Example

Example

1. Find $u(t) * u(t)$
2. Find $u(t-3) * [t^2\delta(t-3) - u(t+2)]$
3. Find $u(t) * u(t) * (t^2 u(t))$
4. Find $u(t) * \delta(7 - t/3)$

(Selected from Exam 1 in Summer 2014)

Solution

1. $$u(t) * u(t) = tu(t)$$

   Use this fact for the following convolution calculations

2. $$u(t-3)*[t^2\delta(t-3)-u(t+2)] = \boxed{9u(t-6)-(t-1)u(t-1)}$$

3. $$y(t) = u(t) * u(t) * (t^2u(t)) = (tu(t)) * (t^2u(t))$$
   $$= \left[\int_0^t \tau^2(t-\tau)\,d\tau\right] u(t)$$
   $$= t\tau^3/3 - \tau^4/4\Big|_{\tau=0}^{t} = t^4/3 - t^4/4 = \boxed{t^4/12u(t)}$$

4. $$u(t) * \delta(7 - t/3) = \boxed{3u(t-21)}$$

Example

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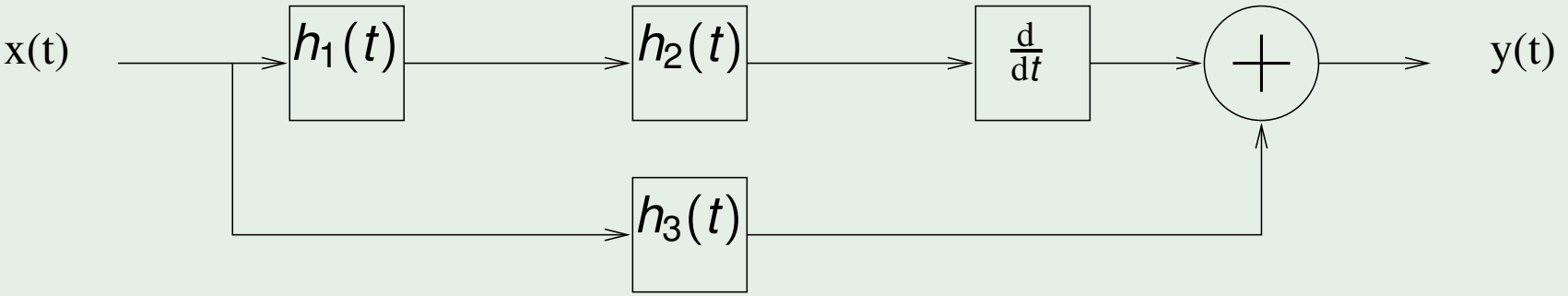
Consider the following CT system composed of LTI CT subsystems.



The impulse response functions of the subsystems are as follows: $h_1(t) = tu(t)$, $h_2(t) = u(t-1)$, $h_3(t) = \delta(t-3)$.

1. (6 points) If this system is LTI, state so and find its overall impulse response. If it is not LTI, explain why (giving a counter example).
2. (4 points) Determine the input-output relationship for the above system, (regardless of whether it is LTI or not).
   (For full credit, your final answer must not contain the letter "h.")

(Selected from Exam 1 in Summer 2014)

Solution (1)

Solution

Parallel and series connections of LTI subsystems always yield LTI systems so the system $\boxed{\text{is LTI.}}$

$$h(t) = \frac{\mathrm{d}}{\mathrm{d}t} h_1(t) * h_2(t) + h_3(t) = \boxed{(t-1)u(t-1) + \delta(t-3).}$$

Solution(2)

Solution

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau = \boxed{x(t-3) + \int_{-\infty}^{t-1} (t-\tau-1)x(\tau)\,d\tau.}$$