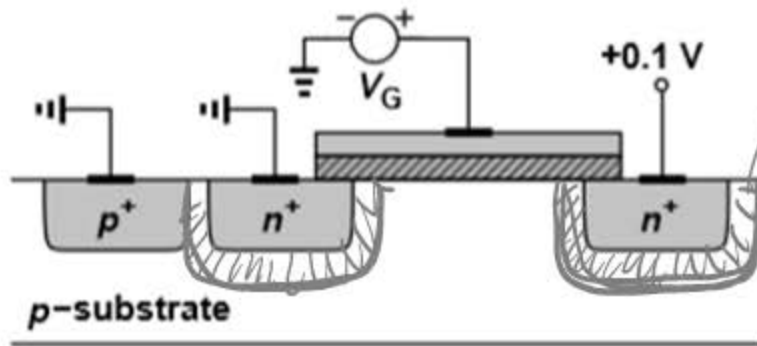
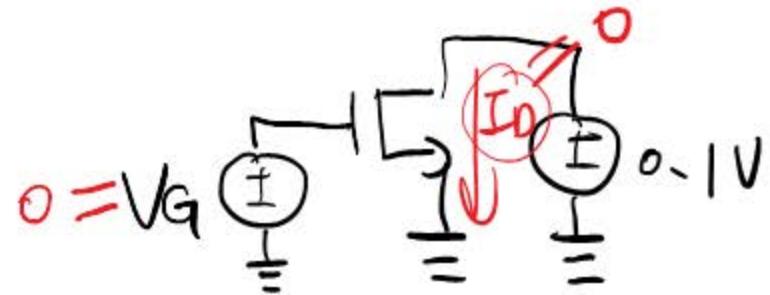


# Threshold Voltage ( $V_{TH}$ ) for NMOS

1

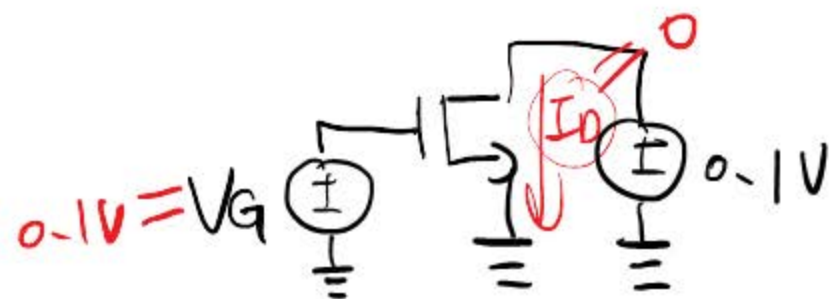
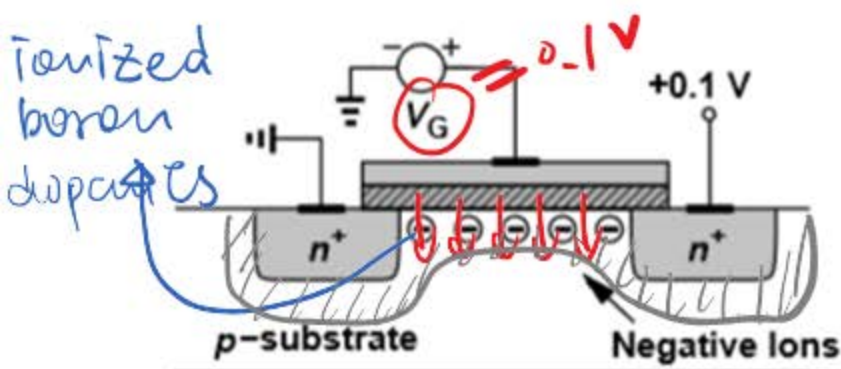


- depletion region.*
- $V_G = 0\text{ V}$
  - No current flow



# Threshold Voltage ( $V_{TH}$ ) for NMOS

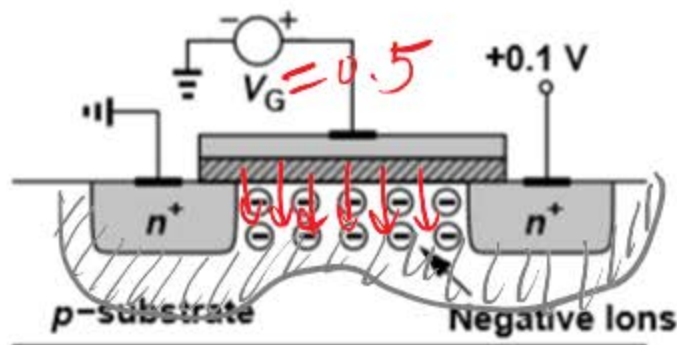
2



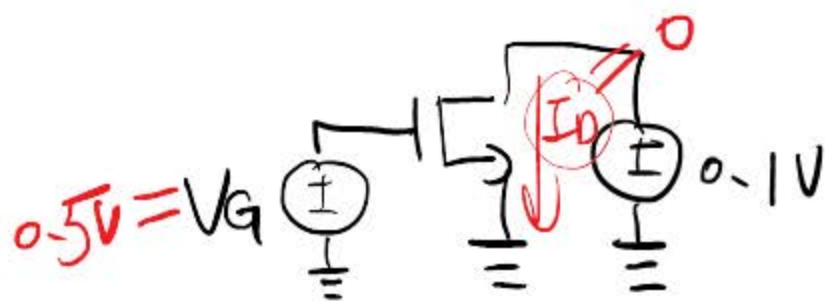
- As  $V_G$  increases from zero, holes in  $p$ -substrate are repelled, leaving negative ions (ionized boron dopants) behind to form a **depletion region**.
- Positive charges are mirrored at the gate.
- No charge carriers (electrons or holes) in the channel, so no current flow.

