Example

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The signal $x(t) = \sum_{n=-\infty}^{\infty} \text{rect}(t - 1/2 - 2n)$ is passed through a filter with frequency response $H(\omega) = 3\,\text{rect}(\omega/\pi)$. Determine the output signal $y(t)$.
(Selected from Midterm Exam 2 of Summer 2014)

Solution

$T_0 = 2$ so $\omega_0 = \pi$, so the harmonics of $x(t)$ are at multiples of π. All frequency components above $\pi/2$ are eliminated by this filter, so only the DC component passes through. $c_0 = 1/2$. Thus

$$\boxed{y(t) = c_0 H(0) = 3/2.}$$

$H(\omega)$ and $H(j\omega)$ are interchangeable notation

$$H(\omega) = H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}\,dt$$

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Consider the following cascade of LTI systems:

$$x(t) \to \boxed{h_1(t)} \to \boxed{h_2(t)} \to y(t),$$

where $h_1(t) = e^{-t}u(t)$ and $h_2(t) = e^{-3t}u(t)$.

1. Find the frequency response of the overall system.
2. Find the linear constant coefficient differential equation that describes this system.

(Selected from Midterm Exam 2 of Summer 2014)

Solution

1. *The overall frequency response is*

$$\boxed{H(\omega) = \frac{1}{j\omega + 1}\frac{1}{j\omega + 3}}$$

2. *The linear constant coefficient differential equation*

$$H(\omega) = \frac{1}{(j\omega + 1)(j\omega + 3)} = \frac{1}{(j\omega)^2 + 4j\omega + 3}$$

so

$$\boxed{3y(t) + 4\frac{\mathrm{d}}{\mathrm{d}t}y(t) + \frac{d^2}{dt^2}y(t) = x(t)}$$