# Diode Circuit

VE311 Electronic Circuits (Fall 2020)

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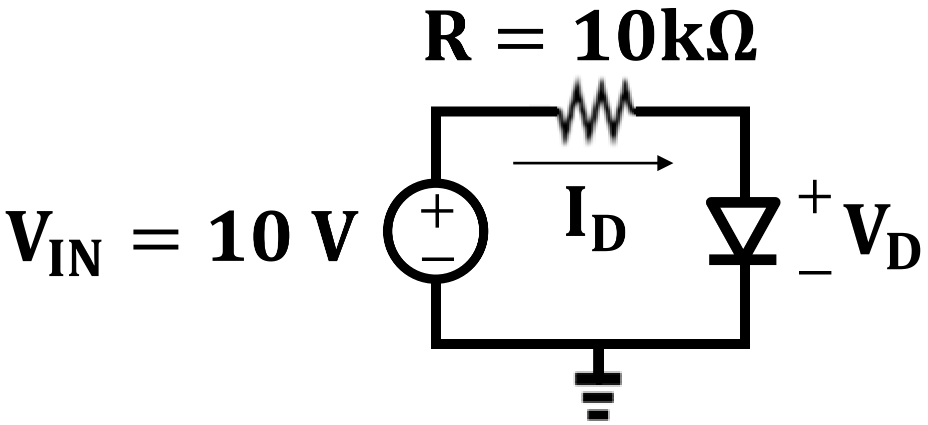
Last Updated on **September 13, 2020**

# Solving Techniques




$$V_{IN} = I_D R + V_D$$

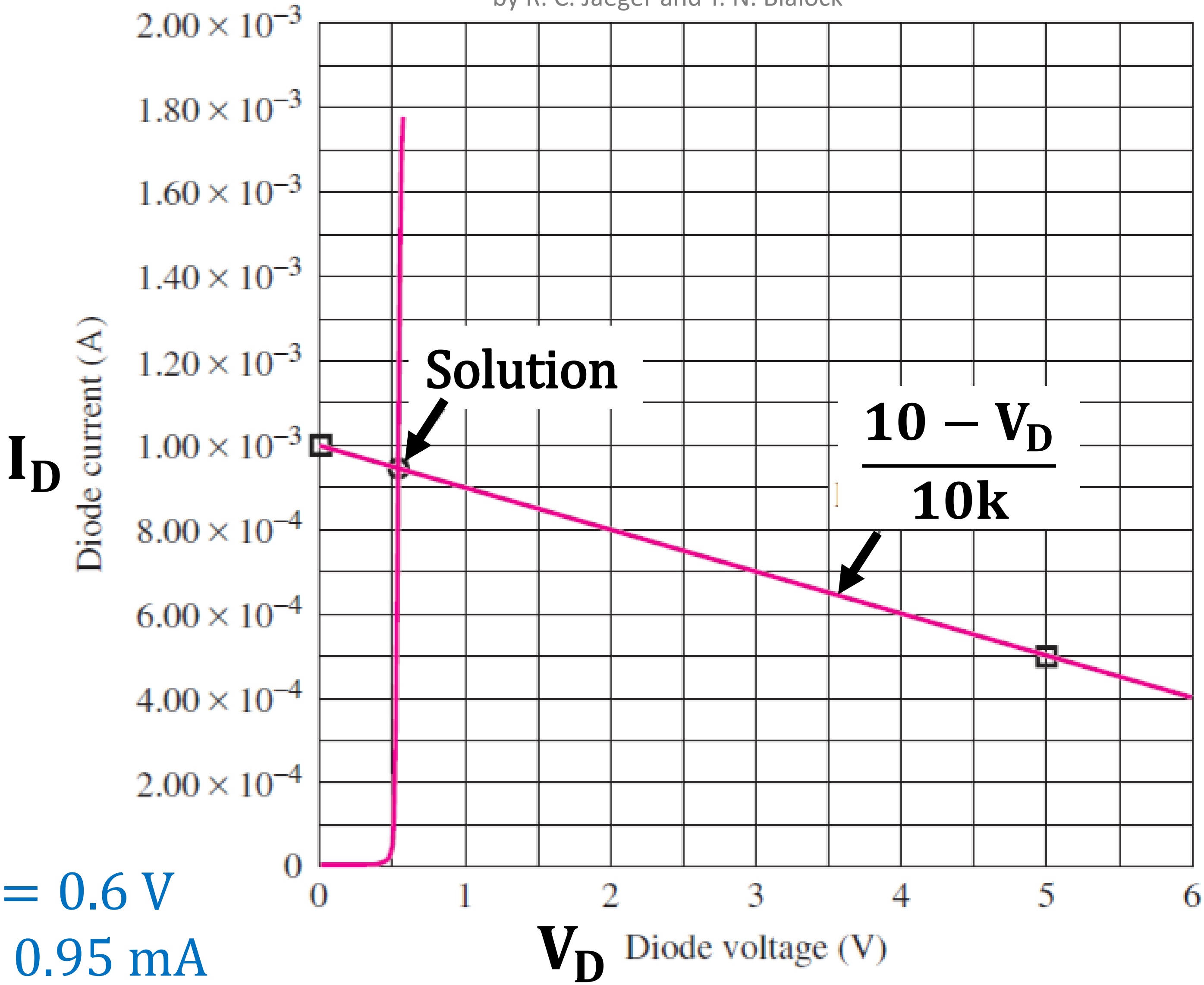
$$I_D = ?$$

$$V_D = ?$$

1. **Graphical analysis**
2. **Mathematical analysis**
3. **Simplified analysis** (ideal diode)
4. **Simplified analysis** (constant voltage drop)

# Graphical Analysis

Source: Microelectronic Circuit Design, 4th Edition, by R. C. Jaeger and T. N. Blalock




$V_D = 0.6\ V$

$I_D = 0.95\ mA$

# Mathematical Analysis

$$V_{IN} = I_D R + V_D$$

$$V_{IN} = \left[I_S\left(e^{\frac{qV_D}{kT}} - 1\right)\right] R + V_D$$

$$10 = \left[10^{-13}\left(e^{\frac{V_D}{0.0258}} - 1\right)\right] 10^4 + V_D$$

Above is a transcendental equation which does not have a close-form **analytical solution.** So, through trial and error, we seek a **numerical solution.**

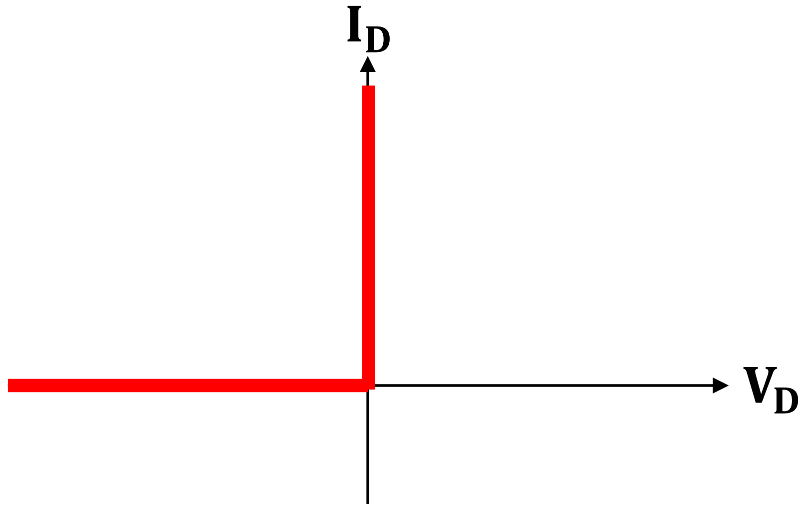
function xd = diode(vd)

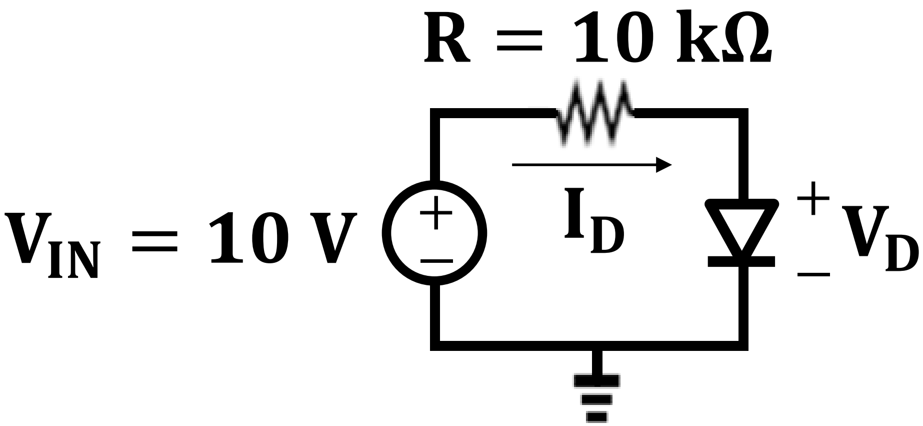
$$xd = 10 - (10^{-9}) * \left(\exp\left(\frac{vd}{0.0258}\right) - 1\right) - vd$$

Use MATLAB to plot xd as a function of vd, and find out a vd that makes xd closest to zero.

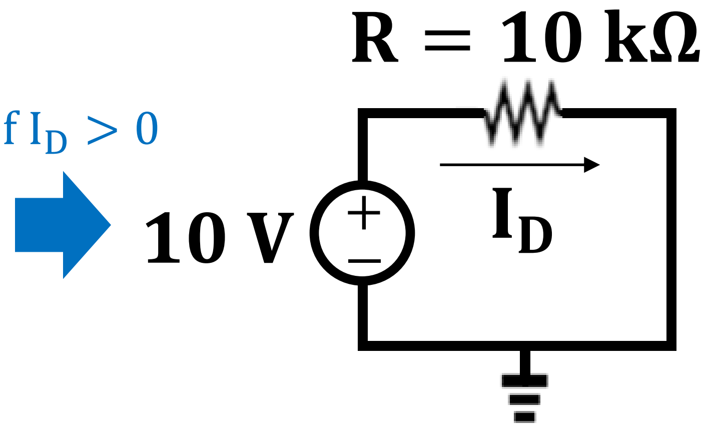
$$V_D = 0.5742\ \mathrm{V}$$
$$I_D = 0.944\ \mathrm{mA}$$