# Threshold Voltage ( $V_{TH}$ ) for NMOS

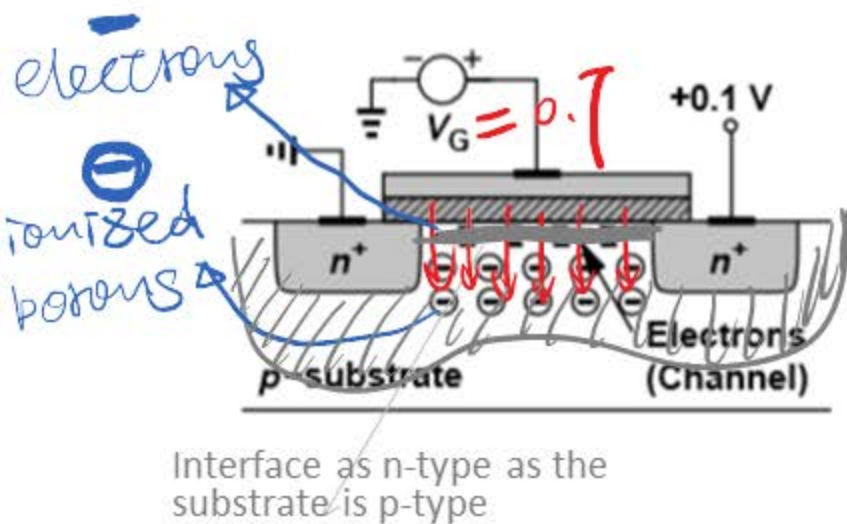
3



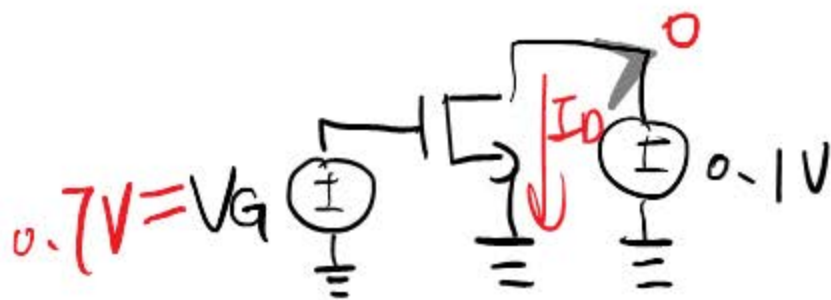
- Higher  $V_G$  further increases the width of the depletion region.

