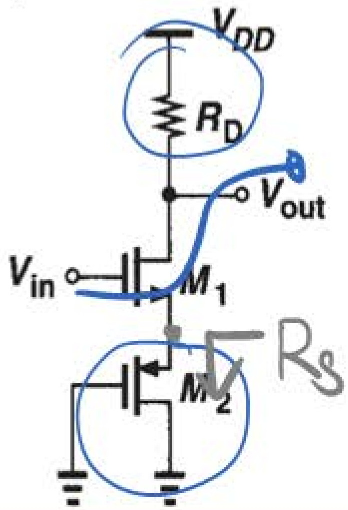
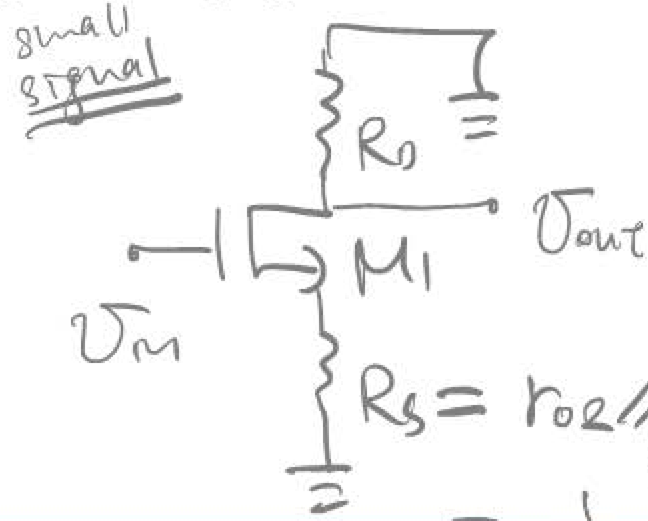
Example

Assuming λ = γ = 0, calculate the small signal voltage gain of the circuit below.



$\Rightarrow$



$$R_S = r_{o2} \,//\, \left(\frac{1}{g_{m2}+g_{mb2}}\right) = \frac{1}{g_{m2}}$$

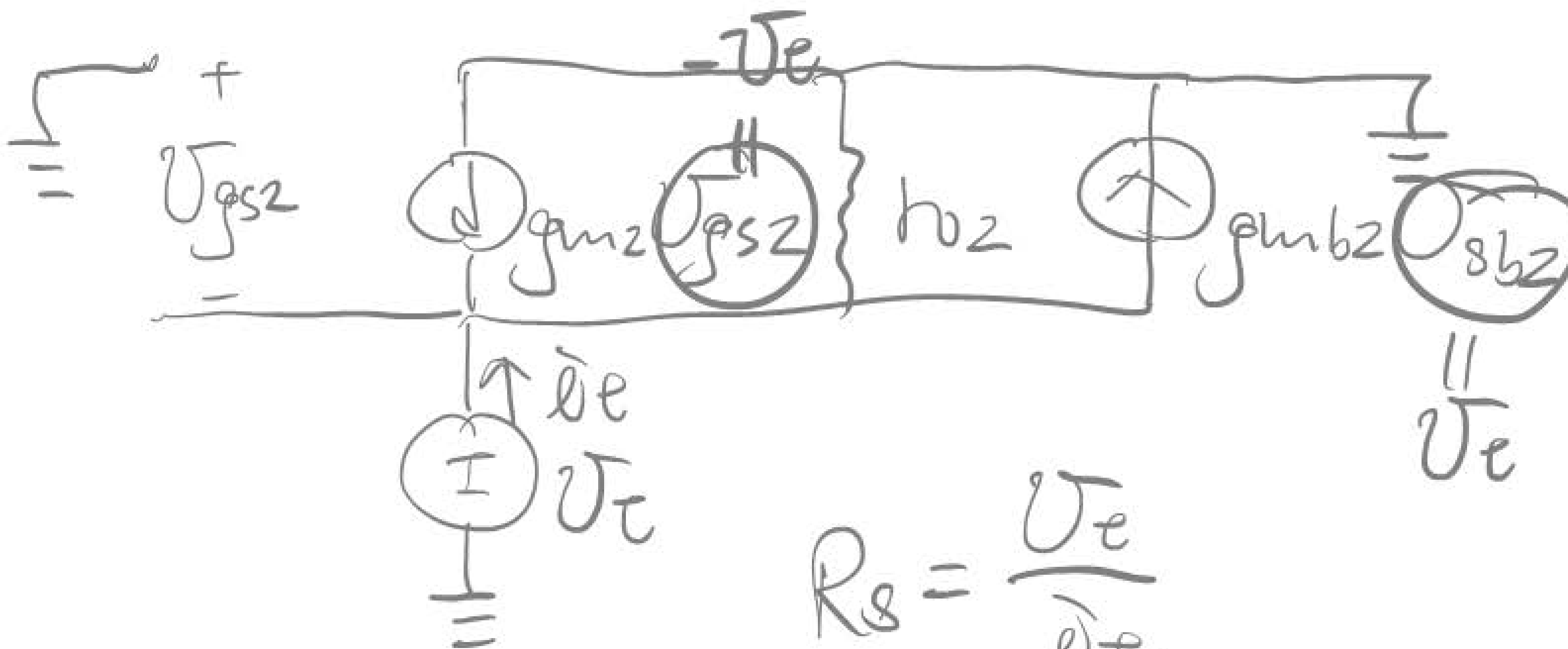
Solution:

$$G_m = -\frac{1}{\frac{1}{gm_1}+\frac{1}{gm_2}} = \frac{-g_{m1} r_{o1}}{\cancel{R_S} + r_{o1} + (g_{m1}+g_{mb1}) r_{o1} R_S}$$

$$= \frac{-g_{m1} \cancel{r_{o1}}}{\cancel{r_{o1}} + g_{m1} \cancel{r_{o1}} R_S}$$

