① Chapter 1

- $y = x(at-b)$

① $d(t) = x(t-b)$

② $y(t) = d(at) = x(at-b)$

- $y = x\left(\frac{t-t_0}{w}\right)$

① $s(t) = x\left(\frac{t}{w}\right)$

② $y(t) = s(t-t_0) = x\left(\frac{t-t_0}{w}\right)$

- Differentiator: $y(t) = \frac{d}{dt}x(t)$
- Integrator: $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$
- A sum of two periodic signal is periodic iff the ratio of their periods is rational
- $x(t) = x_e(t) + x_o(t)$, where $x_e(t) = \frac{1}{2}[x(t)+x(-t)]$ and $x_o(t) = \frac{1}{2}[x(t)-x(-t)]$
- Average value:

$$A = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x(t)dt$$

奇的A为0.

- Energy $E = \int_{-\infty}^{\infty}|x(t)|^2dt$
- Average power:

$$P = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|x(t)|^2dt$$

- 如果E有限($E<\infty$)则$x(t)$为energy signal 并且$P=0$; 如果E无穷, 则P既可有限也可无穷。如果P有限并且非0, 则$x(t)$为power signal.
- Euler's formula:

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

- unit step function:

$$u(t) = \begin{cases}1, & t>0 \text{ or } t\geq 0\\ 0, & t<0\end{cases}$$

- rect(angle) function

$$rect(t) = \begin{cases}1, & -\frac{1}{2}<t<\frac{1}{2}\\ 0, & otherwise\end{cases}$$

$$rect\left(\frac{t-t_0}{T}\right) = \begin{cases}1, & t_0-\frac{T}{2}<t<t_0+\frac{T}{2}\\ 0, & otherwise\end{cases}$$

centered at t_0 with width T.

- unit impluse function
  zero width, infinite height, unit area

$\int_{-\infty}^{\infty}\delta(t-t_0)dt = 1$ for any t_0

$\delta(at+b) = \frac{1}{|a|}\delta\left(t+\frac{b}{a}\right)$ for $a\neq 0$

$\delta(t) = \delta(-t)$, $\delta(t-t_0) = 0$ for $t\neq t_0$

$\delta(t) = \frac{d}{dt}u(t)$, $u(t) = \int_{-\infty}^{t}\delta(\tau)d\tau$

$x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$

$\int_{-\infty}^{\infty}x(t)\delta(t-t_0)dt = x(t_0)$

$t\delta(t)=0$

- Practical implues function

$$\delta_\Delta(t) = \begin{cases}\frac{1}{\Delta}, & 0<t<\Delta\\ 0, & otherwise\end{cases}$$

- Practical almost-step function

$$u_\Delta(t) = \begin{cases}0; & t\leq 0\\ t/\Delta; & 0<t<\Delta\\ 1; & t\geq\Delta\end{cases}$$

$$\frac{d}{dt}u_\Delta(t) = \begin{cases}\frac{1}{\Delta}; & 0<t<\Delta\\ 0; & otherwise\end{cases}$$

$rect(t) = u(t+\frac{1}{2}) - u(t-\frac{1}{2})$
$= u(t+\frac{1}{2})\times u(t-\frac{1}{2})$

- moving average filter:

$$y(t) = \frac{1}{T}\int_{t-T}^{T}x(\tau)d\tau$$

- series connection: $y(t) = T_2[T_1[x(t)]]$
- parallel: $y(t) = T_1[x(t)] + T_2[x(t)]$
- Linear iff $T[a_1x_1(t)+a_2x_2(t)] = a_1T[x_1(t)] + a_2T[x_2(t)]$

线性的property: $T[ax(t)] = aT[x(t)]$

$T[x_1(t)+x_2(t)] = T[x_1(t)] + T[x_2(t)]$

$T[\sum x_k(t)] = \sum T[x_k(t)]$

$T[\int x(t;v)dv] = \int T[x(t;v)]dv$

- BIBO stable

If $\exists M_x$ s.t. $|x(t)|\leq M_x<\infty$ for $\forall t$ then there must $\exists M_y$ s.t. $|y(t)|\leq M_y<\infty$ for $\forall t$.

利于证明: $|a+b|\leq|a|+|b|$

$|\Sigma_n a_n|\leq\Sigma|a_n|$

$|\int f(t)dt|\leq\int|f(t)|dt$

- Invertibility: there $\exists T^{-1}$ s.t. $T^{-1}[T[x(t)]] = x(t)$
- Causal: $y(t)$ depends only on the present and past inputs
- Memoryless: $y(t)$ depends only on the current input $x(t)$

Memoryless 一定 causal

Dynamic (不memorlyss)可能causal 或为不causal.