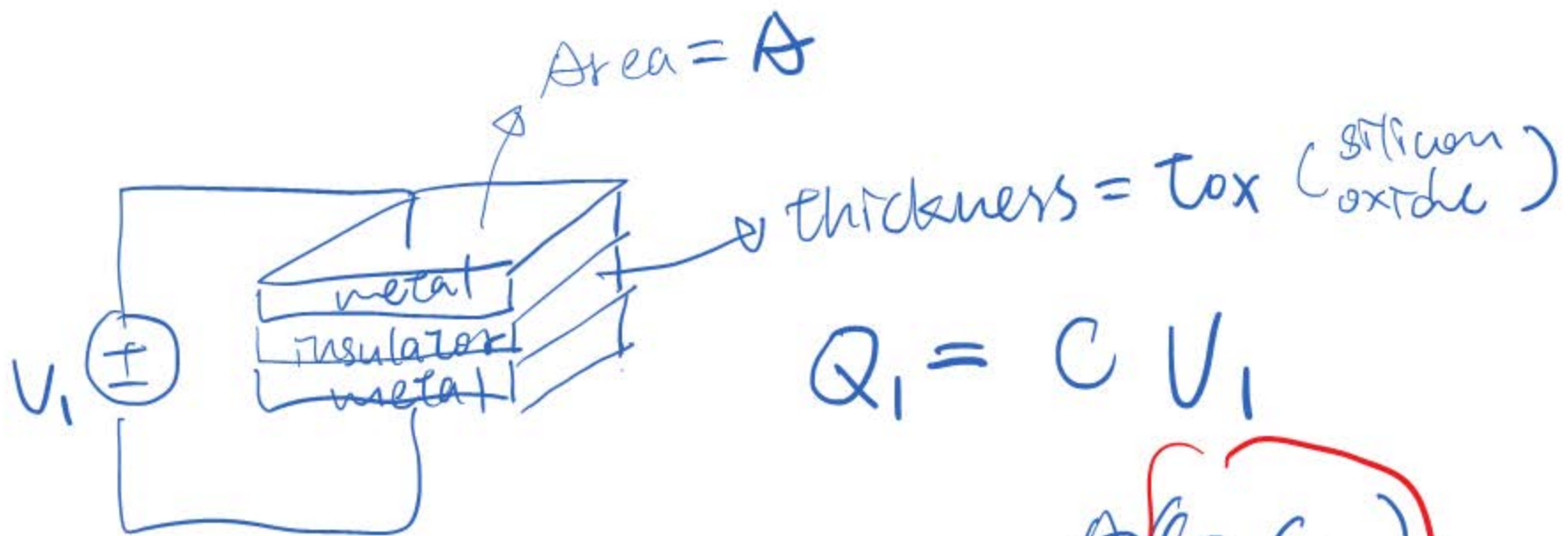


# Threshold Voltage ( $V_{TH}$ ) for NMOS



- When  $V_G$  reaches a sufficiently positive value, a channel of electrons (**inversion layer**) is formed beneath the gate oxide.
- Electrons flow from "source" to "drain". Equivalently, current flows from "drain" to "source".
- The value of  $V_G$  at which the inversion layer forms is the **threshold voltage ( $V_{TH}$ )**.
- If  $V_G$  rises further, the charges in the depletion region remain relatively constant, whereas the charges in the inversion layer increase rapidly.





$$Q_1 = C V_1$$

$$= \frac{A(\epsilon_0 \epsilon_r)}{t_{ox}} V_1$$

$C_{ox}$

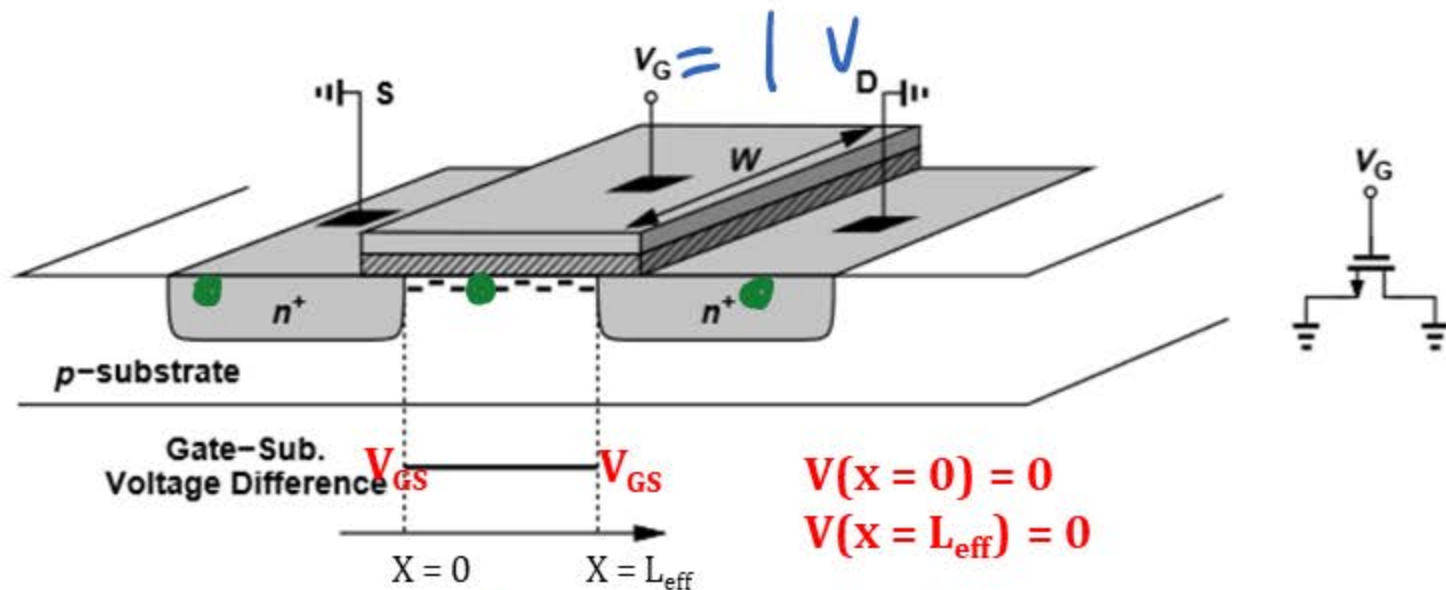
$$\epsilon_0: 8.85 \times 10^{-12} \text{ F/m}$$

$$\epsilon_r: 3.9$$



# I-V Characteristics for NMOS (Triode)

6



For  $V_{GS} \geq V_{TH} = 0.7V$

$$Q = -WL_{eff}C_{ox}(V_{GS} - V_{TH}) \text{ (unit: coulomb)}$$

$$Q_d = -WC_{ox}(V_{GS} - V_{TH}) \text{ (unit: coulomb} \cdot \text{m}^{-1}\text{)}$$

$C_{ox}$  (gate oxide capacitance per unit area)

