BJT and BJT Circuit

Ve311 Electronic Circuits (Summer 2020)

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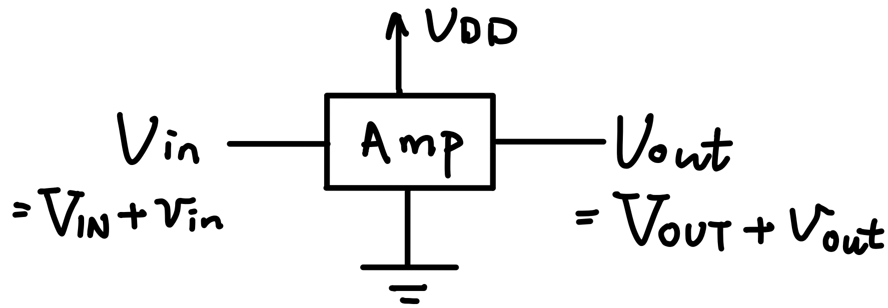
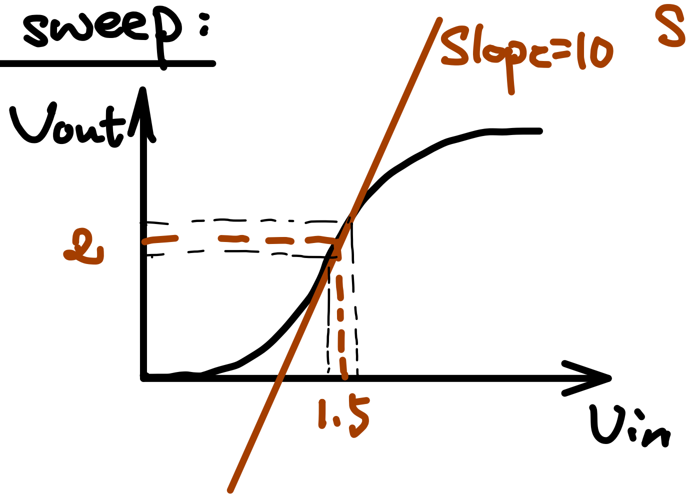
For a general amplifier model:



$$V_{out} = V_{OUT} + A_v \cdot v_{in}$$

DC sweep:

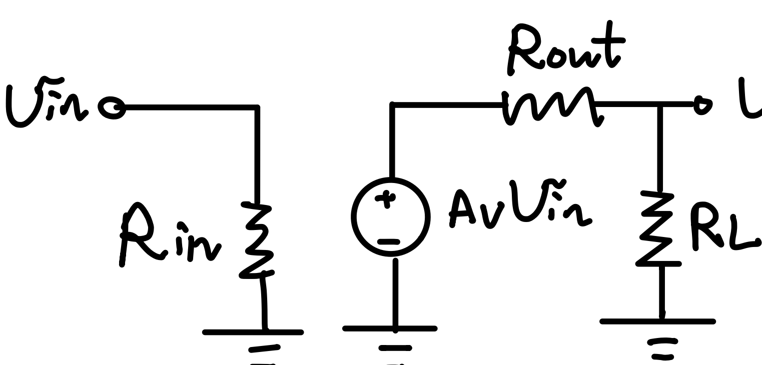


Suppose $V_{in} = 1.5 + \underbrace{0.001\sin(2\pi 60 t)}_{v_{in}}$

$V_{out} = 2 + \underbrace{0.01\sin(2\pi 60 t)}_{v_{out}}$

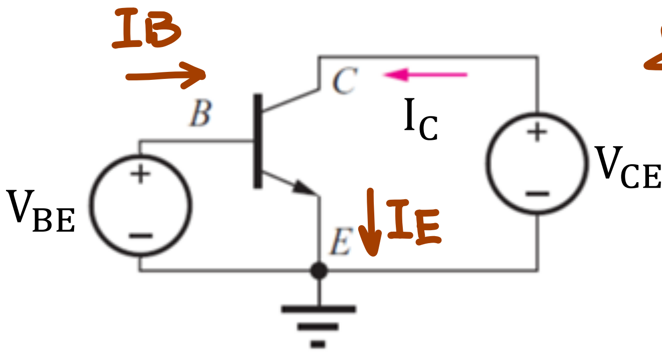
$$A_v = \left.\frac{dV_{out}}{dV_{in}}\right|_{@V_{IN}} = \frac{v_{out}}{v_{in}} = 10$$

Generalized small-signal model: (more important in final exam, no need to understand this deeply for now)



$$V_{out} = A_v \cdot V_{in} \cdot \frac{R_L}{R_{out} + R_L}$$

Summary



$$\boxed{V_{CE} \geq V_{BE}}$$

$\Rightarrow$ **Forward – Active**

Ideal case

$$I_C = I_S\left(e^{\frac{qV_{BE}}{kT}} - 1\right)$$

$$\alpha = \frac{I_C}{I_E} \cong 1$$

$$\beta = \frac{I_C}{I_B} = \frac{\alpha}{1-\alpha}$$

$\Rightarrow$

$$I_C = I_S\left(e^{\frac{qV_{BE}}{kT}} - 1\right)$$

$$\alpha = \frac{I_C}{I_E} = 1$$

$$\beta = \frac{I_C}{I_B} = \infty$$

$I_E = I_B + I_C$

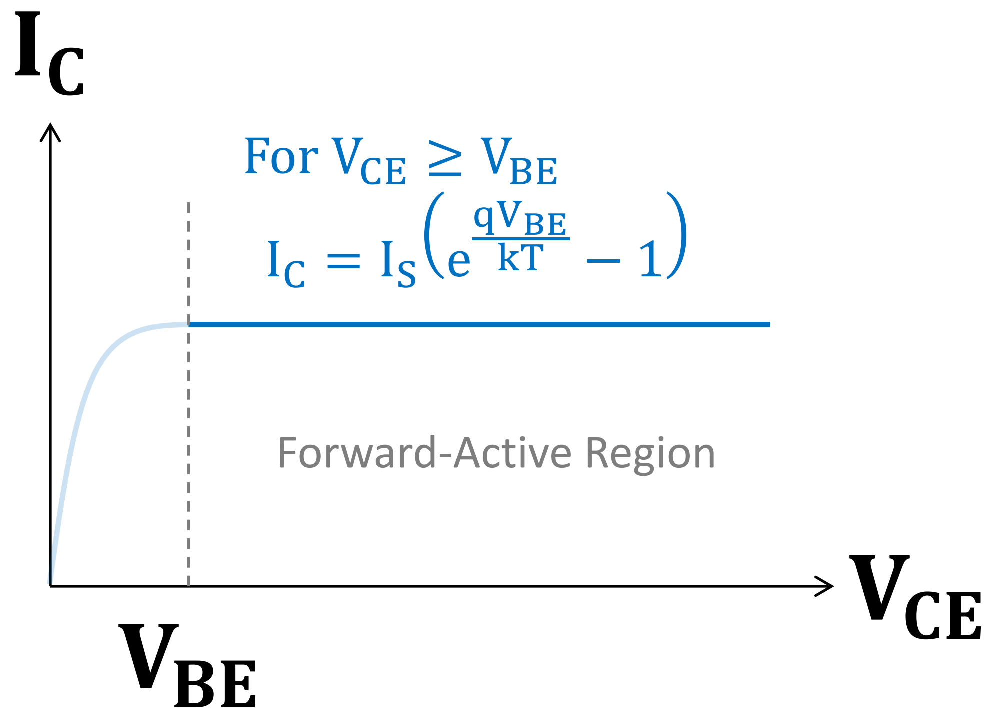
I_S is a constant in the spice model.