- Time invariance

① 求 $y(t-t_0)$, 把所有t换成$t-t_0$

② 求 $y_d(t)$, $x(t)$换成$x_d(t) = x(t-t_0)$

③ 如果 $y_d(t) = y(t-t_0)$ 则符合.

Chapter 2

- $y(t) = \int_{-\infty}^{\infty}x(\tau)h(t-\tau)d\tau$
  $= x(t)*h(t) = (x*h)(t)$
- Shifting property:

$x(t) = \int_{-\infty}^{\infty}x(\tau)\delta(t-\tau)d\tau$

$x(t_0) = \int_{-\infty}^{\infty}x(t)\delta(t-t_0)dt$

- property: $x(t)*h(t) = h(t)*x(t)$

$[x(t)*h_1(t)]*h_2(t) = x(t)*[h_1(t)*h_2(t)]$

$x(t)*[h_1(t)+h_2(t)] = x(t)*h_1(t) + x(t)*h_2(t)$

$x(t)*\delta(t) = x(t)$

$x(t)*\delta(t-t_0) = x(t-t_0)$

$\delta(t-t_0)*\delta(t-t_1) = \delta(t-t_0-t_1)$

If $y(t) = x(t)*h(t)$, then

$x(t-t_0)*h(t-t_1) = y(t-t_0-t_1)$

- An LTI system is causal iff $h(t)=0$ for all $t<0$.
  When causal,

$$y(t) = \int_{-\infty}^{t}x(\tau)h(t-\tau)d\tau$$

- A causal signal is a signal $x(t)$ which is zero for all $t<0$
- If the input to a causal LTI system is a causal signal, the output is simply

$$y(t) = \begin{cases}0, & t<0\\ \int_0^t x(\tau)h(t-\tau)d\tau, & t\geq 0\end{cases}$$

- Memoryless: An LTI system is memoryless iff its impulse response is $h(t) = a\delta(t)$. Otherwise, dynamic response. $y(t) = x(t)*h(t) = ax(t)$
- An LTI sys is BIBO stable iff its impulse response is absolutely integrable, eg. $\int_{-\infty}^{\infty}|h(t)|dt<\infty$
- An LTI sys is invertable iff $\exists$ an inverse sys whose impulse response $h_i(t)$ satisfies the following relationship with $h(t)$

$h(t)*h_i(t) = \delta(t)$

- (Unit) step response

$s(t) = \int_{-\infty}^{\infty}u(t-\tau)h(\tau)d\tau = \int_{-\infty}^{t}h(\tau)d\tau$

If the sys is causal

$s(t) = [\int_0^t h(\tau)d\tau]\,u(t)$

$h(t) = \frac{d}{dt}s(t)$

Chapter 3

- transfer function

$x(t) = e^{st}\xrightarrow{LTI}y(t) = e^{st}H(s)$

$H(s) = \int_{-\infty}^{\infty}h(t)e^{-st}dt$

当$s=j\omega$时, $H(j\omega)$ is called the frequency response, 通常写为$H(\omega)$.

- 找RC电路的H(s)

$h(t) = \alpha e^{-\alpha t}u(t)$, where $\alpha = \frac{1}{RC}$

结果 $H(s) = \frac{1}{1+(RC)s}$

- Euler's identity:

$e^{j\theta} = \cos\theta + j\sin\theta$

$\cos\theta = \frac{1}{2}(e^{j\theta}+e^{-j\theta})$

$\sin\theta = \frac{1}{2}(e^{j\theta}-e^{-j\theta})$

- 具有linearity:

$x(t) = \sum_k C_k e^{j\omega_0kt}\xrightarrow{LTI}$

$y(t) = \sum_k C_k H(j\omega_0k)e^{j\omega_0kt}$

若输入为周期信号, 输出也为周期信号且周期相同

- Periodic signal $x(t)$ with 最小周期T_0有如下FS

$$x(t) = \sum_{k=-\infty}^{\infty}C_k e^{jk\omega_0t} \quad (\text{synthesis equation})$$

$\omega_0 = \frac{2\pi}{T}$, C_k是Fourier coefficients

$k\omega_0$ is called the kth harmonic

$$C_k = \frac{1}{T_0}\int_{T_0}x(t)e^{-jk\omega_0t}dt$$

注: k=0时we get the average value or DC value of the signal

频率 $f = \frac{1}{T}$

Hermitian property:

If $x(t)$ is real, then $C_{-k} = C_k^*$

C_k^*为C_k的共轭, j部分变为原来的相反数.

