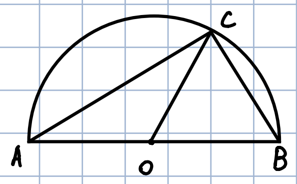
2-11.



$\vec{AC} = \vec{AO} + \vec{OC}$

$\vec{BC} = \vec{BO} + \vec{OC} = -\vec{AO} + \vec{OC}$

$\vec{AC} \cdot \vec{BC} = (\vec{AO} + \vec{OC}) \cdot (-\vec{AO} + \vec{OC})$

$= -AO^2 + \vec{AO} \cdot \vec{OC} - \vec{AO} \cdot \vec{OC} + OC^2$

$= 0$

Therefore $AC \perp BC$ and it is a right angle.

2-17. a). $|\vec{E}| = \left| \vec{a}_R \cdot \frac{25}{(-3)^2+4^2+(-5)^2} \right| = \frac{1}{2}$

$E_x = \frac{1}{2} \times \frac{-3}{\sqrt{(-3)^2+4^2+(-5)^2}} = -0.212$

(b). $\vec{OP} = (-3, 4, -5)$ $\quad \vec{OB} = (2, -2, 1)$

$\cos\theta = \frac{\vec{OP} \cdot \vec{OB}}{|\vec{OP}| \cdot |\vec{OB}|} = -\frac{19}{15\sqrt{2}}$

$\theta = \cos^{-1}\left(-\frac{19}{15\sqrt{2}}\right) = 153.59^\circ$

2-21. a). $x = 2y^2 \Rightarrow dx = 4dy$

$$\int_{P_1}^{P_2} \vec{E} \cdot d\vec{l} = \int_1^2 (4y^2 dy + 2y^2 dy) = 14$$

b). $6y = x + 4 \Rightarrow dx = 6dy$

$$\int_{P_1}^{P_2} \vec{E} \cdot d\vec{l} = \int_1^2 \left(6y\,dy + (6y-4)\,dy\right) = 14$$

Although the line integral between two points from different paths are equal, there is no enough evidence that $\vec{E}$ is a conservative field.

2-26. a). $\nabla \cdot (\vec{a}_R R^n) = \frac{1}{R^2} \frac{\partial}{\partial R} (R^2 \cdot R^n) = (n+2) R^{n-1}$

b). $\nabla \cdot \left(\vec{a}_R \cdot \frac{k}{R^2}\right) = \frac{1}{R^2} \frac{\partial}{\partial R} \left(R^2 \cdot \frac{k}{R^2}\right) = 0$

2-29. Top: $\int_S \vec{A} \cdot d\vec{s} = \int_S (\vec{a}_r r^2 + \vec{a}_z \cdot 8)\, d\vec{s} = \pi \times 5^2 \times 2 \times 4 = 200\pi$

Bottom. $\int_S \vec{A} \cdot d\vec{s} = \int_S (\vec{a}_r \cdot r^2)\, d\vec{s} = 0.$

Side: $\int_S \vec{A} \cdot d\vec{s} = 2\pi \times 5 \times 4 \times 5^2 = 1000\pi$

$200\pi + 0 + 1000\pi = 1200\pi.$

$\nabla \cdot \vec{A} = 3r + 2$

$$\int_V \nabla \cdot \vec{A}\, dv = \int_0^4 \int_0^{2\pi} \int_0^5 (3r+2)\, r\, dr\, d\phi\, dz = 1200\pi.$$

The results are the same, therefore, the divergence theorem holds.

2-33. Let $\vec{E} = \hat{x} a_1 + \hat{y} a_2 + \hat{z} a_3$, $\vec{H} = \hat{x} b_1 + \hat{y} b_2 + \hat{z} b_3$

$$\vec{E} \times \vec{H} = \hat{x}(a_2 b_3 - a_3 b_2) + \hat{y}(a_3 b_1 - a_1 b_3) + \hat{z}(a_1 b_2 - a_2 b_1)$$

$$\nabla \cdot (\vec{E} \times \vec{H}) = \frac{\partial}{\partial x}(a_2 b_3 - a_3 b_2) + \frac{\partial}{\partial y}(a_3 b_1 - a_1 b_3) + \frac{\partial}{\partial z}(a_1 b_2 - a_2 b_1)$$

$$\nabla \times \vec{E} = \hat{x}\left(\frac{\partial a_3}{\partial y} - \frac{\partial a_2}{\partial z}\right) + \hat{y}\left(\frac{\partial a_1}{\partial z} - \frac{\partial a_3}{\partial x}\right) + \hat{z}\left(\frac{\partial a_2}{\partial x} - \frac{\partial a_1}{\partial y}\right)$$

$$\vec{H} \cdot (\nabla \times \vec{E}) = b_1\left(\frac{\partial a_3}{\partial y} - \frac{\partial a_2}{\partial z}\right) + b_2\left(\frac{\partial a_1}{\partial z} - \frac{\partial a_3}{\partial x}\right) + b_3\left(\frac{\partial a_2}{\partial x} - \frac{\partial a_1}{\partial y}\right)$$

