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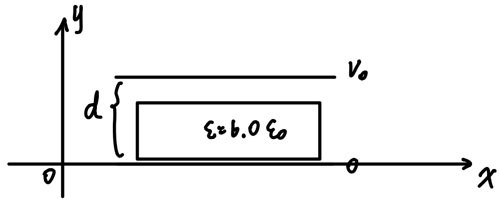
P4-1 a). Since $\nabla^2 V=0$, we can assume that the V in air is $V_{air}=ay+b$, and the V in dielectric is $V_{die}=my+n$.



Then $E=-\nabla V$, we have $\vec{E}_{air}=-\hat{a_y}\cdot a$ and $\vec{E}_{die}=-\hat{a_y}m$.

Moreover, we know that
$$\begin{cases} V_{die}(0)=0 \Rightarrow n=0 \\ V_{air}(d)=ad+b=V_0 \\ V_{die}(0.8d)=V_{air}(0.8d) \Rightarrow a\times 0.8d+b=m\times 0.8d \\ 6.0\varepsilon_0 E_{die}(0.8d)=\varepsilon_0 E_{air}(0.8d) \Rightarrow 6\times 0.8m=0.8a \end{cases}$$

We can get that
$$\begin{cases} a=\frac{3V_0}{d} \\ b=-2V_0 \\ m=\frac{V_0}{2d} \\ n=0 \end{cases}$$

Then, in dielectric slab: $V=\frac{V_0}{2d}y \qquad \vec{E}=-\frac{V_0}{2d}\hat{a_y}$

b). $V=\frac{3V_0}{d}y-2V_0, \quad \vec{E}=-\frac{3V_0}{d}\hat{a_y}$

c). $\rho_{upper}=\frac{3\varepsilon_0 V_0}{d}$

$\rho_{lower}=-\frac{V_0}{2d}\times 6\varepsilon_0=-\frac{3\varepsilon_0 V_0}{2d}$

d). Without slab: $V=\frac{V_0}{d}\cdot y$

$\vec{E}=-\frac{V_0}{d}\hat{a_y}$

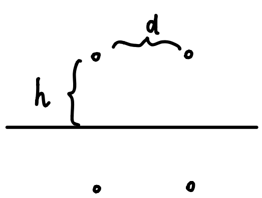
P4-5 a). $V(x,y,z)=\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{x^2+(y-d)^2+z^2}}-\frac{1}{\sqrt{x^2+(y+d)^2+z^2}}\right)$

$\nabla^2 V=\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=0.$

b). $V(x,y,z)=\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_+}-\frac{1}{R_-}\right)+V_0$

c). $\vec{F}=-\hat{a_y}\cdot\frac{Q}{4\pi\varepsilon_0(2d)^2}=\frac{-Q^2}{16\pi\varepsilon_0 d^2}\hat{a_y}$

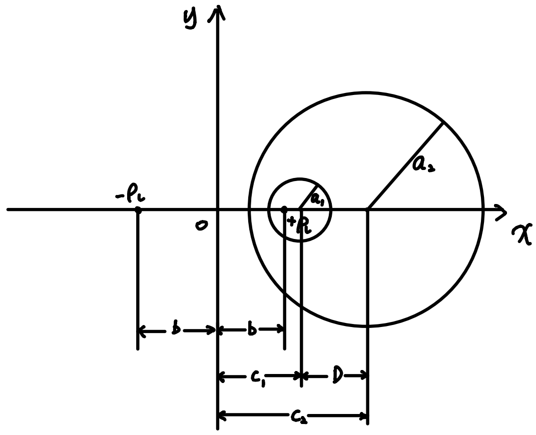
P4-11



$$\Delta V=\frac{\rho}{2\pi\varepsilon_0}\ln\left(\frac{d}{a}\right)+\frac{-\rho}{2\pi\varepsilon_0}\ln\left(\frac{a}{d}\right)+\frac{-\rho}{2\pi\varepsilon_0}\ln\frac{\sqrt{4h^2+d^2}}{2h}+\frac{\rho}{2\pi\varepsilon_0}\ln\frac{2h}{\sqrt{4h^2+d^2}}$$

$$C=\frac{\rho}{\Delta V}=\frac{\pi\varepsilon_0}{\ln\frac{2dh}{a\sqrt{4h^2+d^2}}}$$

P4-14 a). Let l_1 denotes the distance to $+\rho_l$ and l_2 denotes the distance to $-\rho_l$



$$V = \frac{\rho_l}{2\pi\varepsilon_0} \ln\frac{l_2}{l_1}$$

When on the surface of the long wire,

$$l_2 = b+(c_1-a_1) \quad l_1 = b+(c_1-a_1)$$

When on the surface of the circular tunnel,

$$l_2 = b+(c_2-a_2) \quad l_1 = b-(c_2-a_2)$$

Then $V_1 - V_2 = \dfrac{\rho_l}{2\pi\varepsilon_0} \ln\left[\dfrac{b+(c_1-a_1)}{b-(c_1-a_1)} \cdot \dfrac{b-(c_2-a_2)}{b+(c_2-a_2)}\right]$