# Simplified Analysis (Ideal Diode)



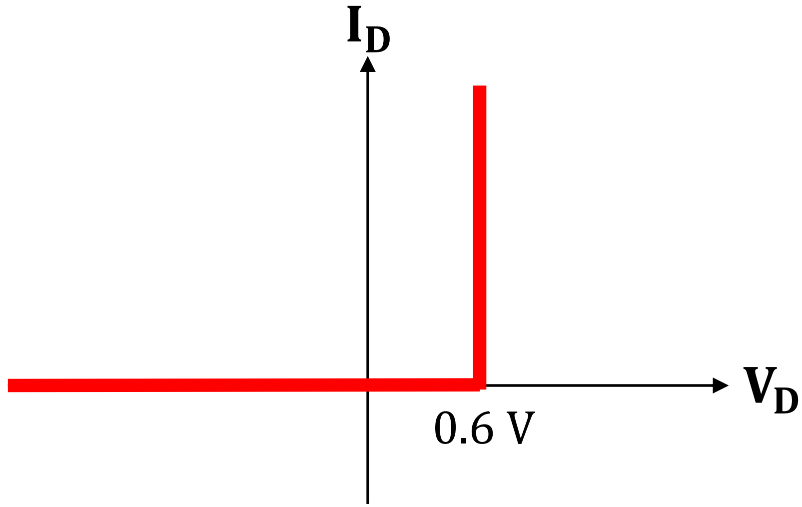

If $I_D > 0$

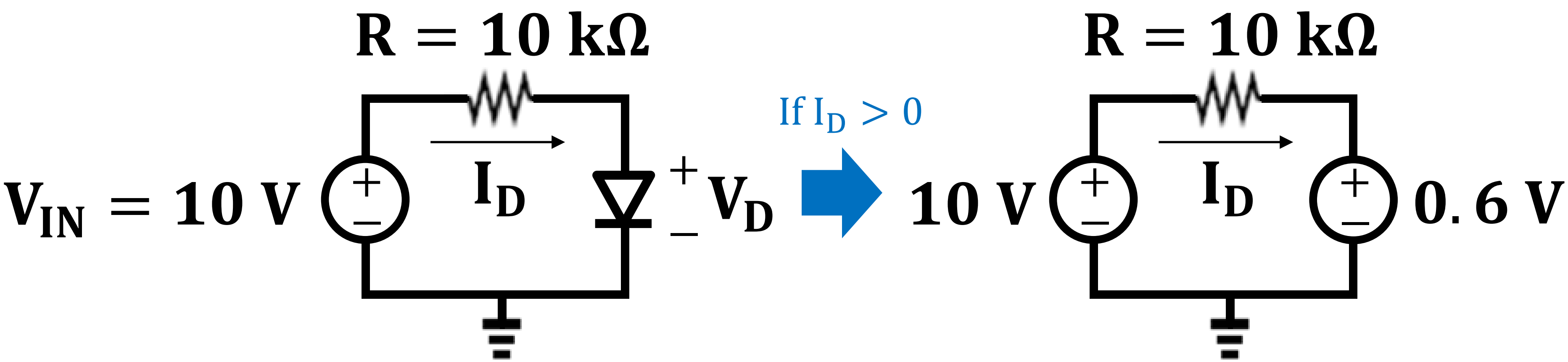



$$V_D = 0\ V$$

$$I_D = \frac{(10 - 0)\ V}{10\ k\Omega} = 1\ mA$$

# Simplified Analysis (Constant Voltage Drop)




$$V_D = 0.6\ V$$

$$I_D = \frac{(10 - 0.6)\ V}{10\ k\Omega} = 0.94\ mA$$

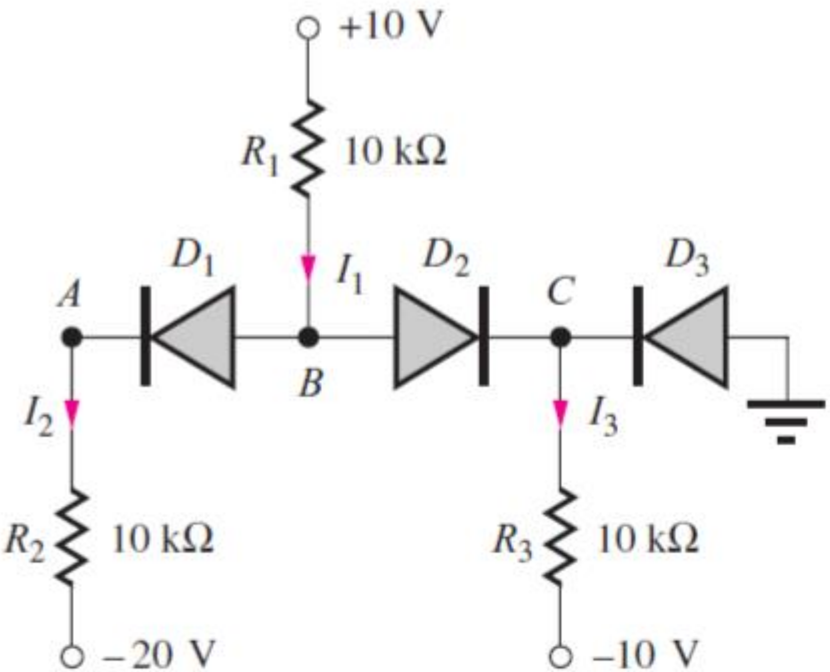
*$V_D$ a power supply?*
*What happens if $V_{IN} = 0.5\ V$?*

# Comparison

| | $V_D$ | $I_D$ |
|---|---|---|
| Graphical Analysis | 0.6 V | 0.95 mA |
| Mathematical Analysis | 0.5742 V | 0.944 mA |
| Ideal Diode Model | 0 V | 1 mA |
| Constant Voltage Drop Model | 0.6 V | 0.94 mA |

# Example

Use constant voltage drop model ($V_{on} = 0.6$ V) to calculate $V_D$ and $I_D$ of each diode.




- Assume no current flowing through $D_3$

$$\frac{10 - V_B}{10k} = \frac{V_B - 0.6 + 20}{10k} + \frac{V_B - 0.6 + 10}{10k}$$

$$V_B = -6.27\text{ V}$$

$$V_C = -6.87\text{ V} \quad \Rightarrow \quad D_3 \text{ in forward bias}$$

**Assumption NOT valid**

- Assume no current flowing through $D_2$

$$\frac{10 - V_B}{10k} = \frac{V_B - 0.6 + 20}{10k}$$

$$V_B = -4.7\text{ V}$$

$$V_C = -0.6\text{ V} \quad \Rightarrow \quad D_2 \text{ indeed in reverse bias}$$

**Assumption valid**

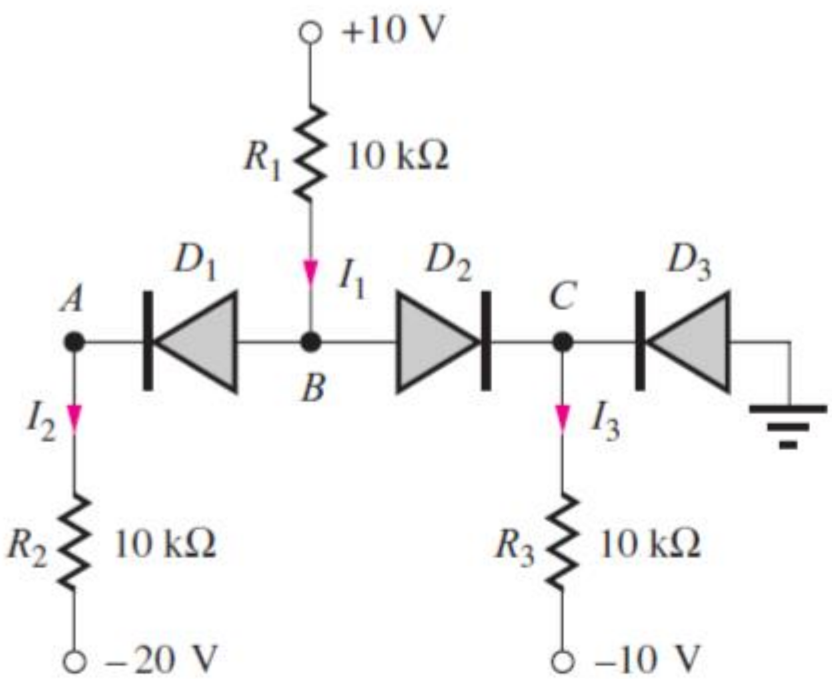
$V_{D1} = 0.6$ V, $I_{D1} = 1.47$ mA

$V_{D2} = -4.1$ V, $I_{D2} = 0$ mA

$V_{D3} = 0.6$ V, $I_{D3} = 0.94$ mA

# Example

Use ideal diode model ($V_{on} = 0$ V) to calculate $V_D$ and $I_D$ of each diode.