- Combined trigonometric form

$$x(t) = C_0 + \sum_{k=1}^{\infty}2|C_k|\cos(k\omega_0t+\theta_k)$$

where $C_k = |C_k|e^{j\theta_k}$, $\theta_k = \angle C_k$

- Trigonometric form

$x(t) = C_0 + 2\sum_{k=1}^{\infty}[A_k\cos(k\omega_0t) - B_k\sin(k\omega_0t)]$, where $A_k = Real(C_k)$, $B_k = Imag(C_k)$

- error signal:

$e_N(t) = x(t) - x_N(t)$

- Amplitude transforms

$y(t) = ax(t)+b$, where $\omega_1 = \omega_0$

then $d_k = \begin{cases}b+aC_0 & k=0\\ aC_k & k\neq 0\end{cases}$

- general time transforms

$y(t) = x(at+b)$, where $\omega_1 = a\omega_0$

then $d_k = C_k e^{jk\omega_0b}$

- time reversel

$y(t) = x(-t) \Rightarrow d_k = C_{-k}$

- time shift

$y(t) = x(t-t_0)$, where $\omega_1 = \omega_0$

$d_k = C_k e^{-jk\omega_0t_0}$

- conjugation

$y(t) = [x(t)]^* \Rightarrow d_k = C_{-k}^*$

- complex modulation

$y(t) = x(t)e^{j\omega_0tN}$,

$d_k = C_{k-N}$

- Differentiation

$y(t) = \frac{d}{dt}x(t) \Rightarrow d_k = jk\omega_0C_k$, k

- Linearity: $x_1(t)$ 和 $x_2(t)$ 周期 T_0 相同, $x(t)=Ax_1(t)+Bx_2(t)$, x_1 的FS系数为 a_k, x_2 的FS系数为 b_k, 则 $x(t)$ 的为 $C_k=Aa_k+Bb_k$
- Multiplication: $x(t)$ 和 $y(t)$ 有相同的周期 T_0 且 $x(t)y(t)$ 的周期也为 T_0, $x(t)\to a_k$, $y(t)\to b_k$, 则 $x(t)y(t)\to C_k=\sum_{l=-\infty}^{\infty}a_l b_{k-l}$
- Parseval's relation for 周期信
$$P=\underbrace{\frac{1}{T_0}\int_{T_0}|x(t)|^2dt}_{\text{average power}}=\sum_{k=-\infty}^{\infty}|C_k|^2$$
- power density spectrum: a plot of $|C_k|^2$ vs $k\omega_0$
- magnitude spectrum: $|C_k|$ vs $k\omega_0$
phase spectrum. $\angle C_k$ vs $k\omega_0$
- $\operatorname{sinc}(x)=\begin{cases}\frac{\sin\pi x}{\pi x} & x\neq 0\\ 1 & x=0\end{cases}$
- $H(j\omega)=\int_{-\infty}^{\infty}h(t)e^{-j\omega t}dt$
$=|H(j\omega)|e^{j\angle H(j\omega)}$
- $z=x+jy$, ploar form: $z=|z|e^{j\theta}$, $|z|=\sqrt{x^2+y^2}$, $\theta=\angle z=\arctan\frac{y}{x}$
- $x(t)=\sum_k C_k e^{jk\omega_0 t}\xrightarrow{LTI}$
$y(t)=\sum_k C_k H(jk\omega_0)e^{jk\omega_0 t}$
- Hermitian symmetry
If $h(t)$ is real, then $H^*(s)=H(s^*)$ and $H(-j\omega)=H^*(j\omega)$
- $x(t)=\sum_k A_k\cos(\omega_k t+\phi_k)$
$\xrightarrow{LTI}y(t)=\sum_k A_k|H(j\omega_k)|\cos(\omega_k t+\phi_k+\angle H(j\omega_k))$
- $x(t)=\sum_k A_k\sin(\omega_k t+\phi_k)$
$\xrightarrow{LTI}y(t)=\sum_k A_k|H(j\omega_k)|\sin(\omega_k t+\phi_k+\angle H(j\omega_k))$
- Filters described by diffeqs
$$\sum_{k=0}^{N}a_k\frac{d^k}{dt^k}y(t)=\sum_{k=0}^{M}b_k\frac{d^k}{dt^k}x(t)$$
$$H(s)=\frac{\sum_{k=0}^{M}b_k s^k}{\sum_{k=0}^{N}a_k s^k}$$

