Ve215 Electric Circuits

Sung-Liang Chen

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Chapter 8

Second-Order Circuits

8.1 Introduction

- In this chapter, we consider circuits containing two storage elements, known as second-order circuits.
- Examples of second-order circuits are shown in Fig. 8.1.

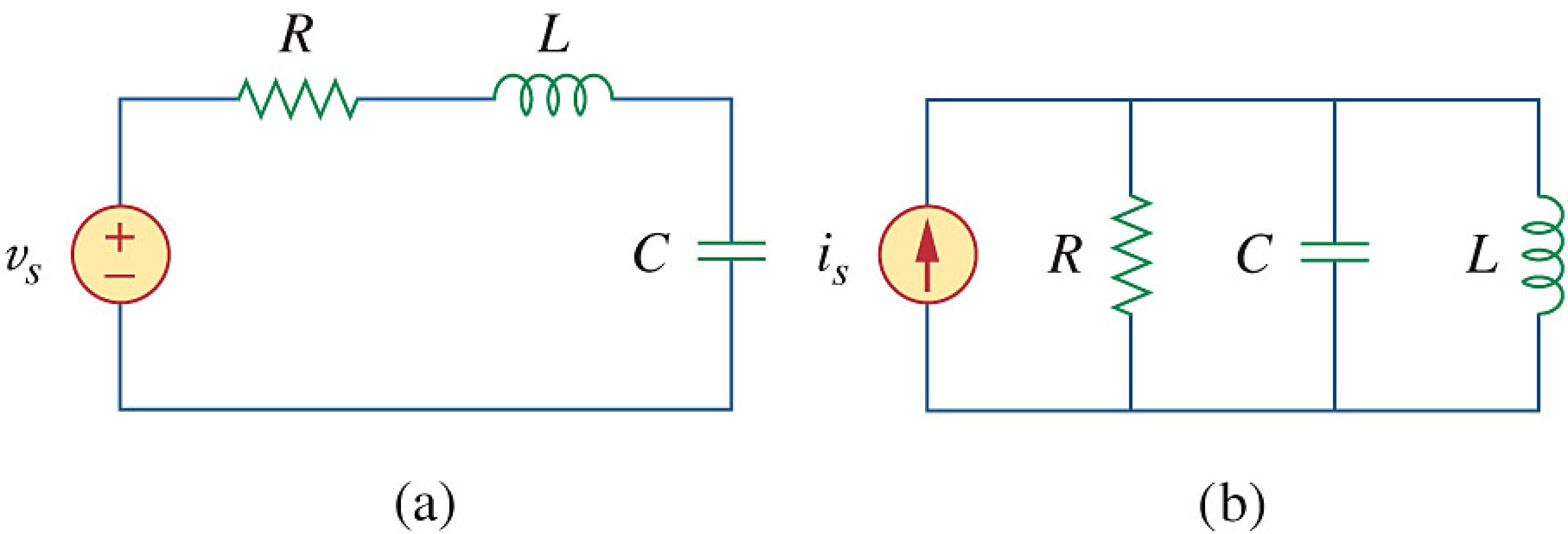


Figure 8.1 Typical examples of second-order circuits: (a) series RLC circuit, (b) parallel RLC circuit.

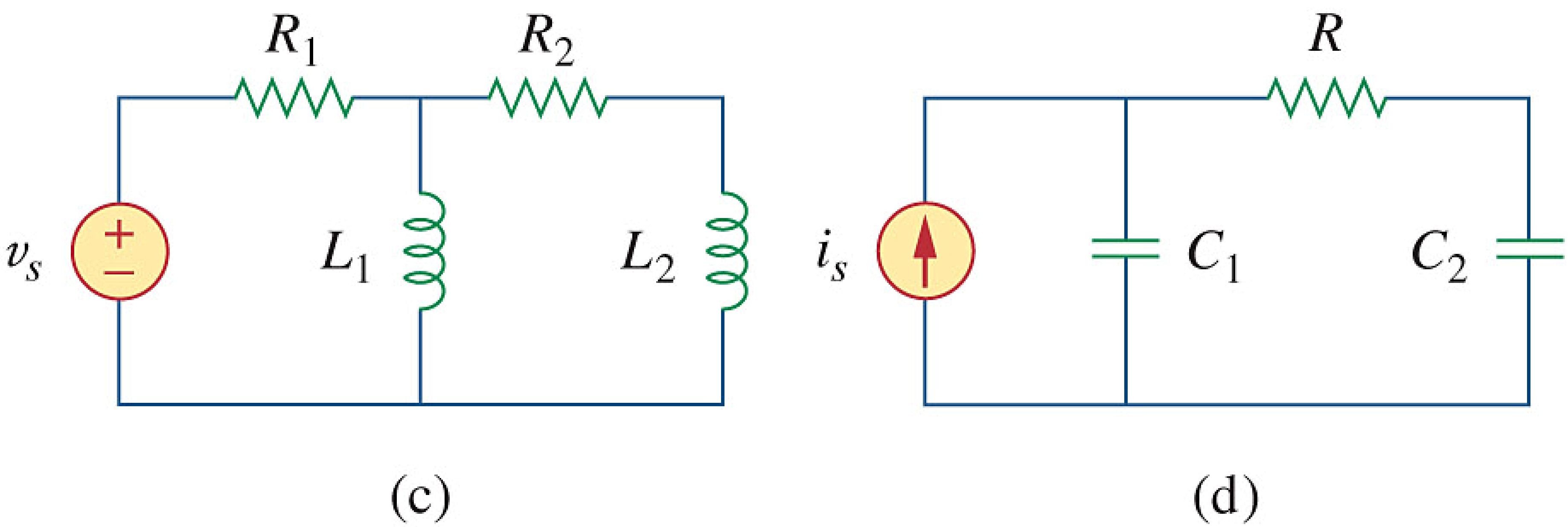


Figure 8.1 Typical examples of second-order circuits: (c) RLL circuit, (d) RCC circuit.

8.2 Finding Initial and Final Values

Example 8.1 The switch in Fig. 8.2 has been closed for a long time. It is open at $t = 0$. Find: (a) $i(0^+)$, $v(0^+)$, (b) $di(0^+)/dt$, $dv(0^+)/dt$, (c) $i(\infty)$, $v(\infty)$.

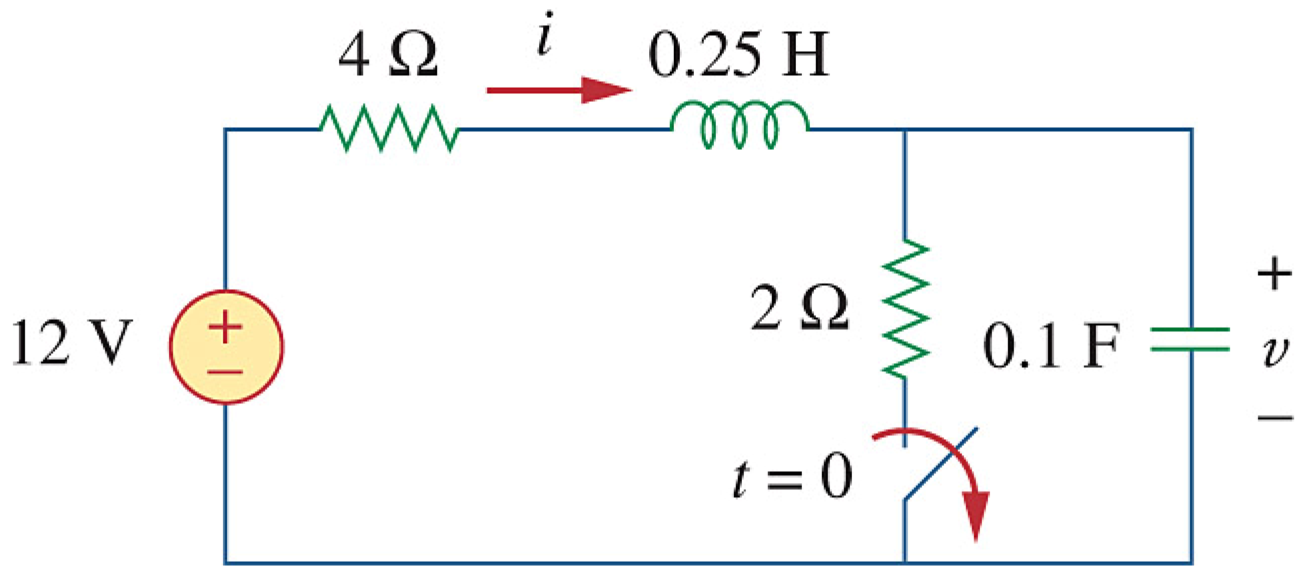


Figure 8.2

(a) $t=0^-$

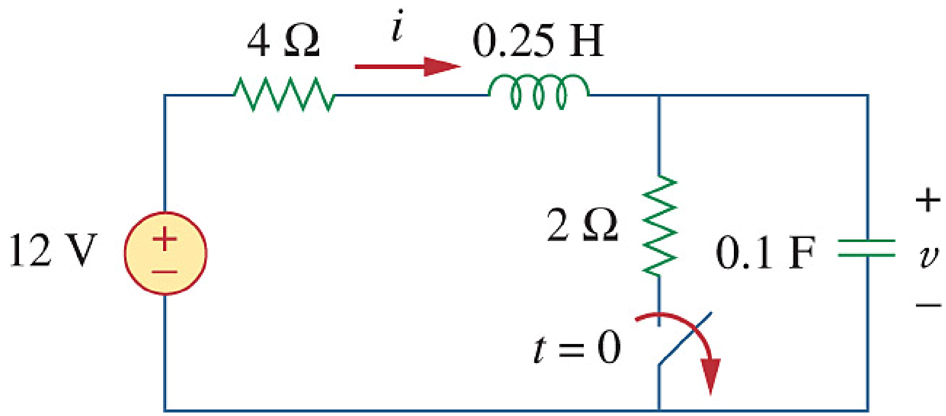


Figure 8.2

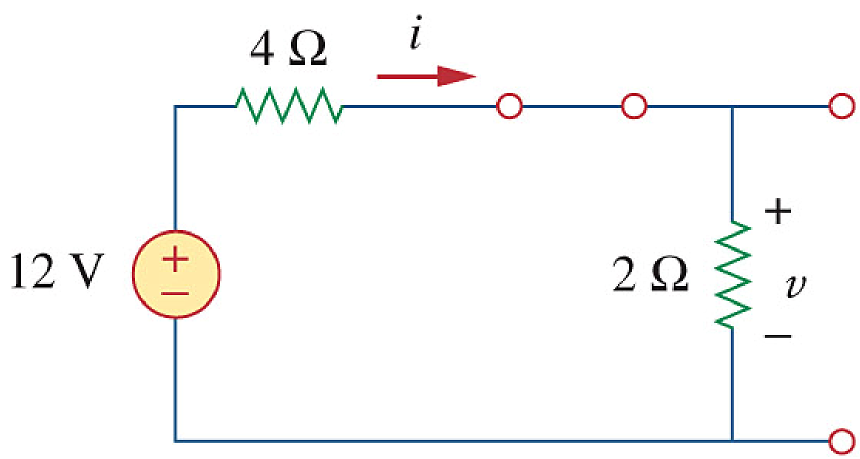


Figure 8.3 (a) Equivalent circuit of that in Fig. 8.2 for $t = 0^-$.