- Assume $D_3$ in reverse bias

$$\frac{10 - V_B}{10k} = \frac{V_B + 20}{10k} + \frac{V_B + 10}{10k}$$

$$V_B = -6.67 \text{ V}$$

$$V_C = -6.67 \text{ V} \quad \Rightarrow \quad D_3 \text{ in forward bias}$$

**Assumption NOT valid**

- Assume $D_2$ in reverse bias

$$\frac{10 - V_B}{10k} = \frac{V_B + 20}{10k}$$

$$V_B = -5 \text{ V}$$

$$V_C = 0 \text{ V} \quad \Rightarrow \quad D_2 \text{ indeed in reverse bias}$$

**Assumption valid**

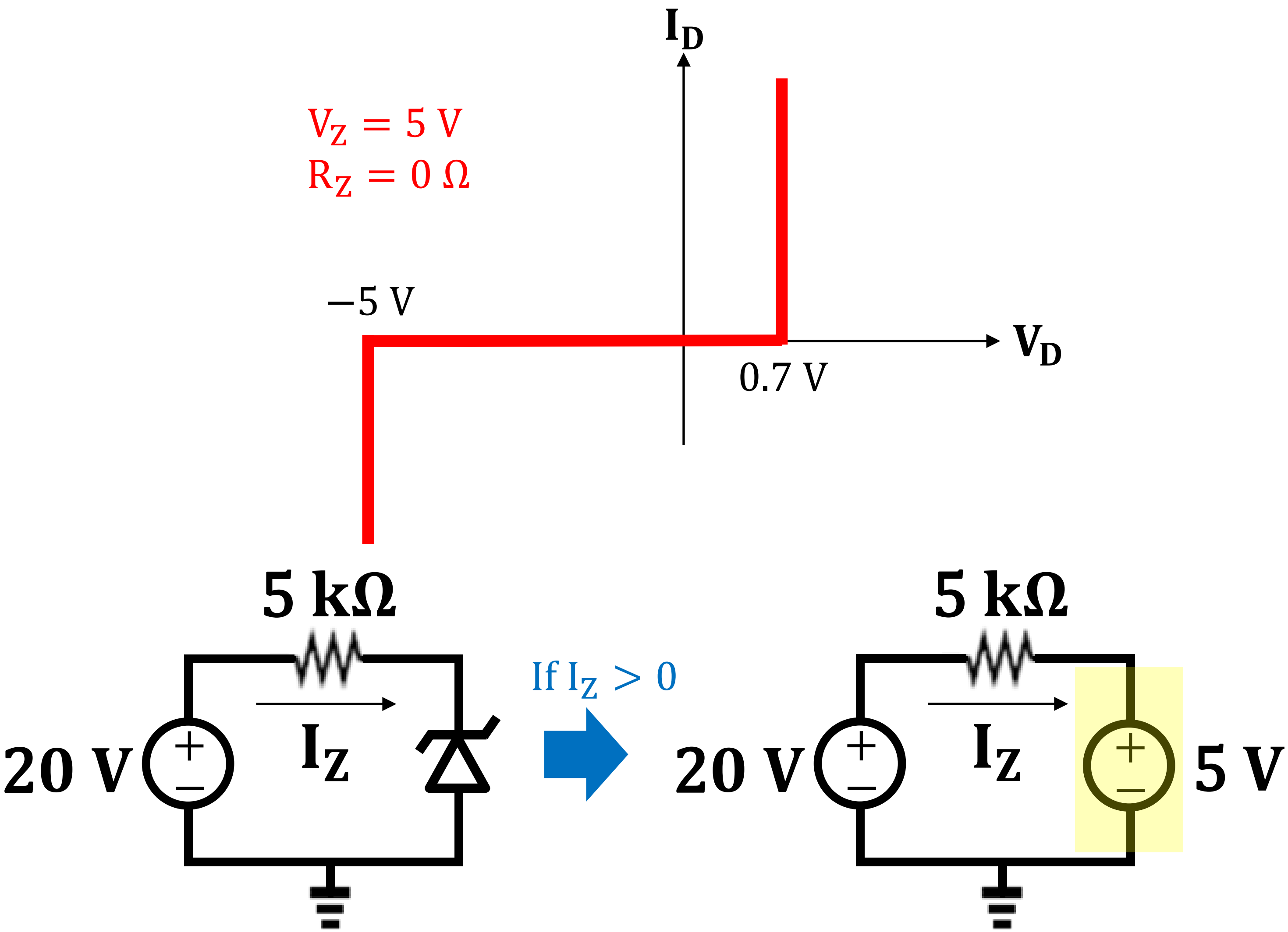
$V_{D1} = 0$ V, $I_{D1} = 1.5$ mA

$V_{D2} = -5$ V, $I_{D2} = 0$ mA

$V_{D3} = 0$ V, $I_{D3} = 1$ mA

# Zener Diode Circuit

# Simplified Analysis for Zener Diode ($R_Z = 0$)

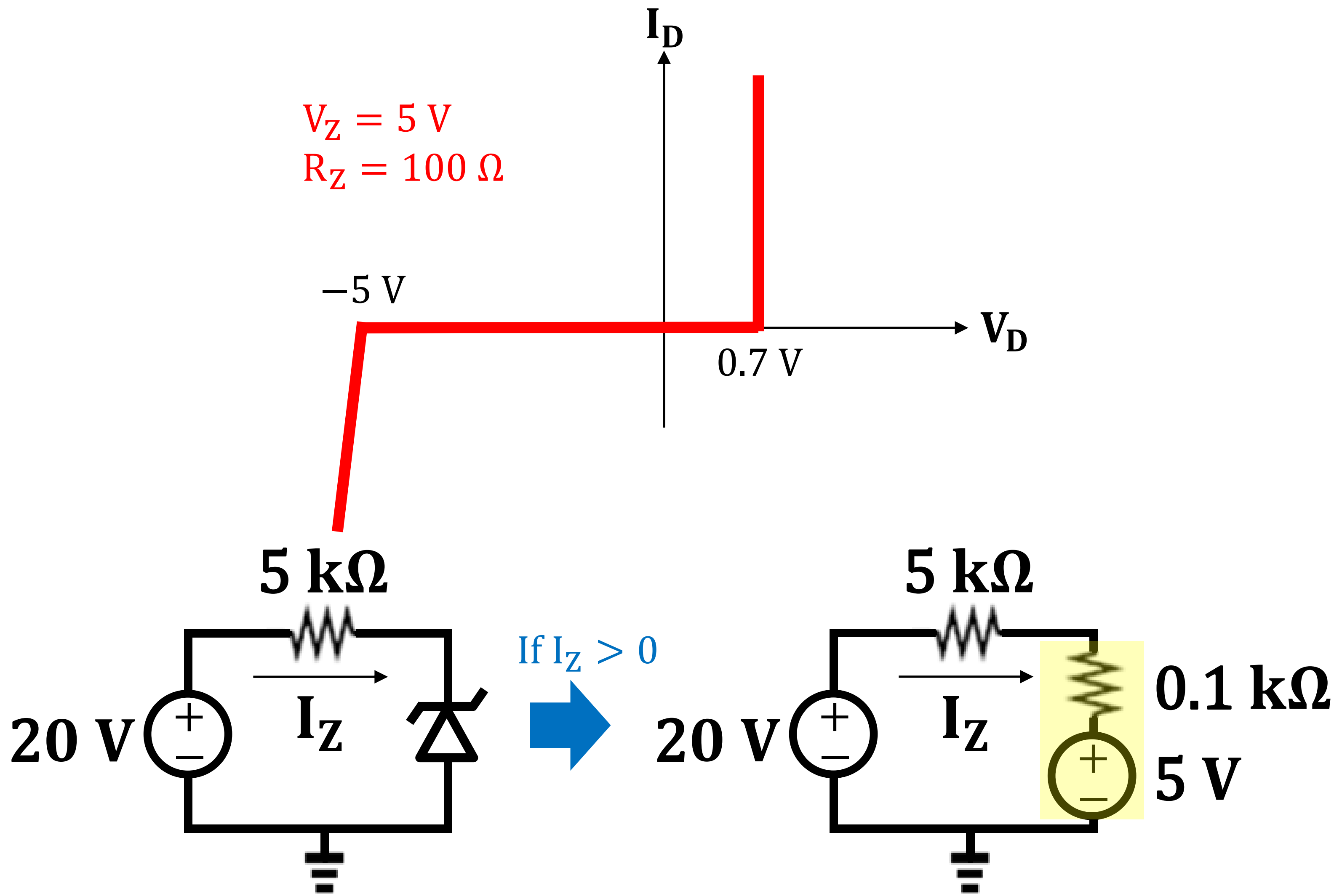



$$I_Z = \frac{20 - 5}{5000} = 3 \times 10^{-3}\ (A)$$

*What happens if $V_{IN} = 1\ V$?*

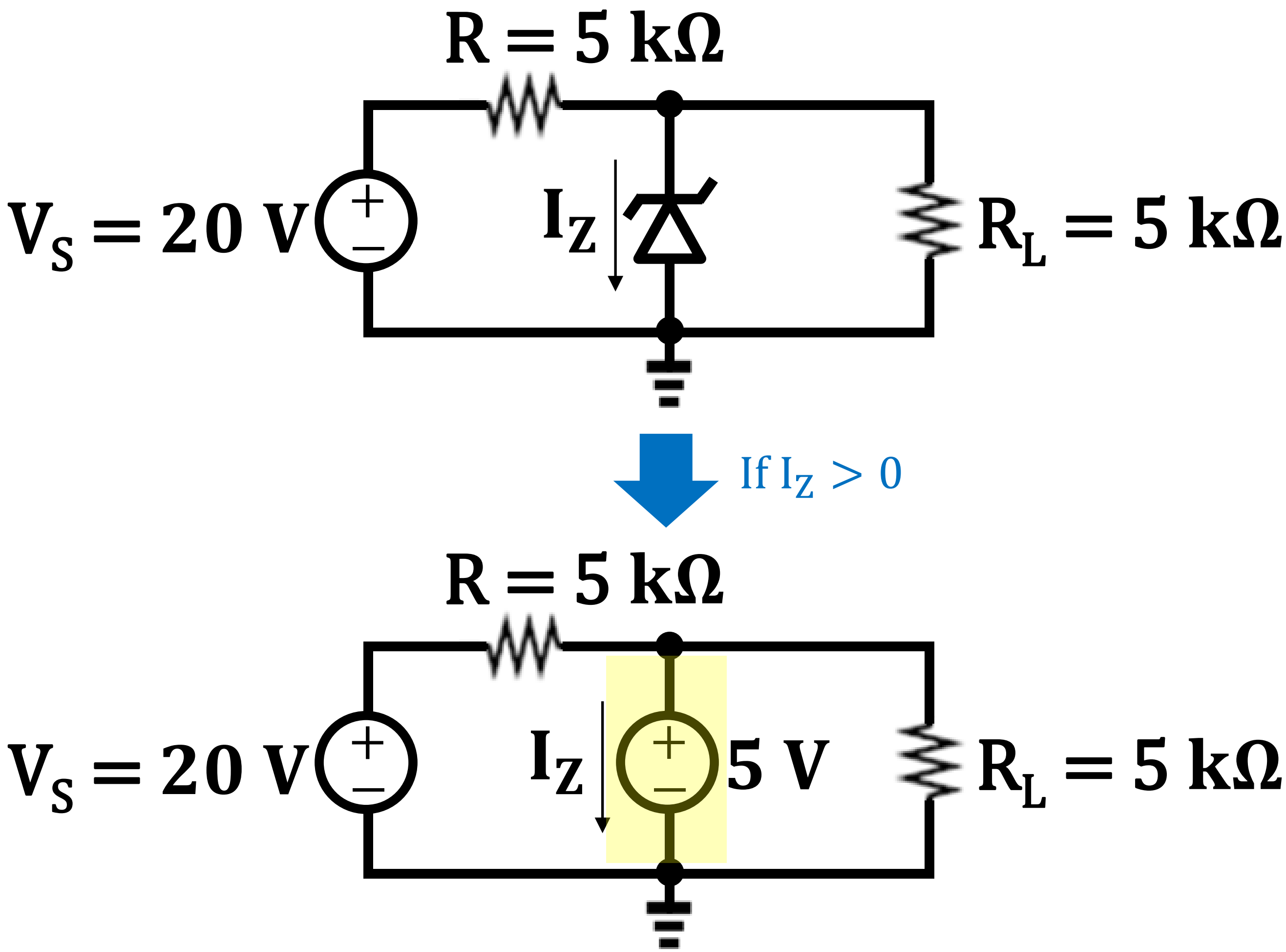
# Simplified Analysis for Zener Diode ($R_Z \neq 0$)






$$I_Z = \frac{20 - 5}{5100} = 2.94 \times 10^{-3}\ (\mathrm{A})$$

# **Voltage Regulator** Using Zener Diode ($R_Z = 0$)

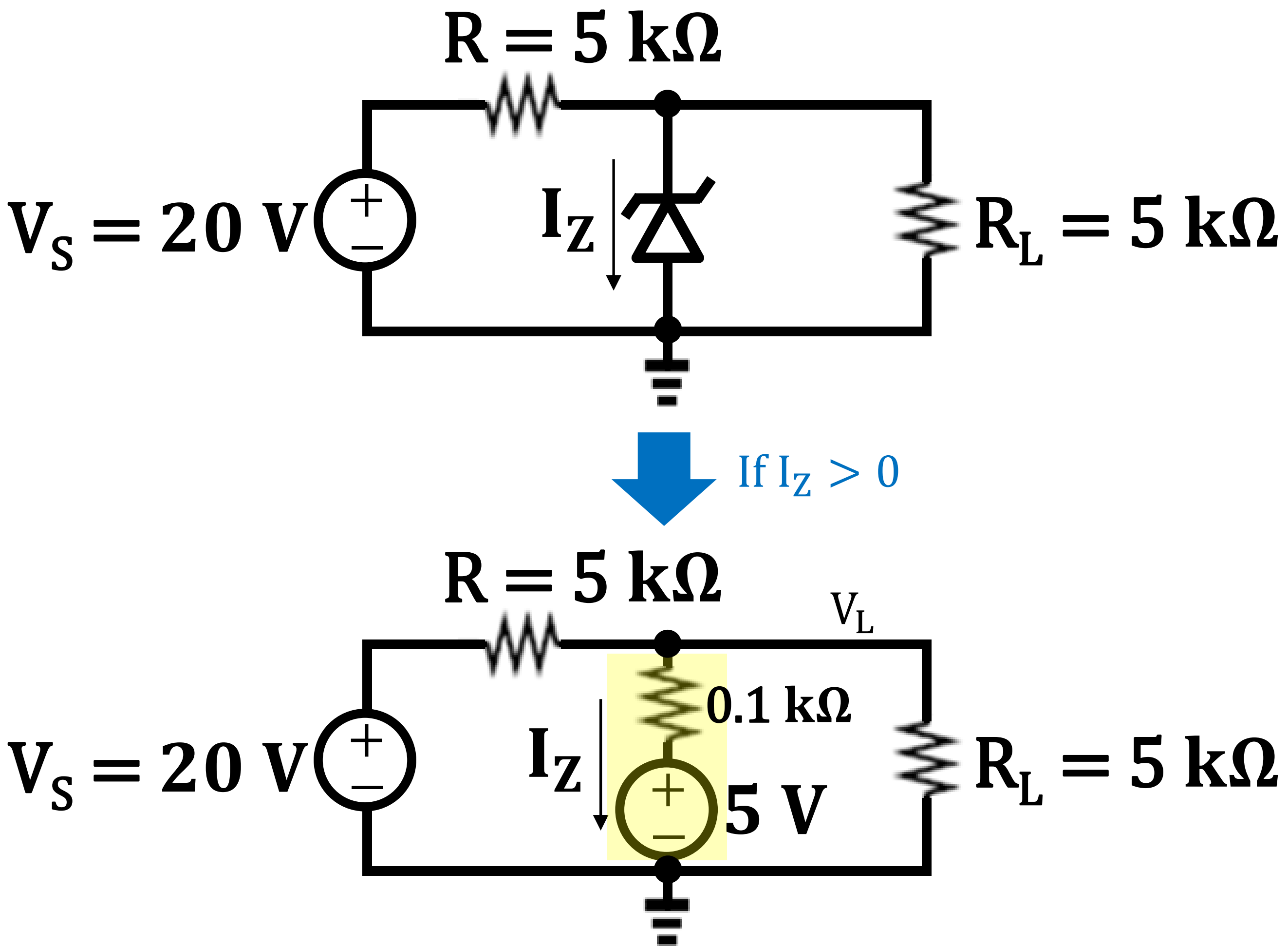



$$I_Z = \frac{20-5}{5k} - \frac{5}{5k} = 2\ \text{mA} > 0$$

As long as the zener diode operates in reverse breakdown region ($I_Z > 0$), a constant voltage (5 V) appears across $R_L$.

*What is the smallest $R_L$ for reverse breakdown to happen? Answer: $R_{L,min} = 1.67\,k\Omega$*

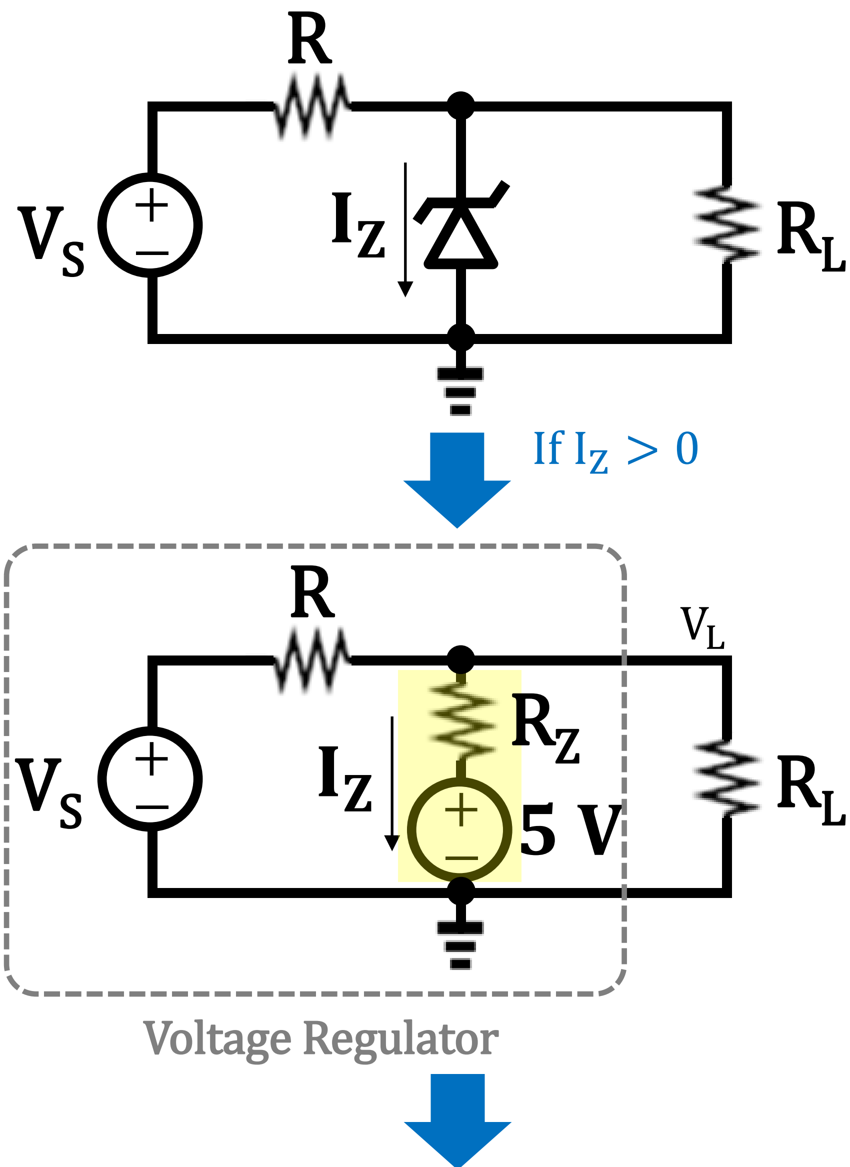
# **Voltage Regulator** Using Zener Diode ($R_Z \neq 0$)




$$\frac{20 - V_L}{5k} = \frac{V_L - 5}{0.1k} + \frac{V_L}{5k} \qquad V_L = 5.1923\ V$$

$$I_Z = \frac{5.19 - 5}{0.1k} = 1.9\ mA\ > 0$$

# Line Regulation and Load Regulation




- **Line Regulation:** how sensitive the output voltage ($V_L$) is to input voltage ($V_S$) changes, when $R_L = \infty$.

