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Q1.

(a) $y_1(t) = x_1(\sin t) + \int_1^3 e^{-\tau^2} x_1(t-\tau)\,d\tau$

$a_1 x_1(t) + a_2 x_2(t) \longrightarrow a_1 x_1(\sin t) + a_2 x_2(\sin t) + \int_1^3 e^{-\tau^2}\left[a_1 x_1(t-\tau) + a_2 x_2(t-\tau)\right] d\tau$

$= a_1 y_1(t) + a_2 y_2(t)$

$\therefore$ $\boxed{\text{Linear}}$

b) $y_d(t) = x(\sin(t) - t_0) + \int_1^3 e^{-\tau^2} x(t - t_0 - \tau)\,d\tau$

$y(t-t_0) = x(\sin(t-t_0)) + \int_1^3 e^{-\tau^2} x(t-t_0-\tau)\,d\tau$

It's obvious that $x(\sin(t) - t_0) \neq x(\sin(t-t_0))$,

thus $y_d(t) \neq y(t-t_0)$

i.e. $x(t) = t$

then $y_d(t) = \sin t - t_0 + \int_1^3 e^{-\tau^2}(t-\tau)\,d\tau$

$\updownarrow \neq$

$y(t-t_0) = \sin(t-t_0) + \int_1^3 e^{-\tau^2}(t-\tau)\,d\tau$

$\therefore$ $\boxed{\text{Not time-invariant}}$

c) Noncausal.

When $t = -\frac{\pi}{2}$, $x(\sin t) = x(-1)$ and $-1 > -\frac{\pi}{2}$, so $y(t)$ may depend on the future time, the system is $\boxed{\text{noncausal}}$.

d) Since the system is noncausal, it must be $\boxed{\text{memory.}}$

e) Let's define $y_1(t) = x(\sin(-t))$

$$y_2(t) = \int_1^3 e^{-\tau^2} x(t-\tau)\, d\tau$$

if $|x(t)| \le M_x$, obviously $|y_1(t)| \le M_x$

Since $|x(t)|$ is bounded, then $e^{-\tau^2} \cdot |x(t-\tau)|$ is also bounded. As the integral is bounded by the value 1 and 3, $|y_2(t)|$ should be bounded.

Therefore, $|y(t)|$ is bounded if $|x(t)|$ is bounded, and thus the system is $\boxed{\text{stable}}$.

2. ① It is $\boxed{\text{linear.}}$

Denote $x(t) = \alpha x_1(t) + \beta x_2(t)$.

$$y(t) = \left(\int_{t-T}^{t} (\alpha x_1(\tau) + \beta x_2(\tau))\,d\tau\right) * u(t)$$

$$= \alpha \int_{t-T}^{t} x_1(\tau)\,d\tau * u(t) + \beta \int_{t-T}^{t} x_2(\tau)\,d\tau * u(t)$$

$$= \alpha y_1(t) + \beta y_2(t).$$

② It's $\boxed{\text{not time invariant.}}$

$$y_d(t) = \left(\int_{t-T}^{t} x(\tau)\,d\tau\right) * u(t).$$

$$= \left(\int_{t-T-d}^{t-d} x(\tau')\,d\tau'\right) * u(t) \qquad (\text{denote } \tau' = \tau - d).$$

But $y(t-d) = \left(\int_{t-T-d}^{t-d} x(\tau)\,d\tau\right) * u(t-d)$

Since $u(t) \neq u(t-d)$, it's not time-invariant.

③ Since $h(t) = u(t) = 0$ for every $t<0$,

then the system is $\boxed{\text{causal.}}$

④. Since $h(t) = u(t)$ cannot be expressed as $h(t) = a\,\delta(t)$,

the system is a $\boxed{\text{memory}}$ system.

⑤ Since $\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{-\infty}^{\infty} |u(t)|\,dt = \int_{0}^{\infty} 1\,dt = \infty$,

the system is $\boxed{\text{unstable.}}$