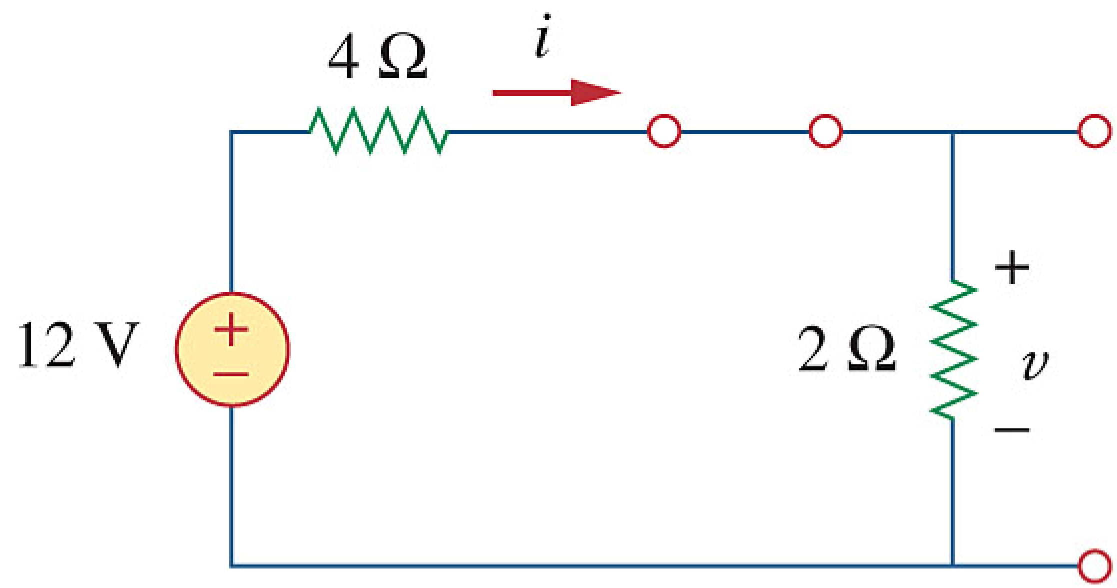


Figure 8.3 (a) Equivalent circuit of that in Fig. 8.2 for $t = 0^-$.

Solution :

(a)

$$i(0^+) = i(0^-) = \frac{12}{4+2} = 2 \text{ (A)}$$

$$v(0^+) = v(0^-) = 2i(0^-) = 4 \text{ (V)}$$

(b) t=0$^+$

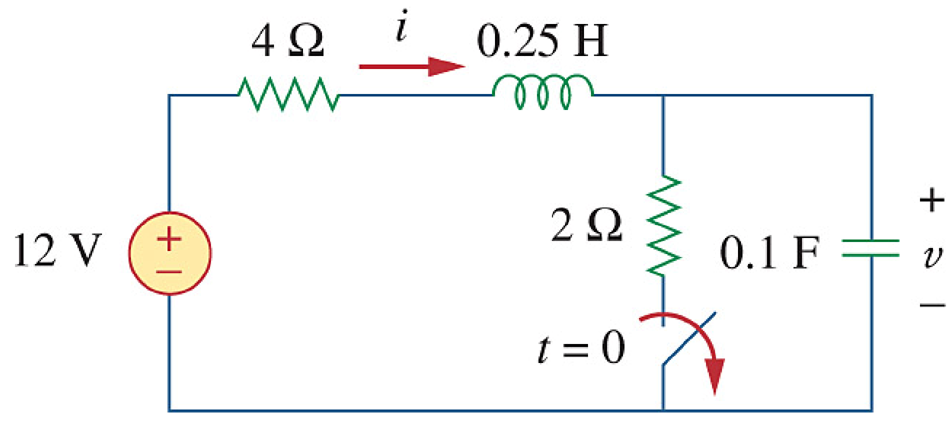


Figure 8.2

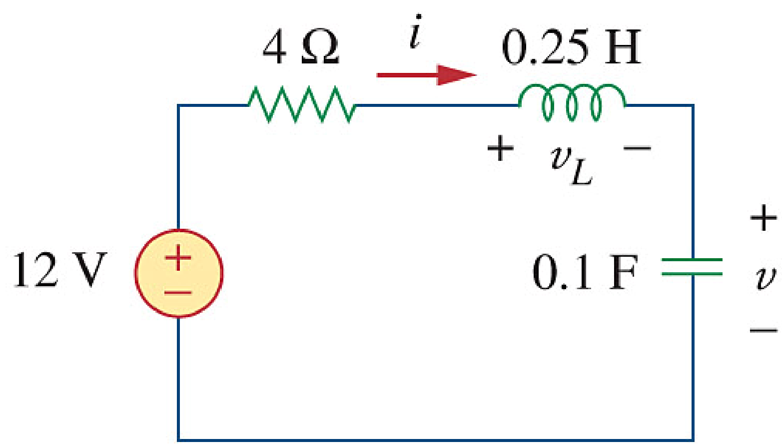


Figure 8.3 (a) Equivalent circuit of that in Fig. 8.2 for $t = 0^+$.

Represent dv/dt or di/dt in terms of v_C and/or i_L

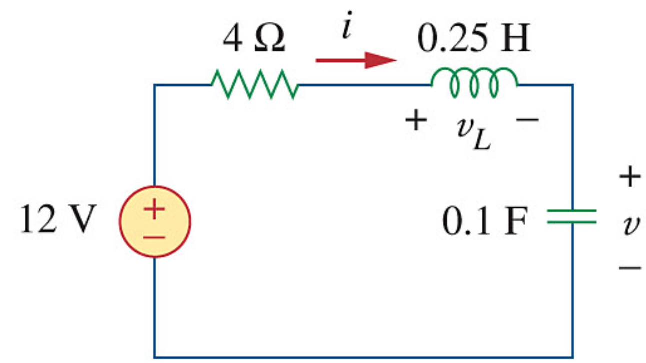


Figure 8.3 (a) Equivalent circuit of that in Fig. 8.2 for $t = 0^+$.

(b)

$$\begin{cases} i = 0.1\dfrac{dv}{dt} \\ 12 = 4i + 0.25\dfrac{di}{dt} + v \end{cases} \Rightarrow \begin{cases} \dfrac{dv}{dt} = \dfrac{i}{0.1} \\ \dfrac{di}{dt} = \dfrac{12 - 4i - v}{0.25} \end{cases}$$

$$dv(0^+)/dt = i(0^+)/0.1 = 2/0.1 = 20 \text{ (V/s)}$$

$$di(0^+)/dt = \left[12 - 4i(0^+) - v(0^+)\right]/0.25$$

$$= \left[12 - 4 \times 2 - 4\right]/0.25 = 0 \text{ (A/s)}$$

(c) t → ∞

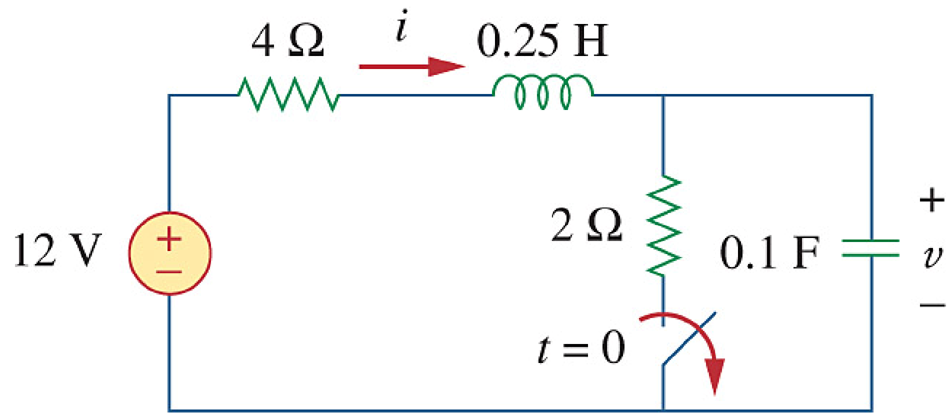


Figure 8.2

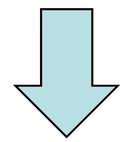


Figure 8.3 (c) Equivalent circuit of that in Fig. 8.2 for t = infinity.

(c)

$$i(\infty) = 0$$

$$v(\infty) = 12\ (\text{V})$$

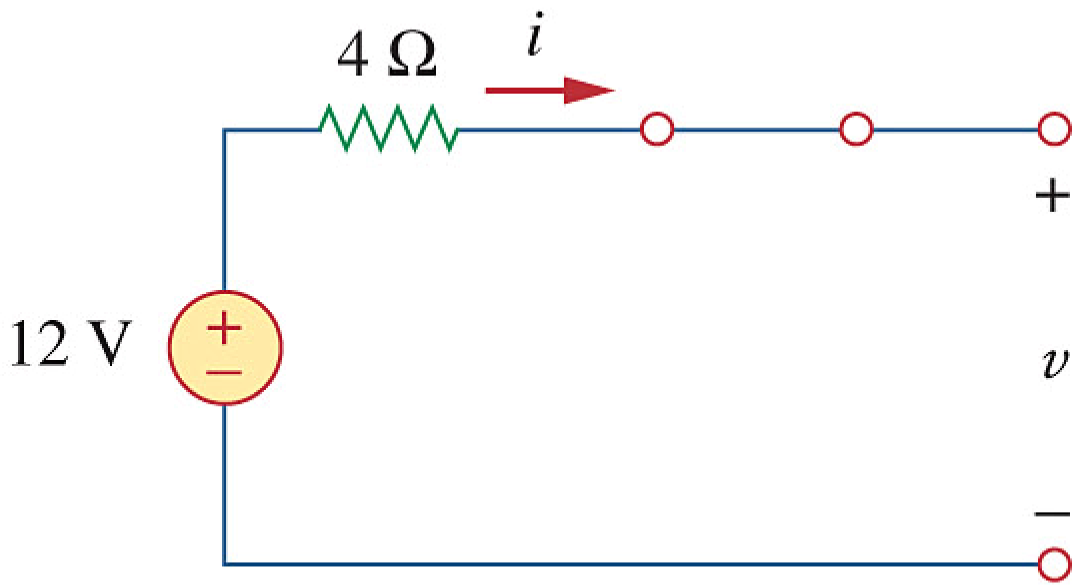


Figure 8.3 (c) Equivalent circuit of that in Fig. 8.2 for t = infinity.

8.3 The Source-Free Series *RLC* Circuit

Consider the circuit shown in Fig. 8.8. The circuit is being excited by the energy initially stored in the capacitor and inductor.

