VE215 Intro to Circuits

Mid 1 Review Class

Content for Mid 1

1. Chapter 1: Basic Concepts
2. Chapter 2: Basic Laws
3. Chapter 3: Methods of Analysis
4. Chapter 4: Circuit Theorems
5. Chapter 5: Operational Amplifiers

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Basic Definition

- The relationship between current i, charge q, and time t:

$$i = \frac{dq}{dt}$$

- The charge transferred between time t_0 and t is obtained by

$$q = \int_{t_0}^{t} i \, dt$$

- The voltage v_{ab} between two points a and b is defined by

$$v_{ab} = \frac{dw}{dq}$$

Direction of Voltage & Current

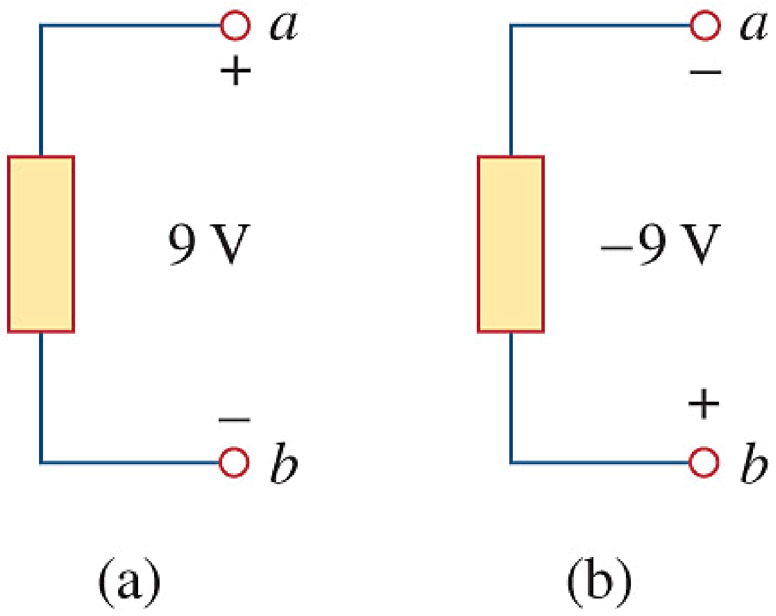


Figure 1.7 Two equivalent representations of the same voltage v_{ab} : (a) point a is 9 V above point b, (b) point b is -9 V above point a.

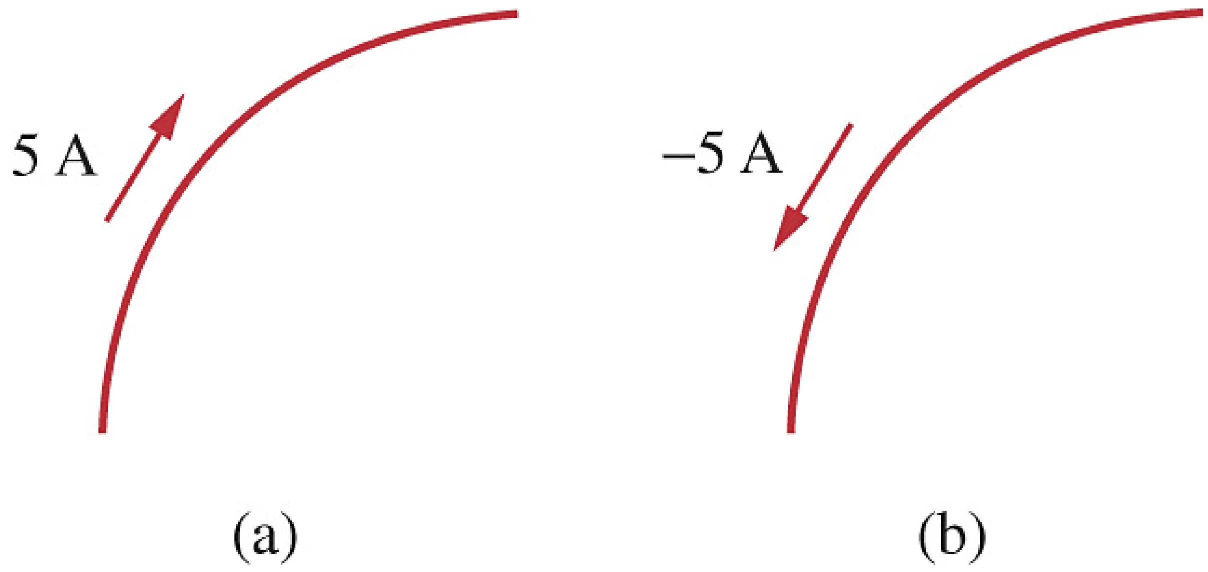


Figure 1.5 Conventional current flow: (a) positive current flow, (b) negative current flow.

Example

- When using *Thevenin's Theorem*, we usually consider V_{Th} as the voltage between point *a* and *b*.
- Be careful in the exam!

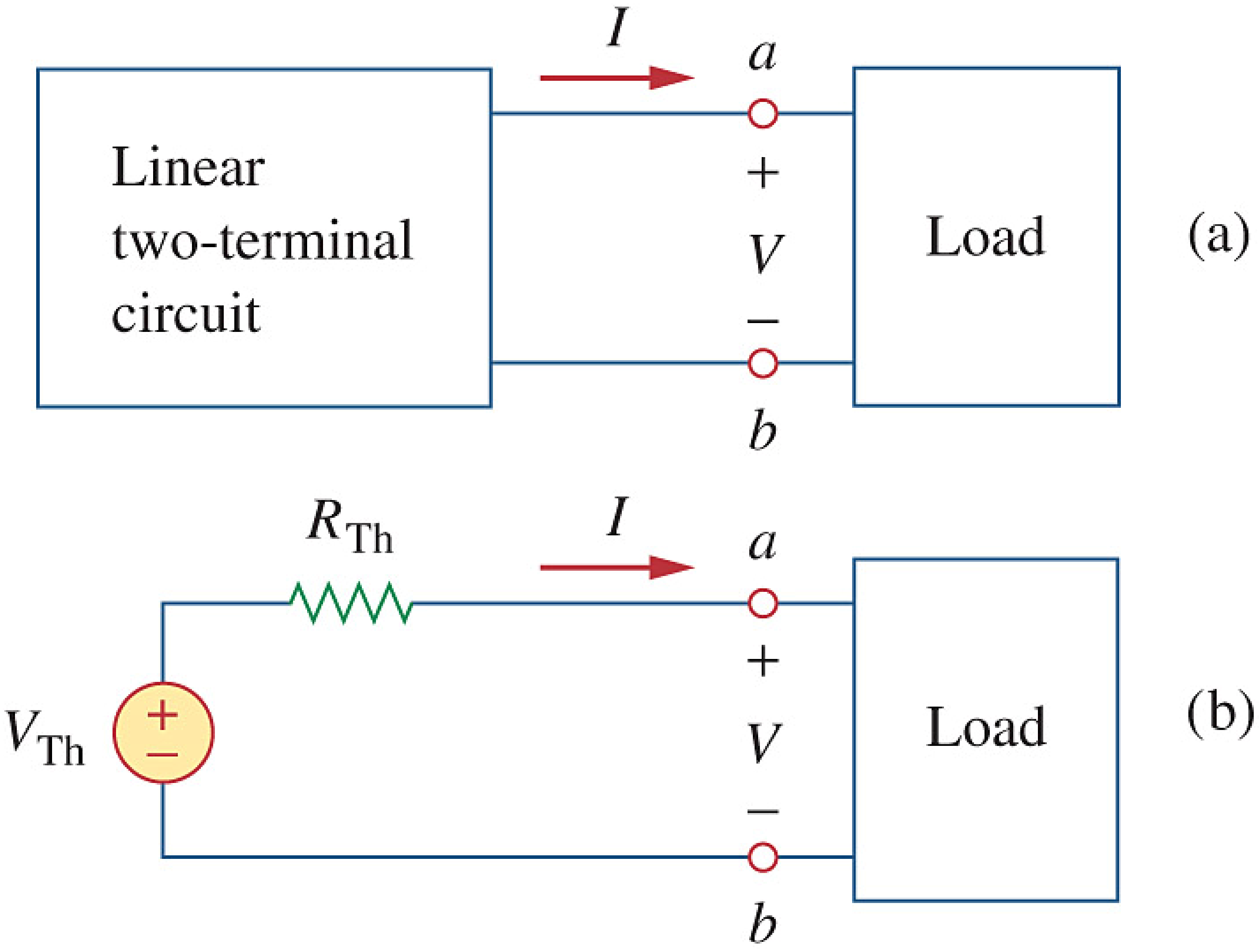


Figure 4.23 Replacing a linear two-terminal circuit by its Thevenin equivalent.

Homework 4 – Prob 4.52

4.52 For the transistor model in Fig. 4.118, obtain the Thevenin equivalent at terminals *a*-*b*.

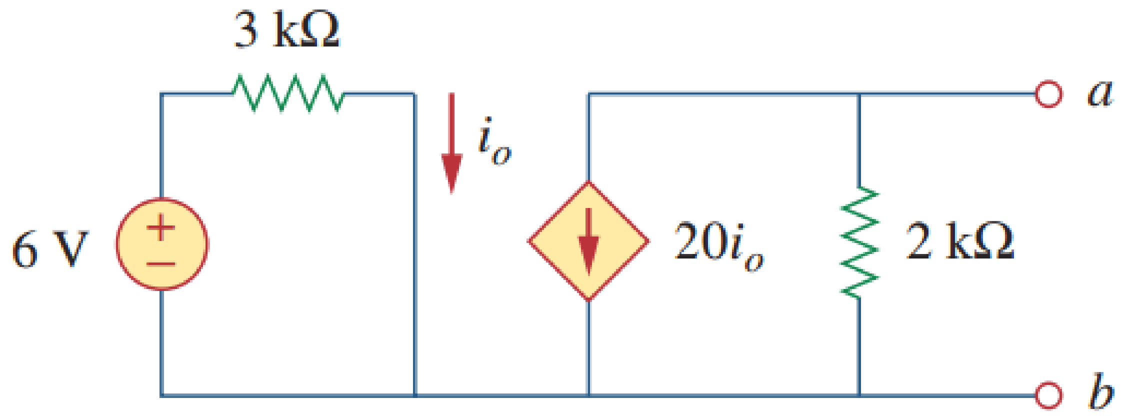


Figure 4.118
For Prob. 4.52.

$$R_{Th} = 2k\ \Omega$$
$$V_{Th} = -80\ V$$

Detailed solution will be coverd in the fourth part.

Basic Definition

- *Power* is the time rate of expending or absorbing energy:

$$p = \frac{dw}{dt}$$

- In circuit, we have the calculation form

$$p = vi$$

- Energy is the capacity to do work

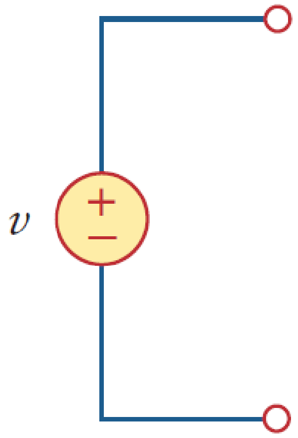
$$w = \int_{t_0}^{t} p\, dt = \int_{t_0}^{t} vi\, dt = vit$$

- The *law of conservation of energy*

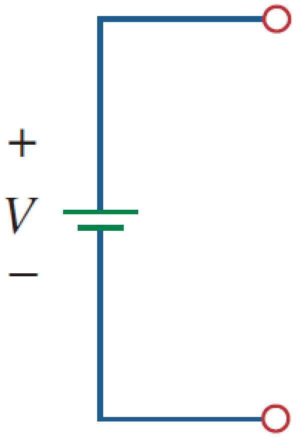
$$\sum p = 0$$

the total power supplied to the circuit must balance the total power absorbed, $i.e. Power\ absorbed = -Power\ supplied.$

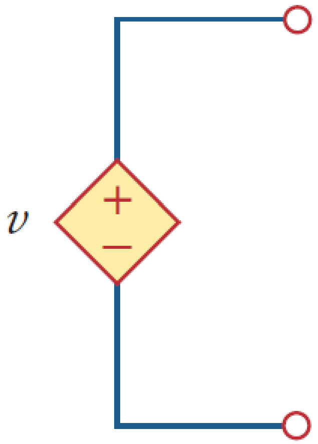
Symbols



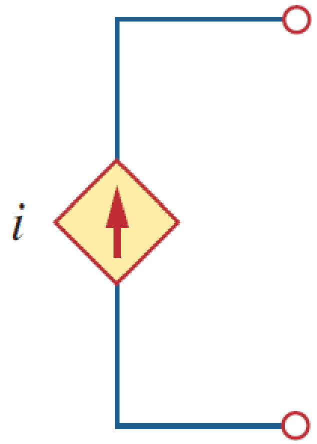
Constant
or
time – varying
voltage



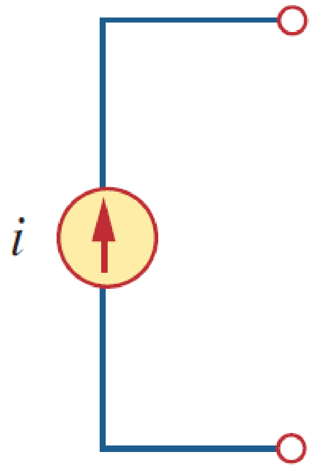
Constant
voltage



Dependent
Voltage
source



Dependent
Current
source



Independent
Current
source

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Basic Definition

- Branch:
- A branch b represents a single element.

- Node:
- A node n is the point connection between two or more branches.

- Loop:
- A loop l is any closed path in a circuit.

$$b = l + n - 1$$

Basic Definition

- Ohm's Law:
- The voltage v across a resistor is proportional to the current i flowing through the resistor.

$$v = i \times R$$

- The power dissipated by a resistor can be expressed in terms of R.

$$p = vi = i^2 R = \frac{v^2}{R}$$

- Conductance

$$G = \frac{1}{R}$$

Basic Definition

- Series
- Two or more elements are in **series** if they exclusively share a single node and consequently carry the same current.

$$R_{total} = \sum R_s$$

- Parallel
- Two or more elements are in **parallel** if they are connected to the same two nodes and consequently have the same voltage across them.

$$\frac{1}{R_{total}} = \sum \frac{1}{R_p}$$

Basic Definition

- The principle of voltage division
- For register in **series**, voltage across each resistor

$$v_n = \frac{R_n}{\sum_{n=1}^{N} R_n} v, \qquad n = 1,2, \dots, N$$

- The principle of current division
- For register in **parallel**, current across each resistor

$$i_n = \frac{G_n}{\sum_{n=1}^{N} G_n} i, \qquad n = 1,2, \dots, N$$

Kirchhoff's current law (KCL)

- Definition:
- The algebraic sum of currents entering a node (or a closed boundary) is zero.

$$\sum_{n=1}^{N} i_n = 0$$

- where n is the number of branches connected to the node and i_n is the n-th current entering (or leaving) the node.

$$\sum i_n = 0$$

$$i_1 - i_2 + i_3 + i_4 - i_5 = 0$$

$$i_1 + i_3 + i_4 = i_2 + i_5$$

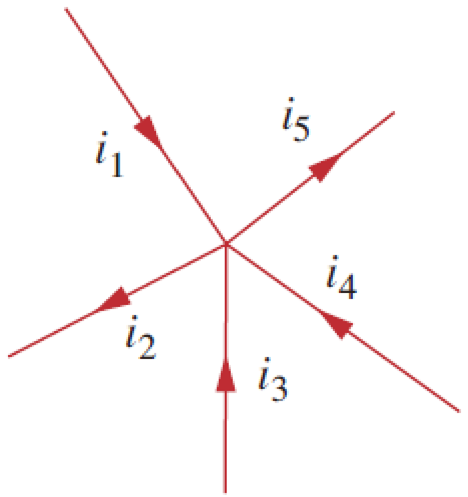


Figure 2.16
Currents at a node illustrating KCL.

