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7-24. $\nabla\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t} \Rightarrow \nabla\times\nabla\times\vec{E} = -\mu\frac{\partial}{\partial t}\nabla\times\vec{H} = -\mu\frac{\partial}{\partial t}\left[\vec{J}+\varepsilon\frac{\partial\vec{E}}{\partial t}\right] = \nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E}$

Then the equation for $\vec{E}$ is $\nabla^2\vec{E} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = \mu\frac{\partial\vec{J}}{\partial t} + \frac{1}{\varepsilon}\nabla\rho$

$\nabla\times\vec{H} = \vec{J}+\varepsilon\frac{\partial\vec{E}}{\partial t} \Rightarrow \nabla\times\nabla\times\vec{H} = \nabla\times\vec{J} + \varepsilon\frac{\partial}{\partial t}\nabla\times\vec{E} = \nabla\times\vec{J} - \mu\varepsilon\frac{\partial^2\vec{H}}{\partial t^2} = \nabla(\nabla\cdot\vec{H}) - \nabla^2\vec{H} = -\nabla^2\vec{H}$

Then the equation for $\vec{H}$ is $\nabla^2\vec{H} - \mu\varepsilon\frac{\partial^2\vec{H}}{\partial t^2} = -\nabla\times\vec{J}$

Also $\frac{\partial}{\partial t} \to j\omega$, $\frac{\partial^2}{\partial t^2} \to -\omega^2$

Then we have $\nabla^2\vec{E} + \omega^2\mu\varepsilon\vec{E} = j\omega\mu\vec{J} + \frac{1}{\varepsilon}\nabla\rho$

$\nabla^2\vec{H} + \omega^2\mu\varepsilon\vec{H} = -\nabla\times\vec{J}$

7-27 $\vec{E} = \hat{a}_\theta\frac{E_0}{R}\sin\theta\cdot e^{-jkR}$

$\nabla\times\vec{E} = \hat{a}_\phi\frac{E_0}{R}\sin\theta\cdot e^{-jkR}\cdot(-jk)$

Also $\nabla\times\vec{E} = -j\omega\mu_0\vec{H}$

We have $\vec{H} = \hat{a}_\phi\frac{kE_0}{\omega\mu_0 R}\sin\theta\cdot e^{-jkR}$

In free space, $k = \omega\sqrt{\mu_0\varepsilon_0}$

Then we have $H = \hat{a}_\phi\frac{E_0}{R}\sqrt{\frac{\varepsilon_0}{\mu_0}}\sin\theta\cos\left(t-\sqrt{\mu_0\varepsilon_0}R\right)$

7-29 $\vec{H} = j\omega\varepsilon_0\nabla\times\vec{\pi}_e$ ①

$\nabla\times\vec{E} = -j\omega\mu_0\vec{H} = \omega^2\mu_0\varepsilon_0\nabla\times\vec{\pi}_e$

$\nabla\times(\vec{E} - k_0^2\vec{\pi}_e) = 0$

Let $\vec{E} - k_0^2\vec{\pi}_e = \nabla V_e$ ②

$\vec{\nabla}\times\vec{H} = j\omega\vec{D} = j\omega(\varepsilon_0\vec{E}+\vec{P}) = j\omega\varepsilon_0\left(\vec{E}+\frac{\vec{P}}{\varepsilon_0}\right)$

Using ①②, we can have

$j\omega\varepsilon_0\nabla\times\nabla\times\vec{\pi}_e = j\omega\varepsilon_0\left(k_0^2\vec{\pi}_e + \nabla V_e + \frac{\vec{P}}{\varepsilon_0}\right)$

Also $j\omega\varepsilon_0\nabla\times\nabla\times\vec{\pi}_e = j\omega\varepsilon_0\left(\nabla\nabla\cdot\vec{\pi}_e - \nabla^2\vec{\pi}_e\right)$

Then $j\omega\varepsilon_0\left(\nabla\nabla\cdot\vec{\pi}_e - \nabla^2\vec{\pi}_e\right) = j\omega\varepsilon_0\left(k_0^2\vec{\pi}_e + \nabla V_e + \frac{\vec{P}}{\varepsilon_0}\right)$

Let $\nabla\cdot\vec{\pi}_e = V_e$, we can have

$\nabla^2\vec{\pi}_e + k_0^2\vec{\pi}_e = -\frac{\vec{P}}{\varepsilon_0}$ ((b) is proved)

From②, we have $\vec{E} = k^2\vec{\pi}_e + \nabla V_e = k_0^2\vec{\pi}_e + \nabla\nabla\cdot\vec{\pi}_e$

$= k^2\vec{\pi}_e + \left(\nabla^2\vec{\pi}_e + \nabla\times\nabla\times\vec{\pi}_e\right)$ (b).

