VE 216 Homework 6 周维凯 518021911039

1. $2\times10^6 Y(s)+10^5 sY(s)+60s^2Y(s)+s^3Y(s)=8\times10^6X(s)-10^4 sX(s)$

$$\frac{Y(s)}{X(s)}=\frac{10^4(800-s)}{s^3+60s^2+10^5s+2\times10^6}$$

For $s^3+60s^2+10^5s+2\times10^6=0$,

we can know that all the poles are in the LHP, therefore, $\boxed{\text{it is stable}}$

2. We know that $s=-2,\ -3,\ -\frac{1}{2}\pm j\frac{\sqrt{3}}{2}$

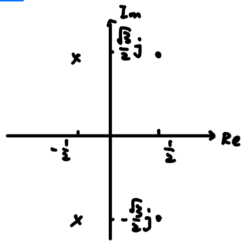
$Re\{s\}>-\frac{1}{2}$, $-2<Re\{s\}<-\frac{1}{2}$, $-3<Re\{s\}<-2$, $Re\{s\}<-3$

Therefore, $\boxed{\text{4 different signals}}$

3. poles: $s=-\frac{1}{2}\pm j\frac{\sqrt{3}}{2}$

zeros: $s=\frac{1}{2}\pm j\frac{\sqrt{3}}{2}$

$\boxed{|X(j\omega)|=1}$



4. $sX(s)=-2Y(s)+1$

$sY(s)=2X(s)$

$$\Rightarrow \boxed{\begin{cases} X(s)=\dfrac{s}{s^2+4} & \text{region of convergence: } Re\{s\}>0 \\ Y(s)=\dfrac{2}{s^2+4} & \text{region of convergence: } Re\{s\}>0 \end{cases}}$$

5. (a). $s^3Y(s)+(1+\alpha)s^2Y(s)+\alpha(\alpha+1)sY(s)+\alpha^2Y(s)=X(s)$

$$H(s)=\frac{Y(s)}{X(s)}=\frac{1}{s^3+(1+\alpha)s^2+\alpha(\alpha+1)s+\alpha^2}$$

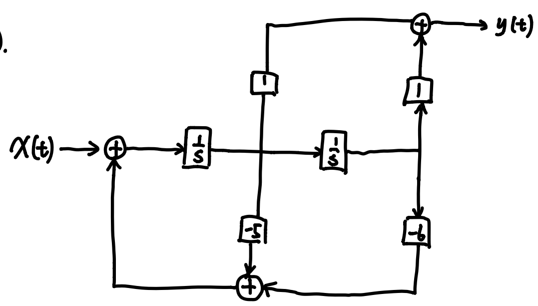
$$G(s)=sH(s)+H(s)=\frac{s+1}{s^3+(1+\alpha)s^2+\alpha(\alpha+1)s+\alpha^2}=\frac{1}{s^2+\alpha s+\alpha^2}$$

$\boxed{\text{If } \alpha=0, \text{ it has one pole, otherwise two poles.}}$

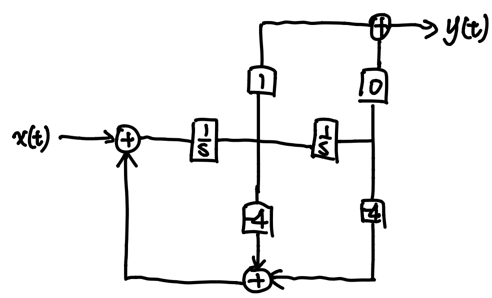
(b). For $H(s)$, it has poles at -1, $-\frac{\alpha}{2}\pm j\frac{\sqrt{3}}{2}\alpha$.

$-\frac{\alpha}{2}<0 \Rightarrow \boxed{\alpha>0}$

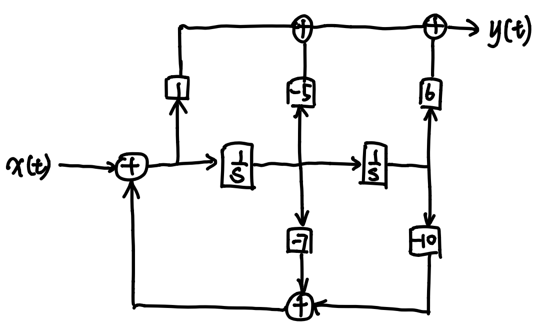
6. (a).



(c).



(b).



7. $sH(s) + 2H(s) = \frac{1}{s+4} + \frac{b}{s}$

$$H(s) = \frac{s + b(s+4)}{s(s+2)(s+4)}$$

$$x(t) = e^{st} \xrightarrow{LTI} y(t) = e^{st} H(s)$$

Therefore $H(2) = \frac{1}{6}$

$$H(2) = \frac{2 + b \times 6}{2 \times 4 \times 6} = \frac{1}{6} \Rightarrow b = 1$$