At $t = 0$,

$$v(0) = V_0, \quad i(0) = I_0$$

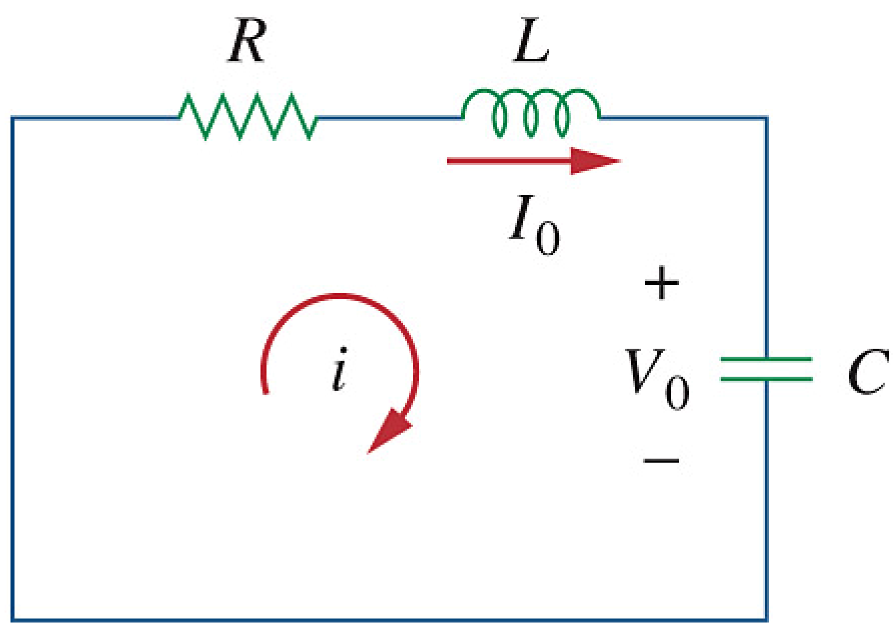


Figure 8.8 A source-free series *RLC* circuit.

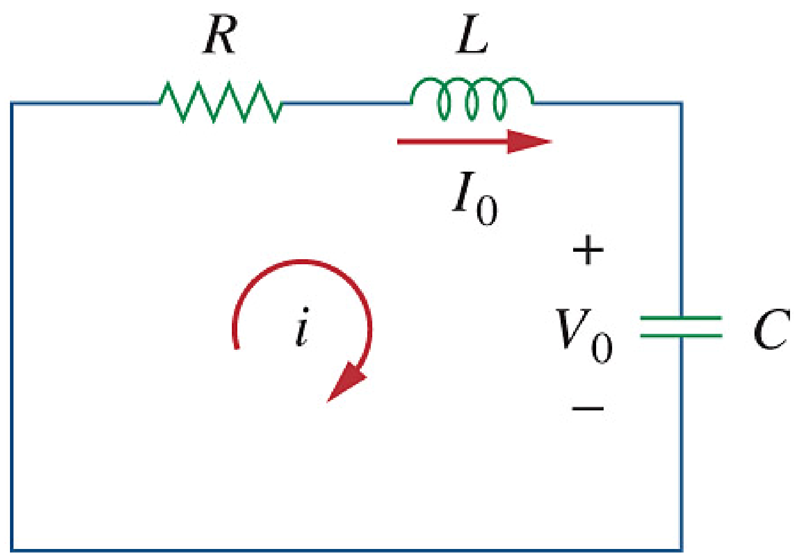


Figure 8.8 A source-free series RLC circuit.

$$iR + L\frac{di}{dt} + v = 0$$

$$iR + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i\,dt = 0$$

Represent the equation in terms of only one parameter i

$$\frac{di}{dt}R + L\frac{d^2 i}{dt^2} + \frac{1}{C}i = 0$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$$

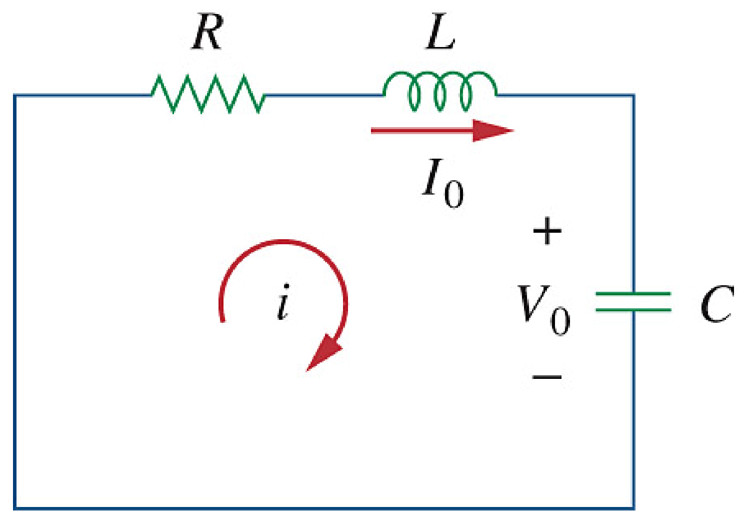


Figure 8.8 A source-free series RLC circuit.

The initial conditions are

$$i(0^+) = i(0^-) = I_0$$

$$i'(0^+) = -\frac{1}{L}\left(i(0^+)R + v(0^+)\right) \longleftarrow \quad iR + L\frac{di}{dt} + v = 0$$

$$= -\frac{1}{L}\left(i(0^-)R + v(0^-)\right)$$

$$= -\frac{1}{L}\left(I_0R + V_0\right)$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$s = \frac{-R/L \pm \sqrt{(R/L)^2 - 4 \times 1 \times \left(1/(LC)\right)}}{2 \times 1}$$

$$= -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$= -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

where

$\alpha = \dfrac{R}{2L}$: neper frequency (damping factor), Np/s (nepers per second)

$\omega_0 = \dfrac{1}{\sqrt{LC}}$: resonant frequency (undamped natural frequency), rad/s

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$, $s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$: natural frequencies, Np/s

Solution 1: Overdamped

There are three types of solutions:

1. If $\alpha > \omega_0$, $s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$, $s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$, we have the *overdamped* case,

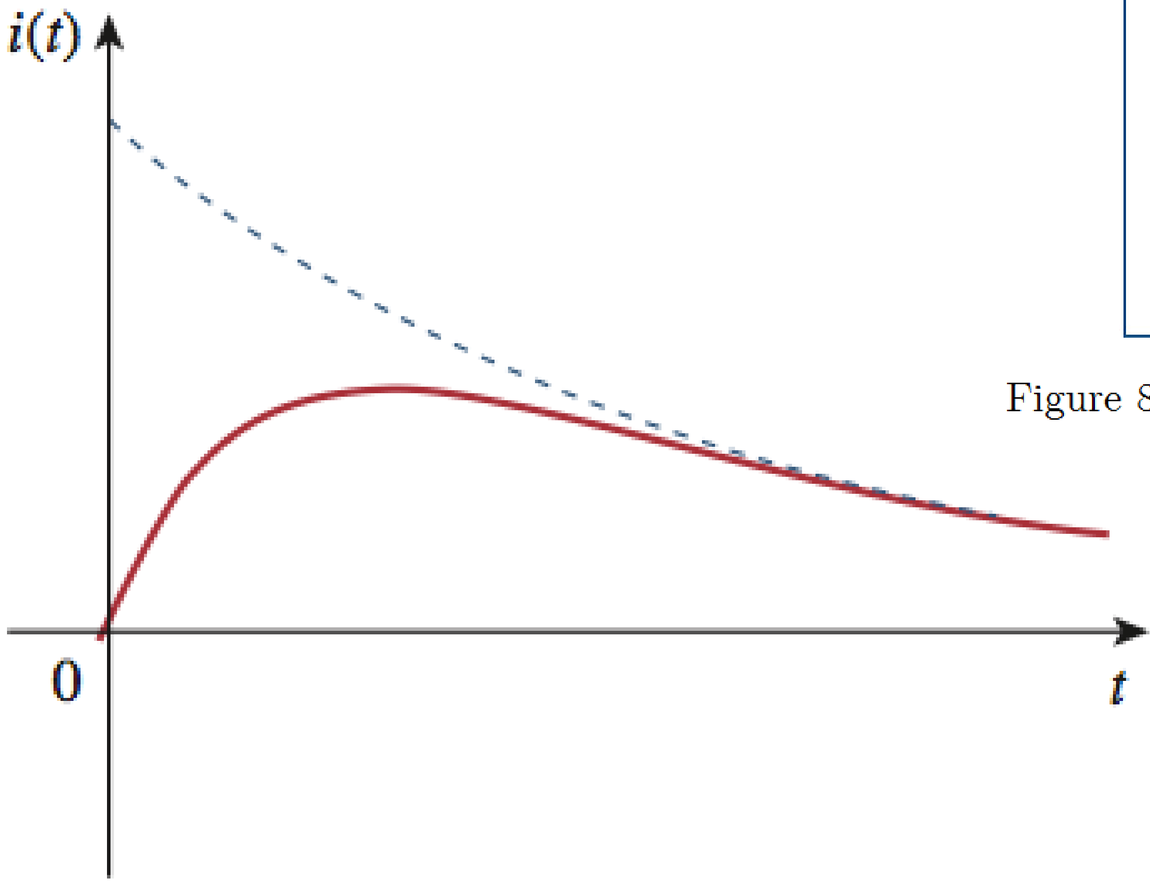
$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

where

$$A_1 = \frac{i'(0^+) - s_2 i(0^+)}{s_1 - s_2}$$

$$A_2 = \frac{s_1 i(0^+) - i'(0^+)}{s_1 - s_2}$$

$S_1 < 0$, $S_2 < 0$
$S_1 \neq S_2$



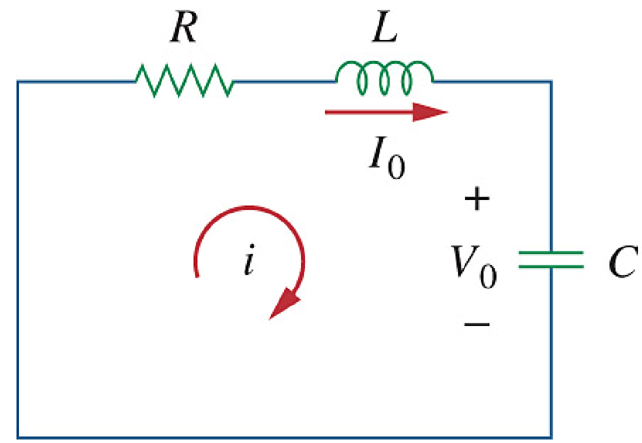


Figure 8.8 A source-free series RLC circuit.

