JOINT INSTITUTE
交大密西根学院

Pledge of Honor
for Online Examinations
Summer Term of 2020

The University of Michigan – Shanghai Jiao Tong University Joint Institute trusts its students to participate in examinations in an honorable and respectful manner, following a spirit of fairness and equality. Cheating, seeking unfair advantage and disturbing the safe and harmonious environment of examinations are contrary to the ethical principles of students of the Joint Institute. The letter and spirit of the Honor Code shall guide the behavior of students, faculty and all members of the Joint Institute.

Therefore, I hereby declare that

1. I will neither give nor receive unauthorized aid during the present examination, nor will I conceal any violations of the Honor Code by others or myself.
2. I confirm that I have read and understood the rules and procedures for examination set out by SJTU. I will follow them to the best of my ability.
3. I understand that violating the rules and procedures for examinations or the Honor Code will lead to administrative and/or academic sanctions.

Signature: 周维W

Student ID: 518021911039

Date: 2020.08.04

1. ①

$$y(t)=\begin{cases}\int_{-\infty}^{\infty} rect\left(\frac{\tau-(t-2)}{2}\right) e^{-(t-\tau)^2} x(\tau)d\tau, x<0 \\ \int_{-\infty}^{\infty} rect\left(\frac{\tau-2}{2}\right) e^{-\tau^2} x(t-\tau)d\tau, x\geq 0.\end{cases}$$

It is not memoryless because it ~~does~~ involves integration and $x(t-\tau)$, so $\boxed{\text{it } \cancel{\text{doesn't}} \text{ is dynamic}}$

$x = a_1 x_1(t) + a_2 x_2(t)$. since $y(t)$ distinguishes between $x<0$ and $x\geq 0$

Therefore, it is $\boxed{\text{not linear}}$

$$y(t-t_0)=\begin{cases}\int_{t-t_0-1}^{t-t_0-3} e^{-(t-t_0-\tau)^2} x(\tau)\,d\tau, x<0. \\ \int_1^3 e^{-\tau^2} x(t-t_0-\tau)\,d\tau, x\geq 0.\end{cases}$$

$$y_d(t)=\begin{cases}\int_{t-1}^{t-3} e^{-(t-\tau)^2} x(\tau-t_0)\,d\tau, x<0. \\ \int_1^3 e^{-\tau^2} x(t-t_0-\tau)\,d\tau, x\geq 0.\end{cases}$$

Let $m=\tau-t_0 \Rightarrow \tau=m+t_0$.
$dm=d\tau$

$$=\begin{cases}\int_{t-t_0-1}^{t-t_0-3} e^{-(t-t_0-m)^2} x(m)\,dm, x<0. \\ \int_1^3 e^{-\tau^2} x(t-t_0-\tau)\,d\tau, x\geq 0.\end{cases}$$

Therefore, $y(t-t_0)=y_d(t)$. It is $\boxed{\text{time invariant}}$

It is $\boxed{\text{causal}}$ because it depends only on present and past input.

It is $\boxed{\text{BIBO stable}}$.

$$|y(t)| \leq \begin{cases}\int_{t-1}^{t-3} e^{-(t-\tau)^2} |x(\tau)|\,d\tau, x<0 \\ \int_1^3 e^{-\tau^2} |x(t-\tau)|\,d\tau, x\geq 0.\end{cases}$$

② $y(t-t_0) = \int_{t-t_0}^{\infty} \int_{-\infty}^{\tau} x(2s)\, ds\, d\tau$

$y_d(t) = \int_{t}^{\infty} \int_{-\infty}^{\tau} x(2s-t_0)\, ds\, d\tau$

let $2m = 2s - t_0 \Rightarrow dm = ds$.

$y_d(t) = \int_{t}^{\infty} \int_{-\infty}^{\tau - t_0} x(2m)\, dm\, d\tau$.

let $n = \tau - t_0$, $\Rightarrow dn = d\tau$

$y_d(t) = \int_{t-t_0}^{\infty} \int_{-\infty}^{n} x(2m)\, dm\, dn = y(t-t_0)$.

Therefore, $\boxed{\text{time-invariant.}}$

It is $\boxed{\text{not BIBO stable}}$. $|y(t)| \leq \int_{t}^{\infty} \int_{-\infty}^{\tau} |x(2s)|\, ds\, d\tau$, ~~that~~ involves infinite integration.

$\boxed{\text{Linear}}$ $x = a_1 x_1(t) + a_2 x_2(t)$

$y(t) = \int_{t}^{\infty} \int_{-\infty}^{\tau} (a_1 x_1(2s) + a_2 x_2(2s))\, ds\, d\tau$

$= a_1 \int_{t}^{\infty} \int_{-\infty}^{\tau} x_1(2s)\, ds\, d\tau + a_2 \int_{t_0}^{\infty} \int_{-\infty}^{\tau} x_2(2s)\, ds\, d\tau$

$= a_1 y_1 + a_2 y_2$

③ $y(t) = x(t)$.

