VE215 Intro to Circuits

Mid 2 Review Class

Content for Mid 2

1. Chapter 6: Capacitors and Inductors
2. Chapter 7: First-Order Circuits
3. Chapter 8: Second-Order Circuits
4. Chapter 9: Sinusoids and Phasors
5. Chapter 10: Sinusoidal Steady-State Analysis

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Capacitor

- A capacitor is a passive element designed to store energy in its electric field.

- The amount of charge stored, represented by q, is given by
$$q = Cv$$

- If $vi > 0$, the capacitor is being charged.

- If $vi < 0$, the capacitor is discharging.

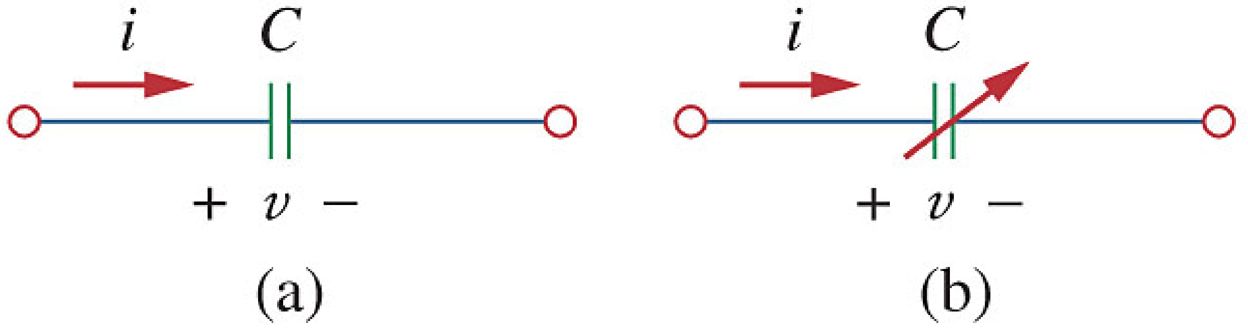


Figure 6.3 Circuit symbols for capacitors: (a) fixed capacitor, (b) variable capacitor.

Capacitor

- The current-voltage relationship

$$i = C\frac{dv}{dt}$$

- The voltage-current relation

$$v = \frac{1}{C}\int_{-\infty}^{t} idt = \frac{1}{C}\int_{t_0}^{t} idt + v(t_0)$$

- The instantaneous power delivered to the capacitor and the energy stored

$$p = vi = v\left(C\frac{dv}{dt}\right)$$

$$w = \frac{1}{2}Cv^2$$

Series and Parallel Capacitor

- The equivalent capacitance of N parallel connected capacitors is

$$C_{eq} = C_1 + C_2 + \cdots + C_N$$

- The equivalent capacitance of N series connected capacitors is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}$$

Inductor

- An inductor is a passive element designed to store energy in its magnetic field.

- The magnetic flux linkage in the inductor, represented by Ψ, is given by
$$\Psi = Li$$

- If $vi > 0$, the inductor is being charged.

- If $vi < 0$, the inductor is discharging.

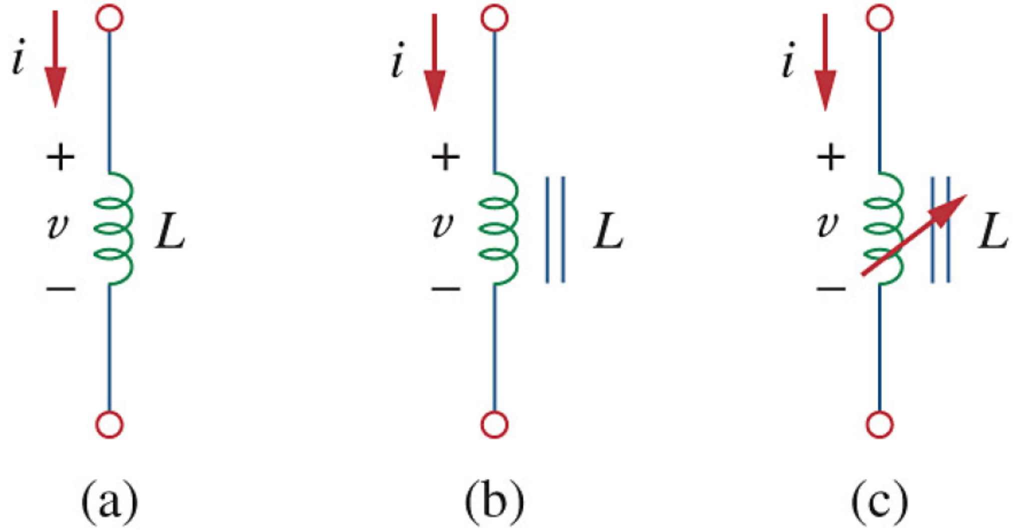


Figure 6.23 Circuit symbols for inductors: air-core, (b) iron-core, (c) variable iron-core.

Inductor

- The voltage-current relationship

$$v = L\frac{di}{dt}$$

- The current-voltage relation

$$i = \frac{1}{L}\int_{-\infty}^{t} v dt = \frac{1}{L}\int_{t_0}^{t} v dt + i(t_0)$$

- The instantaneous power delivered to the inductor and the energy stored

$$p = vi = \left(L\frac{di}{dt} \right) i$$

$$w = \frac{1}{2}Li^2$$

Series and Parallel Inductor

- The equivalent inductance of N parallel connected inductors is

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}$$

- The equivalent inductance of N series connected inductors is

$$L_{eq} = L_1 + L_2 + \cdots + L_N$$

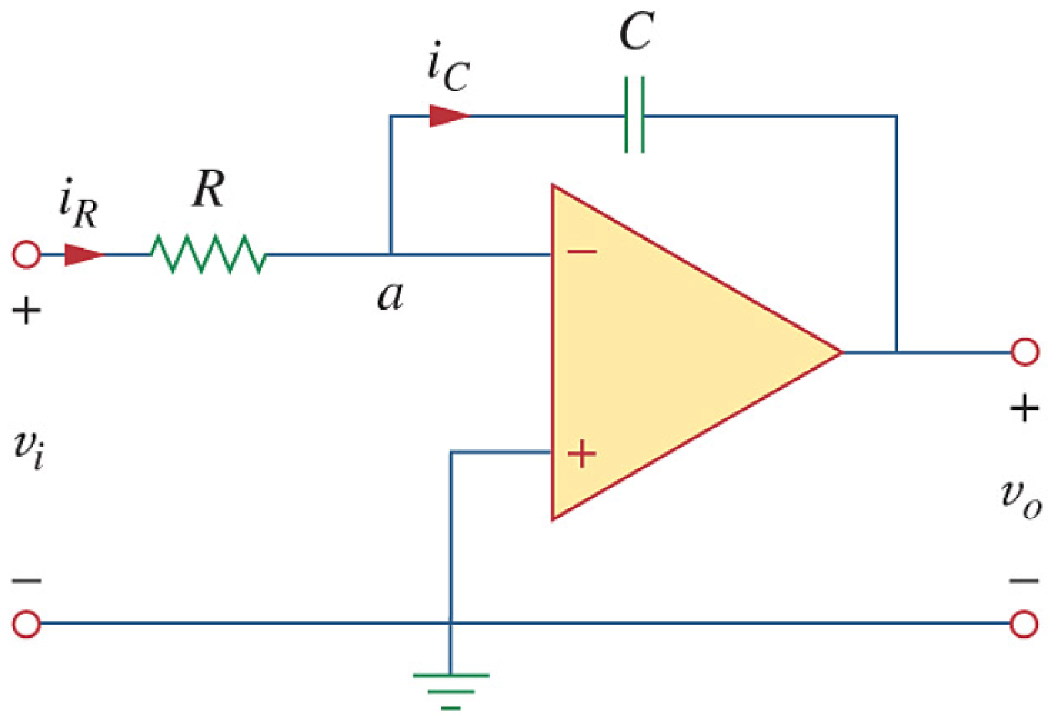


Figure 6.35(b) An integrator circuit.

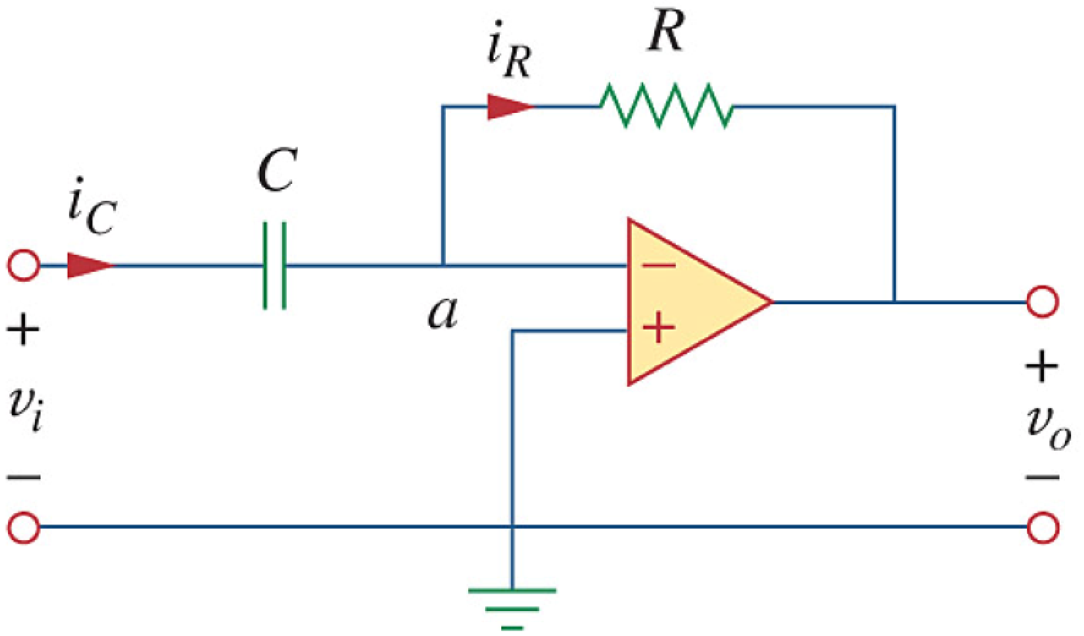


Figure 6.37 A differentiator circuit.

$$v_o = -\frac{1}{RC}\int_0^t v_i dt + v_o(0)$$

If $v_o(0) = 0$, then

$$v_o = -\frac{1}{RC}\int_0^t v_i dt$$

$$v_o = -RC\frac{dv_i}{dt}$$

Application

- Go through again the example in your lecture slide:

Example 6.15 Design an analog computer circuit to solve the differential equation

$$\frac{d^2v_o}{dt^2} + 2\frac{dv_o}{dt} + v_o = 10\sin 4t, \quad t > 0$$

subject to $v_o(0) = -4,\ v_o'(0) = 1,$ where the prime refers to the time derivative.

Method 1: **Main role is $\frac{dv_0}{dt}$!**

First part: summing amplifier for $\frac{dv_0}{dt}$:

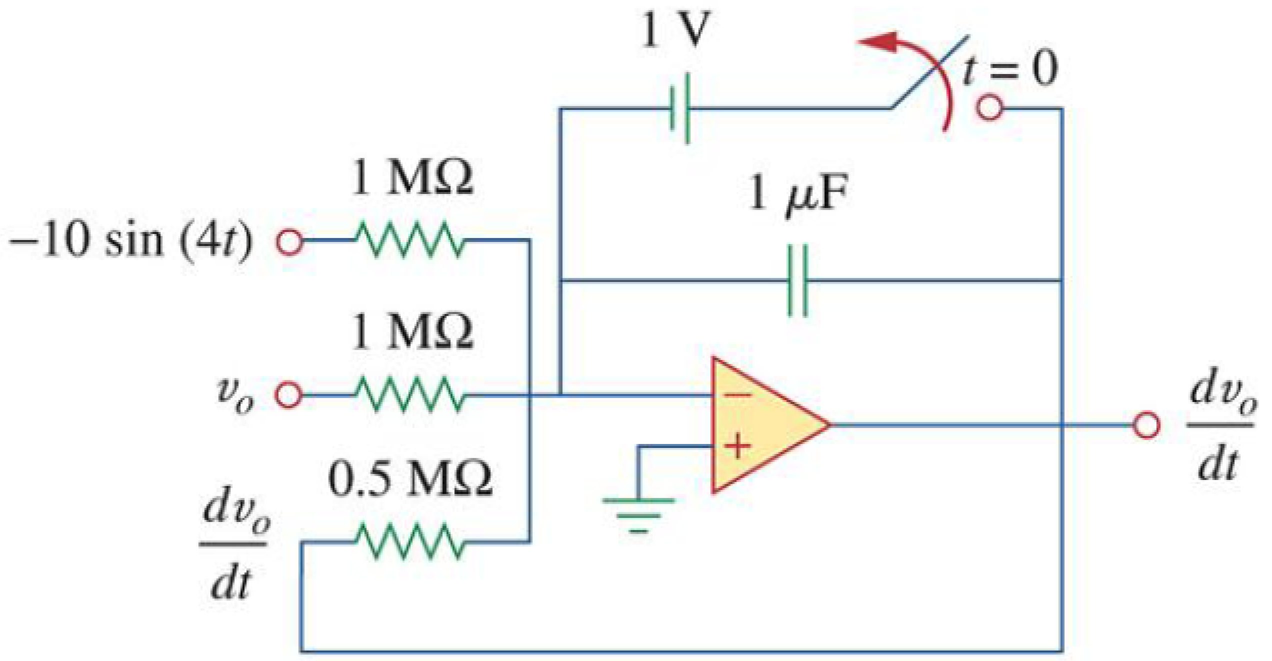


Figure 6.40(a) The "summing integrator" circuit.