Similarily $\vec{E} \cdot (\nabla \times \vec{H}) = a_1\left(\frac{\partial b_3}{\partial y} - \frac{\partial b_2}{\partial z}\right) + a_2\left(\frac{\partial b_1}{\partial z} - \frac{\partial b_3}{\partial x}\right) + a_3\left(\frac{\partial b_2}{\partial x} - \frac{\partial b_1}{\partial y}\right)$

$$\vec{H} \cdot (\nabla \times \vec{E}) - \vec{E} \cdot (\nabla \times \vec{H}) = \frac{\partial}{\partial x}(a_2 b_3 - a_3 b_2) + \frac{\partial}{\partial y}(a_3 b_1 - a_1 b_3) + \frac{\partial}{\partial z}(a_1 b_2 - a_2 b_1)$$

Therefore $\nabla \cdot (\vec{E} \times \vec{H}) = \vec{H} \cdot (\nabla \times \vec{E}) - \vec{E} \cdot (\nabla \times \vec{H})$

2-35 $(\nabla \times \vec{A})_R = \lim_{\Delta S_R \to 0} \frac{1}{\Delta S_R}\left(\oint \vec{A} \cdot d\vec{l}\right)$, where $\Delta S_R = R^2 \sin\theta \, \Delta\theta \Delta\phi$

Side 1: $\vec{A} \cdot d\vec{l} = A_\phi(R, \theta + \frac{\Delta\theta}{2}, \phi) \Delta\phi R \sin(\theta + \frac{\Delta\theta}{2})$,

where $A_\phi(R, \theta + \frac{\Delta\theta}{2}, \phi) = A_\phi(R, \theta, \phi) + \frac{\Delta\theta}{2} \left.\frac{\partial A_\phi}{\partial\theta}\right|_{(R,\theta,\phi)} + \text{H.O.T.}$

$$\int_{\text{side 1}} \vec{A} \cdot d\vec{l} = A_\phi(R, \theta, \phi) R \sin\theta \Delta\phi + \frac{\Delta\theta}{2} \cdot \frac{\partial}{\partial\theta}(\sin\theta A_\phi) R\Delta\phi + \text{H.O.T.} R\Delta\phi$$

Similarily: $\int_{\text{side 3}} \vec{A} \cdot d\vec{l} = A_\phi(R, \theta, \phi) R \sin\theta \Delta\phi - \frac{\Delta\theta}{2} \cdot \frac{\partial}{\partial\theta}(\sin\theta A_\phi) R\Delta\phi + \text{H.O.T.} R\Delta\phi$

Therefore $\int_{\text{side 1 and 3}} \vec{A} \cdot d\vec{l} = \left[\frac{\partial}{\partial\theta}(\sin\theta A_\phi)\right]\Big|_{(R,\theta,\phi)} R\Delta\theta\Delta\phi + \text{H.O.T.}$

Similarily: $\int_{\text{side 2 and 4}} \vec{A} \cdot d\vec{l} = \left[-\frac{\partial}{\partial\phi} A_\theta\right]\Big|_{(R,\theta,\phi)} R\Delta\theta\Delta\phi + \text{H.O.T.}$

Therefore $(\nabla \times \vec{A})_R = \frac{1}{R\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right]$

2-39 a). $\nabla \times \vec{F} = 0 \Rightarrow \hat{x}\left[\frac{\partial(x + c_3 y + c_4 z)}{\partial y} - \frac{\partial(c_2 x - 3z)}{\partial z}\right] + \hat{y}\left[\frac{\partial(x + c_1 z)}{\partial z} - \frac{\partial(x + c_3 y + c_4 z)}{\partial x}\right] + \hat{z}\left[\frac{\partial(c_2 x - 3z)}{\partial x} - \frac{\partial(x + c_1 z)}{\partial y}\right] = 0$

$$\Rightarrow \begin{cases} c_3 + 3 = 0 \\ c_1 - 1 = 0 \\ c_2 - 0 = 0 \end{cases} \Rightarrow \begin{cases} c_1 = 1 \\ c_2 = 0 \\ c_3 = -3 \end{cases}$$

b). $\nabla \cdot \vec{F} = 0 \Rightarrow \frac{\partial(x + c_1 z)}{\partial x} + \frac{\partial(c_2 x - 3z)}{\partial y} + \frac{\partial(x + c_3 y + c_4 z)}{\partial z} = 0$

$$\Rightarrow 1 + c_4 = 0 \Rightarrow c_4 = -1$$

c). $\vec{F} = -\nabla \cdot V \Rightarrow \hat{a}_x(x + z) + \hat{a}_y(-3z) + \hat{a}_z(x - 3y - z) = -\hat{a}_x \frac{\partial V}{\partial x} - \hat{a}_y \frac{\partial V}{\partial y} - \hat{a}_z \frac{\partial V}{\partial z}$

$$V = -\int (x + z)\, dx + f(y, z) = -\frac{1}{2}x^2 - zx + f(y, z)$$

$$V = -\int (-3z)\, dy + f(x, z) = 3yz + f(x, y)$$

$$V = -\int (x - 3y - z)\, dz + f(x, y) = -xz + 3yz + \frac{1}{2}z^2 + f(x, y)$$

Therefore, $V = -\frac{1}{2}x^2 - zx + 3yz + \frac{1}{2}z^2$