UM-SJTU JOINT INSTITUTE
INTRO TO CIRCUITS
(VE 215)

LABORATORY REPORT

LAB 3

TRANSIENT LAB

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1. Introduction [1]

1.1. Objectives

- Apply the theory we learned on the step responses in first- and second-order circuits to series *RC* and *RLC* circuits, which we will build in the lab.
- Build a series *RC* circuit, observe its responses to input square wave signal of varied frequency, and explain them based on the theory we learned
- Relate the observed capacitor voltage and resistor voltage as functions of time to the pre-lab calculations
- Explain the changes of both output waveforms in response to the increase of the frequency of the input square wave signal
- Explain the amplitudes of the capacitor voltage and the resistor voltage related to the amplitude of the input square wave
- Build a series RLC circuit, observe the three types of its responses to input square wave signal, and relate them to the theory you have learned. For the under-damped/over-damped/critically damped response, compare the resistance in the circuit measured in the lab with the critical resistance you calculated in the pre-lab
- Build the simplest second-order circuit, an *LC* tank, and observe oscillations

1.2. Apparatus & Theoretical background

1.2.1. First-order circuits

Theoretically, the transient responses in electric circuits are described by differential equations. The circuits, whose responses obey the first-order differential equation

$$\frac{dx(t)}{dt} + \frac{1}{\tau} \cdot x(t) = f(t)$$

are called **first-order circuits**. Their responses are always monotonic and appear in the form of exponential function

$$x(t) = K_1 \cdot e^{-(\frac{t}{\tau})} + K_2$$

A first-order circuit includes the effective resistance *R* and one energy-storage element, an inductor *L* or a capacitor *C*.

In an *RC* circuit, the time constant is

$$\tau = RC.$$

In an *LC* circuit, the time constant is

$$\tau = \frac{L}{R}.$$

The **fall time** of a signal is defined as the interval between the moment when the signal reaches its 90% and the moment when the signal reaches its 10% level. We should note that the 10% level is reached between 2τ and 3τ. Approximately, we can assume *fall time* $\approx 2.2\tau$. After $t = 5\tau$, the exponent practically equals zero.

1.2.2. Second-order circuits

Many circuits involve two energy-storing elements, both an inductor *L* and a capacitor *C*. Such circuits require a second-order differential equation description

$$\frac{d^2x(t)}{dt^2} + 2 \cdot \alpha \cdot \frac{dx(t)}{dt} + {\omega_0}^2 \cdot x(t) = f(t)$$

thus they are called **second-order circuits**.

We will consider only second-order circuits with one inductor and one capacitor. The differential equation includes two parameters: the damping factor α and the undamped frequency ω_0 which are determined by the circuit and its components.

For example, in the series *RLC* circuit, which you will build and study in this lab,

$$\alpha = \frac{R}{2 \cdot L}, \text{and } \omega_0 = \frac{1}{\sqrt{L \cdot C}},$$

while in the parallel *RLC* circuit,

$$\alpha = \frac{1}{2 \cdot R \cdot C}, \text{and } \omega_0 = \frac{1}{\sqrt{L \cdot C}}.$$

Depending on the two parameters α and ω_0, second-order circuits can exhibit three types of responses.

1.2.2.1. The underdamped response

If $\alpha < \omega_0$,

$$x(t) = e^{-\alpha t}(K_1 \cos(\omega t) + K_2 \sin(\omega t))$$

where $\omega = \sqrt{\omega_0^2 - \alpha^2}$.

The underdamped circuit response involves decaying oscillations, which may last for many periods or for less than one period, depending on the damping ratio $\xi = \frac{\alpha}{\omega_0}$, which for the series *RLC* circuit $\xi = \frac{R}{2L}\sqrt{LC} = \frac{R}{2} \cdot \sqrt{\frac{C}{L}}$. Varying the values of *R*, *L*, *C*, affects the damping ratio ξ.

1.2.2.2. The critically damped response

If $\alpha = \omega_0$,

$$x(t) = e^{-\alpha t}(K_1 + K_2 t)$$

and the circuit has the critically damped response.

The critically damped response does not involve oscillations.

For the series RLC circuits, $\alpha = \omega_0$ corresponds to $\frac{R}{2L} = \frac{1}{\sqrt{LC}}$ or $R = R_{critical} = 2\sqrt{\frac{L}{C}}$.

If $L = 1mH$ and $C = 10nF$, then $R_{critical} \approx 632\Omega$.

1.2.2.3. The critically damped response

If $\alpha > \omega_0$,

$$x(t) = K_1 \cdot e^{s_1 t} + K_2 \cdot e^{s_2 t}$$

where $s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$ and $s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$.

In the series *RLC* circuits, the overdamped solution is obtained if the resistance is larger than the critical resistance, such that $R > R_{critical} = 2 \cdot \sqrt{\frac{L}{C}}$.

