4.2 Effect of the SLM

Certainly, to understand what phase is accumulated to the Electric field, it is necessary to correctly quantify the evolution of the Electric field while passing through the SLM. For this reason, we will construct the Jones matrix of the SLM [13]. And calculate the phase it imparts on two different circular polarization states of light.

We know TNSLM is made up of many layers of birefringent layers. First, lets assume the twisting is linear, i.e. the twist ϕ can be interpreted as

$$\psi(z) = \alpha z \tag{4.4}$$

where z is the distance along the propagation direction and α is contant.For our TNSLM total twist angle, $\psi(d) = \frac{\pi}{2}$; d being the thickness of the SLM. And, we denote the total birefringence of the SLM to be

$$\delta_t = \frac{2\pi}{\lambda}(n_e - n_0)d \tag{4.5}$$

Now let there be N number of layers in the SLM. With these properties, each of the layer will be twisted at an angle $\frac{\phi}{N}$ and the birefringence of that layer is given by $\frac{\delta_t}{N}$. So the full Jones matrix of the SLM is given as:

$$J = R(\psi)\left[\begin{pmatrix} e^{i\frac{2\pi n_0 d}{\lambda N}} & 0 \\ 0 & e^{i\frac{2\pi n_e d}{\lambda N}} \end{pmatrix} R(-\frac{\psi}{N})\right]^N \tag{4.6}$$

Which can be further simplified as:

$$J = R(\psi)\left[\psi_0 R(-\frac{\psi}{N})\right]^N \tag{4.7}$$

where

$$\psi_0 = e^{i\frac{2\pi n_0 d}{\lambda N}} e^{i\frac{\delta_t}{2N}} \begin{pmatrix} e^{-i\frac{\delta_t}{2N}} & 0 \\ 0 & e^{i\frac{\delta_t}{2N}} \end{pmatrix} \tag{4.8}$$

$$\Rightarrow J = e^{i(\frac{2\pi n_0 d}{\lambda}+\frac{\delta_t}{2})} R(\psi) \begin{pmatrix} cos\frac{\psi}{N} e^{-i\frac{\delta_t}{2N}} & sin\frac{\psi}{N} e^{-i\frac{\delta_t}{2N}} \\ sin\frac{\psi}{N} e^{i\frac{\delta_t}{2N}} & cos\frac{\psi}{N} e^{i\frac{\delta_t}{2N}} \end{pmatrix}^N = e^{i(\frac{2\pi n_0 d}{\lambda}+\frac{\delta_t}{2})} R(\psi) \begin{pmatrix} X - iY & Z \\ -Z & X + iY \end{pmatrix} \tag{4.9}$$

with

$$\begin{aligned} X &= cos\left(\sqrt{\psi^2 + (\frac{\delta_t}{2})^2}\right) \\ Y &= \left(\frac{\delta_t/2}{\left(\sqrt{\psi^2 + (\frac{\delta_t}{2})^2}\right)}\right) sin\left(\sqrt{\psi^2 + (\frac{\delta_t}{2})^2}\right) \\ Z &= \left(\frac{\psi}{\left(\sqrt{\psi^2 + (\frac{\delta_t}{2})^2}\right)}\right) sin\left(\sqrt{\psi^2 + (\frac{\delta_t}{2})^2}\right) \end{aligned} \tag{4.10}$$

Here, the phase factor in front of the rotation matrix gives the dynamical phase due to the SLM. So, **dynamical phase**

$$= \left(\frac{2\pi n_0 d}{\lambda} + \frac{\delta_t}{2}\right) \tag{4.11}$$

Now the geometrical phase associated to a particular polarization state due to *J'* is given as $arg\langle\psi|J'|\psi\rangle$ for $|\psi\rangle \in \{|LCP\rangle, |RCP\rangle\}$. where the matrix *J'* is given as:

$$J' = R(\phi) \begin{pmatrix} X - iY & Z \\ -Z & X + iY \end{pmatrix} = R(\phi) U(\delta_t, \psi) \tag{4.12}$$

Here, at this point we face an experimental difficulty that the parameter δ_t i.e. the total birefringence present in *J'* that will be present in thus calculated geometrical phases, cannot be measured directly. For this reason, we record the Mueller matrices of the SLM extract the "effective" birefringence and rotation and calculate back the total birefringence[14].

4.2.1 Rotator Retarder Model

We employ the fact that any unitary matrix **U** has an optically equivalent system consisting of one retarder and one rotator [15]. So we write

$$
\begin{aligned}
& U(\delta_t,\psi) \\
= \ & R(\psi_{eq})Ret(\delta_{eq},\theta_{eq}) \qquad (4.13) \\
= \ & \begin{pmatrix} cos\psi_{eq} & sin\psi_{eq} \\ -sin\psi_{eq} & cos\psi_{eq} \end{pmatrix} \begin{pmatrix} cos\theta_{eq} & -sin\theta_{eq} \\ sin\theta_{eq} & cos\theta_{eq} \end{pmatrix} \begin{pmatrix} e^{-i\frac{\delta_{eq}}{2}} & 0 \\ 0 & e^{i\frac{\delta_{eq}}{2}} \end{pmatrix} \begin{pmatrix} cos\theta_{eq} & sin\theta_{eq} \\ -sin\theta_{eq} & cos\theta_{eq} \end{pmatrix}
\end{aligned}
$$

From this relation and equation 4.12, we have :

$$
\begin{aligned}
X &= cos\frac{\delta_{eq}}{2}cos\psi_{eq} \\
Y &= sin\frac{\delta_{eq}}{2}cos(2\theta_{eq}-\psi_{eq}) \\
Z &= cos\frac{\delta_{eq}}{2}sin\psi_{eq} \qquad (4.14) \\
sin\frac{\delta_{eq}}{2}sin(2\theta_{eq}-\psi_{eq}) &= 0
\end{aligned}
$$

comparing eqns. 4.10 and ~~4.15~~ 4.13, we get δ_t in terms of δ_{eq} and ψ_{eq} as:

$$\delta_t = 2\sqrt{\left[cos^{-1}\left(cos\left(\frac{\delta_{eq}}{2}\right)cos\psi_{eq}\right)\right]^2 - \psi^2} \qquad (4.15)$$