Kirchhoff's voltage law (KVL)

- Definition
- The algebraic sum of all voltages around a closed path (or loop) is zero.

$$\sum_{m=1}^{M} v_m = 0$$

- where M is the number of branches in the loop and v_m is the m-th voltage drop (or rise) in the loop.

$$\sum v_m = 0$$

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

$$v_2 + v_3 + v_5 = v_4 + v_1$$

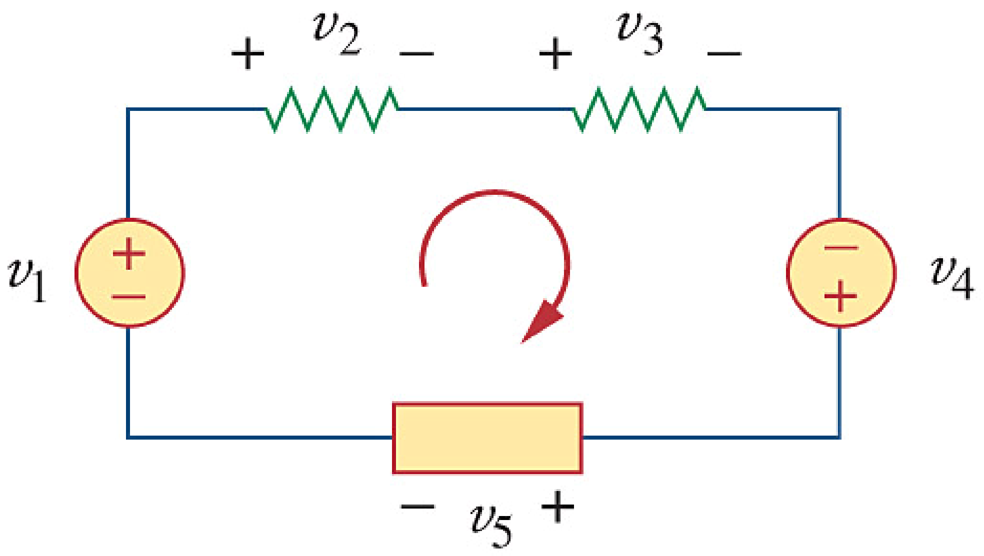


Figure 2.19 A single-loop circuit illustrating KVL.

Wye-Delta Transformations

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \qquad R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

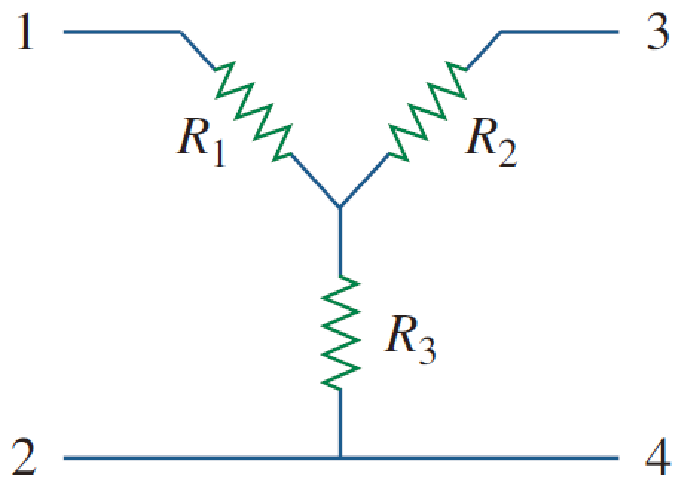
$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \qquad R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

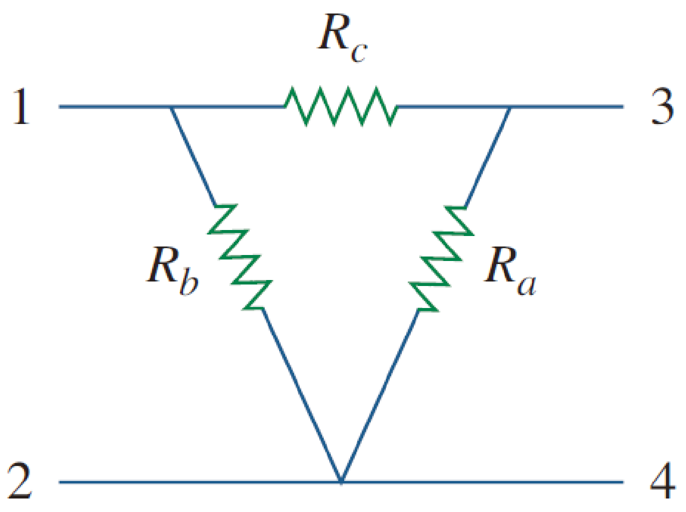
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} \qquad R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

The Y and Δ networks are said to be *balanced* when

$$R_1 = R_2 = R_3 = R_Y, \qquad R_a = R_b = R_c = R_\Delta$$

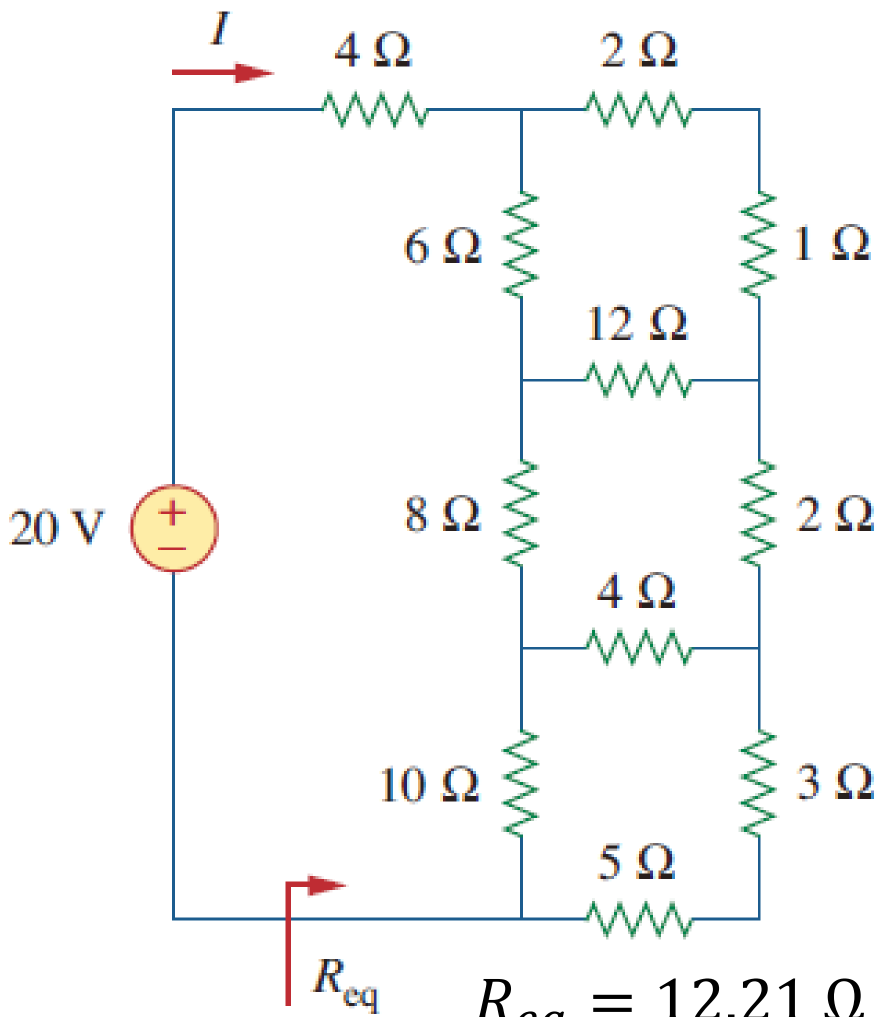
$$R_Y = \frac{R_\Delta}{3} \qquad \text{or} \qquad R_\Delta = 3R_Y$$





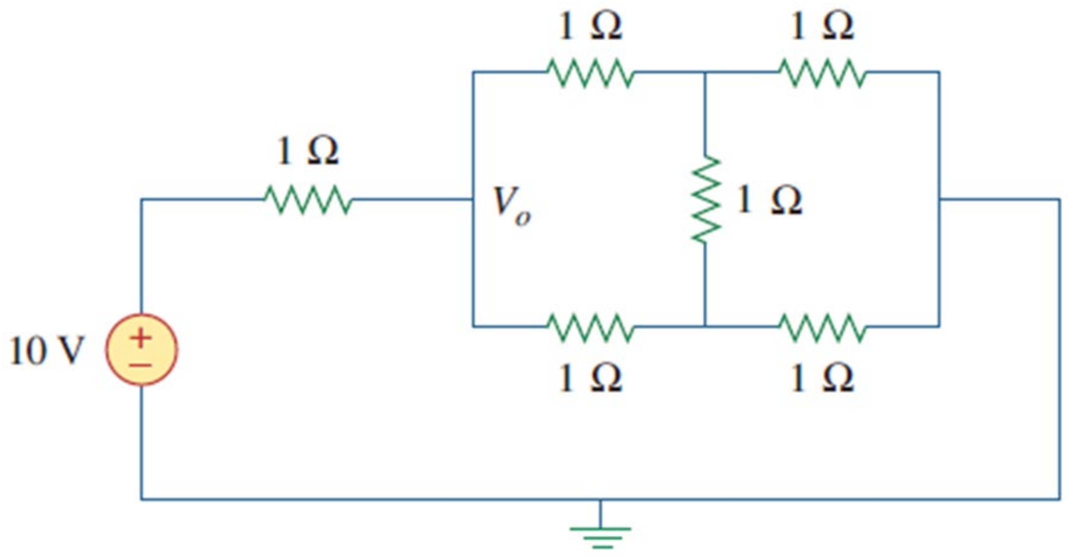
Sample Questions

Find R_{eq} and I in the circuit



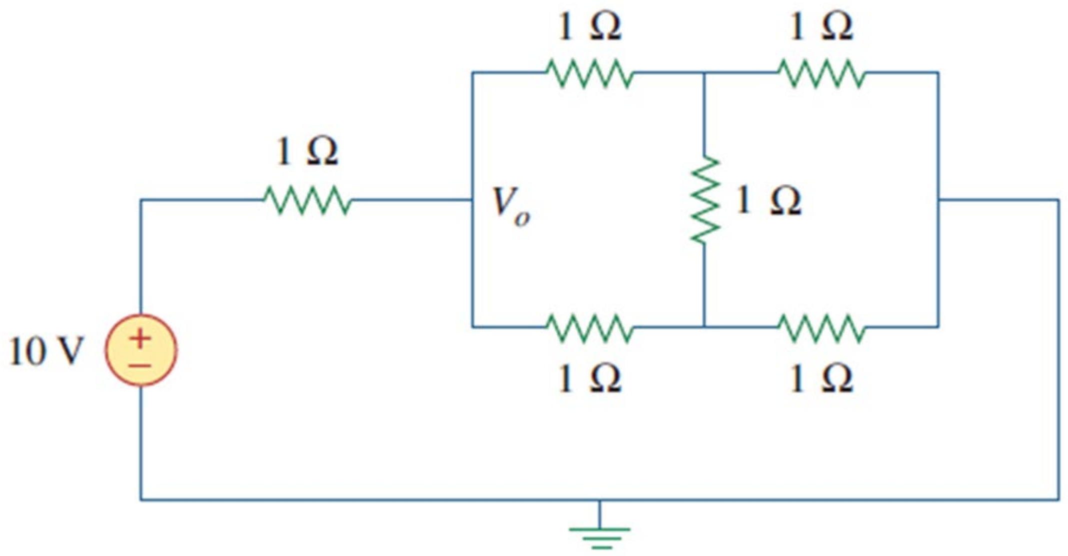
$$R_{eq} = 12.21\ \Omega$$
$$I = 1.638\ A$$

Find V_o in the two-way power divider circuit

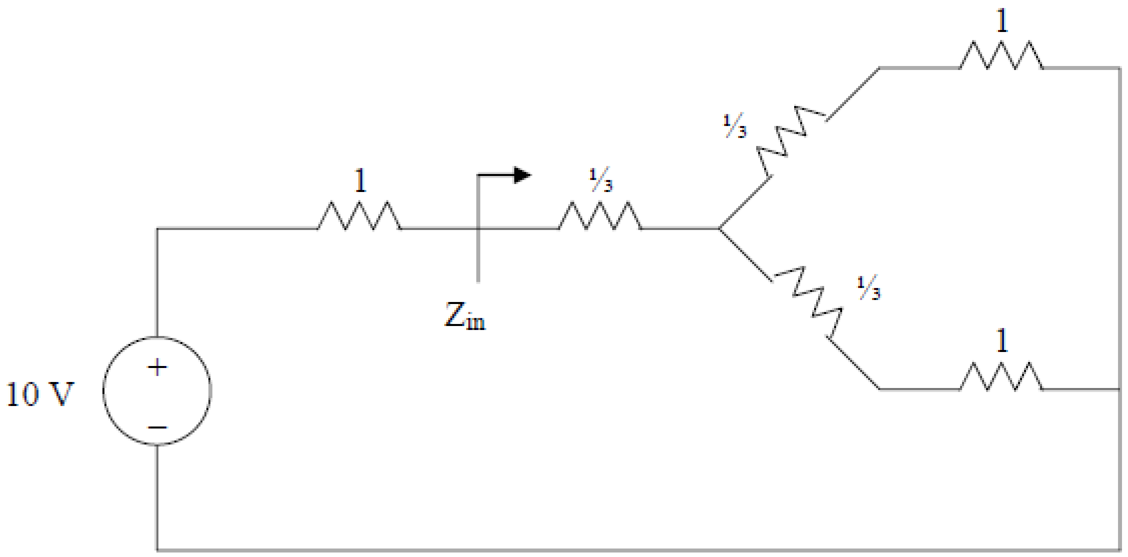


Solutions

Find V_o in the two-way power divider circuit



Converting the delta subnetwork into wye gives the circuit below.



$$Z_{in} = \frac{1}{3} + (1+\frac{1}{3})//(1+\frac{1}{3}) = \frac{1}{3} + \frac{1}{2}(\frac{4}{3}) = 1\,\Omega$$

$$V_o = \frac{Z_{in}}{1+Z_{in}}(10) = \frac{1}{1+1}(10) = \underline{5\text{ V}}$$

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Nodal Analysis

- Given a circuit with nodes without voltage sources, the nodal analysis involves taking the following four steps:
- 1. Select a node as the reference node (commonly called the ground). We flag the chosen reference node with one of the symbols as in Fig. 3.1.
- 2. Assign node voltages v_1, v_2, K, v_{n-1} to the remaining $n - 1$ nodes. A node voltage is defined as the voltage rise from the reference node to a nonreference node.

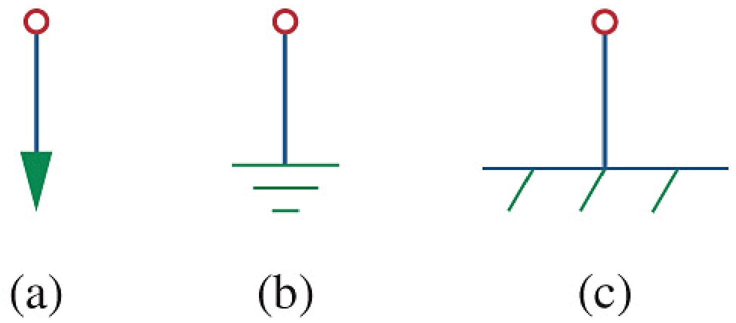


Figure 3.1 Common symbols for indicating a reference node: (a) common ground, (b) ground, (c) chassis ground.

- 3. Apply KCL to each of the nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
- 4. Solve the resulting simultaneous equations to obtain the unknown node voltages.

Nodal Analysis

Plz Write Down the Matrix in the Exam!

