Fabry-Pérot Interferometry
Experiment 03

Sagnik Seth   22MS026   Group:B6

1 Aim

- To measure the distance between the plates of a Fabry-Pérot etalon

- To measure wavelength of unknown laser source using the distance between plates

- To estimate the finesse of a fringe pattern.

2 Apparatus

The required apparatus for performing the experiment are:

1. Laser source

2. Fabry-Perot etalon

3. Power meter

4. Focusing lens

5. Screen

6. Optical table

3 Theory

The experiment is based on the phenomenon of multi-beam from a parallel glass slab. Consider a beam of light falling on a glass slab with refractive index n. This beam will undergo multiple refractions and internal reflections inside the slab, as shown below.

The higher order reflections become important if the reflection coefficient r is not very small. t denotes the fraction of amplitude transmitted on entering the slab and t' denotes the fraction transmitted when leaving the slab.
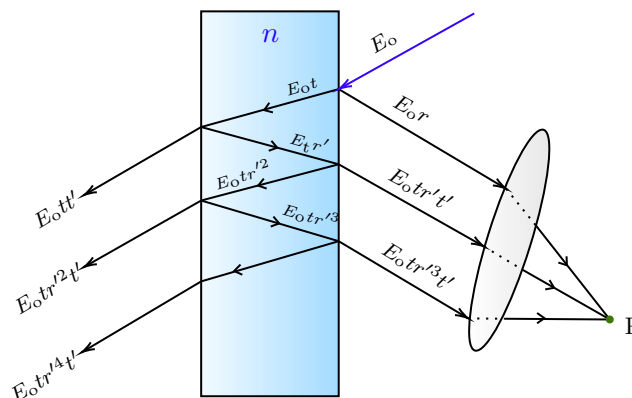

Figure 1: Multibeam Interference from a parallel slab

Consider the parallel reflected rays, each of which bears a fixed phase relation with the other. These rays are mutually coherent and will interfere when brought to focus at P. Suppose that the incident wave is

given by $E_0 e^{i\omega t}$, then the optical field at P is given by:

$$\widetilde{E}_{1\mathrm{r}} = E_0 r e^{i\omega t}$$
$$\widetilde{E}_{1\mathrm{r}} = E_0 r' t t' e^{i(\omega t - \delta)}$$
$$\widetilde{E}_{1\mathrm{r}} = E_0 r'^3 t t' e^{i(\omega t - 2\delta)} \tag{1}$$
$$\vdots$$
$$\widetilde{E}_{1\mathrm{r}} = E_0 r'^{(2N-3)} t t' e^{i(\omega t - (N-1)\delta)}$$

The total reflected field is equal to the sum of these components, which gives us

$$\begin{aligned}\widetilde{E}_{\mathrm{r}} &= \widetilde{E}_{1\mathrm{r}} + \widetilde{E}_{2\mathrm{r}} + \ldots + \widetilde{E}_{N\mathrm{r}} \\ &= E_0 e^{-i\omega t}\left[r + r' t t' e^{-i\delta}\left(1 + r'^2 e^{-i\delta} + \ldots + \left(r'^2 e^{-i\delta}\right)^{N-2}\right)\right]\end{aligned} \tag{2}$$

For $\left| r'^2 \right| < 1$ and $N \to \infty$, the above sum becomes an infinite GP and we obtain,

$$\widetilde{E}_{\mathrm{r}} = E_0 e^{i\omega t}\left[r + \frac{r' t t' e^{-i\delta}}{1 - r'^2 e^{-i\delta}}\right] \tag{3}$$

We know that $r' = -r$ and $t t' + r^2 = 1$, from which we get

$$\widetilde{E}_{\mathrm{r}} = \left[\frac{1 - e^{-i\delta}}{1 - r^2 e^{-i\delta}}\right] E_0 e^{i\omega t} r \tag{4}$$

The reflected flux density at P can be defined as,

$$\mathcal{I}_{\mathrm{r}}^{\mathrm{P}} = \frac{1}{2}\widetilde{E}_{\mathrm{r}}\widetilde{E}_{\mathrm{r}}^* = \frac{1}{2}E_0^2 \frac{2 r^2 (1 - \cos\delta)}{(1 + r^4 - 2 r^2 \cos\delta)} = \frac{\mathcal{F}\sin^2\left(\delta/2\right)}{1 + \mathcal{F}\sin^2\left(\delta/2\right)}\mathcal{I}_i \tag{5}$$

where $\mathcal{I}_i$ is the incident flux and $\mathcal{F} = \left(\frac{2r}{1-r^2}\right)^2$ is defined as the *finesse*. Similarly for transmitted waves we have,

$$\widetilde{E}_{\mathrm{t}} = E_0 e^{i\omega t}\frac{t t'}{1 - r^2 e^{-i\delta}} \implies \mathcal{I}_{\mathrm{t}}^{\mathrm{P}} = \frac{1}{1 + \mathcal{F}\sin^2(\delta/2)}\mathcal{I}_i \tag{6}$$

From this we can observe that $\mathcal{I}_i = \mathcal{I}_{\mathrm{r}} + \mathcal{I}_{\mathrm{t}}$. Maximum transmission intensity occurs when $\sin(\delta/2) = 0 \implies \delta_{\mathrm{M}} = 2\pi M$ for integral values of M while minimum intensity occurs when $\sin(\delta/2) = 1 \implies \delta_{\mathrm{m}} = (2m+1)\pi$. The maximum and minimum values of intensity are respectively,

$$\mathcal{I}_{\mathrm{t}}^{\max} = \mathcal{I}_i \qquad\qquad \mathcal{I}_{\mathrm{t}}^{\min} = \left(\frac{1 - r^2}{1 + r^2}\right)^2 \mathcal{I}_i \tag{7}$$

3.1 Fabry-Pérot Interferometer

Having seen the basics of multi-beam interference, we introduce the principle behind our experimental setup.
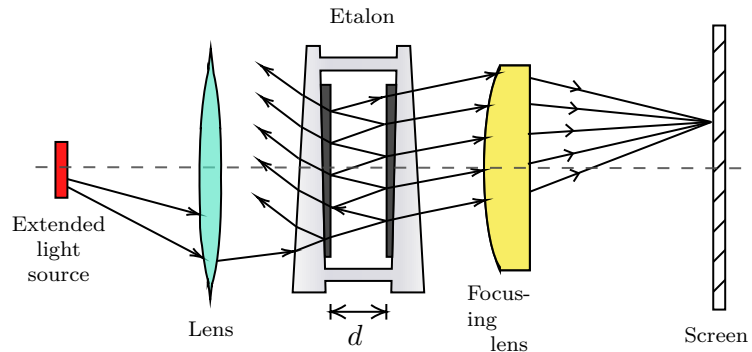

Figure 2: Schematic diagram of the Fabry-Perot experimental set-up

4 Data and Calculations

4.1 Distance between Plates

4.2 Wavelength of Unknown Laser

4.3 Finesse Calculation

5 Error Analysis

6 Discussion and Conclusion