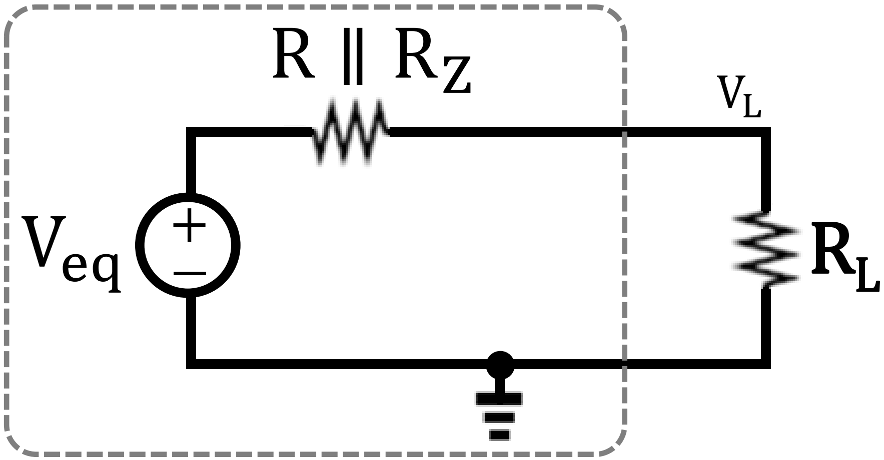
$$\text{Line Regulation} = \frac{dV_L}{dV_S} = \frac{R_Z}{R + R_Z}$$

- **Load Regulation:** output impedance of the voltage regulator.

$$\text{Load Regulation} = \frac{dV_L}{dI_L} = R \parallel R_Z$$

# Thevinen Equivalent Circuit of Voltage Regulator






Voltage Regulator

$$V_{eq} = 5 + \frac{V_S - 5}{R + R_Z} R_Z$$

$$V_L = V_{eq} \frac{R_L}{(R \parallel R_Z) + R_L}$$

Numerical Test: $V_S = 20\ \mathrm{V},\ R = 5\ \mathrm{k\Omega},\ R_Z = 0.1\ \mathrm{k\Omega},\ V_Z = 5\mathrm{V},\ R_L = 5\ \mathrm{k\Omega}$

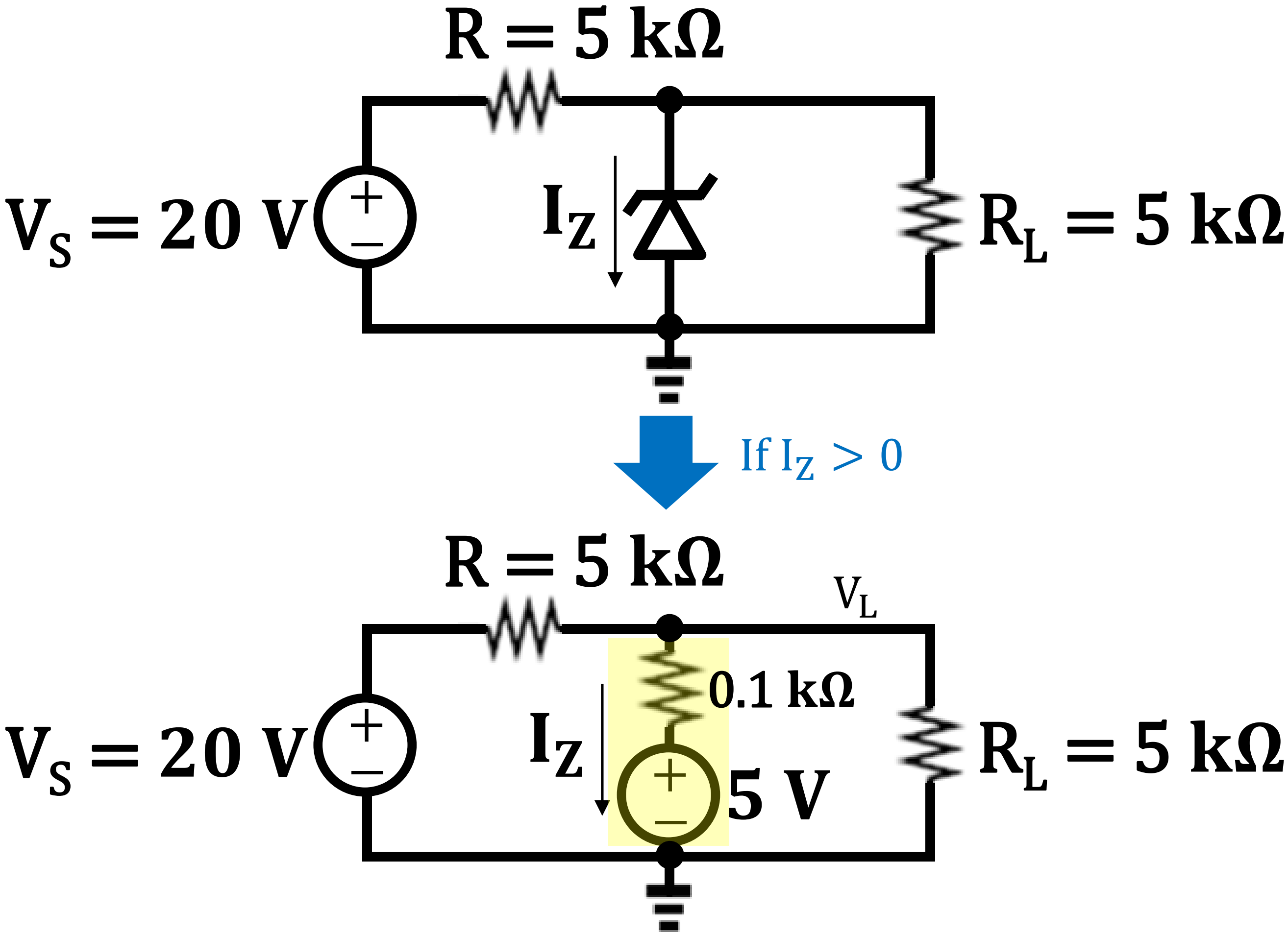
$$V_{eq} = 5 + \frac{20 - 5}{5\mathrm{k} + 0.1\mathrm{k}} 0.1\mathrm{k} = 5.2941\ (\mathrm{V})$$

$$V_L = 5.2941 \times \frac{5k}{\frac{5k \times 0.1k}{5k + 0.1k} + 5k} = 5.1923\ (V)$$

The result exactly the same as on page 14.

# Example

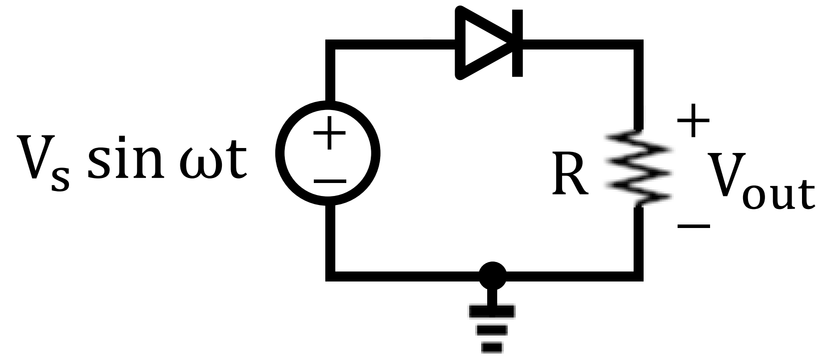
What are the line and load regulations for the circuit below?




$$\text{Line Regulation} = \frac{0.1\text{k}}{5\text{k} + 0.1\text{k}} = 19.6\ \text{mV/V} \qquad \text{Load Regulation} = 5\text{k} \parallel 0.1\text{k} = 98\ \Omega$$

# Half-Wave Rectifier

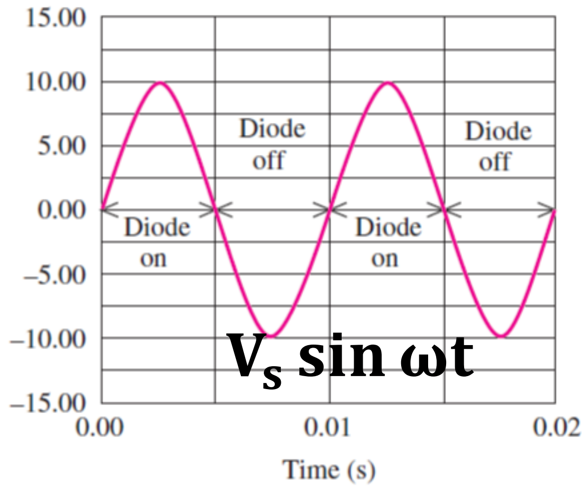
# **Half-Wave Rectifier** with Resistive Load



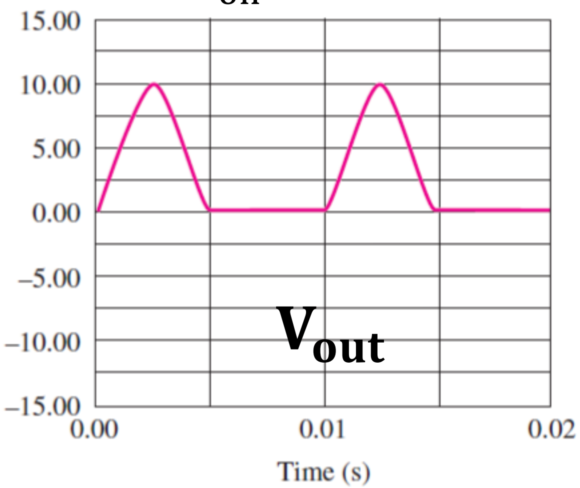

$V_s = 10\ \mathrm{V}$

$$\omega = 2\pi f = 2\pi \frac{1}{0.01} = 200\pi\ (\mathrm{rad/sec})$$

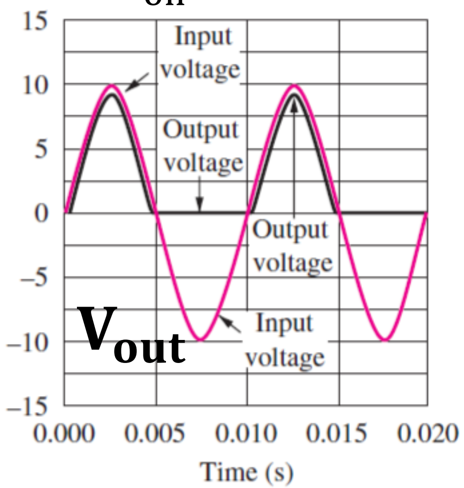
**$V_{out}$ is not DC.**



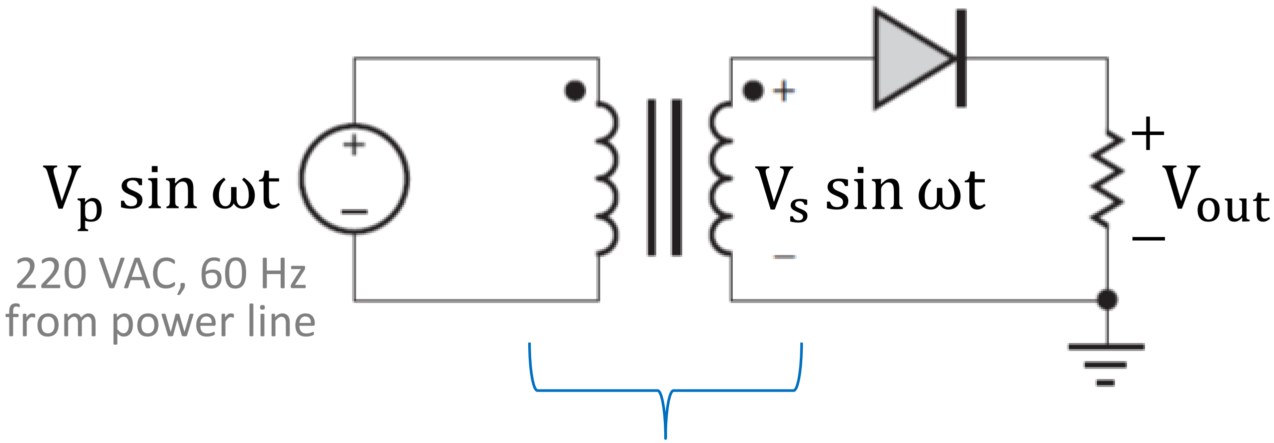

$V_{on} = 0\ \mathrm{V}$



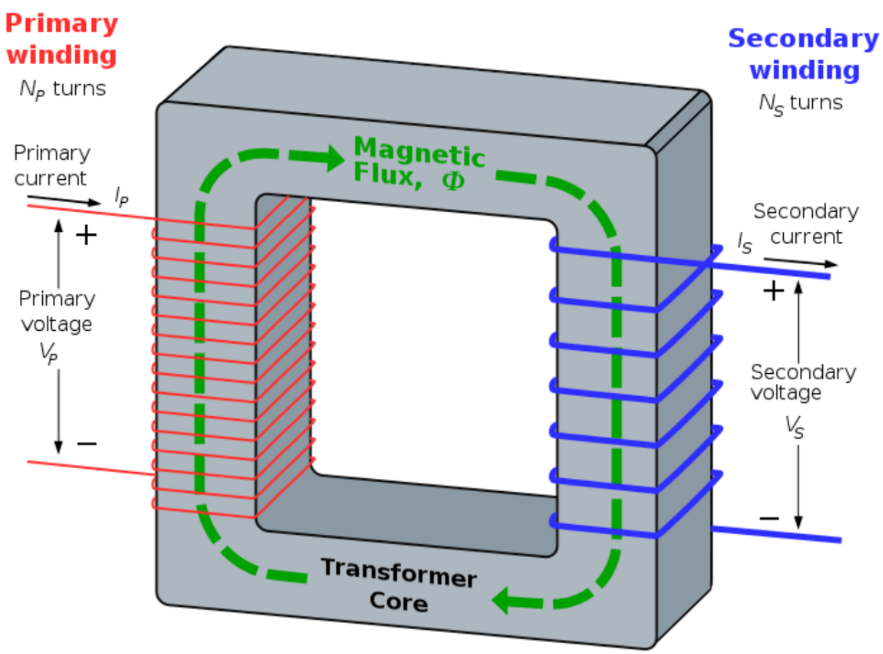

$V_{on} = 0.7\ \mathrm{V}$



# Transformer




**The output of an ideal transformer can be represented as an ideal voltage source.**




Source: Daniels, A (1976). Introduction to Electrical Machines. Macmillan Publishers.

