VE 216 Lab 3 PreLab 周维凯 518021911039

3.1.1
$$\begin{cases} V_B = V_D = V_o \\ \text{At node A: } \dfrac{V_A - V_i}{R} + \dfrac{V_A - V_D}{R} = \dfrac{V_B - V_A}{\frac{1}{sC_1}} \\ \text{At node D: } \dfrac{V_A - V_D}{R} = \dfrac{V_D}{\frac{1}{sC_2}} \end{cases}$$

$$\Rightarrow \frac{V_o}{V_i} = \frac{\frac{1}{R^2 C_1 C_2}}{s^2 + \frac{2}{RC_1} s + \frac{1}{R^2 C_1 C_2}}$$

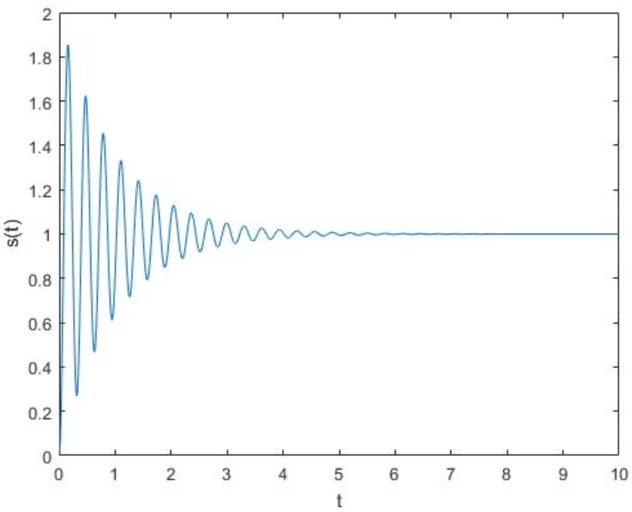
Therefore, $a_1 = \dfrac{1}{R^2 C_1 C_2}$, $a_2 = \dfrac{2}{RC_1}$, $a_3 = \dfrac{1}{R^2 C_1 C_2}$

3.1.2 From $-1 \pm j\sqrt{399}$, we know $s^2 + 2s + 400 = 0$

$$C_2 = \frac{1}{R^2 C_1 a_3} = \frac{1}{(10\times10^3)^2 \times 100\times10^{-6} \times 400} = 2.5\times10^{-7} F = 0.25\ \mu F$$

3.1.3
$$V_o(s) = \frac{400}{(s^2+2s+400)s} = \frac{1}{s} + \frac{-s-2}{s^2+2s+400}$$

$$V_o(t) = \left[1 - e^{-t}\left(\cos\sqrt{399}\,t + \frac{\sqrt{399}}{399}\sin\sqrt{399}\,t\right)\right]u(t)$$



3.2 For Eq. (3.2.2), we know
$$\left.\begin{array}{l} V_- = V_+ = 0 \\ \dfrac{V_- - V_i}{R_1} = \dfrac{V_o - V_-}{R_f} \end{array}\right\} \Rightarrow \frac{V_o}{V_i} = -\frac{R_f}{R_1}.$$
Therefore, $\dfrac{V_o(s)}{V_i(s)} = K_P$, where $K_P = -\dfrac{R_f}{R_1}$

For Eq. (3.2.3), we know
$$\left.\begin{array}{l} V_- = V_+ = 0 \\ \dfrac{V_- - V_i}{\frac{1}{sC}} = \dfrac{V_o - V_-}{R} \end{array}\right\} \Rightarrow \frac{V_o}{V_i} = -RCs.$$
Therefore, $\dfrac{V_o(s)}{V_i(s)} = K_D s$, where $K_D = -RC$

For Eq. (3.2.4), we know
$$\left.\begin{array}{l} V_- = V_+ = 0 \\ \dfrac{V_- - V_i}{R} = \dfrac{V_o - V_-}{\frac{1}{sC}} \end{array}\right\} \Rightarrow \frac{V_o}{V_i} = -\frac{1}{RCs}.$$
Therefore, $\dfrac{V_o(s)}{V_i(s)} = \dfrac{K_I}{s}$, where $K_I = -\dfrac{1}{RC}$

3.3.1 $$Y(s) = P(s)\left(C_1(s)\left(X(s)-Y(s)\right) - C_2(s)Y(s)\right)$$

$$G_{cl}(s) = \frac{Y(s)}{X(s)} = \frac{C_1(s)P(s)}{1+P(s)\left(C_1(s)+C_2(s)\right)}$$

3.3.2 $$G_{cl}(s) = \frac{K_P P(s)}{1+(K_P+K_D s)P(s)}$$

3.3.3 $$P(s) = \frac{400}{s^2+2s+400}$$

$$G_{cl}(s) = \frac{400K_P}{s^2+2s+400+400(K_P+K_D s)} = \frac{400K_P}{s^2+(2+400K_D)s+(1+K_P)400}$$