Therefore $\boxed{H(s) = \frac{2s+4}{s(s+2)(s+4)} = \frac{2}{s(s+4)}}$

8. $H_1(s) = G_1 \frac{s-1}{s+3} \Rightarrow H_1(0) = -\frac{G_1}{3} = 1 \Rightarrow G_1 = -3 \Rightarrow H_1(s) = \frac{3(1-s)}{s+3}$

$$H_2(s) = G_2 \frac{s-2}{s+2} \Rightarrow H_2(0) = -G_2 = 1 \Rightarrow G_2 = -1 \Rightarrow H_2(s) = \frac{2-s}{s+2}$$

$$Y(s) = \frac{1}{s}\left(H_1(s) + H_2(s)\right) = \frac{3(1-s)}{s(s+3)} + \frac{2-s}{s(s+2)} = \frac{2}{s} + \frac{-2}{s+2} + \frac{-4}{s+3}$$

$$\boxed{y(t) = 2u(t) - 2e^{-2t}u(t) - 4e^{-3t}u(t)}$$

9. (a). $\boxed{X(s) = \frac{1}{s+1} \quad Re\{s\} > -1 \qquad H(s) = \frac{1}{s+2} \quad Re\{s\} > -2}$

(b). $\boxed{Y(s) = X(s)H(s) = \frac{1}{(s+1)(s+2)} \quad Re\{s\} > -1}$

(c). $Y(s) = \frac{1}{s+1} - \frac{1}{s+2}$

$$\boxed{y(t) = e^{-t}u(t) - e^{-2t}u(t)}$$

(d). $y(t) = x(t) * h(t)$

$$= \int_{-\infty}^{\infty} e^{-\tau} u(\tau) e^{-2(t-\tau)} u(t-\tau) \, d\tau$$

$$= u(t) \int_0^t e^{-2t+\tau} \, d\tau$$

$$= \boxed{\left(e^{-t} - e^{-2t}\right) u(t)}$$

10. $x(t) = e^{-|t|} = e^{-t}u(t) + e^{t}u(-t)$

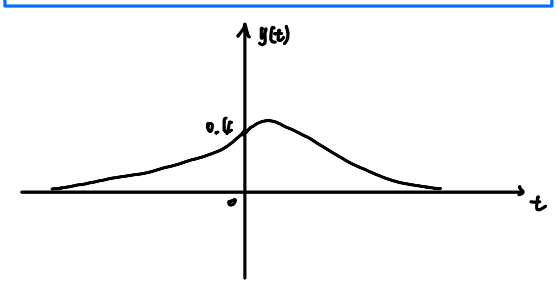
$$X(s) = \frac{1}{s+1} - \frac{1}{s-1}$$

$$Y(s) = X(s)H(s) = \left(\frac{1}{s+1} - \frac{1}{s-1}\right)\frac{s+1}{s^2+2s+2} = \frac{-2}{(s-1)(s^2+2s+2)}$$

$$Y(s) = \frac{-\frac{2}{5}}{s-1} + \frac{\frac{2}{5}s + \frac{6}{5}}{s^2+2s+2}$$

$$= \frac{-\frac{2}{5}}{s-1} + \frac{2}{5}\left(\frac{s+1}{(s+1)^2+1}\right) + \frac{4}{5}\frac{1}{(s+1)^2+1}$$

$$\boxed{y(t) = \frac{2}{5}e^{t}u(-t) + \frac{2}{5}e^{-t}\cos t\, u(t) + \frac{4}{5}e^{-t}\sin t\, u(t)}$$



11. (a). $\frac{Y_1(s)}{X(s)} = H_1(s) = \frac{1}{s^2+3s+2} \Rightarrow X(s) = (s^2+3s+2)\,Y_1(s)$

$\frac{Y(s)}{X(s)} = H(s) = \frac{2s^2+4s-6}{s^2+3s+2}$

$\Rightarrow Y(s) = (2s^2+4s-6)\,Y_1(s)$

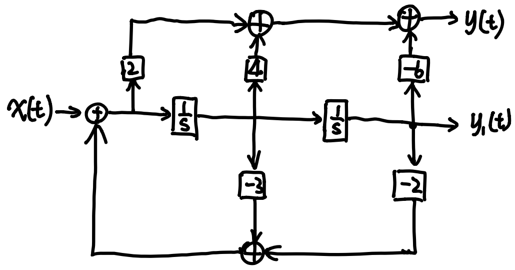
$$\boxed{y(t) = 2\frac{d^2 y_1(t)}{dt^2} + 4\frac{dy_1(t)}{dt} - 6\,y_1(t)}$$

(b). $\boxed{f(t) = \frac{dy_1(t)}{dt}}$

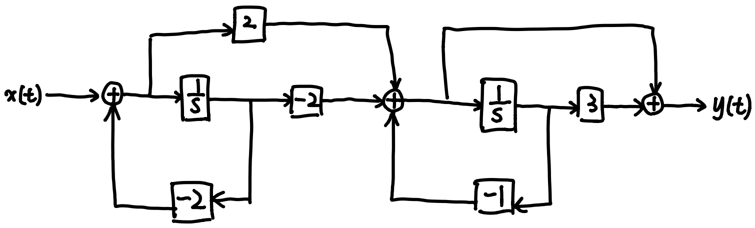
(c). $\boxed{e(t) = \frac{d^2 y_1(t)}{dt^2}}$

(d). $\boxed{y(t) = 2e(t) + 4f(t) - 6y_1(t)}$

(e).



(f).



(g).