$$R_{out} = R_D$$

$$A_v = G_m R_{out}$$

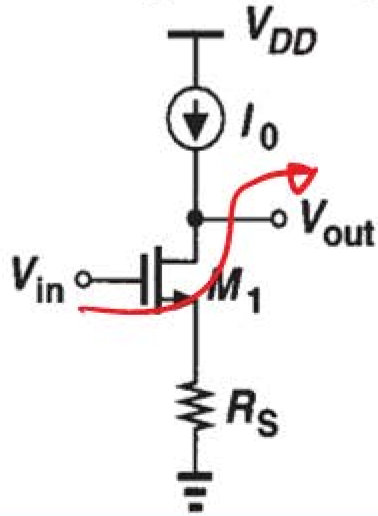


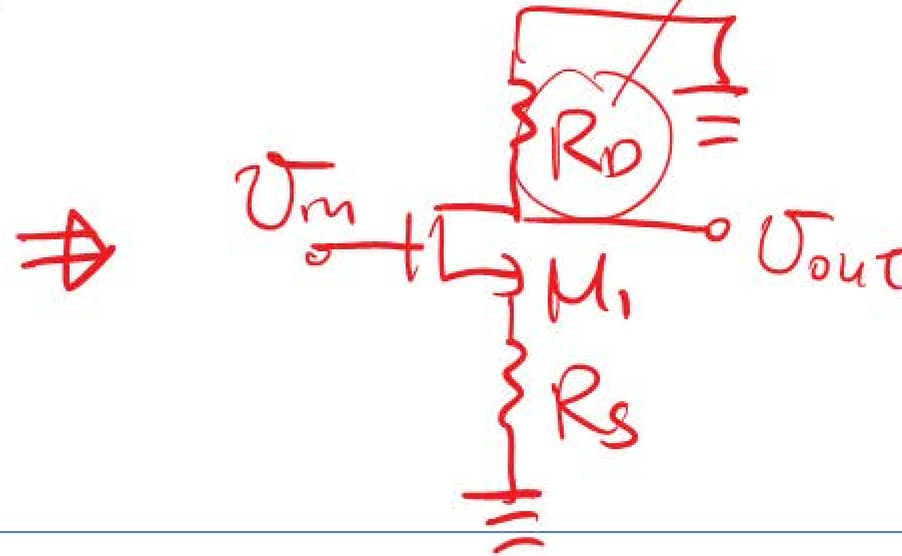
$$R_s = \frac{v_t}{i_t}$$

$$= r_{o2} \parallel \left(\frac{1}{g_{m2} + g_{mb2}} \right)$$

Example

Calculate the small signal voltage gain of the circuit below.





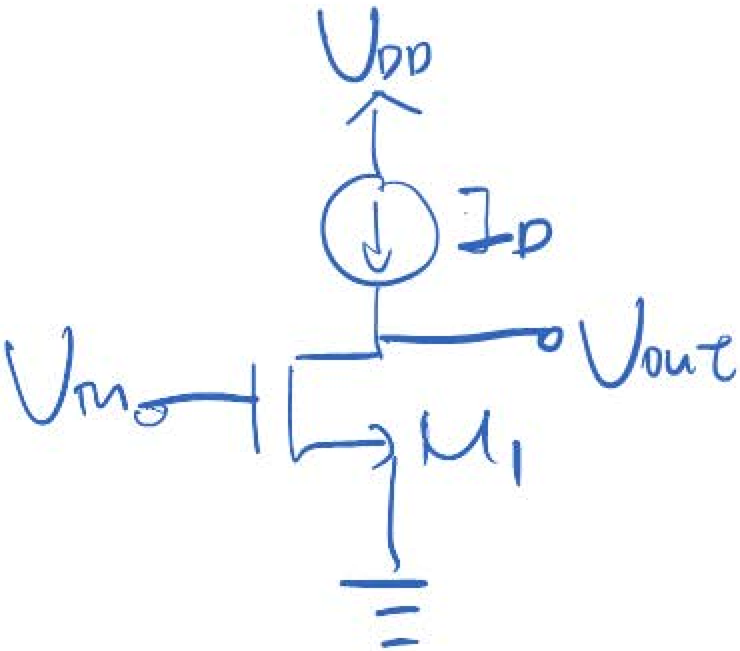
Solution:

$$G_m = \frac{-gm_1 r_{o1}}{r_{o1} + R_S + (gm_1 + gmb_1) r_{o1} R_S}$$

$$R_{out} = r_{o1} + R_S + (gm_1 + gmb_1) r_{o1} R_S$$

$$A_v = G_m R_{out} = -gm_1 r_{o1}$$

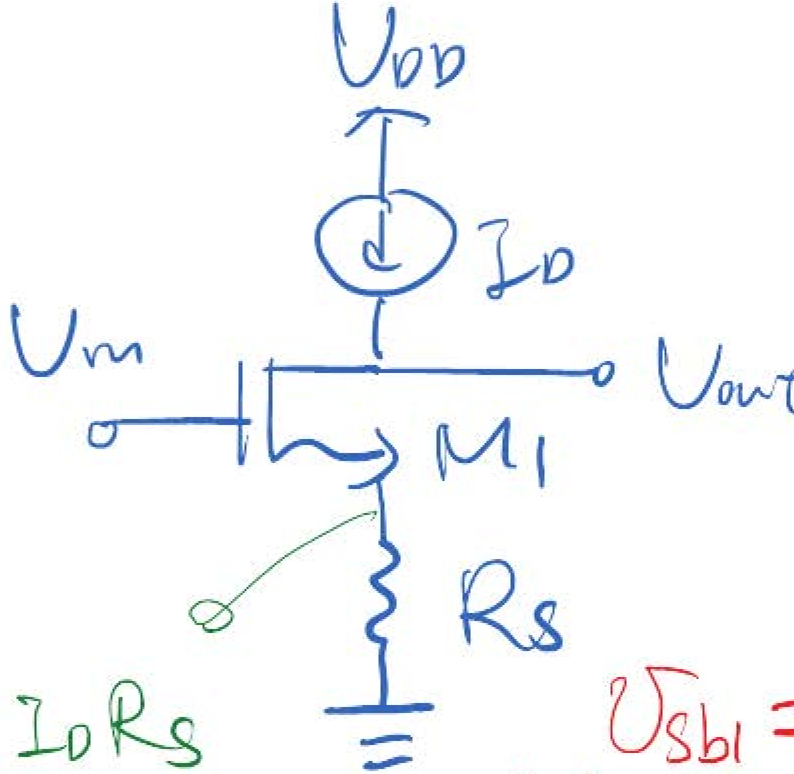
- I_o is ideal current source → Voltage across R_s is constant → M_1 source shorted to ground
- R_D replaced by current source → Nonlinearity issue arises again