Second part: integrator and inverting for v_0:

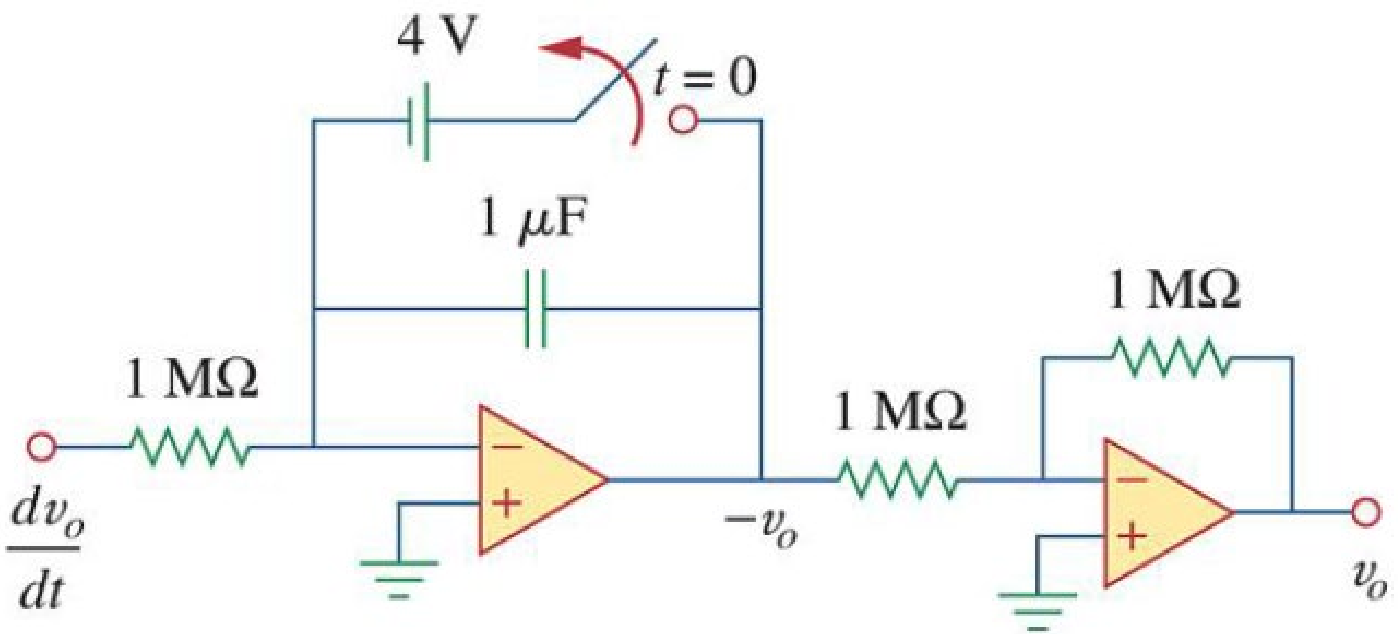


Figure 6.40(b) The cascading of an integrator and an inverting amplifier.

Last step: connect back v_0 to the first part:

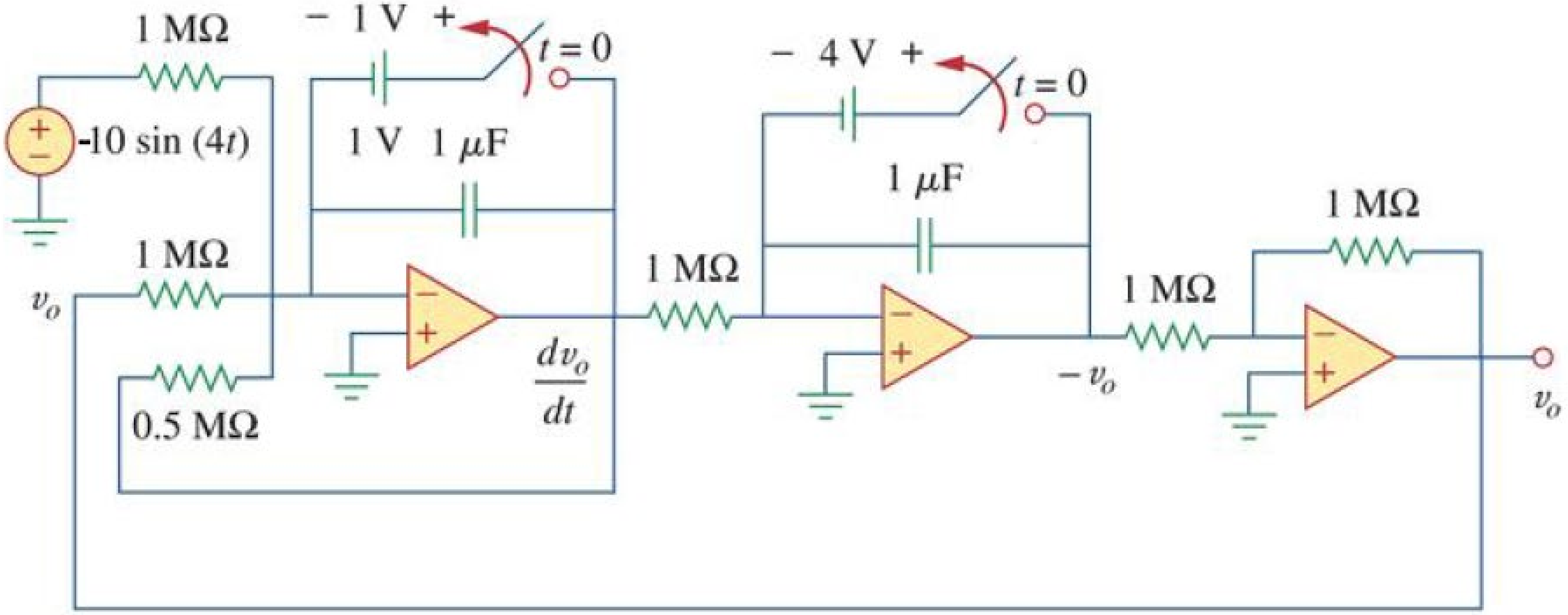
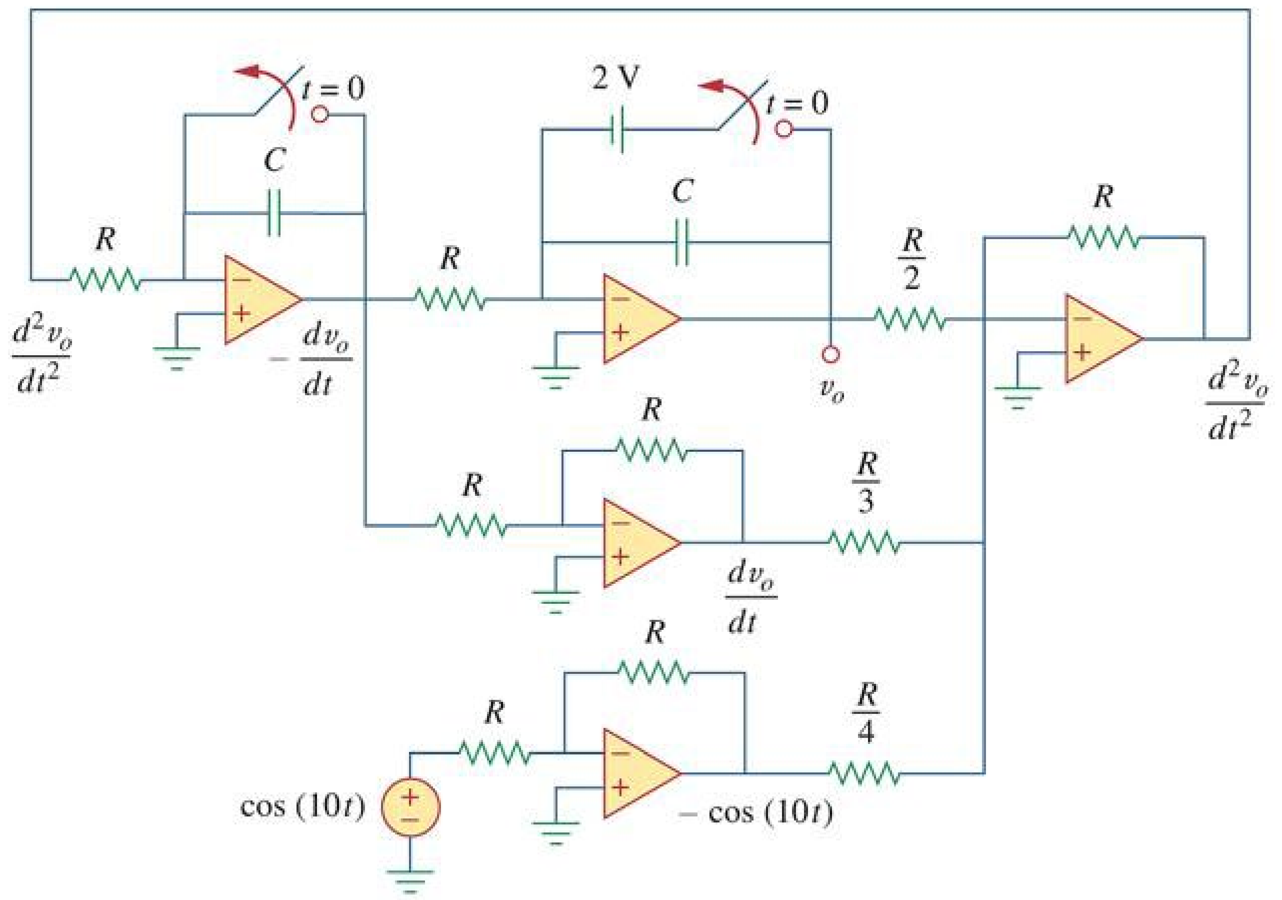


Figure 6.40(c)

Method 2: **Main role is $\frac{d^2 v_0}{dt^2}$!**
Similar to Method 1. You may try after class and pick whatever you like!



Suppose we desire the solution $x(t)$ of the equation

$$a\frac{d^2x}{dt^2} + b\frac{dx}{dt} + cx = f(t), \qquad t > 0 \tag{6.38}$$

where a, b, and c are constants, and $f(t)$ is an arbitrary forcing function. The solution is obtained by first solving the highest-order derivative term. Solving for d^2x/dt^2 yields

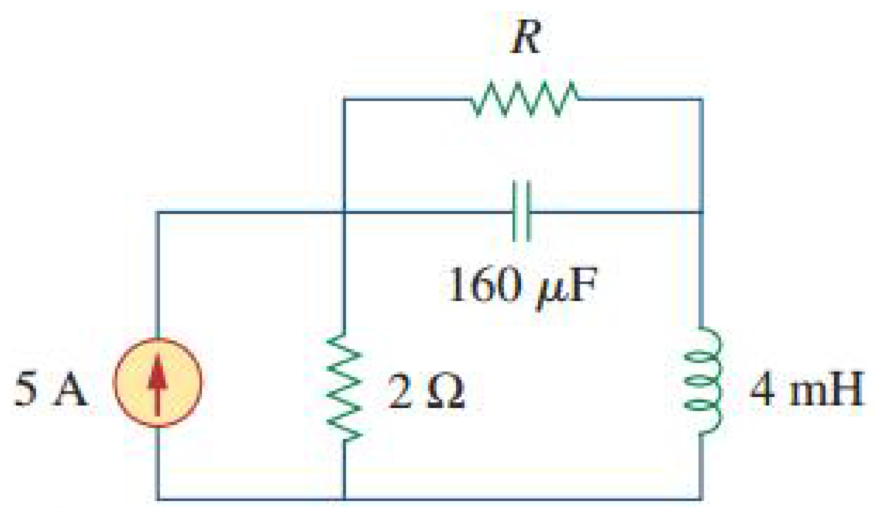
$$\frac{d^2x}{dt^2} = \frac{f(t)}{a} - \frac{b}{a}\frac{dx}{dt} - \frac{c}{a}x \tag{6.39}$$

To obtain dx/dt, the d^2x/dt^2 term is integrated and inverted. Finally, to obtain x, the dx/dt term is integrated and inverted. The forcing function is injected at the proper point. Thus, the analog computer for solving Eq. (6.38) is implemented by connecting the necessary summers, inverters, and integrators. A plotter or oscilloscope may be used to view the output x, or dx/dt, or d^2x/dt^2, depending on where it is connected in the system.

Sample Question

Ex.1

For the circuit in Fig. 6.70, calculate the value of R that will make the energy stored in the capacitor the same as that stored in the inductor under dc conditions.



Ex.2

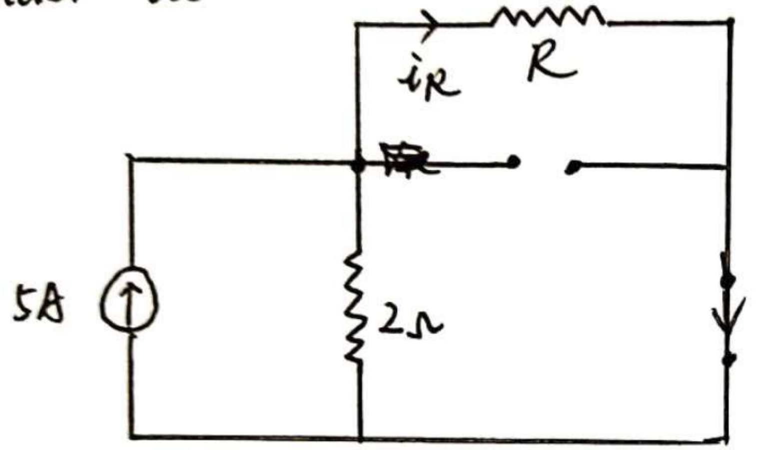
Design an analog computer to simulate

$$\frac{d^2v_o}{dt^2} + 2\frac{dv_o}{dt} + v_o = 10 \sin 2t$$

where $v_0(0) = 2$ and $v_0'(0) = 0$.

Ex. 1.