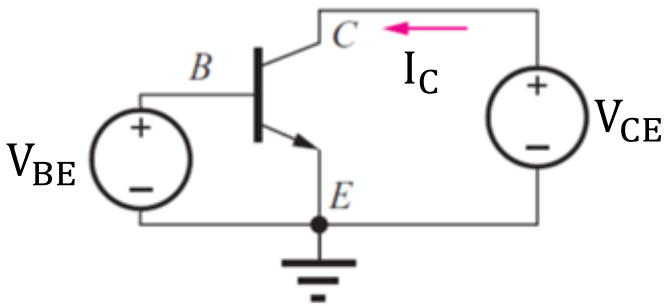
完全理想：门电压 V_{BE} 一定，$I_C = I_E$ 固定
第一种非理想：门漏电
第二种非理想：随着 $V_{CE}\uparrow$，$I_C\uparrow \Rightarrow$ Early effect
集电压对集电流有影响

I_C vs V_{CE} and I_C vs V_{BE} in Forward-Active Region

I_C vs V_{CE} (not considering Early Effect)

At given V_{BE}, DC sweep V_{CE}





I_C vs V_{CE} (considering Early Effect)

At given V_{BE}, DC sweep V_{CE}

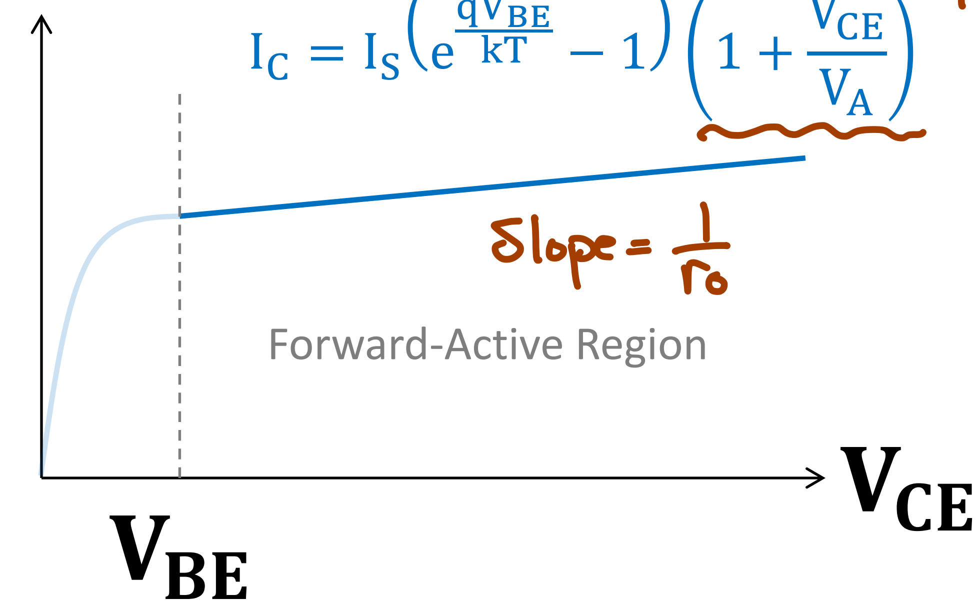
V_A is a constant
$= VAF$ in Pspice model

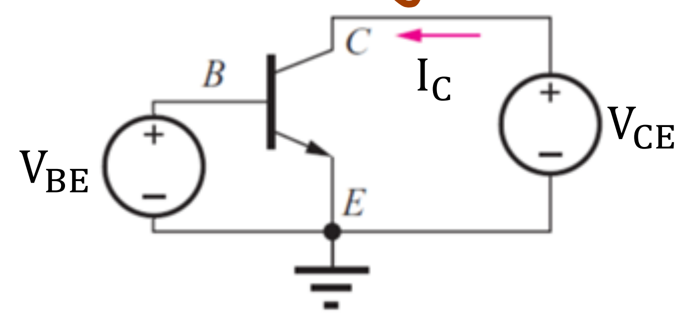
If $V_A \to \infty$,
no early-effect

For $V_{CE} \geq V_{BE}$

$$I_C = I_S\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\underbrace{\left(1 + \frac{V_{CE}}{V_A}\right)}$$

Slope $= \frac{1}{r_o}$





V_A is a constant in the spice model.

