Discrete Mathematics, Summer 2020 - Final Exam

Exercise 1. Let M = {a, b, c} and let ⊆ be the usual subset relation. Draw the Hasse diagram for the partially ordered set (P(M), ⊆) , where P(M) is the power set of M.
(3 Marks)

Exercise 2. If G = (V, E) is a simple graph, then we define the complement of G as the graph G' := (V, E'), where

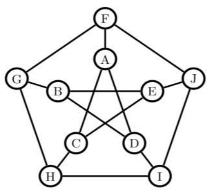
E'= {{x, y}: x not equal to y, {x, y} not in E}.

In other words, G' is the simple graph having the same vertices as G and containing precisely those edges that are not E. Suppose that G has n vertices. Prove that if both G and G' are planar, then n ≤ 10.
(4 Marks)

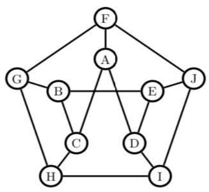
Exercise 3. Find the chromatic number of the **Petersen** graph. Prove your answer by exhibiting a k-coloring and showing that fewer colors will **not** be sufficient.
(3 Marks)

Exercise 4. The **crossing number** k ∈ **N** of a graph is the minimum number of edges in the graph such that the graph with these edges deleted has a planar representation. The crossing number of a planar graph is zero. Find the crossing number of the **Petersen** graph. Prove your answer by showing which edges need to be deleted and giving the corresponding planar representation. Show also that deleting fewer edges will not be sufficient. (You do **not** need to formally prove that the planar representation is isomorphic to the original representation with deleted edges.)
(3 Marks)

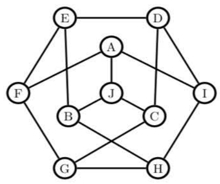
Exercise 5. Consider the following three graphs:



Graph I



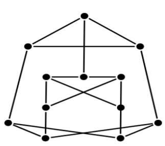
Graph II

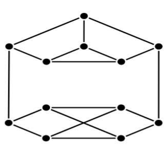


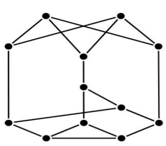
Graph III

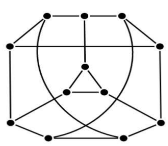
Determine which of these graphs are isomorphic (I and II?, II and III?, I and III?). If they are, give the graph isomorphism and prove that it is an isomorphism. If they are not, prove this.
(6 Marks)

Exercise 6. If two graphs are isomorphic, they either both have a hamiltonian circuit or they both do not have such a circuit. Consider the following graphs:


A


B


C


D

Each of the graphs above has a single simple circuit of length 3; in each of the graphs, delete the vertices in the circuit and all edges incident to them, obtaining subgraphs A′, B′, C′, D′. Which of these subgraphs have a hamiltonian circuit? Draw the circuit in those that have one. Show that the graphs among A′, B′, C′, D′ with a hamiltonian circuit are **not** isomorphic to each other.
(2+3+3 Marks)