If a circuit with only independent current sources has N nonreference nodes, the node-voltage equations can be written as

$$\begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1N} \\ G_{21} & G_{22} & \cdots & G_{2N} \\ \vdots & \vdots & & \vdots \\ G_{N1} & G_{N2} & \cdots & G_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}$$

G_{kk} = Sum of the conductances connected to node k

$G_{kj} = G_{jk}$ = Negative of the sum of the conductances directly connecting nodes k and j, $k \neq j$

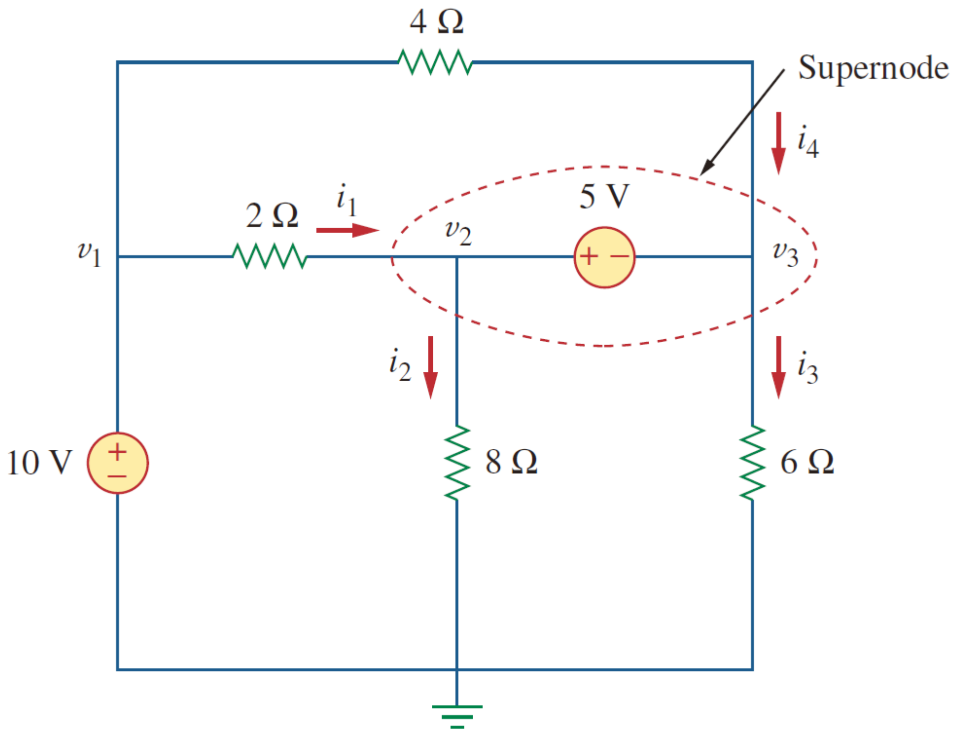
v_k = Unknown voltage at node k

i_k = Sum of all independent current sources directly connected to node k, with currents entering the node treated as positive

Nodal Analysis with the voltage sources

In some cases, there are voltage sources in the circuits. We may adopt the following two principles when we faced with such cases.
1. If a voltage source is connected between the reference node and a non-reference node, the node voltage is known.
2. If a voltage source is connected between two non-reference nodes, the two non-reference nodes form a **Supernode** .The supernode provides a constraint on the two node voltages.
3. A supernode is formed by enclosing a voltage source between two non-reference nodes and any elements connected in parallel with it. Mention that we usually adopt the KCL for the whole **whole super nodes**.
It sounds might be slightly abstract. You may consider the examples to understand the method.

Example



1. According to the principle I: we can know that the voltage of node 1 v_1 is 10 volts.
2. According to the principle II: we can know that the voltage difference between node 2 and node 3: $v_2 - v_3$ is 5 volts.
3. Adopt KCL to the supernode formed by node2 and node3(marked by the red circle), we can have the following equation:

$$\frac{v_2 - v_1}{2} + \frac{v_2 - 0}{8} + \frac{v_3 - v_1}{4} + \frac{v_3 - 0}{6} = 0 \tag{1}$$

Mesh Analysis

- Steps to Determine Mesh Currents for circuit with n Meshes without Current Sources :
- 1. Assign mesh currents i_1, i_2,…, i_n to the n meshes. (**ATTENTION: the mesh current for all meshes are always clockwise!**)
- 2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
- 3. Solve the resulting n simultaneous equations to get the mesh currents.

Mesh Analysis

Plz Write Down the Matrix in the Exam!

If a circuit with only independent voltage sources has N meshes, the node-current equations can be written as

$$\begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1N} \\ R_{21} & R_{22} & \cdots & R_{2N} \\ \vdots & \vdots & & \vdots \\ R_{N1} & R_{N2} & \cdots & R_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}$$

R_{kk} = Sum of the resistances in mesh k

$R_{kj} = R_{jk}$ = Negative of the sum of the resistances in common with meshes k and j, $k \neq j$

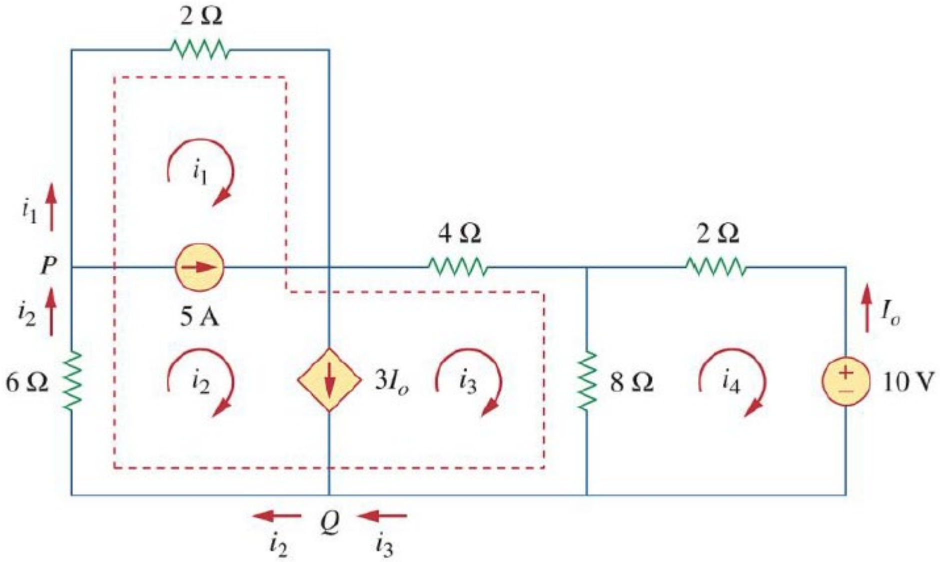
i_k = Unknown mesh current for mesh k in the clockwise direction

v_k = Sum taken clockwise of all independent voltage sources in mesh k, with voltage rise treated as positive

Mesh Analysis with Current Sources

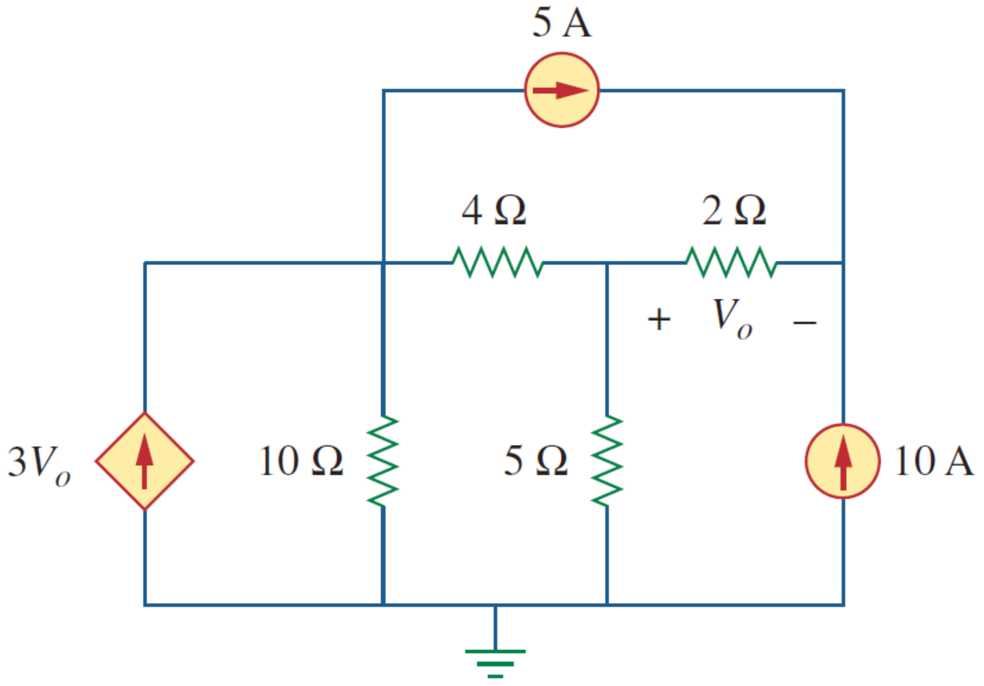
Similar to the Nodal Analysis with Voltage sourses, we can also generate three principles in general:

1. When a current source is only in one mesh, the mesh current of that mesh is just the value of the current source.
2. When a current source locate in between two meshes, then the two meshes forms a **supermesh**. We can get the difference of two meshes currents.
3. We usually apply the KVL to the whole supermesh. The total voltage of a supermesh is zero.(Metion that more than two meshes can also form a supermesh. We will discuss that case in the following example.)

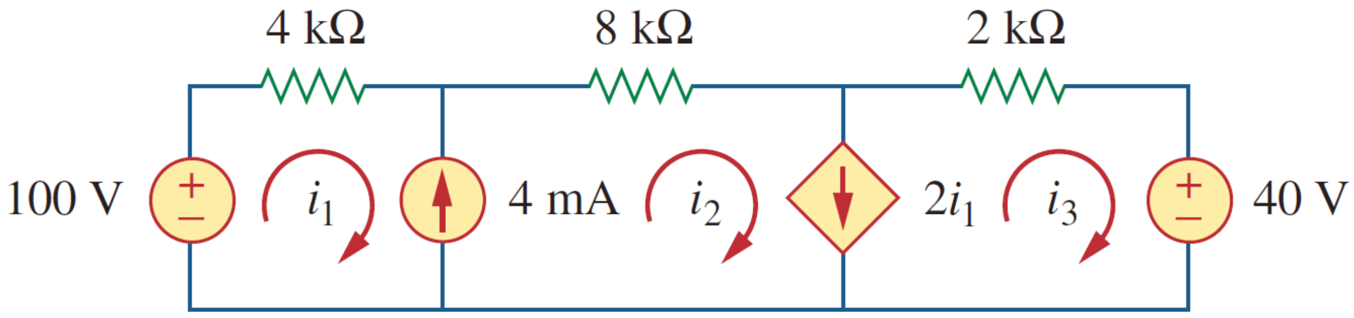


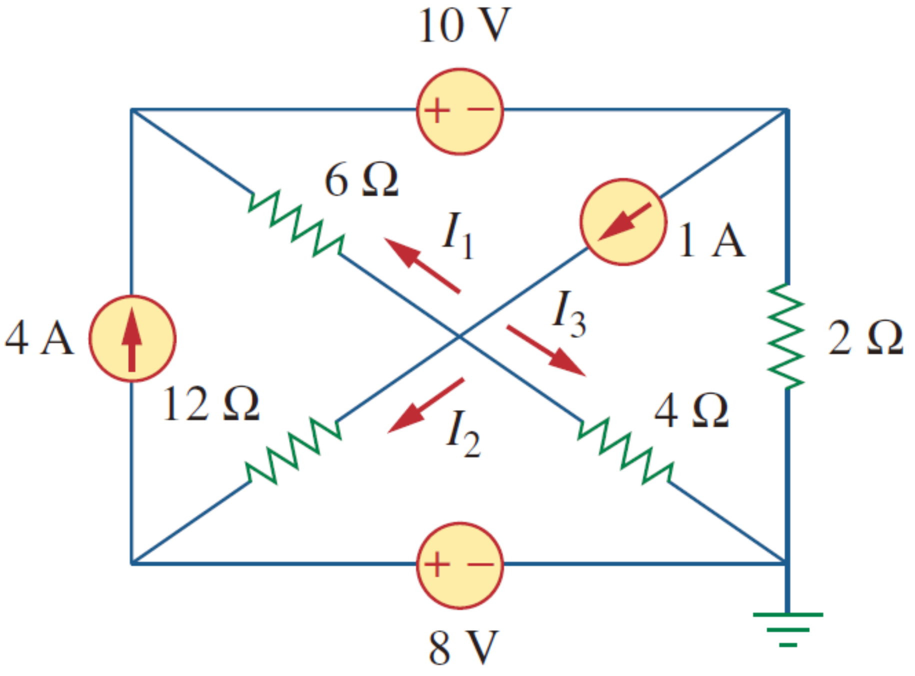
Sample Questions

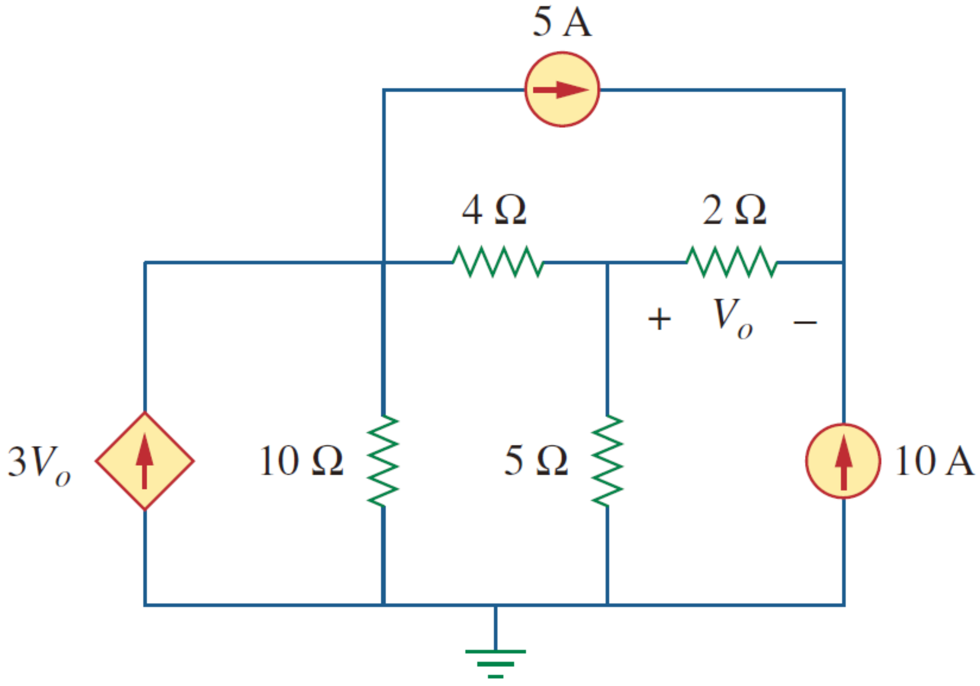
Obtain the node-voltage equations for the circuit