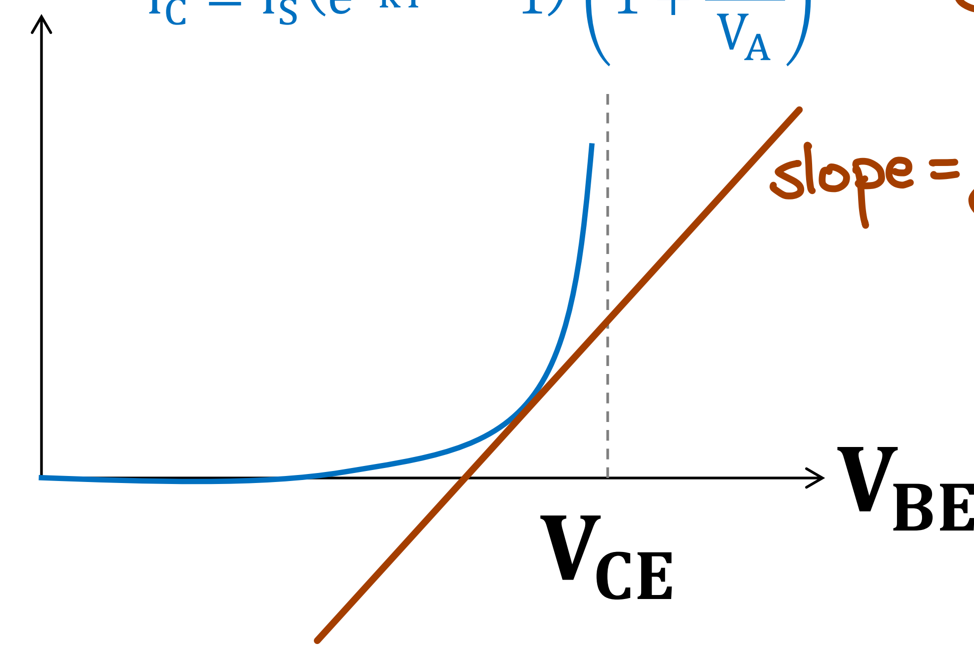
I_C vs V_{BE}

At given V_{CE}, DC sweep V_{BE}

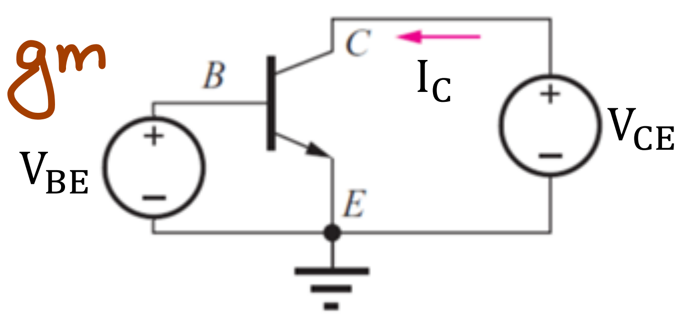
For $V_{CE} \geq V_{BE}$

$$I_C = I_S\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right)$$

$$g_m = \frac{dI_C}{dV_{BE}} \cong \frac{I_C}{kT/q}$$



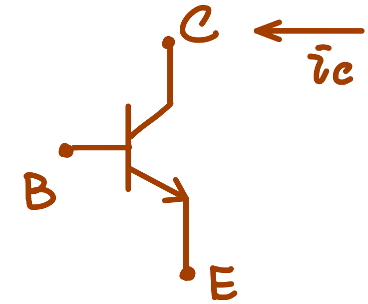
slope = g_m



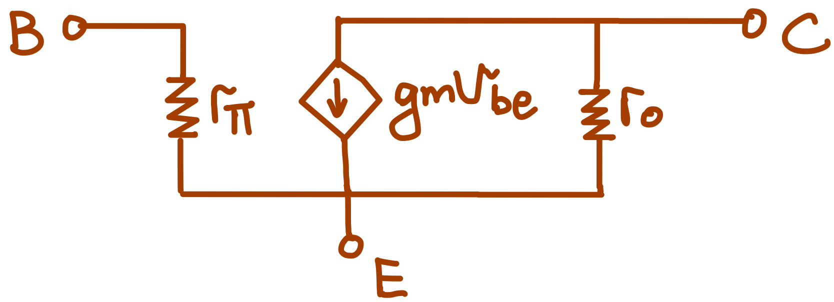
Small-Signal Model

Conclusion first:

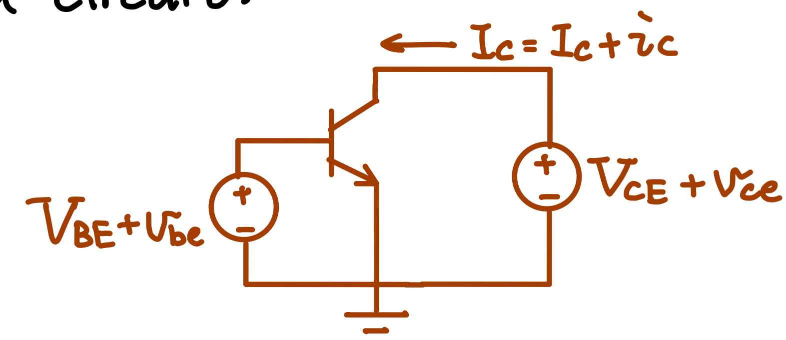
For a BJT as :



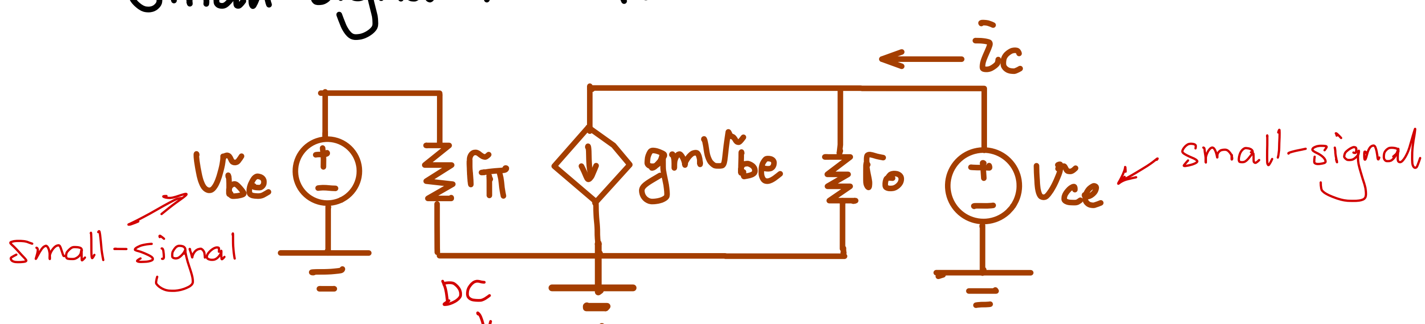
Its small signal model is :



Typical circuit:



Small signal model:



用DC求

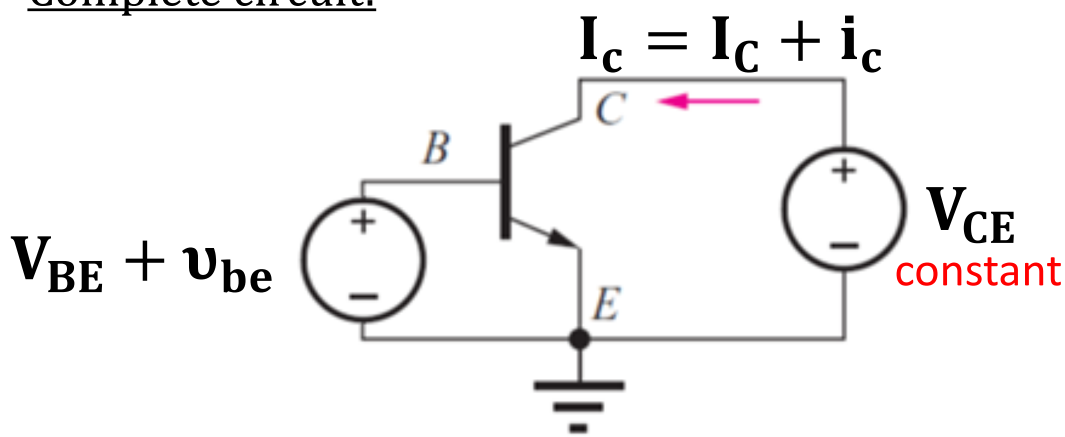
$$
\begin{cases}
I_c = I_s\left(e^{\frac{q\overset{\text{DC}}{V_{BE}}}{kT}} - 1\right)\left(1 + \frac{\overset{\text{DC}}{V_{CE}}}{V_A}\right) \\
g_m \simeq \frac{I_c}{kT/q} \\
r_o \simeq \frac{V_A}{I_c} \quad \leftarrow \text{VAF in Pspice model} \\
r_\pi = \frac{\beta}{g_m} \quad \leftarrow \text{BF in Pspice model}
\end{cases}
$$

If no early-effect.

