UM-SJTU JOINT INSTITUTE
INTRO TO CIRCUITS
(VE 215)

LABORATORY REPORT

LAB 5

FILTER LAB

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1. Introduction [1]

1.1. Objectives

- Learn about four types of filters – Low-Pass, High-Pass, Band-Pass, and Band-reject.
- Learn about transfer functions.
- Predict the theoretical result and make comparison with lab data.

1.2. Theoretical background & Apparatus

1.2.1. Filter

Filters are everywhere in our lives. The circuits built to operate on signals usually apply filters. For example, telephone lines pass the sounds at frequencies between about 100Hz and 3kHz and practically blocks all other frequencies.

1.2.2. Transfer function

Mathematically, the transfer function is used to analyze what the circuit did to the signal:

$$\text{transfer function} = \frac{Output\ signal}{Input\ signal}$$

This function can also be expressed as

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)}$$

The magnitude of the transfer function is called "voltage gain", often measured as the ratio of the peak-to-peak (ppk) voltages:

$$|H(\omega)| = \left|\frac{V_{out}(\omega)}{V_{in}(\omega)}\right| = \frac{V_{out,ppk}(\omega)}{V_{in,ppk}(\omega)}$$

It is convenient to express and plot the magnitude of the transfer function on the logarithmic scale using decibels:

$$|H(\omega)|_{dB} = 20\, log_{10}\left(\frac{V_{out,ppk}(\omega)}{V_{in,ppk}(\omega)}\right)$$

Since both ppk voltages are always positive, the transfer function magnitude is positive and thus can always be converted to decibels. The use of decibels allows us to review data over a broad range.

1.2.3. Types of filters

The figure below shows four main families of filters (Figure 1).

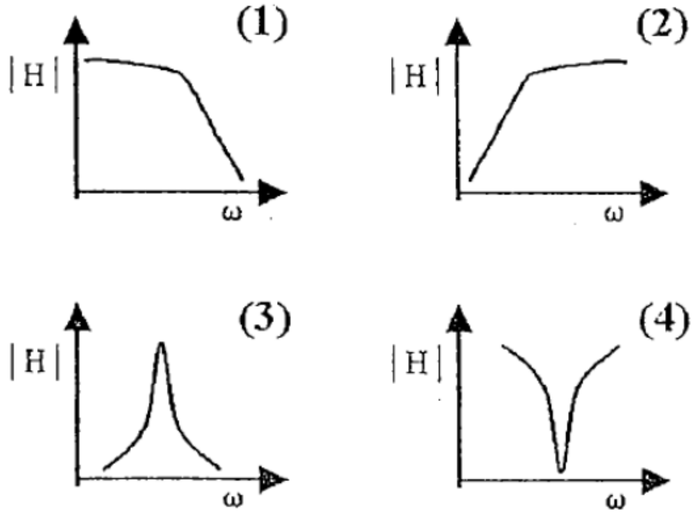

Figure 1. Four main families of filters

They are (1) Low-Pass, (2) High-Pass, (3) Band-Pass and (4) Band-reject (also called band-stop or notch) respectively.

The table below summarizes the characteristics of ideal filters (Table 1).

Summary of the characteristics of ideal filters.

Type of Filter	$H(0)$	$H(\infty)$	$H(\omega_c)$ or $H(\omega_0)$
Lowpass	1	0	$1/\sqrt{2}$
Highpass	0	1	$1/\sqrt{2}$
Bandpass	0	0	1
Bandstop	1	1	0

ω_c is the cutoff frequency for lowpass and highpass filters; ω_0 is the center frequency for bandpass and bandstop filters.

Table 1. Summary of the characteristics of ideal filters

Filter circuits, which we are going to build in this lab, contain resistors, capacitors, and inductors. They are all passive filters.

1.2.4. High-pass filter

The high-pass filter we are going to build uses a capacitor and a resistor (Figure 2)

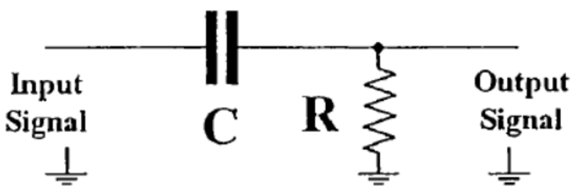

Figure 2. A typical structure of high-pass filter

For the high-pass filter,

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC}$$

We should note that $H(0)=0$, $H(\infty)=1$. Hence, it would only let high frequency pass.