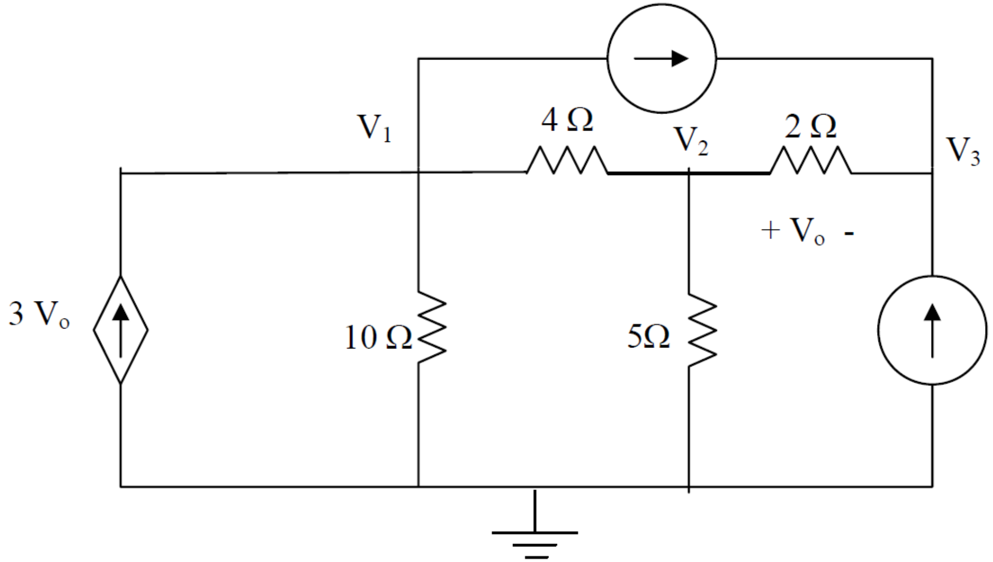


Find the mesh currents i_1, i_2, and i_3 in the network







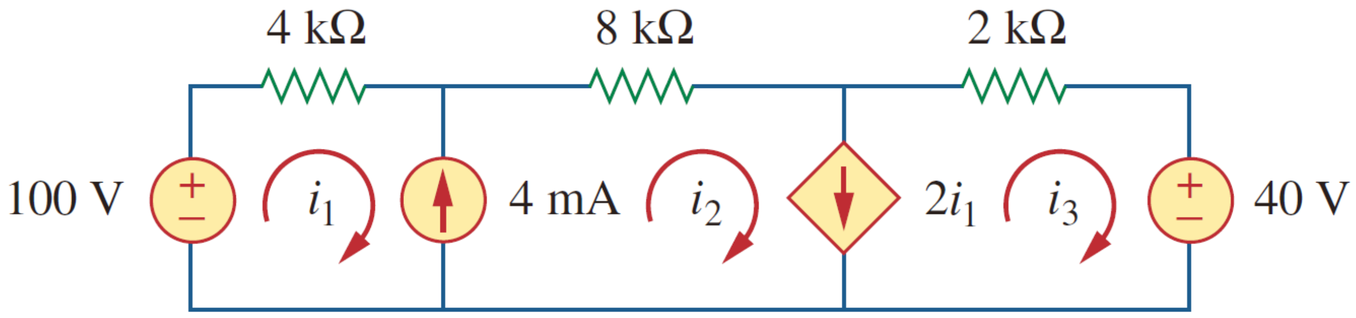


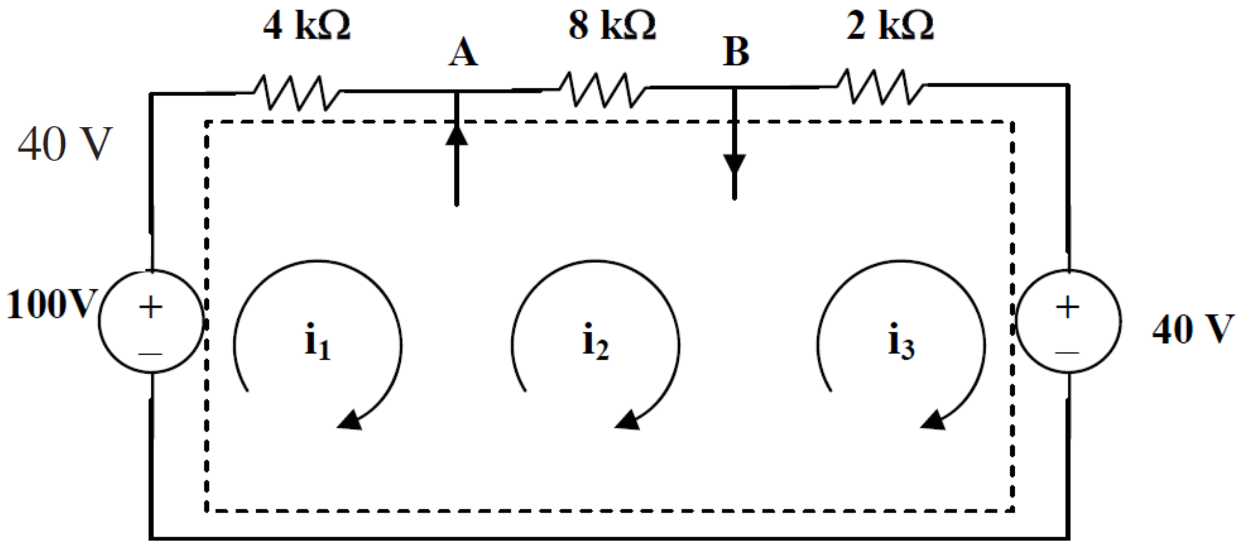
$$\begin{bmatrix} 0.35 & -0.25 & 0 \\ -0.25 & 0.95 & -0.5 \\ 0 & -0.5 & 0.5 \end{bmatrix} V = \begin{bmatrix} 3V_0 - 5 \\ 0 \\ 15 \end{bmatrix}$$

Since we actually have four unknowns and only three equations, we need a constraint equation.

$$V_o = V_2 - V_3$$

$$\begin{bmatrix} 0.35 & -3.25 & 3 \\ -0.25 & 0.95 & -0.5 \\ 0 & -0.5 & 0.5 \end{bmatrix} V = \begin{bmatrix} -5 \\ 0 \\ 15 \end{bmatrix}$$





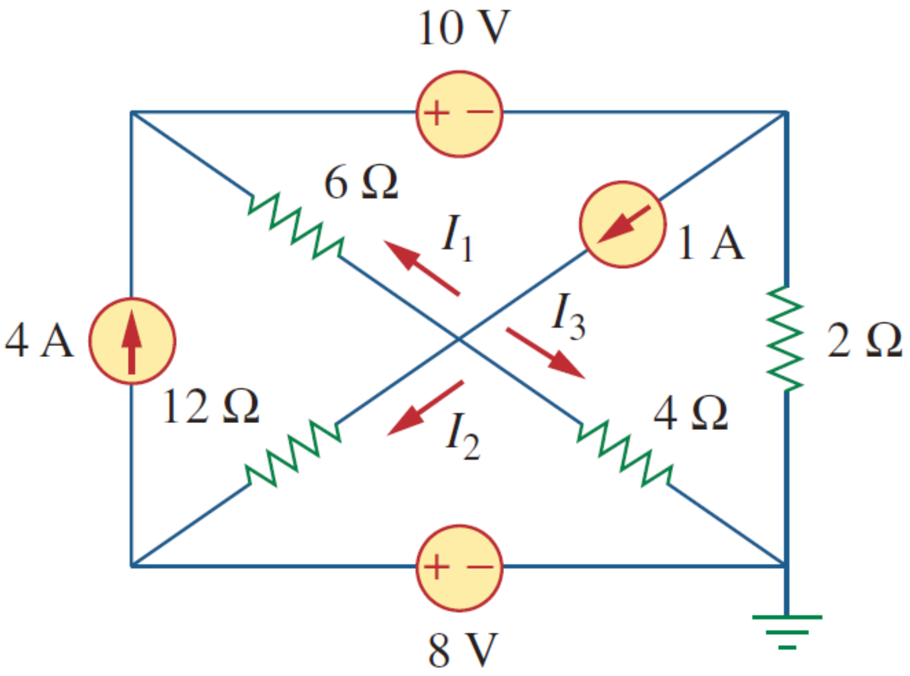
We have a supermesh. Let all R be in kΩ, i in mA, and v in volts.

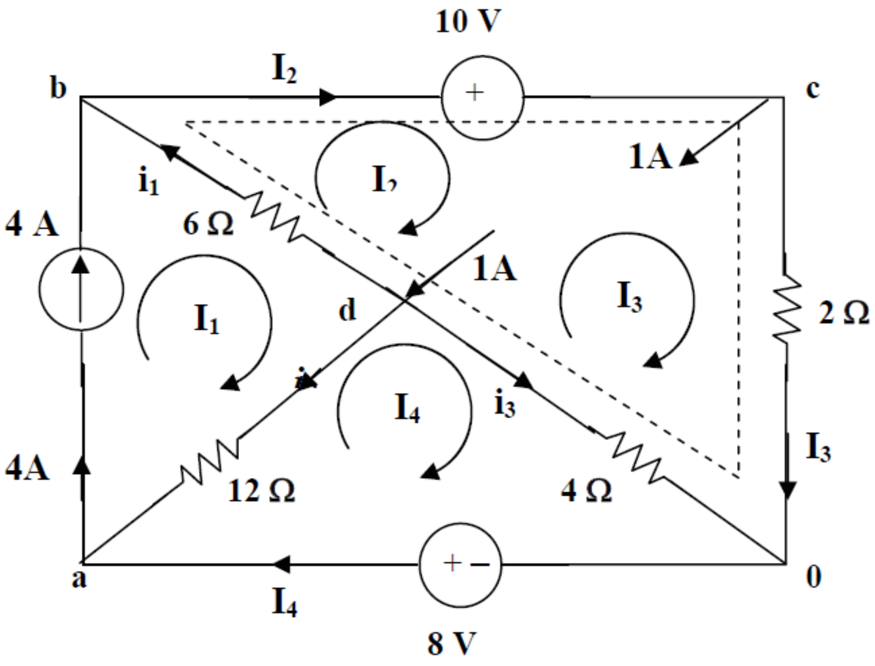
For the supermesh, $-100 + 4i_1 + 8i_2 + 2i_3 + 40 = 0$ or $30 = 2i_1 + 4i_2 + i_3$ (1)

At node A, $i_1 + 4 = i_2$ (2)

At node B, $i_2 = 2i_1 + i_3$ (3)

Solving (1), (2), and (3), we get i_1 = **2 mA**, i_2 = **6 mA**, and i_3 = **2 mA**.





It is evident that $I_1 = 4$ (1)

For mesh 4, $12(I_4 - I_1) + 4(I_4 - I_3) - 8 = 0$ (2)

For the supermesh $6(I_2 - I_1) + 10 + 2I_3 + 4(I_3 - I_4) = 0$

or $-3I_1 + 3I_2 + 3I_3 - 2I_4 = -5$ (3)

At node c, $I_2 = I_3 + 1$ (4)

Solving (1), (2), (3), and (4) yields, $I_1 = 4A$, $I_2 = 3A$, $I_3 = 2A$, and $I_4 = 4A$

At node b, $i_1 = I_2 - I_1 = \underline{\mathbf{-1A}}$

At node a, $i_2 = 4 - I_4 = \underline{\mathbf{0A}}$

At node 0, $i_3 = I_4 - I_3 = \underline{\mathbf{2A}}$

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Basic Definition

- Linearity:
- A linear circuit is one whose **output** (also called response) is linearly related to its **input** (also called excitation).
- The linearity property is a combination of both the **homogeneity** (scaling) property and the **additivity** property.

- Superposition:
- Superposition principle is based on additivity. It states that whenever a linear system is excited, or driven, by more than one independent source, **the total response is the sum of the individual responses**.

Superposition Principle Steps

- To apply the superposition principle, we must keep two things in mind:
- – We consider **only one independent source** at a time while all other independent sources are turned off. This implies that we replace every **voltage source by 0 V** (or a short circuit), and every **current source by 0 A** (or an open circuit).
- – Dependent sources are left intact because they are controlled by circuit variables.
- Add all the outputs up.

4.12 Determine v_o in the circuit of Fig. 4.80 using the superposition principle.

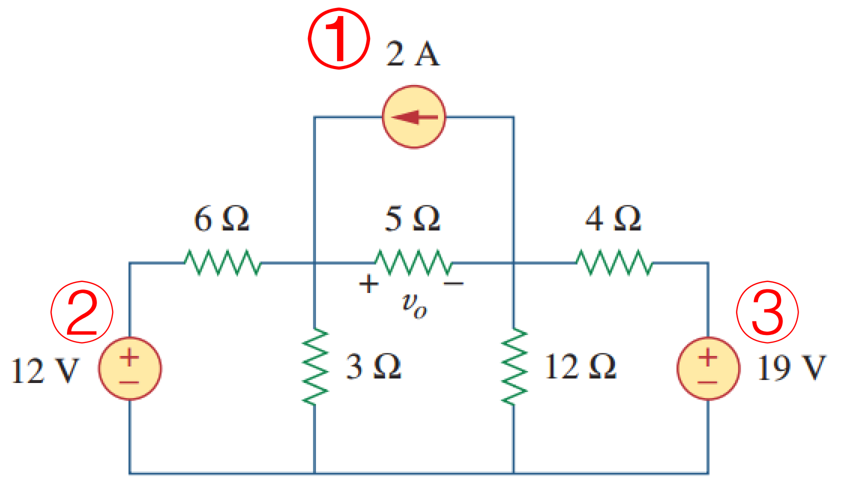
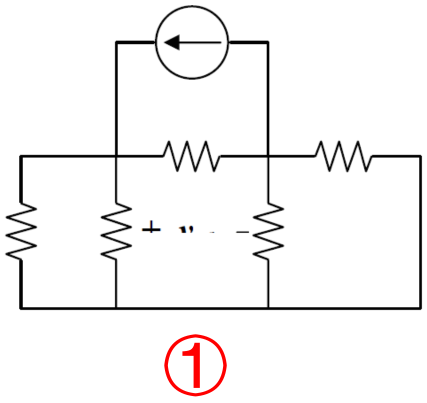
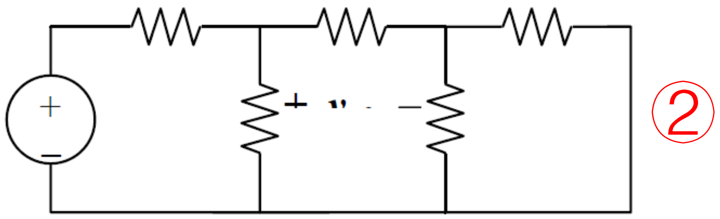
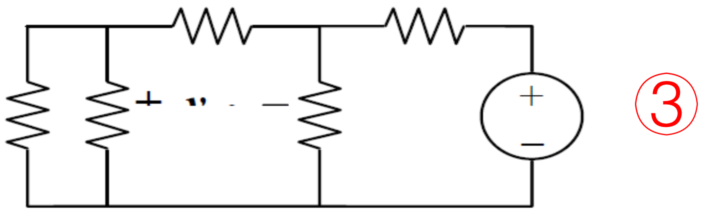


Figure 4.80 For Prob. 4.12.

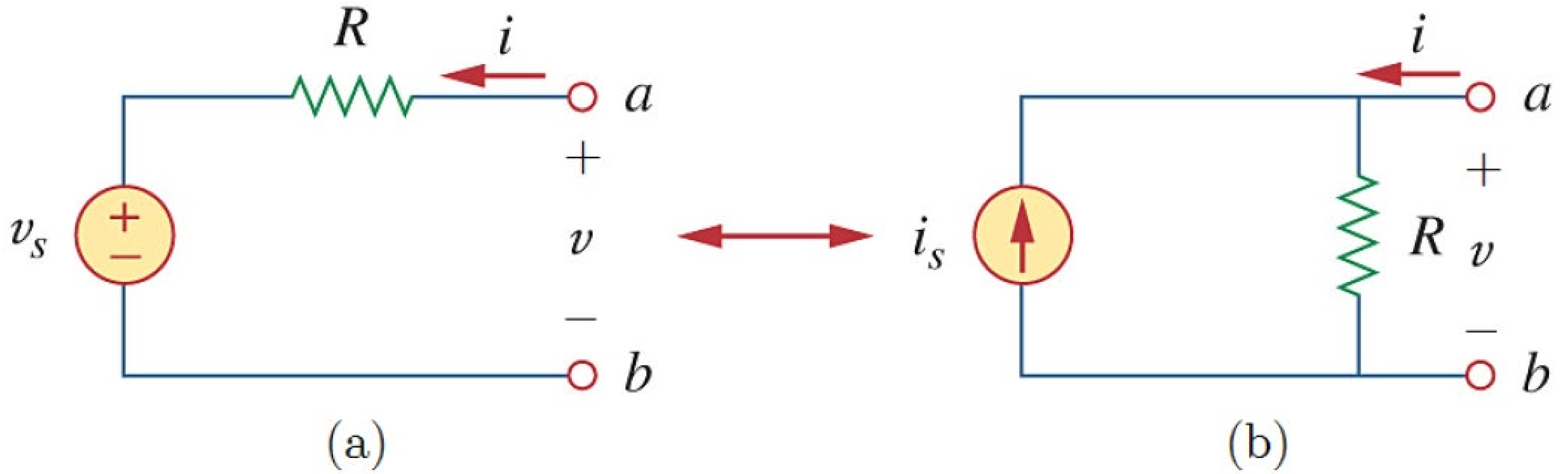






Plz Analyze the Sources separately!