$V_A \to \infty$, $r_o \to \infty$

Hybrid-π Model (how to get gm and r_π)

Complete circuit:

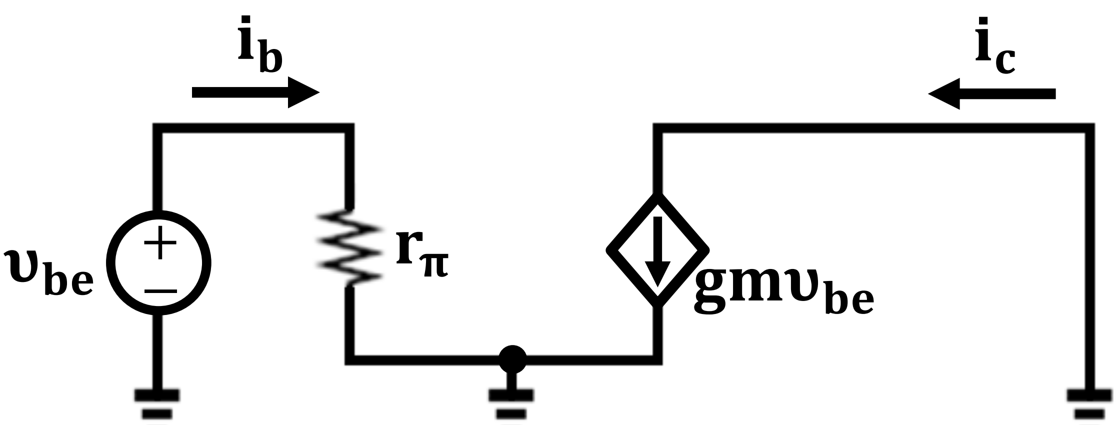


$$V_{CE} \geq V_{BE}$$

$\Rightarrow$ Forward – Active

$$\boxed{I_C = I_S\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right)}$$

Small-signal circuit:

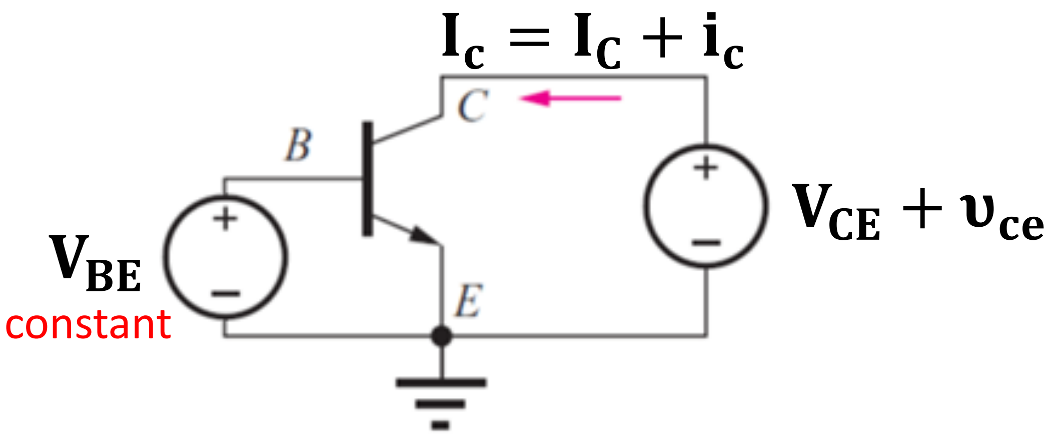


$$r_\pi = \frac{dV_{BE}}{dI_B} = \frac{1}{\frac{dI_C}{\beta dV_{BE}}} = \frac{1}{\frac{gm}{\beta}} = \frac{\beta}{gm}$$

$$gm = \frac{dI_C}{dV_{BE}} \cong \frac{I_C}{kT/q}$$

Hybrid-π Model (how to get r_o)

Complete circuit:

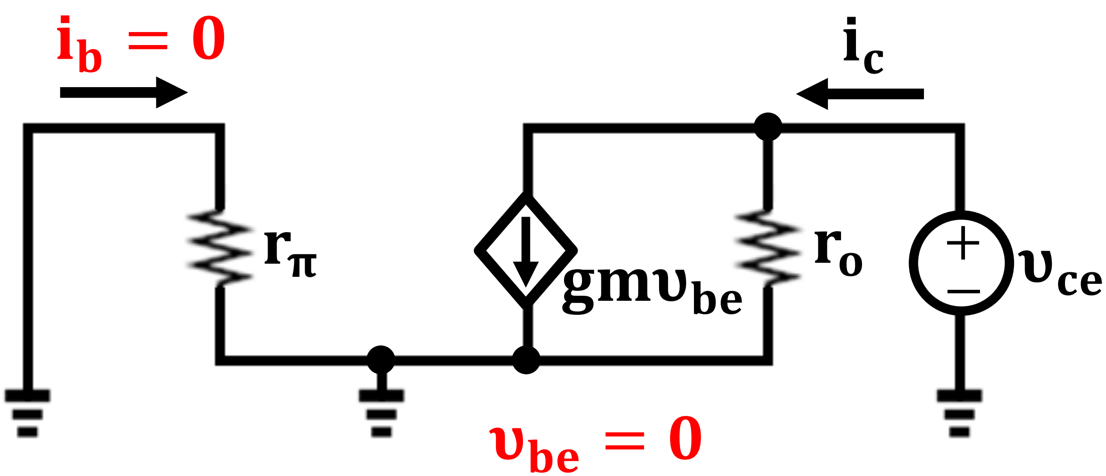


$V_{CE} \geq V_{BE}$

$\Rightarrow$ Forward – Active

$$\boxed{I_C = I_S\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right)}$$

Small-signal circuit:



$$r_\pi = \frac{1}{\frac{dI_B}{dV_{BE}}} = \frac{1}{\frac{dI_C}{\beta dV_{BE}}} = \frac{1}{\frac{gm}{\beta}} = \frac{\beta}{gm}$$

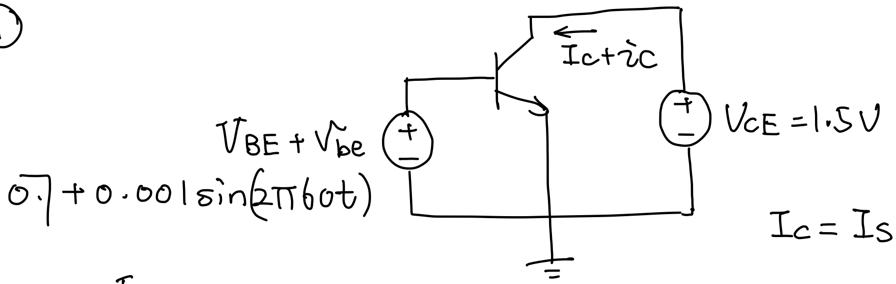
$$gm = \frac{dI_C}{dV_{BE}} \cong \frac{I_C}{kT/q}$$

$$r_o = \frac{1}{\frac{dI_C}{dV_{CE}}} \cong \frac{V_A}{I_C}$$

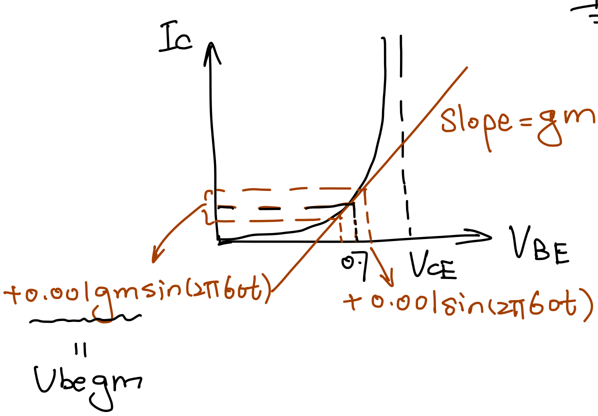
Note: Why do we have r_o, r_π, g_m in small signal model?

