Homework 6 Solution

Problem 1 [5]

The characteristic polynomial is:

$$s^3 + 60s^2 + 10^5 s + 2 \cdot 10^6 = 0$$

Use Matlab to help solve this equation

```
roots([1 60 10^5 2*10^6])

ans =

1.0e+02 *

-0.1992 + 3.1432i
-0.1992 - 3.1432i
-0.2016 + 0.0000i
```

We can see that the poles are all in LHP, so the system is **stable**.

Problem 2 [5]

We may find different signal with the given Laplace transform by choosing different regions of convergence , the poles of the given Laplace transform are

$$s_0 = -2 \ s_1 = -3 \ s_2 = -\frac{1}{2} + \frac{\sqrt{3}}{2} j \ s_3 = -\frac{1}{2} - \frac{\sqrt{3}}{2} j$$

Based on the locations of the locations of these poles , we my choose form the following regions of convergence:

1. $$Re\{s\} > -\frac{1}{2}$$
2. $$-2 < Re\{s\} < -\frac{1}{2}$$
3. $$-3 < Re\{s\} < -2$$
4. $$Re\{s\} < -3$$

Therefore ,we may find four different signals the given Laplace transform.

Problem 3 [5]

X(s) has poles at $s = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$ and $-\frac{1}{2} - j\frac{\sqrt{3}}{2}$. X(s) has zeros at $s = \frac{1}{2} + j\frac{\sqrt{3}}{2}$ and $s = \frac{1}{2} - j\frac{\sqrt{3}}{2}$. We can get $\|X(j\omega)\| = 1$.

Problem 4 [5]

Taking the Laplace transform of both sides of the two differential equations, we have

$$sX(s) = -2Y(s) + 1$$

$$sY(s) = 2X(s)$$

Solving for X(s) and Y(s), we obtain

$$X(s) = \frac{s}{s^2+4}$$

$$Y(s) = \frac{2}{s^2+4}$$

The region of convergence for both X(s) and Y(s) is $Re\{s\} > 0$ because both are right-hand signals.

Problem 5 [10]

Taking the Laplace transform of both sides of the given differential equations ,we obtain

$$Y(s)[s^3 + (1+\alpha)s^2 + \alpha(1+\alpha)s + \alpha^2] = X(s)$$

therefore,

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^3 + (1+\alpha)s^2 + \alpha(1+\alpha)s + \alpha^2}$$

1. Taking the Laplace transform of both sides of the given equation, we have

$$G(s) = sH(s) + H(s)$$

   Substituting for H(s) from above,

$$G(s) = \frac{1}{s^2 + \alpha s + \alpha^2}$$

   Therefore, G(s) has 2 poles.

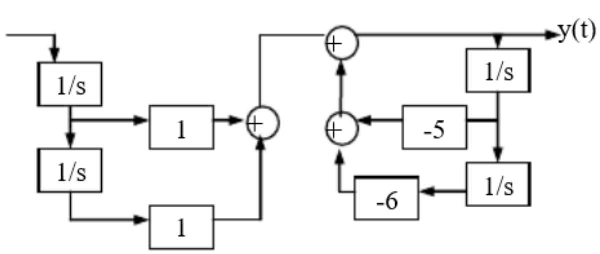
2. we know that

$$H(s) = \frac{1}{(s+1)(s^2 + \alpha s + \alpha^2)}$$

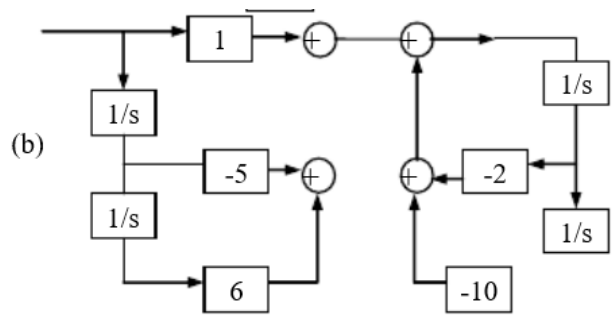
   Therefore, H(s) has poles at -1, $\alpha(-\frac{1}{2} + j\frac{\sqrt{3}}{2})$, and $\alpha(-\frac{1}{2} - j\frac{\sqrt{3}}{2})$. If the system has to be stable, then the real part of the poles has to be less than zero. For this to be true, we require that $-\frac{\alpha}{2} < 0$, $\alpha > 0$.

Problem 6 [10]

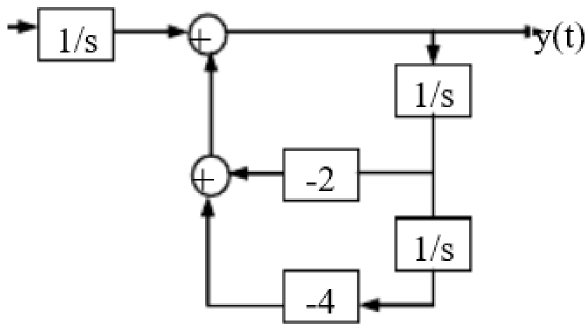
1. Figure1



2. Figure2



3. Figure3



Problem 7 [10]