$V_{SB1} = 0$

$v_{sb1} = 0$

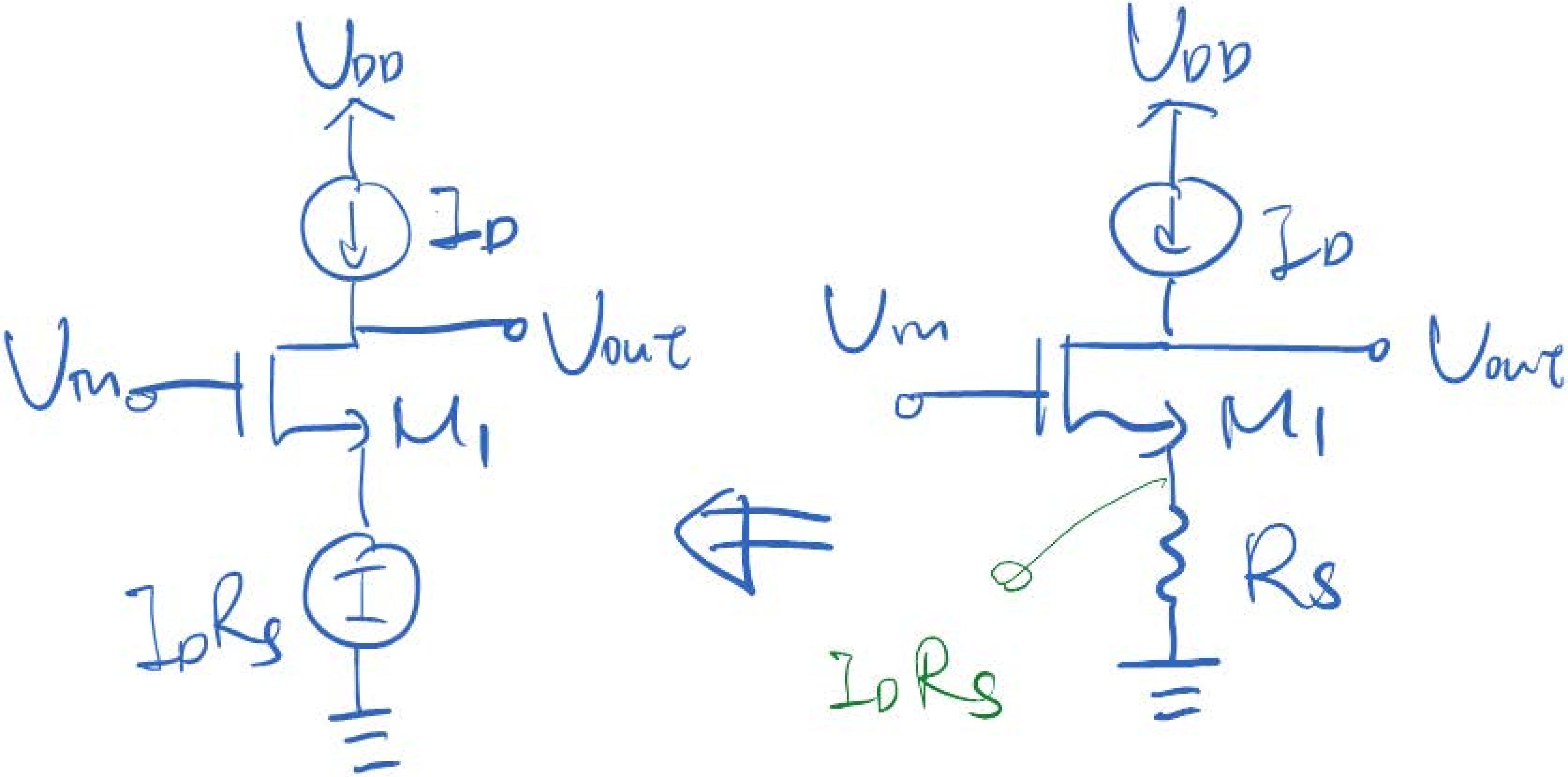
$A_v = -g_{m1} r_{o1}$



$v_{sb1} = 0$

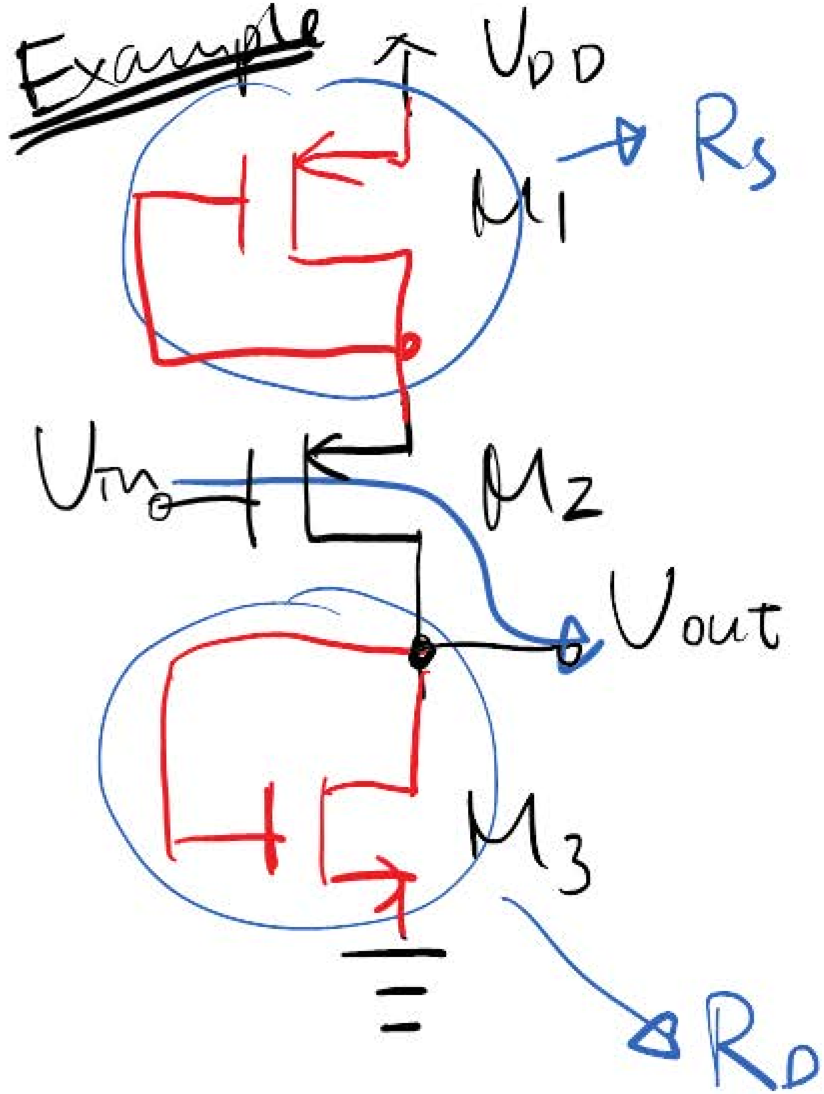
$V_{SB1} = I_D R_S$

$A_v = -g_{m1} r_{o1}$



$$A_v = -g_{m1} r_{o1}$$

Example



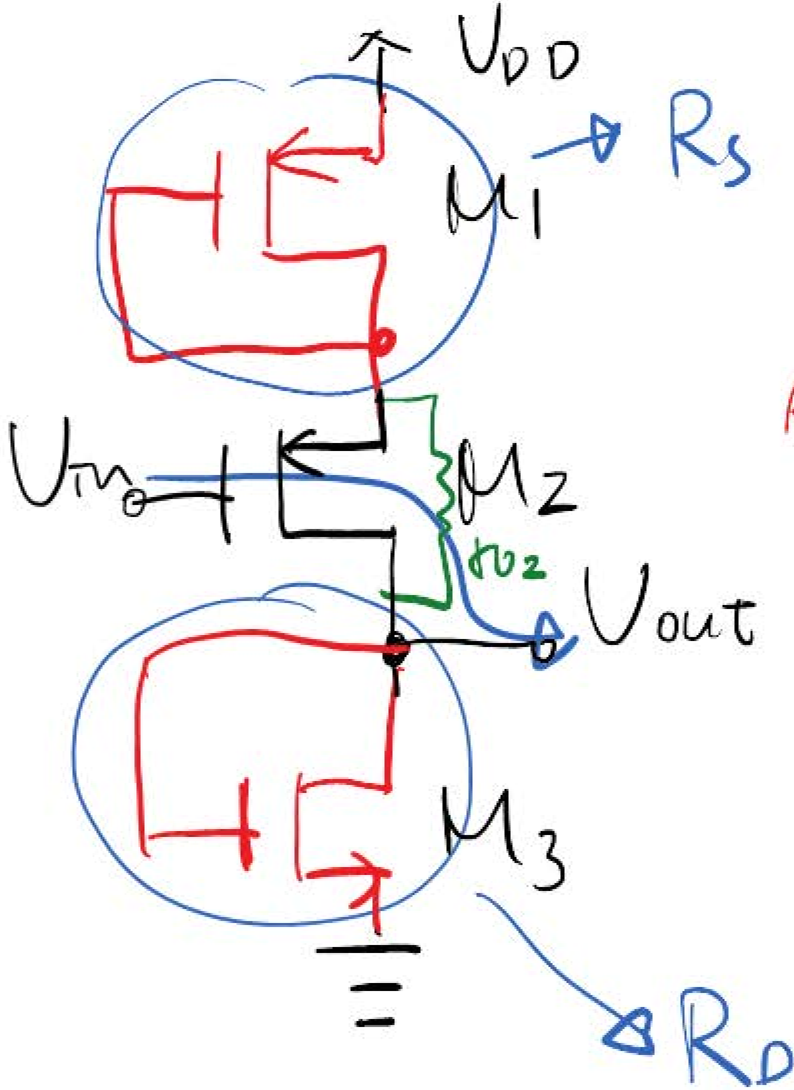
$G_m = ?$ $R_{out} = ?$

$\lambda = 0, \gamma \neq 0$

$R_S = r_{o1} \parallel \frac{1}{g_{m1}} = \frac{1}{g_{m1}}$

$R_D = \frac{1}{g_{m3}}$

$$G_m = \frac{-g_{m2} r_{o2}}{r_{o2} + R_S + (g_{m2} + g_{mb2}) r_{o2} R_S} = \frac{-g_{m2}}{1 + (g_{m2} + g_{mb2}) R_S}$$



