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P5-7 a). $e^{-(\sigma/\varepsilon)t} = \frac{\rho}{\rho_0} = 0.01$

$t = \frac{\ln 100}{\sigma/\varepsilon} = 4.88 \times 10^{-12}$ s

b). $W_i = \frac{\varepsilon}{2} \int_{v'} E_i^2 \, dv' = \frac{2\pi \rho_0 b^3}{45 \varepsilon} e^{-2(\sigma/\varepsilon)t}$

$\frac{W_i}{(W_i)_0} = e^{-2(\sigma/\varepsilon)t} = 10^{-4}$

heat loss

c). $W = \frac{\varepsilon_0}{2} \int_b^{\infty} E_0^2 4\pi R^2 dR = \frac{Q_0^2}{8\pi \varepsilon_0 b} = 45$ kJ

It doesn't change with time.

P5-10 a). Let $\sigma(y) = ay + b$

$$\begin{cases} \sigma(0) = b = \sigma_1 \\ \sigma(d) = ad + b = \sigma_2 \end{cases} \Rightarrow \begin{cases} a = \frac{\sigma_2 - \sigma_1}{d} \\ b = \sigma_1 \end{cases}$$

$\sigma(y) = \frac{\sigma_2 - \sigma_1}{d} y + \sigma_1$

$E = \frac{J}{\sigma}$

$V = -\int_0^d E \, dy = \frac{J d}{\sigma_2 - \sigma_1} \ln \frac{\sigma_2}{\sigma_1}$

$I = J S$

$R = \frac{V}{I} = \frac{d}{(\sigma_2 - \sigma_1) S} \ln \frac{\sigma_2}{\sigma_1}$

b). Upper plate: $\rho = \varepsilon_0 E = \frac{\varepsilon_0 J}{\sigma_2}$

Since $V = \frac{J d}{\sigma_2 - \sigma_1} \ln \frac{\sigma_2}{\sigma_1} \Rightarrow J = \frac{V(\sigma_2 - \sigma_1)}{d \ln \frac{\sigma_2}{\sigma_1}}$

$\rho = \frac{\varepsilon_0 V (\sigma_2 - \sigma_1)}{\sigma_2 d \ln \frac{\sigma_2}{\sigma_1}}$

lower plate: $\rho = -\frac{\varepsilon_0 V (\sigma_2 - \sigma_1)}{\sigma_1 d \ln \frac{\sigma_2}{\sigma_1}}$

c). $\rho = \frac{d}{dy} \varepsilon_0 E = -\varepsilon_0 J \frac{d}{dy} \frac{1}{\frac{\sigma_2 - \sigma_1}{d} y + \sigma_1} = \varepsilon_0 J \cdot \frac{\frac{\sigma_2 - \sigma_1}{d}}{\left(\frac{\sigma_2 - \sigma_1}{d} y + \sigma_1\right)^2}$

P5-16 $\vec{J} = \vec{a_R} \frac{I}{4\pi R^2} = \sigma \vec{E}$

$V = -\int_{R_2}^{R_1} E \, dR = -\int_{R_2}^{R_1} \frac{I}{4\pi R^2 \sigma_0 (1 + \frac{k}{R})} dR = \frac{I}{4\pi \sigma_0 k} \cdot \ln \frac{R_2 (R_1 + k)}{R_1 (R_2 + k)}$

$R = \frac{V}{I} = \frac{1}{4\pi \sigma_0 R} \ln \frac{R_2 (R_1 + k)}{R_1 (R_2 + k)}$

