VE230 HW1

Due: Wednesday 27th May, 2020

P.2–11 Prove that an angle inscribed in a semicircle is a right angle.

P.2–17 A field is expressed in spherical coordinates by $\mathbf{E} = \mathbf{a}_R(25/R^2)$.

a) Find $|\mathbf{E}|$ and E_x at the point $P(-3, 4, -5)$.

b) Find the angle that $\mathbf{E}$ makes with the vector $\mathbf{B} = \mathbf{a}_x 2 - \mathbf{a}_y 2 + \mathbf{a}_z$ at point P.

P.2–21 Given a vector function $\mathbf{E} = \mathbf{a}_x y + \mathbf{a}_y x$, evaluate the scalar line integral $\int \mathbf{E} \cdot d\boldsymbol{\ell}$ from $P_1(2, 1, -1)$ to $P_2(8, 2, -1)$

a) along the parabola $x = 2y^2$,

b) along the straight line joining the two points.

Is this $\mathbf{E}$ a conservative field?

P.2–26 Find the divergence of the following radial vector fields:

a) $f_1(\mathbf{R}) = \mathbf{a}_R R^n$,

b) $f_2(\mathbf{R}) = \mathbf{a}_R \dfrac{k}{R^2}$.

P.2–29 For vector function $\mathbf{A} = \mathbf{a}_r r^2 + \mathbf{a}_z 2z$, verify the divergence theorem for the circular cylindrical region enclosed by $r = 5$, $z = 0$, and $z = 4$.

P.2–33 For two differentiable vector functions $\mathbf{E}$ and $\mathbf{H}$, prove that

$$\nabla \cdot (\mathbf{E} \times \mathbf{H}) = \mathbf{H} \cdot (\nabla \times \mathbf{E}) - \mathbf{E} \cdot (\nabla \times \mathbf{H}).$$

P.2–35 Use the definition in Eq. (2–126) to derive the expression of the $\mathbf{a}_R$-component of $\nabla \times \mathbf{A}$ in spherical coordinates for a vector field $\mathbf{A} = \mathbf{a}_R A_R + \mathbf{a}_\theta A_\theta + \mathbf{a}_\phi A_\phi$.

$$\boxed{(\nabla \times \mathbf{A})_u = \mathbf{a}_u \cdot (\nabla \times \mathbf{A}) = \lim_{\Delta s_u \to 0} \frac{1}{\Delta s_u}\left(\oint_{C_u} \mathbf{A} \cdot d\boldsymbol{\ell}\right),} \qquad (2\text{–}126)$$

P.2–39 Given a vector function $\mathbf{F} = \mathbf{a}_x(x + c_1 z) + \mathbf{a}_y(c_2 x - 3z) + \mathbf{a}_z(x + c_3 y + c_4 z)$.

a) Determine the constants c_1, c_2, and c_3 if $\mathbf{F}$ is irrotational.

b) Determine the constant c_4 if $\mathbf{F}$ is also solenoidal.

c) Determine the scalar potential function V whose negative gradient equals $\mathbf{F}$.