$G_m = ?$ $\quad R_{out} = ?$

$\lambda = 0, \ \gamma \neq 0$

Assume all transistors in sat.

$R_S = r_{o1} \,//\, \frac{1}{g_{m1}} = \frac{1}{g_{m1}}$

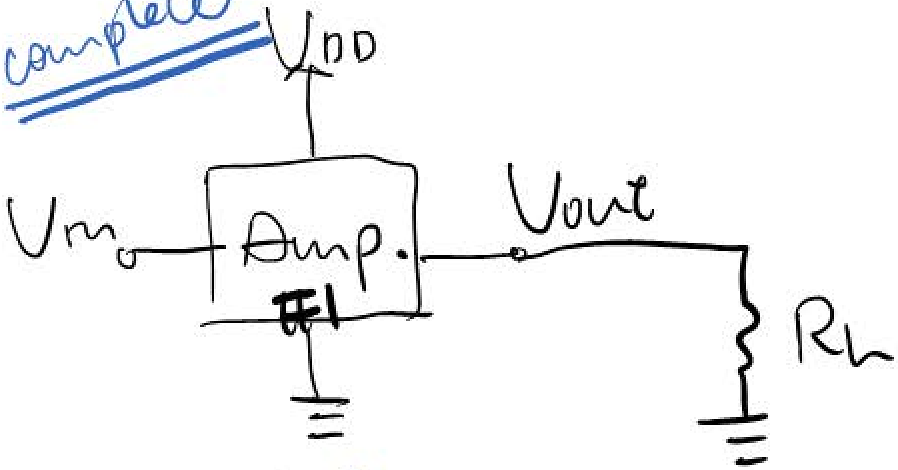
$R_D = \frac{1}{g_{m3}}$

$$R_{out} = R_D \,//\, \Big[r_{o2} + R_S + \underbrace{\boxed{(g_{m2} + g_{mb2}) r_{o2}}}_{\text{intrinsic gain of } M_2} R_S \Big] = \frac{1}{g_{m3}}$$

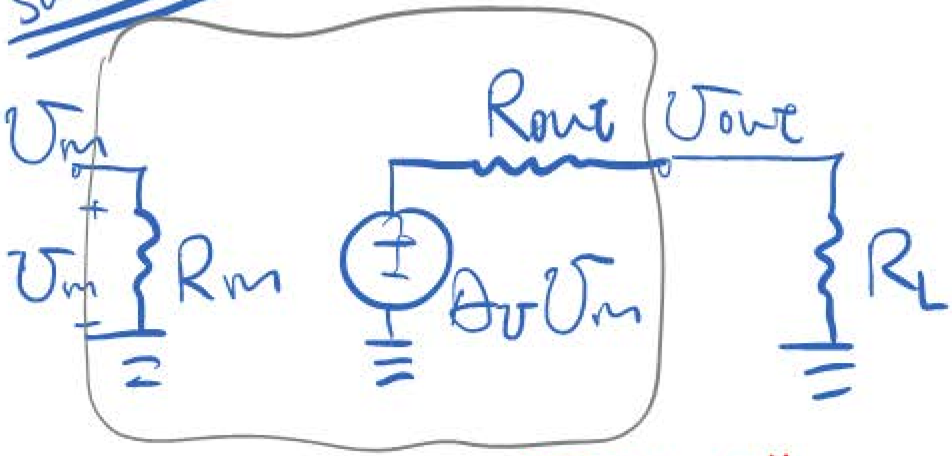
Common-Drain

Source Follower (Buffer)