3.4.1 $$S = -(1+200K_D) \pm \sqrt{(1+200K_D)^2-400(1+K_P)}$$

(a). $\Delta > 0$

$$(1+200K_D)^2 - 400(1+K_P) > 0$$

$$K_P < \frac{(1+200K_D)^2}{400} - 1$$

(b). $\Delta = 0$

$$K_P = \frac{(1+200K_D)^2}{400} - 1$$

(c). $\Delta < 0$

$$K_P > \frac{(1+200K_D)^2}{400} - 1$$

3.4.2 $$Y(s) = \frac{1}{s}\cdot\frac{400K_P}{s^2+(2+400K_D)s+(1+K_P)400} = \frac{A_1}{s} + \frac{A_2}{s-s_1} + \frac{A_3}{s-s_2}$$

Therefore, $y(t) = \left(A_1 + A_2 e^{s_1 t} + A_3 e^{s_2 t}\right)u(t)$

We can get that $A_1 = \frac{400K_P}{s_1 s_2}$, $A_2 = \frac{400K_P}{s_1(s_1-s_2)}$, $A_3 = \frac{400K_P}{s_2(s_2-s_1)}$

$$\frac{dy}{dt} = \frac{400K_P}{s_1-s_2}\left(e^{s_1 t} - e^{s_2 t}\right) > 0 \quad \text{for all } t>0$$

Therefore, $y(t)$ is monotonically increasing function

$$\lim_{t\to\infty} y(t) = A = \frac{400K_P}{400(1+K_P)} = \frac{K_P}{1+K_P}$$

3.4.3 $$Y(s) = \frac{1}{s}\cdot\frac{400K_P}{s^2+(2+400K_D)s+(1+K_P)400} = \frac{A_1}{s} + \frac{A_2}{s-s_1} + \frac{A_3}{(s-s_1)^2}$$

Therefore, $y(t) = \left(A_1 + A_2 e^{s_1 t} + A_3 t e^{s_1 t}\right)u(t)$

We can get that $A_1 = \frac{400K_P}{s_1^2}$, $A_2 = -\frac{400K_P}{s_1^2}$, $A_3 = \frac{400K_P}{s_1}$

$$\frac{dy}{dt} = 400K_P t e^{s_1 t} > 0 \text{ for all } t>0$$

Therefore, $y(t)$ is monotonically increasing function

$$\lim_{t\to\infty} y(t) = A = \frac{400K_P}{400(1+K_P)} = \frac{K_P}{1+K_P}$$

3.4.4 Let $s_1 = a + jb$, $S_2 = a - jb$

$$Y(s) = \frac{400K_p}{s(s-a-jb)(s-a+jb)} = \frac{400K_p}{s(s^2-2as+a^2+b^2)}$$

$$y(t) = 400K_p\, u(t)\left(\frac{1}{a^2+b^2} - \frac{1}{a^2+b^2} e^{at}\left(\cosh(jbt) + j\frac{a}{b}\sinh(jbt)\right)\right)$$

$$= 400K_p\, u(t)\left(\frac{1}{a^2+b^2} - \frac{1}{b\sqrt{a^2+b^2}} e^{at} \cos\left(bt + \arctan\frac{a}{b}\right)\right)$$

Therefore, it can be written as $y(t) = (A_1 + B_1 e^{at}\cos(bt+\theta))\,u(t)$,

where $A_1 = \frac{400K_p}{a^2+b^2}$, $B_1 = -\frac{400K_p}{b\sqrt{a^2+b^2}}$, $\theta = \arctan\frac{a}{b}$

$$\frac{dy(t)}{dt} = 0 \Rightarrow t = \frac{\pi}{b}$$

$$y(t)_{max} = y\left(\frac{\pi}{b}\right) = \frac{K_p}{K_p+1}\left[1 + e^{\frac{\pi a}{b}}\right]$$

$$\lim_{t\to\infty} y(t) = \frac{400K_p}{a^2+b^2} = \frac{400K_p}{400+400K_p} = \frac{K_p}{1+K_p}$$

3.5.1 $V_A = 0$

$$I_{A-R_2} = \frac{X(s)}{R_2}$$

$$I_{down} = \frac{-Y(s)}{\frac{1}{\frac{1}{R_2}+sC}} = \frac{-Y(s)(1+R_2Cs)}{R_2}$$

$$I_{A-R_3} = \frac{-W}{R_3}$$

$$\frac{X(s)}{R_2} - \frac{Y(s)(1+R_2Cs)}{R_2} - \frac{W}{R_3} = 0$$

$$W = \frac{R_3}{R_2}(X(s) - Y(s)) - R_3Cs\,Y(s)$$

Therefore, $K_P = \frac{R_3}{R_2}$, $K_D = R_3C$