

$$1) X(\omega) = 2\pi (\delta(\omega+45) + \delta(\omega+35) + \delta(\omega+25) + \delta(\omega+15) + \delta(\omega+5) \\ + \delta(\omega-5) + \delta(\omega-15) + \delta(\omega-25) + \delta(\omega-35) + \delta(\omega-45))$$

$$h(t) = \frac{20}{2\pi} \text{sinc}(\frac{20}{2\pi}t) + \frac{40}{2\pi} \text{sinc}(\frac{40}{2\pi}t) + \frac{60}{2\pi} \text{sinc}(\frac{60}{2\pi}t)$$

$$H(\omega) = \text{rect}(\frac{\omega}{20}) + \text{rect}(\frac{\omega}{40}) + \text{rect}(\frac{\omega}{60})$$

$$\text{Thus, } X_1(\omega) = X(\omega) \cdot H(\omega)$$

$$= \boxed{6\pi(\delta(\omega-5) + \delta(\omega+5)) + 4\pi(\delta(\omega-15) + \delta(\omega+15)) \\ + 2\pi(\delta(\omega-25) + \delta(\omega+25))}$$

$$2) W_s = 40, T_s = \frac{\pi}{20}.$$

$$X_s(\omega) = \frac{20}{\pi} \cdot \sum_{k=-\infty}^{+\infty} X_1(\omega - kW_s) = \frac{20}{\pi} \cdot \sum_{k=-\infty}^{+\infty} X_1(\omega - 40k)$$

Since  $H_2(\omega) = \text{rect}(\frac{\omega}{80})$ , we only need to find  $\omega \in [-40, 40]$ .

$$Z(\omega) = \frac{20}{\pi} \cdot 2\pi \cdot \left[ 3(\delta(\omega-5) + \delta(\omega+5)) + 3(\delta(\omega-15) + \delta(\omega+15)) \right. \\ \left. + 3(\delta(\omega-25) + \delta(\omega+25)) + 3(\delta(\omega-35) + \delta(\omega+35)) \right] \\ = 120(\delta(\omega-5) + \delta(\omega+5) + \delta(\omega-15) + \delta(\omega+15) + \delta(\omega-25) + \delta(\omega+25) \\ + \delta(\omega-35) + \delta(\omega+35))$$

$$\text{Thus, } Z(t) = \frac{60}{\pi} \cdot (e^{-5jt} + e^{5jt} + e^{-15jt} + e^{15jt} + e^{-25jt} + e^{25jt} + e^{-35jt} + e^{35jt}) \\ = \boxed{\frac{120}{\pi} \cdot (\cos(5t) + \cos(15t) + \cos(25t) + \cos(35t))}$$