complete



small-signal

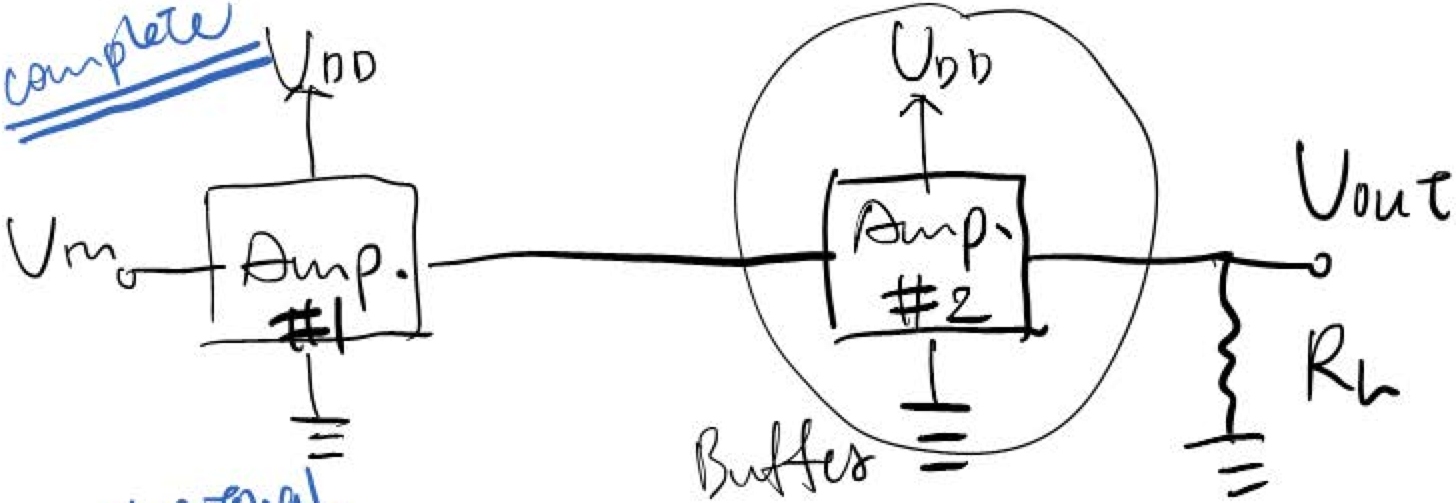


small

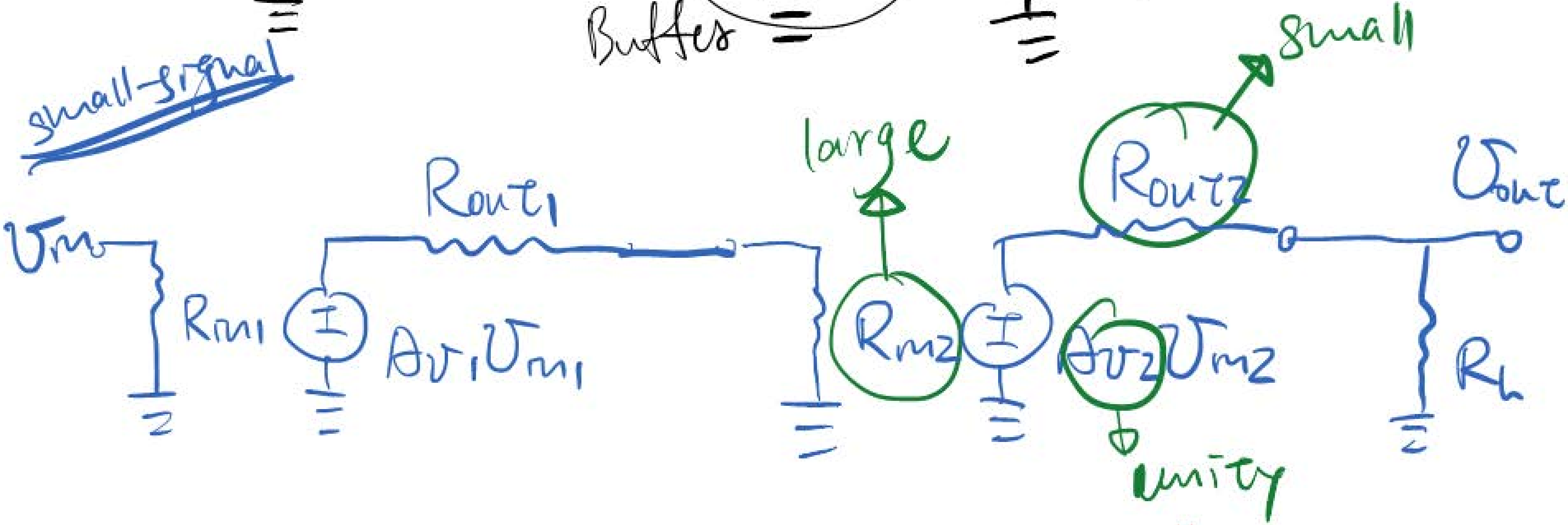
$$\frac{V_{out}}{V_{in}} = A_v \cdot \frac{R_L}{R_{out}+R_L} \Rightarrow A_v \text{ consumed by the factor } \frac{R_L}{R_{out}+R_L}$$

large

complete

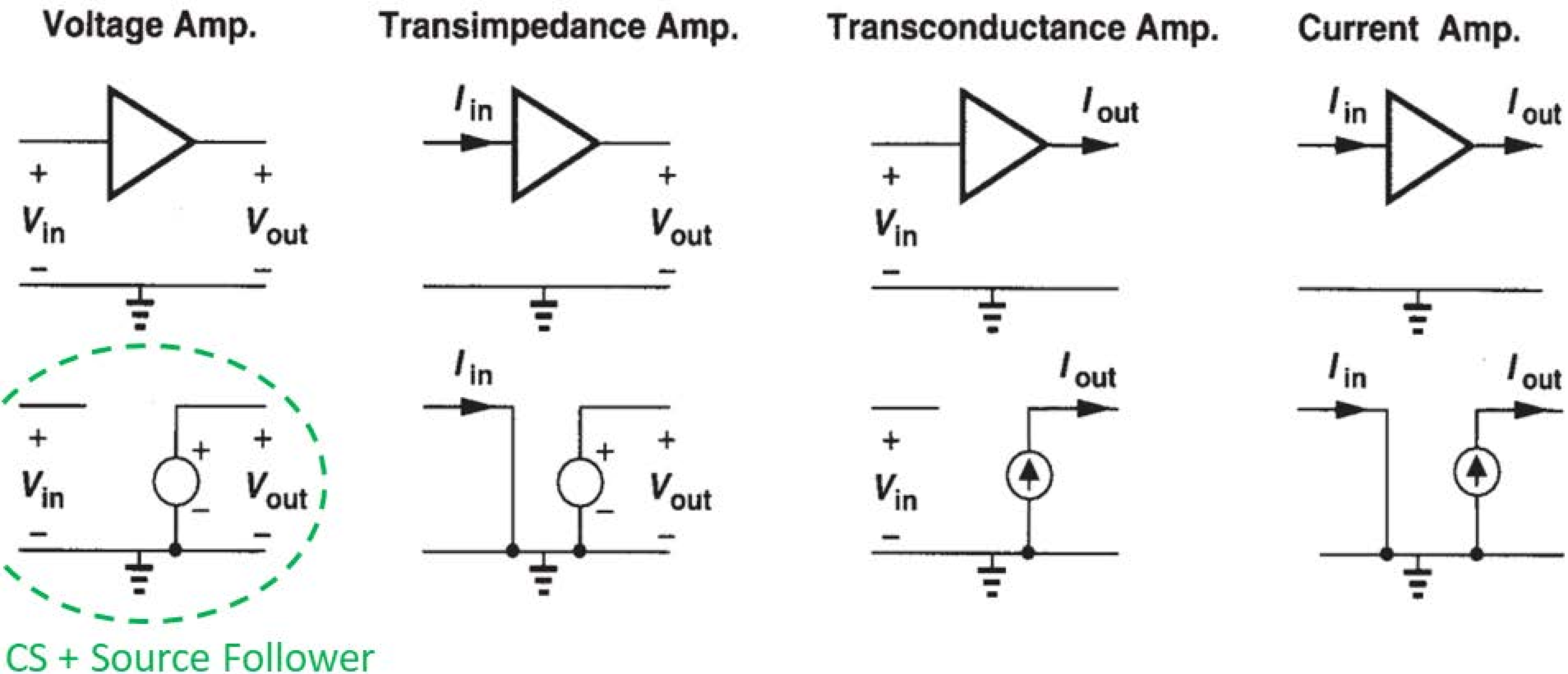


small-signal

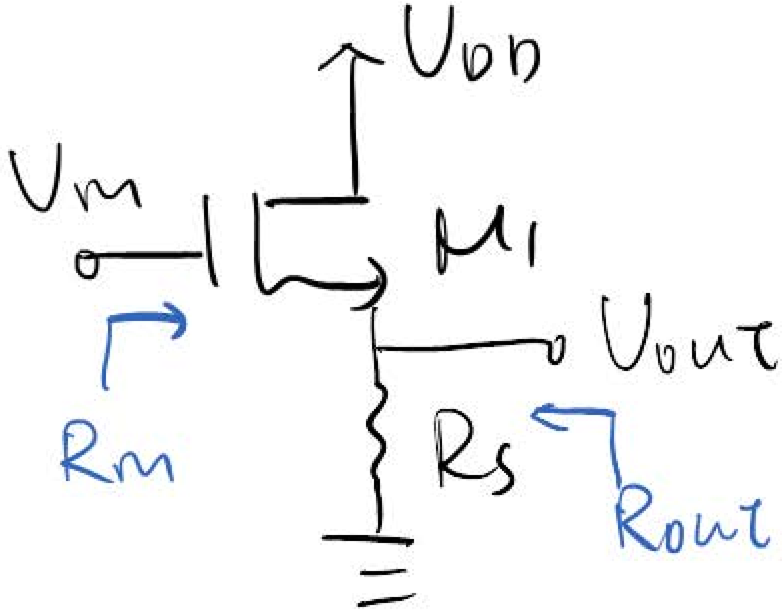


$$\frac{V_{out}}{V_{in}} = A_{V1}\frac{R_{in2}}{\underbrace{R_{out1}}_{\text{large}}+R_{in2}}\, A_{V2}\frac{\overbrace{R_L}^{\text{small}}}{R_{out2}+R_L}$$