We know that from the text book P. 168-169.

$$\begin{cases} b^2 = c_1^2 - a_1^2 = c_2^2 - a_2^2 \\ c_2 - c_1 = D \end{cases} \Rightarrow \begin{cases} c_1 = \frac{1}{2D}(a_2^2 - a_1^2 - D) \\ c_2 = \frac{1}{2D}(a_2^2 - a_1^2 + D) \end{cases}$$

$$b = \sqrt{c_1^2 - a_1^2} = \frac{1}{2D}\sqrt{(a_2^2 - a_1^2 - D)^2 - 4D^2 a_1^2}$$

$$C = \frac{\rho_l}{V_1 - V_2} = \frac{2\pi\varepsilon_0}{\ln\left[\dfrac{b+(c_1-a_1)}{b-(c_1-a_1)} \cdot \dfrac{b-(c_2-a_2)}{b+(c_2-a_2)}\right]}$$

b). $$F = \frac{\rho_l^2}{2\pi\varepsilon_0 (2b)^2} = \frac{\rho_l^2 D^2}{2\pi\varepsilon_0\left[(a_2^2 - a_1^2 - D)^2 - 4D^2 a_1^2\right]}$$

P4-17 Suppose the point is (x, y, z).

$$V_1 = \frac{1}{4\pi\varepsilon_1\varepsilon_0}\left(\frac{Q}{\sqrt{(x-d)^2+y^2+z^2}} - \frac{Q_1}{\sqrt{(x+d)^2+y^2+z^2}}\right)$$

$$V_2 = \frac{1}{4\pi\varepsilon_2\varepsilon_0}\left(\frac{Q+Q_1}{\sqrt{(x-d)^2+y^2+z^2}}\right)$$

$$\nabla^2 V_1 = \nabla^2 V_2 = 0.$$

$$\varepsilon_0\varepsilon_1 \frac{\partial V_1}{\partial x}\bigg|_{x=0} = \varepsilon_0\varepsilon_2 \frac{\partial V_2}{\partial x}\bigg|_{x=0}.$$

$$\frac{Q-Q_1}{\varepsilon_1} = \frac{Q+Q_2}{\varepsilon_2}$$

$$Q_1 = Q_2 = \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + \varepsilon_1} Q$$

P4-23 a). $V = a\phi + b$

$$\left.\begin{array}{l} V(0) = b = 0 \\ V(\alpha) = a\alpha + b = V_0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} a = \frac{V_0}{\alpha} \\ b = 0 \end{array}\right.$$

Therefore, $V = \frac{V_0}{\alpha} \cdot \phi$ for $0 < \phi < \alpha$

b) $$\left.\begin{array}{l} V(\alpha) = a\alpha + b = V_0 \\ V(2\pi) = a \cdot 2\pi + b = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} a = -\frac{V_0}{2\pi - \alpha} \\ b = \frac{2\pi V_0}{2\pi - \alpha} \end{array}\right.$$

Therefore, $V = -\frac{V_0}{2\pi - \alpha}\phi + \frac{2\pi V_0}{2\pi - \alpha}$

P4-28 a). $V(b, \theta) = V_0$

$V(R, \theta) = -E_0 R \cos\theta + V_0$ for $R \geq b$.

$$V(R,\theta) = \frac{B_0}{R} + \left(\frac{B_1}{R^3} - E_0\right)\cos\theta - \sum_{n=2}^{\infty} B_n R^{-(n+1)} P_n(\cos\theta)$$

$B_0 = bV_0$

$B_1 = E_0 b^3$

$B_n = 0$ when $n \geq 2$

$$V(R,\theta) = \frac{bV_0}{R} + \left(\frac{E_0 b^3}{R^3} - E_0\right) R\cos\theta.$$

b). $$\vec{E}_R = -\frac{\partial V}{\partial R} = \frac{bV_0}{R^2} + \left(\frac{2E_0 b^3}{R^3} + E_0\right)\cos\theta$$

$$\vec{E}_\theta = -\frac{\partial V}{\partial \theta} = \left(\frac{E_0 b^3}{R^3} - E_0\right) R\sin\theta$$

$$\vec{E}(R,\theta) = \hat{a}_R\left(\frac{bV_0}{R^2} + \left(\frac{2E_0 b^3}{R^3} + E_0\right)\cos\theta\right) + \hat{a}_\theta\left(\frac{E_0 b^3}{R^3} - E_0\right) R\sin\theta$$