- total harmonic distortion (THD)
$$THD=\frac{\text{avg. powe in DC \& harmonics}}{\text{avg. signal power}}\times 100\%$$
$$=\left[1-\frac{\text{avg. power in fundametal}}{\text{avg. signal power}}\right]\times 100\%$$

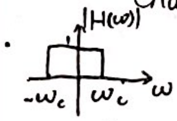
Chapter 4

$$F(\omega)=\int_{-\infty}^{\infty}f(t)e^{-j\omega t}dt$$
$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)e^{j\omega t}d\omega$$
$$\operatorname{sinc}(x)=\begin{cases}1, & x=0\\ \frac{\sin\pi x}{\pi x}, & x\neq 0\end{cases}$$
- $\operatorname{rect}\left(\frac{t}{\tau}\right)\xleftrightarrow{F}\tau\operatorname{sinc}\left(\tau\frac{\omega}{2\pi}\right)$
- $\delta(t-t_0)\xleftrightarrow{F}e^{-j\omega t_0}$
- $1\xleftrightarrow{F}\delta\left(\frac{\omega}{2\pi}\right)=2\pi\delta(\omega)$
- $e^{-at}u(t)\xleftrightarrow{F}\frac{1}{j\omega+a}\quad(a>0)$

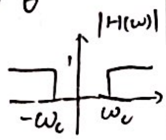
$\operatorname{sgn}(t)=\begin{cases}1, & t>0\\ -1, & t<0\end{cases}=u(t)-u(-t)=2u(t)-1$
- $\operatorname{sgn}(t)\xleftrightarrow{F}\frac{2}{j\omega}$
- $x(t)=\sum_{k=-\infty}^{\infty}C_k e^{jk\omega_0 t}\xleftrightarrow{F}X(\omega)=\sum_{k=-\infty}^{\infty}C_k 2\pi\delta(\omega-k\omega_0)$
- $e^{j\omega_0 t}\xleftrightarrow{F}2\pi\delta(\omega-\omega_0)$
- Linearity: 线性
$f(t)=a_1f_1(t)+a_2f_2(t)\xleftrightarrow{F}F(\omega)=a_1F_1(\omega)+a_2F_2(\omega)$
- $x(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT_0)=\sum_{k=-\infty}^{\infty}\frac{1}{T_0}e^{jk\omega_0 t}$
$\xleftrightarrow{F}X(\omega)=\sum_{k=-\infty}^{\infty}\frac{1}{T_0}2\pi\delta(\omega-k\omega_0)=\sum_{k=-\infty}^{\infty}\omega_0\delta(\omega-k\omega_0)$
- $\cos\omega_0 t\xleftrightarrow{F}\pi\delta(\omega-\omega_0)+\pi\delta(\omega+\omega_0)$
- $\cos(\omega_0 t+\phi)\xleftrightarrow{F}\pi e^{j\phi}\delta(\omega-\omega_0)+\pi e^{-j\phi}\delta(\omega+\omega_0)$
- Time transform property:
$f(at+b)\xleftrightarrow{F}\frac{1}{|a|}e^{j\omega b/a}F(\omega/a)$
- Time-shift
$f(t-t_0)\xleftrightarrow{F}e^{-j\omega t_0}F(\omega)$
- Time-scale
$f(at)\xleftrightarrow{F}\frac{1}{|a|}F\left(\frac{\omega}{a}\right)$
$f(-t)\xleftrightarrow{F}F(-\omega)$ time-reversal
- $x(t)$ even: $x(-t)=x(t)$ then $X(\omega)=X(-\omega)$
- conjugation: $f^*(t)\xleftrightarrow{F}F^*(-\omega)$
$f(t)$ real: $f(t)=f^*(t)$, then $F(\omega)=F^*(-\omega)$, Hermitian symmetric
$\angle F(\omega)=\angle F^*(-\omega)=-\angle F(-\omega)$

