Lab 3 Transient Lab

VE215 Intro to Circuits

Fall 2014

1 Lab goals

1. Apply the theory you learned on the step responses in first- and second-order circuits to series RC and RLC circuits, which you will build in the lab.

2. Build a series RC circuit, observe its responses to input square wave signal of varied frequency, and explain them based on the theory you learned:

- Relate the observed capacitor voltage and resistor voltage as functions of time to your pre-lab calculations
- Explain the changes of both output waveforms in response to the increase of the frequency of the input square wave signal
- Explain the amplitudes of the capacitor voltage and the resistor voltage related to the amplitude of the input square wave

3. Build a series RLC circuit, observe the three types of its responses to input square wave signal, and relate them to the theory you have learned. For the under-damped/ over-damped/ critical damped response, compare the resistance in the circuit measured in the lab with the critical resistance you calculated in the pre-lab.

4. Build the simplest second-order circuit, an LC tank, and observe oscillations.

2 Theoretical background

2.1 First-order circuits

Theoretically, the transient responses in electric circuits are described by differential equations. The circuits, whose responses obey the first-order differential equation

$$\frac{dx(t)}{dt} + \frac{1}{\tau} \cdot x(t) = f(t)$$

are called **first-order circuits**. Their responses are always monotonic and appear in the form of exponential function

$$x(t) = K_1 \cdot e^{-(\frac{t}{\tau})} + K_2$$

A first-order circuit includes the effective resistance R and one energy-storage element, an inductor L or a capacitor C.

In an RC circuit, the time constant is

$$\tau = RC.$$

In an LC circuit, the time constant is

$$\tau = \frac{L}{R}.$$

The **fall time** of a signal is defined as the interval between the moment when the signal reaches its 90% and the moment when the signal reaches its 10% level. Note that the 10% level is reached between 2τ and 3τ. Approximately, you can assume $falltime \approx 2.2\tau$. After $t = 5\tau$, the exponent practically equals zero.

2.2 Second-order circuits

Many circuits involve two energy-storing elements, both an inductor L and a capacitor C. Such circuits require a second-order differential equation description

$$\frac{d^2x(t)}{dt^2} + 2 \cdot \alpha \cdot \frac{dx(t)}{dt} + \omega_0^2 \cdot x(t) = f(t)$$

thus they are called **second-order circuits**.

We will consider only second-order circuits with one inductor and one capacitor. The differential equation includes two parameters: the damping factor α and the undamped frequency ω_0 which are determined by the circuit and its components.

For example, in the series RLC circuit, which you will build and study in this lab,

$$\alpha = \tfrac{R}{2 \cdot L}, \text{ and } \omega_0 = \tfrac{1}{\sqrt{L \cdot C}},$$

while in the parallel RLC circuit,

$$\alpha = \tfrac{1}{2 \cdot R \cdot C}, \text{ and } \omega_0 = \tfrac{1}{\sqrt{L \cdot C}}.$$

Depending on the two parameters α and ω_0, second-order circuits can exhibit three types of responses.

2.2.1 The underdamped response

If $\alpha < \omega_0$,

$$x(t) = e^{-\alpha t}(K_1 cos(\omega t) + K_2 sin(\omega t))$$

where $\omega = \sqrt{\omega_0^2 - \alpha^2}$.

The underdamped circuit response involves decaying oscillations, which may last for many periods or for less than one period, depending on the damping ratio $\xi = \frac{\alpha}{\omega_0}$, which for the series RLC circuit $\xi = \frac{R}{2L}\sqrt{LC} = \frac{R}{2} \cdot \sqrt{\frac{C}{L}}$. Varying the values of R, L, C, affects the damping ratio ξ.

2.2.2 The critically damped response

If $\alpha = \omega_0$,

$$x(t) = e^{-\alpha t}(K_1 + K_2 t)$$

and the circuit has the critically damped response.

The critically damped response does not involve oscillations.