Under dc:



$$i_R = 5 \cdot \frac{2}{R+2} = \frac{10}{R+2} = i_L$$

$$V_C = V_R = i_R \cdot R = \frac{10R}{R+2}$$

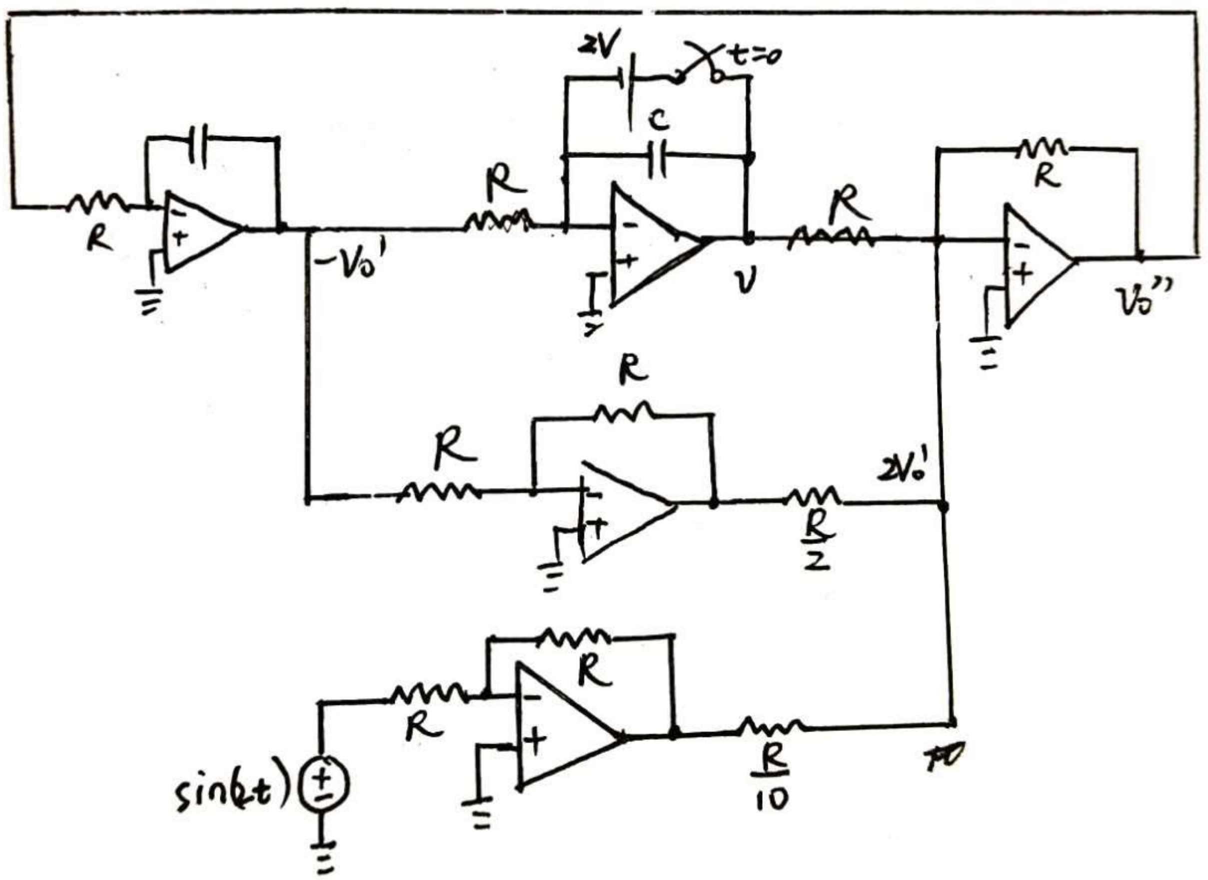
$$W_C = \frac{1}{2} C V_C^2 = 80\times10^{-6} \times \frac{100R^2}{(R+2)^2}$$

$$W_L = \frac{1}{2} L i_L^2 = 2\times10^{-3} \times \frac{100}{(R+2)^2}$$

$$W_C = W_L \Rightarrow R = 5\Omega$$

Ex. 2:

$$\frac{d^2 v_0}{dt^2} = 10\sin 2t - 2v_0' - v_0, \quad v_0(0)=2, \ v_0'(0)=0$$



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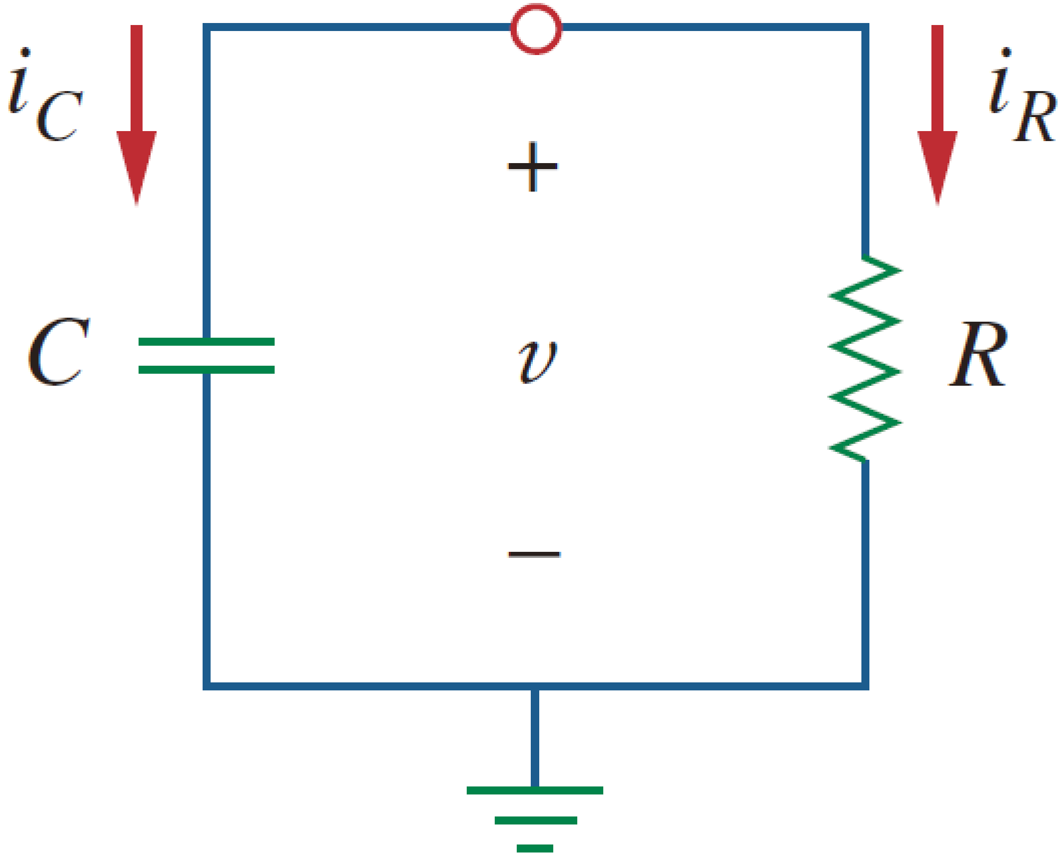
The Source-Free RC Circuit



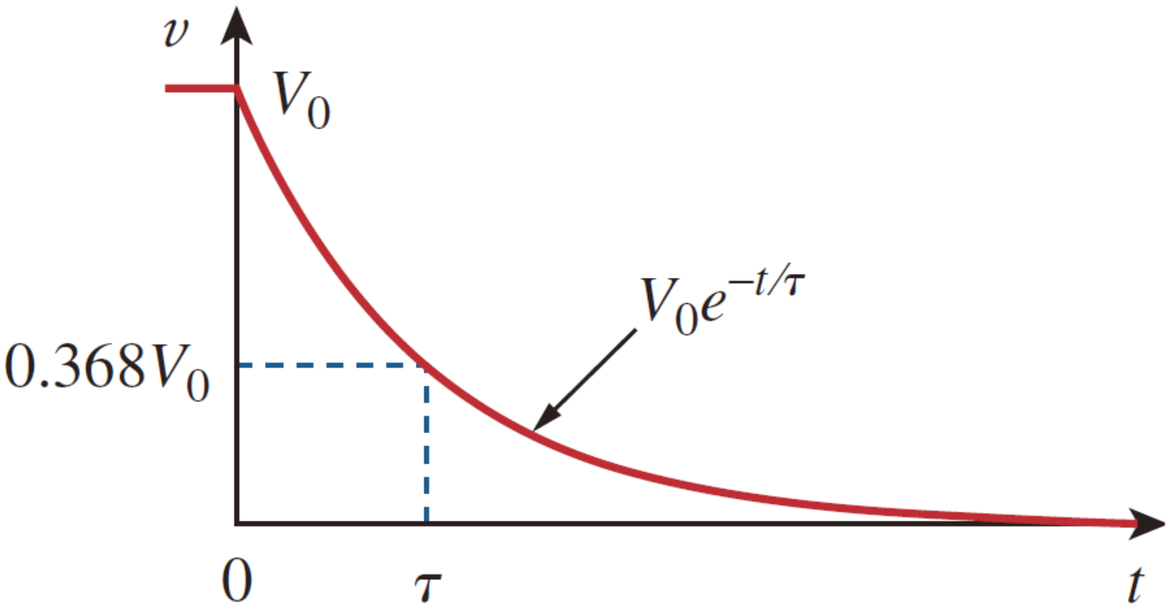
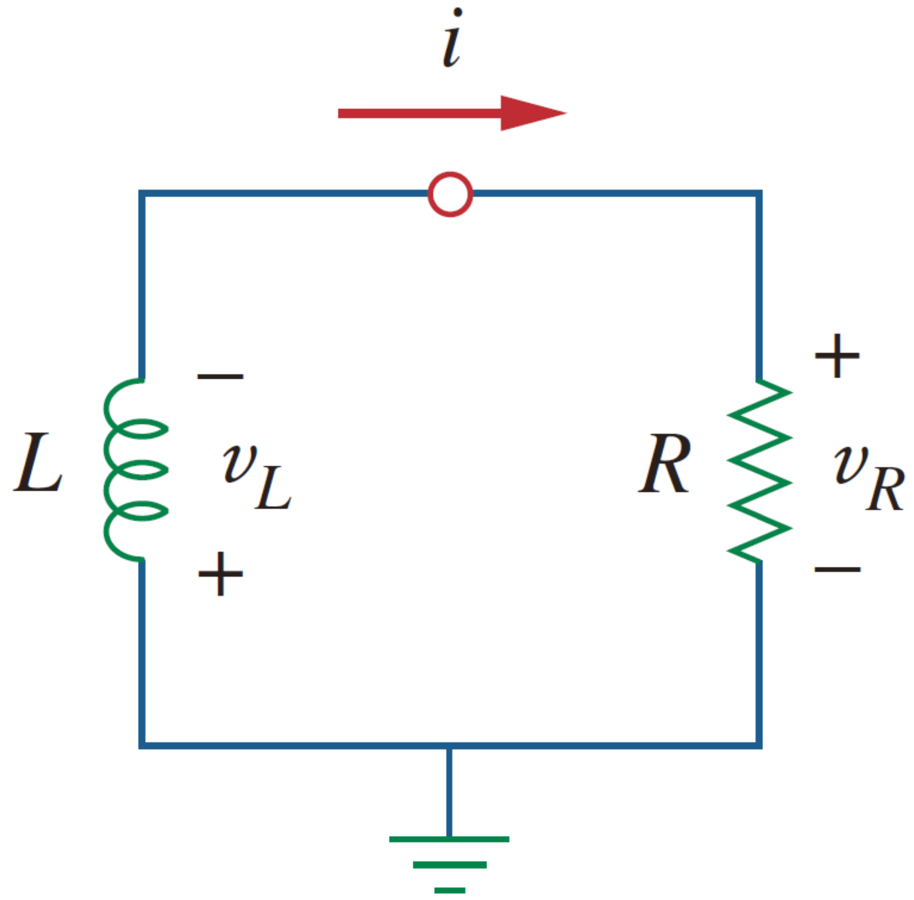


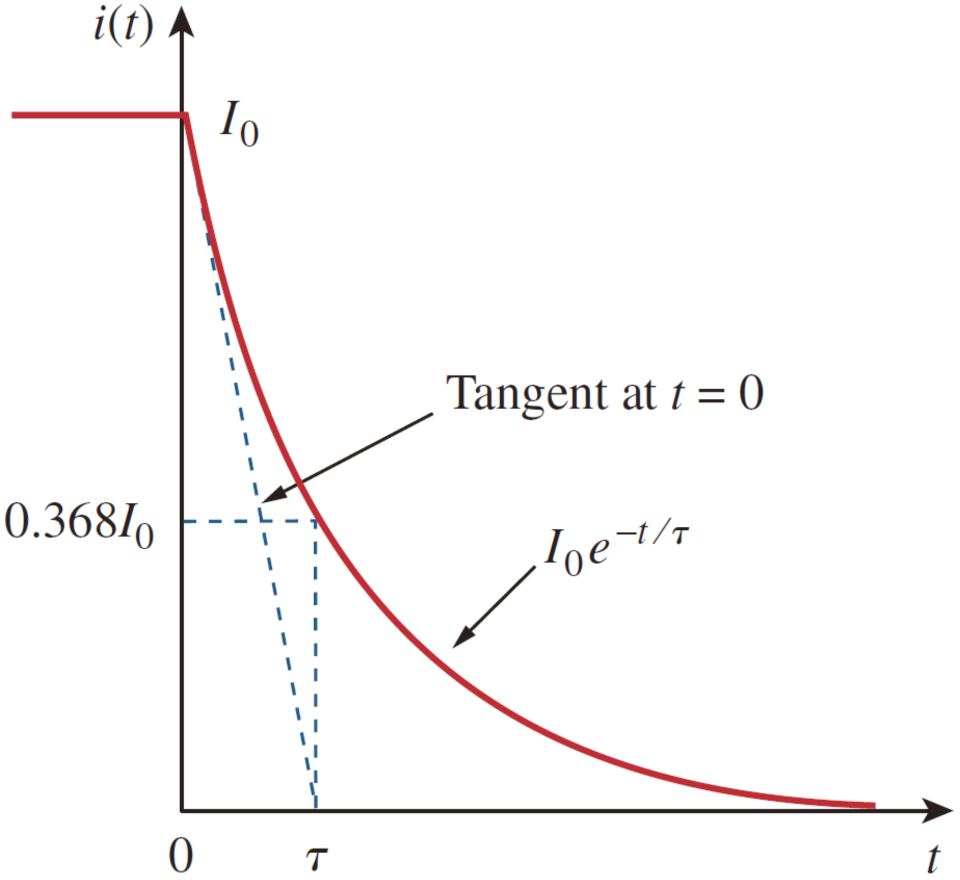
Figure 7.2
The voltage response of the *RC* circuit.

$$\tau = RC$$

$$v(t) = V_0 e^{-t/\tau}$$

The Source-Free RL Circuit





$$\tau = \frac{L}{R}$$

$$i(t) = I_0 e^{-t/\tau}$$

Singularity Functions

- Unit step $u(t)$:
- $u(t-t_0)$ (Delay) and $u(t\ +\ t_0)$ (Advance):

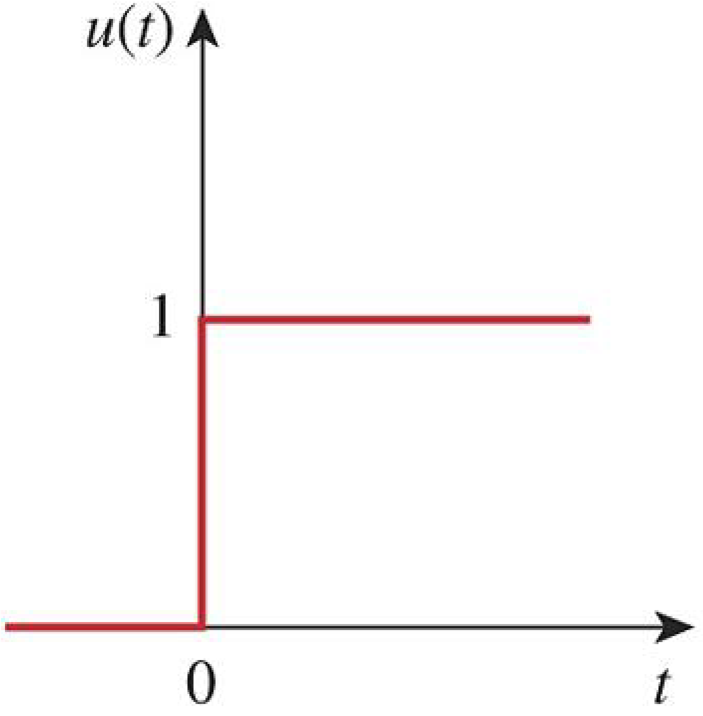


Figure 7.23 The unit step function.