We should notice that the larger resistance corresponds to the longer delay, and even the faster decay has a much longer fall time than the critically damped response.

One of the most interesting features of series *RLC* circuits is that increasing the resistance above the critical value results in much longer fall time, or longer delays of responses in digital circuits. Among all monotonic responses, the critically damped is the fastest.

1.2.3. Apparatus

The experimental setup of this experiment consists of DC source, protoboards, function generator, oscilloscope, resistors, slide rheostat and so on.

2. Measurements [2]

2.1. First-order circuit

$R_1 = 1\text{k}\Omega$, $C = 0.1\mu F$ are used in this part. First, we turn on the function generator. And we set a square wave at 1 V_{ppk} and 100 Hz. Then, we apply it to the circuit as the input signal. Second, we monitor the input signal in Channel 1 and the output in Channel 2 of the oscilloscope. And finally complete the table.

For the **fastest** circuit response, we should set $\boldsymbol{R_P = 0}$, and set the oscilloscope:

a) Vertical scale for the input signal 200mV/div;
b) Vertical scale for the output signal 200mV/div;
c) Horizontal scale 5ms/div;

For the **slowest** circuit response, we should set $\boldsymbol{R_P = 10k\Omega}$, and set the oscilloscope:

a) Vertical scale for the input signal 200mV/div;
b) Vertical scale for the output signal 50mV/div;
c) Horizontal scale 5ms/div;

2.2. Second-order circuit

$L = 1mH$, $R_2 = 100\Omega$, $R_P = 10k\Omega$ and $C = 820pF$ are used in this part

a) On the function generator, we should set a square wave at **1** $\boldsymbol{V_{ppk}}$ and **10 kHz** as the input signal.
b) We should vary $\boldsymbol{R_P}$ to generate three kinds of plot on the oscilloscope. (Under-damped, critically damped, over-damped response).
c) We should observe and save the graph from the oscilloscope.
d) We should record: **Fall time** and **Rise time**, the time interval between the neighboring peaks, Δt, and the resistance of the potentiometer, $\boldsymbol{R_P}$.

3. Results & discussion

3.1. First-order circuit

According to the procedure described in 2.1., we can get Table 1 for the first-order as shown below.

The setting of the potentiometer corresponding to the …	**Fastest** circuit response	**Slowest** circuit response
Peak-to-peak voltage of the **input** square wave, $\boldsymbol{V_{ppk}}$ [V]	0.960	0.952
Peak-to-peak voltage of the **output** square wave, $\boldsymbol{V_{ppk}}$ [V]	0.864	0.832
Period of the **input** square wave, T [ms]	10.0	10.0
Rise time of the **output** waveform, [ms]	0.320	1.89
Fall time of the **output** waveform, [ms]	0.280	1.88

Table 1. First-order circuit

Based on Table 1, we can calculate the time constant using the fastest fall time of the output waveform:

$$\tau_1 \approx \frac{falltime}{2.2} = \frac{0.280 \times 10^{-3}}{2.2} = 1.27 \times 10^{-4}\ [s],$$

which corresponds to the $R_P = 0$. Theoretically, the value should be

$$\tau_{theoretical1} = (R_1 + R_P) \times C = 1000 \times 0.1 \times 10^{-6} = 1 \times 10^{-4}\ [s].$$

Then, the relative error can be calculated as

$$\frac{1.27 \times 10^{-4} - 1 \times 10^{-4}}{1 \times 10^{-4}} \times 100\% = 27.0\%$$

We find that the relative error is huge. Considering the way how we operate, we may contribute it to the precision of the instrument. Moreover, when time is small, it is hard to measure it accurately. Besides, the wave on the oscillator has width, which will increase the difficulty of measuring.

We can calculate the time constant using the slowest fall time of the output waveform:

$$\tau_2 \approx \frac{falltime}{2.2} = \frac{1.88 \times 10^{-3}}{2.2} = 8.55 \times 10^{-4}\ [s],$$

which corresponds to the $R_P = 10k\Omega$. Theoretically, the value should be

$$\tau_{theoretical2} = (R_1 + R_P) \times C = (1000 + 10000) \times 0.1 \times 10^{-6} = 1.1 \times 10^{-3}\ [s].$$

Then, the relative error can be calculated as

$$\frac{1.1 \times 10^{-3} - 8.55 \times 10^{-4}}{1.1 \times 10^{-3}} \times 100\% = 22.3\%$$

We find that the relative error is huge. Similarly, considering the way how we operate, we may contribute it to the precision of the instrument. Moreover, when time is small, it is hard to measure it accurately. Besides, the wave on the oscillator has width, which will increase the difficulty of measuring.

3.2. Second-order circuit

According to the procedure described in 2.2., we can get Table 2 for the second-order as shown below.