$|F(\omega)|=|F^*(-\omega)|=|F(-\omega)|$
- If $f(t)$ is real and even, $F(\omega)$ is also real and even. If $f(t)$ is real and odd, $F(\omega)$ is purely imaginary and odd.
- Duality: $f(t)\xleftrightarrow{F}F(\omega)$
then: $x(t)=F(t)\xleftrightarrow{F}X(\omega)=2\pi f(-\omega)$
- Time differentiation
$\frac{d^k}{dt^k}f(t)\xleftrightarrow{F}(j\omega)^k F(\omega)$
- Frequency differentiation
$(-jt)^n f(t)\xleftrightarrow{F}\frac{d^n}{d\omega^n}F(\omega)$
- $\omega=0$ (DC) value
$F(0)=\int_{-\infty}^{\infty}f(t)dt$
- $t=0$ value $f(0)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)d\omega$
- convolution:
$y(t)=h(t)*x(t)\xleftrightarrow{F}Y(\omega)=H(\omega)X(\omega)$
- time integration
$\int_{-\infty}^{t}f(\tau)d\tau\xleftrightarrow{F}\frac{F(\omega)}{j\omega}+\pi F(0)\delta(\omega)$
- PFE method
- Parseval's relation
$E=\int_{-\infty}^{\infty}|x(t)|^2dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2d\omega$
$|X(\omega)|$: energy density spectrum
- Time-domain multiplication
$f_1(t)f_2(t)\xleftrightarrow{F}\frac{1}{2\pi}F_1(\omega)*F_2(\omega)$
- Frequency shift (complex modulation)
$e^{j\omega_0 t}f(t)\xleftrightarrow{F}F(\omega-\omega_0)$
inductor: $j\omega L$ capacitor: $\frac{1}{j\omega C}$

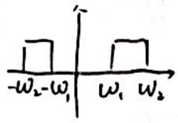
Chapter 6



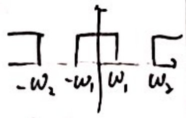
low pass filter
ω_c is cut off signal



high pass filter



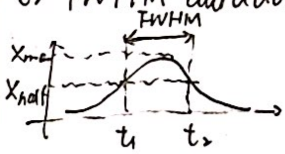
band pass filter



bandstop

- $Y(\omega)=H(\omega)X(\omega)\Rightarrow|Y(\omega)|\cdot e^{j\angle Y(\omega)}=|H(\omega)|e^{j\angle H(\omega)}|X(\omega)|e^{j\angle X(\omega)}$
phase: addition $\angle Y(\omega)=\angle H(\omega)+\angle X(\omega)$
magnitude. multiplication $|Y(\omega)|=|H(\omega)||X(\omega)|$
magnitude on a logarithmic scale
addition. $\lg|Y(\omega)|=\lg|H(\omega)|+\lg|X(\omega)|$
- $20\log_{10}(\cdot)$ decibles (dB)
- bode plots: plots of $20\log_{10}|H(\omega)|$ and $\angle H(\omega)$ versus $\log_{10}\omega$

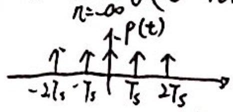
- For symmetric spectra, root mean-squared bandwidth or RMS bandwidth is defined as:
$$\omega_{rms}=\sqrt{\frac{\int_{-\infty}^{\infty}\omega^2|X(\omega)|^2d\omega}{\int_{-\infty}^{\infty}|X(\omega)|^2d\omega}}$$
not required to calculate
- fort time-limited signal, absolute time duration
$\tau=t_2-t_1$
- for non-time-limited signals, an frequently-used measure is the full-width at half maximum or FWHM duration



- For symmetric signals, root mean-squared time duration or RMS time duration is
$$\tau_{rms}=\sqrt{\frac{\int_{-\infty}^{\infty}t^2|x(t)|^2dt}{\int_{-\infty}^{\infty}|x(t)|^2dt}}$$
- $\omega_{rms}\tau_{rms}\geq\frac{1}{2}$ inversely related

Chapter 7

- Ideal periodic sampling or uniform sampling is defined by $x[n]=x(nT_s)$, $n=0,\pm1,\pm2,\cdots$. T_s is the sampling period or sampling interval. $\omega_s/2\pi=1/T_s$ is called the sampling rate or the sampling frequency.
- Dirac comb/ideal sampling function/impulse train
$p(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT_s)$



$x_s(t)=x(t)p(t)=\sum_{n=-\infty}^{\infty}x(nT_s)\delta(t-nT_s)$
$p(t)\xleftrightarrow{F}P(\omega)=\sum_{k=-\infty}^{\infty}\frac{2\pi}{T_s}\delta(\omega-k\omega_s)$
$X_s(\omega)=\frac{1}{T_s}\sum_{k=-\infty}^{\infty}X(\omega-k\omega_s)$
- Bandlimited signal: whose spectrum is nonzero only over finite interval.