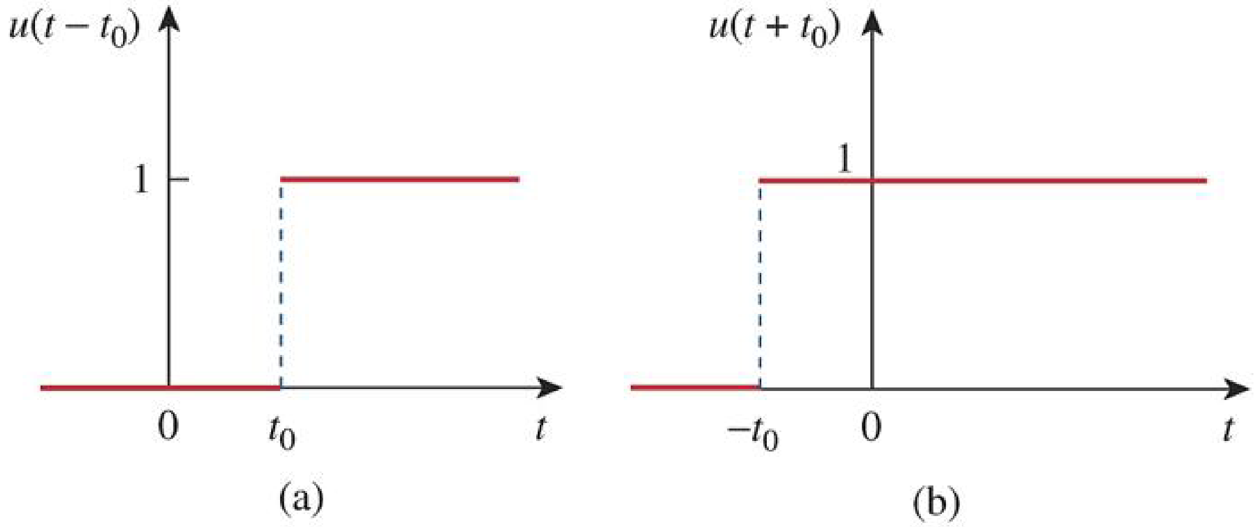


Figure 7.24 (a) The delayed version of the unit step, (b) the advanced version of the unit step.

Singularity Functions

- Unit step $\delta(t)$:

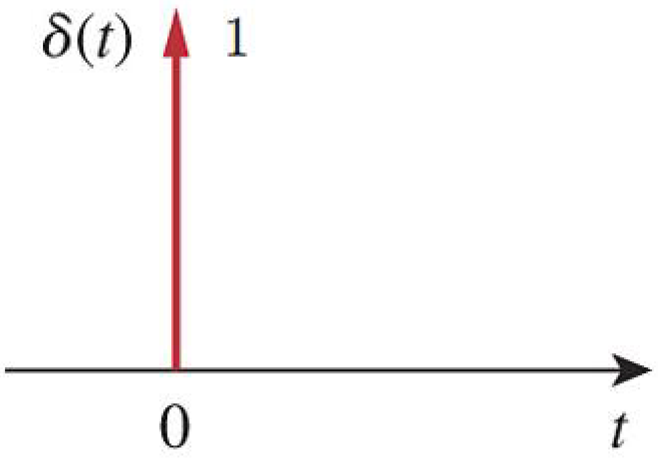


Figure 7.27 The unit impulse function.

$$\int_{-\infty}^{\infty} \delta(t)dt = 1.$$

$$\int_{-\infty}^{\infty} f(t)\delta(t - t_0)dt = f(t_0).$$

- Unit ramp $r(t)$:

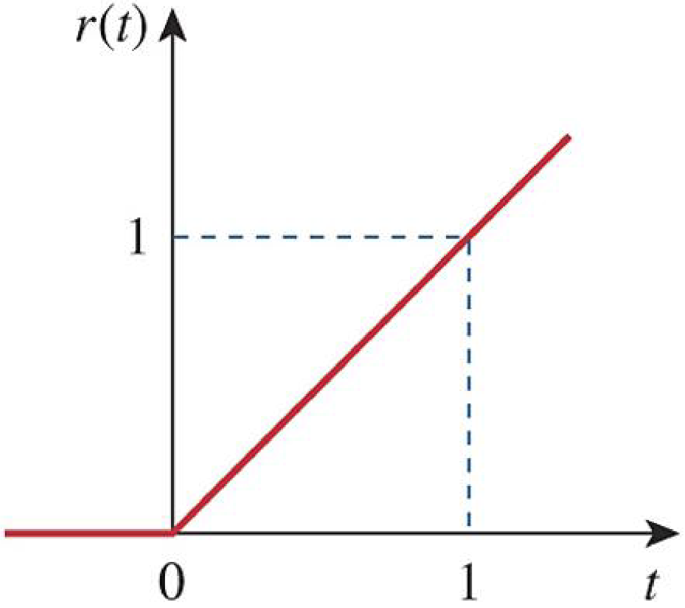


Figure 7.29 The unit ramp function.

$$\delta(t) = \frac{du(t)}{dt},$$

$$r(t) = \int_{-\infty}^{t} u(\tau)d\tau = tu(t).$$

Step Response of an RC Circuit

- Excitation: step function, a V_S or an I_S .
- Assume the initial voltage of capacitor is V_0.
- Try to derive v expression.

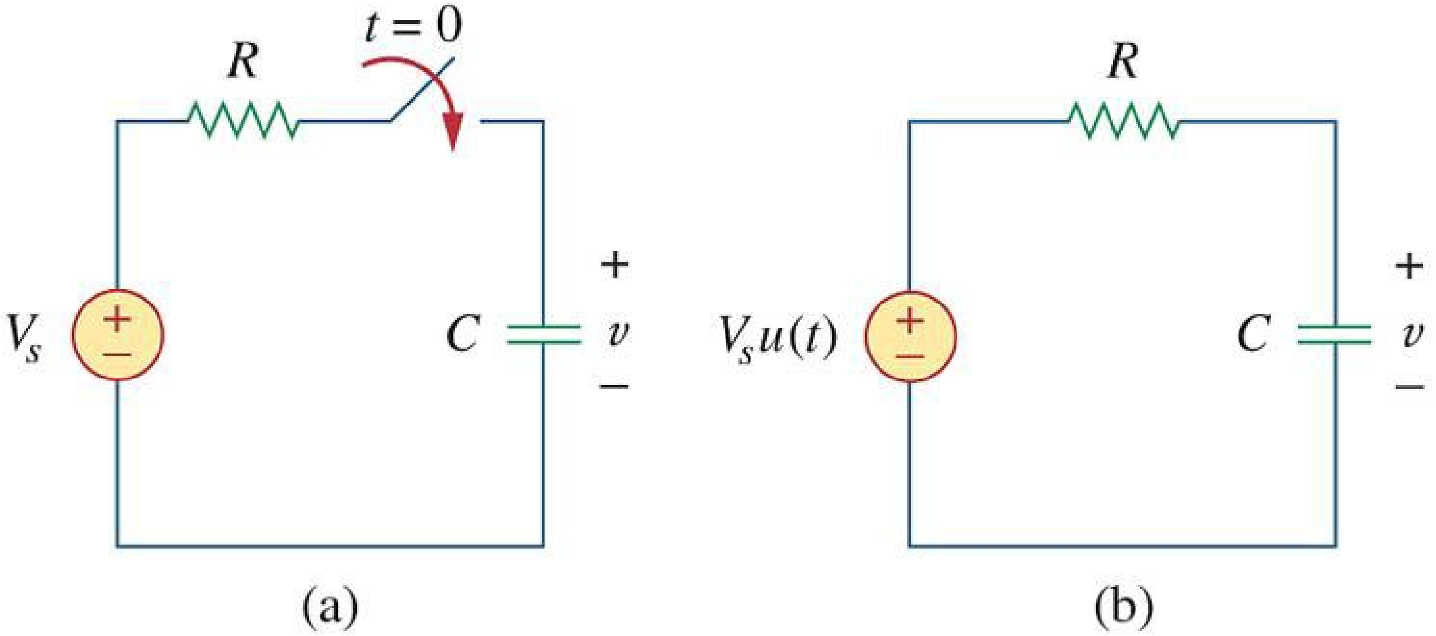


Figure 7.40 An RC circuit with voltage step input.

- Initial condition:

$$v(0^+) = v(0^-) = V_0.$$

- Equation:

$$RC\frac{dv}{dt} + v = V_S.$$

- Result:

$$v(t) = V_S + (V_0 - V_S)e^{-\frac{t}{\tau}}$$

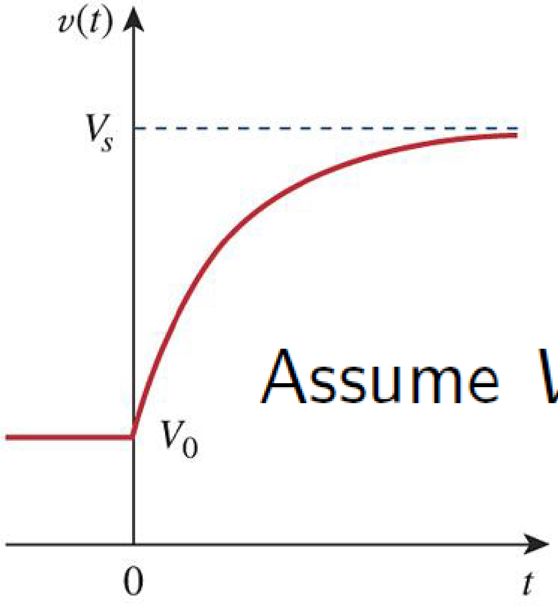
$$t > 0$$

Step Response of an RC Circuit

Complete response:

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau},$$

where v(0) is the initial voltage, and $v(\infty)$ is the final value.



Assume $V_s > V_0 > 0$

Figure 7.41 Response of an RC circuit.

Thus, to find the response of a first-order RC circuit requires three things:

1. The initial capacitor voltage $v(0^+)$.
2. The final capacitor voltage $v(\infty)$.
3. The time constant $\tau = RC$.

This is know as the *three - factor method*.

Step Response of an RL Circuit

- Excitation: step function, a V_S or an I_S .
- Assume the initial current of inductor is I_0.
- Try to derive i expression.

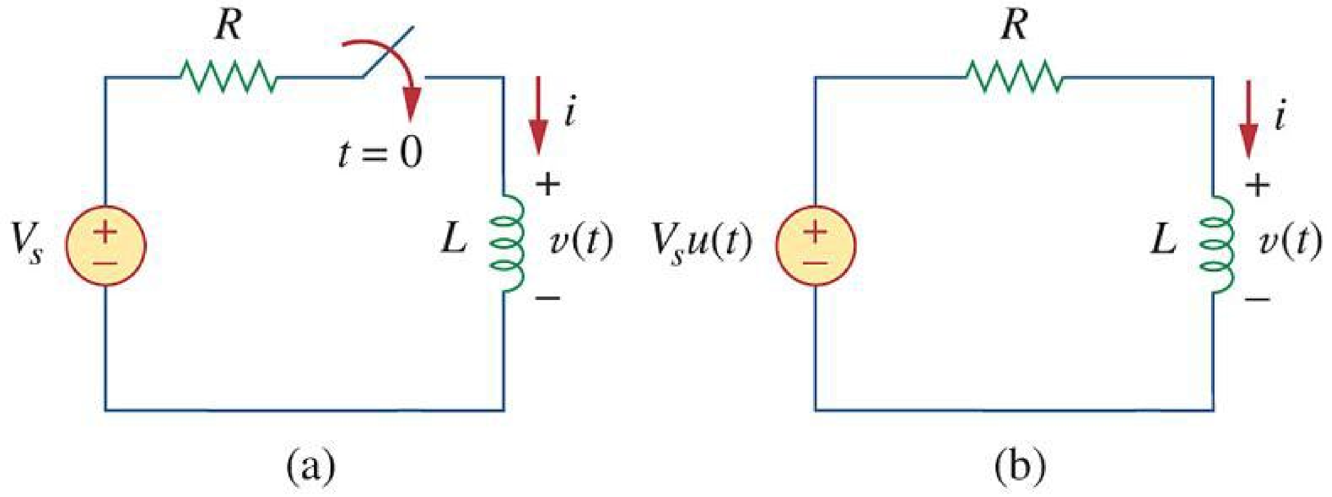


Figure 7.48 An RL circuit with a step input voltage.

- Initial condition:

$$i(0^+) = i(0^-) = I_0.$$

- Equation:

$$L\frac{di}{dt} + iR = V_S.$$

- Result:

$$i(t) = \frac{V_S}{R} + \left(I_0 - \frac{V_S}{R}\right)e^{-\frac{t}{\tau}}$$

$$t > 0$$

Step Response of an RC Circuit

Complete response:

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau},$$

where i(0) is the initial current, and $i(\infty)$ is the final value.