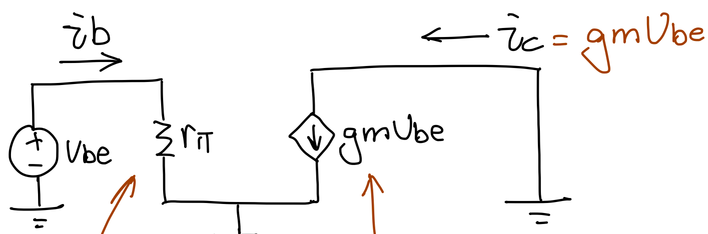
(Skip this page if you already understand)

①



$$I_c = I_s\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right)$$





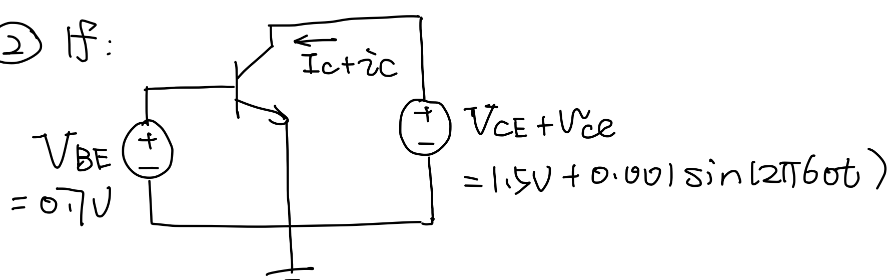
So, here's a dependent current source that simulates $i_c = g_m v_{be}$

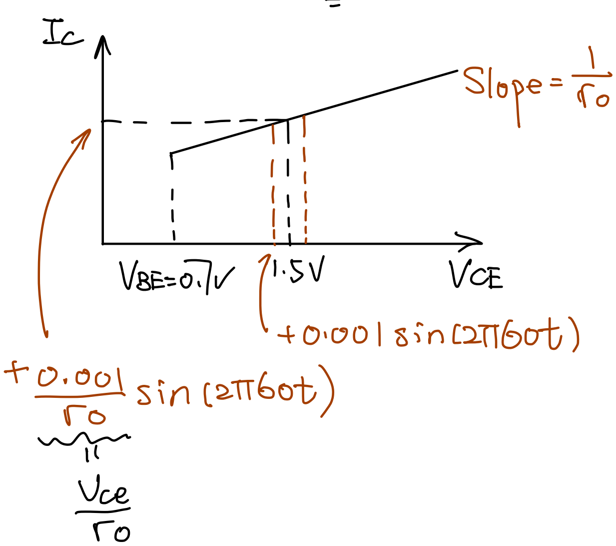
Since $\beta = \frac{I_c}{I_B} = 100$

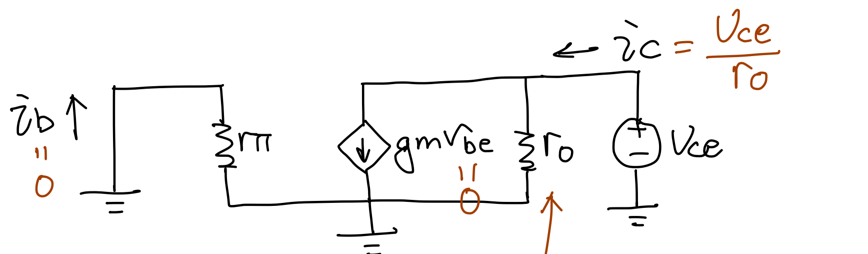
If $v_{be} \uparrow \Rightarrow i_c \uparrow \Rightarrow i_B \uparrow$

"feels like" r_π exists

② If:





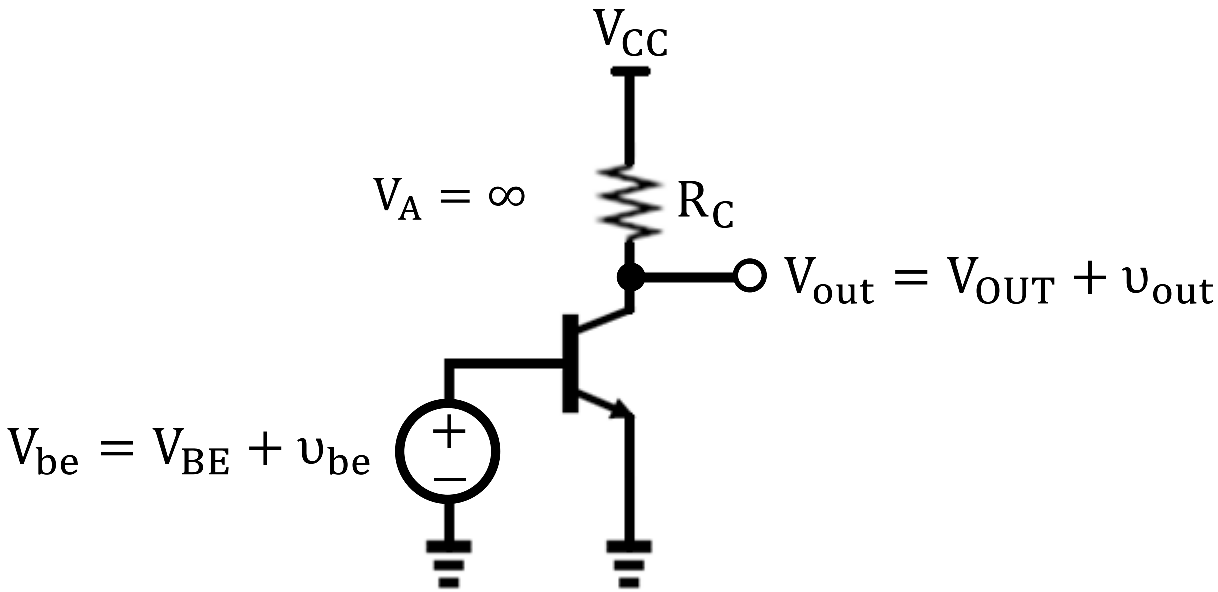


since $i_c = \frac{v_{ce}}{r_o}$

"feels like" r_o exists here

Common-Emitter Amplifier

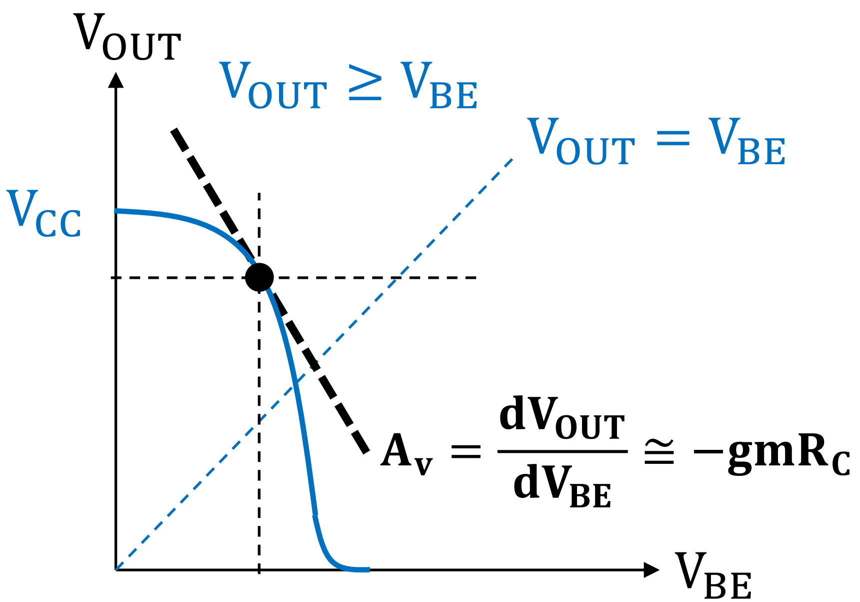
Common-Emitter Amplifier ($V_A = \infty$)



- **DC Analysis**

$$V_{OUT} = V_{CC} - I_C R_C$$
$$= V_{CC} - \frac{AqD_n n_i^2}{N_a W_B}\left(e^{\frac{qV_{BE}}{kT}} - 1\right)R_C$$

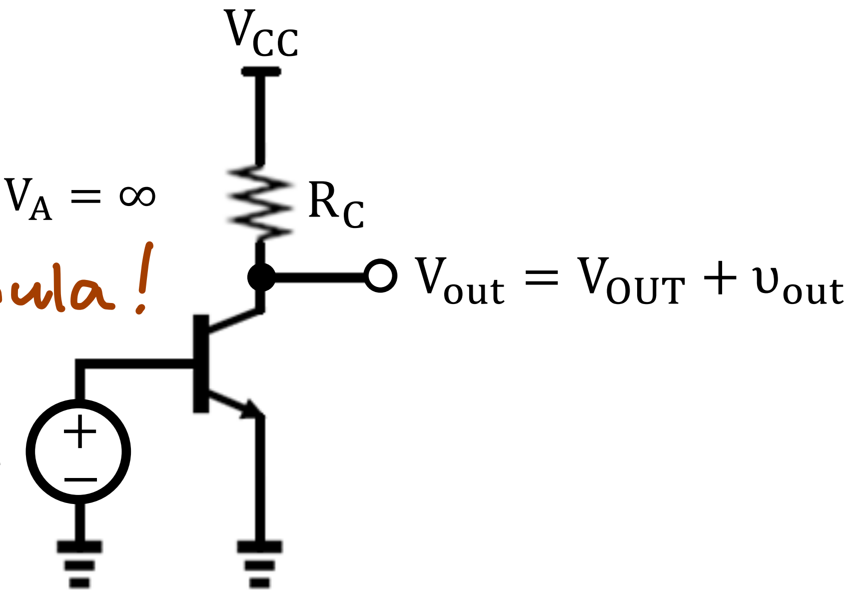
$$\boxed{A_v = \frac{dV_{OUT}}{dV_{BE}} \cong -\frac{I_C}{kT/q}R_C = -gmR_C}$$



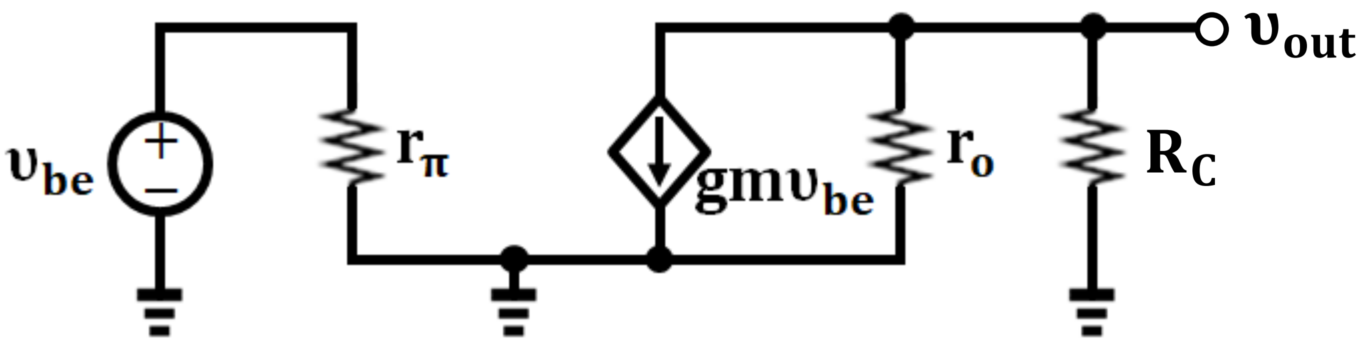
Common-Emitter Amplifier ($V_A = \infty$)

$A_v = \frac{dV_{OUT}}{dV_{BE}} = \boxed{\frac{v_{out}}{v_{be}}}$

Use small-signal model to calculate Av's formula!