Ideal Amplifier



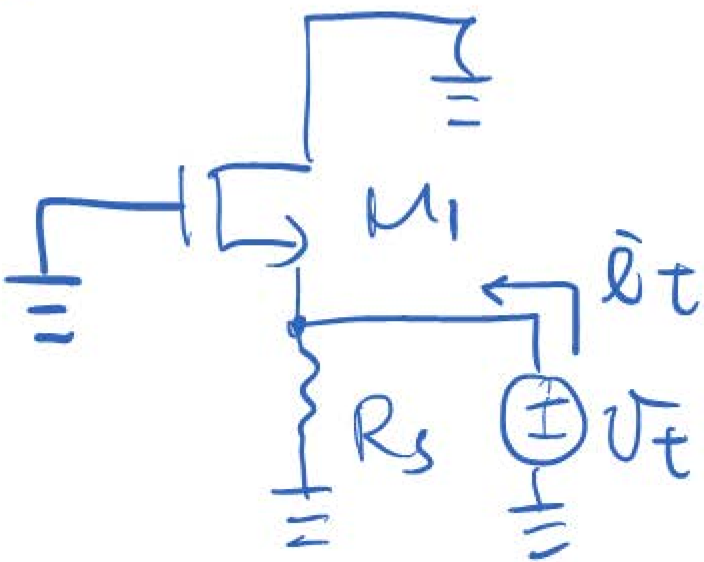
- For driving a low impedance load, source follower, as a buffer, provides **no gain** but **large input impedance** and **low output impedance**.