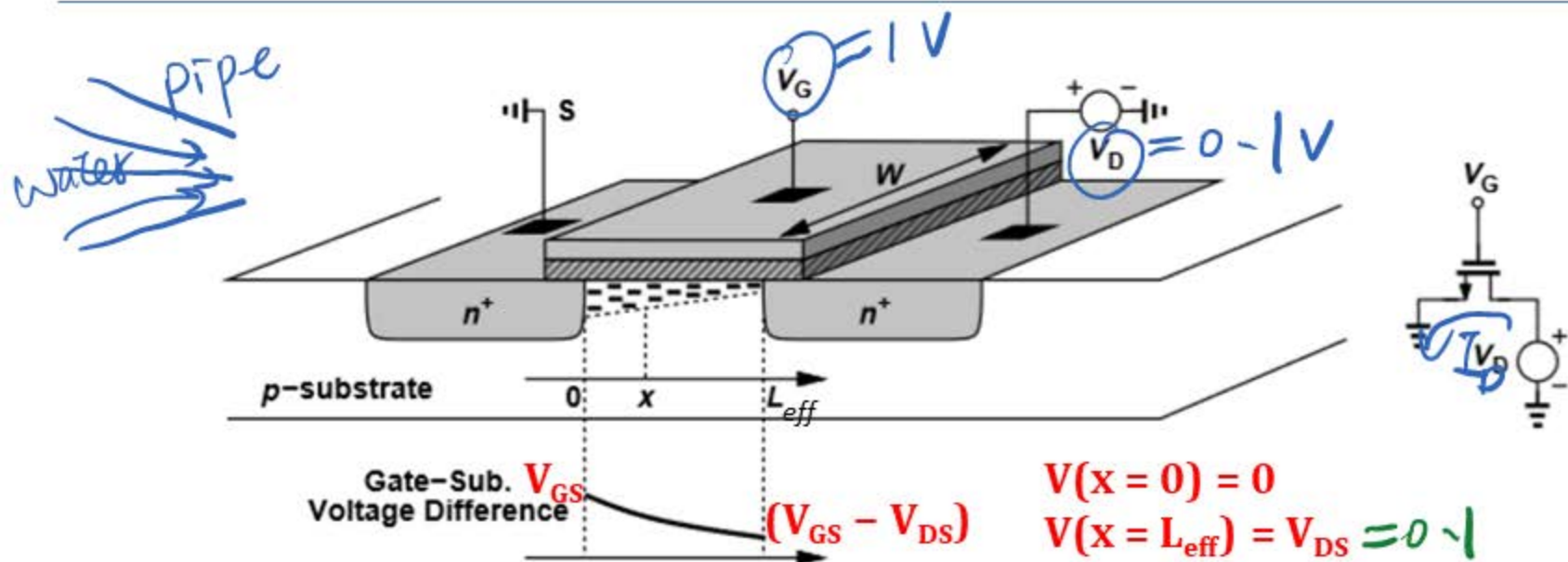
$$= \epsilon_{\text{silicon oxide}} / t_{ox}$$

$$= [8.85 \times 10^{-12} \text{ (F/m)} \times 3.9] / t_{ox}$$

(F/m<sup>2</sup>)

# I-V Characteristics for NMOS (Triode)

7



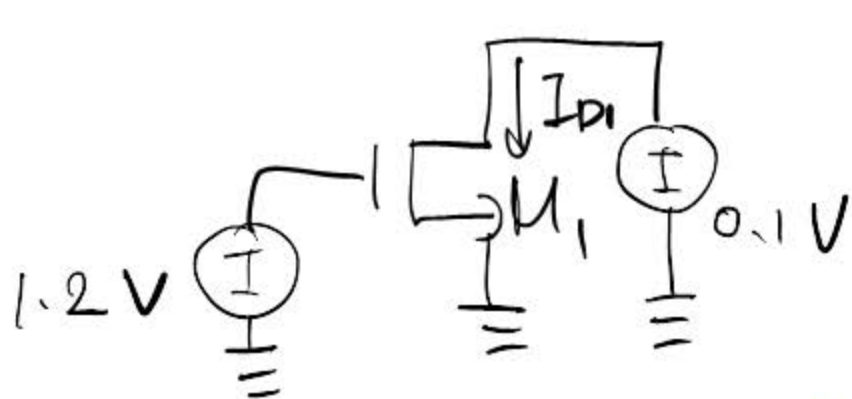
$$I_D = Q_d \times v = Q_d \times (\mu_n \mathcal{E}) = -WC_{ox}[V_{GS} - V_{TH} - V(x)] \times (\mu_n \mathcal{E}) \quad \mathcal{E} = -dV(x)/dx$$

$$= WC_{ox}[V_{GS} - V_{TH} - V(x)] \times \mu_n \times \frac{dV(x)}{dx}$$

$$\int_{x=0}^{x=L_{eff}} I_D \cdot dx = \int_{V(0)=0}^{V(L)=V_{DS}} \mu_n C_{ox} W [V_{GS} - V_{TH} - V(x)] \cdot dV(x)$$

$I_D$ : constant along channel

$$I_D = \mu_n C_{ox} \frac{W}{L_{eff}} [(V_{GS} - V_{TH})V_{DS} - \frac{1}{2}V_{DS}^2]$$



$$\mu_1 = \frac{20 \mu\text{m}}{5 \mu\text{m}} = \frac{W_{\text{drawn}}}{L_{\text{drawn}}}$$

$$I_{D1} = \underbrace{350}_{\mu\text{m}} \left( \frac{\text{cm}^2}{\text{V}\cdot\text{sec}} \right) C_{ox} \left[ \frac{20 \times 10^{-6}}{(5 \times 10^{-6}) - 2(0.08 \times 10^{-6})} \right]$$

$$\left[ (1.2 - 0.7) 0.1 - \frac{1}{2} 0.1^2 \right]$$

$$I_D = \mu_n C_{ox} \frac{W}{L_{\text{eff}}} \left[ (V_{GS} - V_{TH}) V_{DS} - \frac{1}{2} V_{DS}^2 \right]$$

