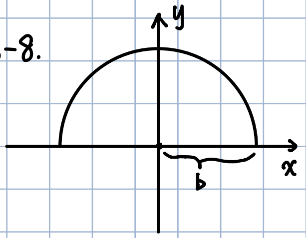
VE 230 Hw2 周维凯 518021911039

P3-8.



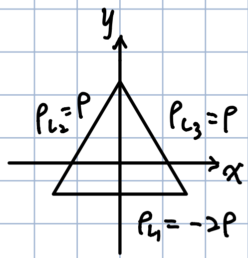
$Q = \rho_L \times l$

$dQ = \rho_L \, dl$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int_L \hat{a}_R \frac{\rho_L}{b^2} dl' = \frac{1}{4\pi\varepsilon_0} \int_0^{\pi} -\hat{a}_y \frac{\rho_L}{b^2} \sin\theta \, b \, d\theta = -\hat{a}_y \frac{\rho_L}{2\pi\varepsilon_0 b}$$

Therefore, the magnitude is $\frac{|\rho_L|}{2\pi\varepsilon_0 b}$. When $\rho_L > 0$, the direction is $-y$; $\rho_L < 0$, it is $+y$.

P3-9



Let $\rho_{L_1} = -2\rho$, $\rho_{L_2} = \rho$, $\rho_{L_3} = \rho$.

$$E_1 = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{1}{4\pi\varepsilon_0} \cdot \frac{\rho}{\left(x^2 + \frac{L^2}{12}\right)} \cdot \frac{\frac{L}{2\sqrt{3}}}{\sqrt{x^2 + \frac{L^2}{12}}} dx = \frac{3\rho}{\pi\varepsilon_0 L}$$

Similarly, $E_2 = E_3 = \frac{3\rho}{2\pi\varepsilon_0 L}$

$$\vec{E} = \frac{3\rho}{\pi\varepsilon_0 L} \hat{a}_y + \frac{3\rho}{2\pi\varepsilon_0 L} \left(\frac{\sqrt{3}}{2} \hat{a}_x - \frac{1}{2} \hat{a}_y + \frac{\sqrt{3}}{2} \hat{a}_x - \frac{1}{2} \hat{a}_y \right)$$

$$= \frac{3\rho}{2\pi\varepsilon_0 L} \hat{a}_y$$

P3-12 a). When $r<a$, the Q enclosed is 0, therefore $E=0$

When $a<r<b$, $2\pi r h E = \frac{2\pi a h \rho_{sa}}{\varepsilon_0} \Rightarrow E = \frac{a\rho_{sa}}{r\varepsilon_0}$

When $r>b$, $2\pi r h E = \frac{2\pi a h \rho_{sa} + 2\pi b h \rho_{sb}}{\varepsilon_0} \Rightarrow E = \frac{a\rho_{sa} + b\rho_{sb}}{r\varepsilon_0}$

Therefore, $\vec{E} = \begin{cases} 0 & r<a \\ \frac{a\rho_{sa}}{r\varepsilon_0} \vec{r} & a<r<b \\ \frac{a\rho_{sa}+b\rho_{sb}}{r\varepsilon_0} \vec{r} & r>b \end{cases}$, where $\vec{r}$ is direction along the radius.

b). $a\rho_{sa} + b\rho_{sb} = 0 \Rightarrow \frac{a}{b} = -\frac{\rho_{sb}}{\rho_{sa}}$

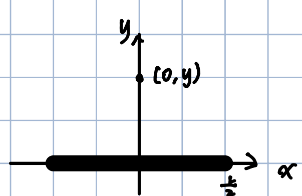
P3-13 a). $dx = 4y\,dy$

$$W = \int_1^2 2\times10^{-6} \times (y \cdot 4y\,dy + 2y^2 dy) = 28 \ \mu J$$

b). $y = \frac{1}{6}x + \frac{2}{3} \Rightarrow dx = 6dy$

$$W = \int_1^2 2\times10^{-6} \times \left(y \cdot 6dy + (6y-4)dy \right) = 28 \ \mu J$$

P3-16



a) Let $V=0$ at infinity.

$$V = 2\int_0^{\frac{L}{2}} \frac{\rho_L}{4\pi\varepsilon_0\sqrt{y^2+x^2}} dx = \frac{\rho_L}{2\pi\varepsilon_0} \left(\ln\left(\sqrt{\tfrac{1}{4}L^2+y^2} - \tfrac{1}{2}L\right) - \ln y \right)$$

where y is the distance from the x-axis.

b) $$\vec{E} = \hat{a}_y \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\rho_L}{4\pi\varepsilon_0(x^2+y^2)} \cdot \frac{y}{\sqrt{x^2+y^2}} dx = \frac{L\rho_L}{4\pi\varepsilon_0 y\sqrt{\frac{L^2}{4}+y^2}} \hat{a}_y$$

c). $-\nabla V = -\frac{\rho_L L}{4\pi\varepsilon_0 y\sqrt{\frac{L^2}{4}+y^2}}$

P3-19 Let the center of the bottom of the tube be the origin.

$$\rho = \frac{Q}{2\pi b h}$$

a).
$$V = \int_0^h \int_0^{2\pi} \frac{1}{4\pi\varepsilon_0} \frac{\rho}{\sqrt{b^2 + (z-x)^2}}\, b\, d\theta\, dx = \frac{Q}{4\pi\varepsilon_0 h} \ln \frac{z + \sqrt{b^2 + z^2}}{(z-h) + \sqrt{b^2 + (z-h)^2}}$$

$$E = -\frac{\partial V}{\partial z}\hat{a}_z = \frac{Q}{4\pi\varepsilon_0 h}\left(\frac{1}{\sqrt{b^2 + (z-h)^2}} - \frac{1}{\sqrt{z^2 + b^2}}\right)\hat{a}_z$$

b). $V = \int_0^h \int_0^{2\pi} \frac{1}{4\pi\varepsilon_0} \frac{\rho}{\sqrt{b^2 + (x-z)^2}}\, b\, d\theta\, dx.$ Therefore, the answers are the same.