2) aliasing: overlap of the spectral replicates

- sampling theorem: Let $x(t)$ be a band-limited signal with $X(\omega)=0$ for $|\omega|>\omega_{max}$. Then $x(t)$ is uniquely determined by its samples $x[n]=x(nT_s)$, $n=0,\pm1,\cdots$ if $\omega_s>2\omega_{max}$ where $\omega_s=\frac{2\pi}{T_s}$ and $2\omega_{max}$ is called Nyquist rate
- to recover $x(t)$ from $x_s(t)$, we need a filter with frequency response: $H(\omega)=T_s\,\mathrm{rect}\left(\frac{\omega}{2\omega_c}\right)$, where $\omega_{max}<\omega_c<\omega_s-\omega_{max}$. usually $\omega_c=\frac{\omega_s}{2}=\omega_{max}$

$$h(t)=\frac{\omega_c T_s}{\pi}\,\mathrm{sinc}\left(\frac{\omega_c}{\pi}t\right)$$

- $X_r(\omega)=H(\omega)X_s(\omega)$, $H(\omega)=T_s\,\mathrm{rect}\left(\frac{\omega}{2\omega_c}\right)$ — ω_c的范围 $(\omega_{max}, \omega_s-\omega_{max})$

$$x_r(t)=\sum_{n=-\infty}^{\infty}x(nT_s)\frac{\omega_c T_s}{\pi}\cdot\mathrm{sinc}\left(\frac{\omega_c(t-nT_s)}{\pi}\right)$$

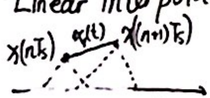
$$H(\omega)\overset{F}{\longleftrightarrow}h(t)=\frac{\omega_c T_s}{\pi}\,\mathrm{sinc}\left(\frac{\omega_c t}{\pi}\right)$$

Usually, we approximate that $\omega_c=\frac{\omega_s}{2}$, $\omega_s=\frac{2\pi}{T_s}$, then

$$x_r(t)=\sum_{n=-\infty}^{\infty}x(nT_s)\,\mathrm{sinc}\left(\frac{t-nT_s}{T_s}\right)$$

This is known as sinc interpolation

- Linear interpolation



$$x_1(t)=\left(1-\frac{t-nT_s}{T_s}\right)x(nT_s)+\frac{t-nT_s}{T_s}\cdot x((n+1)T_s)$$

$$x_1(t)=\sum_{n=-\infty}^{\infty}x(nT_s)\,\mathrm{tri}\left(\frac{t-nT_s}{T_s}\right)$$

$$=\mathrm{tri}\left(\frac{t}{T_s}\right)*\sum_{n=-\infty}^{\infty}x(nT_s)\delta(t-nT_s)$$

$$=\mathrm{tri}\left(\frac{t}{T_s}\right)*x_s(t)$$

$$h_1(t)=\mathrm{tri}\left(\frac{t}{T_s}\right)\overset{F}{\longleftrightarrow}H_1(\omega)=T_s\,\mathrm{sinc}^2\left(\frac{T_s\omega}{2\pi}\right)=T_s\,\mathrm{sinc}^2\left(\frac{\omega}{\omega_s}\right)$$

$$X_1(\omega)=\sum_{k=-\infty}^{\infty}\mathrm{sinc}^2\left(\frac{\omega}{\omega_s}\right)X(\omega-k\omega_s)$$

- zero-order hold interpolation

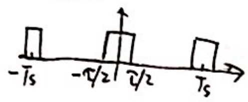
the impluse response of the zero-order hold filter is

$$h_2(t)=\mathrm{rect}\left(\frac{t}{T_s}-\frac{1}{2}\right)$$

$$h_2(t)\overset{F}{\longleftrightarrow}H_2(\omega)=T_s\,\mathrm{sinc}\left(\frac{\omega T_s}{2\pi}\right)e^{-j\omega T_s/2}=T_s\sin\left(\frac{\omega}{\omega_s}\right)e^{-j\omega T_s/2}$$

- rectangular pulse train

$$p(t)=\sum_{n=-\infty}^{\infty}\mathrm{rect}\left(\frac{t-nT_s}{\tau}\right)\quad(\tau\le T_s)$$



$$p(t)=\sum_{k=-\infty}^{\infty}c_k e^{jk\omega_s t}$$

$$c_k=\frac{1}{T_s}\int_{-T_s/2}^{T_s/2}p(t)e^{-jk\omega_s t}dt$$

$$x_s(t)=x(t)p(t)=\sum_{k=-\infty}^{\infty}c_k\left[x(t)e^{jk\omega_s t}\right]$$

$$X_s(\omega)=\sum_{k=-\infty}^{\infty}c_k X(\omega-k\omega_s)$$

- $\sum_{n=-\infty}^{\infty}x(nT_s)\,\mathrm{sinc}\left(\frac{t-nT_s}{T_s}\right)\overset{F}{\longleftrightarrow}\sum_{n=-\infty}^{\infty}x(nT_s)e^{-j\omega nT_s}\,\mathrm{rect}\left(\frac{T_s\omega}{2\pi}\right)$