1. no oscillation
2. region 1: i(t) changes due to initially stored energy in L and C
3. region 2: steady state value should be 0 due to “zero input response”
4. $\alpha \uparrow$ (more damping) ➔ reaches steady state faster

Solution 2: Critically damped

2. If $\alpha = \omega_0$, $s_1 = s_2 = -\alpha$, we have the *critically damped* case,

$$i(t) = (B_1 t + B_2)e^{-\alpha t}$$

where

$$B_1 = i'(0^+) + \alpha i(0^+)$$

$$B_2 = i(0^+)$$

$S_1 < 0$, $S_2 < 0$
$S_1 = S_2$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = 0$$

$$\alpha = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}} \qquad \alpha = \omega_0$$

$$\frac{d^2 i}{dt^2} + 2\alpha\frac{di}{dt} + \alpha^2 i = 0$$

or

$$\frac{d}{dt}\left(\frac{di}{dt} + \alpha i\right) + \alpha\left(\frac{di}{dt} + \alpha i\right) = 0$$

If we let

Reduced to 1st order DE

$$f = \frac{di}{dt} + \alpha i$$

then Eq. (8.16) becomes

$$\frac{df}{dt} + \alpha f = 0$$

which is a first-order differential equation with solution $f = A_1 e^{-\alpha t}$, where A_1 is a constant. Equation (8.17) then becomes

$$\frac{di}{dt} + \alpha i = A_1 e^{-\alpha t}$$

or

$$e^{\alpha t}\frac{di}{dt} + e^{\alpha t}\alpha i = A_1 \tag{8.18}$$

This can be written as

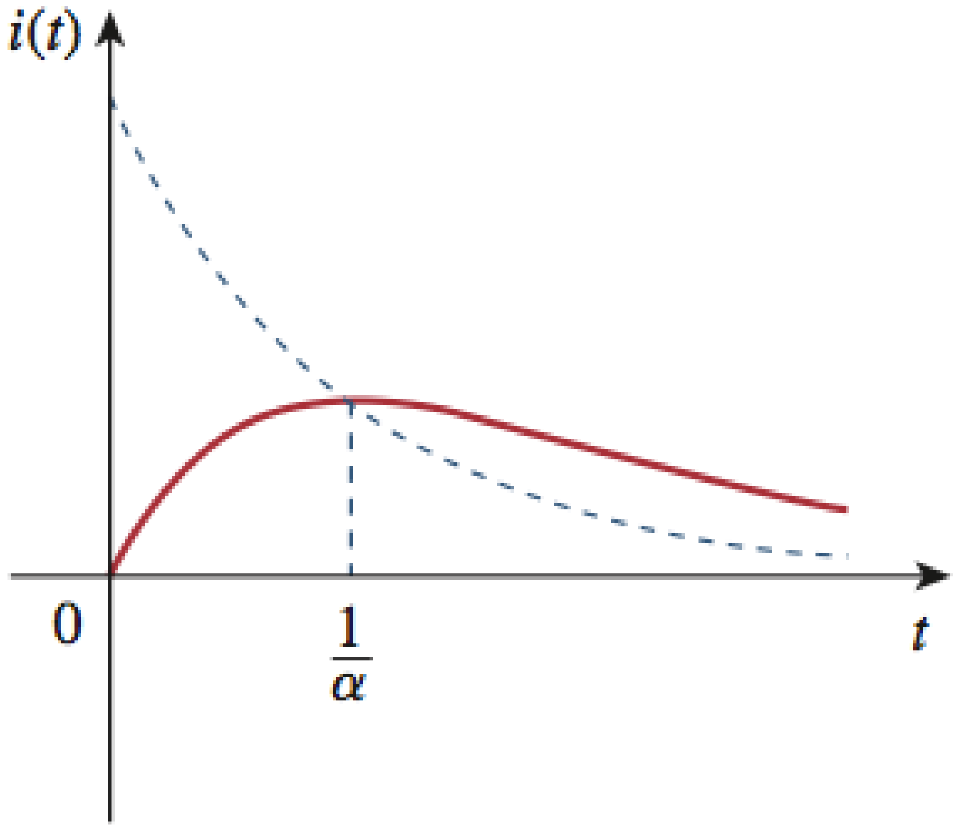
$$\frac{d}{dt}(e^{\alpha t} i) = A_1 \tag{8.19}$$

Integrating both sides yields

$$e^{\alpha t} i = A_1 t + \underbrace{A_2}_{\text{Integration constant}}$$

or

$$i = (A_1 t + A_2)e^{-\alpha t} \tag{8.20}$$



1. no oscillation
2. region 1: i(t) reaches a maximum value at t = $1/\alpha$
3. region 2: decays all the way to zero
4. $\alpha \uparrow$ (more damping) ➔ reaches steady state faster

Solution 3: Underdamped

3. If $\alpha < \omega_0$, $s_1 = -\alpha + j\omega_d$, $s_2 = -\alpha - j\omega_d$,

where $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$, which is called the *damping frequency*, we have the *underdamped* case,

$$i(t) = e^{-\alpha t}\underline{(C_1 \cos \omega_d t + C_2 \sin \omega_d t)}$$

where

$$C_1 = i(0^+)$$

$$C_2 = \frac{i'(0^+) + \alpha i(0^+)}{\omega_d}$$

S_1, S_2 are complex conjugates

$$\begin{aligned} i(t) &= A_1 e^{-(\alpha - j\omega_d)t} + A_2 e^{-(\alpha + j\omega_d)t} \\ &= e^{-\alpha t}(A_1 e^{j\omega_d t} + A_2 e^{-j\omega_d t}) \end{aligned} \tag{8.23}$$

Using Euler's identities,

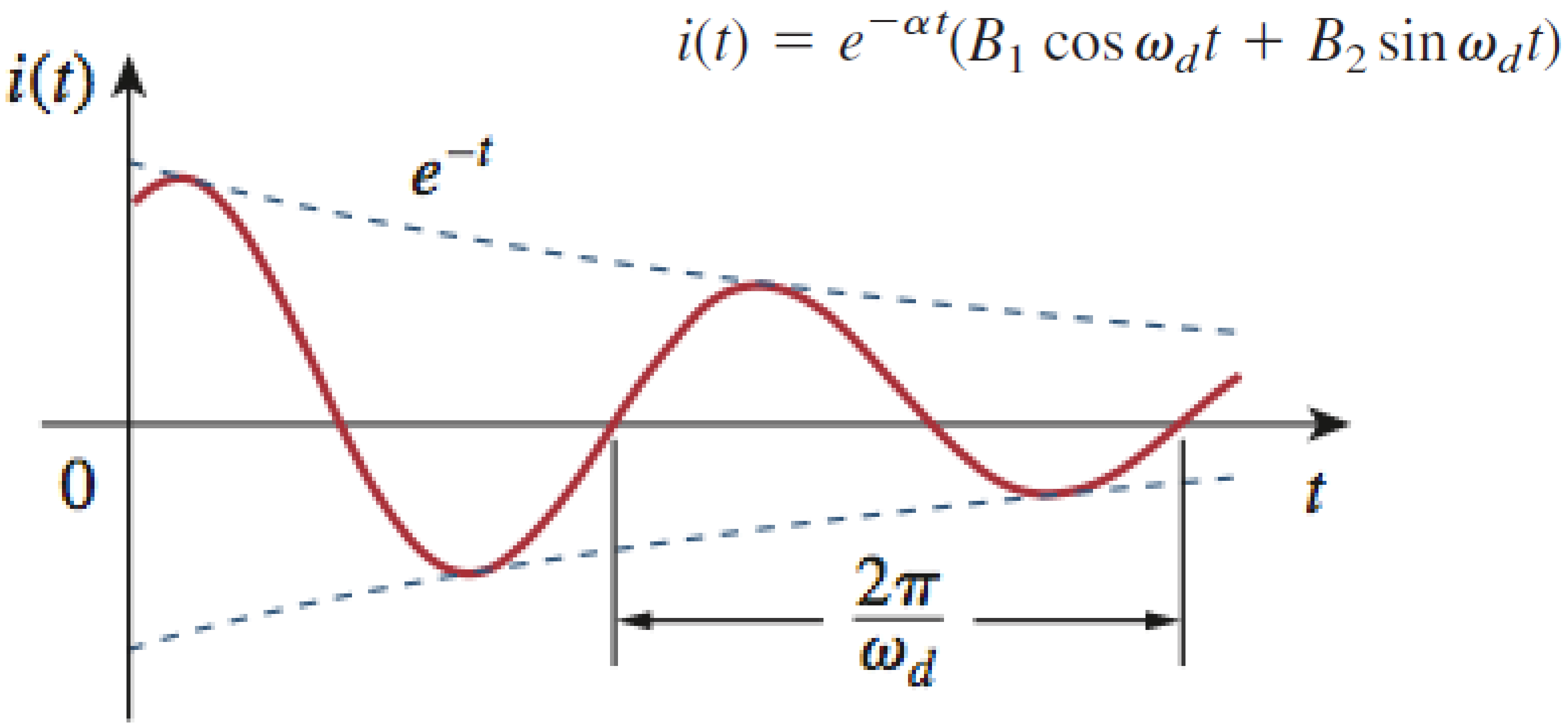
$$e^{j\theta} = \cos\theta + j\sin\theta, \qquad e^{-j\theta} = \cos\theta - j\sin\theta \tag{8.24}$$

we get

$$\begin{aligned} i(t) &= e^{-\alpha t}[A_1(\cos\omega_d t + j\sin\omega_d t) + A_2(\cos\omega_d t - j\sin\omega_d t)] \\ &= e^{-\alpha t}[(A_1 + A_2)\cos\omega_d t + j(A_1 - A_2)\sin\omega_d t] \end{aligned} \tag{8.25}$$

Replacing constants $(A_1 + A_2)$ and $j(A_1 - A_2)$ with constants B_1 and B_2, we write

$$\boxed{i(t) = e^{-\alpha t}(B_1\cos\omega_d t + B_2\sin\omega_d t)} \tag{8.26}$$



1. Oscillatory response
2. $\alpha \uparrow$ (more damping) ➔ reaches steady state faster
3. α: envelope
4. ω_d: oscillation frequency

Once the inductor current $i(t)$ is found, other circuit quantities can be found,

$$v_R(t) = i(t)R$$

$$v_L(t) = L\frac{di(t)}{dt}$$

$$v_C(t) = \frac{1}{C}\int_0^t i(t)dt + v_C(0)$$

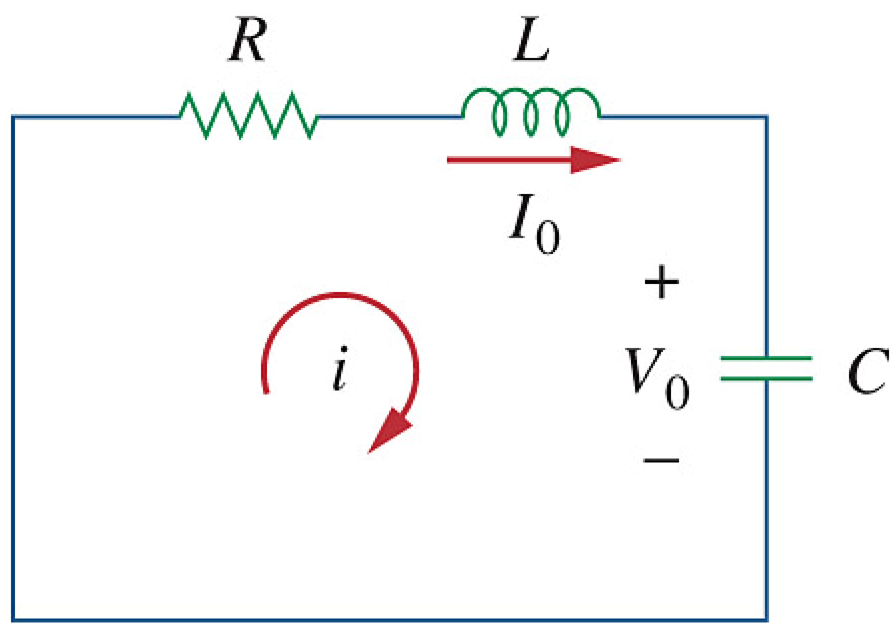


Figure 8.8 A source-free series RLC circuit.

Practice Problem 8.4 The circuit in Fig. 8.12 has reached steady state at $t = 0^{-}$. If the make-before-break switch moves to position b at $t = 0$, calculate $i(t)$ for $t > 0$.