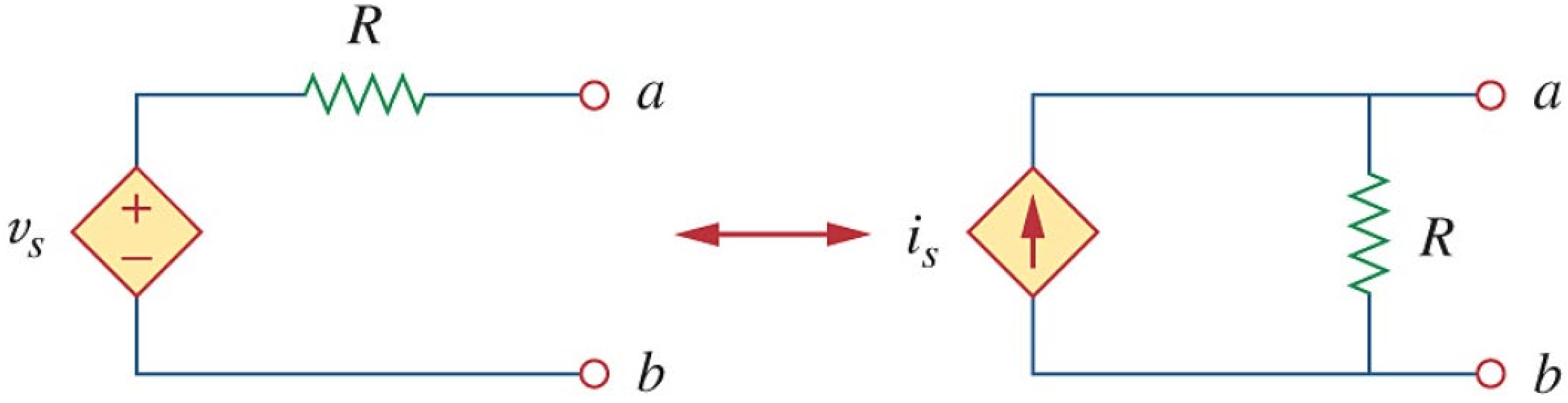
Source Transformation



$$v_s = i_s R \quad \text{or} \quad i_s = \frac{v_s}{R}$$

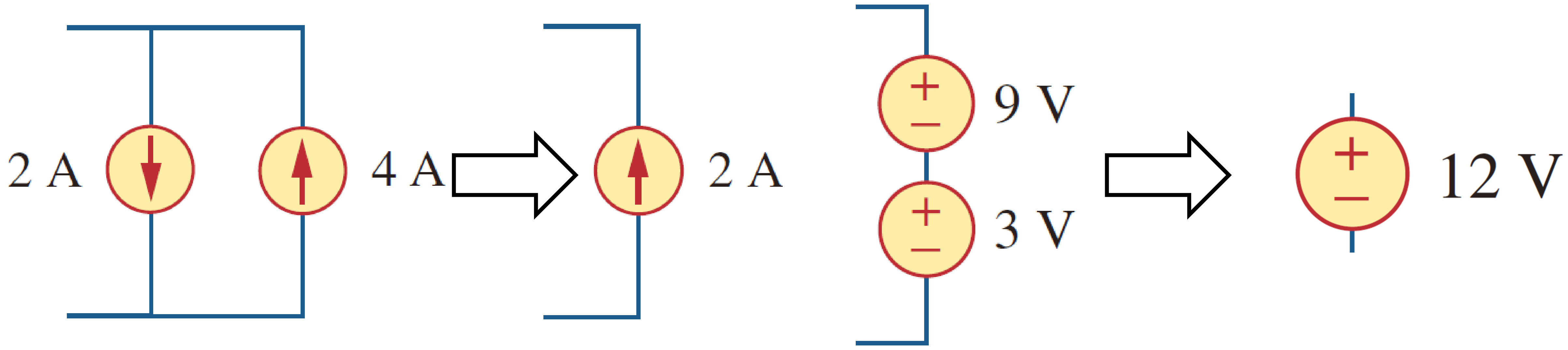
Replacing a voltage source in series with a resistor by a current source in parallel with a resistor, or vice versa.

Source Transformation (dependent sources)



Source transformation also applies to dependent sources, provided we carefully handle the dependent variable.

Source Transformation (special)



Source transformation also applies to dependent sources, provided we carefully handle the dependent variable.

Thevenin’s Theorem & Norton’s Theorem

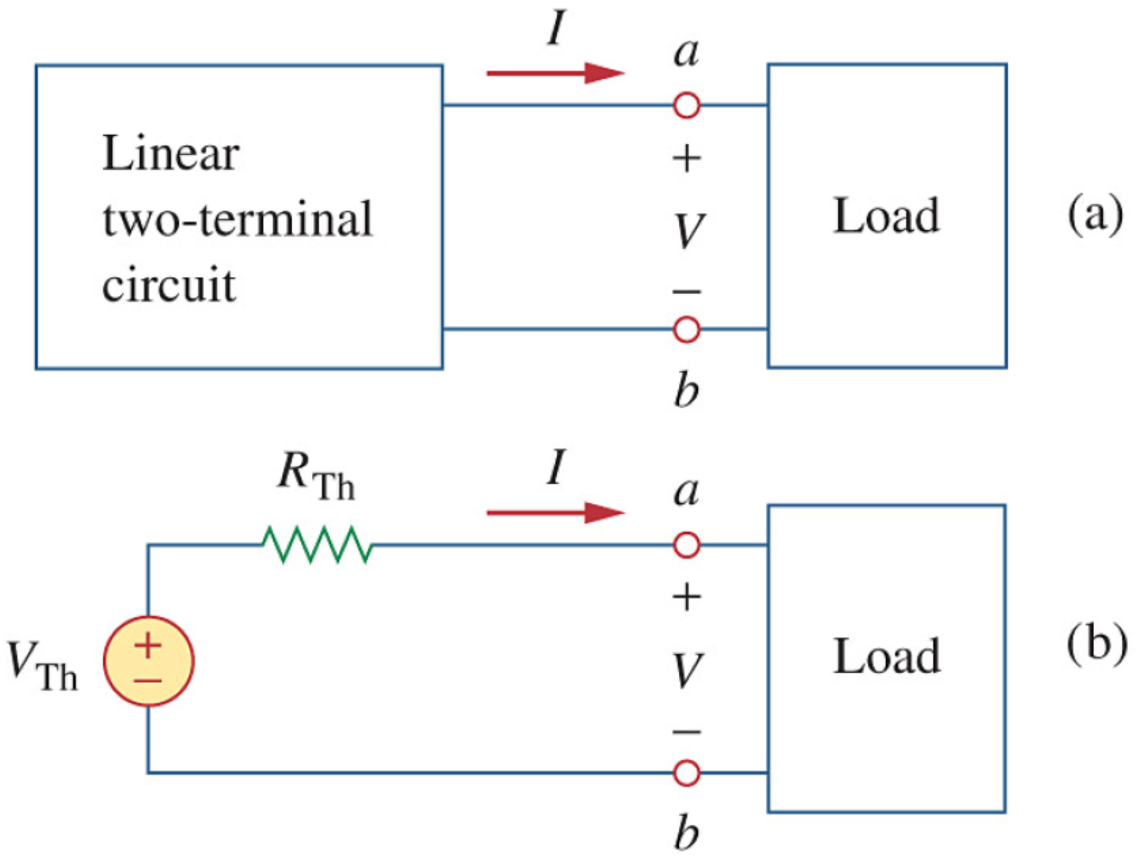


Figure 4.23 Replacing a linear two-terminal circuit by its Thevenin equivalent.

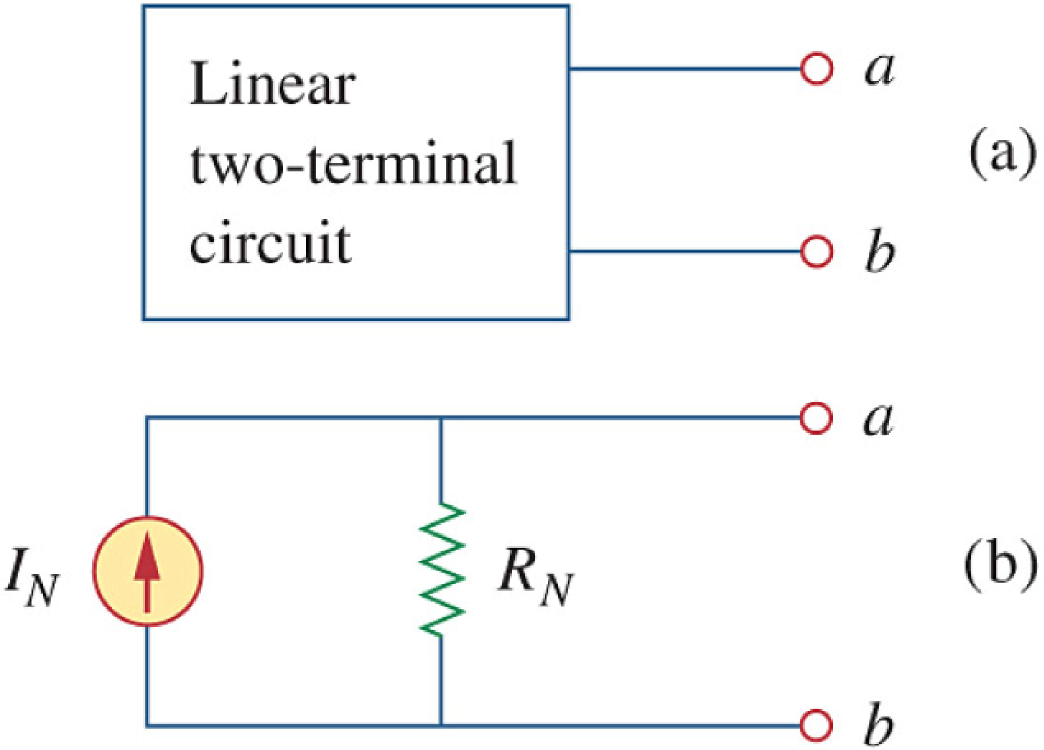
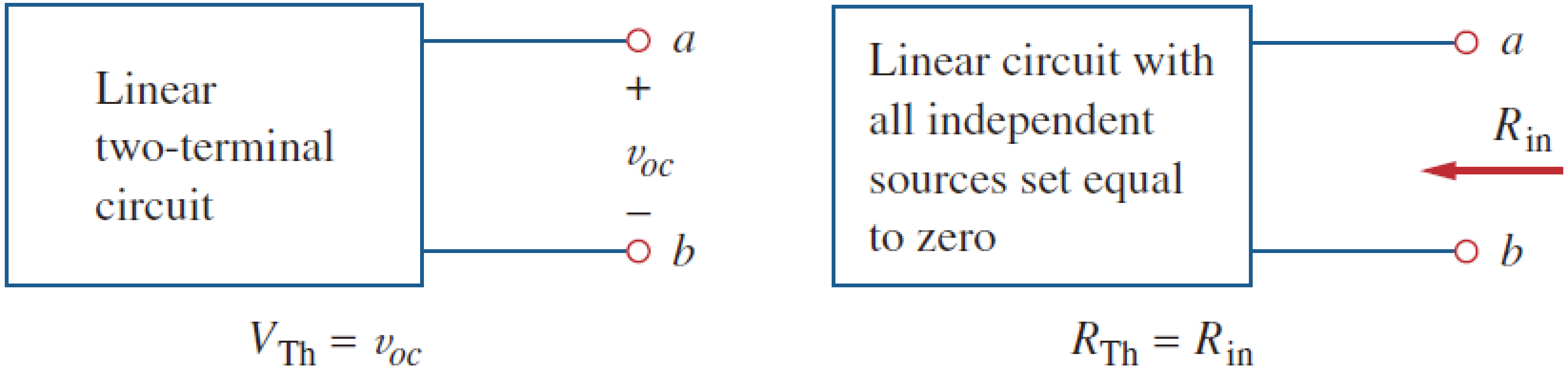


Figure 4.37 (a) Original circuit, (b) Norton equivalent circuit.

Thevenin's Theorem

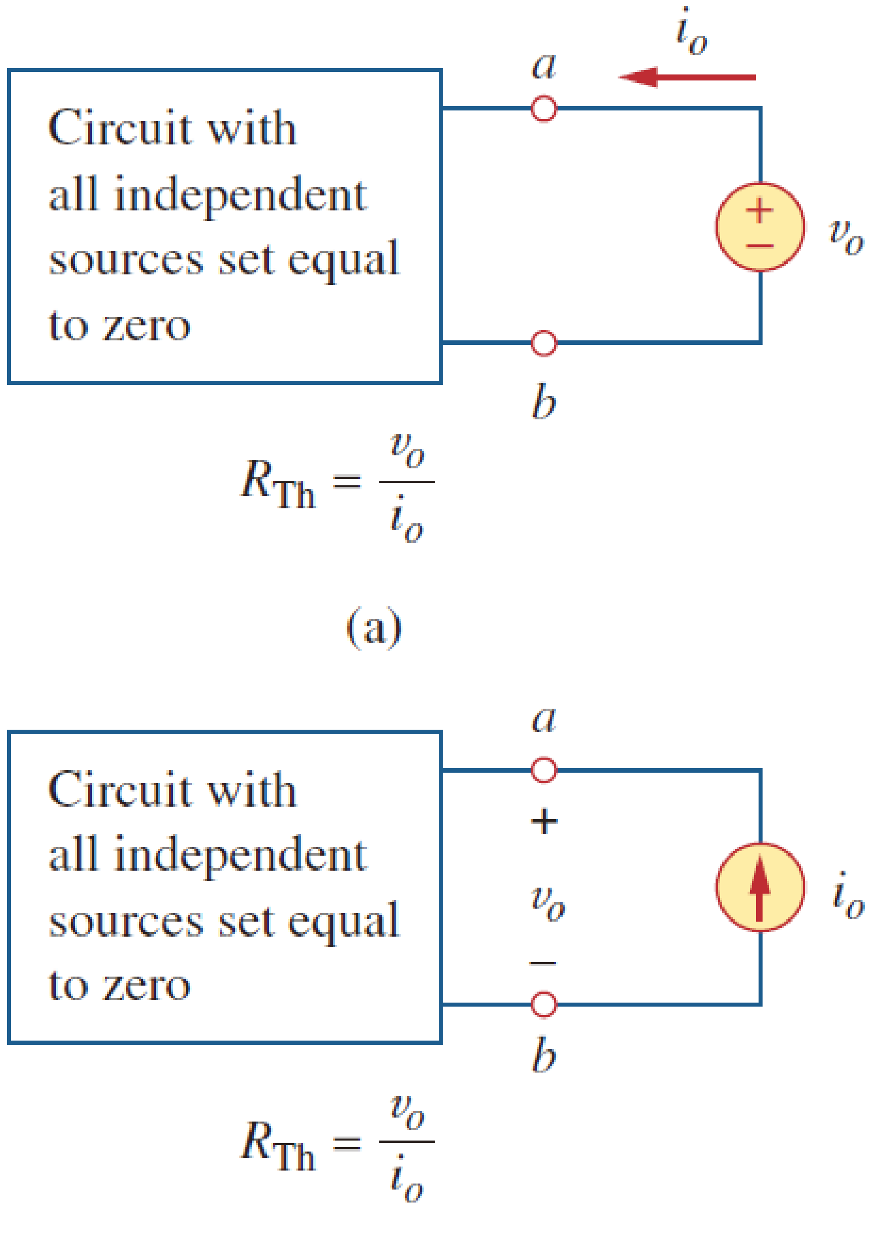
- To apply this idea in finding the Thevenin resistance R_{Th}, we need to consider two cases.
- **CASE 1** If the network has no dependent sources, we turn off all independent sources. R_{Th} is the input resistance of the network looking between terminals a and b.



(a) (b)

V_{Th} is the open-circuit voltage across the terminals

- **CASE 2** If the network has dependent sources, we turn off all independent sources. As with superposition, dependent sources are not to be turned off because they are controlled by circuit variables. We apply a voltage source at terminals a and b and determine the resulting current . Then $R_{Th} = {}^{v_0}/_{i_0}$. Alternatively, we may insert a current source i_0 at terminals $a - b$ and find the terminal voltage . Again $R_{Th} = {}^{v_0}/_{i_0}$. Either of the two approaches will give the same result. In either approach we may assume any value of v_0 and i_0.