Chapter 8

- Modulation property

$$e^{j\omega_0 t}f(t)\overset{F}{\longleftrightarrow}F(\omega-\omega_0)$$

$$f(t)\cos\omega_0 t\overset{F}{\longleftrightarrow}\frac{F(\omega-\omega_0)+F(\omega+\omega_0)}{2}$$

- Antenna length requirement

it should be longer than $\lambda/10$.

λ是波长, $\lambda=\frac{c}{f}$

- $x(t)$: modulating signal (audio)

$c(t)$: carrier signal (高频率 cos 函数)

$c(t)=\cos(\omega_c t+\theta_c)$

ω_c是carrier frequency

$$y(t)=x(t)c(t)\overset{F}{\longleftrightarrow}Y(\omega)=\frac{1}{2}\left[e^{j\theta_c}X(\omega-\omega_c)+e^{-j\theta_c}X(\omega+\omega_c)\right]$$

- restore to baseband (synchronous)

$$y(t)\rightarrow\otimes\rightarrow w(t)\rightarrow\boxed{h(t)}\rightarrow x(t)$$

$\cos(\omega_c t+\theta_c)$ — low-pass filter

$$W(t)=y(t)\cos(\omega_c t+\theta_c)$$

$$W(\omega)=\frac{1}{2}\left[e^{j\theta_c}Y(\omega-\omega_c)+e^{-j\theta_c}Y(\omega+\omega_c)\right]$$

$$=\frac{1}{4}e^{2j\theta_c}X(\omega-2\omega_c)+\frac{1}{2}X(\omega)+\frac{1}{4}e^{-2j\theta_c}X(\omega+2\omega_c)$$

- DSB/WC−AM

$$y(t)=(A+x(t))\cos(\omega_c t)$$

$$Y(\omega)=A\pi[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)]+\frac{1}{2}[X(\omega-\omega_c)+X(\omega+\omega_c)]$$

$m(t)=A+\hat{x}(t)$ → approximation

$y(t)\rightarrow$ envelop detector $\rightarrow m(t)\rightarrow$ DC blocking filter $\rightarrow\hat{x}(t)$

IF filter: $\omega_{IF}/2\pi=455$ kHz

Chapter 9

- Bilateral Laplace transform

$$X(s)=\int_{-\infty}^{\infty}x(t)e^{-st}dt$$

$s=\sigma+j\omega$ is a complex variable with real part σ and imaginary part ω.

$$x(t)\overset{L}{\longleftrightarrow}X(s)$$

- The unilateral laplace transform

$$X_+(s)=\int_{0^-}^{\infty}x(t)e^{-st}dt$$

$$e^{-at}u(t)\overset{L}{\longleftrightarrow}\frac{1}{s+a},\ \mathrm{real}\{s\}>\mathrm{real}\{-a\}$$

- ROC: region of convergence

$$\left\{s:\int_{-\infty}^{\infty}|x(t)|e^{-\mathrm{real}\{s\}t}dt<\infty\right\}$$

$$-e^{-at}u(-t)\overset{L}{\longleftrightarrow}\frac{1}{s+a},\ \mathrm{real}\{s\}<\mathrm{real}\{a\}$$

- If ROC includes $j\omega$ 轴

$$X(\omega)=X(s)|_{s=j\omega}=X(j\omega)=\int_{-\infty}^{\infty}x(t)e^{-j\omega t}dt$$