Solution :

$$i(0^{+}) = i(0^{-}) = \frac{50}{10} = 5 \text{ (A)}$$

$$v(0^{+}) = v(0^{-}) = 0 \text{ (V)}$$

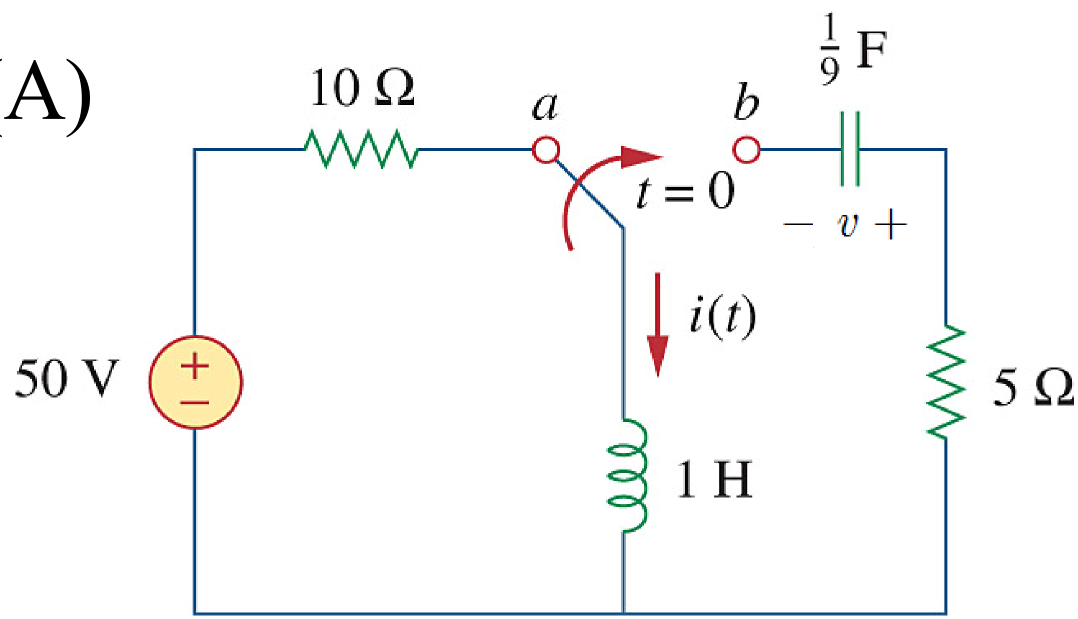


Figure 8.12

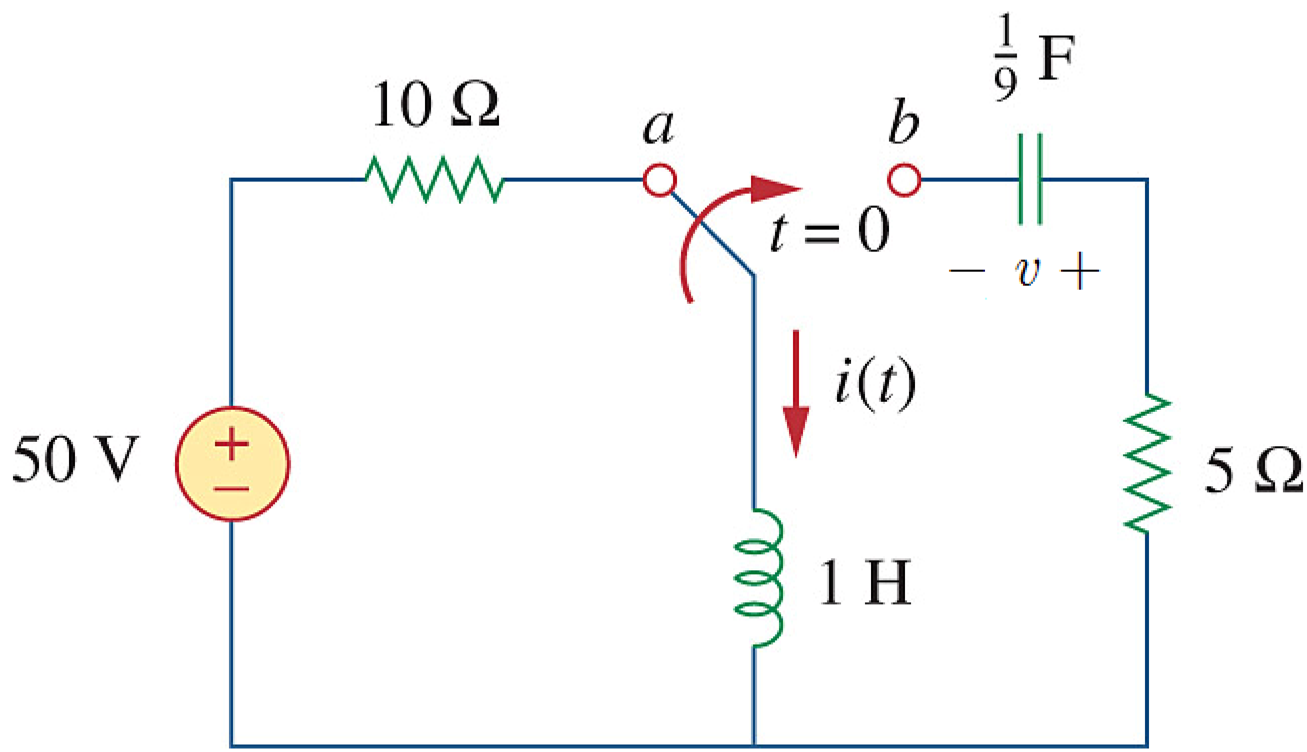


Figure 8.12

$$1\times\frac{di(t)}{dt}+i(t)\times 5+v(t)=0,\ i(t)=\frac{1}{9}\frac{dv(t)}{dt}$$

$$i'(0^+)=-5i(0^+)-v(0^+)=-5\times 5-0$$

$$=-25\ (\mathrm{A/s})$$

$$1\times\frac{d^2i(t)}{dt^2}+\frac{di(t)}{dt}\times 5+\frac{dv(t)}{dt}=0$$

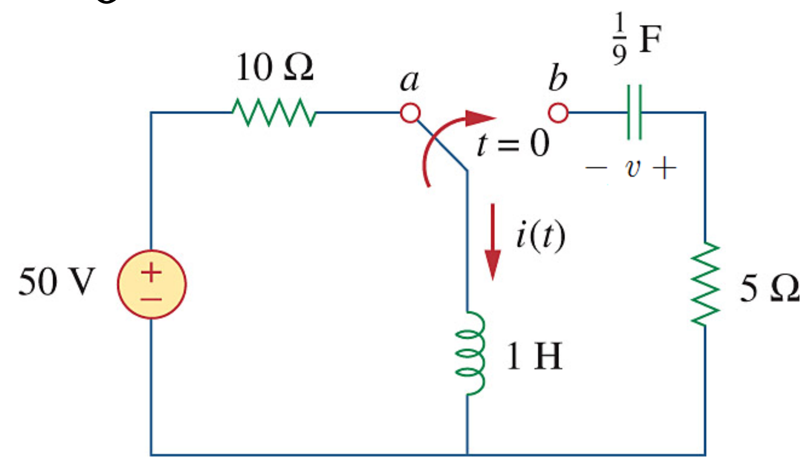


Figure 8.12

$$1\times\frac{d^2i(t)}{dt^2}+\frac{di(t)}{dt}\times 5+\frac{1}{1/9}i(t)=0$$

$$\frac{d^2i(t)}{dt^2}+5\frac{di(t)}{dt}+9i(t)=0$$

$$s=\frac{-5\pm\sqrt{5^2-4\times1\times9}}{2\times1}=\frac{-5\pm j\sqrt{11}}{2}$$

$$i(t)=e^{-2.5t}(A_1\cos\frac{\sqrt{11}}{2}t+A_2\sin\frac{\sqrt{11}}{2}t)$$

$$i(0^+)=A_1\Rightarrow A_1=i(0^+)=5$$

$$i'(0^+)=-2.5A_1+\frac{\sqrt{11}}{2}A_2\Rightarrow A_2=\frac{i'(0^+)+2.5A_1}{\sqrt{11}/2}$$

$$=\frac{-25+2.5\times5}{\sqrt{11}/2}=-\frac{25}{\sqrt{11}}$$

$$i(t)\approx e^{-2.5t}(5\cos1.6583t-7.5378\sin1.6583t)\ (\mathrm{A})$$

Steps for source-free 2^{nd} order circuit

1. Plot the circuit at $t<0$, find initial conditions, $i(0^+)$, $v(0^+)$
2. Plot the circuit at $t>0$, express di/dt or dv/dt in terms of i_L and v_c, find initial conditions $di(0^+)/dt$, $dv(0^+)/dt$
3. Express the circuit in 2^{nd} order D.E. with only one parameter (either i or v) and solve it.
4. Solve the coefficients using initial conditions.

8.4 The Source-Free Parallel *RLC* Circuit

Consider the circuit shown in Fig. 8.13. The circuit is being excited by the energy initially stored in the capacitor and inductor. At $t = 0$,

$$v(0) = V_0, \quad i(0) = I_0$$

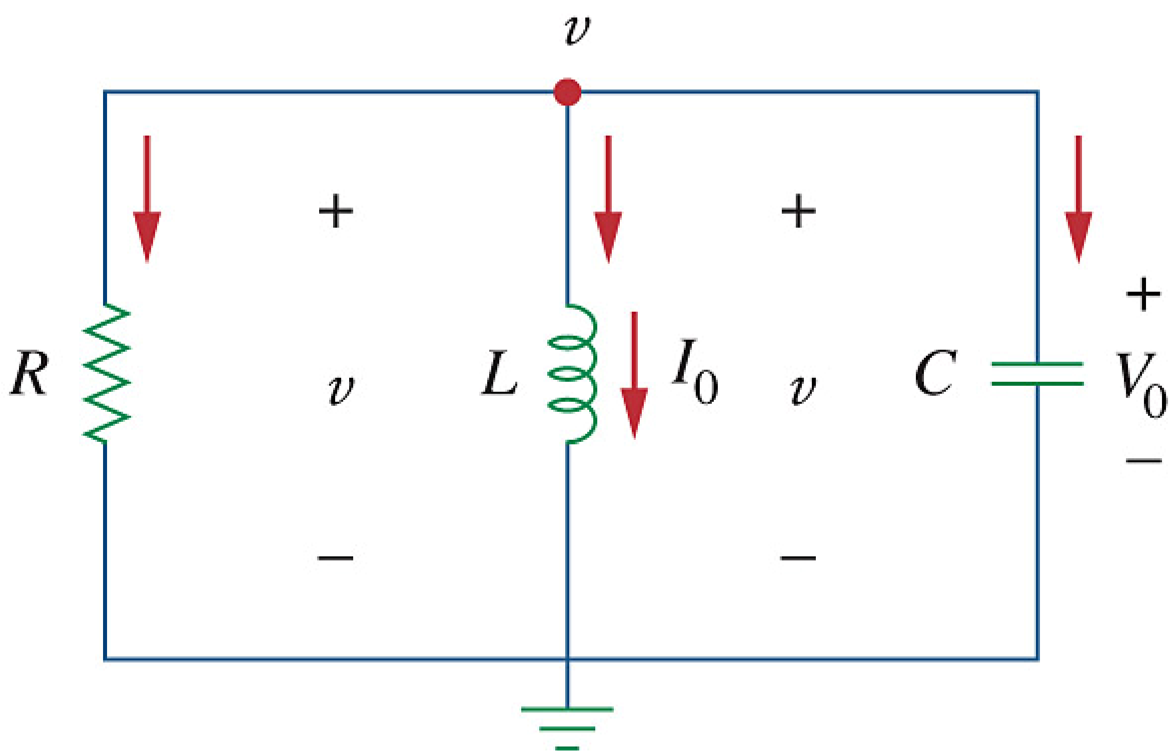


Figure 8.13 A source-free parallel RLC circuit.