If $x(t) = e^{(2t)}$ produces $y(t) = \frac{1}{6}e^(2t)$, then $H(2) = \frac{1}{6}$. Also, by taking the Laplace transform of both sides of the given differential equation we get

$$H(s) = \frac{s + b(s+4)}{s(s+4)(s+2)}$$

Since $H(2) = \frac{1}{6}$, we may deduce that $b = 1$. Therefore

$$H(s) = \frac{2}{s(s+4)}$$

Problem 8 [10]

$$H_1(s) = G_1 \frac{s-1}{s+3}$$

$$H_2(s) = G_2 \frac{s-2}{s+2}$$

Since they all have unit DC gains, we can get $G_1 = -3$ and $G_2 = -1$
$H_1(s) = -3\frac{s-1}{s+3}$, $H_2(s) = -\frac{s-2}{s+2}$

$$u(t) \xrightarrow{\mathscr{L}} \frac{1}{s}$$

$$Y(s) = \frac{-3(s-1)}{s(s+3)} + \frac{-(s-2)}{s(s+2)}$$

$$Y(s) = \frac{1}{s} + \frac{-4}{s+3} + \frac{1}{s} + \frac{-2}{s+2}$$

$$y(t) = 2u(t) - 4e^{-3t}u(t) - 2e^{-2t}u(t)$$

Problem 9 [10]

(a)

$$x(t) = e^{-t}u(t) \xrightarrow{\mathscr{L}} X(s) = \frac{1}{s+1} \quad ROC : real\{s\} > -1$$

$$h(t) = e^{-2t}u(t) \xrightarrow{\mathscr{L}} H(s) = \frac{1}{s+2} \quad ROC : real\{s\} > -2$$

(b)

$$y(t) = x(t) * h(t) \xrightarrow{\mathscr{L}} Y(s) = X(s)H(s) = \frac{1}{(s+1)(s+2)}$$

(c)

$$Y(s) = \frac{1}{(s+1)(s+2)} = \frac{1}{s+1} - \frac{1}{s+2}$$

$$y(t) = e^{-t}u(t) - e^{-2t}u(t)$$

(d)

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} e^{-\tau}u(\tau)e^{-2(t-\tau)}u(t-\tau)d\tau$$

$$y(t) = \int_{0}^{t} e^{-\tau}e^{-2(t-\tau)}d\tau \quad t > 0$$

$$y(t) = e^{-2t+\tau}|_0^t u(t)$$

$$y(t) = e^{-t}u(t) - e^{-2t}u(t)$$

Problem 10 [10]

$$x(t) = e^{-|t|} = e^{-t}u(t) + e^{t}u(-t)$$

$$X(s) = \frac{1}{s+1} - \frac{1}{s-1} = \frac{-2}{(s+1)(s-1)}, \quad -1 < \Re\{s\} < 1$$

We also have

$$H(s) = \frac{s+1}{s^2+2s+2}$$

The poles of $H(s)$ are $-1 \pm j$, and since $h(t)$ is causal, we have that the ROC of $H(s)$ is $\Re\{s\} > -1$

$$Y(s) = H(s)X(s) = \frac{-2}{(s^2+2s+2)(s-1)}, \quad \text{ROC} : -1 < \Re\{s\} < 1$$

Rewrite it as

$$Y(s) = -\frac{2/5}{s-1} + \frac{2}{5}\left[\frac{s+1}{(s+1)^2+1}\right] + \frac{4}{5}\left[\frac{1}{(s+1)^2+1}\right]$$

we get,

$$y(t) = \frac{2}{5}e^{t}u(-t) + \frac{2}{5}e^{-t}\cos(t)u(t) + \frac{4}{5}e^{-t}\sin(t)u(t)$$

Problem 11 [20]

(a)

$$y_1(t) \xrightarrow{\mathscr{L}} Y_1(s) = H_1(s)X(s)$$

$$\frac{dy_1(t)}{dt} \xrightarrow{\mathscr{L}} sY_1(s) = sH_1(s)X(s)$$

$$\frac{d^2y_1(t)}{dt^2} \xrightarrow{\mathscr{L}} s^2Y_1(s) = s^2H_1(s)X(s)$$

$$y(t) \xrightarrow{\mathscr{L}} Y(s) = H(s)X(s)$$

$$Y(s) = 2s^2Y_1(s) + 4sY_1(s) - 6Y_1(s)$$

$$y(t) = 2\frac{d^2y_1(t)}{dt^2} + 4\frac{dy_1(t)}{dt} - 6y_1(t)$$

(b)

$$f(t) = \frac{dy_1(t)}{dt}$$

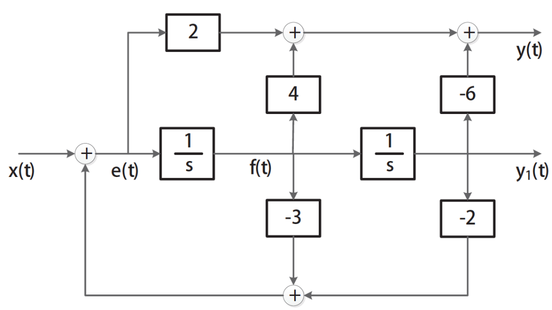
(c)

$$e(t) = \frac{d^2y_1(t)}{dt^2}$$

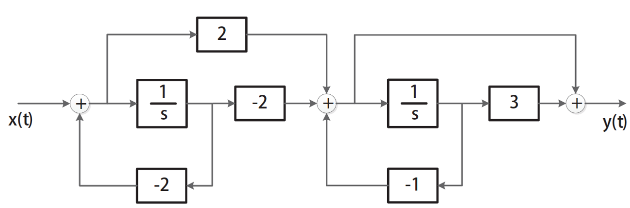
(d)

$$y(t) = 2e(t) + 4f(t) - 6y_1(t)$$

(e) Figure



(f) Figure



(g) Figure