当且仅当ROC包括$j\omega$轴时才有FT

- If the laplace transform of a signal $x(t)$ has the form $X(s)=\frac{N(s)}{D(s)}$

$$=\frac{b_m s^m+\cdots+b_1 s+b_0}{a_n s^n+\cdots+a_1 s+a_0}$$

$$=G\frac{(s-z_1)\cdots(s-z_m)}{(s-p_1)\cdots(s-p_n)}$$

→ zeros 用"o"表示 → poles 用"x"

we say that it is rational

G是gain $=\frac{b_m}{a_n}$

ROC is bounded by poles

- finite duration → entire s-plane
- right-sided → right half plane
- left sided → left half plane
- two-sided → vertical strip
- If $x(t)$ is finite duration and absolutely integrable then ROC $=\mathbb{C}$
- If $x(t)$ is right-sided signal, i.e. $x(t)=0$ for $t<T_1$ (constant) then ROC of $x(t)$ will be a RHP of $\mathrm{real}\{s\}>\sigma_0$
- If $X(s)$ has a rational form, then if $x(t)$ is right-sided, the RHP of ROC will be to the 最右边的 pole
- left-sided 也是同样
- If $X(s)$ is rational, then the ROC will have the form $\sigma_1<\mathrm{real}\{s\}<\sigma_2$ for some $\sigma_1<\sigma_2$. In fact the
- some important Laplace transform pairs see summary 3
- $x(t)=e^{\sigma t}\frac{1}{2\pi}\int_{-\infty}^{\infty}X(\sigma+j\omega)\cdot e^{j\omega t}dt$

or $x(t)=\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}X(s)e^{st}dt$

where $s=\sigma+j\omega$, $ds=jd\omega$

σ is any fixed number that lies in the ROC

$$e^{-t}\sin(t)u(t)\overset{L}{\longleftrightarrow}\frac{1}{(s+1)^2+1},\ \mathrm{real}\{s\}>-1$$

- Stability

For a system with a rational system function $H(s)$, it is stable iff the ROC of $H(s)$ includes the $j\omega$ 轴

- For a system with a rational system function $H(s)$, it is causal iff the ROC is RHP

- If it is known to be causal, then it is also stable iff all of its poles lie within the LHP
- Function with $m>n$ are all unstable
- A diffeq system is stable iff all roots of its characteristic polynomial are in the LHP ($\sigma<0$)
关于y的 characteristic ...
- If the LT of signal is rational, we have

$$H(s)=G\frac{(s-z_1)\cdots(s-z_m)}{(s-p_1)\cdots(s-p_n)}$$

frequency response

$$H(\omega)=H(s)|_{s=j\omega}=G\frac{(j\omega-z)\cdots}{(j\omega-p_1)\cdots}$$

magnitude response

$$|H(\omega)|=|G|\frac{|j\omega-z_1|\cdots|j\omega-z_m|}{|j\omega-p_1|\cdots|j\omega-p_n|}$$

phase response

$$\angle H(\omega)=\angle G+\angle(j\omega-z_1)+\cdots+\angle(j\omega-z_m)-\angle(j\omega-p_1)-\cdots-\angle(j\omega-p_n)$$

- Linearity

$x_1(t)\overset{L}{\longleftrightarrow}X_1(s)$, $x_2(t)\overset{L}{\longleftrightarrow}X_2(s)$
ROC$_1$ ROC$_2$

$x(t)=a_1x_1(t)+a_2x_2(t)\overset{L}{\longleftrightarrow}$
$X(s)=a_1X_1(s)+a_2X_2(s)$

the new ROC is at least 至和 ROC$_1$与ROC$_2$交集一样大

- differentiation

$\frac{d}{dt}x(t)\overset{L}{\longleftrightarrow}sXs$

the new ROC 至少和以前一样大

- convolution

$y(t)=x(t)*h(t)\overset{L}{\longleftrightarrow}Y(s)$
$=H(s)X(s)$

the new ROC 至少和 $X(s)$与 $H(s)$ ROC的交集一样大

- time shift

$x(t-t_0)\overset{L}{\longleftrightarrow}e^{-t_0s}X(s)$

和$X(s)$的 ROC一样大

- modulation

$e^{s_0t}x(t)\overset{L}{\longleftrightarrow}X(s-s_0)$

$ROC_{new}=ROC_{old}+real\{s_0\}$

$e^{j\omega_0t}x(t)\overset{L}{\longleftrightarrow}X(s-j\omega_0)$

ROC不变

- time scaling

$x(at)\overset{L}{\longleftrightarrow}\frac{1}{|a|}X(\frac{s}{a})$

$ROC_{new}=a\,ROC_{old}$

- differentiation is s-domain

$-tx(t)\overset{L}{\longleftrightarrow}\frac{d}{ds}X(s)$

ROC不变

- running integration in time

$\int_{-\infty}^{t}x(\tau)d\tau=x(t)*u(t)$
$\overset{L}{\longleftrightarrow}\frac{1}{s}X(s)$

ROC_{new} must contain $ROC_{old}\cap\{real\{s\}>0\}$