	Resistance, R_P [Ω]	Rise time, [ms]	Fall time, [ms]	Time interval, Δt [ms]
Underdamped	34.7	1780	1710	0.1
	201.6	2400	2380	0.1
Critically damped	984	3600	3570	0.1
Overdamped	2233	6840	6340	0.1
	5190	13200	13100	0.1

Table 2. Second-order circuit

Based on Table 2, we can get the resistance for critically damped is 984 Ω.

According to the theory we learnt, when it is critically damped, $\alpha = \omega_0$, then we may get that

$$\frac{R}{2 \cdot L} = \frac{1}{\sqrt{L \cdot C}},$$

$$R = \frac{2 \cdot L}{\sqrt{L \cdot C}},$$

$$R_P = \frac{2 \cdot L}{\sqrt{L \cdot C}} - R_2 = 2108.6\ \Omega.$$

The relative error is

$$\frac{2108.6 - 984}{2108.6} \times 100\% = 53.3\%,$$

which is very huge. First, we may think this is because of the wrong connection of the circuit. But the circuit is approved by the TA. And when we see the value of the overdamped cases, we may think that we may find the critically damped point wrong because the resistance for the overdamped cases is larger than $2108.6\ \Omega$, which is correct and the difference of resistance for critically damped and overdamped is also huge. Therefore, we may think that we find the critically damped point wrong.

However, we forgot to save the graph from the oscilloscope, which is a huge pity for this experiment. Theoretically, we should have the graphs like below for the underdamped, critically damped and overdamped cases.

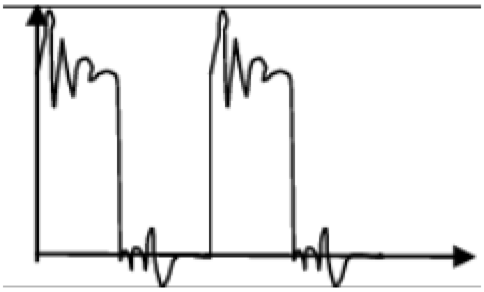

Figure 1. Underdamped [3]

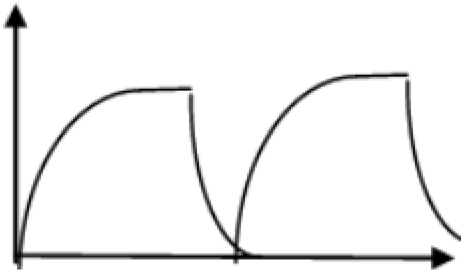

Figure 2. Critically damped [3]

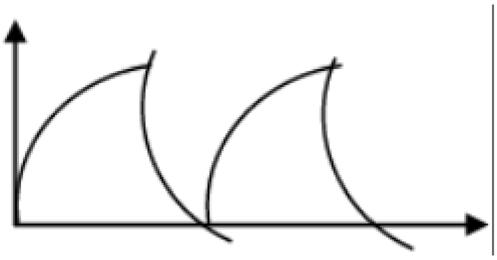

Figure 3. Overdamped [3]

4. Conclusions [1]

In this experiment, we apply the theory we learned on the step responses in first- and second-order circuits to series *RC* and *RLC* circuits, which we will build in the lab; we build a series *RC* circuit, observe its responses to input square wave signal of varied frequency, and explain them based on the theory we learned; we relate the observed capacitor voltage and resistor voltage as functions of time to the pre-lab calculations; we explain the changes of both output waveforms in response to the increase of the frequency of the input square wave signal; we explain the amplitudes of the capacitor voltage and the resistor voltage related to the amplitude of the input square wave; we build a series RLC circuit, observe the three types of its responses to input square wave signal, and relate them to the theory you have learned. For the under-damped/over-damped/critically damped response, compare the resistance in the circuit measured in the lab with the critical resistance you calculated in the pre-lab; we build the simplest second-order circuit, an *LC* tank, and observe oscillations.

First, we built the first-order circuit *RC* and recorded peak-to-peak voltage of the Input square wave, peak-to-peak voltage of the Output square wave, period of the Input square wave, rise time of the output waveform and fall time of the output waveform. The relative error we get in the experiment are around 20-25%, and we contribute it to the precision of the instrument. Moreover, when time is small, it is hard to measure it accurately. Besides, the wave on the oscillator has width, which will increase the difficulty of measuring.

Then, we built the second-order *RLC* circuit and recorded the detailed valued of resistors, rise time, fall time and time interval for underdamped, critically damped and overdamped cases. The critically damped point we get is wrong in the lab. Besides, we forgot to save the graph form the oscilloscope, which is a really pity.

5. References

[1] Lab 3_Transient Lab_Manual.pdf

[2] Lab 3_Transient Lab_DataSheet.pdf

[3] Physics Lab Repot of Nanchang University
https://wenku.baidu.com/view/fc3c8a7ecdbff121dd36a32d7375a417866fc1a4.html