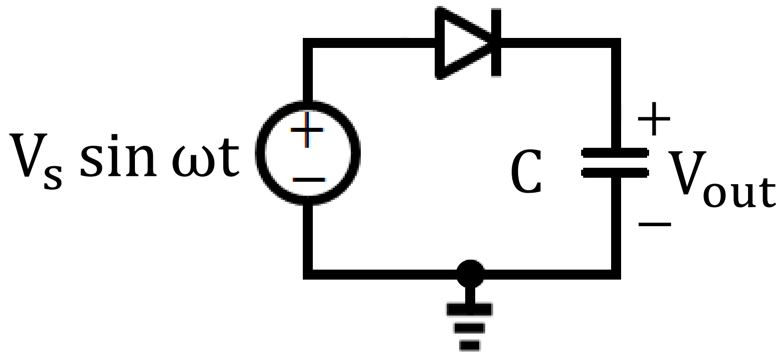
Working Principle:
By Faraday's law of induction

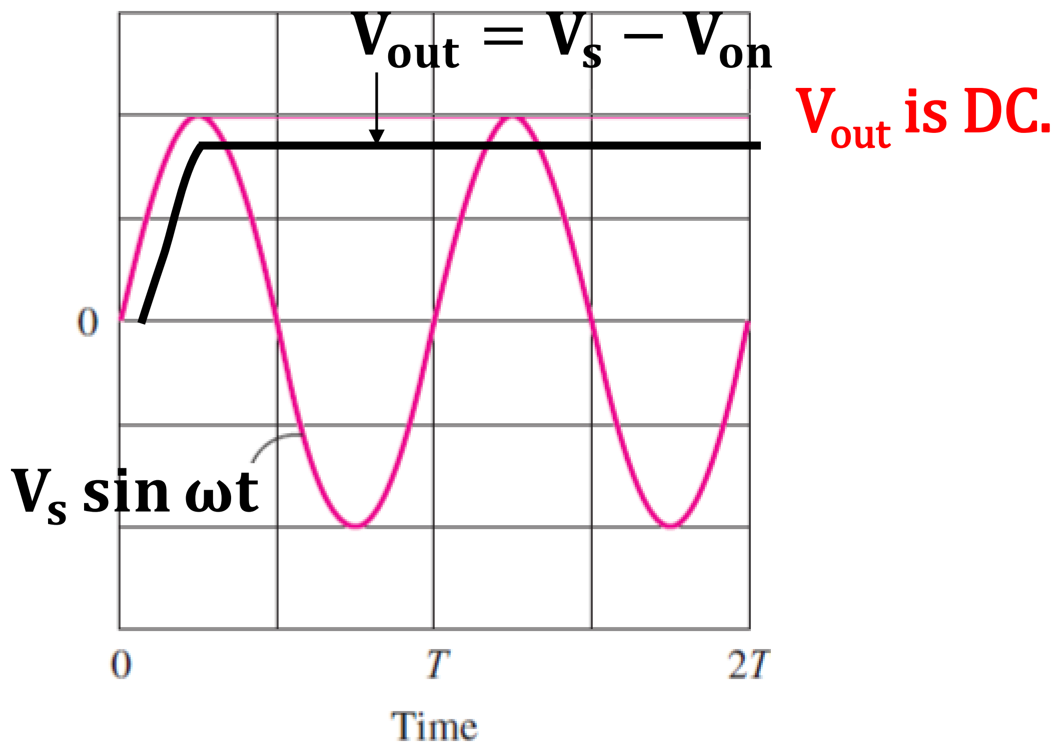
$$V_s = -N_s \frac{d\emptyset}{dt}$$

$$V_p = -N_p \frac{d\emptyset}{dt}$$

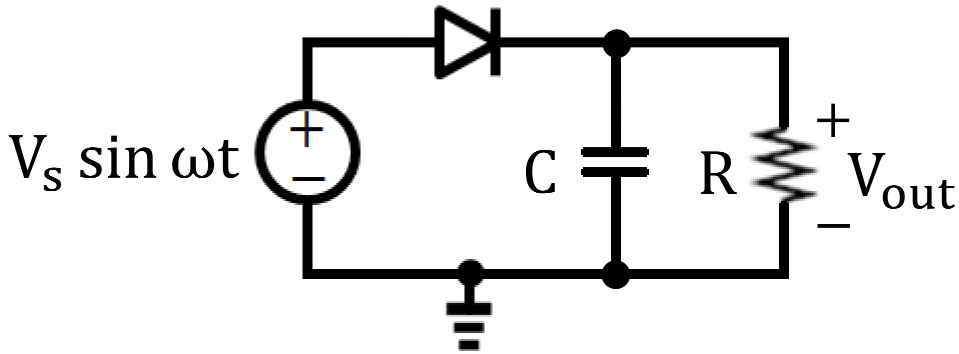
$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

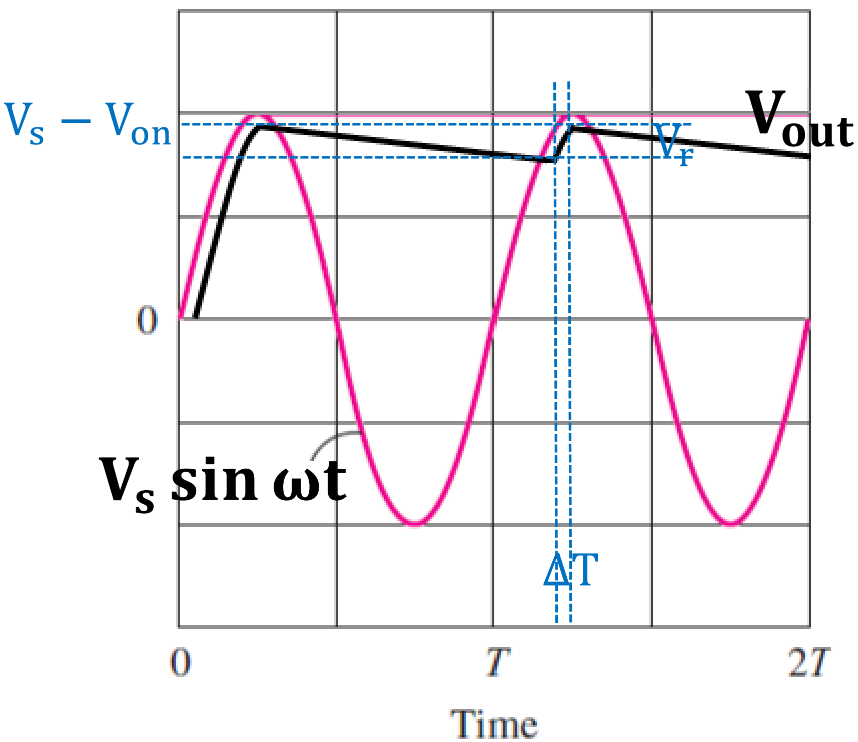
# **Half-Wave Rectifier** with Capacitive Load



# Half-Wave Rectifier with RC Load (I)




$$V_{dc} = \boxed{\mathbf{V_s - V_{on}}}$$

$$I_{dc} = \frac{V_{dc}}{R}$$

ripple voltage

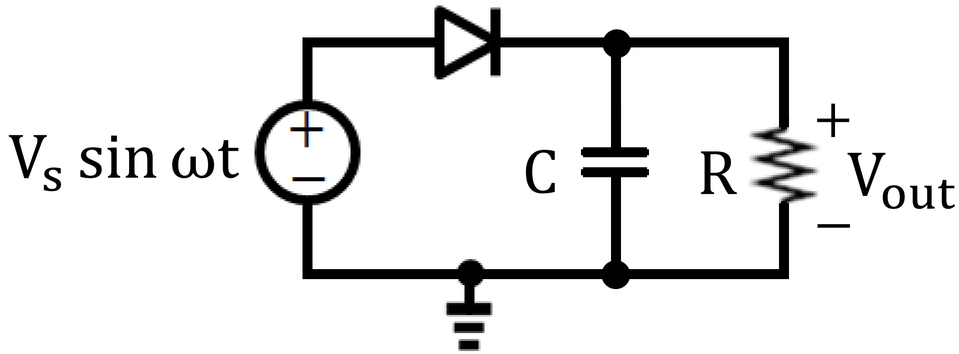
$$V_r = (V_s - V_{on})\left(1 - e^{-\frac{T-\Delta T}{RC}}\right)$$

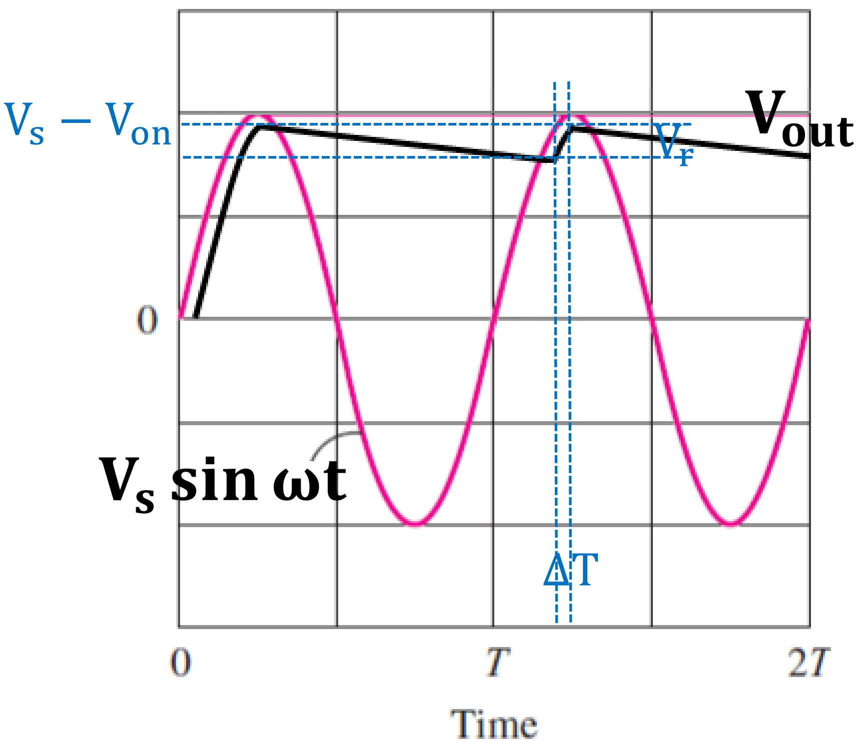
$$\cong (V_s - V_{on})\left(\frac{T-\Delta T}{RC}\right) \quad if\,(T-\Delta T) \ll RC$$

$$\cong \boxed{\mathbf{(V_s - V_{on})\left(\frac{T}{RC}\right)}} \quad if\,\Delta T \ll T$$

# Half-Wave Rectifier with RC Load (II)






**conduction angle and interval**

$$V_s \sin\left[\omega\left(\frac{5T}{4} - \Delta T\right)\right] - V_{on} = (V_s - V_{on}) - V_r$$

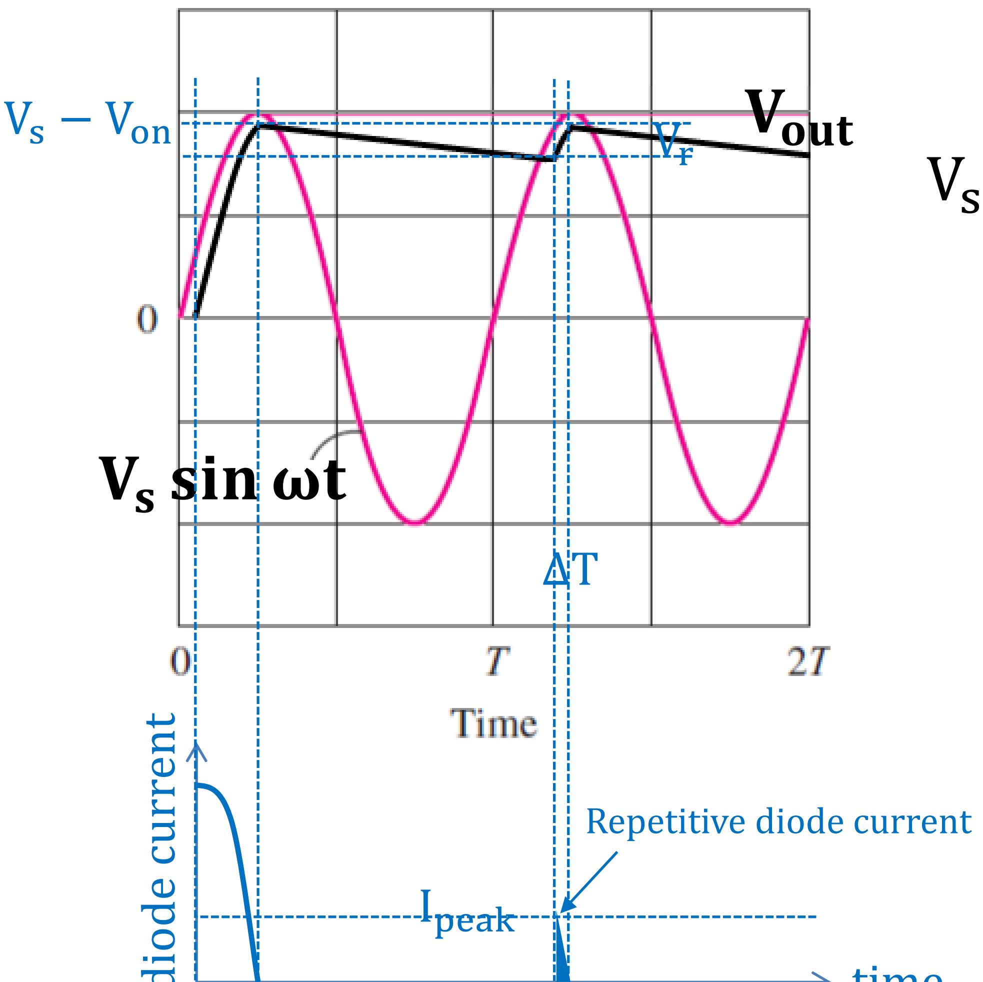
$$V_s \sin\left(\frac{5\pi}{2} - \theta_c\right) - V_{on} = (V_s - V_{on}) - V_r$$