$$\frac{v}{R} + i + C\frac{dv}{dt} = 0$$

$$\frac{v}{R} + \frac{1}{L}\int_{-\infty}^{t} v\,dt + C\frac{dv}{dt} = 0$$

$$\frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v + C\frac{d^2v}{dt^2} = 0$$

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0$$

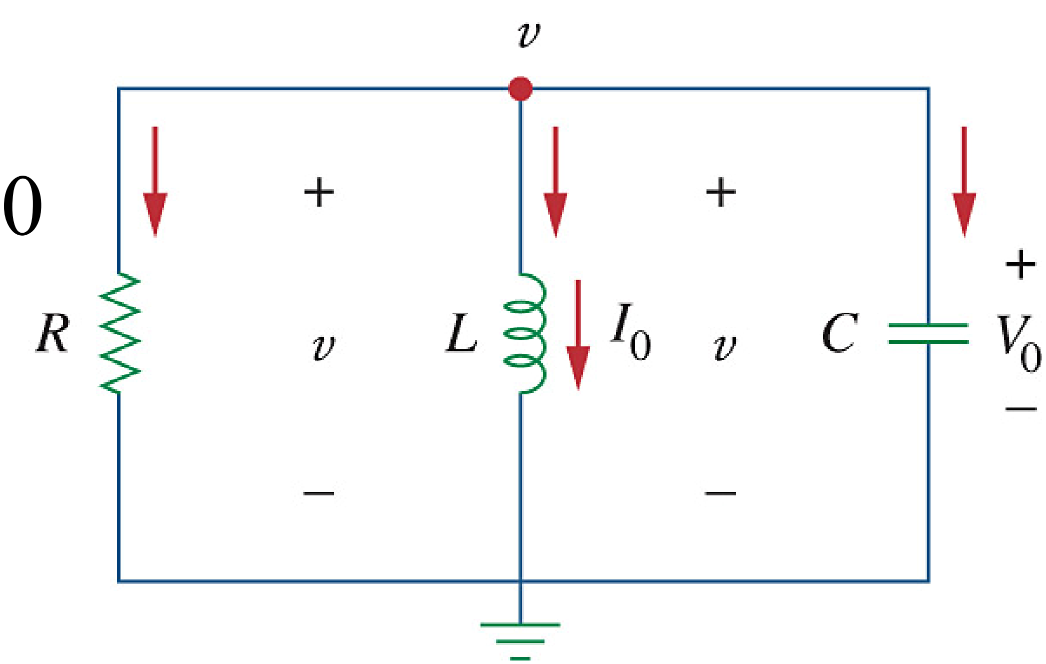


Figure 8.13 A source-free parallel *RLC* circuit.

The initial conditions are

$$v(0^+) = v(0^-) = V_0$$

$$v'(0^+) = -\frac{1}{C}\left(v(0^+)/R + i(0^+)\right) \longleftarrow \frac{v}{R} + i + C\frac{dv}{dt} = 0$$

$$= -\frac{1}{C}\left(v(0^-)/R + i(0^-)\right)$$

$$= -\frac{1}{C}\left(V_0/R + I_0\right)$$

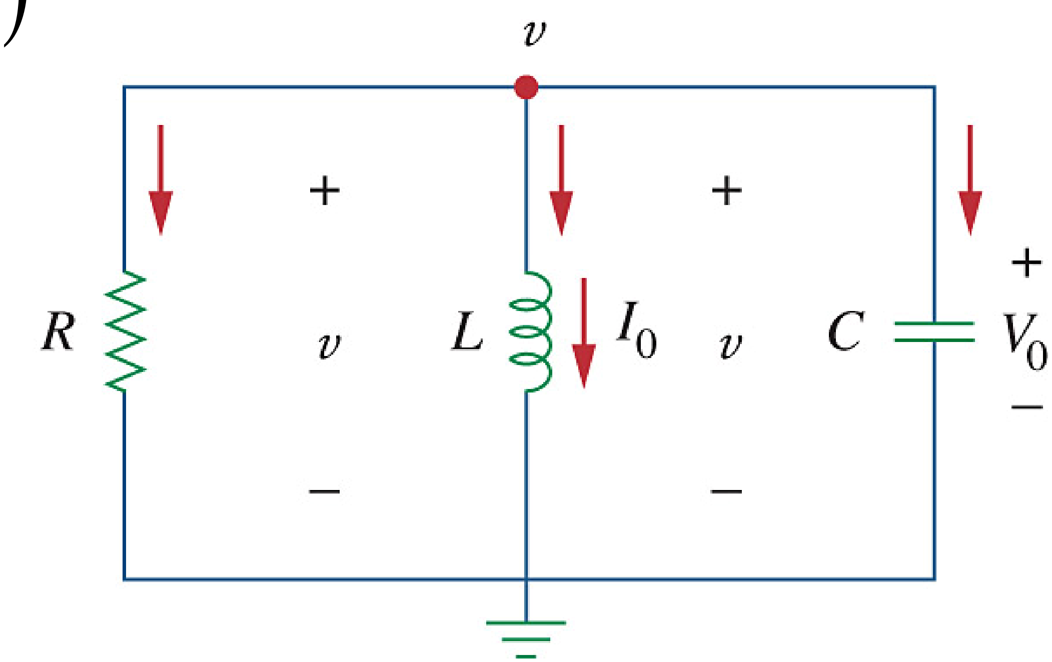


Figure 8.13 A source-free parallel RLC circuit.

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$s = \frac{-1/(RC) \pm \sqrt{1/(RC)^2 - 4\times 1\times\left(1/(LC)\right)}}{2\times 1}$$

$$= -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$= -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

where

$\alpha = \dfrac{1}{2RC}$: neper frequency (damping factor), Np/s (nepers per second)

$\omega_0 = \dfrac{1}{\sqrt{LC}}$: resonant frequency (undamped natural frequency), rad/s

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$, $s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$: natural frequencies, Np/s

There are three types of solutions:

1. If $\alpha > \omega_0$, $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$, we have the *overdamped* case,

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

2. If $\alpha = \omega_0$, $s_1 = s_2 = -\alpha$, we have the *critically damped* case,

$$v(t) = (B_1 t + B_2) e^{-\alpha t}$$

3. If $\alpha < \omega_0$, $s_1 = -\alpha + j\omega_d$, $s_2 = -\alpha - j\omega_d$, we have the *underdamped* case,

$$v(t) = e^{-\alpha t}(C_1 \cos \omega_d t + C_2 \sin \omega_d t)$$

Once the capacitor voltage $v(t)$ is found, other circuit quantities can be found,

$$i_R(t) = \frac{v(t)}{R}$$

$$i_L(t) = \frac{1}{L}\int_0^t v(t)dt + i_L(0)$$

$$i_C(t) = C\frac{dv(t)}{dt}$$

Example 8.6 Find $v(t)$ for $t > 0$ in the *RLC* circuit of Fig. 8.15.

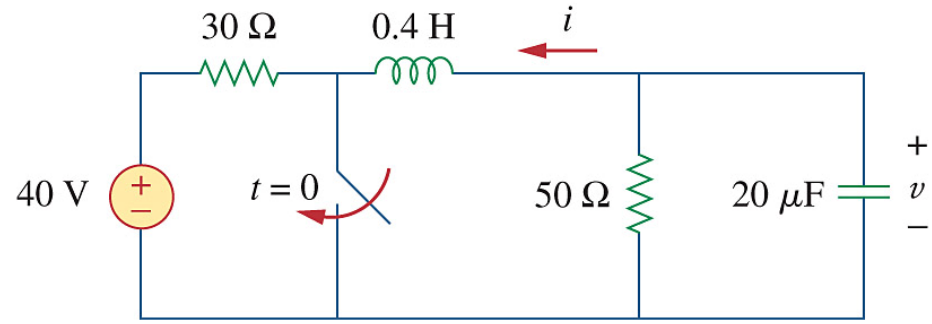


Figure 8.15

Solution :