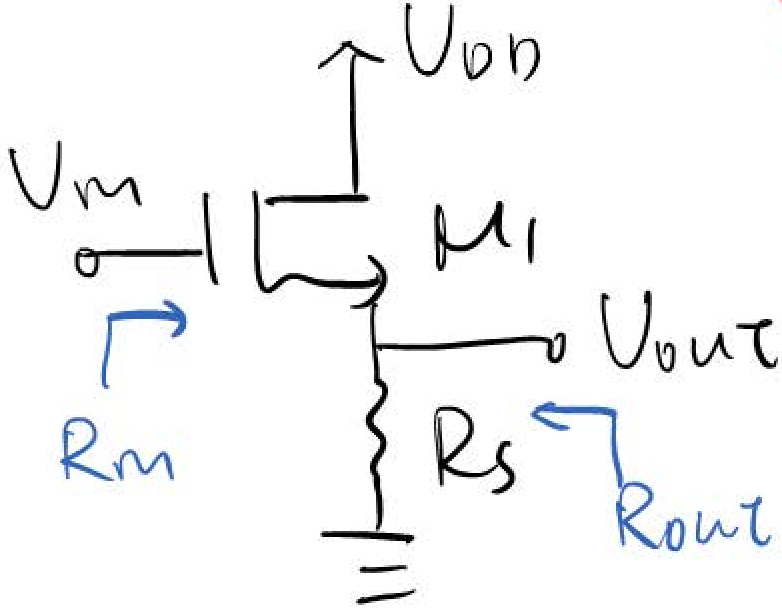
$$R_{in} = \infty$$

$$R_{out} = R_s \,//\, r_{o1} \,//\, \left(\frac{1}{g_{m1} + g_{mb1}}\right)$$

small-signal



$$R_{out} = \frac{v_t}{i_t}$$



$\lambda \neq 0,\ \gamma \neq 0$

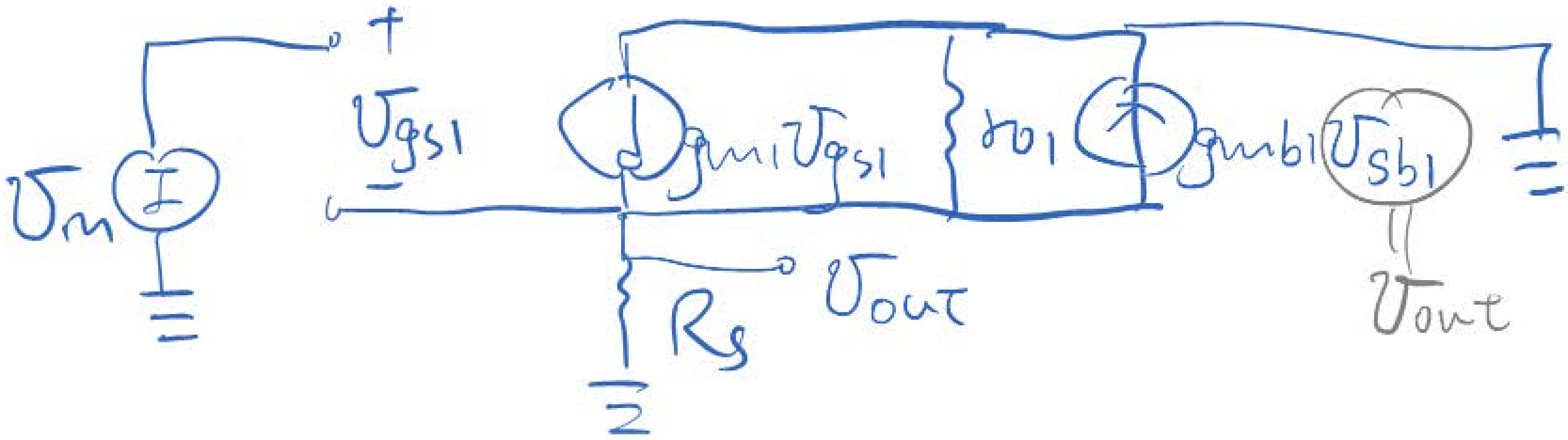
$$R_{in} = \infty$$

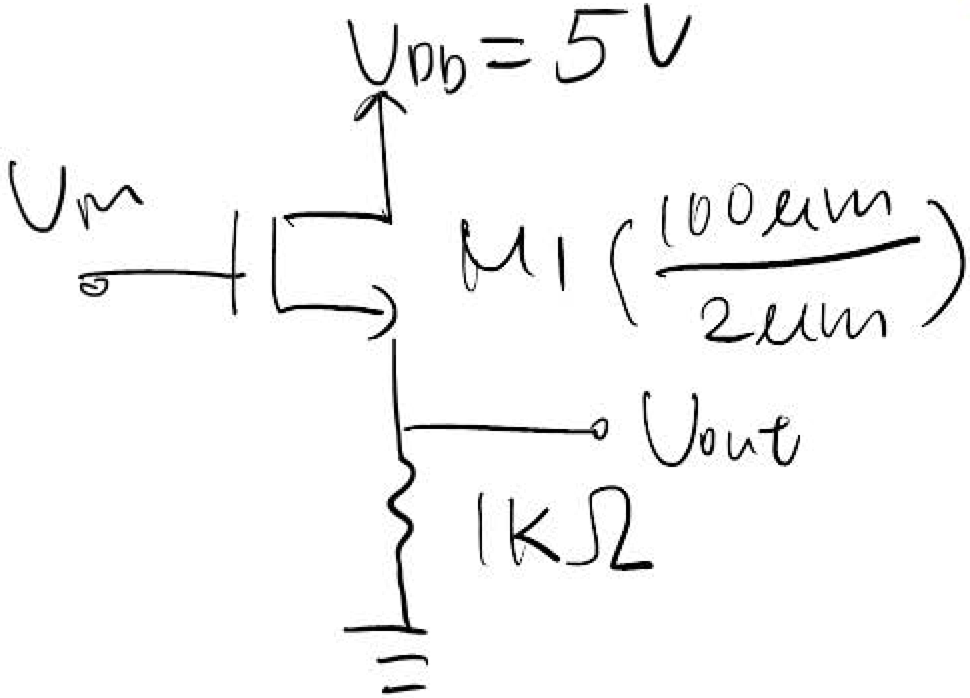
$$R_{out} = R_S \,//\, r_{o1} \,//\, \left(\frac{1}{g_{m1}+g_{mb1}}\right)$$

$$A_v = \frac{v_{out}}{v_{in}}$$

small-signal

$$\frac{v_{out}}{R_S} + (v_{out} - v_{in})\, g_{m1} + \frac{v_{out}}{r_{o1}} + g_{mb1} v_{out} = 0$$





$\lambda \neq 0,\ \gamma \neq 0$

1° $V_{OUT} = ?$

DC biasing analysis.

M_1 must be in sat.

$$\frac{V_{OUT}}{1k} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L_{eff}}\right)\cdot(3 - V_{OUT} - V_{TH})^2\cdot[1 + \lambda(5 - V_{out})]$$

$V_{in} = 3 + 0.001\sin(2\pi 100t)$

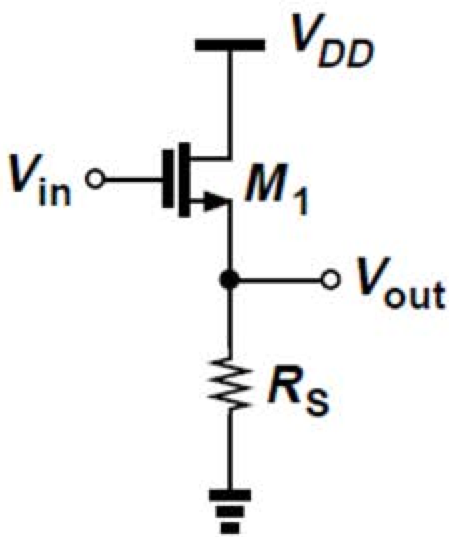
$V_{out} = ?$

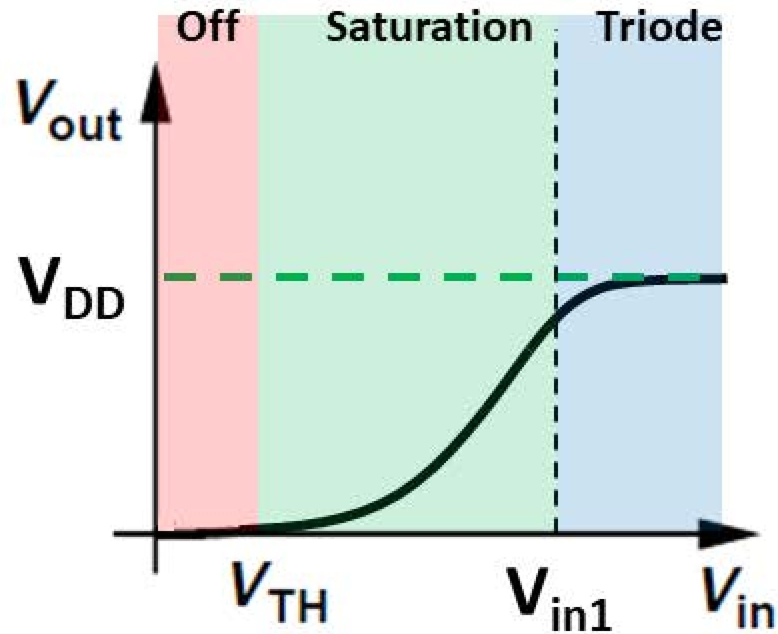
Source Follower

DC Analysis

$\lambda = 0$

$\gamma \neq 0$





$$V_{DD} - V_{out} = V_{in1} - V_{out} - V_{TH}$$

$$\rightarrow V_{in1} = V_{DD} + V_{TH}$$

- $V_{in} < V_{TH} \rightarrow M_1$ **Off**

$$V_{out} = 0$$

- $V_{in1} > V_{in} > V_{TH} \rightarrow M_1$ **in Saturation**

$$R_S \frac{1}{2} \mu_n C_{ox} \frac{W}{L_{eff}} (V_{in} - V_{out} - V_{TH})^2 = V_{out}$$

- $V_{in} > V_{in1} \rightarrow M_1$ **in Triode**

$$R_S \mu_n C_{ox} \frac{W}{L_{eff}} \left[(V_{in} - V_{out} - V_{TH})(V_{DD} - V_{out}) - \frac{1}{2}(V_{DD} - V_{out})^2 \right] = V_{out}$$

Source Follower

DC Analysis

$V_{out} = V_{SB}$.

$\lambda = 0$

$\gamma \neq 0$

- **$V_{in1} > V_{in} > V_{TH} \rightarrow M_1$ in Saturation**

$$R_S \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{out} - V_{TH})^2 = V_{out}$$

$$R_S \frac{1}{2}\mu_n C_{ox}\frac{W}{L}2(V_{in} - V_{out} - V_{TH})\left(1 - \frac{\partial V_{out}}{\partial V_{in}} - \frac{\partial V_{TH}}{\partial V_{in}}\right) = \frac{\partial V_{out}}{\partial V_{in}}$$

$$R_S \underbrace{\boxed{\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{out} - V_{TH})}}_{= gm}\left(1 - \frac{\partial V_{out}}{\partial V_{in}} - \underbrace{\frac{\partial V_{TH}}{\partial V_{out}}}_{= \frac{\partial V_{TH}}{\partial V_{SB}} = \eta = \frac{\gamma}{2\sqrt{2\Phi_F + V_{SB}}}}\frac{\partial V_{out}}{\partial V_{in}}\right) = \frac{\partial V_{out}}{\partial V_{in}}$$

$$\boxed{A_v = \frac{gmR_S}{1 + gmR_S(1+\eta)} = \frac{gmR_S}{1 + (gm + gmb)R_S} \approx \frac{1}{1+\eta}}$$