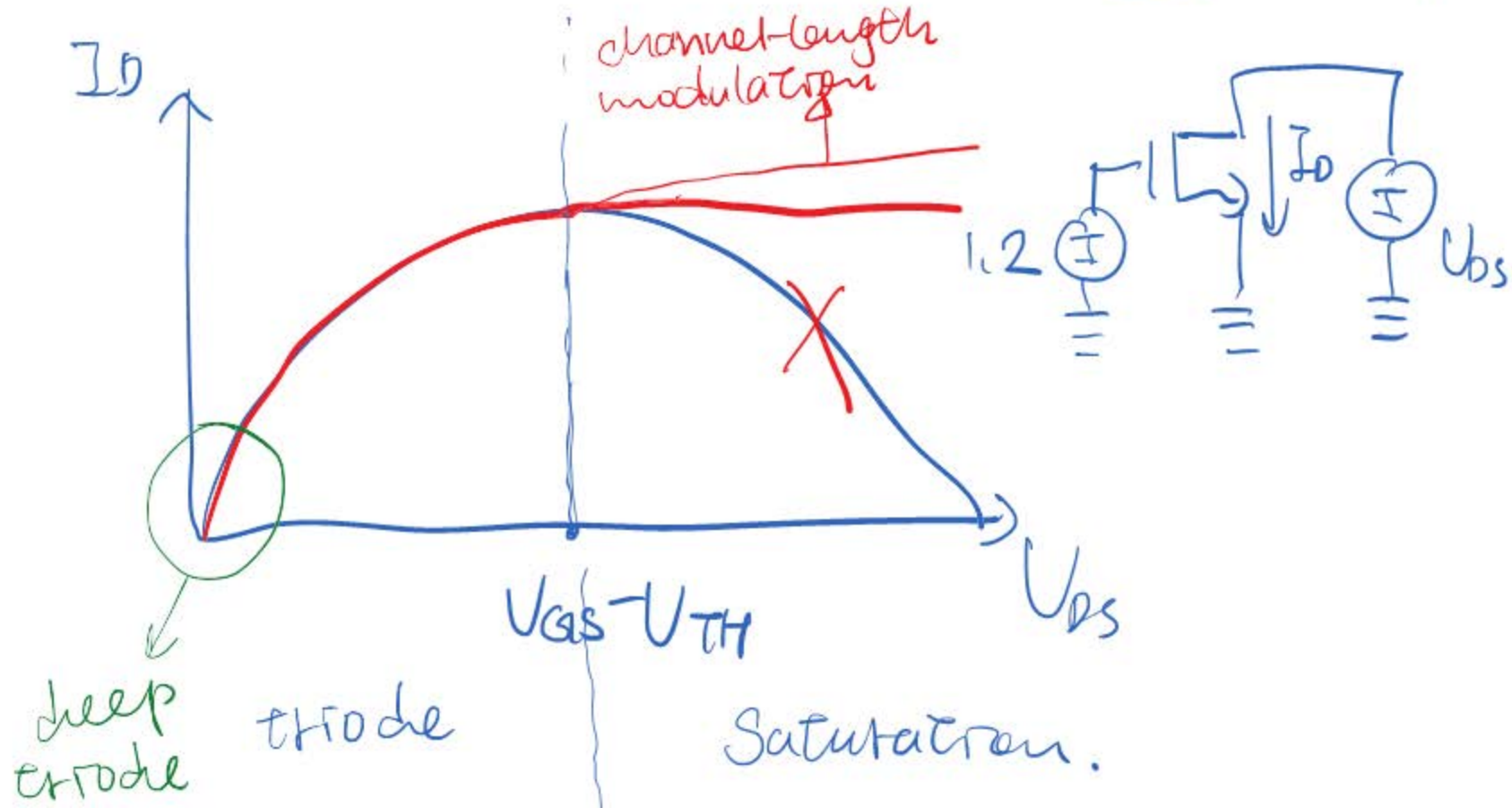
$$L_{\text{drawn}} - 2LD$$

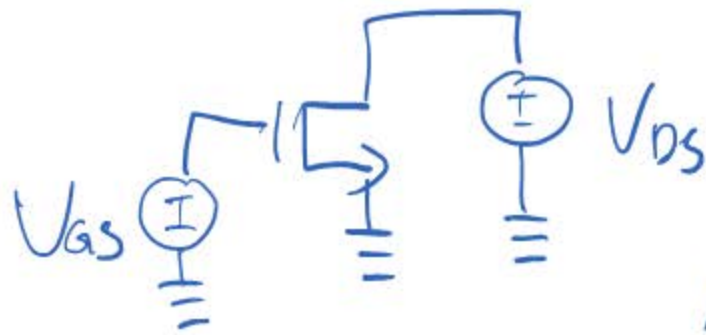


$$I_D = \boxed{\text{constant}} V_{DS} \quad \text{can be neglected if } V_{DS} \text{ small}$$

