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7.2 $\int_s \vec{B}\cdot d\vec{s}$

$$= \int_0^{0.6} 3\cos(5\pi 10^7 t - \frac{2}{3}\pi x)\cdot(0.2\,dx)\times10^{-6}$$

$$= -\frac{0.09}{\pi}\times10^{-6}\left[\sin(5\pi\times10^7 t - 0.4\pi) - \sin(5\pi\times10^7 t)\right] \quad Wb$$

$$V = -\frac{d}{dt}\left(\int_s \vec{B}\cdot d\vec{s}\right) = 4.5\left[\cos(5\pi\times10^7 t - 0.4\pi) - \cos(5\pi\times10^7 t)\right] \quad V$$

$$i = \frac{V}{2R} = 0.15\left[\cos(5\pi\times10^7 t - 0.4\pi) - \cos(5\pi\times10^7 t)\right] \quad A.$$

7.6 a). $\Phi = B(t)\,\pi r^2$

$$V = iR_r = \frac{d\Phi}{dt} \Rightarrow i\left(\frac{2\pi r}{\sigma h(dr)}\right) = \frac{d\Phi}{dt}$$

Therefore, $i = \frac{\sigma h}{2\pi r}\cdot dr\cdot\frac{dB(t)}{dt}\cdot\pi r^2 = \frac{\sigma h r}{2}\cdot dr\cdot\frac{dB(t)}{dt}$

$$dP = i^2R = \frac{\pi\sigma h r^3}{2}\cdot dr\cdot\left(\frac{dB(t)}{dt}\right)^2$$

$$P = \int\left[\frac{\pi\sigma h r^3}{2}\left(\frac{dB}{dt}\right)^2\right]dr = \frac{\pi\sigma h a^4}{8}\left(\frac{dB}{dt}\right)^2 = \frac{\pi\sigma h a^4}{8}B_0^2\omega^2\cos^2\omega t$$

$$P_{Avg} = \frac{\pi\sigma h a^4}{16}B_0^2\omega^2$$

b). $S = \frac{0.95\pi a^2}{N} = \pi b^2 \Rightarrow b = a\sqrt{\frac{0.95}{N}}$

$$P_1 = N\frac{\pi\sigma h}{8}\left(a\sqrt{\frac{0.95}{N}}\right)^4 B_0^2\omega^2\cos^2\omega t = \frac{\pi\sigma h}{8N}\left(a\sqrt{0.95}\right)^2 B_0^2\omega^2\cos^2\omega t = \frac{0.95^2}{N}P_{Avg}$$

7-11 $\nabla\cdot(\nabla\times\vec{E}) = \nabla\cdot\left(-\frac{\partial\vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla\cdot\vec{B})$

$$0 = -\frac{\partial}{\partial t}(\nabla\cdot\vec{B})$$

$$0 = \nabla\cdot\vec{B}$$

$$\nabla\cdot(\nabla\times\vec{H}) = \nabla\cdot\left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right) = \nabla\cdot\vec{J} + \frac{\partial}{\partial t}\nabla\cdot\vec{D} = -\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial t}\nabla\cdot\vec{D}$$

$$0 = -\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial t}\nabla\cdot\vec{D} = \frac{\partial}{\partial t}(\rho - \nabla\cdot\vec{D})$$

$$\rho = \nabla\cdot\vec{D}$$

7-12 $\nabla^2 A - \mu\varepsilon\frac{\partial^2 A}{\partial t^2} = -\mu J \Rightarrow J = -\frac{1}{\mu}\left(\nabla^2 A - \mu\varepsilon\frac{\partial^2 A}{\partial t^2}\right)$

$$\nabla^2 V - \mu\varepsilon\frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon} \Rightarrow \rho = -\varepsilon\left(\nabla^2 V - \mu\varepsilon\frac{\partial^2 V}{\partial t^2}\right)$$

$$\nabla\cdot J + \frac{\partial\rho}{\partial t} = -\frac{1}{\mu}\nabla^2\left(\nabla\cdot A + \mu\varepsilon\frac{\partial V}{\partial t}\right) + \varepsilon\frac{\partial^2}{\partial t^2}\left(\nabla\cdot A + \mu\varepsilon\frac{\partial V}{\partial t}\right)$$

Since $\nabla\cdot A + \mu\varepsilon\frac{\partial V}{\partial t} = 0$, we have $\nabla\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0 \Rightarrow \nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$

7-16 $\nabla\times\vec{H}=\vec{J}+\frac{\partial\vec{D}}{\partial t} \Rightarrow \nabla\times\left(\frac{\vec{B}}{\mu}\right)=\vec{J}+\varepsilon\frac{\partial\vec{E}}{\partial t}$

$$\Rightarrow \nabla\times\left(\frac{\nabla\times\vec{A}}{\mu}\right)=\vec{J}+\varepsilon\frac{\partial\left(-\nabla V-\frac{\partial A}{\partial t}\right)}{\partial t}$$

$$\nabla\times\frac{(\nabla\times\vec{A})}{\mu}=\vec{J}-\varepsilon\nabla\frac{\partial V}{\partial t}-\varepsilon\frac{\partial^2\vec{A}}{\partial t^2}$$

Since $\nabla\cdot(\varepsilon\vec{A})+\mu\varepsilon^2\frac{\partial V}{\partial t}=0 \Rightarrow \frac{\partial V}{\partial t}=\frac{-\nabla\cdot(\varepsilon\vec{A})}{\mu\varepsilon^2}$

Then $\nabla\times\frac{(\nabla\times\vec{A})}{\mu}=\vec{J}-\varepsilon\nabla\left(\frac{-\nabla\cdot(\varepsilon\vec{A})}{\mu\varepsilon^2}\right)-\varepsilon\frac{\partial^2\vec{A}}{\partial t^2}$

$-\nabla\times\left(\frac{1}{\mu}\nabla\times\vec{A}\right)+\varepsilon\nabla\left(\frac{1}{\mu\varepsilon^2}\nabla\cdot(\varepsilon\vec{A})\right)-\varepsilon\frac{\partial^2\vec{A}}{\partial t^2}=-\vec{J}$, wave equation for vector potential

$\nabla\cdot\vec{D}=\rho$

$\nabla\cdot(\varepsilon\nabla V)+\frac{\partial}{\partial t}\nabla\cdot(\varepsilon\vec{A})=-\rho$

$\frac{1}{\varepsilon}\nabla(\varepsilon\nabla V)-\mu\varepsilon\frac{\partial^2 V}{\partial t^2}=-\frac{\rho}{\varepsilon}$ wave equation for scalar potential

7-17 a). $E_{1t}=E_{2t}$

$B_{1n}=B_{2n}$

b). $\vec{a}_{n2}\times(\vec{H}_1-\vec{H}_2)=\vec{J}$

$\vec{a}_{n2}\cdot(\vec{D}_1-\vec{D}_2)=\rho$

7-20 $\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right)-\mu\varepsilon\frac{\partial^2 V}{\partial t^2}=0.$

$V(R,t)=\frac{1}{R}U(R,t)$

$\frac{\partial^2 U}{\partial R^2}=\mu\varepsilon\frac{\partial^2 U}{\partial t^2}$