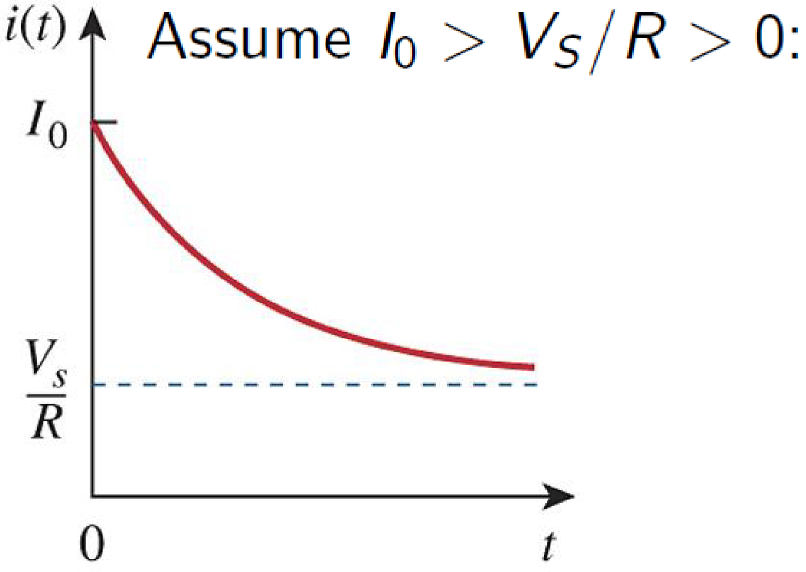
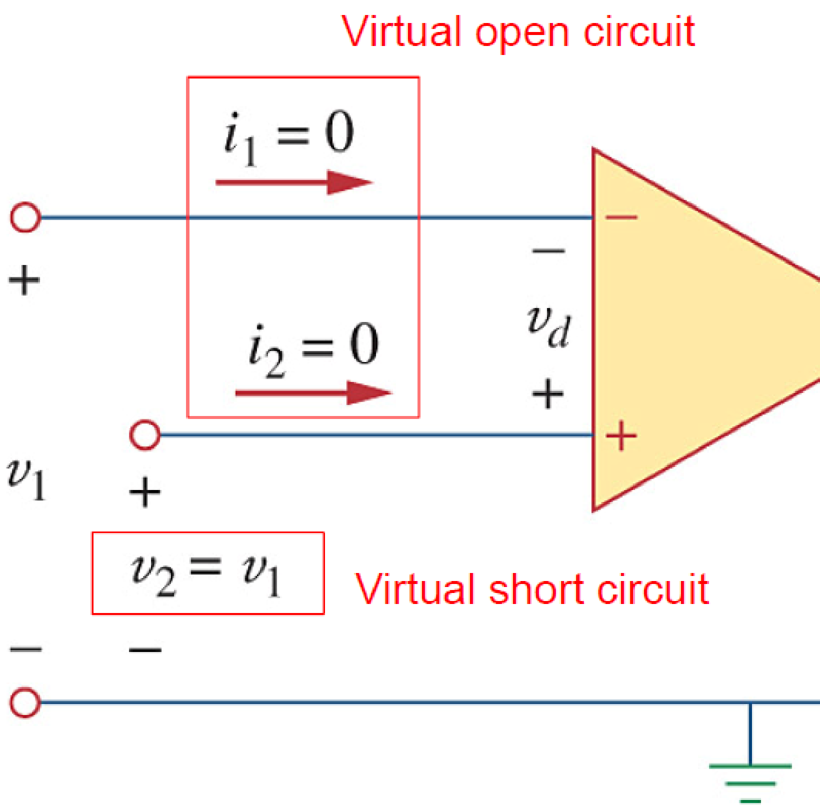


Figure 7.49 Total response of the *RL* circuit.

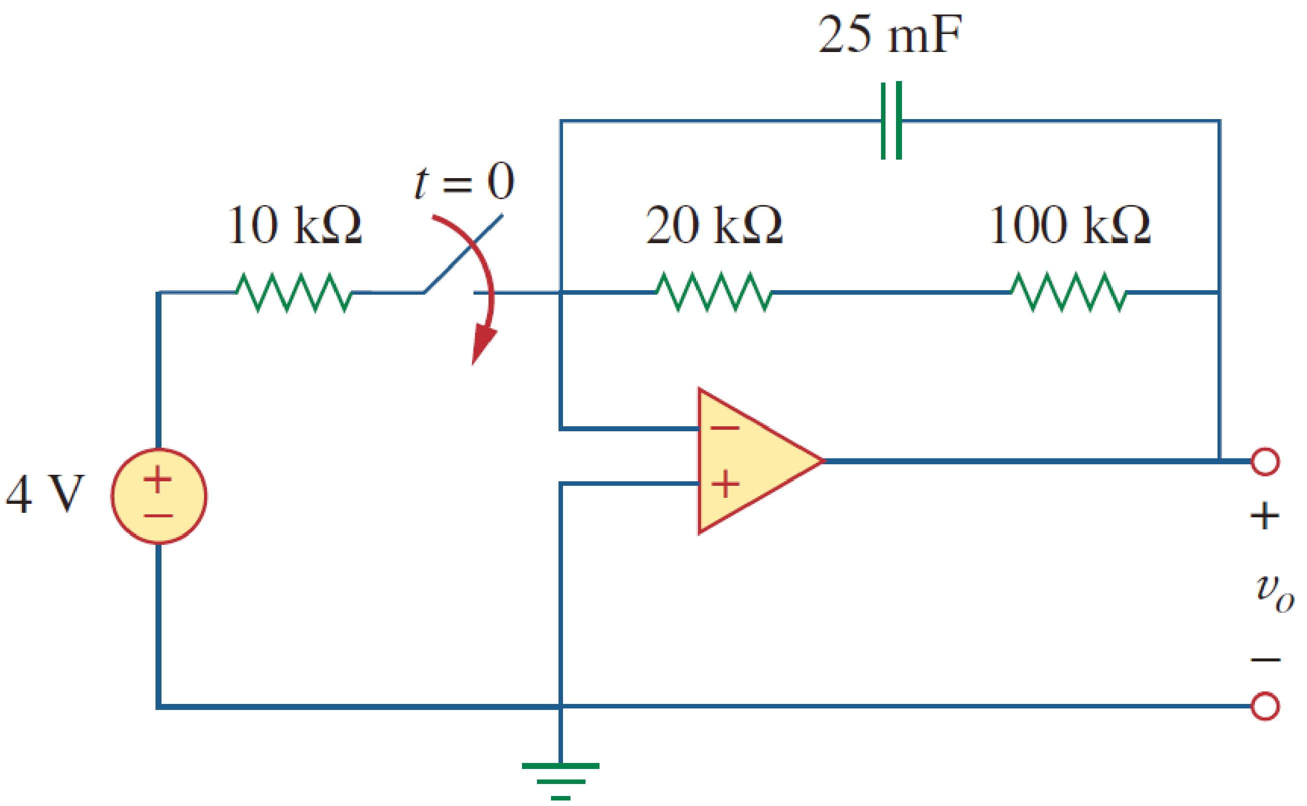
Thus, to find the response of a first-order *RL* circuit requires three things:

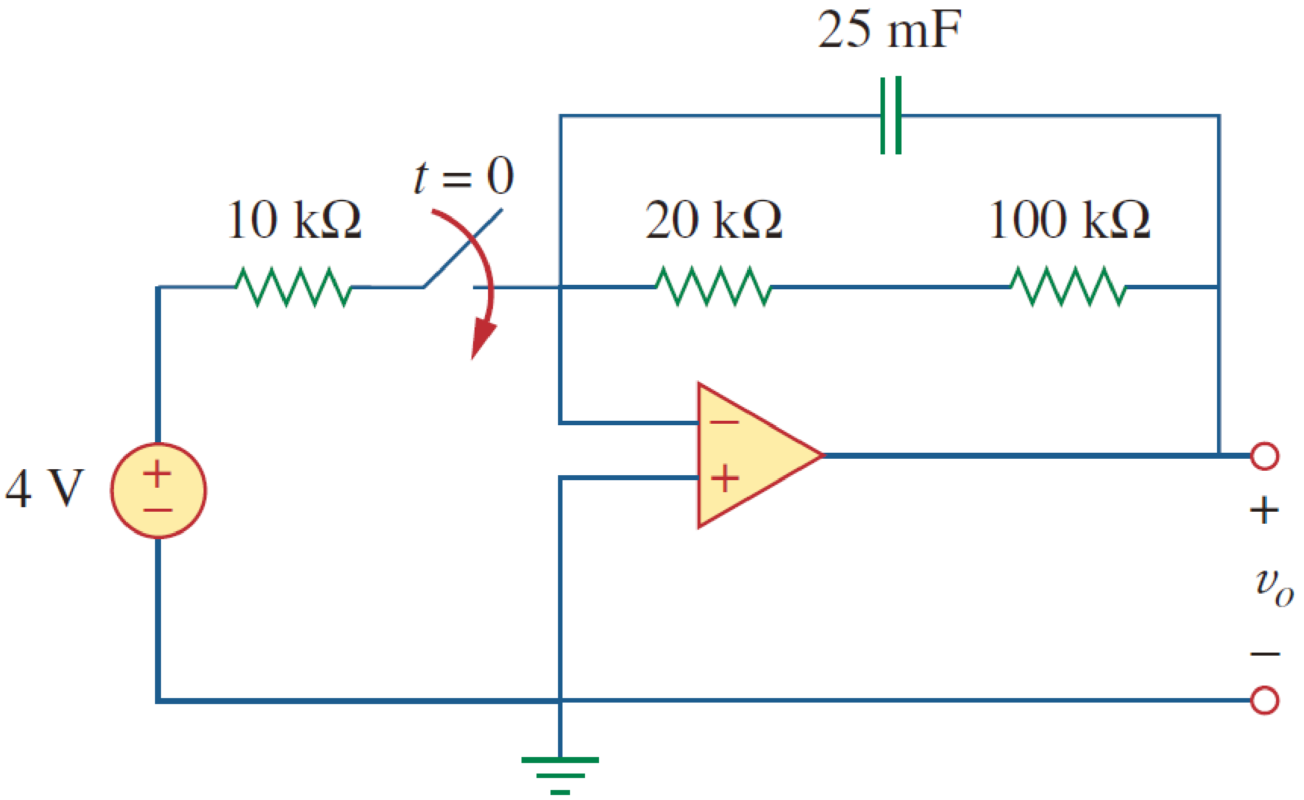
1. The initial inductor current $i(0^+)$.
2. The final inductor current $i(\infty)$.
3. The time constant $\tau = L / R$.

First-Order Op Amp Circuits



For the op amp circuit in Fig. 7.134, find $v_o(t)$ for $t > 0$.





Let v_x be the capacitor voltage.

For $t < 0$, $v_x(0) = 0$

$$v_o(\infty) = \frac{-4}{10}(20+100) = -48$$

$$R_{th} = 20+100 = 120\ k\Omega, \qquad \tau = R_{th}C = (120\times10^{3})(25\times10^{-3}) = 3000$$

$$v_o(t) = v_o(\infty) + \left[v_o(0) - v_o(\infty)\right]e^{-t/\tau}$$

$$v_o(t) = -48\left(1-e^{-t/3000}\right)V = \mathbf{48(e^{-t/3000}-1)u(t)V}$$

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Definition

- The circuit is now containing two storage elements, a capacitor and an inductor, thus known as second-order circuits.

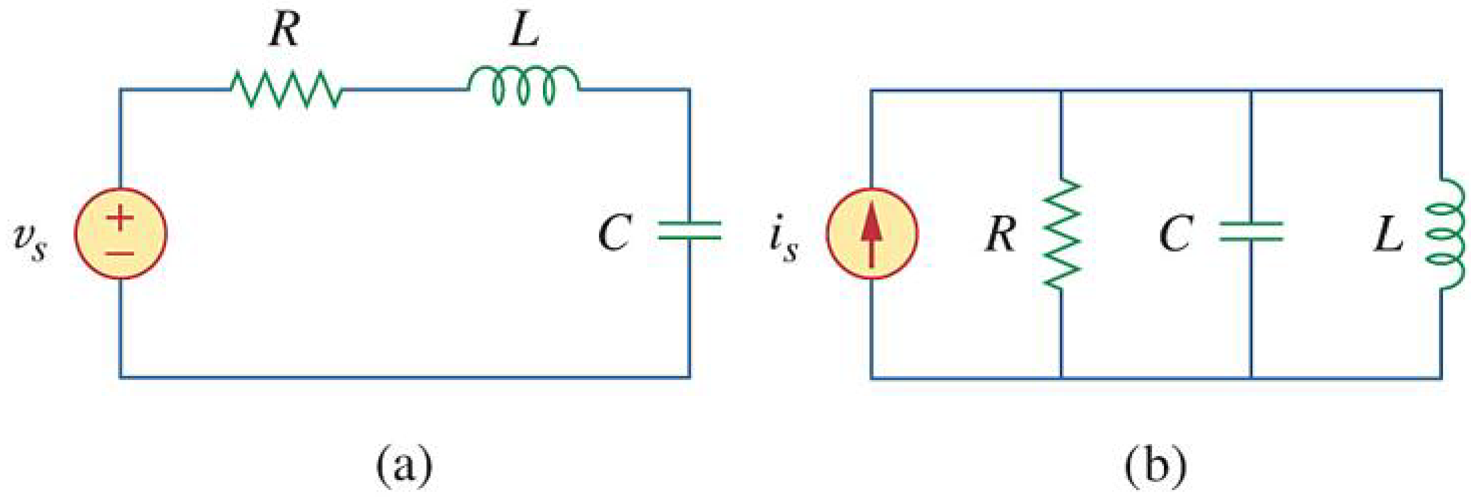


Figure 8.1 Typical examples of second-order circuits: (a) series RLC circuit, (b) parallel RLC circuit.

Solving steps: source-free

- Plot the circuit at $t < 0$, find initial conditions, $i(0^+)$, $v(0^+)$.
- Plot the circuit at $t > 0$, express $\frac{di}{dt}$ or $\frac{dv}{dt}$ in terms of i_L and v_c , find initial conditions $\frac{di(0^+)}{dt}$, $\frac{dv(0^+)}{dt}$.
- Express the circuit in 2nd order D.E. with only one parameter (either i or v) and solve it.
- Solve the coefficients using initial conditions.
- (Series connection: prefer to use i.
- Parallel connection: prefer to use v.)

Solving steps: source-free

- When $t < 0$, the circuit is at steady state, use dc conditions for $i(0^-)$, $v(0^-)$.
- When $t > 0$, use KCL and KVL to get 1st ODE for $\frac{di(0^+)}{dt}$ and $\frac{dv(0^+)}{dt}$.
- When $t \to \infty$, the circuit is at steady state, use dc conditions again for $i(+\infty)$, $v(+\infty)$.

Solving steps: source-free (series)

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = 0$$

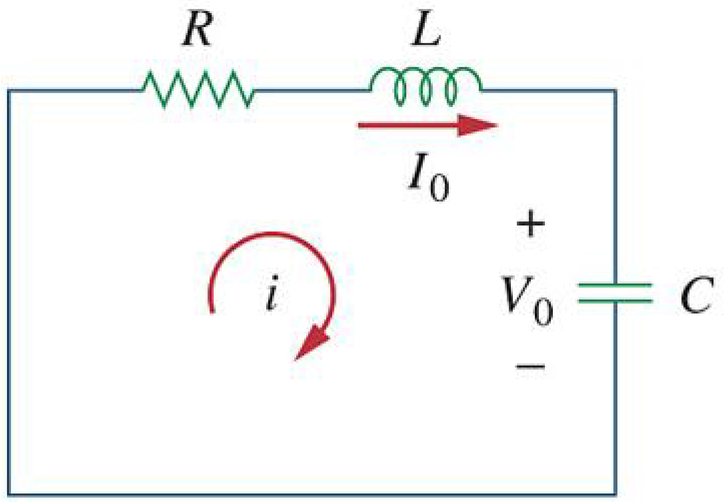


Figure 8.8 A source-free series RLC circuit.

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

Solution: $s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$,

where $\alpha = \frac{R}{2L}$, $\omega_0 = \frac{1}{\sqrt{LC}}$.