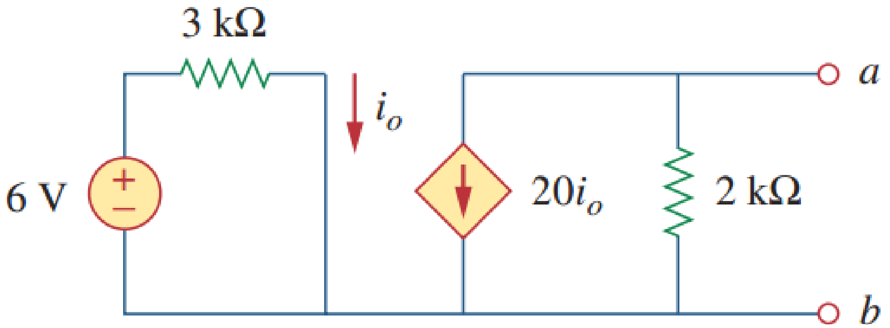


(a)

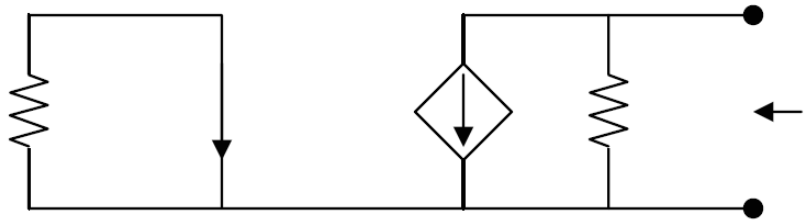
(b)

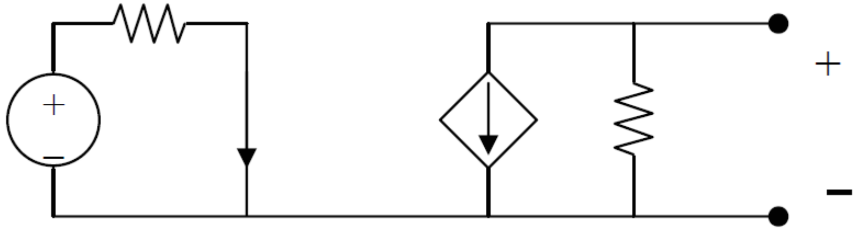
Homework 4 – Prob 4.52

4.52 For the transistor model in Fig. 4.118, obtain the Thevenin equivalent at terminals *a*-*b*.



For R_{Th}, consider the circuit in Fig. (a).





For Fig. (a), $I_o = 0$, hence the current source is inactive and

$$R_{Th} = \underline{\mathbf{2\ k\ ohms}}$$

For V_{Th}, consider the circuit in Fig. (b).

$$I_o = 6/3k = 2\ mA$$

$$V_{Th} = (-20I_o)(2k) = -20x2x10^{-3}x2x10^3 = \underline{\mathbf{-80\ V}}$$

Norton's Theorem

$$R_N = R_{\text{Th}}$$

I_N is the short-circuit current.

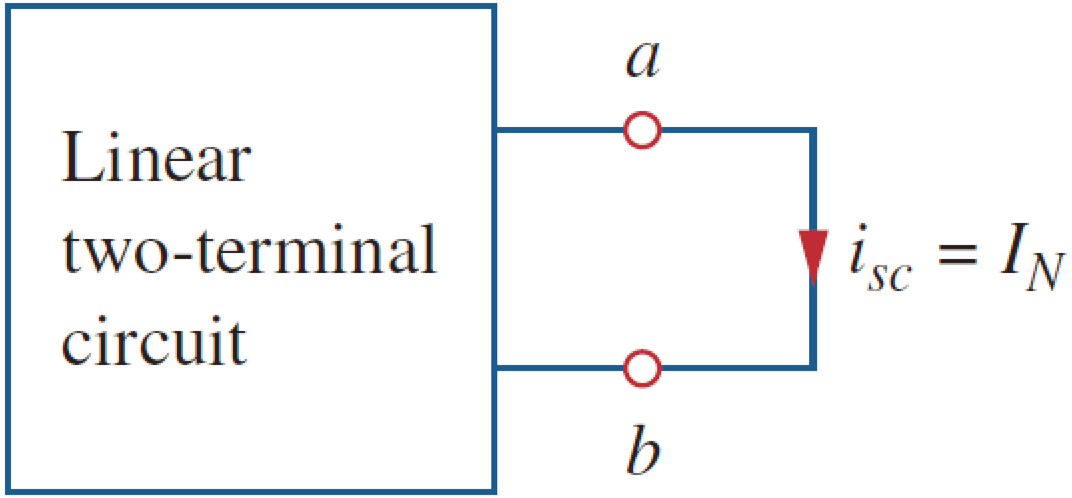
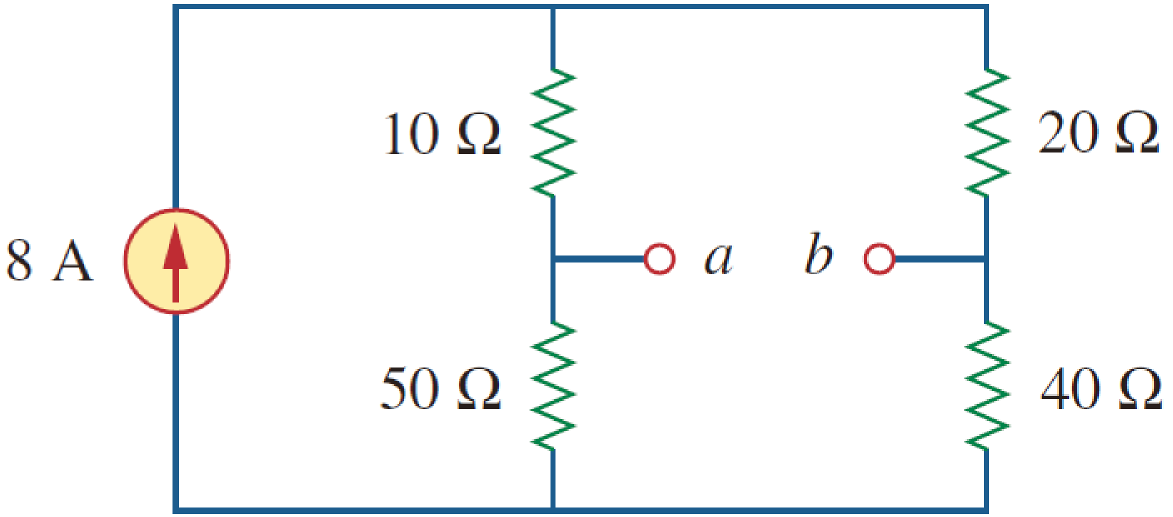


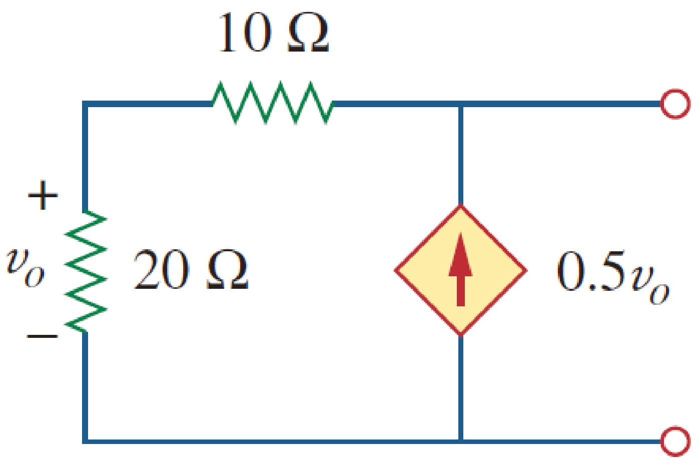
Figure 4.38
Finding Norton current I_N.

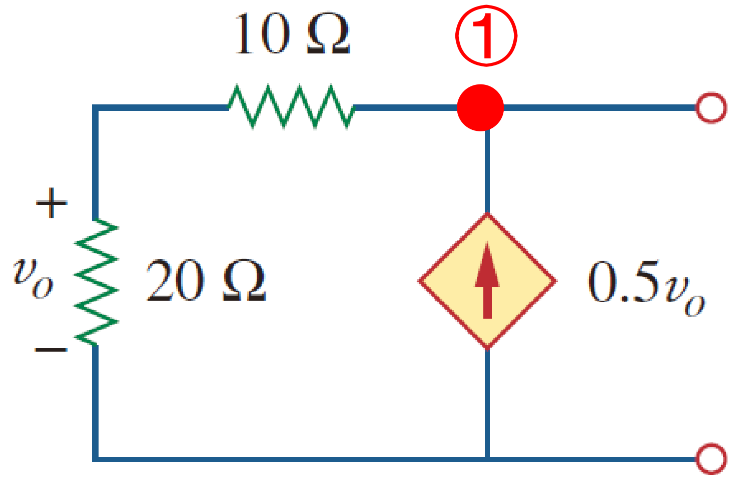
Sample Questions

Determine the Thevenin and Norton equivalents at terminals *a*-*b* of the circuit



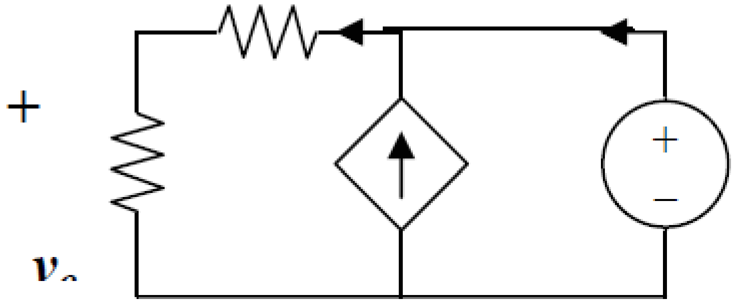
Find the Norton equivalent for the circuit





Because there are no independent sources, $I_N = I_{sc} =$ **0 A**

R_N can be found using the circuit below.

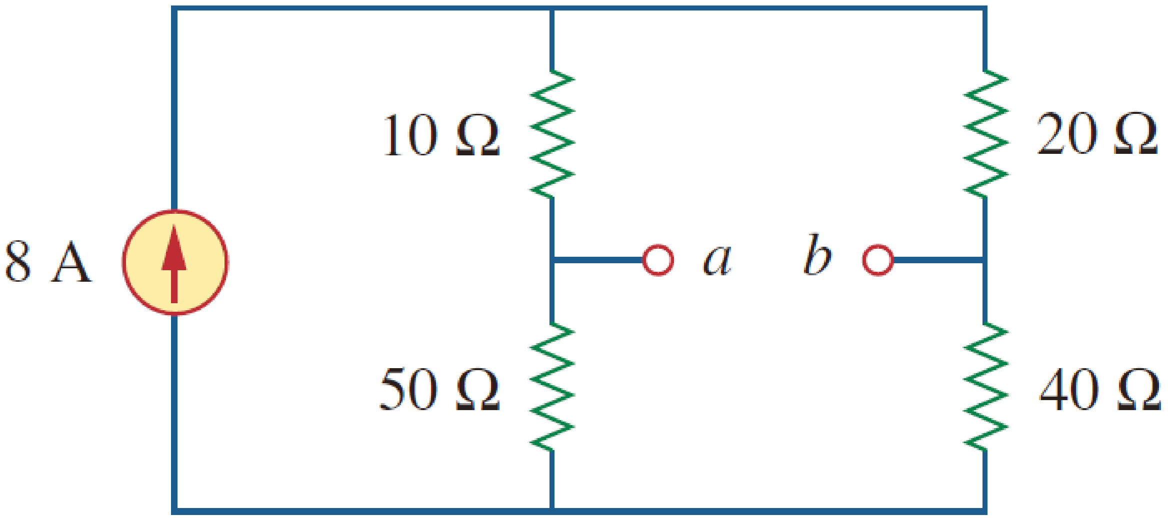


Applying KCL at node 1, $v_1 = 1$, and $v_o = (20/30)v_1 = 2/3$

$i_o = (v_1/30) - 0.5v_o = (1/30) - 0.5x2/3 = 0.03333 - 0.33333 = -0.3$ A.

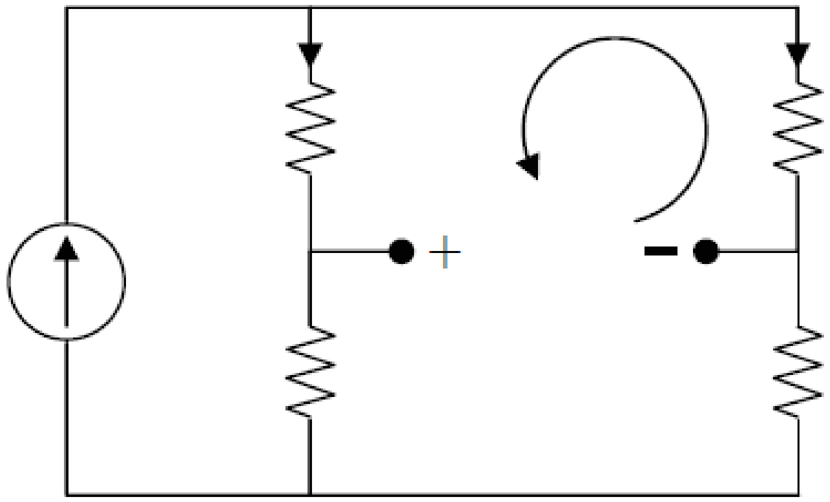
Hence,

$R_N = 1/(-0.3) =$ **–3.333 ohms**



$R_{Th} = (10 + 20)||(50 + 40) \;\; 30||90 = \underline{\mathbf{22.5\ ohms}}$

To find V_{Th}, consider the circuit below.

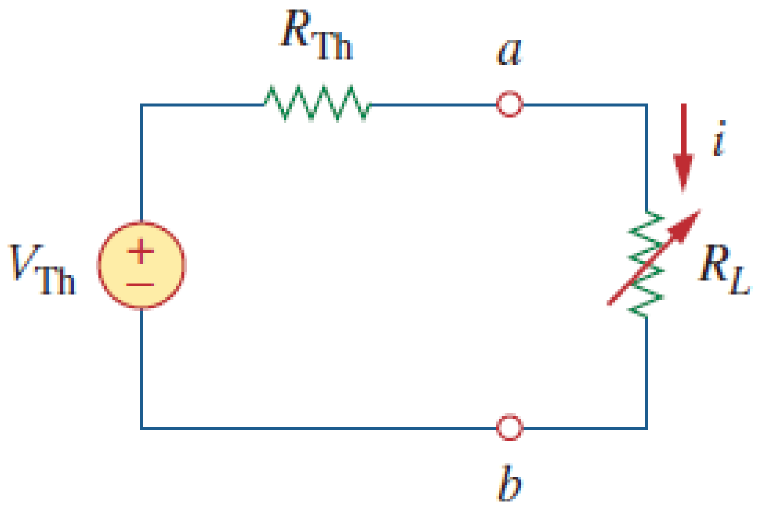


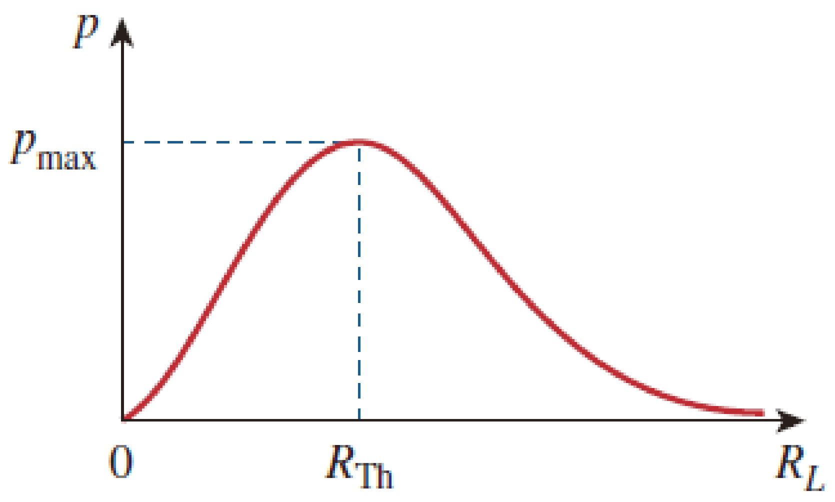
$i_1 = i_2 = 8/2 = 4, \;\; 10i_1 + V_{Th} - 20i_2 = 0, \;\text{or}\; V_{Th} = 20i_2 - 10i_1 = 10i_1 = 10x4$

$V_{Th} = \underline{\mathbf{40V}}$, and $I_N = V_{Th}/R_{Th} = 40/22.5 = \underline{\mathbf{1.7778\ A}}$

Maximum Power Transfer

- **Maximum power** is transferred to the load when the **load resistance equals the Thevenin resistance** as seen from the load ($R_L = R_{Th}$).





