VE 216 Lab 1 Pre-lab 周维凯 518021911039

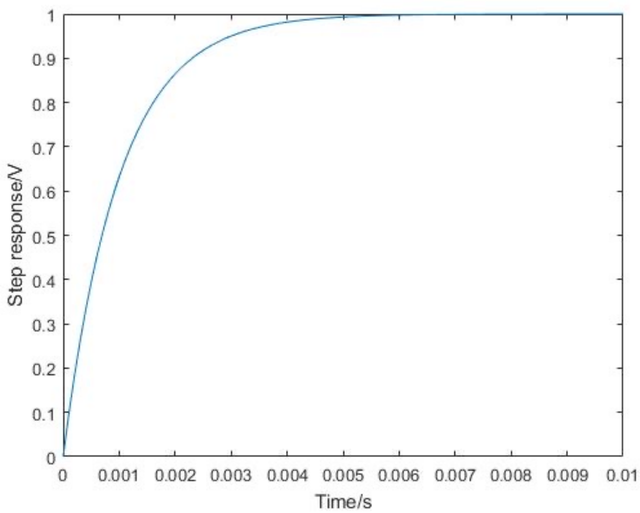
4.1 (a). For $t>0$, $V_{in}(t)=RC\frac{d}{dt}(1-e^{-\frac{t}{RC}})+(1-e^{-\frac{t}{RC}})=RC(\frac{1}{RC}e^{-\frac{t}{RC}})+1-e^{-\frac{t}{RC}}=1$

For $t<0$, $V_{in}(t)=RC\frac{d}{dt}(0)+0=0$

Therefore, $RC\frac{d}{dt}y_{step}(t)+y_{step}(t)=\begin{cases}1, & t>0\\ 0, & t<0\end{cases}=u(t)$

(b).

```
t=0:0.0001:0.01;
y=1-exp(-t./0.001);
plot(t,y)
xlabel('Time/s')
ylabel('Step response/V')
axis([0 0.01 0 1])
```



4.2 $h(t)=\frac{dy_{step}(t)}{dt}=\frac{1}{RC}e^{-\frac{t}{RC}}\cdot u(t)$

4.3 (a). When $V_{in}=u(t)$, we have $V_{out}=(1-e^{-\frac{t}{RC}})u(t)$

Therefore, for this problem, we can have

$$V_{out}=\frac{b}{\Delta}\left(y_{step}(t)-y_{step}(t-\Delta)\right)=\frac{b}{\Delta}\left((1-e^{-\frac{t}{RC}})u(t)-(1-e^{-\frac{t-\Delta}{RC}})u(t-\Delta)\right)$$

$$=\begin{cases}0, & t<0\\ \frac{b}{\Delta}(1-e^{-\frac{t}{RC}})u(t), & 0<t<\Delta\\ \frac{b}{\Delta}(e^{\frac{\Delta-t}{RC}}-e^{-\frac{t}{RC}}), & t>\Delta\end{cases}$$

Optional: $h(t)*V_{in}(t)=\int_{-\infty}^{\infty}h(\tau)\,v_{in}(t-\tau)\,d\tau=\frac{b}{RC\Delta}\int_{-\infty}^{\infty}e^{-\frac{t-\tau}{RC}}\cdot u(t-\tau)\cdot rect\frac{t-\frac{\Delta}{2}}{\Delta}\,d\tau$

$$=\begin{cases}0, & t<0\\ \frac{b}{\Delta}(1-e^{-\frac{t}{RC}})u(t), & 0<t<\Delta\\ \frac{b}{\Delta}(e^{\frac{\Delta-t}{RC}}-e^{-\frac{t}{RC}}), & t>\Delta\end{cases}$$

(b).

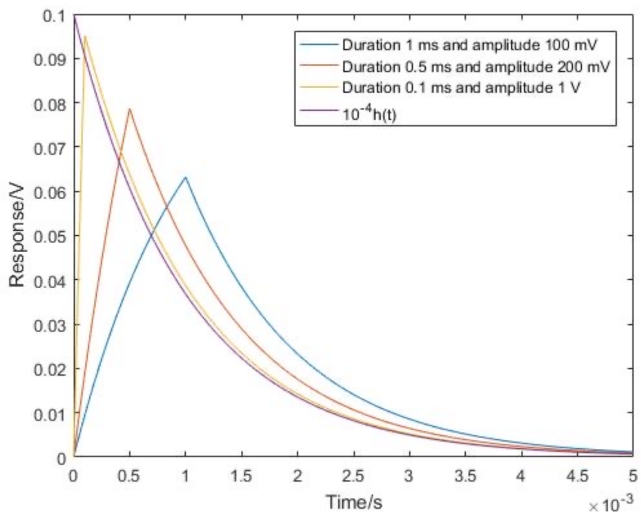
$$\lim_{\Delta \to 0} \frac{1}{\Delta}\left(\left(1-e^{-\frac{t}{RC}}\right)u(t) - \left(1-e^{-\frac{t-\Delta}{RC}}\right)u(t-\Delta)\right)$$