Solving steps: source-free (parallel)

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0$$

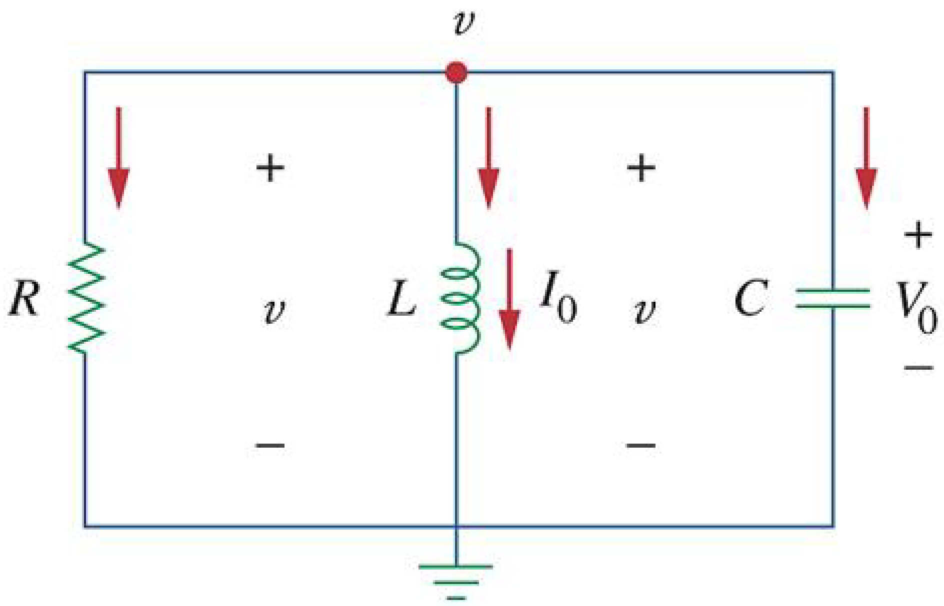


Figure 8.13 A source-free parallel *RLC* circuit.

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

Solution: $s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$,

where $\alpha = \frac{1}{2RC}$, $\omega_0 = \frac{1}{\sqrt{LC}}$.

Solving steps: source-free

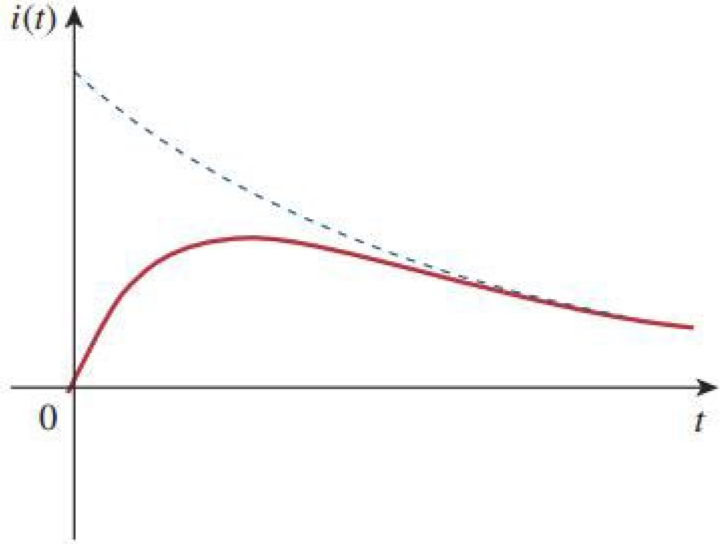
Overdamped $\alpha > \omega_0$:

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$, $s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

$i(t) = A_1 e^{-s_1 t} + A_2 e^{-s_2 t}$.

$$A_1 = \frac{i'(0^+) - s_2 i(0^+)}{s_1 - s_2}$$

$$A_2 = \frac{s_1 i(0^+) - i'(0^+)}{s_1 - s_2}$$



Solving steps: source-free

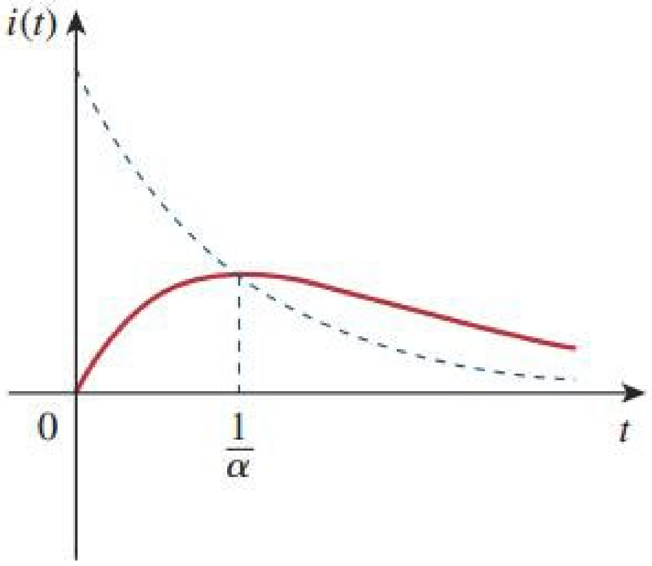
Critically damped $\alpha = \omega_0$:

$s_1 = -\alpha = s_2 = -\alpha$

$i(t) = B_1 t e^{-\alpha t} + B_2 e^{-\alpha t}.$

$$B_1 = i'(0^+) + \alpha i(0^+)$$

$$B_2 = i(0^+)$$



Solving steps: source-free

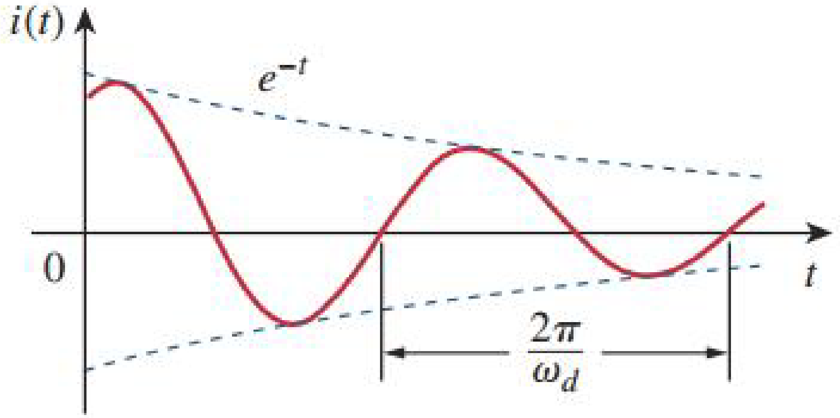
Underdamped $\alpha < \omega_0$:

$s_1 = -\alpha + j\omega_d$, $s_2 = -\alpha - j\omega_d$, $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$ (damping frequency)

$i(t) = e^{-\alpha t}(C_1 cos\omega_d t + C_2 sin\omega_d t)$.

$$C_1 = i(0^+)$$

$$C_2 = \frac{i'(0^+) + \alpha i(0^+)}{\omega_d}$$



Solving steps: step-input

- Plot the circuit at $t < 0$, find initial conditions, $i(0^+)$, $v(0^+)$.
- Plot the circuit at $t > 0$, express $\frac{di}{dt}$ or $\frac{dv}{dt}$ in terms of i_L and v_c , find initial conditions $\frac{di(0^+)}{dt}$, $\frac{dv(0^+)}{dt}$.
- Express the circuit in 2nd order D.E. with only one parameter (either i or v) and solve it.
- Plot the circuit at $t \to \infty$, find steady state values $i(+\infty)$, $v(+\infty)$ (or just solve forced response).
- Solve the coefficients using initial conditions.

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = V_s$$

$$\frac{d^2 i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{1}{LC} i = \frac{I_s}{LC}$$

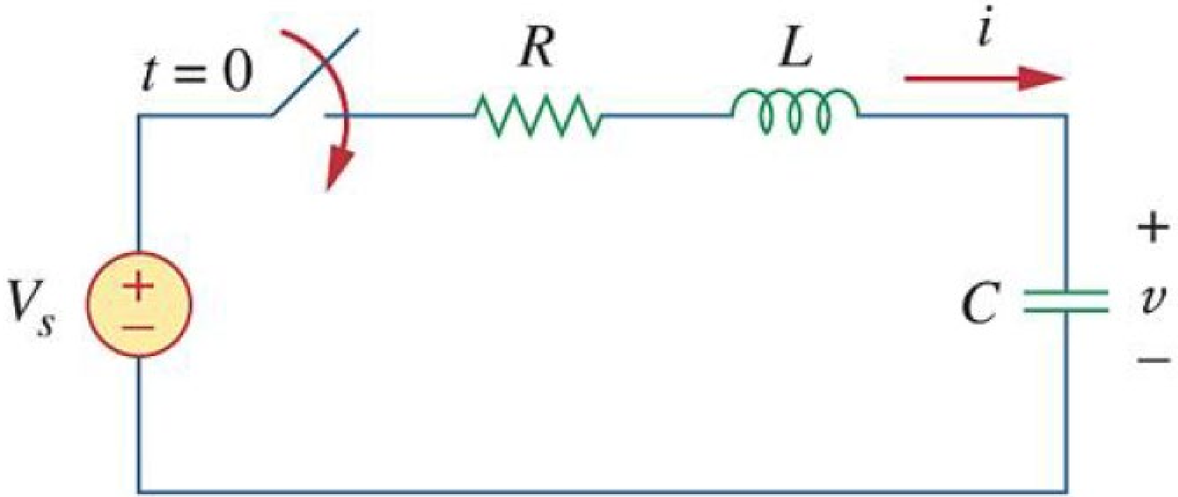


Figure 8.18 Step voltage applied to a series RLC circuit.

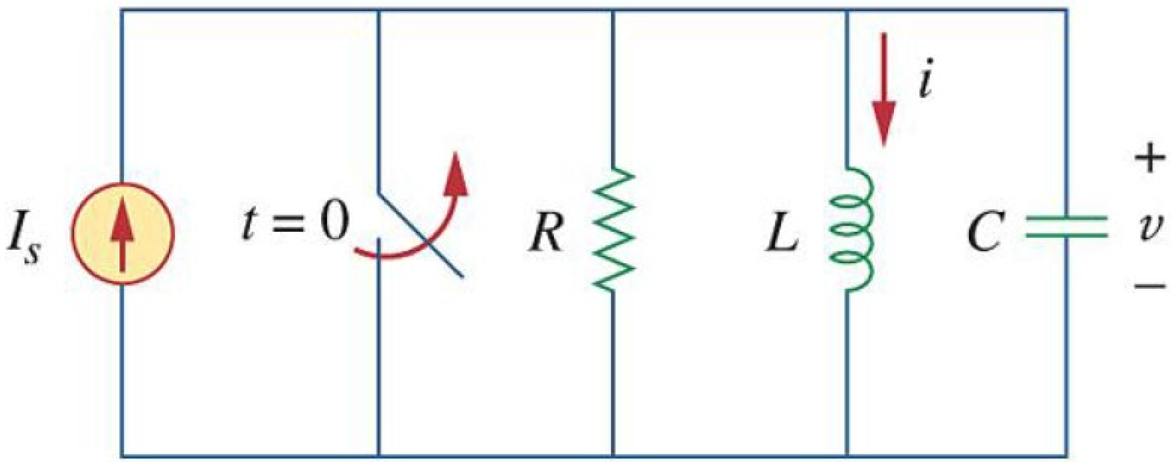


Figure 8.22 Parallel RLC circuit with an applied current.

General 2nd Order Circuits

- Find the initial condition $x(0)$ and $\frac{dx(t)|_{t=0}}{dt}$ and the final value $x(\infty)$.
- Turn off the independent sources and find the form of the transient response $x_t(t)$ by applying KCL and KVL.
- Obtain the steady-state as $x_{ss}(t) = x(\infty)$
- $x(t) = x_t(t) + x_{ss}(t)$

Duality

Find the dual of a given circuit by taking the following three steps.

- Place a node at the center of each mesh of the given circuit. Place the reference node (the ground) of the dual circuit outside the given circuit.
- Draw lines between the nodes such that each line crosses an element. Replace that element by its dual.
- To determine the polarity of voltage sources and direction of current sources, follow this rule: A voltage source that produces a positive (clockwise) mesh current has as its dual a current source whose reference direction is from the ground to the non-reference node.

TABLE 8.1 Dual Pairs

Resistance	Conductance
Inductance	Capacitance
Voltage	Current
Voltage source	Current source
Node	Mesh
Series path	Parallel path
Open circuit	Short circuit
KVL	KCL
Thevenin	Norton

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Definition

- A sinusoid is a signal that has the form of the sine or cosine function.

$$v(t) = V_m \sin(\omega t + \phi)$$

V_m: amplitude

ω: angular frequency

ϕ: initial phase

- A phasor is a complex number that represents the amplitude and phase of a sinusoid.
- $v(t) = V_m \sin(\omega t + \phi) = Re\left(V_m e^{j(\omega t + \phi)}\right) = Re\left(V_m e^{j\phi} e^{j\omega t}\right) = Re(\tilde{V} e^{j\omega t})$
- where $\tilde{V} = V_m e^{j\phi} = V_m \angle \phi$ is the phasor representation of the sinusoid $v(t)$.

$$v(t) = V_m \cos(\omega t + \phi) \Leftrightarrow \tilde{V} = V_m \angle \phi$$

Phasor Relationships for Circuit Elements

- Ohm's law:
- If the current through a resistor is $i(t) = I_m\cos(\omega t + \phi)$, the voltage across it

$$v = iR = RI_m\cos(\omega t + \phi)$$

- The phasor form of this voltage is

$$\tilde{V} = RI_m\angle\phi$$

- The phasor representation of the current is

$$\tilde{I} = I_m\angle\phi$$

$$\tilde{V} = R\tilde{I}$$

$$\tilde{V} = R\tilde{I}$$

$$\tilde{V} = RI_m\angle\phi \qquad \tilde{I} = I_m\angle\phi$$

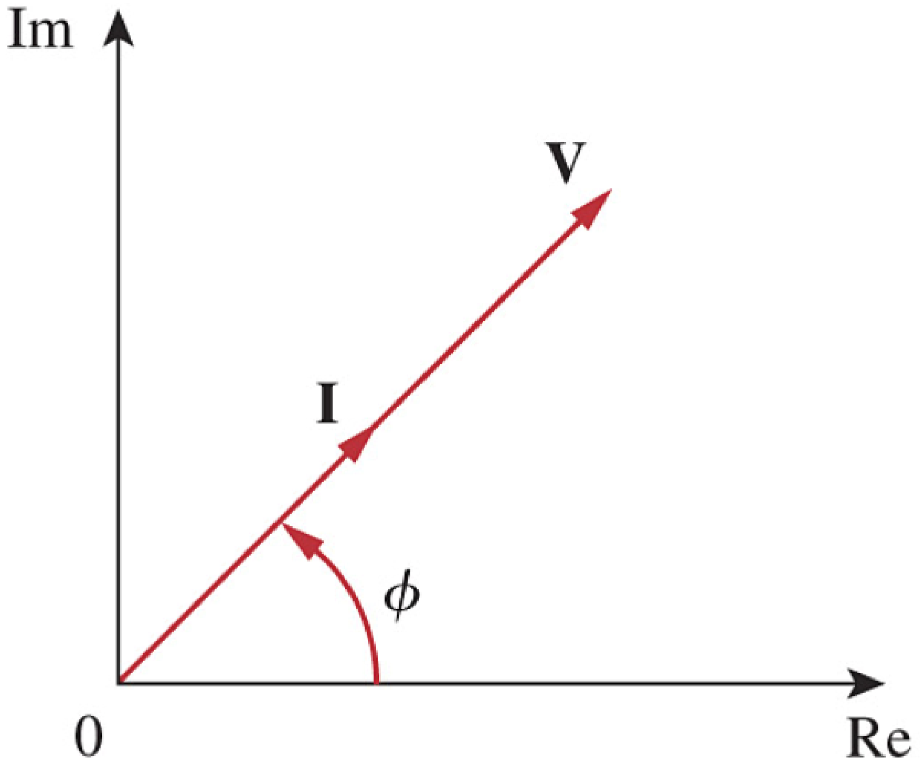

Figure 9.10 Phasor diagram for the resistor.