1.2.5. Low-pass filter

The low-pass filter we are going to build uses a capacitor and a resistor (Figure 3).

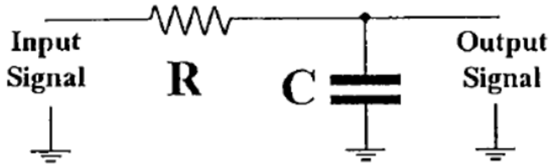

Figure 3. A typical structure of low-pass filter

For the low-pass filter,

$$H(\omega)=\frac{V_{out}(\omega)}{V_{in}(\omega)}=\frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}}=\frac{1}{1+j\omega RC}$$

We should note that $H(0)=1$, $H(\infty)=0$. It would only let low frequency pass.

1.2.6. Band-pass filter

The band-pass filter we are going to build uses a capacitor, an inductor and a resistor (Figure 4).

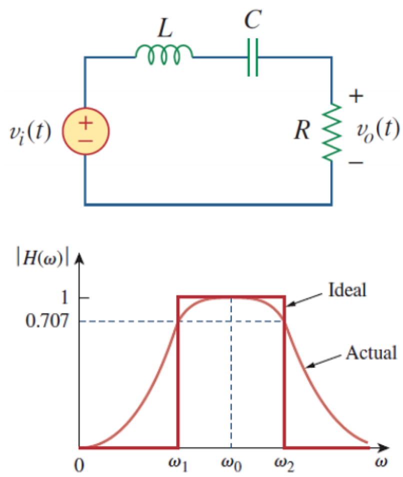

Figure 4. A typical structure of low-pass filter and the its function

For the band-pass filter,

$$H(\omega)=\frac{V_{out}(\omega)}{V_{in}(\omega)}=\frac{R}{R+j\left(\omega L-\frac{1}{\omega C}\right)}$$

We should note that $H(0)=0$, $H(\infty)=0$. The band-pass filter passes a band of frequencies centered on the center frequency ω_0, which is given by $\omega_0=1/\sqrt{LC}$.

1.2.7. Band-stop filter

The band-stop filter we are going to build uses a capacitor, an inductor and a resistor (Figure 5).

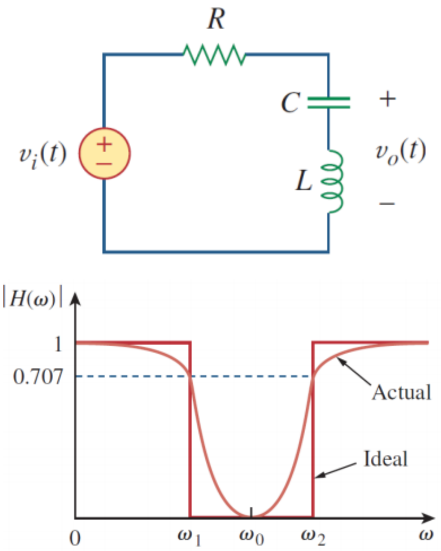

Figure 5. A typical structure of band-stop filter and the its function

For the band-stop filter,

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{j\left(\omega L - \frac{1}{\omega C}\right)}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

We should note that $H(0) = 0$, $H(\infty) = 0$ .the band-stop filter rejects a band of frequencies centered on the center frequency ω_0, which is given by $\omega_0 = 1/\sqrt{LC}$.

1.2.8. Apparatus

The experimental setup consists of function generator, oscilloscope and so on.

2. Measurements [2]

2.1. According to the pre-lab assignments, we are supposed to fill in the Expected Data columns in the tables before the lab.

2.2. During the lab:

1) We should construct the circuit for each type of filter. Resister: R = 982Ω; Capacitor: C = 0.1μF; Inductor: L = 1mH.
2) We should set the input signal in the function generator to be **Sine Wave** with amplitude of **5** $\boldsymbol{V_{ppk}}$ and change the frequency accordingly.
3) We should use the oscilloscope to detect the **amplitudes of the Input and Output signals**. Then, we should record them respectively in the first two column in the tables.
4) Additionally, for the **Band-reject Filter**, when the frequency approach the critical frequency at which the **Transfer Function Magnitude** reaches its minimum, the

Output Signal Amplitude changes rapidly. For a more accurate result, we can (but not strictly required to) add some more rows to record the data.

