical (V); + 45^0 polarization (P) and 135^0 (- 45^0) polarization (M).

Determination of linear retardance δ and optical rotation Ψ of the SLM

(i) Keep the input polarizer P1 at horizontal (H) polarization state (input Stokes vector $[1\ 1\ 0\ 0]^T$).

Perform the four required intensity measurements (I) by orienting the analyzer (P2) at the positions corresponding to the four linear polarization states: I_H, horizontal; I_V, vertical; I_P, $+45^0$; I_M, 135^0 (-45^0) linear polarization. Construct the output Stokes vector

$$\mathbf{S_H} = \begin{bmatrix} I \\ Q \\ U \\ \end{bmatrix}_H = \begin{bmatrix} I_H + I_V \\ I_H - I_V \\ I_P - I_M \\ \end{bmatrix}$$

Represent the measured Stokes vector in normalized form (normalize with respect to the first element).

(ii) Perform the same measurement by keeping the input polarizer P1 at $+ 45^0$ (P) polarization state (input Stokes vector $[1\ 0\ 1\ 0]^T$). Determine the out put Stokes vector $\mathbf{S_P}$.

(iii) Use Eq. (19) to determine the magnitude of linear retardance δ (in radian) and optical rotation ψ of the SLM for a given grey scale level (voltage) set at the input monitor of the SLM.

(iv) Repeat the measurements for three different grey scale levels (e.g, at 25%, 50 % and 100 %) set at the input monitor of the SLM and one for power off state of the SLM

(v) Plot the variation of δ and ψ as a function of grey scale value.

Tabulation of Stokes vector measurements

Input polarization states	Measured intensities (Photodetector current)				Measured output Stokes vectors	Linear retardance δ (rad)	Optical rotation (rad)
	I_H	I_V	I_P	I_M			
Horizontal (H, 0^0)					$\mathbf{S_H} = \begin{bmatrix} 1 \\ Q/I \\ U/I \\ \end{bmatrix}_H$; $I = I_H + I_V$		
$+ 45^0$ (P polarization)					$\mathbf{S_P} = \begin{bmatrix} 1 \\ Q/I \\ U/I \\ \end{bmatrix}_P$; $I = I_H + I_V$		

Comment on the sources of errors in the measurements.

Suggested reading

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