Step 1

$$v(0^+) = v(0^-) = 40 \times \frac{50}{30+50} = 25 \text{ (V)}$$

$$i(0^+) = i(0^-) = -\frac{40}{30+50} = -0.5 \text{ (A)}$$

Step 2

$$v'(0^+) = -\frac{1}{C}\left(v(0^+)/R + i(0^+)\right)$$

$$= -\frac{1}{20\times 10^{-6}}\left(25/50 + (-0.5)\right) = 0 \text{ (V/s)}$$

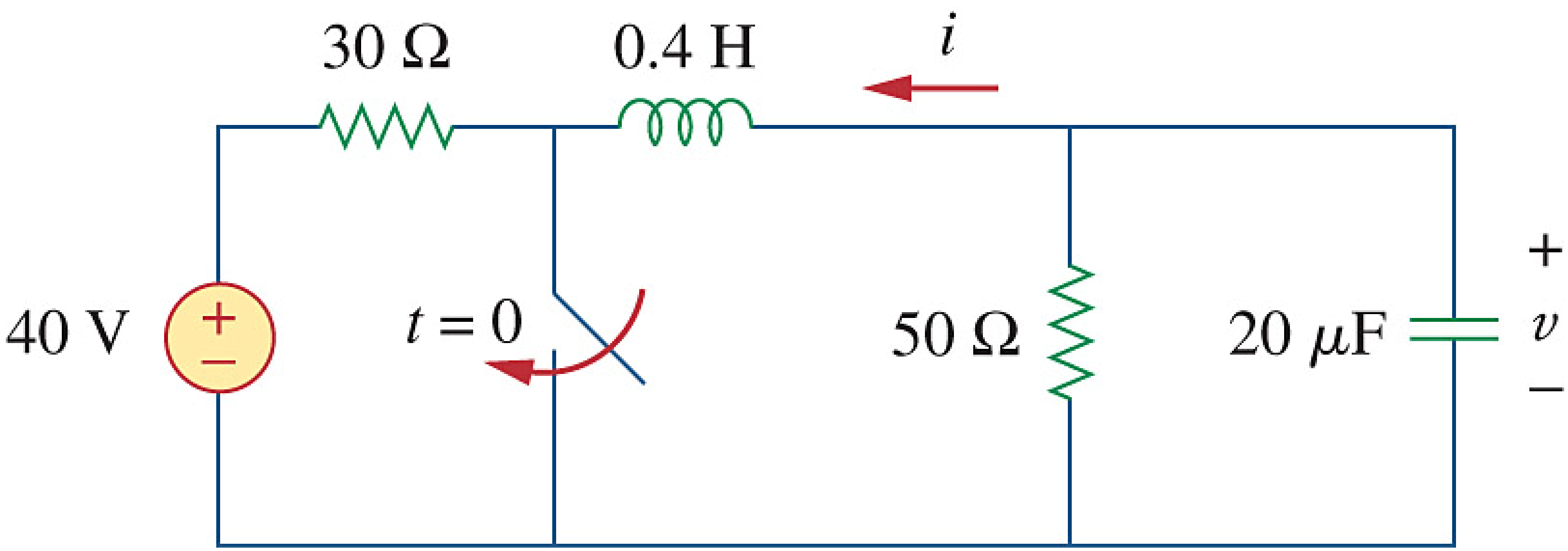


Figure 8.15

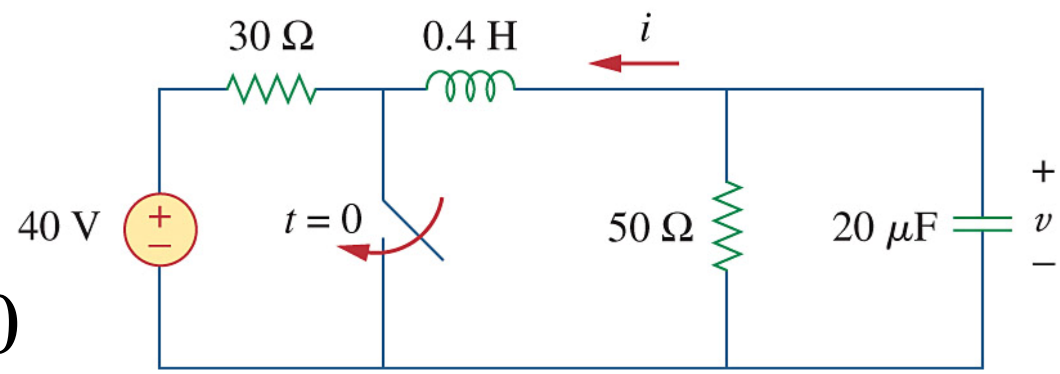


Figure 8.15

$$\frac{d^2v}{dt^2}+\frac{1}{RC}\frac{dv}{dt}+\frac{1}{LC}v=0$$

$$\frac{1}{LC}=\frac{1}{0.4\times 20\times 10^{-6}}=125000$$

$$\frac{1}{RC}=\frac{1}{50\times 20\times 10^{-6}}=1000$$

$$\frac{d^2v}{dt^2}+1000\frac{dv}{dt}+125000v=0$$

$$s_{1,2}=\frac{-1000\pm\sqrt{1000^2-4\times 1\times 125000}}{2\times 1}$$

Step 3

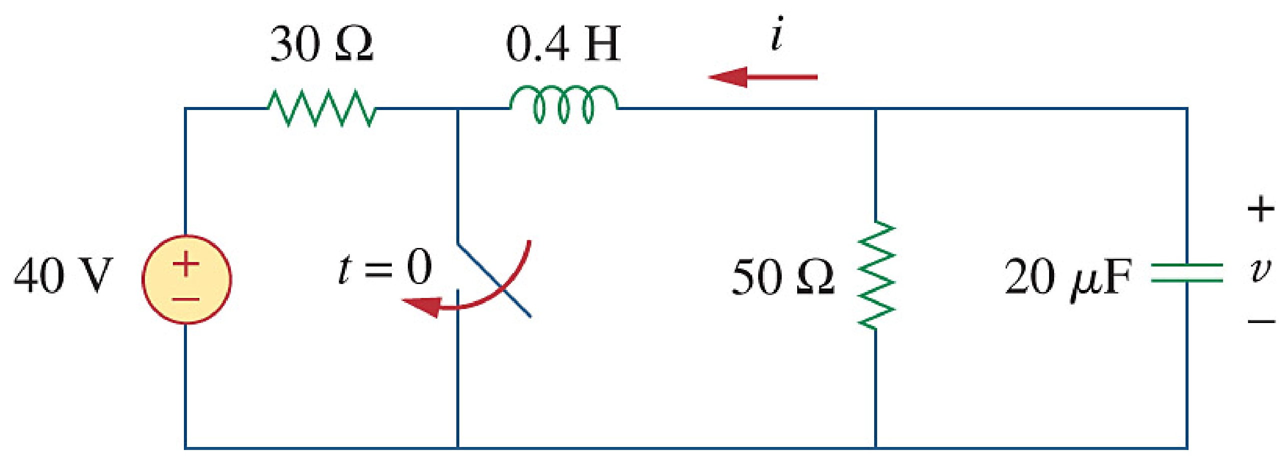


Figure 8.15

$$s_1 \approx -146.4466,\ s_2 \approx -853.5534$$

$$v(t) = A_1 e^{-146.4466t} + A_2 e^{-853.5534t}$$

Step 3

$$v(0^+) = A_1 + A_2 = 25$$

$$v'(0^+) = -146.4466 A_1 - 853.5534 A_2 = 0$$

$$A_1 \approx 30.1777,\ A_2 \approx -5.1777$$

$$v(t) \approx 30.18 e^{-146.45t} - 5.18 e^{-853.55t} \ \text{(V)}$$

Step 4

8.5 Series *RLC* Circuit with Step Input

Consider the circuit in Fig. 8.18. For $t > 0$,

$$\begin{cases} V_s = iR + L\dfrac{di}{dt} + v \\ i = C\dfrac{dv}{dt} \end{cases}$$

$$LC\frac{d^2v}{dt^2} + RC\frac{dv}{dt} + v = V_s$$

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{1}{LC}v = \frac{1}{LC}V_s$$

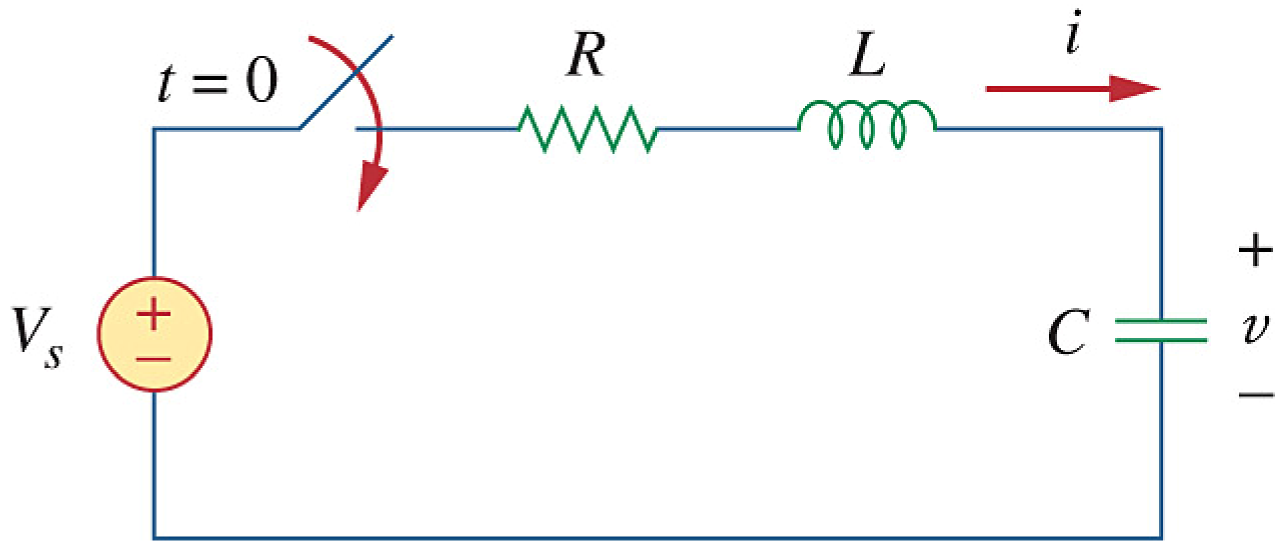


Figure 8.18 Step voltage applied to a series *RLC* circuit.

It can be shown that the solution has three possible forms:

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + V_s$$

(Overdamped)

$$v(t) = (A_1 + A_2 t)e^{-\alpha t} + V_s$$

(Critically damped)

$$v(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t) + V_s$$

(Underdamped)

Example 8.7 For the circuit in Fig. 8.19, find $v(t)$ for $t > 0$. Consider these cases: $R = 5\ \Omega$, $R = 4\ \Omega$, $R = 1\ \Omega$.

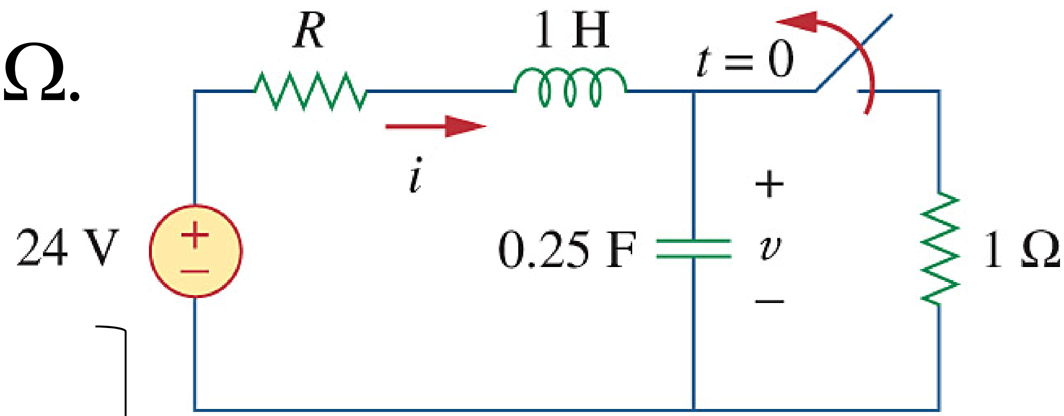


Figure 8.19

Solution :

$$\left.\begin{aligned} i(0^+) &= i(0^-) = \frac{24}{R+1} \\ v(0^+) &= v(0^-) = 24 \times \frac{1}{R+1} \end{aligned}\right\} \text{Step 1}$$

$$\left. i(t) = 0.25\frac{dv(t)}{dt} \Rightarrow v'(0^+) = \frac{1}{0.25}i(0^+) \right\} \text{Step 2}$$