2.3. After the lab, we should calculate with the experimental data for the "**Transfer function magnitude**" and "**Transfer function magnitude, in dB**" columns.

3. Results & discussion [2]

I) Low-pass filter

According to the procedure described in 2.2., we can get the table below (Table 2). The theoretical value uses $H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}} = \frac{1}{1+j\omega RC}$ and $H(\omega)_{\text{in dB}} = 20\log_{10}H(\omega)$ to calculate.

Frequency	Input signal amplitude, V_{ppk}	Output signal amplitude, (m) V_{ppk}	Transfer function magnitude	Expected transfer function magnitude	Transfer function magnitude, in dB	Expected transfer function magnitude, in dB
1 MHz	4.8	32	0.0067	0.0016	-43.5218	-55.8058
100 kHz	4.8	137	0.0285	0.0162	-30.8904	-35.8070
50 kHz	4.9	253	0.0516	0.0324	-25.7415	-29.7898
10 kHz	4.9	1200	0.2449	0.1600	-12.2203	-15.9184
5 kHz	4.9	2200	0.4490	0.3084	-6.9555	-10.2191
1 kHz	5.1	4900	0.9608	0.8510	-0.3475	-1.4010
500 Hz	5.2	5100	0.9808	0.9556	-0.1687	-0.3948

Table 2. Data for low-pass filters

One of the pictures taken during this process is shown below (Figure 6).

Figure 6. One of the pictures taken for low-pass filters

Using the first measurement as an example, we can calculate the transfer function magnitude and transfer function magnitude, in dB as follows

$$\text{transfer function magnitude} = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{32 \times 10^{-3}}{4.8} = 0.0067$$

$$\text{transfer function magnitude, in dB} = 20\log_{10} 0.0067 = -43.5218$$

Then, we can calculate the rest measurements and finish Table 2.

And using the first measurement, we can calculate the relative error as follows

$$relative\ error_{\text{transfer function magnitude}} = \frac{0.0067 - 0.0016}{0.0016} \times 100\% = 318.75\%$$

$$relative\ error_{\text{transfer function magnitude, in dB}} = \frac{-43.5218 + 55.8058}{55.8058} \times 100\% = 22.01\%$$

Then, we can get the Table 3 below for the rest data.

Frequency	Relative error for transfer function magnitude	Relative error for transfer function magnitude, in dB
1 MHz	318.75%	22.01%
100 kHz	75.93%	13.73%
50 kHz	59.26%	13.59%
10 kHz	53.06%	23.23%
5 kHz	45.59%	31.94%
1 kHz	12.90%	75.20%
500 Hz	2.64%	57.27%

Table 3. Relative error for measurements

From this table, we can know that the relative error is large. We may think that this is because that (1) the filter is not ideal; (2) there is resistance in the wire; (3) the connection is not good; (4) the precision of the instrument is not high.

II) High-pass filter

According to the procedure described in 2.2., we can get the table below (Table 4). The theoretical value uses $H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC}$ and $H(\omega)_{\text{in dB}} = 20\log_{10} H(\omega)$ to calculate.

Frequency	Input signal amplitude, V_{ppk}	Output signal amplitude, (m) V_{ppk}	Transfer function magnitude	Expected transfer function magnitude	Transfer function magnitude, in dB	Expected transfer function magnitude, in dB
1 MHz	4.7	5600	1.1915	1.0000	1.5218	0.0000
100 kHz	4.8	5600	1.1667	0.9999	1.3389	-0.0011
50 kHz	4.9	5600	1.1429	0.9995	1.1598	-0.0045
10 kHz	4.9	5500	1.1224	0.9871	1.0033	-0.1126
5 kHz	4.9	5110	1.0429	0.9513	0.3645	-0.4339
1 kHz	5.1	2490	0.4882	0.5251	-6.2274	-5.5951
500 Hz	5.2	1330	0.2558	0.2948	-11.8430	-10.6095
100 Hz	5.1	279	0.0547	0.0616	-25.2393	-24.2097

Table 4. Data for high-pass filters

One of the pictures taken during this process is shown below (Figure 7).

