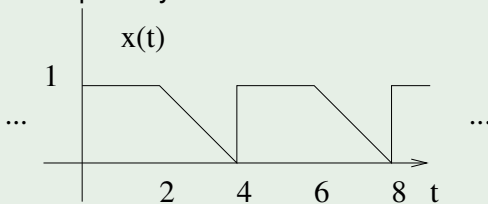


# Example

## Example

Find the Fourier series representation of the signal  $x(t)$  pictured below. Express your results in a real form.



# Solution (1)

## Method 1

$$T_0 = 4 \implies \omega_0 = \pi/2.$$

- $c_0 = (1/4)[2 + 1] = 3/4.$
- For  $k \neq 0$

$$\begin{aligned} c_k &= 1/4 \int_0^2 1 e^{-jk\omega_0 t} dt + 1/4 \int_2^4 (2 - t/2) e^{-jk\omega_0 t} dt \\ &= \frac{1}{j2\pi k} + \begin{cases} 0, & k \text{ even} \\ \frac{-1}{k^2\pi^2}, & k \text{ odd}, \end{cases} \end{aligned}$$

$$x(t) = 3/4 + \sum_{k=1}^{\infty} \frac{1}{\pi k} \sin\left(k\frac{\pi}{2}t\right) + \sum_{k=1 \text{ odd}}^{\infty} \frac{-2}{k^2\pi^2} \cos\left(k\frac{\pi}{2}t\right).$$

# Solution (2)

## Method 2

$$\frac{d}{dt}x(t) == \sum_{n=-\infty}^{\infty} \delta(t - 4n) - \frac{1}{2} \sum_{n=-\infty}^{\infty} \text{rect}\left(\frac{t - 3 - 4n}{2}\right)$$

The FS coefficients of  $y(t) = \frac{d}{dt}x(t)$  are

$k \neq 0$

$$\begin{aligned} d_k &= \frac{1}{T_0} \int_{T_0} y(t) e^{-jk\omega_0 t} dt = \frac{1}{4} \int_0^4 y(t) e^{-jk\omega_0 t} dt \\ &= \frac{1}{4} \int_0^4 \delta(t) e^{-jk\omega_0 t} dt - \frac{1}{8} \int_2^4 e^{-jk\omega_0 t} dt \\ &= \frac{1}{4} - \frac{1}{4} - \frac{1}{-jk\omega_0} e^{-jk\omega_0 t} \Big|_2^4 \\ &= \frac{1}{4} - \frac{1}{8jk\omega_0} (e^{-4jk\omega_0} - e^{-2jk\omega_0}) \end{aligned}$$

## Solution (3)

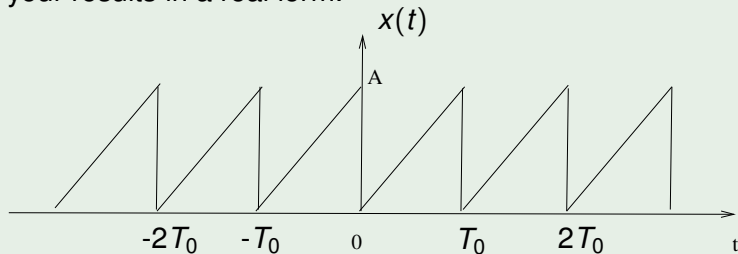
Since  $d_k = jk\omega_0 c_k$ ,

$$\begin{aligned} c_k &= \frac{1}{jk\omega_0} d_k = \frac{1}{4jk\omega_0} - \frac{1}{8(jk\omega_0)^2} (e^{-4jk\omega_0} - e^{-2jk\omega_0}) \\ &= \frac{1}{j2\pi k} + \frac{1}{j2\pi^2 k^2} (e^{-j2\pi k} - e^{-j\pi k}) \quad (\omega_0 = \pi/2) \\ &= \frac{1}{j2\pi k} + \begin{cases} 0, & k \text{ even} \\ \frac{-1}{k^2\pi^2}, & k \text{ odd}, \end{cases} \end{aligned}$$