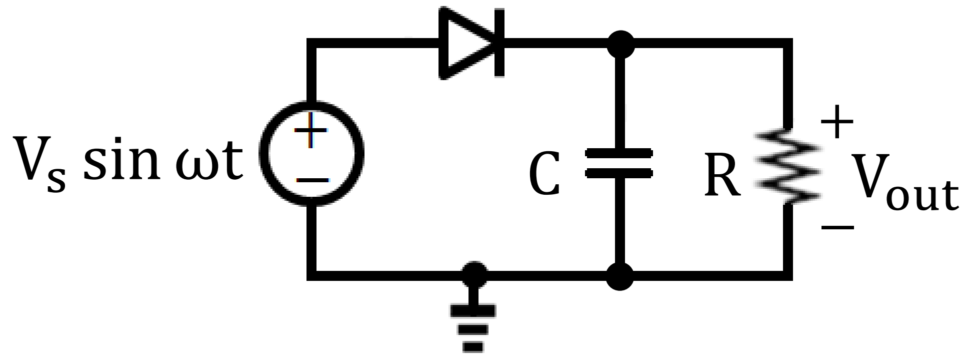
$$V_s \cos \theta_c = V_s - V_r$$

$$\cos \theta_c = \frac{V_s - V_r}{V_s} \cong 1 - \frac{{\theta_c}^2}{2} \quad \textit{if } \theta_c \textit{ very small}$$

$$\boxed{\boldsymbol{\theta_c = \sqrt{\frac{2V_r}{V_s}}}} \qquad \boxed{\boldsymbol{\Delta T = \frac{\theta_c}{\omega} = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_s}}}}$$

# Half-Wave Rectifier with RC Load (III)




**The charge filled on C during $\Delta T$ is discharged during $T - \Delta T$.**

$$Q \cong \frac{I_{peak}\Delta T}{2} = I_{dc}(T - \Delta T) \cong I_{dc}T$$

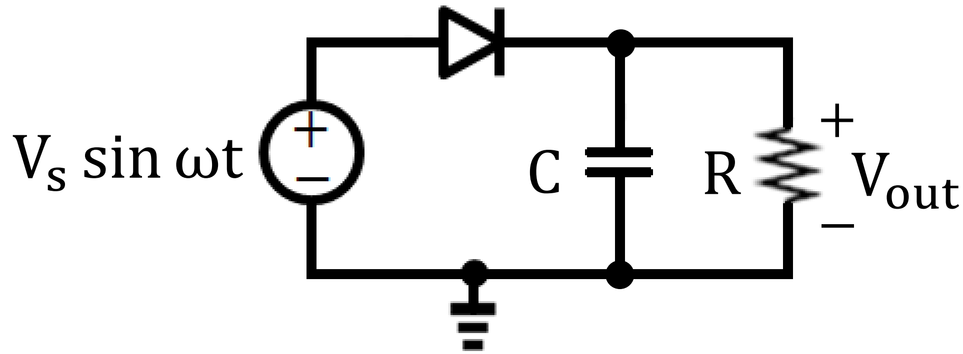
$$\boxed{\mathbf{I_{peak} = \frac{2I_{dc}T}{\Delta T}}}$$

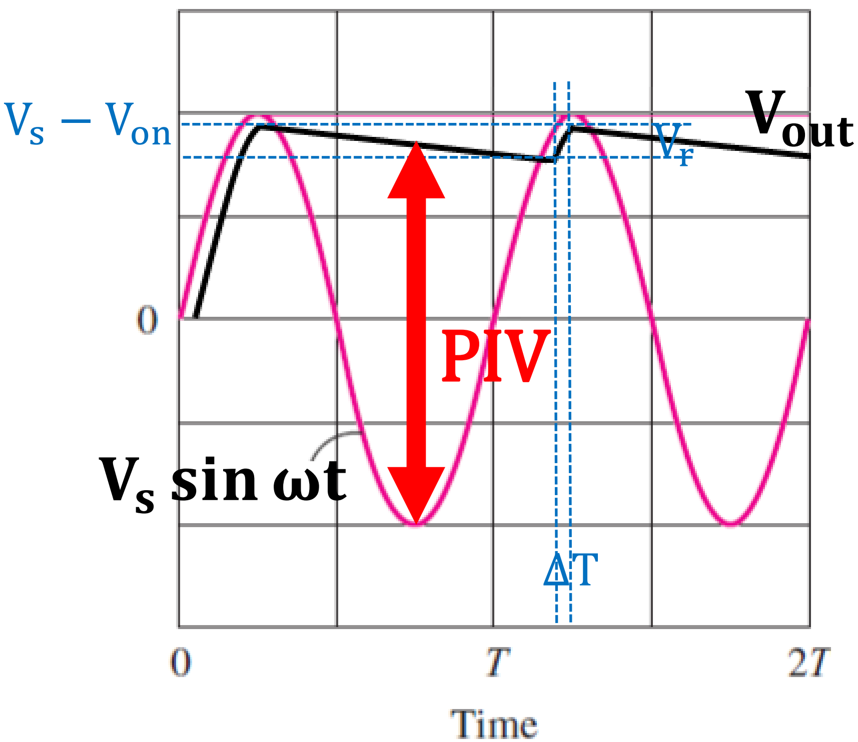
$$\boxed{\mathbf{I_{surge} = C\frac{d(V_s \sin \omega t - V_{on})}{dt} = \omega C V_s} \quad \text{if } t = 0}$$

During charging period ($\Delta T$), almost all diode current goes to C.

$$\left|\frac{1}{SC}\right| = \frac{1}{2\pi\frac{1}{T}C} = \frac{T}{2\pi C} \ll R \quad \text{if } RC \gg T$$

# Half-Wave Rectifier with RC Load (IV)




**Peak-inverse-voltage (PIV) $\cong 2V_s - V_{on}$**

If too large, the diode breaks down.

# Example

Find the value of the dc output voltage, dc output current, ripple voltage, conduction interval, conduction angle and diode peak current for a half-wave rectifier driven from a transformer having a secondary voltage of 12.6 $V_{rms}$ (60 Hz) with $R = 15\,\Omega$ and $C = 25{,}000\ \mu F$. Assume $V_{on} = 1$ V.

$$V_{dc} = 12.6\sqrt{2} - 1 = 16.8\ (V)$$

$$I_{dc} = \frac{16.8}{15} = 1.12\ (A)$$

$$V_r \cong V_{dc}\frac{T}{RC} = 16.8\frac{\frac{1}{60}}{15 \times 25000 \times 10^{-6}} = 0.747\ (V)$$

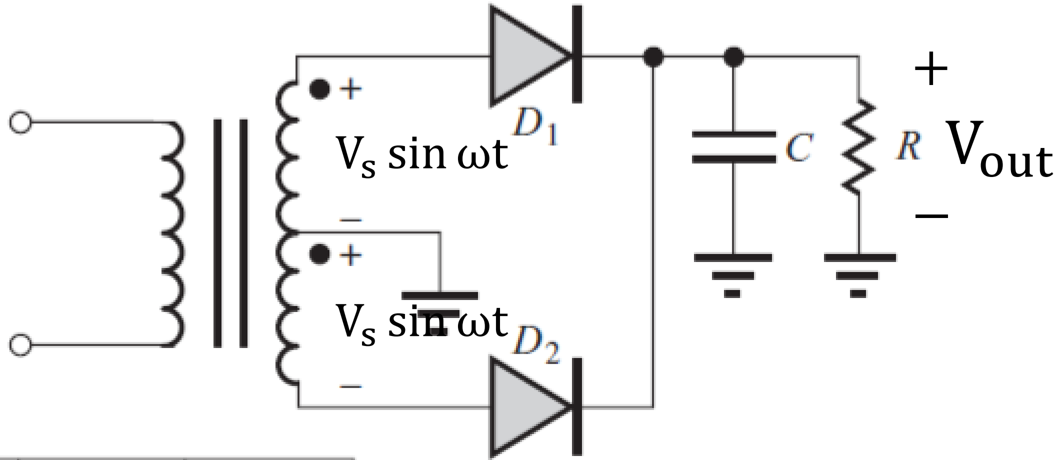
$$\theta_c \cong \sqrt{\frac{2V_r}{V_s}} = \sqrt{\frac{2 \times 0.747}{12.6 \times \sqrt{2}}} = 0.29\ (rad)\ \text{or}\ 16.6^\circ$$

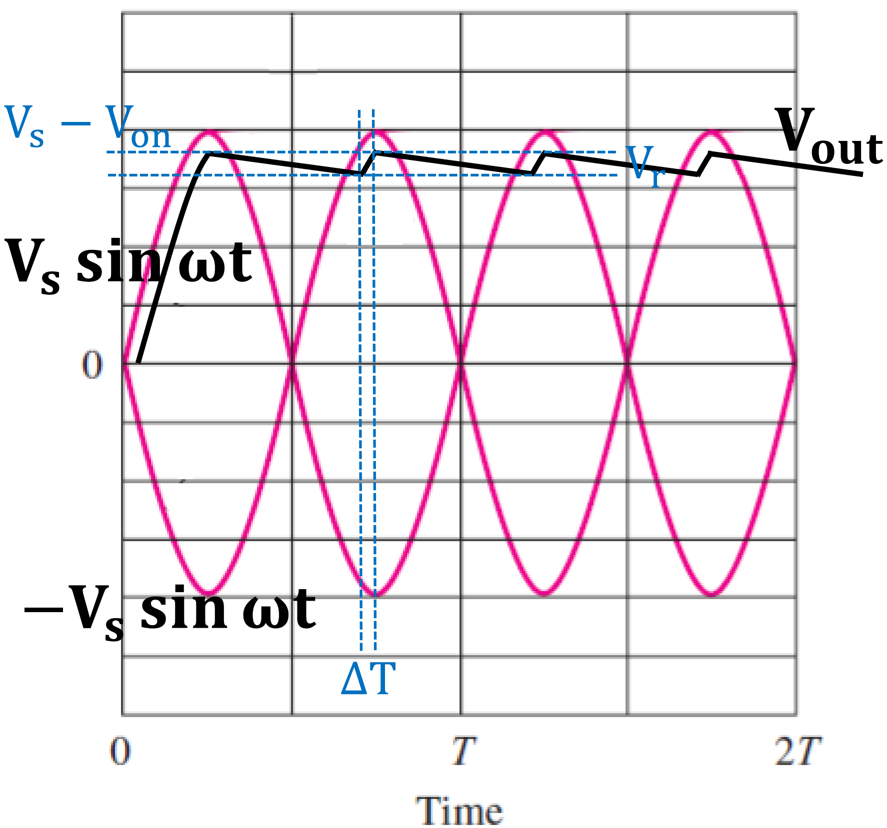
$$\Delta T \cong \frac{\theta_c}{\omega} = \frac{0.29}{2\pi \times 60} = 7.69 \times 10^{-4}\ (sec)$$

$$I_{peak} = \frac{2 \times 1.12 \times \frac{1}{60}}{7.69 \times 10^{-4}} = 48.6\ (A)$$

- Make sure all assumptions are valid.
- Since R is small (15 Ω), C needs to be large (25,000 μF) to maintain a low $V_r$.
- The diode must be able to handle these repetitively high peak currents.