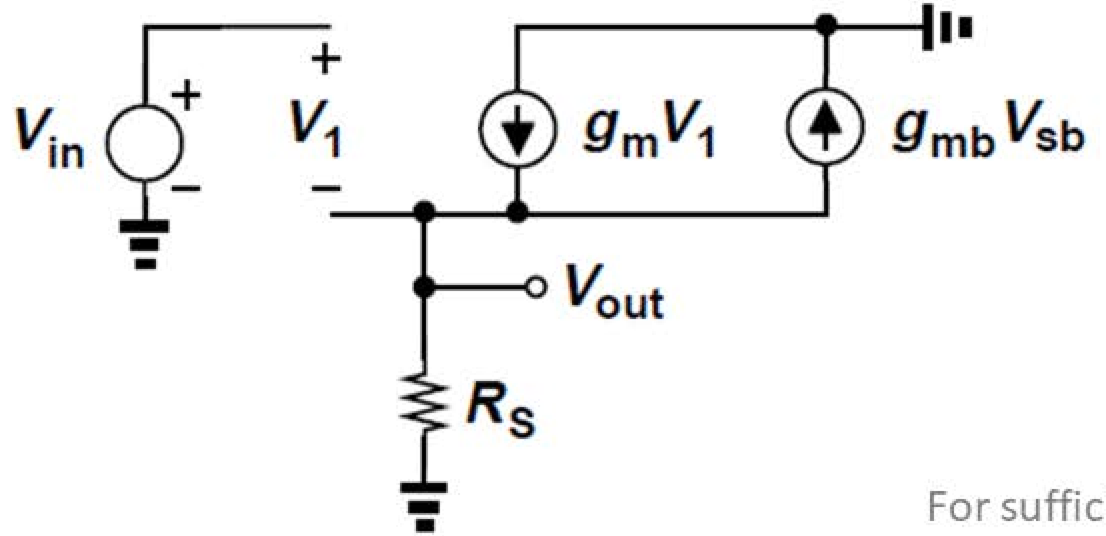
If $(gm + gmb)R_S >> 1$

Source Follower

Small-signal Analysis

$\lambda = 0$

$\gamma \neq 0$



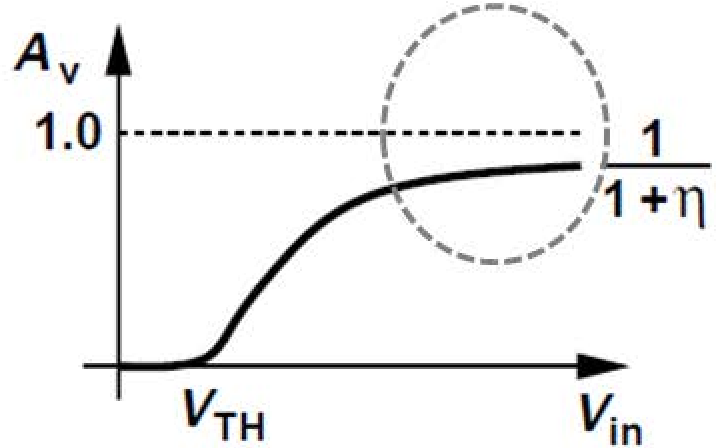
$$G_m = gm$$

$$R_{out} = R_S \parallel \left(\frac{1}{gm + gmb}\right)$$

$$\boxed{A_v = \frac{gmR_S}{1 + (gm + gmb)R_S} \approx \frac{1}{1 + \eta}}$$

If $(gm + gmb)R_S >> 1$

For sufficiently large V_{in}, I_D and thus gm.

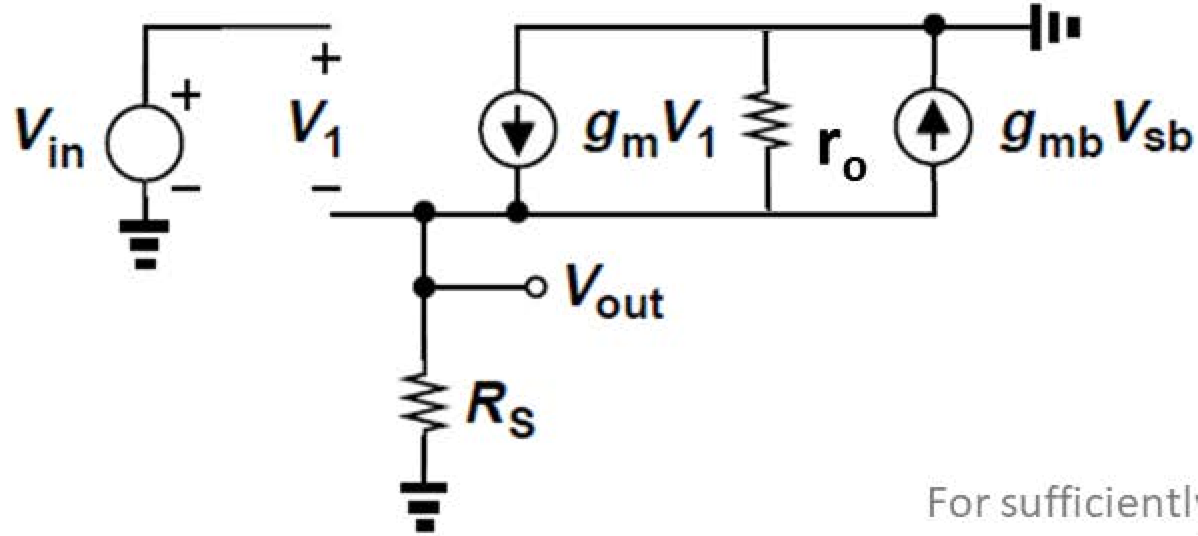


Source Follower

Small-signal Analysis

$\lambda \neq 0$

$\gamma \neq 0$

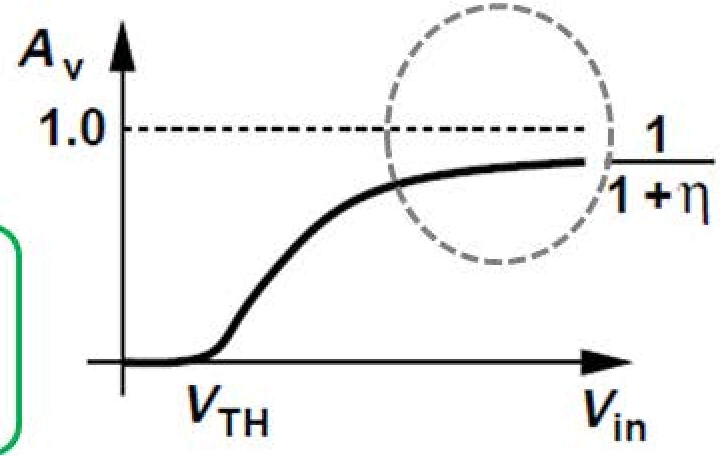


$$G_m = gm$$

$$R_{out} = r_o \parallel R_S \parallel \left(\frac{1}{gm + gmb}\right)$$

$$\boxed{A_v = \frac{gmr_oR_S}{r_o + R_S + (gm + gmb)r_oR_S} \approx \frac{1}{1 + \eta}}$$

For sufficiently large V_{in}, I_D and thus gm.



If $(gm + gmb)r_oR_S \gg r_o$ and R_S