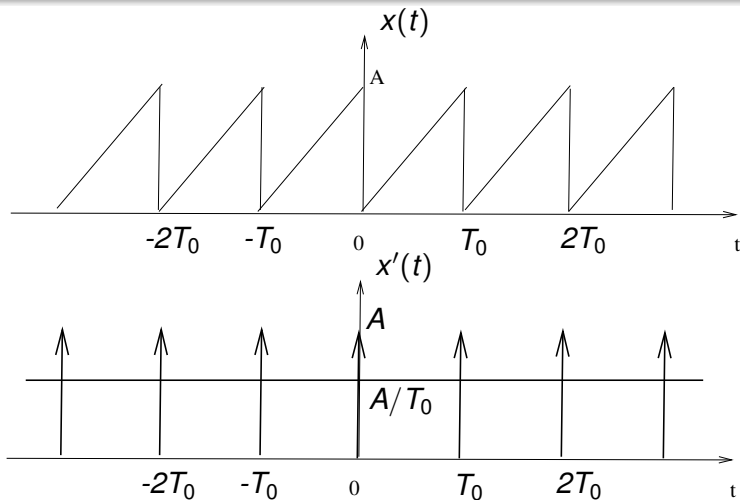
# Example

## Example

Find the FS of  $x(t)$  using the differentiation property. Express your results in a real form.

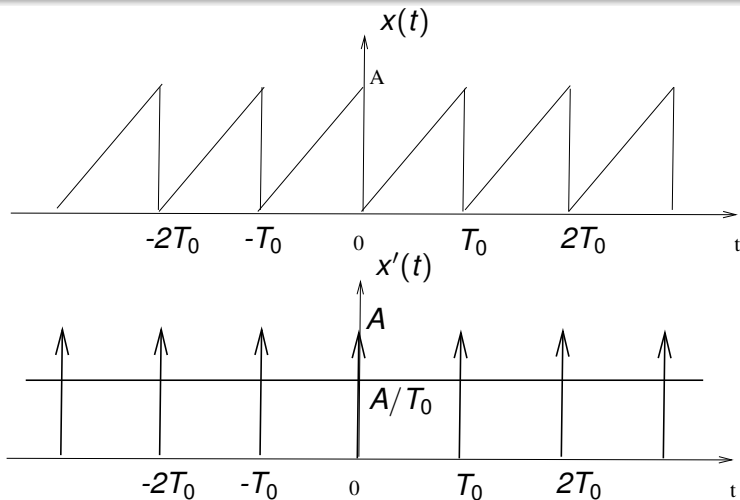


## Solution (1)



$$x'(t) = \frac{A}{T_0} + g(t), \quad g(t) = A \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$

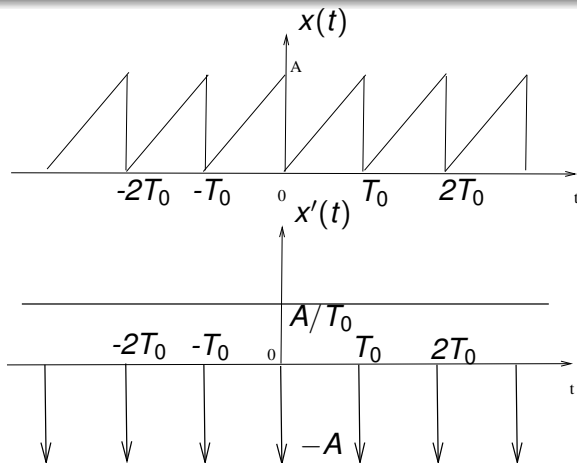
## Solution (1)



$$x'(t) = \frac{A}{T_0} + g(t), \quad g(t) = A \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$

*Wrong!*

## Solution (2)



$$x'(t) = \frac{A}{T_0} - g(t), \quad g(t) = A \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$



