Homework 7

P.7–2 The circuit in Fig. 7–10 is situated in a magnetic field

$$\mathbf{B} = \mathbf{a}_z 3 \cos(5\pi 10^7 t - \tfrac{2}{3}\pi x) \qquad (\mu\text{T}).$$

Assuming $R = 15\ (\Omega)$, find the current i.

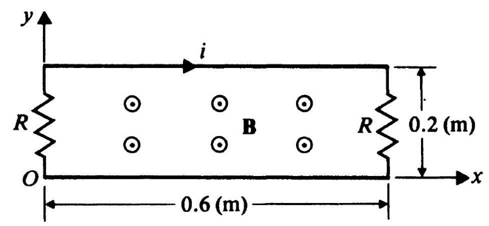
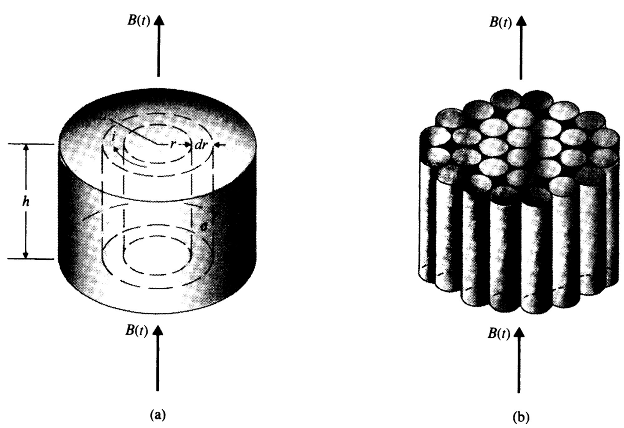


FIGURE 7–10
A circuit in a time-varying magnetic field (Problem P.7–2).



P.7–6 A suggested scheme for reducing eddy-current power loss in transformer cores with a circular cross section is to divide the cores into a large number of small insulated filamentary parts. As illustrated in Fig. 7–12, the section shown in part (a) is replaced by that in part (b). Assuming that $B(t) = B_0 \sin \omega t$ and that N filamentary areas fill 95% of the original cross-sectional area, find

a) the average eddy-current power loss in the section of core of height h in Fig. 7–12(a),

b) the total average eddy-current power loss in the N filamentary sections in Fig. 7–12(b).

The magnetic field due to eddy currents is assumed to be negligible. (*Hint:* First find the current and power dissipated in the differential circular ring section of height h and width dr at radius r.)

P.7–11 Derive the two divergence equations, Eqs. (7–53c) and (7–53d), from the two curl equations, Eqs. (7–53a) and (7–53b), and the equation of continuity, Eq. (7–48).

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{7-53a}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \tag{7-53b}$$

$$\nabla \cdot \mathbf{D} = \rho, \tag{7-53c}$$

$$\nabla \cdot \mathbf{B} = 0. \tag{7-53d}$$

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}. \tag{7-48}$$

P.7–12 Prove that the Lorentz condition for potentials as expressed in Eq. (7–62) is consistent with the equation of continuity.

$$\boxed{\nabla \cdot \mathbf{A} + \mu\epsilon \frac{\partial V}{\partial t} = 0,} \tag{7-62}$$

P.7–14 Substitute Eqs. (7–55) and (7–57) in Maxwell's equations to obtain wave equations for scalar potential V and vector potential $\mathbf{A}$ for a linear, isotropic but inhomogeneous medium. Show that these wave equations reduce to Eqs. (7–65) and (7–63) for simple media. (*Hint:* Use the following gauge condition for potentials in an inhomogeneous medium:

$$\nabla \cdot (\epsilon \mathbf{A}) + \mu\epsilon^2 \frac{\partial V}{\partial t} = 0.) \tag{7-117}$$

$$\boxed{\mathbf{B} = \nabla \times \mathbf{A} \qquad (\mathrm{T}).} \tag{7-55}$$

$$\boxed{\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} \qquad (\mathrm{V/m}).} \tag{7-57}$$

$$\boxed{\nabla^2 \mathbf{A} - \mu\epsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu \mathbf{J}.} \tag{7–63}$$

$$\boxed{\nabla^2 V - \mu\epsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon},} \tag{7–65}$$

P.7–17 Discuss the relations

a) between the boundary conditions for the tangential components of **E** and those for the normal components of **B**,

b) between the boundary conditions for the normal components of **D** and those for the tangential components of **H**.

P.7–20 Prove by direct substitution that any twice differentiable function of $(t - R\sqrt{\mu\epsilon})$ or of $(t + R\sqrt{\mu\epsilon})$ is a solution of the homogeneous wave equation, Eq. (7–73).

$$\frac{\partial^2 U}{\partial R^2} - \mu\epsilon \frac{\partial^2 U}{\partial t^2} = 0. \tag{7–73}$$