$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

the power delivered to the load

$$p_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

Content for Mid 1

1. Chapter 1: Basic Concepts
2. Chapter 2: Basic Laws
3. Chapter 3: Methods of Analysis
4. Chapter 4: Circuit Theorems
5. Chapter 5: Operational Amplifiers

Basic Concept

- The op amp is an electronic unit that behaves like a voltage-controlled voltage source.

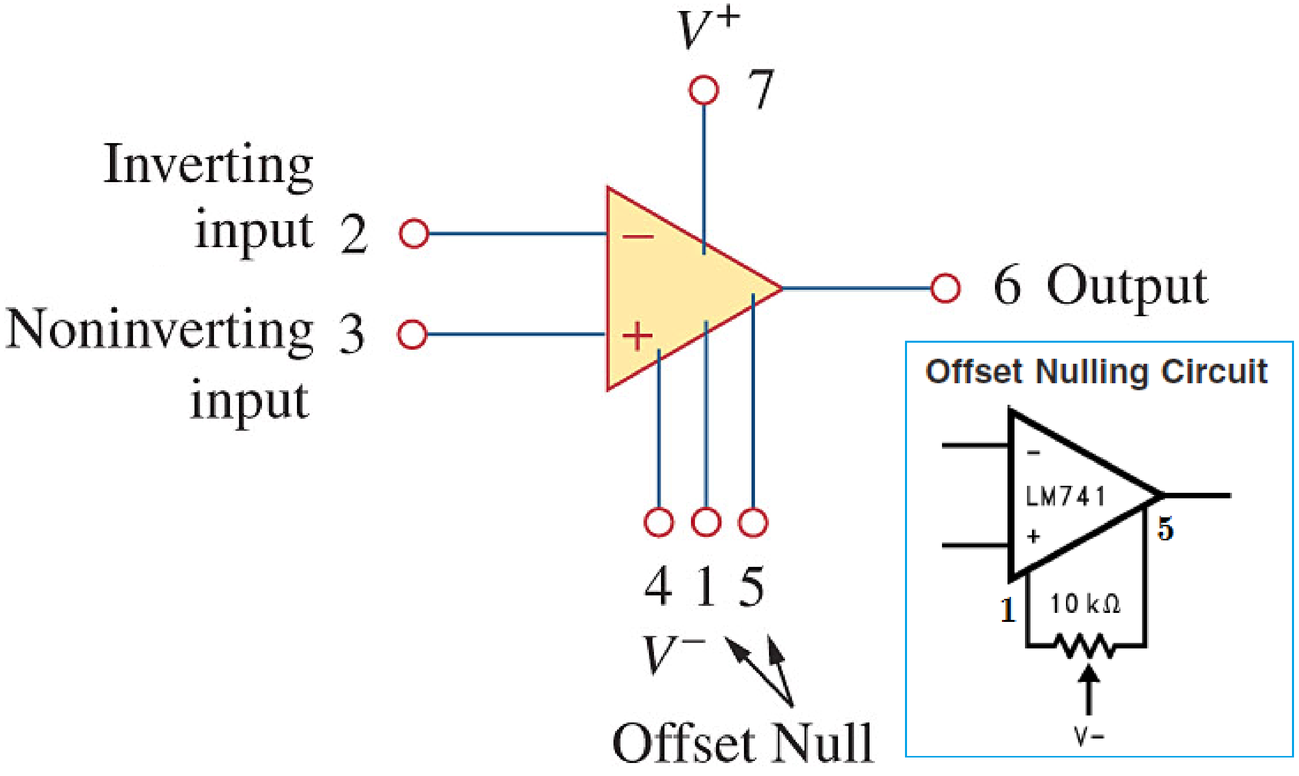


Figure 5.2(b) A typical op amp: circuit symbol.

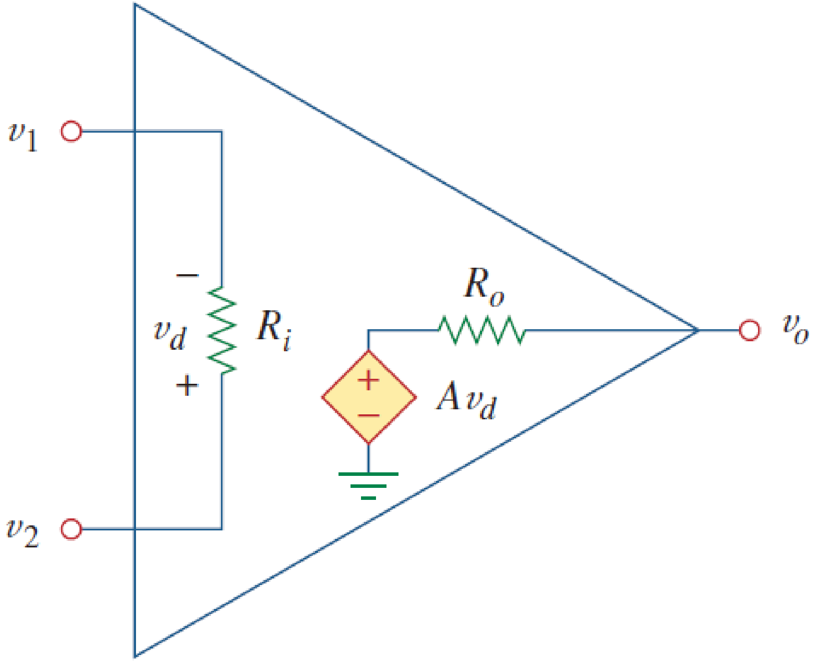


Figure 5.4
The equivalent circuit of the nonideal op amp.

$$\cancel{v_o = Av_d = A(v_2 - v_1)}$$

A is called the *open-loop voltage gain*

$$A = \frac{output}{input}, i.e. A_v = \frac{v_{out}}{v_{in}}, A_i = \frac{i_{out}}{i_{in}}$$

Basic Concept

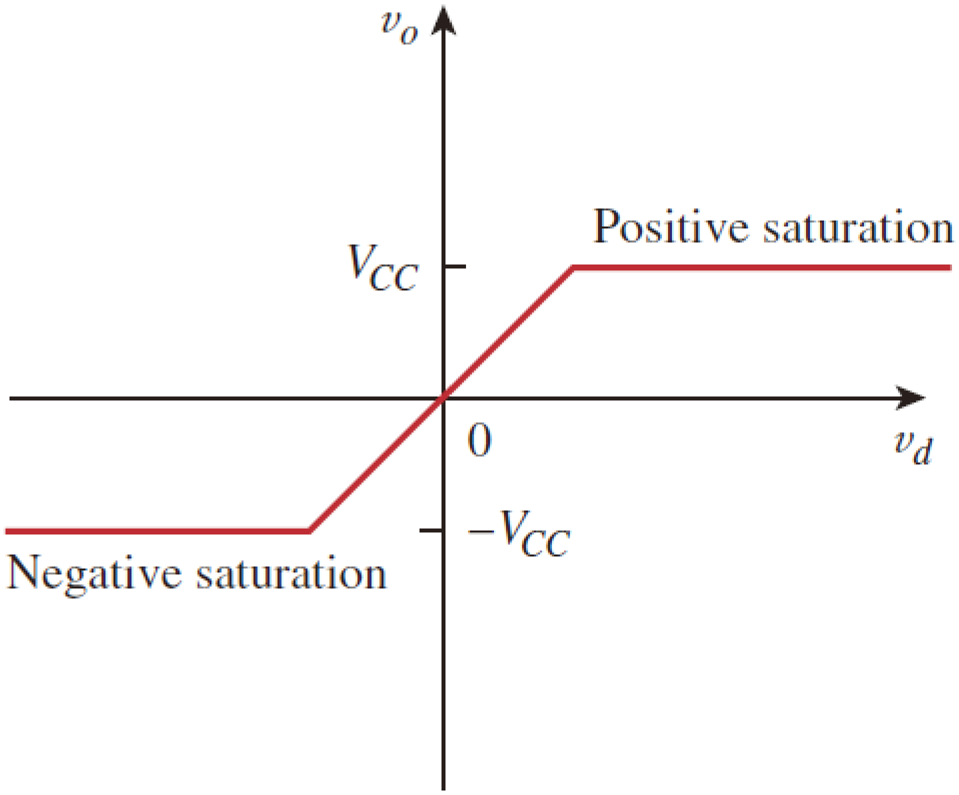


Figure 5.5
Op amp output voltage v_o as a function of the differential input voltage v_d.

A practical limitation of the op amp is that the magnitude of its output voltage cannot exceed $|V_{CC}|$. In other words, the output voltage is dependent on and is limited by the power supply voltage. Figure 5.5 illustrates that the op amp can operate in three modes, depending on the differential input voltage v_d:

1. Positive saturation, $v_o = V_{CC}$.
2. Linear region, $-V_{CC} \leq v_o = Av_d \leq V_{CC}$.
3. Negative saturation, $v_o = -V_{CC}$.

If we attempt to increase v_d beyond the linear range, the op amp becomes saturated and yields $v_o = V_{CC}$ or $v_o = -V_{CC}$. Throughout this book, we will assume that our op amps operate in the linear mode. This means that the output voltage is restricted by

$$-V_{CC} \leq v_o \leq V_{CC} \tag{5.4}$$

Basic Concept

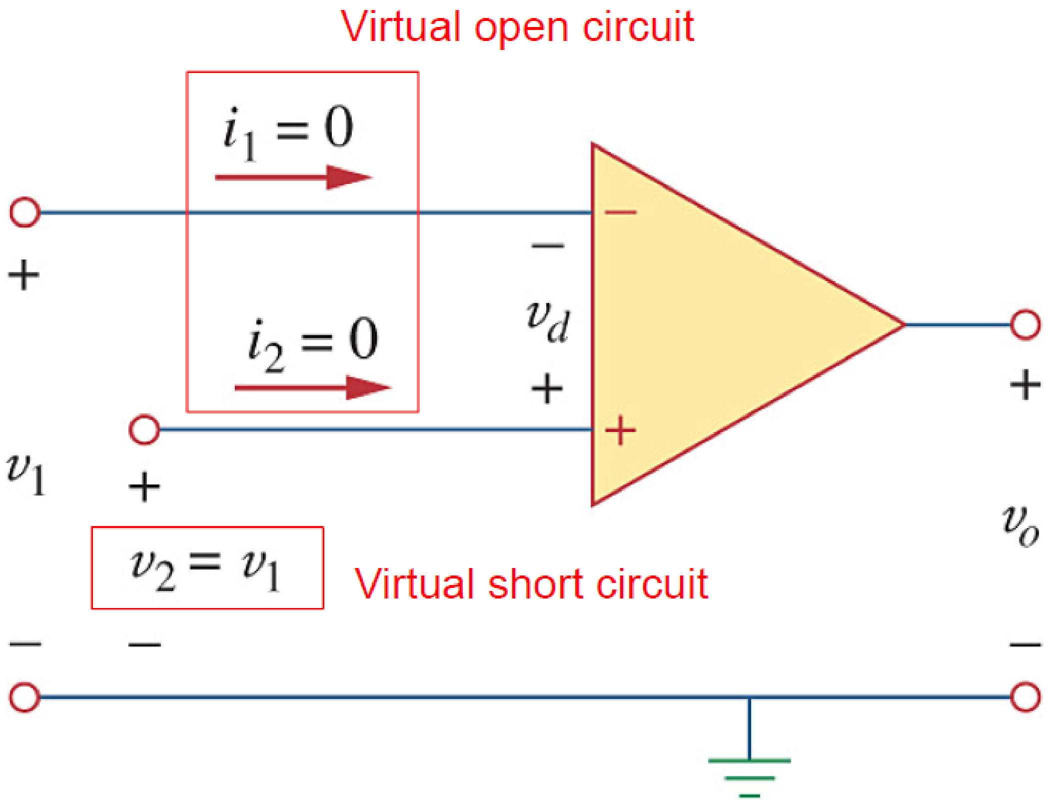
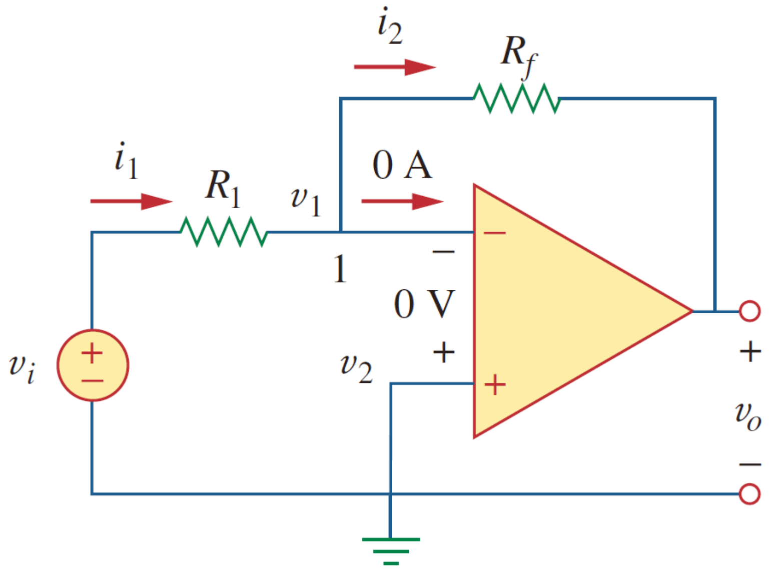


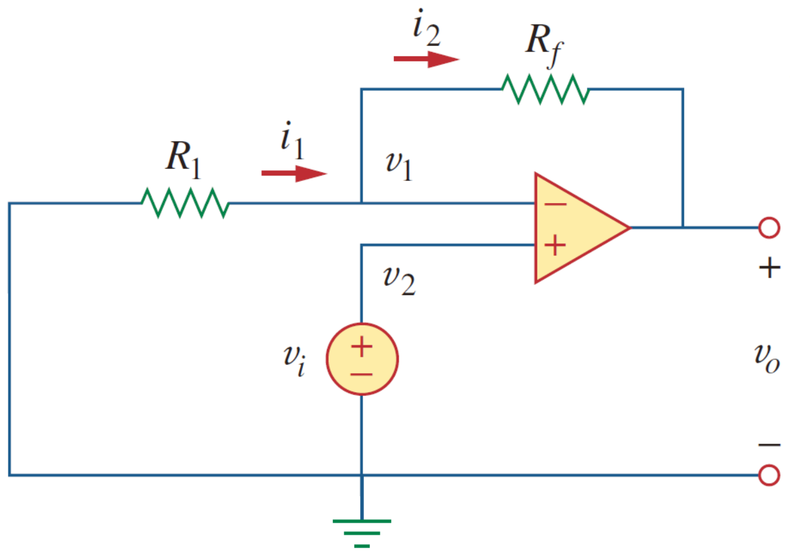
Figure 5.8
Ideal op amp model.