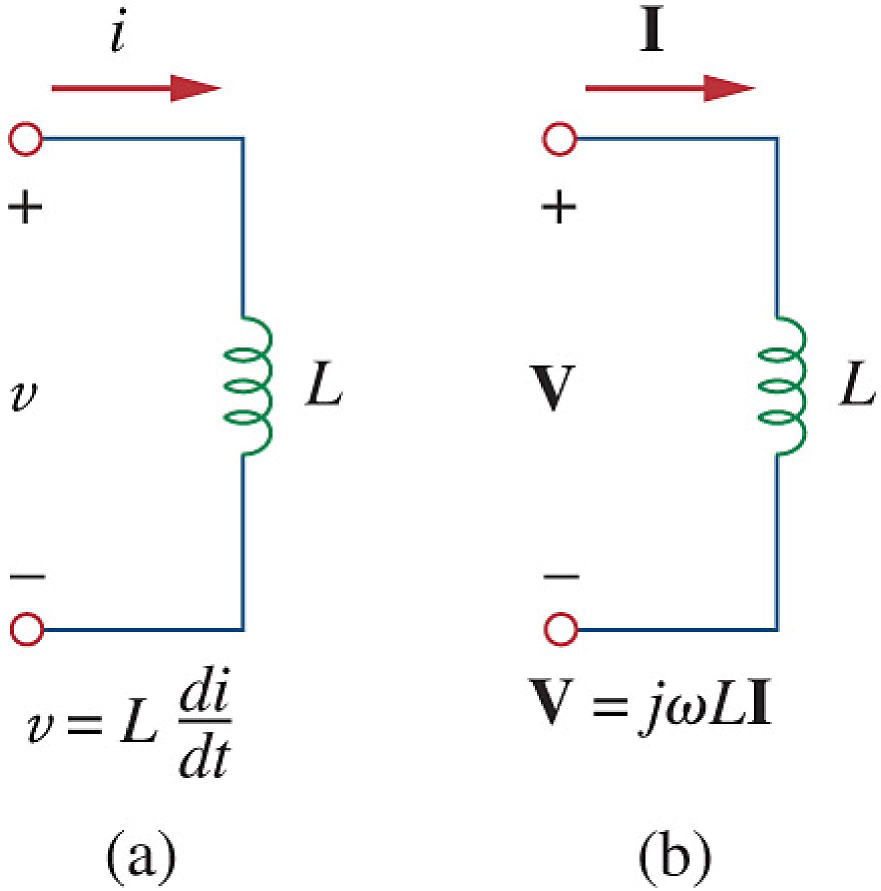

Figure 9.11 Voltage-current relations for an inductor in the (a) time domain, (b) frequency domain.

$$\tilde{I} = I_m \angle\phi \qquad \tilde{V} = \omega L I_m \angle(\phi + 90^\circ) = j\omega L\tilde{I}$$

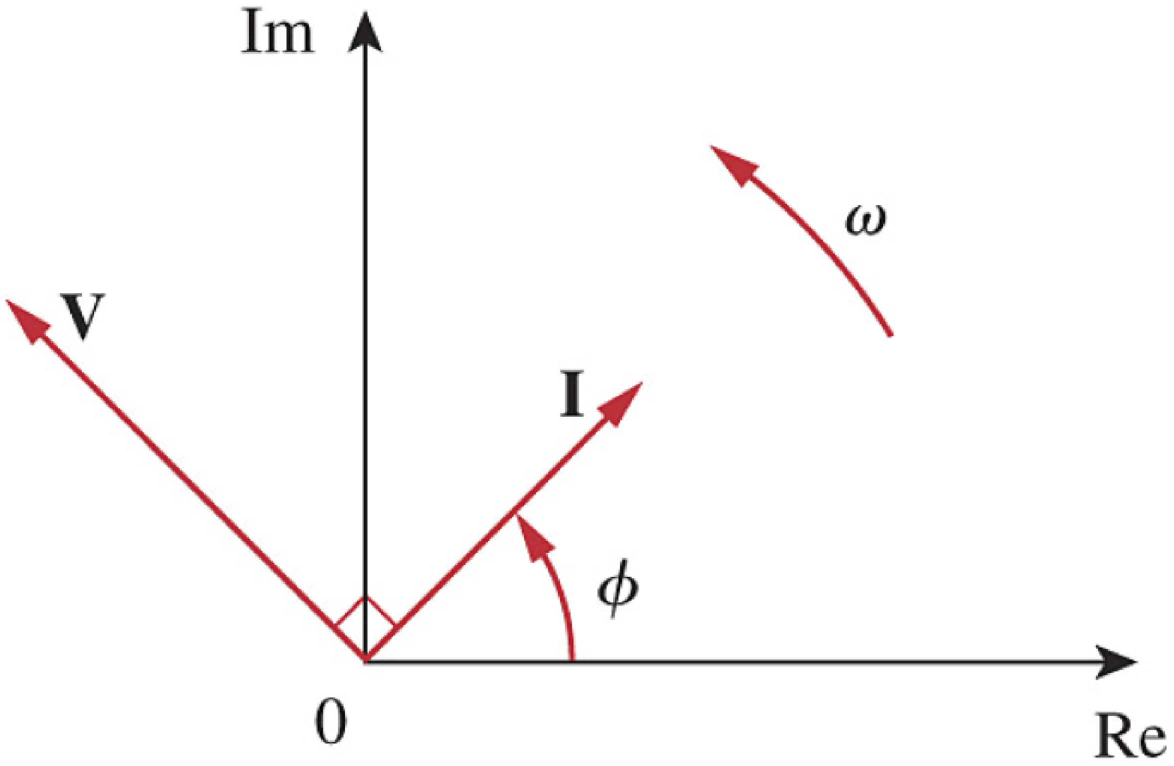

Figure 9.12 Phasor diagram for the inductor.

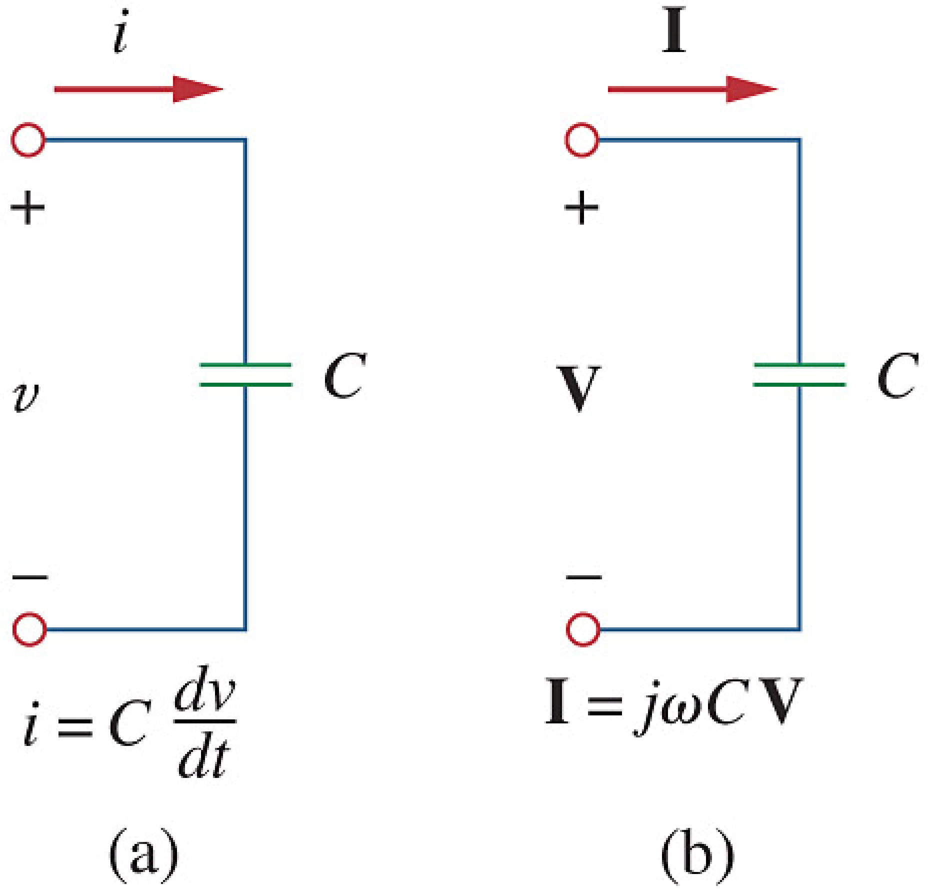

Figure 9.13 Voltage-current relations for a capacitor in the (a) time domain, (b) frequency domain.

$$\tilde{V} = V_m \angle \underline{\phi} \qquad \tilde{I} = \omega C V_m \angle \underline{(\phi + 90°)} = j\omega C \tilde{V}$$

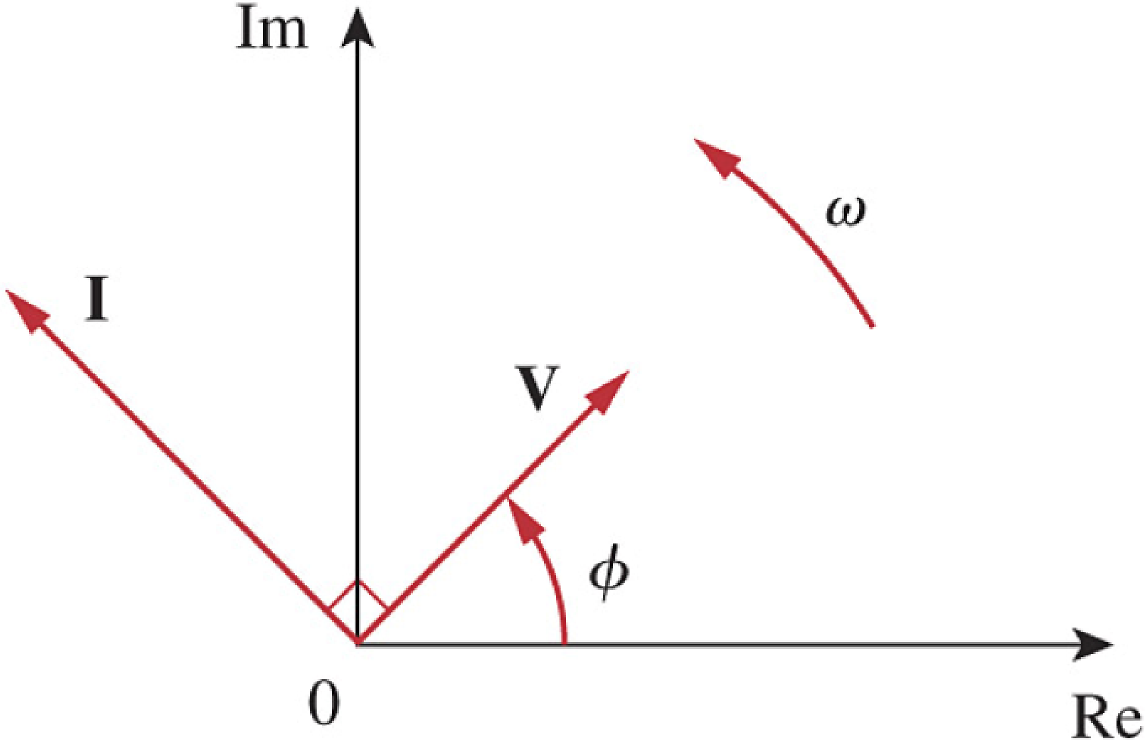

Figure 9.14 Phasor diagram for the capacitor.

Summary of voltage - current relationships

Element	Time domain	Frequency domain
R	$v = Ri$	$\tilde{V} = R\tilde{I}$
L	$v = L\dfrac{di}{dt}$	$\tilde{V} = j\omega L\tilde{I}$
C	$i = C\dfrac{dv}{dt}$	$\tilde{V} = \dfrac{1}{j\omega C}\tilde{I}$

Impedance and Admittance

- The impedance Z of a circuit is the ratio of the phasor voltage $\tilde{V}$ to the phasor current $\tilde{I}$ measured in ohms (Ω).

$$Z = \frac{\tilde{V}}{\tilde{I}}$$

- For the three passive elements, we have

$$Z = R,\ Z = j\omega L, \text{ and } Z = \frac{1}{j\omega C}$$

- The admittance Y of a circuit is the ratio of phasor current $\tilde{I}$ to the the phasor voltage $\tilde{V}$measured in siemens (S).

$$Y = \frac{\tilde{I}}{\tilde{V}} = \frac{1}{Z}$$

Impedance and Admittance

- A complex number Z can be written in rectangular form as

$$Z = x + jy$$

x: the real part of Z,

y: the imaginary part of Z.