It is static, stable, and time-invariant.

2.

① ~~若~~

$$x_1(t)c_1(t) \Longleftrightarrow \frac{1}{2}\left[X_1(\omega-1000) + X_1(\omega+1000)\right] = \frac{1}{2}\left[\text{tri}\left(\frac{\omega-1000}{10}\right) + \text{tri}\left(\frac{\omega+1000}{10}\right)\right]$$

$$c_2(t) = \sin(1000t) = \cos\left(1000t + \frac{\pi}{2}\right)$$

$$x_2(t)c_2(t) \Longleftrightarrow \frac{1}{2}\left[e^{j\frac{\pi}{2}} X_2(\omega-1000) + e^{-j\frac{\pi}{2}} X_2(\omega+1000)\right]$$

$$= \frac{1}{2}\left[j\,\text{rect}\left(\frac{\omega-1000}{20}\right) - j\,\text{rect}\left(\frac{\omega+1000}{20}\right)\right]$$

$$Y(\omega) = \frac{1}{2}\left[\text{tri}\left(\frac{\omega-1000}{10}\right) + \text{tri}\left(\frac{\omega+1000}{10}\right) + j\,\text{rect}\left(\frac{\omega-1000}{20}\right) - j\,\text{rect}\left(\frac{\omega+1000}{20}\right)\right]$$

② $$W_1(\omega) = \frac{1}{2}\left[Y(\omega-1000) + Y(\omega+1000)\right]$$

$$c_4(t) = \cos\left(1010t + \frac{\pi}{2}\right)$$

~~则~~ $$W_2(\omega) = \frac{1}{2}\left[j\,Y(\omega-1010) - j\,Y(\omega+1010)\right]$$

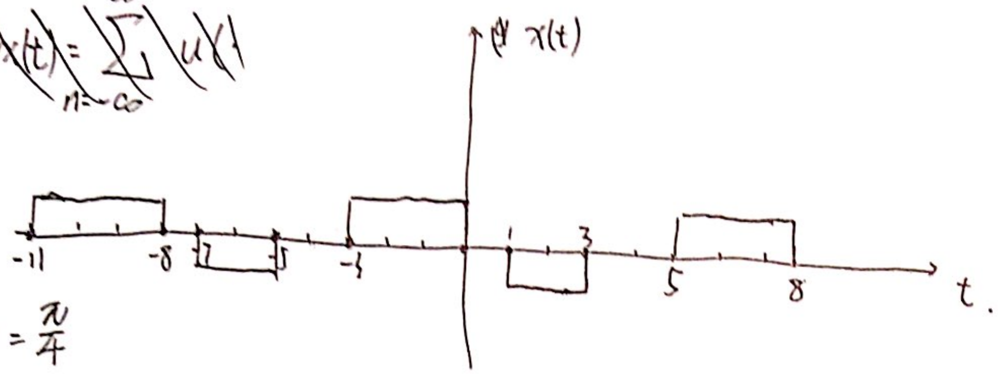
$$W_1(\omega) = \frac{1}{4}\left[\text{tri}\left(\frac{\omega-2000}{10}\right) + 2\,\text{tri}\left(\frac{\omega}{10}\right) + j\,\text{rect}\left(\frac{\omega-2000}{20}\right) \cancel{+ j\,\text{rect}\left(\frac{\omega}{20}\right)} + \text{tri}\left(\frac{\omega+2000}{10}\right) - j\,\text{rect}\left(\frac{\omega+2000}{20}\right)\right]$$

$$Z_1(\omega) = \frac{1}{2}\,\text{tri}\left(\frac{\omega}{10}\right)$$

$$W_2(\omega) = \frac{1}{4}\left(j\,\text{tri}\left(\frac{\omega-2010}{10}\right) + j\,\text{tri}\left(\frac{\omega-10}{10}\right) - \text{rect}\left(\frac{\omega-2010}{20}\right) + \text{rect}\left(\frac{\omega-10}{20}\right) - j\,\text{tri}\left(\frac{\omega+10}{10}\right) - j\,\text{tri}\left(\frac{\omega+2010}{10}\right) + \text{rect}\left(\frac{\omega+10}{20}\right) - \text{rect}\left(\frac{\omega+2010}{20}\right)\right]$$

$$Z_2(\omega) = \frac{1}{4}\left(j\,\text{tri}\left(\frac{\omega-10}{10}\right) + \text{rect}\left(\frac{\omega-10}{20}\right) - j\,\text{tri}\left(\frac{\omega+10}{10}\right) + \text{rect}\left(\frac{\omega+10}{20}\right)\right].$$