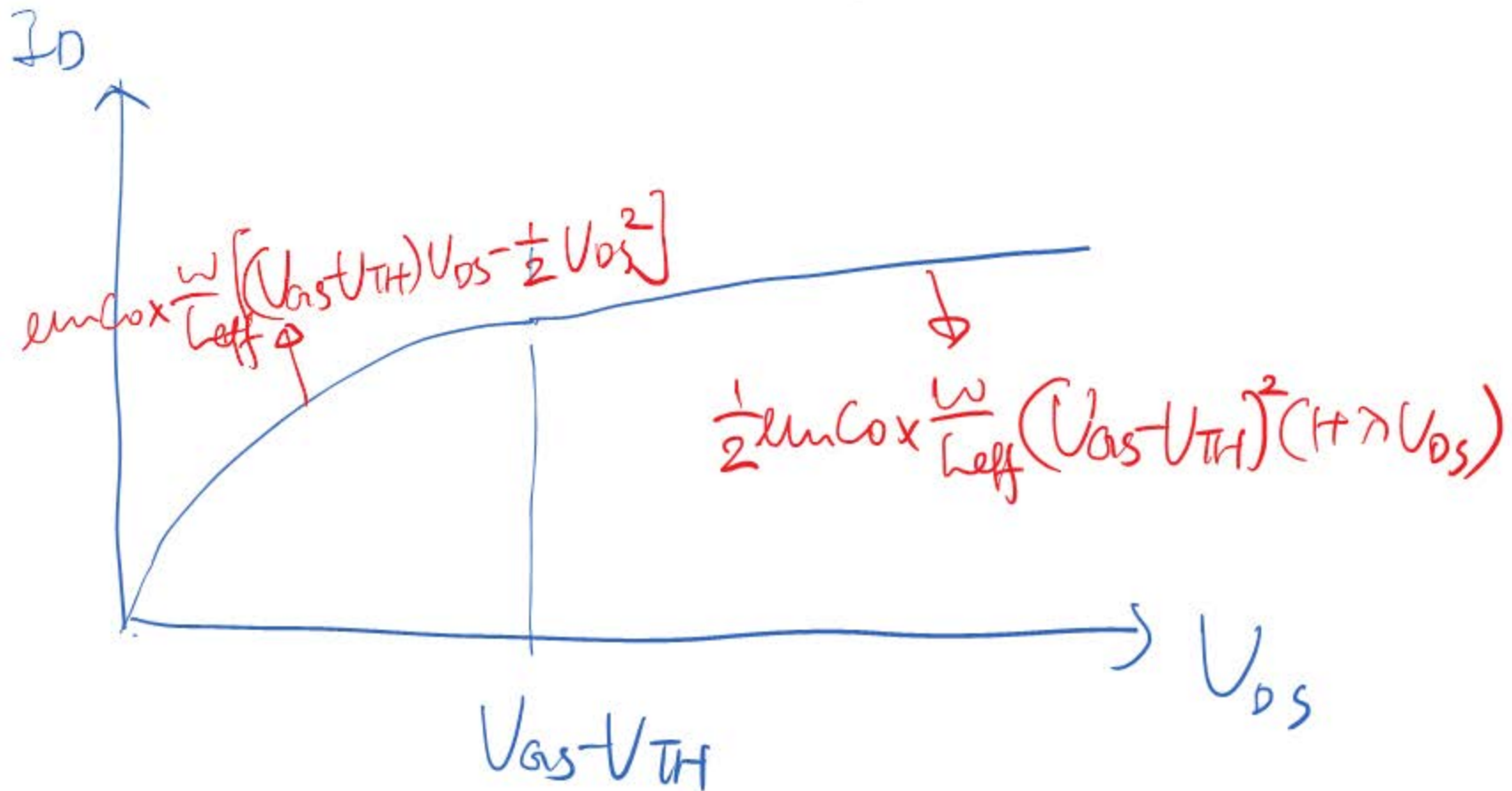
$$I_D = \mu_n C_{ox} \frac{W}{L_{eff}} [(V_{GS} - V_{TH})V_{DS} - \frac{1}{2} V_{DS}^2]$$

