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P3-22 a). $\rho_{ps} = \vec{P}\cdot\hat{a_n} = \frac{1}{2}P_0 L$.

By symmetry, it is the same over all the surface.

$\rho_p = -\nabla\cdot\vec{P} = -3P_0$

b). $Q = \int_S \rho_{ps}\, ds + \int_V \rho_p\, dV = 6\times L^2\times\rho_{ps} + L^3\times\rho_p = 0$

P3-23 The point on the surface is (R, θ, ϕ).

Let $\vec{P} = P\hat{a_z}$

$\rho_{ps} = \vec{P}\cdot\hat{a_n} = -(P\hat{a_z})\cdot(\hat{a_R}) = -P\cos\theta$

$d\vec{E_z} = \frac{P\cos\theta}{4\pi\varepsilon_0 R^2}\cos\theta\,\hat{a_z}$

$\vec{E_z} = \hat{a_z}\int_0^{2\pi}\int_0^{\pi}\frac{P\cos^2\theta}{4\pi\varepsilon_0 R^2}R^2\sin\theta\, d\theta\, d\phi = \frac{P}{3\varepsilon_0}\hat{a_z} = \frac{1}{3\varepsilon_0}\vec{P}$.

P3-25 $\vec{E_1} = \hat{a_x}2y - \hat{a_y}3x + \hat{a_z}5$

$\vec{E_{1t}} = \vec{E_{2t}} = \hat{a_x}2y - \hat{a_y}3x$

$\vec{D_{1n}} = \vec{D_{2n}} \Rightarrow \varepsilon_1\vec{E_{1n}} = \varepsilon_2\vec{E_{2n}}$

$\vec{E_{2n}} = \frac{2}{3}\vec{E_{1n}} = \frac{10}{3}\hat{a_z}$

$\vec{E_2} = \hat{a_x}2y - \hat{a_y}3x + \hat{a_z}\frac{10}{3}$

$\vec{D_2} = 3\varepsilon_0\vec{E_2} = \varepsilon_0(\hat{a_x}6y - \hat{a_y}9x + \hat{a_z}10)$

Since this is only on the $z=0$ plane, we cannot determine $\vec{E_2}$ and $\vec{D_2}$ at any point in region 2.

P3-28 $E_{2t} = E_{2n}$

$E_{2t} = E_{1t} = 3$

$D_{2n} = \varepsilon_r\varepsilon_0 E_{2n} = 5\varepsilon_0$

Therefore, $\varepsilon_r = \frac{5}{3}$.

P3-32 $\vec{D} = \vec{a_r}\frac{Q}{2\pi r}$

for $r_i < r < b$, $\vec{E_1} = \frac{\vec{D}}{\varepsilon_0\varepsilon_{r1}} = \vec{a_r}\frac{Q}{2\pi r\varepsilon_0\varepsilon_{r1}}$

for $b < r < r_o$, $\vec{E_2} = \frac{\vec{D}}{\varepsilon_0\varepsilon_{r2}} = \vec{a_r}\frac{Q}{2\pi r\varepsilon_0\varepsilon_{r2}}$

$V = \int_{r_i}^{b}\vec{E_1}\,d\vec{r} + \int_b^{r_o}\vec{E_2}\,d\vec{r} = \frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{\varepsilon_{r1}}\ln\left(\frac{b}{r_i}\right) + \frac{1}{\varepsilon_{r2}}\ln\left(\frac{r_o}{b}\right)\right]$

$$C = \frac{Q}{V} = \frac{2\pi\varepsilon_0}{\frac{1}{\varepsilon_{r1}}\ln\left(\frac{b}{r_i}\right) + \frac{1}{\varepsilon_{r2}}\ln\left(\frac{r_o}{b}\right)}$$

P3-43 Within dt, they are charged with dq.

Then $dV = \frac{dq}{C} \Rightarrow dq = C\,dV$

$$dW_e = V\,dq$$

$$W_e = \int_0^V V\,dq = C\int_0^V V\,dV = \frac{1}{2}CV^2$$

Also $C = \frac{Q}{V}$, $W_e = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}$