$$= \lim_{\Delta \to 0} \frac{1}{\Delta}\left(1-e^{-\frac{t}{RC}} - 1 + e^{-\frac{t-\Delta}{RC}}\right)u(t)$$

$$= \lim_{\Delta \to 0} \frac{1}{\Delta} e^{\frac{-t}{RC}}\left(e^{\frac{\Delta}{RC}} - 1\right) u(t)$$

$$= e^{-\frac{t}{RC}} u(t) \lim_{\Delta \to 0} \frac{e^{\frac{\Delta}{RC}}-1}{\Delta} = e^{-\frac{t}{RC}} u(t) \lim_{\Delta \to 0} \frac{\frac{1}{RC} e^{\frac{\Delta}{RC}}}{1} = \frac{1}{RC} e^{-\frac{t}{RC}} u(t)$$

(c).



4.4 For $u(t)-u(t-0.01)$, we have $V_{out1} = \left(1-e^{-1000t}\right)u(t) - \left(1-e^{-1000(t-0.01)}\right)u(t-0.01)$

For $-2.2\left(u(t-0.011)-u(t-0.016)\right)$, we have $V_{out2} = -2.2\left(1-e^{-1000(t-0.011)}\right)u(t-0.011) + 2.2\left(1-e^{-1000(t-0.016)}\right)u(t-0.016)$

For $200t\left(u(t-0.011)-u(t-0.016)\right)$, we first consider $t\,u(t-t_0)$

$$t\,u(t-t_0) * 1000e^{-1000t}u(t) = \int_{-\infty}^{\infty} \tau\, u(\tau-t_0)\, 1000 e^{-1000(t-\tau)} u(t-\tau)\, d\tau = u(t-t_0)\int_{t_0}^{t} \tau\, 1000\, e^{-1000(t-\tau)} d\tau$$

$$= u(t-t_0)\frac{1}{1000}\left(1000t - 1 - (1000t_0 - 1)e^{1000(t_0-t)}\right)$$

Then, we get $V_{out3} = 200\left(u(t-0.011)\frac{1}{1000}\left(1000t-1-10e^{-1000(t-0.011)}\right) - u(t-0.016)\frac{1}{1000}\left(1000t-1-16e^{-1000(t-0.016)}\right)\right)$

Therefore, $V_{out} = V_{out1} + V_{out2} + V_{out3} = \left(1-e^{-1000t}\right)u(t) - \left(1-e^{-1000(t-0.01)}\right)u(t-0.01) - 2.2\left(1-e^{-1000(t-0.011)}\right)u(t-0.011) +$
$2.2\left(1-e^{-1000(t-0.016)}\right)u(t-0.016) + \frac{1}{5}\left(1000t-1-10e^{-1000(t-0.011)}\right)u(t-0.011) - \frac{1}{5}\left(1000t-1-16e^{-1000(t-0.016)}\right)u(t-0.016)$

4.5

$$H(j\omega) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1+jR\omega C}$$

Optional: $\Gamma(j\omega) = \int_{-\infty}^{\infty} \frac{1}{RC} e^{-\frac{1}{RC}t} \cdot e^{-j\omega t} dt = \frac{1}{RC\left(\frac{1}{RC}+j\omega\right)} = \frac{1}{1+jRC\omega} = H(j\omega)$

4b $|H(j\omega)| = \left|\frac{1}{1+j\omega RC}\right|$

f_c	$\|H(j2\pi f_c)\|$	$\angle H(j2\pi f_c)$	τ_d
50Hz	0.9540	$-17.4406°$	0.9689 ms
200Hz	0.6227	$-51.4881°$	0.7151 ms
500 Hz	0.3033	$-72.3432°$	0.4019 ms
1000Hz	0.1572	$-80.9569°$	0.2249 ms
5000Hz	0.0318	$-88.1768°$	0.0490 ms