DC sweep  $I_D$  v.s.  $V_{DS}$  @ given  $V_{GS} = 1.2V > V_{TH}$





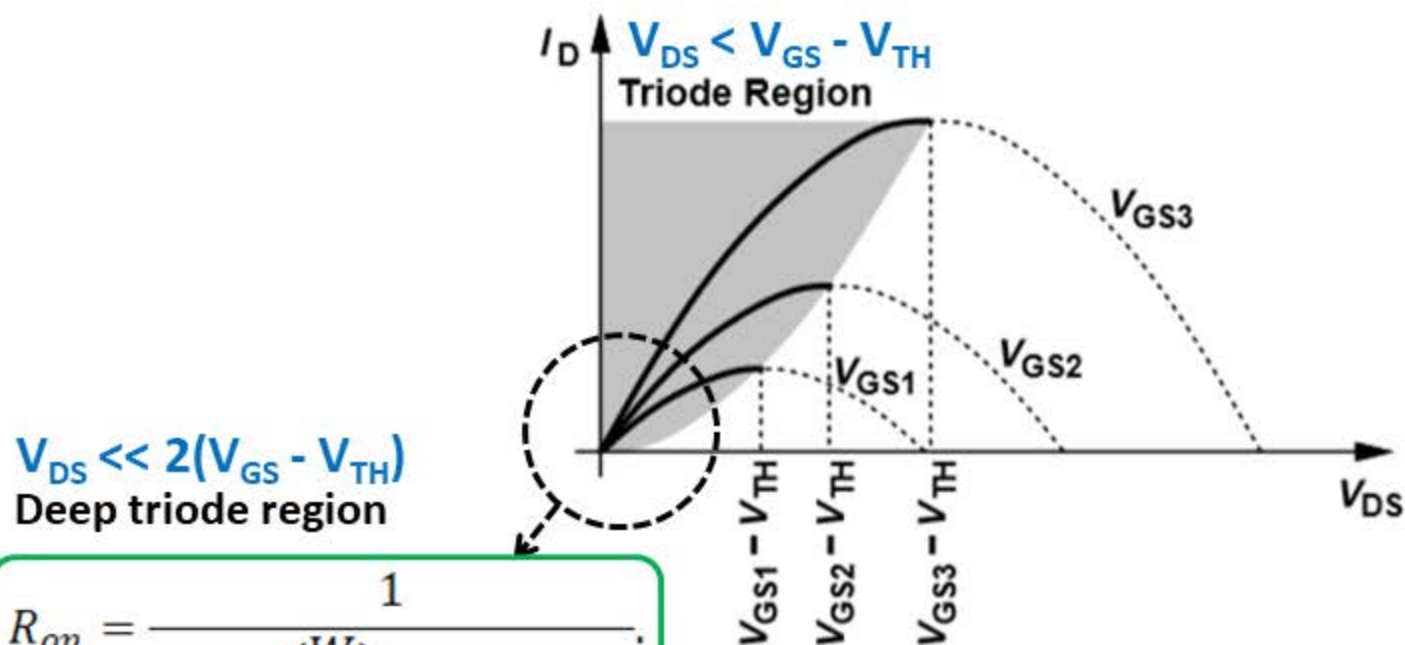
When  $V_{DS} > V_{TH}$   
At given  $V_{GS}$



# I-V Characteristics for NMOS (Triode)

$$I_D = \mu_n C_{ox} \frac{W}{L_{eff}} \left[ (V_{GS} - V_{TH}) V_{DS} - \frac{1}{2} V_{DS}^2 \right]$$

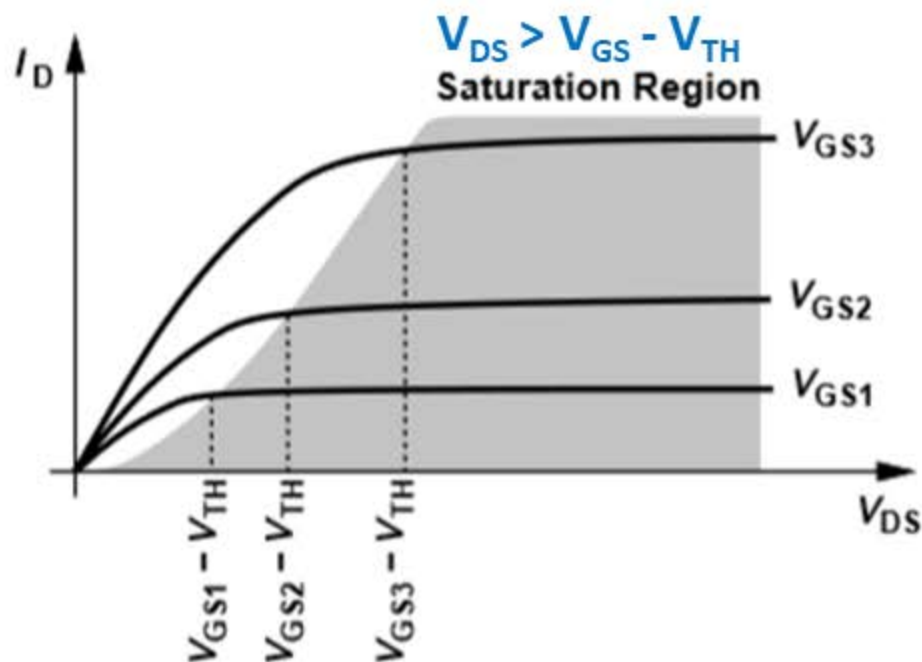
$$I_{D,max} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L_{eff}} (V_{GS} - V_{TH})^2 \quad V_{DS} = V_{GS} - V_{TH}$$



- For digital circuit, MOSFET, as a switch, usually operates in deep triode region.
- This is why reducing  $t_{ox}$  and  $L_{eff}$  can improve speed.