# Full-Wave Rectifier (I)




$$V_{dc} = \boxed{\mathbf{V_s - V_{on}}}$$

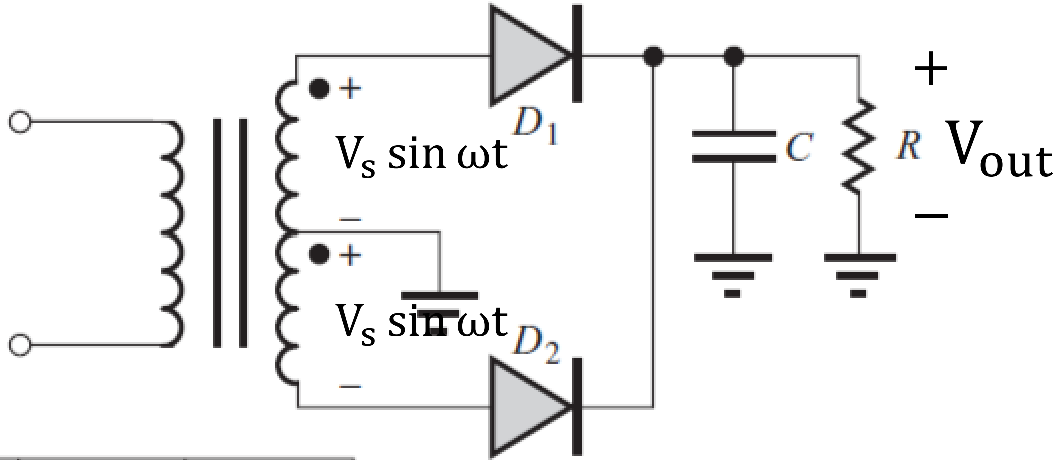
$$I_{dc} = \frac{V_{dc}}{R}$$

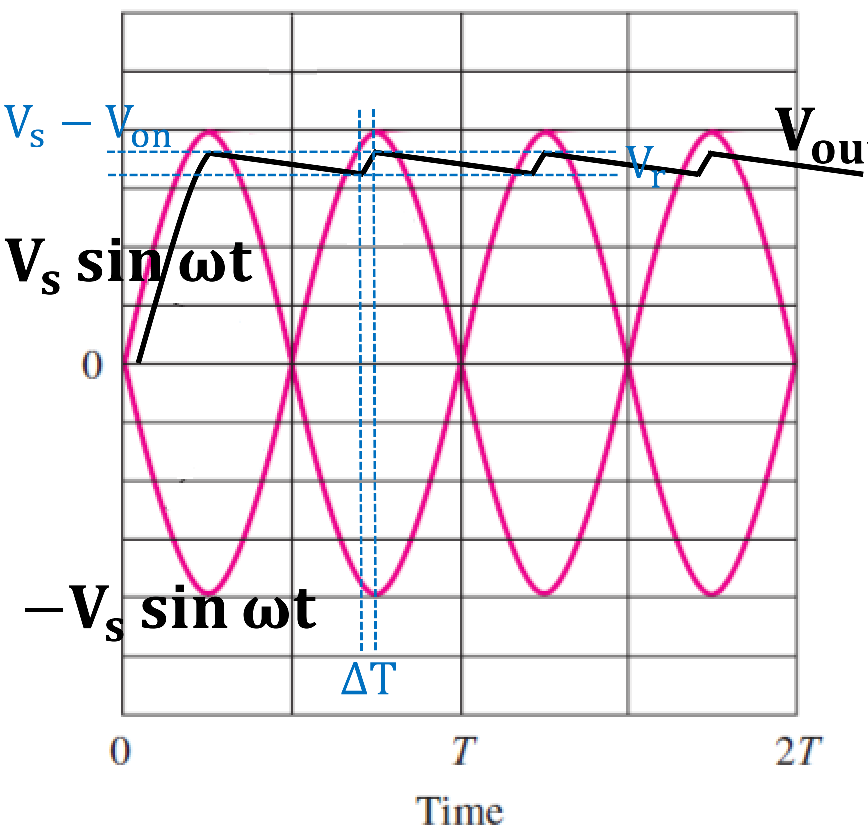
$$V_r = (V_s - V_{on})\left(1 - e^{-\frac{T/2 - \Delta T}{RC}}\right)$$

$$\cong (V_s - V_{on})\left(\frac{T/2 - \Delta T}{RC}\right) \quad if\left(\frac{T}{2} - \Delta T\right) \ll RC$$

$$\cong \boxed{\mathbf{(V_s - V_{on})\left(\frac{T}{2RC}\right)}} \quad if\,\Delta T \ll \frac{T}{2}$$

# Full-Wave Rectifier (II)




$$-V_s \sin\left[\omega\left(\frac{3T}{4} - \Delta T\right)\right] - V_{on} = (V_s - V_{on}) - V_r$$

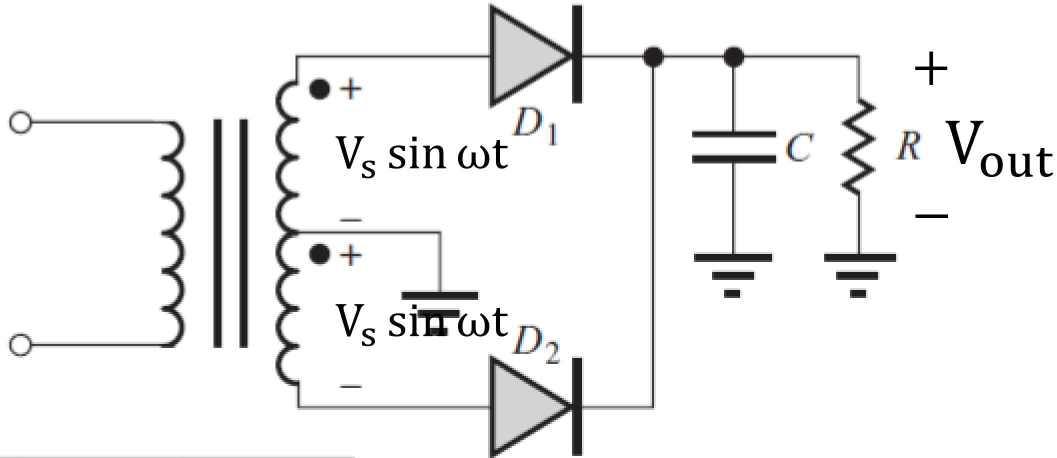
$$-V_s \sin\left(\frac{3\pi}{2} - \theta_c\right) - V_{on} = (V_s - V_{on}) - V_r$$

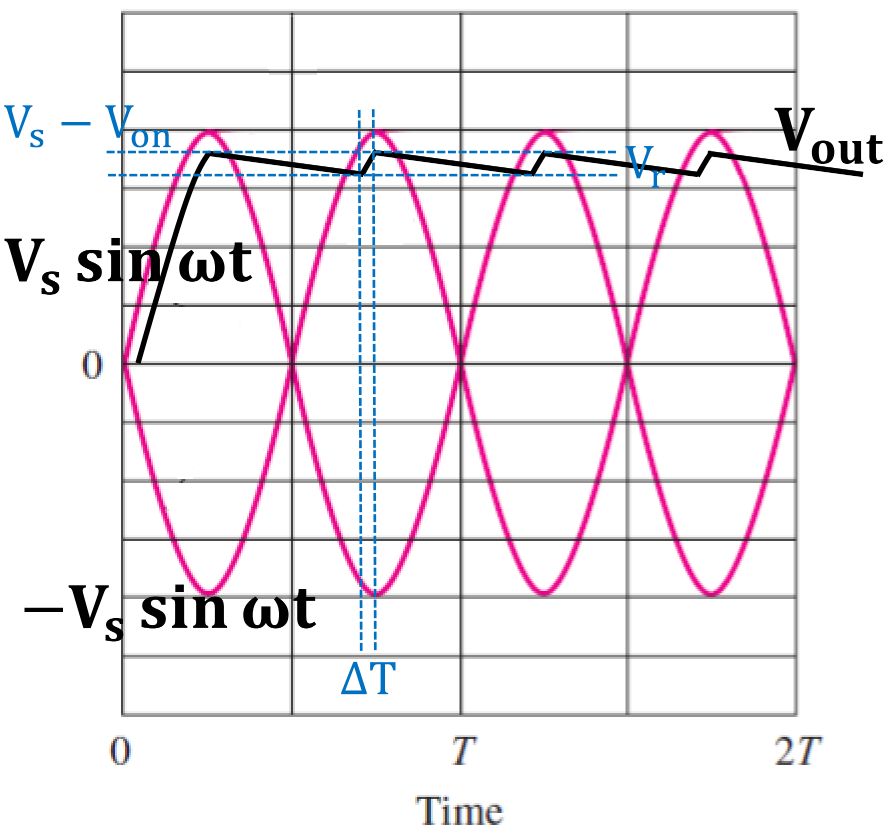
$$V_s \cos \theta_c = V_s - V_r$$

$$\cos \theta_c = \frac{V_s - V_r}{V_s} \cong 1 - \frac{{\theta_c}^2}{2} \quad \textit{if } \theta_c \textit{ very small}$$

$$\boxed{\boldsymbol{\theta_c = \sqrt{\frac{2V_r}{V_s}}}} \qquad \boxed{\boldsymbol{\Delta T = \frac{\theta_c}{\omega} = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_s}}}}$$

# Full-Wave Rectifier (III)



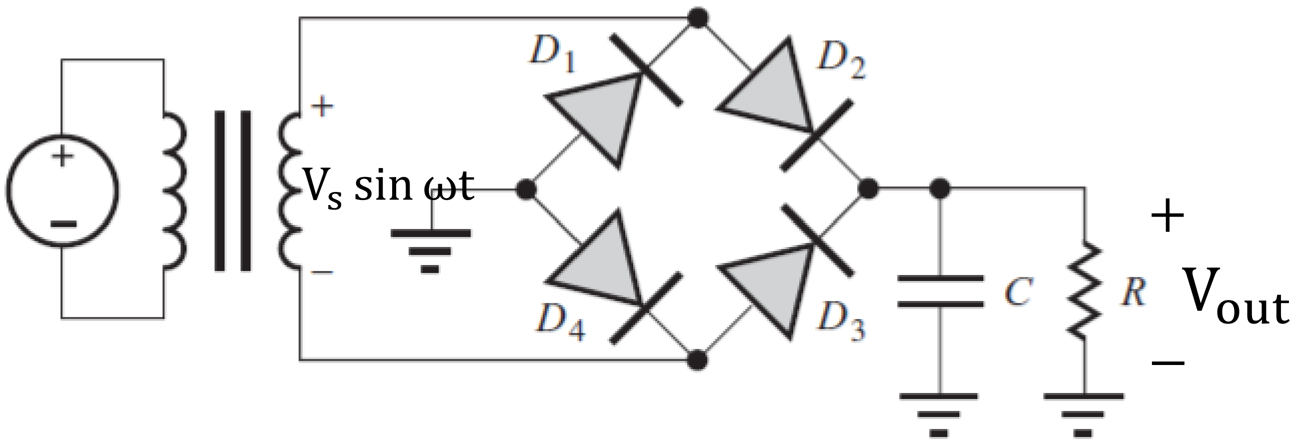

$$Q \cong \frac{I_{peak}\Delta T}{2} = I_{dc}\left(\frac{T}{2} - \Delta T\right) \cong I_{dc}\frac{T}{2}$$

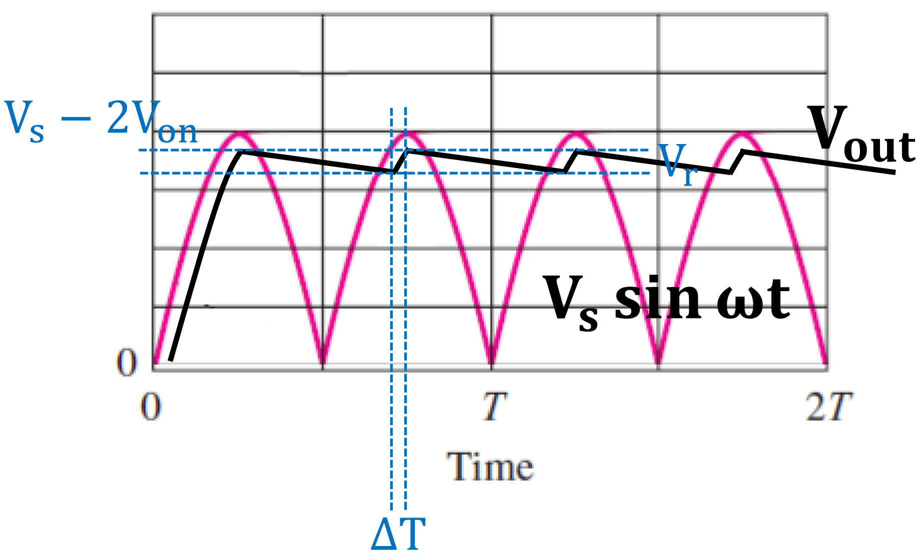
$$\boxed{\mathbf{I_{peak} = \frac{I_{dc}T}{\Delta T}}}$$

$$\boxed{\mathbf{I_{surge} = \omega C V_s}}$$

$$\boxed{\mathbf{PIV = 2V_s - V_{on}}}$$

# Full-Wave Bridge Rectifier (I)




$$V_{dc} = \boxed{\mathbf{V_s - 2V_{on}}}$$

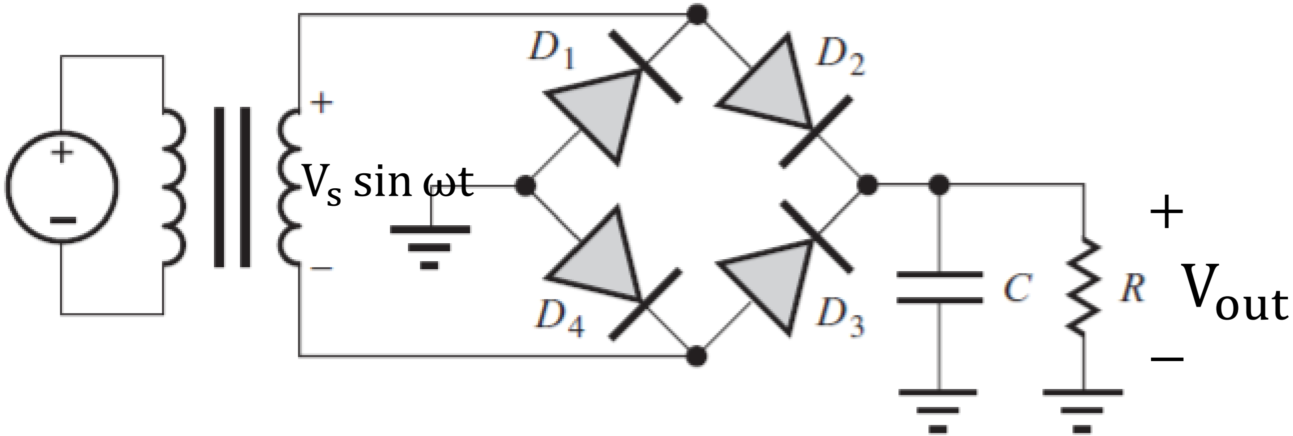
$$I_{dc} = \frac{V_{dc}}{R}$$

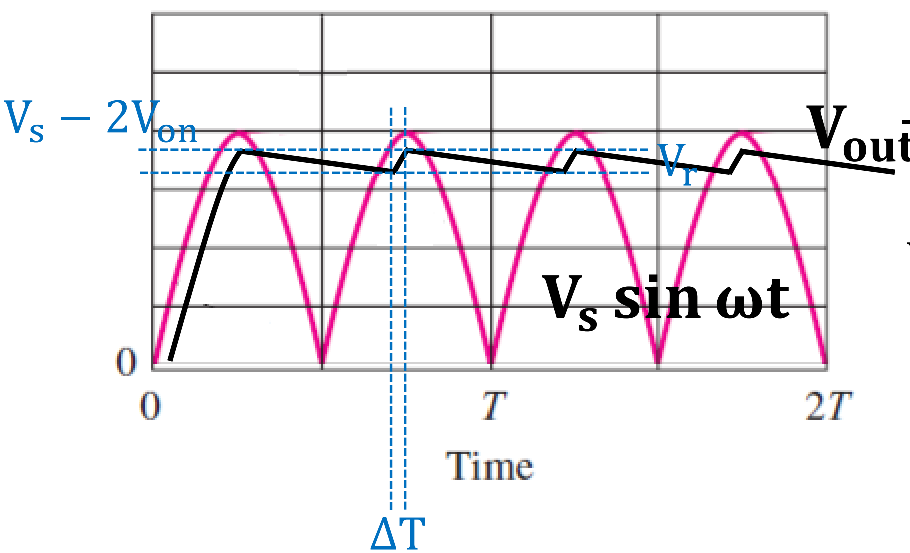
$$V_r = (V_s - 2V_{on})\left(1 - e^{-\frac{T/2 - \Delta T}{RC}}\right)$$