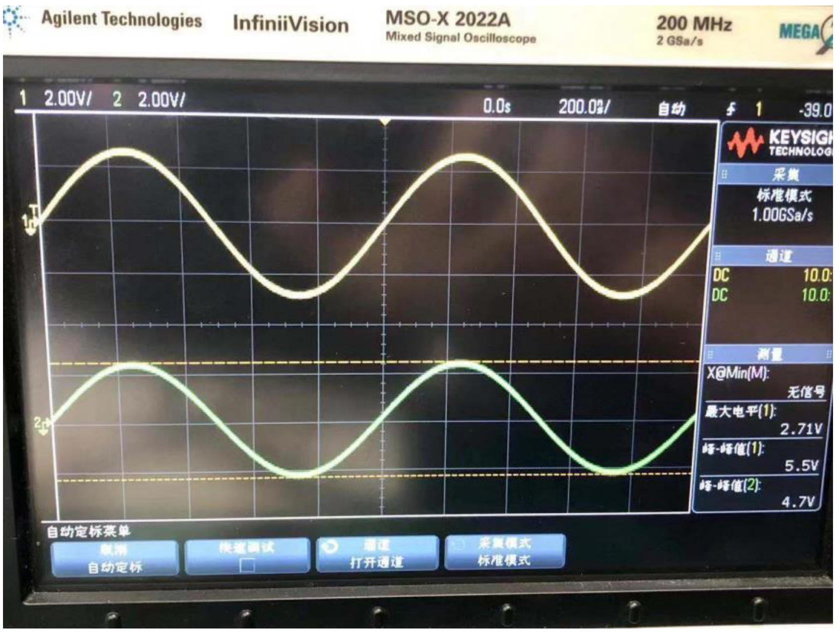

Figure 7. One of the pictures taken for high-pass filters

Using the first measurement as an example, we can calculate the transfer function magnitude and transfer function magnitude, in dB as follows

$$\text{transfer function magnitude} = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{5600 \times 10^{-3}}{4.7} = 1.1915$$

$$\text{transfer function magnitude, in dB} = 20\log_{10}1.1915 = 1.5218$$

Then, we can calculate the rest measurements and finish Table 3.

And using the first measurement, we can calculate the relative error as follows

$$relative\ error_{\text{transfer function magnitude}} = \frac{1.1915 - 1.0000}{1.0000} \times 100\% = 19.15\%$$

$$relative\ error_{\text{transfer function magnitude, in dB}} = \frac{1.5218 - 0}{0} \times 100\% = \infty$$

Then, we can get the Table 5 below for the rest data.

Frequency	Relative error for transfer function magnitude	Relative error for transfer function magnitude, in dB
1 MHz	19.15%	∞
100 kHz	16.68%	121818.18%
50 kHz	14.35%	25873.33%
10 kHz	13.71%	991.03%
5 kHz	9.63%	184.01%
1 kHz	7.03%	11.30%
500 Hz	13.23%	11.63%
100 Hz	11.20%	4.25%

Table 5. Relative error for measurements

From this table, we can know that some of the relative errors are large. We may think that this is because that (1) the filter is not ideal; (2) there is resistance in the wire; (3) the connection is not good; (4) the precision of the instrument is not high. Besides, we find some cases that transfer function magnitude is larger than 1. We think this is mainly due to the non-ideal filter.

III) Band-pass filter

According to the procedure described in 2.2., we can get the table below (Table 6). The theoretical value uses $H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{R}{R+j\left(\omega L - \frac{1}{\omega C}\right)}$ and $H(\omega)_{\text{in dB}} = 20\log_{10}H(\omega)$ to calculate.

Frequency	Input signal amplitude, V_{ppk}	Output signal amplitude, (m) V_{ppk}	Transfer function magnitude	Expected transfer function magnitude	Transfer function magnitude, in dB	Expected transfer function magnitude, in dB
1 MHz	5.0	580	0.1160	0.1545	-18.7108	-16.2240
500 kHz	5.0	1650	0.3300	0.2986	-9.6297	-10.4981
100 kHz	4.9	4900	1.0000	0.8484	0.0000	-1.4275
50 kHz	4.8	5400	1.1250	0.9610	1.0231	-0.3455
10 kHz	4.8	5500	1.1458	0.9952	1.1824	-0.0418
1 kHz	5.1	2410	0.4725	0.5265	-6.5111	-5.5716
500 Hz	5.1	1290	0.2529	0.2951	-11.9396	-10.6009

Table 6. Data for band-pass filters

One of the pictures taken during this process is shown below (Figure 8).