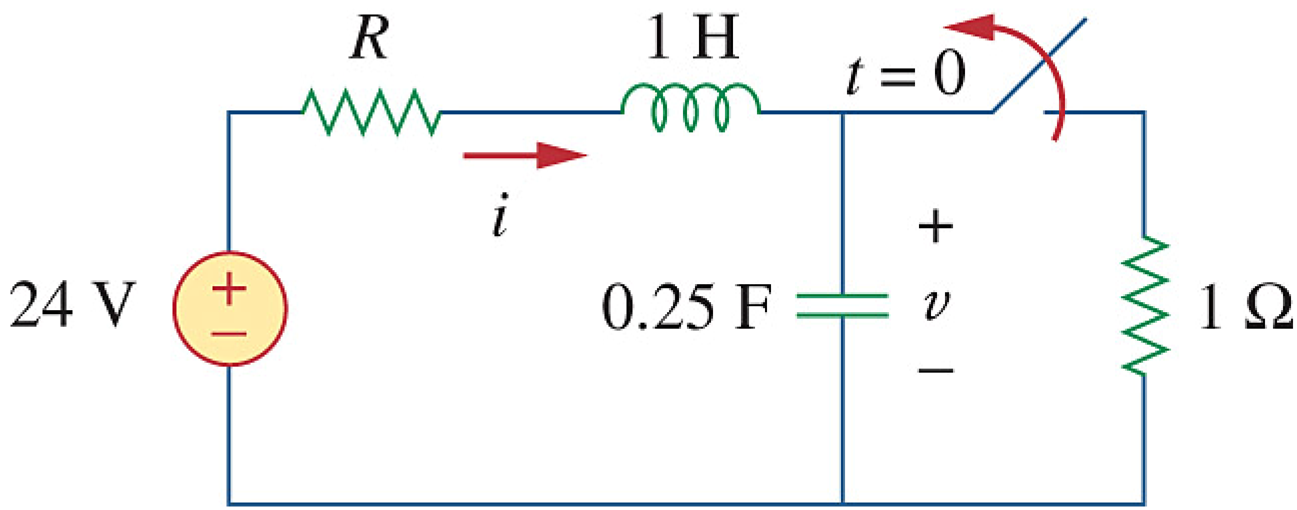


Figure 8.19

$$\left.\frac{d^2v}{dt^2}+\frac{R}{L}\frac{dv}{dt}+\frac{1}{LC}v=\frac{1}{LC}V_s\right\}\text{ Step 3 \& 4}$$

$$\boxed{\text{(a) } R = 5\ \Omega}$$

$$i(0^+) = 4\text{ A},\ v(0^+) = 4\text{ V},\ v'(0^+) = 16\text{ V/s}$$

$$\frac{d^2v}{dt^2}+5\frac{dv}{dt}+4v=96$$

$$s^2+5s+4=0 \Rightarrow s_1=-1,\ s_2=-4$$

$$v_n(t) = A_1e^{-t} + A_2e^{-4t}$$

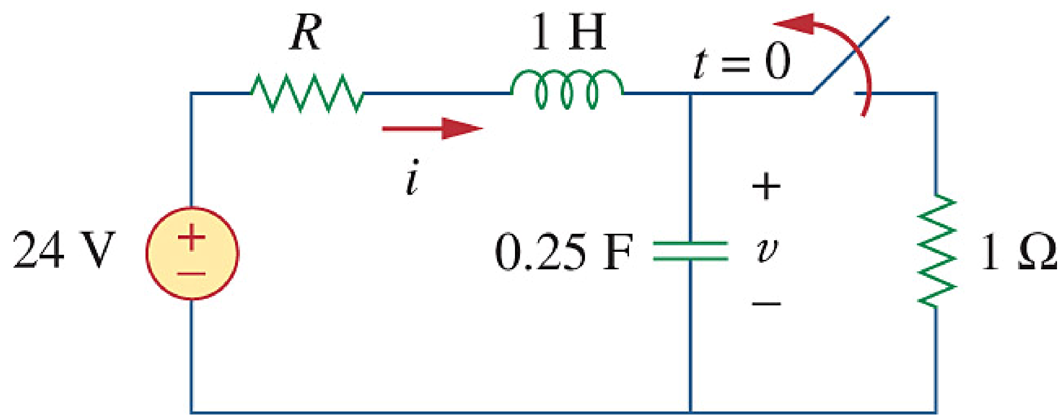


Figure 8.19

$$v_f(t) = B \Rightarrow B = 96/4 = 24$$

$$v(t) = v_n(t) + v_f(t) = A_1 e^{-t} + A_2 e^{-4t} + 24$$

$$v(0^+) = A_1 + A_2 + 24 = 4$$

$$v'(0^+) = -A_1 - 4A_2 = 16$$

$$A_1 = -\frac{64}{3},\ A_2 = \frac{4}{3}$$

$$v(t) = -\frac{64}{3}e^{-t} + \frac{4}{3}e^{-4t} + 24 \text{ (V)}$$

$$i(t) = \frac{16}{3}e^{-t} - \frac{4}{3}e^{-4t} \text{ (A)}$$

$$\boxed{\text{(b) } R = 4\ \Omega}$$

$$i(0^+) = 4.8 \text{ A}, v(0^+) = 4.8 \text{ V}, v'(0^+) = 19.2 \text{ V/s}$$

$$\frac{d^2v}{dt^2} + 4\frac{dv}{dt} + 4v = 96$$

$$s^2 + 4s + 4 = 0 \Rightarrow s_1 = s_2 = -2$$

$$v_n(t) = (A_1 + A_2 t)e^{-2t}$$

$$v_f(t) = B \Rightarrow B = 96/4 = 24$$

$$v(t) = v_n(t) + v_f(t) = (A_1 + A_2 t)e^{-2t} + 24$$

$$v(0^+) = A_1 + 24 = 4.8$$

$$v'(0^+) = A_2 - 2A_1 = 19.2$$

$$A_1 = A_2 = -19.2$$

$$v(t) = (-19.2 - 19.2t)e^{-2t} + 24 \text{ (V)}$$

$$i(t) = 4.8(1 + 2t)e^{-2t} \text{ (A)}$$

$$\boxed{\text{(c) } R = 1\ \Omega}$$

$$i(0^+) = 12 \text{ A}, v(0^+) = 12 \text{ V}, v'(0^+) = 48 \text{ V/s}$$

$$\frac{d^2v}{dt^2} + \frac{dv}{dt} + 4v = 96$$

$$s^2 + s + 4 = 0 \Rightarrow s_{1,2} = -\frac{1}{2} \pm j\frac{\sqrt{15}}{2}$$

$$v_n(t) = e^{-t/2}(A_1 \cos\frac{\sqrt{15}}{2}t + A_2 \sin\frac{\sqrt{15}}{2}t)$$

$$v_f(t) = B \Rightarrow B = 96/4 = 24$$

$$v(t) = v_n(t) + v_f(t)$$

$$= e^{-t/2}(A_1 \cos\frac{\sqrt{15}}{2}t + A_2 \sin\frac{\sqrt{15}}{2}t) + 24$$

$$v(0^+) = A_1 + 24 = 12$$

$$v'(0^+) = -\frac{1}{2}A_1 + \frac{\sqrt{15}}{2}A_2 = 48$$

$$A_1 = -12,\ A_2 = \frac{84}{\sqrt{15}} \approx 21.689$$

$$v(t) = e^{-t/2}(-12\cos\frac{\sqrt{15}}{2}t + \frac{84}{\sqrt{15}}\sin\frac{\sqrt{15}}{2}t) + 24\ \text{(V)}$$

$$i(t) = e^{-t/2}(12\cos\frac{\sqrt{15}}{2}t + \frac{12}{\sqrt{15}}\sin\frac{\sqrt{15}}{2}t)\ \text{(A)}$$

	(a)	(b)	(c)
R	5Ω	4Ω	1Ω
α	2.5	2	0.5
v_f	24V	24V	24V
	Overdamped	Critically damped	Underdamped

Figure 8.20 plots the responses for the three cases. From this figure, we observe that the critically damped response approaches the step input of 24 V the fastest.

Overall, the speed to reach steady state value:
Critically damped > Overdamped > Underdamped

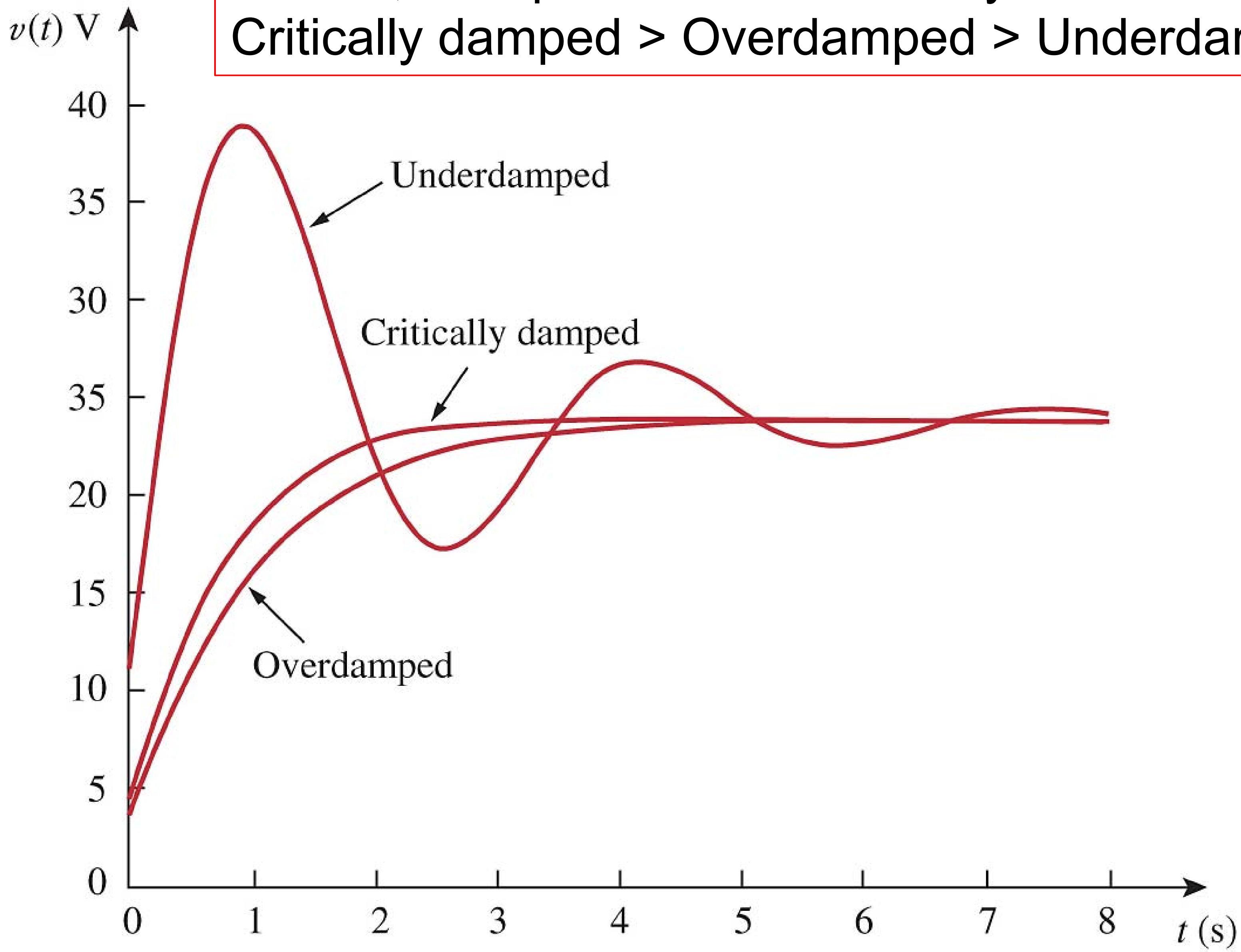


Figure 8.20

Steps for 2nd order circuit with *step input*

1. Plot the circuit at $t<0$, find initial conditions, $i(0^+)$, $v(0^+)$
2. Plot the circuit at $t>0$, express di/dt or dv/dt in terms of i_L and v_c, find initial conditions $di(0^+)/dt$, $dv(0^+)/dt$
3. Express the circuit in 2nd order D.E. with only one parameter (either i or v) and solve it.
4. Plot the circuit at $t \rightarrow \infty$, find steady state values $i(\infty)$, $v(\infty)$ (or just solve forced response)
5. Solve the coefficients using initial conditions.