Example

Example

Classify the following CT systems by circling one choice in each column of categories. Justify your answers.

- $y(t) = x(t-2) + \int_{t-1}^{t-3} e^{-(t-\tau)^2} x(\tau)\, d\tau$ (selected from Exam 1 in Summer 2015)
- $(2 + \sin(t))x(t)$

static	linear	time invariant	causal	BIBO stable
dynamic	nonlinear	time varying	noncausal	unstable
unsure	unsure	unsure	unsure	unsure

Solution

Solution

- $y(t) = x(t-2) + \int_{t-1}^{t-3} e^{-(t-\tau)^2} x(\tau)\, d\tau$

  dynamic, linear, time-invariant, causal, stable

- $(2 + \sin(t))x(t)$

  static, linear, time-variant, causal, stable

Hint :

1. Prove $y(t) = \int_{t-1}^{t+3} e^{-(t-\tau)^2} x(\tau)\, d\tau$ is Time-Invariant By Definition :

- $x(t) \xrightarrow{T} y(t) \dots\dots> y(t-t_0) = \int_{t-t_0-1}^{t-t_0+3} e^{-(t-t_0-\tau)^2} x(\tau)\, d\tau$

- $x_d = x(t-t_0) \xrightarrow{T} y_d(t) = \int_{t-1}^{t+3} e^{-(t-\tau)^2} x_d(\tau)\, d\tau = \int_{t-1}^{t+3} e^{-(t-\tau)^2} \cdot x(\tau-t_0)\, d\tau$. let $k = \tau - t_0 \Rightarrow \tau = k + t_0$

  $= \int_{t-t_0-1}^{t-t_0+3} e^{-(t-t_0-k)} \cdot x(k)\, dk$ ↳ $\in (t-t_0-1,\ t-t_0+3)$

$\Rightarrow y(t-t_0) = y_d(t)$. Time-Invariant .

2. $y(t) = (2 + \sin(t)) \cdot x(t)$.

Note that :

- $x(t) \Rightarrow$ static. casual
- $\sin(t) \Rightarrow$ time-varying