3. ① $\cancel{x(t)=\sum_{n=-\infty}^{\infty} u(t}$



$\omega_0 = \frac{2\pi}{8} = \frac{\pi}{4}$

$$C_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}\,dt = \frac{1}{8}\left(\int_2 -e^{-jk\omega_0 t}\,dt + \int_3 e^{-jk\omega_0 t}\,dt\right)$$

$$= \frac{1}{8}\left(-\frac{1}{-jk\omega_0}e^{-jk\omega_0 t}\Big|_1^3 + \frac{1}{-jk\omega_0}e^{-jk\omega_0 t}\Big|_5^8\right)$$

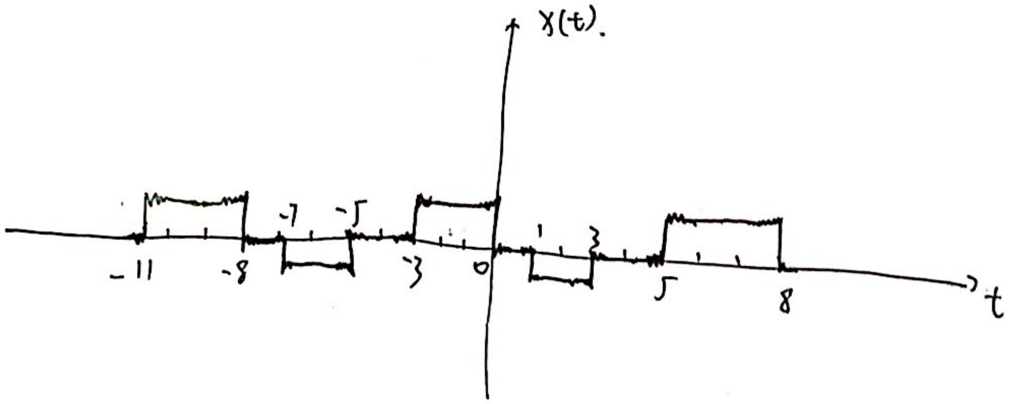
$$= \frac{1}{8}\left(\frac{1}{jk\omega_0}\left(e^{-j3k\omega_0} - e^{-jk\omega_0}\right) - \frac{1}{jk\omega_0}\left(-e^{-j5k\omega_0} + e^{-j8k\omega_0}\right)\right)$$

$$= \frac{1}{8jk\omega_0}\left(e^{-j3k\omega_0} - e^{-jk\omega_0} + e^{-j5k\omega_0} - e^{-j8k\omega_0}\right) \quad (k \neq 0).$$

$k=0.$ $C_k = \frac{1}{8}\left(\int_1^3 (-1)\,dt + \int_5^8 (1)\,dt\right) = \frac{1}{8}(-2+3) = \frac{1}{8}$

$$x(t) = \frac{1}{8} + \sum_{k\neq 0} \frac{1}{8jk\omega_0}\left(e^{-j3k\omega_0} - e^{-jk\omega_0} + e^{-j5k\omega_0} - e^{-j8k\omega_0}\right)\cdot e^{jk\omega_0 t}, \text{ where } \omega_0 = \frac{\pi}{4}.$$

②



4. ① $H_1(\omega) = \text{rect}\left(\frac{\omega}{200}\right)$

$H_2(\omega) = \text{rect}\left(\frac{\omega}{100}\right)$

5. $H_1(s) = G_1 \frac{(s-3j)(s+3j)}{s+3}$, $H_1(0) = 1 = G_1 \frac{9}{3} \Rightarrow G_1 = \frac{1}{3}$

$H_3(s) = G_3 \frac{(s-2)}{(s+2)}$, $H_3(0) = 1 = -G_3 \Rightarrow G_3 = -1$.

$H_1(s) = \frac{(s-3j)(s+3j)}{3(s+3)}$ $H_3(s) = -\frac{(s-2)}{(s+2)}$

$H_5(s) = s$, $H_2(s) = \frac{3}{s^2+9}$ real$\{s\} > 0$.

6. ① $(s+9)Y(s) = sX(s) + 4X(s) + X(s)Z(s)$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s+4+Z(s)}{s+9} = \frac{\frac{1}{3+s}+1+s+4}{s+9}$$

$$= \frac{\frac{1}{6}}{s+3} - \frac{\frac{1}{6}}{s+9} + 1 - \frac{4}{s+9}$$

$$h(t) = \frac{1}{6}e^{-3t}u(t) - \frac{25}{6}e^{-9t}u(t) + \delta(t).$$

②