# I-V Characteristics for NMOS (Saturation)

12

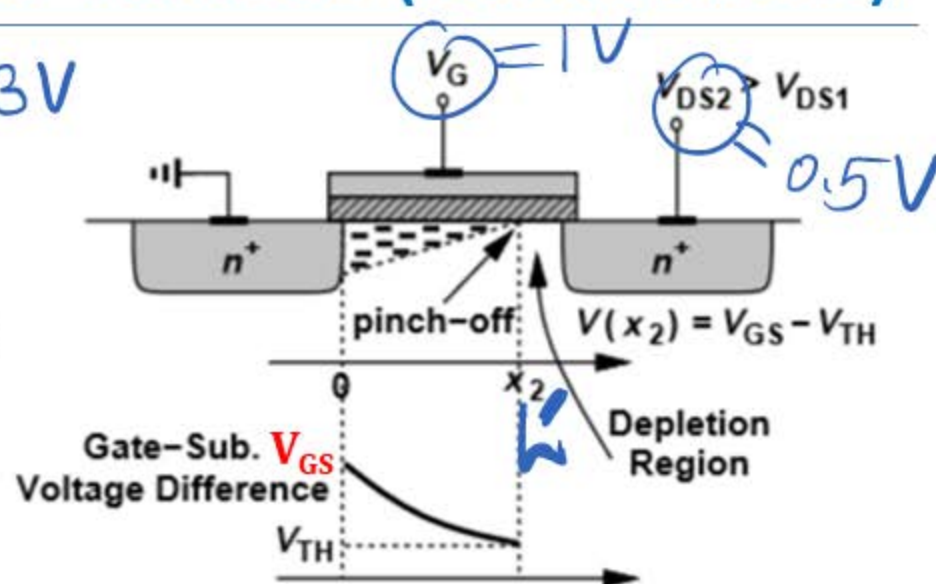
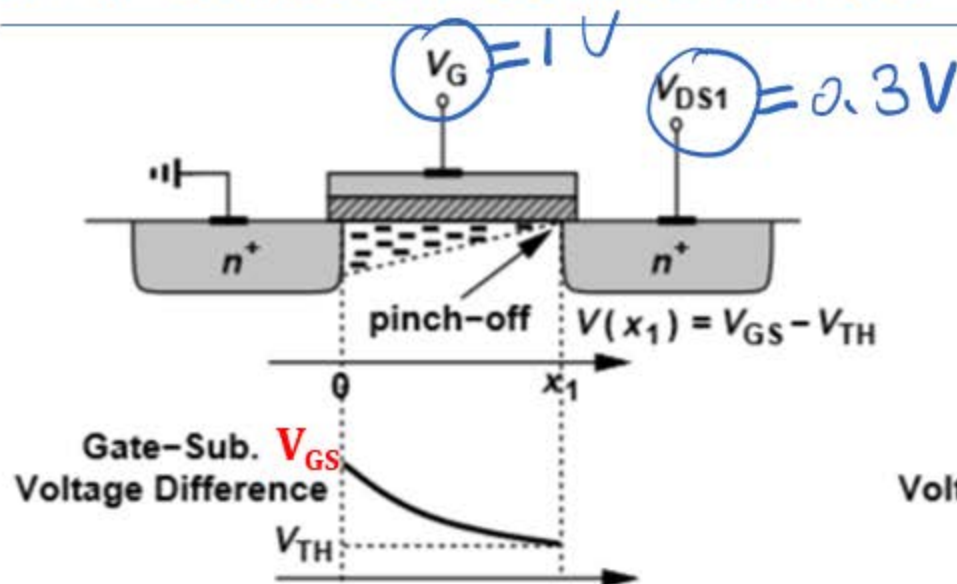


- In reality, for  $V_{DS} > V_{GS} - V_{TH}$ ,  $I_D$  becomes relatively constant.
- $V_{DS} = V_{GS} - V_{TH}$  is the minimum value for the NMOS to operate in saturation region.



# I-V Characteristics for NMOS (Saturation)

13



$$\int_{x=0}^{x=L'} I_D \cdot dx = \int_{V(0)=0}^{V(L')=V_{GS}-V_{TH}} \mu_n C_{ox} W [V_{GS} - V_{TH} - V(x)] \cdot dV(x)$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L'} (V_{GS} - V_{TH})^2$$

$I_D$ : constant along channel

$L'$ : the point at which  $Q_d$  drops to zero

$V_{GS} - V_{TH}$ : the overdrive voltage

- Electron velocity ( $v = I_D / Q_d$ ) becomes tremendously high at the pinch off point ( $Q_d \rightarrow 0$ ), such that electrons shoot through the depletion region and arrive at the drain terminal.



# Channel-Length Modulation

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L'} (V_{GS} - V_{TH})^2$$



$$L' = L_{eff} - \Delta L$$

$$\frac{1}{L'} = \frac{1}{L_{eff} - \Delta L} = \frac{1}{L_{eff}} \cdot \frac{1}{1 - \frac{\Delta L}{L_{eff}}} \approx \frac{1}{L_{eff}} \cdot \left(1 + \frac{\Delta L}{L_{eff}}\right)$$

*small*

$$\begin{aligned} I_D &= \frac{1}{2} \mu_n C_{ox} \frac{W}{L_{eff}} (V_{GS} - V_{TH})^2 \left(1 + \frac{\Delta L}{L_{eff}}\right) \\ &= \frac{1}{2} \mu_n C_{ox} \frac{W}{L_{eff}} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS}) \end{aligned}$$

*$\lambda = 0$  means don't consider "channel-length modulation"*

# Channel-Length Modulation

$$\begin{aligned} r_o &= \frac{\partial V_{DS}}{\partial I_D} = 1 / \frac{\partial I_D}{\partial V_{DS}} \\ &= \frac{1}{\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 \cdot \lambda} \\ &\approx \frac{1}{I_D \cdot \lambda} \end{aligned}$$

# NMOS vs PMOS