- Exponential form:

$$Z = re^{j\phi}$$

- Polar form:

$$Z = r\angle\phi$$

$$r = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1}\frac{y}{x}$$

Impedance Combinations

$$Z_{eq} = \frac{\tilde{V}}{\tilde{I}} = \frac{\sum_{i=1}^{N} \tilde{V}_i}{\tilde{I}} = \sum_{i=1}^{N} \frac{\tilde{V}_i}{\tilde{I}} = \sum_{i=1}^{N} Z_i$$

$$Y_{eq} = \frac{\tilde{I}}{\tilde{V}} = \frac{\sum_{i=1}^{N} \tilde{I}_i}{\tilde{V}} = \sum_{i=1}^{N} \frac{\tilde{I}_i}{\tilde{V}} = \sum_{i=1}^{N} Y_i$$

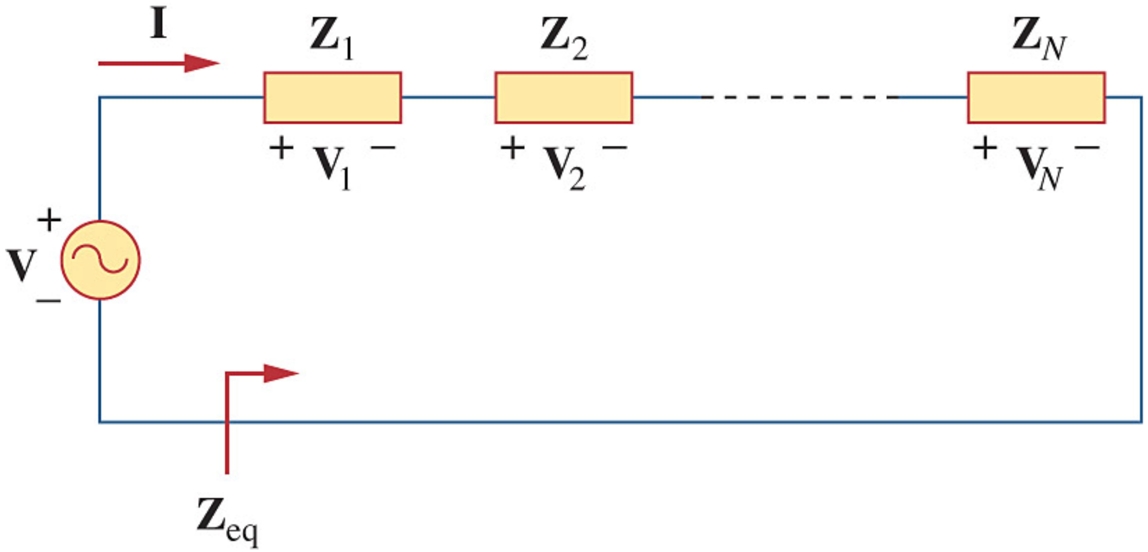

Figure 9.18 N impedances in series.

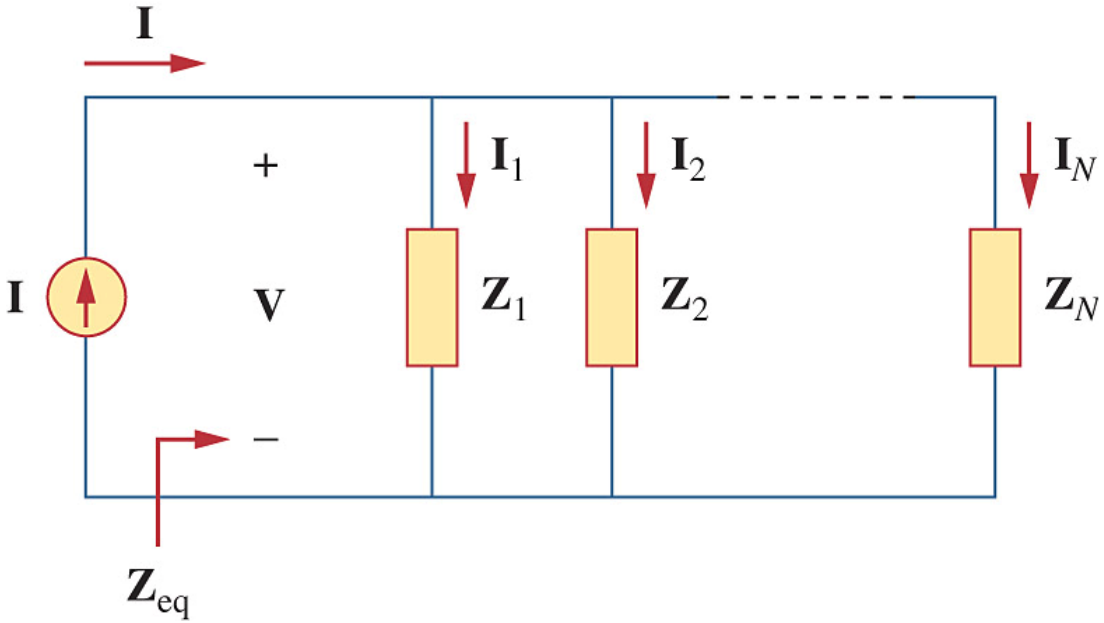

Figure 9.20 N impedances in parallel.

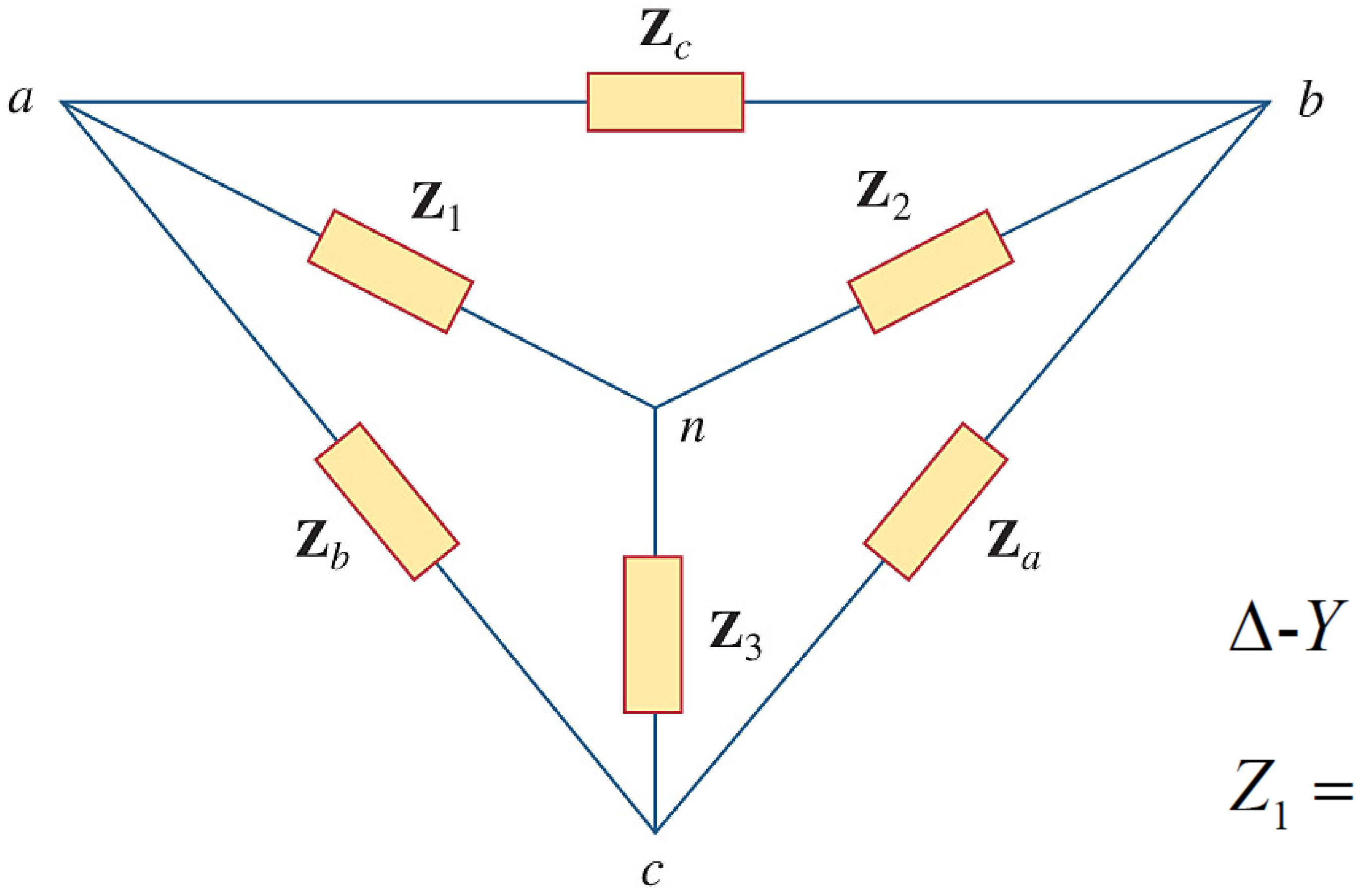

Figure 9.22 Superimposed wye and delta networks.

Y-Δ conversion:

$$Z_a = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1}$$

$$Z_b = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_2}$$

$$Z_c = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3}$$

Δ-Y conversion:

$$Z_1 = \frac{Z_bZ_c}{Z_a + Z_b + Z_c}$$

$$Z_2 = \frac{Z_cZ_a}{Z_a + Z_b + Z_c}$$

$$Z_3 = \frac{Z_aZ_b}{Z_a + Z_b + Z_c}$$

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Steps to Analyze AC Circuits:

1. Transform the circuit to the phasor domain.
2. Find the circuit output using nodal analysis, mesh analysis, superposition, etc.
3. Transform the resulting phasor to the time domain.

Step 1 is not necessary if the problem is specified in the frequency domain. In step 2, the analysis is performed in the same manner as dc circuit analysis except that complex numbers are involved

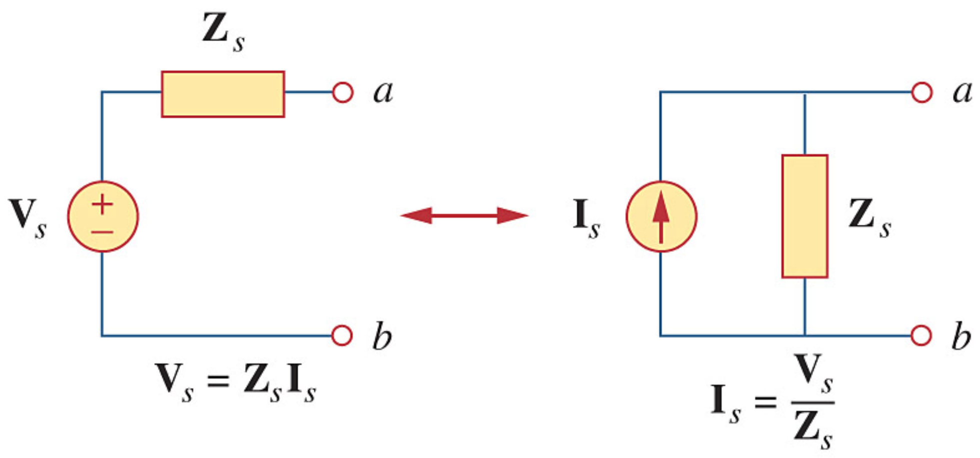

Figure 10.16 Source transformation.

Oscillator

An oscillator is a circuit that produces an ac waveform as output when powered by a dc supply.

In order for sine wave oscillators to sustain oscillations, they must meet the Barkhausen criteria :

1. The overall gain of the oscillator must be unity or greater.
2. The overall phase shift must be zero.

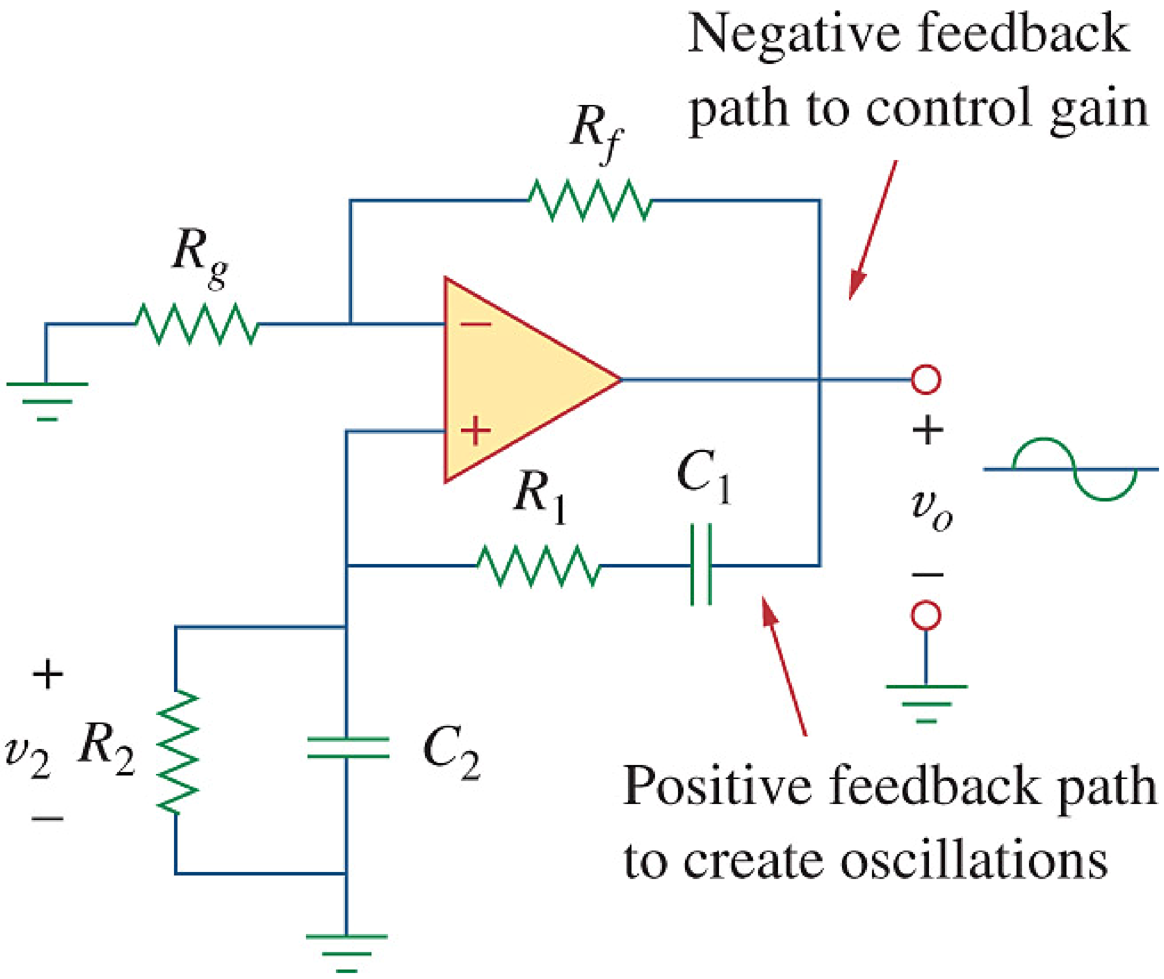

Figure 10.42 Wien-bridge oscillator.

$$\frac{V_2}{V_0} = \frac{\omega R_2 C_1}{\omega(R_2C_1 + R_1C_1 + R_2C_2) + \boxed{j(\omega^2 R_1R_2C_1C_2 - 1)}}$$

- To satisfy the second Barkhausen criterion,
- $\frac{V_2}{V_0}$ must be real

1. Unity gain (or greater)
2. Zero phase shift

Thanks for Listening!