$= \nabla\times\nabla\times\vec{\pi}_e - \frac{\vec{P}}{\varepsilon_0}$ ((a) is solved)

8-7 Let $\alpha = \omega t - kz$

then $\vec{E} = \hat{a}_x E_{10} \sin\alpha + \hat{a}_y E_{20} \sin(\alpha + \psi) = \hat{a}_x E_x + \hat{a}_y E_y$

We have $E_{10} \sin\alpha = E_x \Rightarrow \sin\alpha = \dfrac{E_x}{E_{10}} \Rightarrow \cos\alpha = \sqrt{1 - \left(\dfrac{E_x}{E_{10}}\right)^2}$

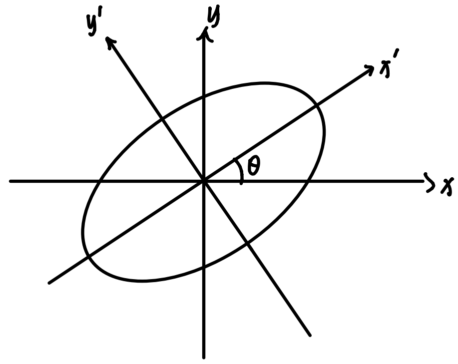
$$E_{20} \sin(\alpha + \psi) = E_y \Rightarrow \sin(\alpha + \psi) = \frac{E_y}{E_{20}} = \sin\alpha\cos\psi + \cos\alpha\sin\psi$$

$$= \frac{E_x}{E_{10}}\cos\psi + \sqrt{1 - \left(\frac{E_x}{E_{10}}\right)^2}\,\sin\psi$$

Then we have $\left(\dfrac{E_y}{E_{20}} - \dfrac{E_x}{E_{10}}\cos\psi\right)^2 = \left[1 - \left(\dfrac{E_x}{E_{10}}\right)^2\right]\sin^2\psi$

$$\left(\frac{E_y}{E_{20}}\right)^2 + \left(\frac{E_x}{E_{10}}\cos\psi\right)^2 - 2\frac{E_x E_y}{E_{10}E_{20}}\cos\psi = \sin^2\psi - \left(\frac{E_x}{E_{10}}\right)^2 \sin^2\psi$$

$$\left(\frac{E_y}{E_{20}\sin\psi}\right)^2 + \left(\frac{E_x}{E_{10}\sin\psi}\right)^2 - 2\frac{E_x E_y}{E_{10}E_{20}}\frac{\cos\psi}{\sin^2\psi} = 1$$



$E_x' = E_x\cos\theta + E_y\sin\theta$

$E_y' = -E_x\sin\theta + E_y\cos\theta$

$$\left(\frac{E_x\cos\theta + E_y\sin\theta}{a}\right)^2 + \left(\frac{-E_x\sin\theta + E_y\cos\theta}{b}\right)^2 = 1$$

$$E_x^2\underbrace{\left(\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}\right)}_{=\frac{1}{E_{10}^2\sin^2\psi}} + E_y^2\underbrace{\left(\frac{\sin^2\theta}{a^2} + \frac{\cos^2\theta}{b^2}\right)}_{=\frac{1}{E_{20}^2\sin^2\psi}} - 2E_xE_y\underbrace{\sin\theta\cos\theta\left(\frac{1}{b^2} - \frac{1}{a^2}\right)}_{=\frac{\cos\psi}{E_{10}E_{20}\sin^2\psi}} = 1$$

Then $\theta = \dfrac{1}{2}\tan^{-1}\left(\dfrac{2E_{10}E_{20}\cos\psi}{E_{10}^2 - E_{20}^2}\right)$

$$a = \sqrt{\frac{2}{\frac{1}{E_{10}^2}(1+\sec 2\theta) + \frac{1}{E_{20}^2}(1-\sec 2\theta)}}\;\sin\psi$$

$$b = \sqrt{\frac{2}{\frac{1}{E_{10}^2}(1-\sec 2\theta) + \frac{1}{E_{20}^2}(1+\sec 2\theta)}}\;\sin\psi$$

If $E_{20} = E_{10} = E_0$, then $\theta = 45^\circ$, $a = \sqrt{2}\,E_0\cos\frac{\psi}{2}$, $b = \sqrt{2}\,E_0\sin\frac{\psi}{2}$

8-9 $k_c = \beta - j\alpha$

$k_c^2 = \beta^2 - \alpha^2 - 2j\alpha\beta$

$= \omega^2 \mu \varepsilon_c = \omega^2 \mu \varepsilon \left(1 - j\frac{\sigma}{\omega\varepsilon}\right)$

Then $\beta^2 - \alpha^2 = Re(k_c^2) = \omega^2 \mu \varepsilon$

$\beta^2 - \alpha^2 = |k_c^2| = \omega^2 \mu \varepsilon \sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2}$

$\Rightarrow \alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}} \cdot \left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]^{\frac{1}{2}}$

$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}} \cdot \left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]^{\frac{1}{2}}$

8-15. a). $u_g = \frac{d\omega}{d\beta} = \frac{d}{d\beta}(\beta u_p) = u_p + \beta \frac{du_p}{d\beta}$

b). $\lambda = \frac{2\pi}{\beta}$, $\frac{d\lambda}{d\beta} = -\frac{2\pi}{\beta^2} = -\frac{\lambda}{\beta}$

$u_g = u_p + \beta\left(\frac{du_p}{d\lambda} \frac{d\lambda}{d\beta}\right) = u_p - \lambda \frac{du_p}{d\lambda}$