P5-22 a). $V = \frac{V_0}{a} x$

b). $\vec{E} = -\nabla V = -\hat{a}_x \frac{V_0}{a}$

$\vec{J} = \sigma \vec{E} = -\hat{a}_x \frac{V_0 \sigma}{a}$

P6-2 a). $\vec{F} = m\vec{a} = -e(\vec{E} + \vec{u}\times\vec{B})$

$$\vec{a} = -\frac{e}{m}(\vec{E} + \vec{u}\times\vec{B})$$

$$\left.\begin{aligned} \frac{\partial u_x}{\partial t} &= 0 \\ \frac{\partial u_y}{\partial t} &= -\frac{e}{m}B_0 u_x \\ \frac{\partial u_z}{\partial t} &= -\frac{e}{m}(E_0 - B_0 u_y) \end{aligned}\right\} \Rightarrow \begin{cases} u_x = 0 \\ u_y = \left(u_0 - \frac{E_0}{B_0}\right)\cos\frac{eB_0}{m}t + \frac{E_0}{B_0} \\ u_z = \left(\frac{E_0}{B_0} - u_0\right)\sin\frac{eB_0}{m}t \end{cases}$$

If injected at $(0,0,0)$, then $\begin{cases} x = 0 \\ y = \dfrac{u_0 - \frac{E_0}{B_0}}{\frac{e}{m}B_0}\sin\frac{eB_0}{m}t + \frac{E_0}{B_0}t \\ z = \dfrac{\frac{E_0}{B_0} - u_0}{\frac{e}{m}B_0}\left(1 - \cos\frac{eB_0}{m}t\right) \end{cases}$

b). $\frac{\partial u_x}{\partial x} = \frac{e}{m}B_0 u_y$

$\frac{\partial u_y}{\partial x} = -\frac{e}{m}B_0 u_x$

$\frac{\partial u_z}{\partial t} = \frac{e}{m}E_0$

P6-12 a). Let midway be $x=0$.

$$B = \frac{N\mu_0 I b^2}{2\left(\left(\frac{d}{2}+x\right)^2 + b^2\right)^{\frac{3}{2}}} + \frac{N\mu_0 I b^2}{2\left(\left(\frac{d}{2}-x\right)^2 + b^2\right)^{\frac{3}{2}}}$$

When $x=0$, $B = \frac{N\mu_0 I b^2}{\left(\frac{d^2}{4} + b^2\right)^{\frac{3}{2}}}$

b). $\frac{dB}{dx} = \frac{N\mu_0 I b^2}{2}\left(-\frac{\frac{3}{2}\times 2\left(\frac{d}{2}+x\right)}{\left(\left(\frac{d}{2}+x\right)^2+b^2\right)^{\frac{5}{2}}} + \frac{\frac{3}{2}\times 2\left(\frac{d}{2}-x\right)}{\left(\left(\frac{d}{2}-x\right)^2+b^2\right)^{\frac{5}{2}}}\right)$

When $x=0$, $\frac{dB}{dx} = 0$.

c). $\frac{dB^2}{d^2x} = -\frac{3N\mu_0 I b^2}{2}\left(-\frac{\left(\left(\frac{d}{2}+x\right)^2+b^2\right)^{\frac{5}{2}} - \left(\frac{d}{2}+x\right)\frac{5}{2}\left(\left(\frac{d}{2}+x\right)^2+b^2\right)^{\frac{3}{2}}\cdot 2\left(\frac{d}{2}+x\right)}{\left(\left(\frac{d}{2}+x\right)^2+b^2\right)^5} + \frac{\left(\left(\frac{d}{2}-x\right)^2+b^2\right)^{\frac{5}{2}} + \left(\frac{d}{2}-x\right)\frac{5}{2}\left(\left(\frac{d}{2}-x\right)^2+b^2\right)^{\frac{3}{2}}\cdot 2\left(\frac{d}{2}-x\right)}{\left(\left(\frac{d}{2}-x\right)^2+b^2\right)^5}\right)$

At $x=0$, $\frac{dB^2}{d^2x} = 0 \Rightarrow b = d$

P6-24 a). $V_m = \frac{I}{4\pi}\int_0^{2\pi}\int_0^b \frac{z}{(z^2+a^2)^{\frac{3}{2}}}\, a\, da\, d\phi$

$= \frac{I}{2}\left(1 - \frac{z}{\sqrt{z^2+b^2}}\right)$

b). $-\mu_0\nabla V_m = -\mu_0\hat{a}_z\frac{\partial V_m}{\partial z} = \hat{a}_z\frac{\mu_0 I b^2}{2(z^2+b^2)^{\frac{3}{2}}}$