Example

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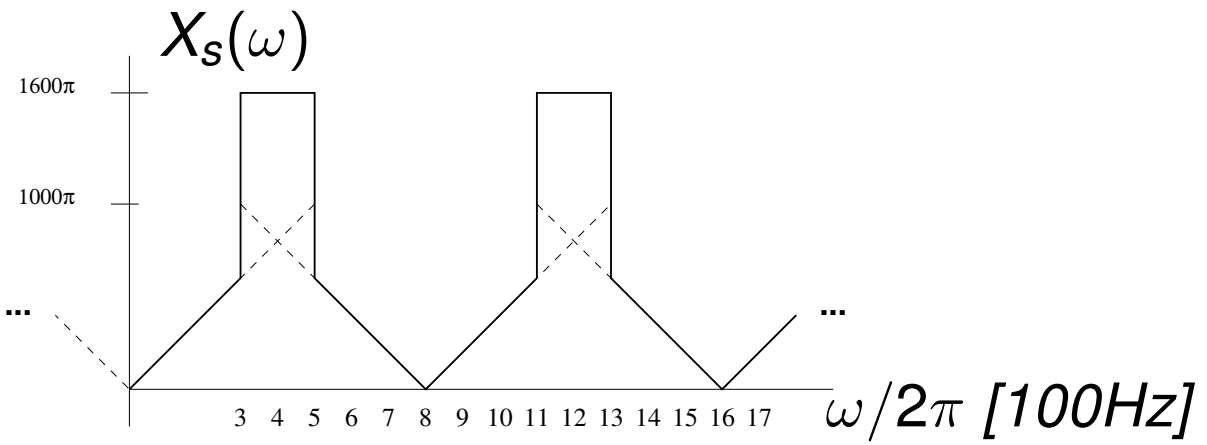
1. The signal with spectrum $X(\omega) = |\omega| \operatorname{rect}\left(\frac{\omega}{2000\pi}\right)$ is sampled at a 800Hz sampling rate (using an ideal impulse train). Find an analytical expression for the spectrum $X_s(\omega)$ of the sampled signal $x_s(t) = x(t)p(t)$, and then carefully sketch $X_s(\omega)$.
2. Part of the original signal spectrum will be aliased. Specify the impulse response of an ideal filter that would extract the *unaliased* portion of the spectrum.
3. Determine the minimum sampling rate that would eliminate aliasing.

Solution (1)

$$X_s(\omega) = \sum_{k=-\infty}^{\infty} \frac{1}{T_s} X(\omega - k\omega_s), \quad \textit{where}\ \omega_s/2\pi = 800Hz$$

so

$$\boxed{X_s(\omega) = \sum_{k=-\infty}^{\infty} \frac{2\pi}{T_s} |\omega/2\pi - k800| \,\mathrm{rect}\left(\frac{\omega/2\pi - k800}{1000}\right)}$$



Solution (2)

The part from ± 300Hz is alias free. So an ideal filter with

$$H(\omega) = \text{rect}\left(\frac{\omega}{1200\pi}\right)$$

would select those frequencies, which has impulse response

$$\boxed{h(t) = 600\,\text{sinc}(600t)\,.}$$

Solution (3)

Since

$$\omega_{\max} = 1000\pi$$

the minimum sampling rate is

$$\boxed{\omega_s/2\pi = 2\omega_{\max}/2\pi = 1000Hz.}$$

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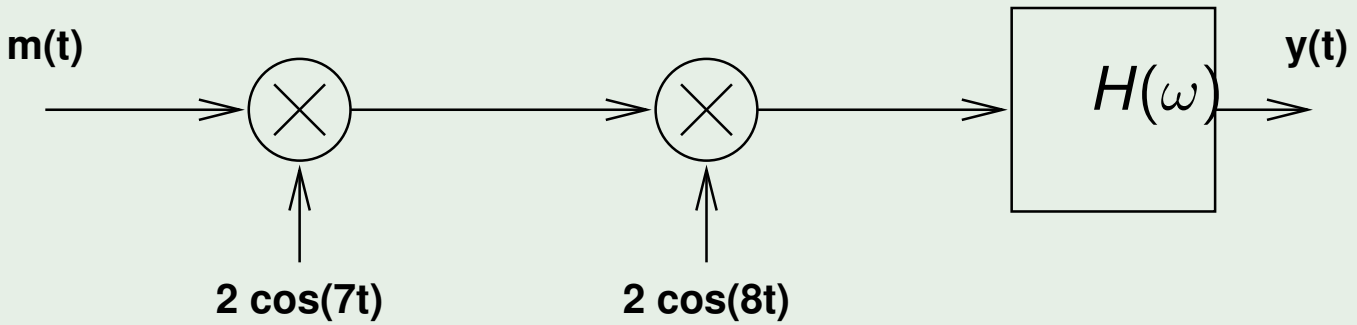
This problem explores the effect of a mis-tuned AM receiver in the following analog communication system.



Assuming that $M(\omega) = \text{tri}(\omega/2)$ and $H(\omega) = \text{rect}(\omega/4)$, carefully draw the output spectrum $Y(\omega)$.
(Selected from Final Exam in Summer 2014.)

Solution

$$Y(\omega) = [M(\omega - 1) + M(\omega + 1)]\,\mathrm{rect}(\omega/4)$$

$$= [\mathrm{tri}((\omega - 1)/2) + \mathrm{tri}((\omega + 1)/2)]\,\mathrm{rect}(\omega/4)$$