Homework 5

You don't need to do P.5-6.

P.5–6 Lightning strikes a lossy dielectric sphere—$\epsilon = 1.2\,\epsilon_0$, $\sigma = 10$ (S/m)—of radius 0.1 (m) at time $t = 0$, depositing uniformly in the sphere a total charge 1 (mC). Determine, for all t,

a) the electric field intensity both inside and outside the sphere,

b) the current density in the sphere.

P.5–7 Refer to Problem P.5–6.

a) Calculate the time it takes for the charge density in the sphere to diminish to 1% of its initial value.

b) Calculate the change in the electrostatic energy stored in the sphere as the charge density diminishes from the initial value to 1% of its value. What happens to this energy?

c) Determine the electrostatic energy stored in the space outside the sphere. Does this energy change with time?

P.5–10 The space between two parallel conducting plates each having an area S is filled with an inhomogeneous ohmic medium whose conductivity varies linearly from σ_1 at one plate ($y = 0$) to σ_2 at the other plate ($y = d$). A d-c voltage V_0 is applied across the plates as in Fig. 5–11. Determine

a) the total resistance between the plates,

b) the surface charge densities on the plates,

c) the volume charge density and the total amount of charge between the plates.

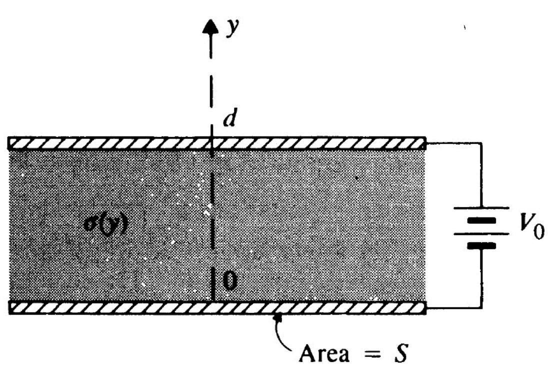

FIGURE 5–11
Inhomogeneous ohmic medium with conductivity $\sigma(y)$ (Problem P.5–10).

P.5–16 Determine the resistance between two concentric spherical surfaces of radii R_1 and R_2 ($R_1 < R_2$), assuming that a material of conductivity $\sigma = \sigma_0(1 + k/R)$ fills the space between them. (*Note:* Laplace's equation for V does not apply here.)

P.5–22 Assume a rectangular conducting sheet of conductivity σ, width a, and height b. A potential difference V_0 is applied to the side edges, as shown in Fig. 5–14. Find

a) the potential distribution,

b) the current density everywhere within the sheet. (*Hint:* Solve Laplace's equation in Cartesian coordinates subject to appropriate boundary conditions.)

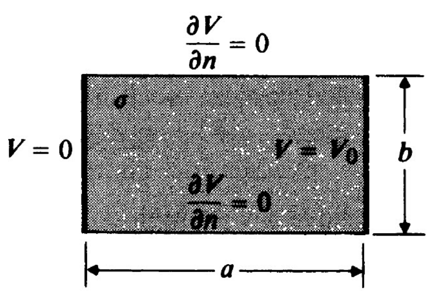

FIGURE 5–14
A conducting sheet (Problem P.5–22).

P.6–2 An electron is injected with a velocity $\mathbf{u}_0 = \mathbf{a}_y u_0$ into a region where both an electric field $\mathbf{E}$ and a magnetic field $\mathbf{B}$ exist. Describe the motion of the electron if

a) $\mathbf{E} = \mathbf{a}_z E_0$ and $\mathbf{B} = \mathbf{a}_x B_0$,

b) $\mathbf{E} = -\mathbf{a}_z E_0$ and $\mathbf{B} = -\mathbf{a}_z B_0$.

Discuss the effect of the relative magnitudes of E_0 and B_0 on the electron paths in parts (a) and (b).

P.6–12 Two identical coaxial coils, each of N turns and radius b, are separated by a distance d, as depicted in Fig. 6–39. A current I flows in each coil in the same direction.

a) Find the magnetic flux density $\mathbf{B} = \mathbf{a}_x B_x$ at a point midway between the coils.

b) Show that dB_x/dx vanishes at the midpoint.

c) Find the relation between b and d such that d^2B_x/dx^2 also vanishes at the midpoint.

Such a pair of coils are used to obtain an approximately uniform magnetic field in the midpoint region. They are known as ***Helmholtz coils***.

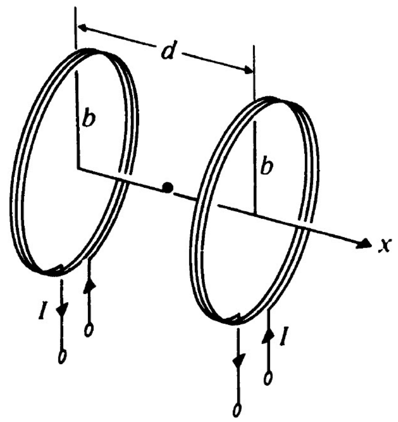

FIGURE 6–39
Helmholtz coils (Problems P.6–12).

P.6–24 Do the following by using Eq. (6–224):

a) Determine the scalar magnetic potential at a point on the axis of a circular loop having a radius b and carrying a current I.

b) Obtain the magnetic flux density $\mathbf{B}$ from $-\mu_0 \nabla V_m$, and compare the result with Eq. (6–38).

$$V_m = -\frac{I}{4\pi}\Omega, \qquad (6\text{–}224)$$

$$\boxed{\mathbf{B} = \mathbf{a}_z \frac{\mu_0 I b^2}{2(z^2 + b^2)^{3/2}} \quad \text{(T)}.} \qquad (6\text{–}38)$$