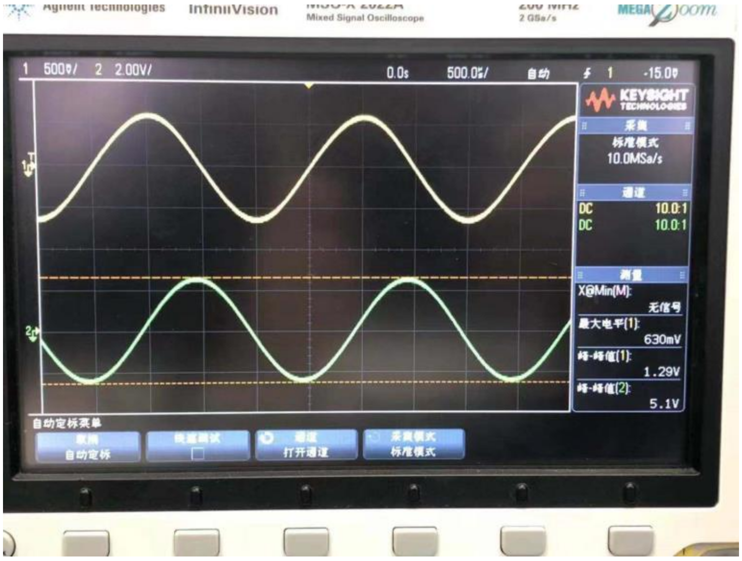

Figure 8. One of the pictures taken for band-pass filters

Using the first measurement as an example, we can calculate the transfer function magnitude and transfer function magnitude, in dB as follows

$$\text{transfer function magnitude} = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{580 \times 10^{-3}}{5.0} = 0.1160$$

$$\text{transfer function magnitude, in dB} = 20\log_{10}0.1160 = -18.7108$$

Then, we can calculate the rest measurements and finish Table 4.

And using the first measurement, we can calculate the relative error as follows

$$relative\ error_{\text{transfer function magnitude}} = \frac{0.1545 - 0.1160}{0.1545} \times 100\% = 24.92\%$$

$$relative\ error_{\text{transfer function magnitude, in dB}} = \frac{-16.2240 + 18.7108}{16.2240} \times 100\% = 15.33\%$$

Then, we can get the Table 7 below for the rest data.

Frequency	Relative error for transfer function magnitude	Relative error for transfer function magnitude, in dB
1 MHz	24.92%	15.33%
500 kHz	10.52%	8.27%
100 kHz	17.87%	100.00%
50 kHz	17.07%	396.12%
10 kHz	15.13%	2928.71%
1 kHz	10.26%	16.86%
500 Hz	14.30%	12.63%

Table 7. Relative error for measurements

From this table, we can know that some of the relative errors are large. We may think that this is because that (1) the filter is not ideal; (2) there is resistance in the wire; (3) the connection is not good; (4) the precision of the instrument is not high. Besides, we find some cases that transfer function magnitude is larger than 1. We think this is mainly due to the non-ideal filter.

IV) Band-reject filter

According to the procedure described in 2.2., we can get the table below (Table 8). The theoretical value uses $H(\omega)=\frac{V_{out}(\omega)}{V_{in}(\omega)}=\frac{j\left(\omega L-\frac{1}{\omega C}\right)}{R+j\left(\omega L-\frac{1}{\omega C}\right)}$ and $H(\omega)_{\text{in dB}}=20\log_{10}H(\omega)$ to calculate.

Frequency	Input signal amplitude, V_{ppk}	Output signal amplitude, (m) V_{ppk}	Transfer function magnitude	Expected transfer function magnitude	Transfer function magnitude, in dB	Expected transfer function magnitude, in dB
1 MHz	5.0	5800	1.1600	0.9880	1.2892	-0.1049
500 kHz	5.0	5600	1.1200	0.9544	0.9844	-0.4056
300 kHz	5.0	5200	1.0400	0.8863	0.3407	-1.0481
200 kHz	4.9	4600	0.9388	0.7860	-0.5488	-2.0911
100 kHz	4.8	2930	0.6104	0.5292	-4.2875	-5.5282
50 kHz	4.8	1490	0.3104	0.2763	-10.1611	-11.1721
10 kHz	4.9	840	0.1714	0.0976	-15.3183	-20.2092
5 kHz	4.9	1970	0.4020	0.2804	-7.9146	-11.0435
1 kHz	5.1	4900	0.9608	0.8501	-0.3475	-1.4105
500 Hz	5.2	5100	0.9808	0.9555	-0.1687	-0.3956

Table 8. Data for band-reject filters

One of the pictures taken during this process is shown below (Figure 9).