Inverting Amplifier



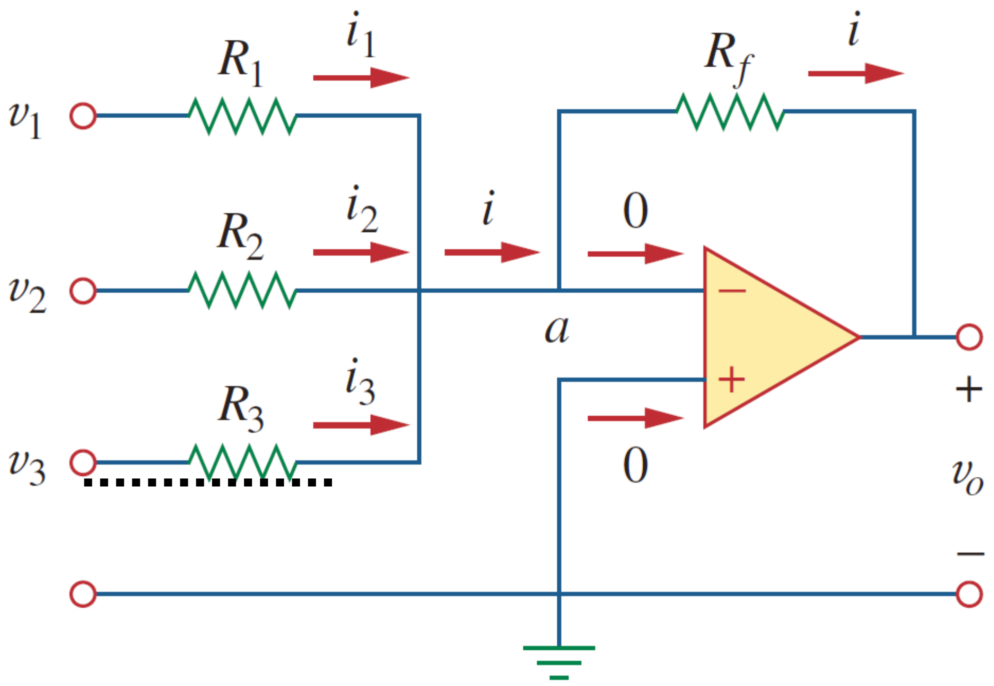
$$v_o = -\frac{R_f}{R_1} v_i$$

Noninverting Amplifier



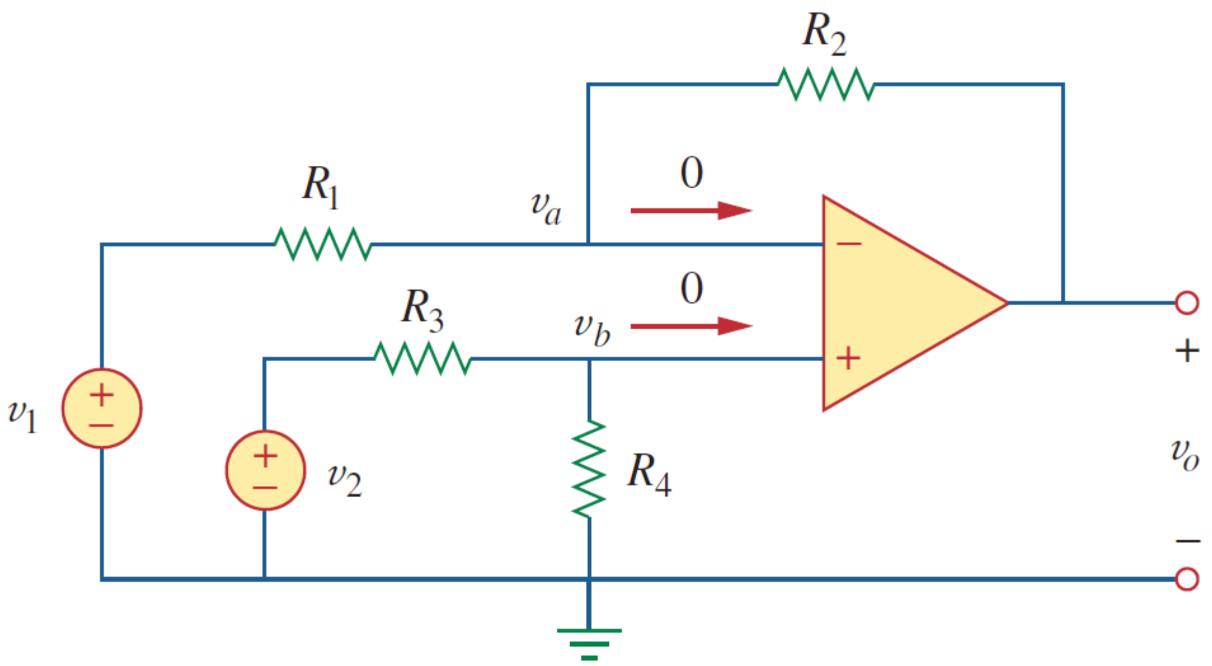
$$v_o = \left(1 + \frac{R_f}{R_1}\right) v_i$$

Summing Amplifier



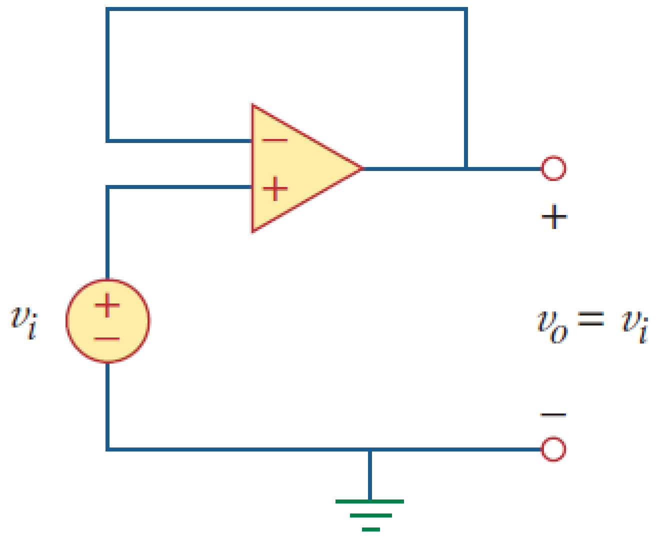
$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)$$

Difference Amplifier

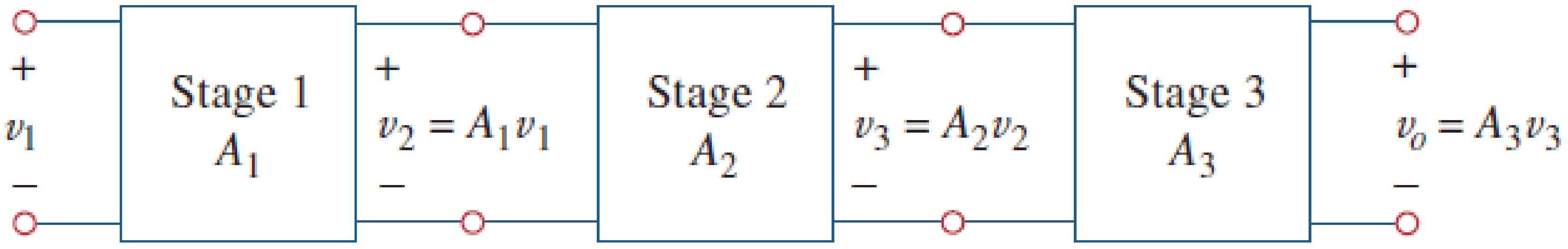


$$v_o = \frac{R_2(1 + R_1/R_2)}{R_1(1 + R_3/R_4)}v_2 - \frac{R_2}{R_1}v_1$$

The voltage follower.



Cascaded Op Amp Circuits



$$A = A_1 A_2 A_3$$

Sample Questions

Figure 5.105 displays a two-op-amp instrumentation amplifier. Derive an expression for v_o in terms of v_1 and v_2. How can this amplifier be used as a subtractor?

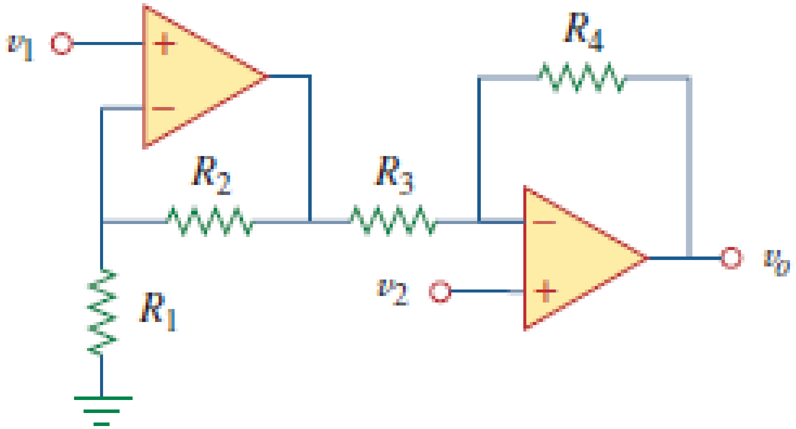


Figure 5.105

Find v_o in the op amp circuit of Fig. 5.92.

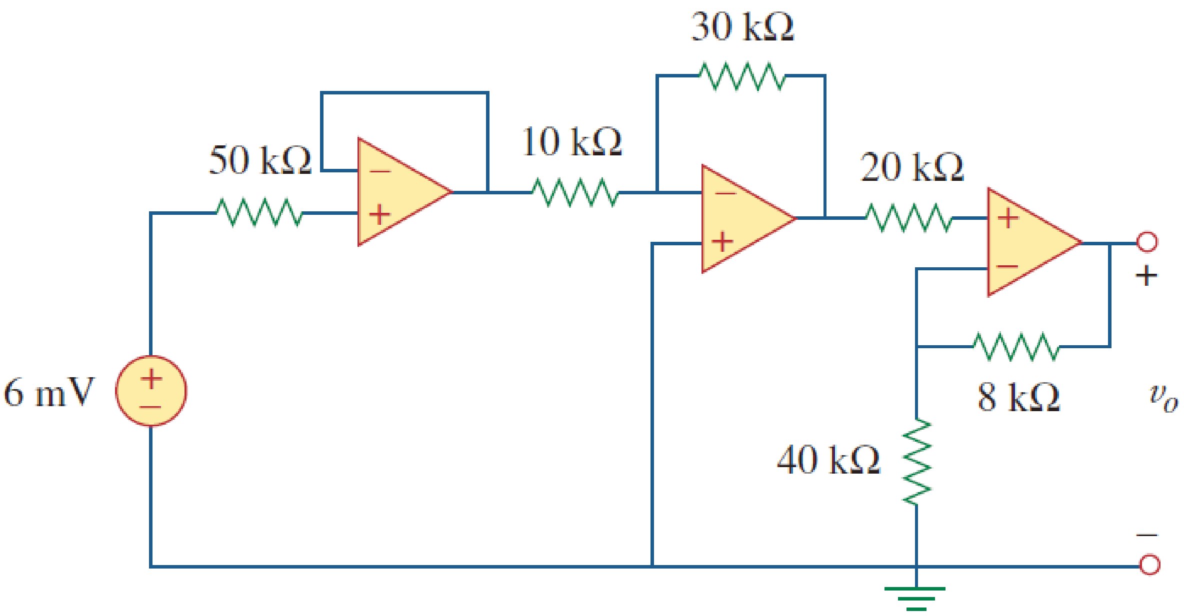
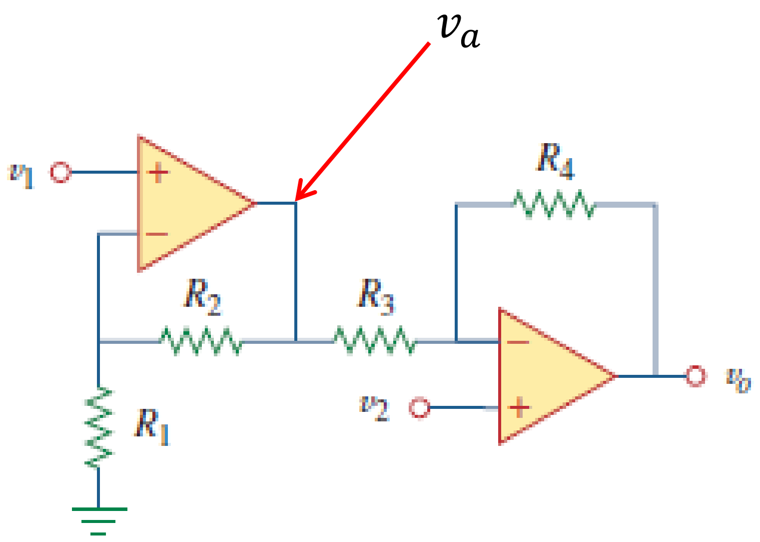


Figure 5.92

Figure 5.105 displays a two-op-amp instrumentation amplifier. Derive an expression for v_o in terms of v_1 and v_2. How can this amplifier be used as a subtractor?



5.105

The output, v_a, of the first op amp is,

$$v_a = (1 + (R_2/R_1))v_1 \qquad (1)$$

Also,

$$v_o = (-R_4/R_3)v_a + (1 + (R_4/R_3))v_2 \qquad (2)$$

Substituting (1) into (2),

$$v_o = (-R_4/R_3)\,(1 + (R_2/R_1))v_1 + (1 + (R_4/R_3))v_2$$

Or,

$$v_o = \underline{\mathbf{(1 + (R_4/R_3))v_2 - (R_4/R_3 + (R_2R_4/R_1R_3))v_1}}$$

If $R_4 = R_1$ and $R_3 = R_2$, then,

$$v_o = (1 + (R_4/R_3))(v_2 - v_1)$$

which is a subtractor with a gain of $\underline{\mathbf{(1 + (R_4/R_3))}}$.

Find v_o in the op amp circuit of Fig. 5.92.

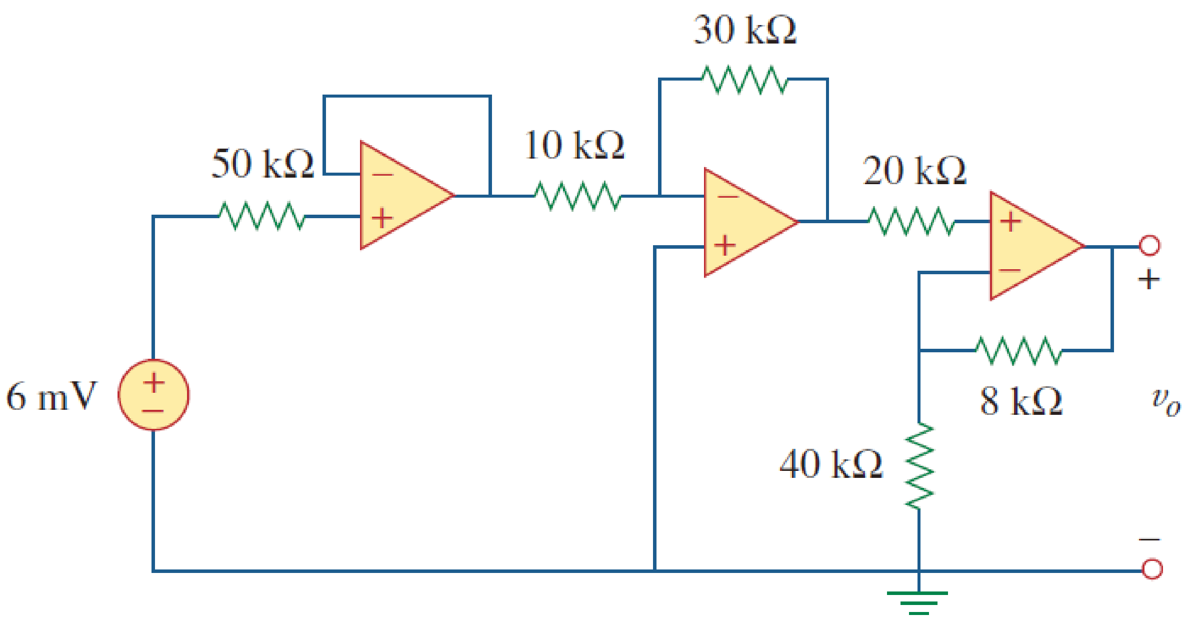


Figure 5.92

The output of the first op amp (to the left) is 6 mV. The second op amp is an inverter so that its output is

$$v_o' = -\frac{30}{10}(6\text{mV}) = \text{-}18\text{ mV}$$

The third op amp is a noninverter so that

$$v_o' = \frac{40}{40+8}v_o \quad \longrightarrow \quad v_o = \frac{48}{40}v_o' = \underline{-21.6\text{ mV}}$$

Thanks for Listening!