For the series RLC circuits, $\alpha = \omega_0$ corresponds to $\frac{R}{2L} = \frac{1}{\sqrt{LC}}$ or $R = R_{critical} = 2\sqrt{\frac{L}{C}}$.

If $L = 1mH$ and $C = 10nF$, then $R_{critical} \approx 632\Omega$.

2.2.3 The overdamped response

If $\alpha > \omega_0$,

$$x(t) = K_1 \cdot e^{s_1 t} + K_2 \cdot e^{s_2 t}$$

where $s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$ and $s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$.

In the series RLC circuits, the overdamped solution is obtained if the resistance is larger that the critical resistance, such that $R > R_{critical} = 2 \cdot \sqrt{\frac{L}{C}}$.

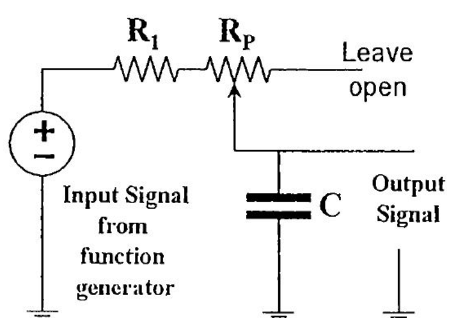
Notice that the larger resistance corresponds to the longer delay, and even the faster decay has a much longer fall time than the critically damped response.

One of the most interesting features of series RLC circuits is that increasing the resistance above the critical value results in much longer fall time, or longer delays of responses in digital circuits. Among all monotonic responses, the critically damped is the fastest.

3 Pre-lab assignments

1. Calculate the time constant τ and fall time values for the maximal and minimal resistances (the two limit positions of the potentiometer) in the circuit on this diagram using the following nominal values:

$$R_1 = 1k\Omega,\ R_P = 10k\Omega,\ C = 100nF = 0.1\mu F.$$

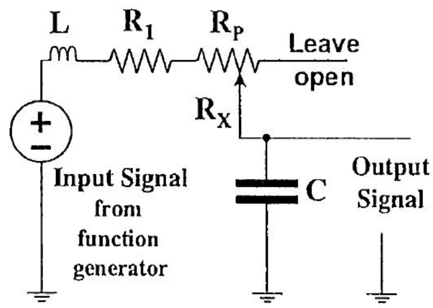


For simplicity, assume $Falltime = 2.2\tau$

2. In the circuit shown on this diagram, assume:

$$L = 1mH,\ R_1 = 100\Omega,\ R_P = 10k\Omega,\ C = 820pF.$$

Depending on the position of the potentiometer's tap, R_X varies from zero to R_P.



Calculate:

A. The range of R_X that ensures under-damped response
B. The range of R_X that ensures critically damped response
C. The range of R_X that ensures over-damped response

3. In this lab, you will use a function generator to produce a square wave. The function generator voltage abruptly changes from $+V_0$ to $-V_0$ and back again, which is quite similar to the switch flipped back and forth. Therefore, the impulse response of the circuit can be studied. Assume the circuit in question 1 is being studied, and the your function generator sets a square wave at $1Vppk$ (peak-to-peak). For circuits with $\tau_m in$ and $\tau_m ax$, calculate the fastest frequency that allows the output signal to reach saturation.

4. Prove that for a first-order circuit, the rise time/ fall time is approximately 2.2 times of the time constant τ.

5. Simulate the circuit in question 1 with *PSpice*, assuming the smallest time constant $\tau_m in$. Analyze the transient response of the circuit, with proper frequency as you calculated in question 3. Generate plots of both input and output signal waveforms. Label the intervals in which rise time and fall time are defined.(Hint: Refer to section 7.8 and 8.9 for instructions of transients analysis with *PSpice*.)

6. Generate plots for the three types of transient responses, assuming the circuit in question 2 is being studied. You can do it with MATLAB or by simulating the circuit in *PSpice*. Both input and output and output signal waveforms should be monitored. Suppose that the function generator produces a square wave at $1Vppk$ at $10kHz$.