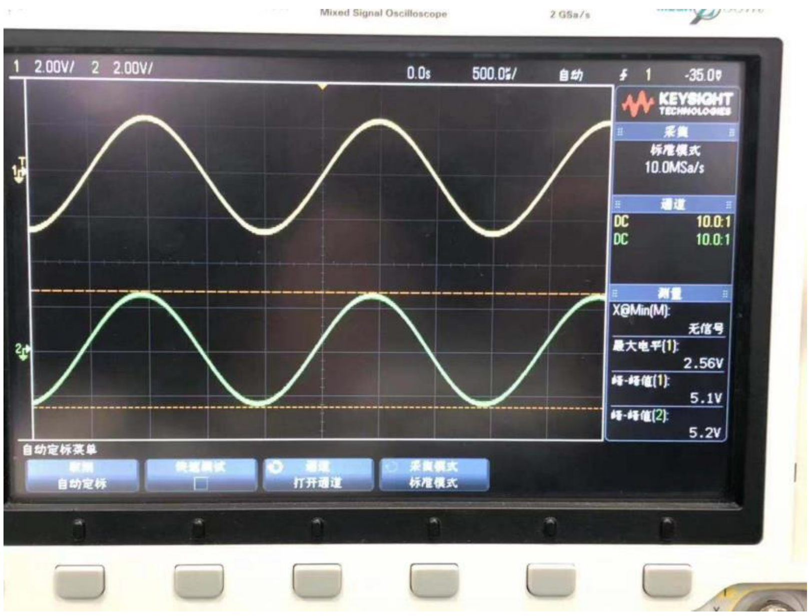

Figure 9. One of the pictures taken for band-reject filters

Using the first measurement as an example, we can calculate the transfer function magnitude and transfer function magnitude, in dB as follows

$$\text{transfer function magnitude} = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{5800 \times 10^{-3}}{5.0} = 1.1600$$

$$\text{transfer function magnitude, in dB} = 20\log_{10} 1.1600 = 1.2892$$

Then, we can calculate the rest measurements and finish Table 5.

And using the first measurement, we can calculate the relative error as follows

$$relative\ error_{\text{transfer function magnitude}} = \frac{1.1600 - 0.9880}{0.9880} \times 100\% = 17.41\%$$

$$relative\ error_{\text{transfer function magnitude, in dB}} = \frac{1.2892 + 0.1049}{0.1049} \times 100\% = 1328.98\%$$

Then, we can get the Table 9 below for the rest data.

Frequency	Relative error for transfer function magnitude	Relative error for transfer function magnitude, in dB
1 MHz	17.41%	1328.98%
500 kHz	17.35%	342.70%
300 kHz	17.34%	132.51%
200 kHz	19.44%	73.76%
100 kHz	15.34%	22.44%
50 kHz	12.34%	9.05%
10 kHz	75.61%	24.20%
5 kHz	43.37%	28.33%
1 kHz	13.02%	75.36%
500 Hz	2.65%	57.36%

Table 9. Relative error for measurements

From this table, we can know that some of the relative errors are large. We may think that this is because that (1) the filter is not ideal; (2) there is resistance in the wire; (3) the connection is not good; (4) the precision of the instrument is not high. Besides, we find some cases that transfer function magnitude is larger than 1. We think this is mainly due to the non-ideal filter.

4. Conclusions [1]

In this experiment, we learn about four types of filters – Low-Pass, High-Pass, Band-Pass, and Band-reject. We also learn about transfer functions and predict the theoretical result and make comparison with lab data. We measured the transfer function magnitudes for the four types. All of the objectives have been achieved. Therefore, the experiment is quite successful.

In our calculation, we find that some of the relative errors are large. We may think that this is because that (1) the filter is not ideal; (2) there is resistance in the wire; (3) the connection is not good; (4) the precision of the instrument is not high. Besides, we find some cases that transfer function magnitude is larger than 1. We think this is mainly due to the non-ideal filter.

5. References

[1] Lab5_Filter Lab_Manual
[2] Lab5_Filter Lab_DataSheet