$$\cong (V_s - 2V_{on})\left(\frac{T/2 - \Delta T}{RC}\right) \quad if \left(\frac{T}{2} - \Delta T\right) \ll RC$$

$$\cong \boxed{\mathbf{(V_s - 2V_{on})\left(\frac{T}{2RC}\right)}} \quad if\ \Delta T \ll \frac{T}{2}$$

# Full-Wave Bridge Rectifier (II)




$$-V_s \sin\left[\omega\left(\frac{3T}{4} - \Delta T\right)\right] - 2V_{on} = (V_s - 2V_{on}) - V_r$$

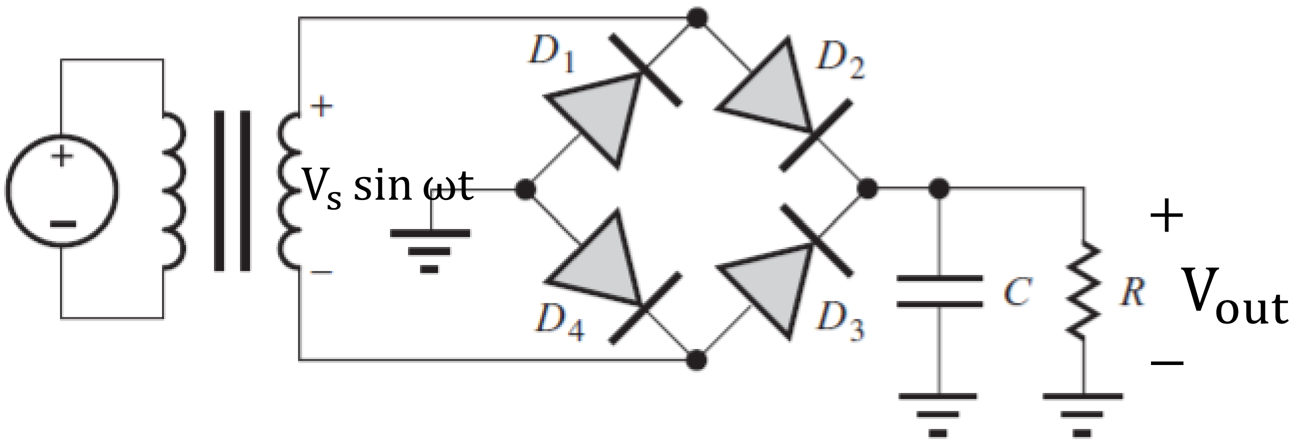
$$-V_s \sin\left(\frac{3\pi}{2} - \theta_c\right) - 2V_{on} = (V_s - 2V_{on}) - V_r$$

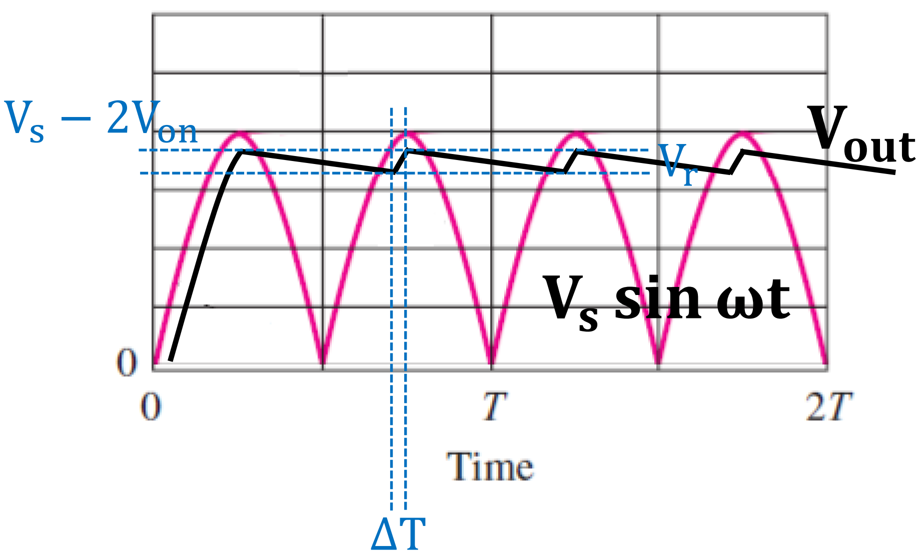
$$V_s \cos \theta_c = V_s - V_r$$

$$\cos \theta_c = \frac{V_s - V_r}{V_s} \cong 1 - \frac{{\theta_c}^2}{2} \quad \textit{if } \theta_c \textit{ very small}$$

$$\boxed{\boldsymbol{\theta_c = \sqrt{\frac{2V_r}{V_s}}}} \qquad \boxed{\boldsymbol{\Delta T = \frac{\theta_c}{\omega} = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_s}}}}$$

# Full-Wave Bridge Rectifier (III)




$$Q \cong \frac{I_{peak}\Delta T}{2} = I_{dc}\left(\frac{T}{2} - \Delta T\right) \cong I_{dc}\frac{T}{2}$$

$$\boxed{\mathbf{I_{peak} = \frac{I_{dc}T}{\Delta T}}}$$

$$\boxed{\mathbf{I_{surge} = \omega C V_s}}$$

$$\boxed{\mathbf{PIV = V_s - V_{on}}}$$

# Example

Design a full-wave bridge rectifier to provide a dc output voltage 15 V with no more than 1 percent ripple at a load current of 2A. ($V_{on} = 1$ V, $T = 1/60$ sec)

$$V_{dc} = 15\ (V)$$
$$V_r < 0.15\ (V)$$
$$I_{dc} = 2\ (A)$$
$$\text{Load resistance} = 15/2 = 7.5\ (\Omega)$$

The required transformer voltage $V_s = 15 + 2 = \mathbf{17\ (V)\ or\ \frac{17}{\sqrt{2}}\ (V_{rms})}$

$$V_r \cong (V_s - 2V_{on})\left(\frac{T}{2RC}\right) = 15\left(\frac{1}{2 \times 60 \times 7.5 \times C}\right) = 0.15 \Rightarrow \mathbf{C = 0.111\ (F)}$$

$$\Delta T = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_s}} = \frac{1}{2\pi \times 60}\sqrt{\frac{2 \times 0.15}{17}} = \mathbf{0.352 \times 10^{-3}\ (sec)}$$

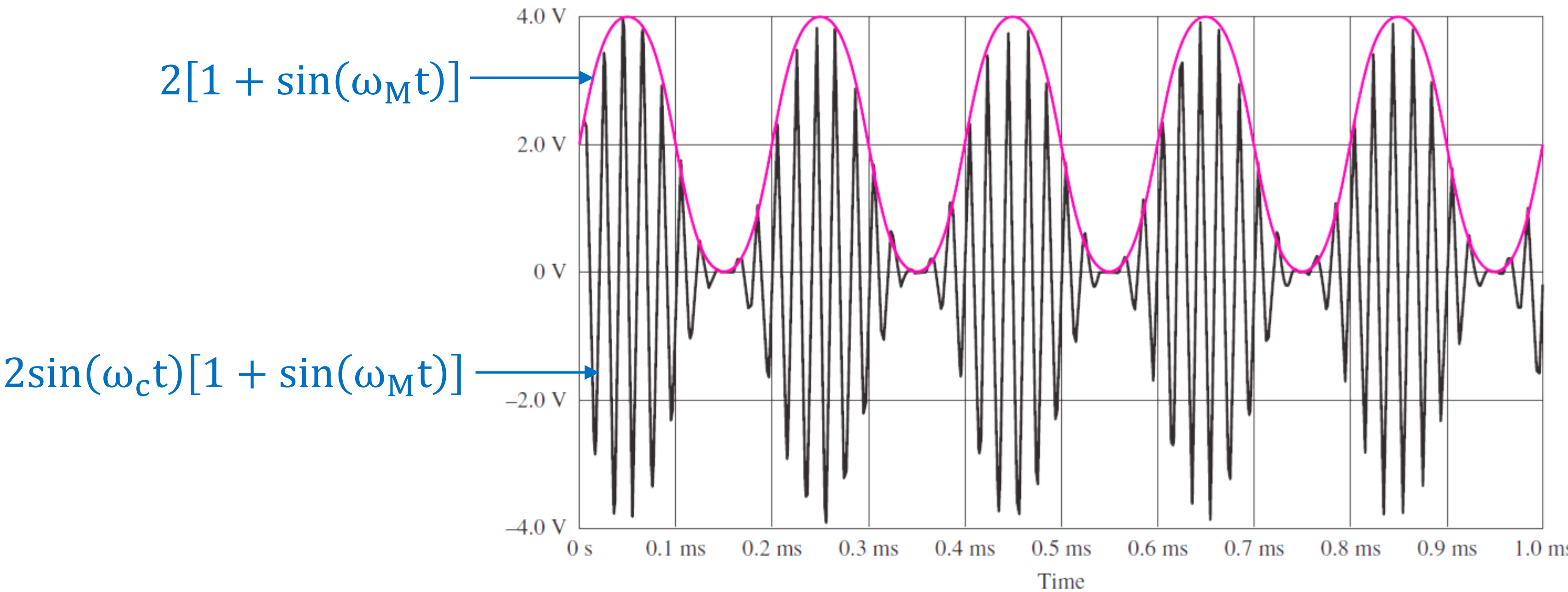
$$I_{peak} = \frac{I_{dc}T}{\Delta T} = \frac{2 \times \frac{1}{60}}{0.352 \times 10^{-3}} = \mathbf{94.7\ (A)}$$

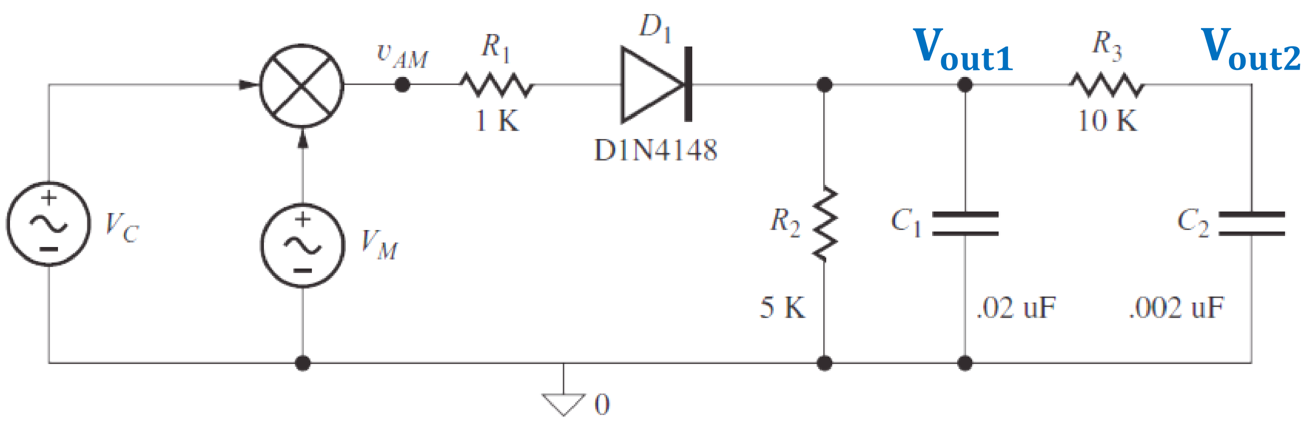
$$I_{surge} = \omega C V_s = 2\pi \times 60 \times 0.111 \times 17 = \mathbf{711\ (A)}$$

Make sure the diodes can handle these large currents

# Example

An amplitude modulated (AM) signal is shown below. The envelope of the AM signal contains the information being transmitted, and the envelope can be recovered using a single half-wave rectifier.



# Example

An amplitude modulated (AM) signal is shown below. The envelope of the AM signal contains the information being transmitted, and the envelope can be recovered using a single half-wave rectifier.