## Solution (2)

The FS coefficients for  $g(t)$  is

$$d'_k = \frac{1}{T_0} \underbrace{\int_{T_0} \delta(t) e^{-jk\omega_0 t} dt}_{=1 \text{ sifting property}} = \frac{1}{T_0}, \quad \forall k$$

$$g(t) = A \sum_{n=-\infty}^{\infty} \delta(t - nT_0) = A \sum_{k=-\infty}^{\infty} \frac{1}{T_0} e^{jk\omega_0 t}$$

$$x'(t) = \frac{A}{T_0} - A \sum_{k=-\infty}^{\infty} \frac{1}{T_0} e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} d_k e^{jk\omega_0 t}$$

$$d_k = \begin{cases} 0, & k = 0 \\ -A\frac{1}{T_0}, & k \neq 0 \end{cases}$$

## Solution (2)

The FS coefficients for  $g(t)$  is

$$d'_k = \frac{1}{T_0} \underbrace{\int_{T_0} \delta(t) e^{-jk\omega_0 t} dt}_{=1 \text{ sifting property}} = \frac{1}{T_0}, \quad \forall k$$

$$g(t) = A \sum_{n=-\infty}^{\infty} \delta(t - nT_0) = A \sum_{k=-\infty}^{\infty} \frac{1}{T_0} e^{jk\omega_0 t}$$

$$x'(t) = \frac{A}{T_0} - A \sum_{k=-\infty}^{\infty} \frac{1}{T_0} e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} d_k e^{jk\omega_0 t}$$

$$d_k = \begin{cases} 0, & k = 0 \\ -A\frac{1}{T_0}, & k \neq 0 \end{cases}$$

## Solution (2)

The FS coefficients for  $g(t)$  is

$$d'_k = \frac{1}{T_0} \underbrace{\int_{T_0} \delta(t) e^{-jk\omega_0 t} dt}_{=1 \text{ sifting property}} = \frac{1}{T_0}, \quad \forall k$$

$$g(t) = A \sum_{n=-\infty}^{\infty} \delta(t - nT_0) = A \sum_{k=-\infty}^{\infty} \frac{1}{T_0} e^{jk\omega_0 t}$$

$$x'(t) = \left( \frac{A}{T_0} \right) A \sum_{k=-\infty}^{\infty} \frac{1}{T_0} e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} d_k e^{jk\omega_0 t}$$
$$d_k = \begin{cases} 0, & k = 0 \\ -A\frac{1}{T_0}, & k \neq 0 \end{cases}$$

## Solution (3)

$$c_k = \begin{cases} \frac{1}{T_0} \int_0^{T_0} x(t) e^{-jk\omega_0 t} dt = \frac{1}{T_0} \int_0^{T_0} \frac{A}{T_0} t dt = \frac{A}{2}, & k = 0 \\ \frac{d_k}{jk\omega_0} = -\frac{A}{T_0 jk\omega_0} = j\frac{A}{2\pi k}, & k \neq 0 \end{cases}$$

$$x(t) = \frac{A}{2} + \sum_{k=-\infty, k \neq 0}^{\infty} j\frac{A}{2\pi k} e^{jk\omega_0 t}$$

$$x(t) = \frac{d}{dt} \chi(t)$$

## Solution (3)

$$c_k = \begin{cases} \frac{1}{T_0} \int_0^{T_0} x(t) e^{-jk\omega_0 t} dt = \frac{1}{T_0} \int_0^{T_0} \frac{A}{T_0} t dt = \frac{A}{2}, & k = 0 \\ \frac{d_k}{jk\omega_0} = -\frac{A}{T_0 jk\omega_0} = j\frac{A}{2\pi k}, & k \neq 0 \end{cases}$$

$$x(t) = \frac{A}{2} + \sum_{k=-\infty, k \neq 0}^{\infty} j\frac{A}{2\pi k} e^{jk\omega_0 t}$$

## Solution (4)

$$\begin{aligned}x(t) &= \frac{A}{2} + \sum_{k=1}^{\infty} j \left[ \frac{A}{2\pi k} e^{jk\omega_0 t} - \frac{A}{2\pi k} e^{-jk\omega_0 t} \right] \\&= \frac{A}{2} - \frac{A}{\pi} \sum_{k=1}^{\infty} \frac{e^{jk\omega_0 t} - e^{-jk\omega_0 t}}{2jk} \\&= \boxed{\frac{A}{2} - \frac{A}{\pi} \sum_{k=1}^{\infty} \frac{1}{k} \sin(k\omega_0 t)}\end{aligned}$$