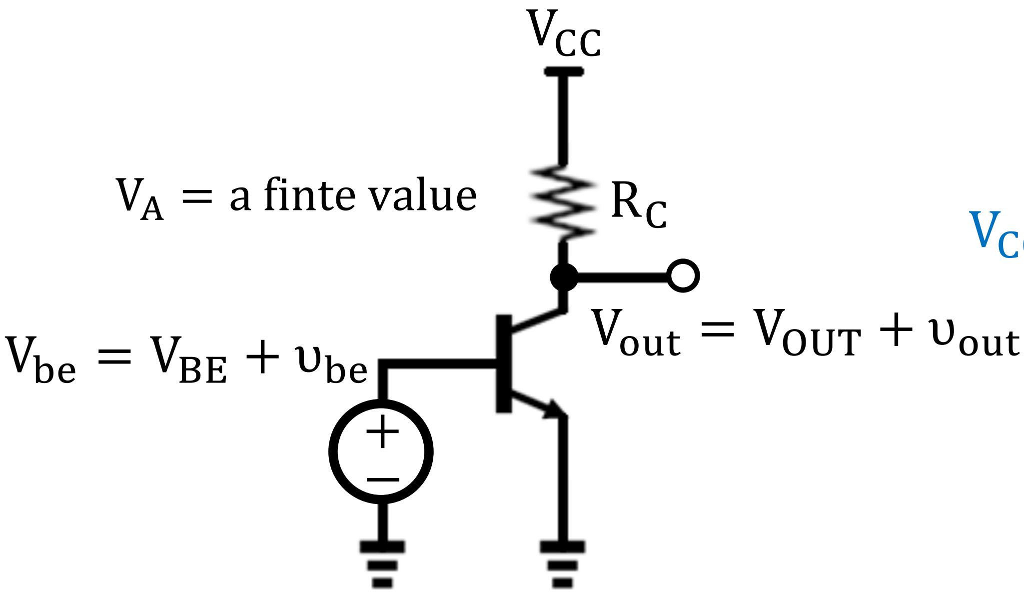


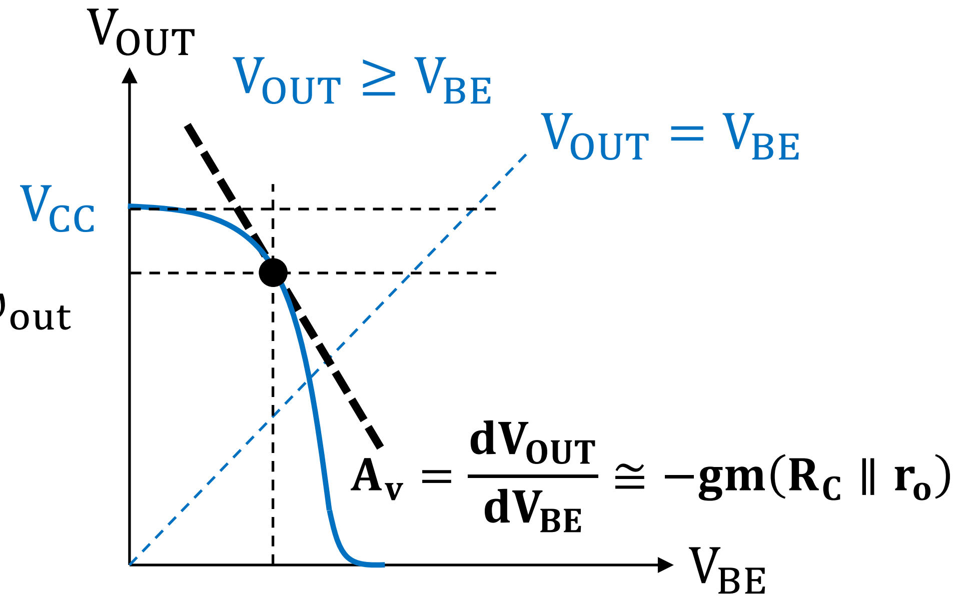
- **Small-Signal Analysis**



$$\boxed{A_v = \frac{\upsilon_{out}}{\upsilon_{be}} = -gm(R_C \parallel r_o) = -gmR_C}$$ (since $r_o = \infty$)

Common-Emitter Amplifier (V_A = a finite value)





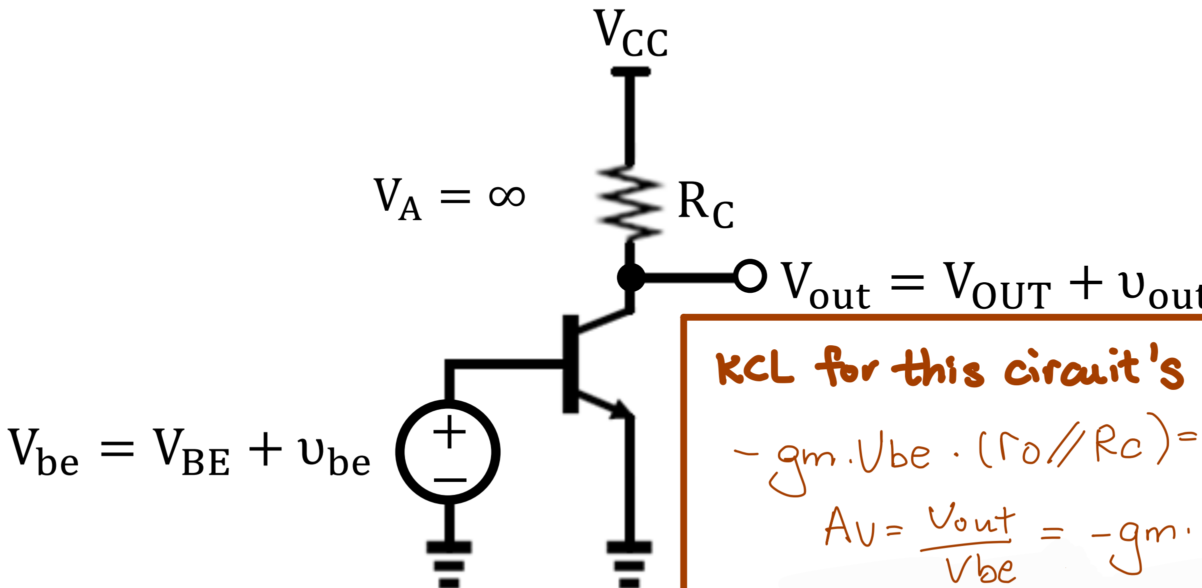
- **DC Analysis**

$$V_{OUT} = V_{CC} - I_C R_C = V_{CC} - I_S\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\left(1 + \frac{V_{OUT}}{V_A}\right)R_C$$

$$\frac{dV_{OUT}}{dV_{BE}} = -\frac{q}{kT} I_S e^{\frac{qV_{BE}}{kT}}\left(1 + \frac{V_{OUT}}{V_A}\right)R_C - I_S\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\frac{1}{V_A}\frac{dV_{OUT}}{dV_{BE}}R_C \cong -gmR_C - \frac{1}{r_o}\frac{dV_{OUT}}{dV_{BE}}R_C$$

$$\boxed{A_v = \frac{dV_{OUT}}{dV_{BE}} \cong -gm(R_C \parallel r_o)}$$

Common-Emitter Amplifier (V_A = a finite value)

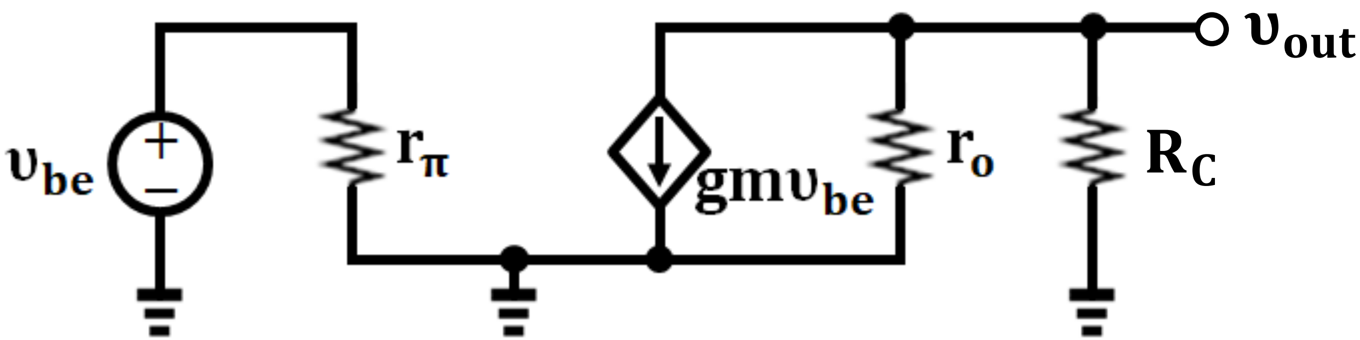


KCL for this circuit's Av:

$-g_m \cdot V_{be} \cdot (r_o // R_c) = V_{out}$

$A_v = \frac{V_{out}}{V_{be}} = -g_m \cdot (r_o // R_c)$

- Small-Signal Analysis



对不同circuit求Av用
KCL分析small-signal
circuit

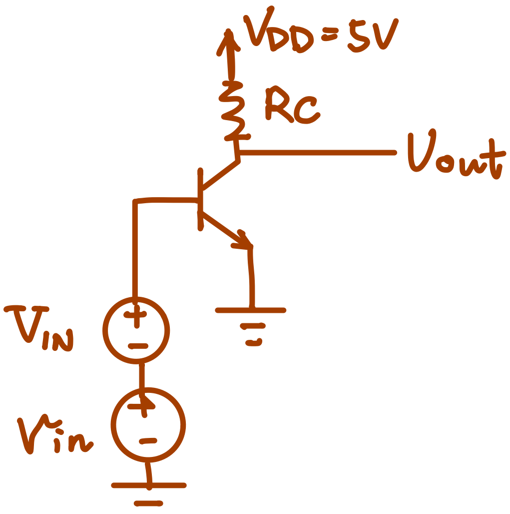
$$\boxed{A_v = \frac{\upsilon_{out}}{\upsilon_{be}} = -g_m(R_C \parallel r_o)}$$

Convert to small-signal model:

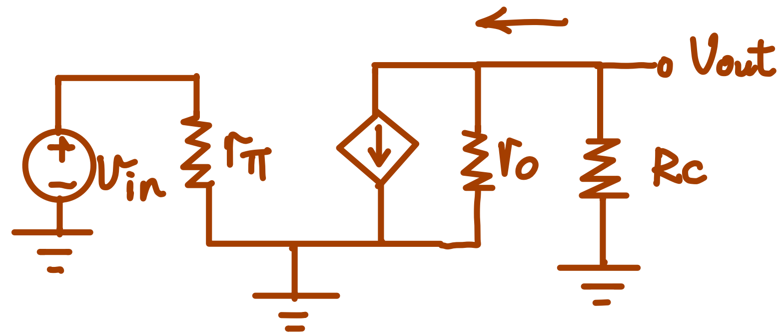
- DC voltage → short to small-signal ground
- DC current → open circuit
- R → still R
- capacitor → short circuit

Ex:

Complete circuit:

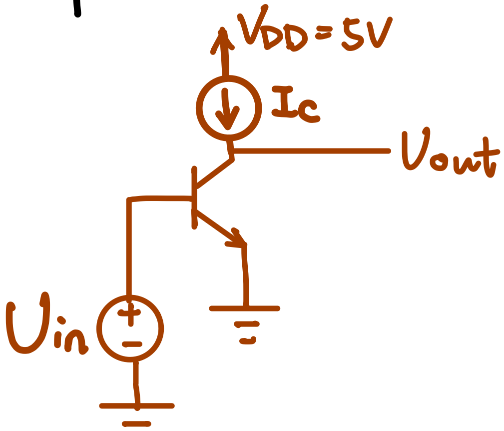


Small-signal:

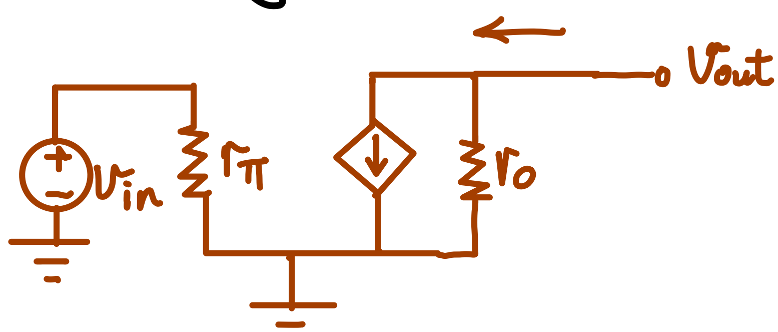


$$A_v = \frac{v_{out}}{v_{in}} = -g_m(r_o // R_C)$$

Complete circuit:

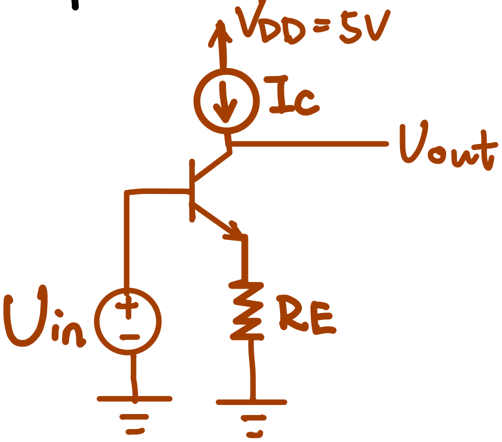


Small-signal:

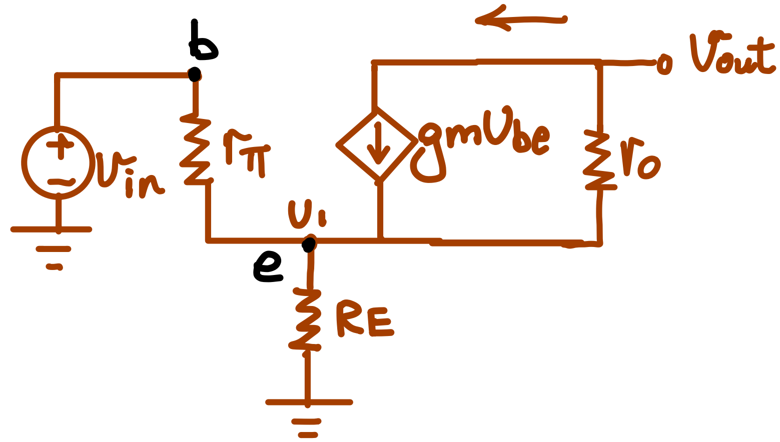


$$A_v = \frac{v_{out}}{v_{in}} = -g_m r_o$$

Complete circuit:



* Small-signal:



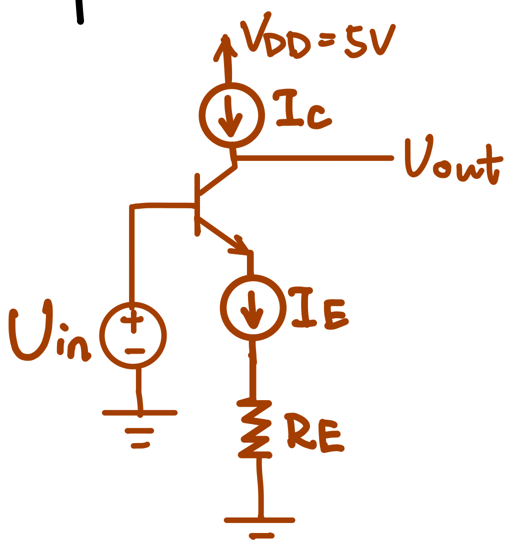
* mind Vbe used in gmVbe doesn't necessarily equals to Vin !

KCL:

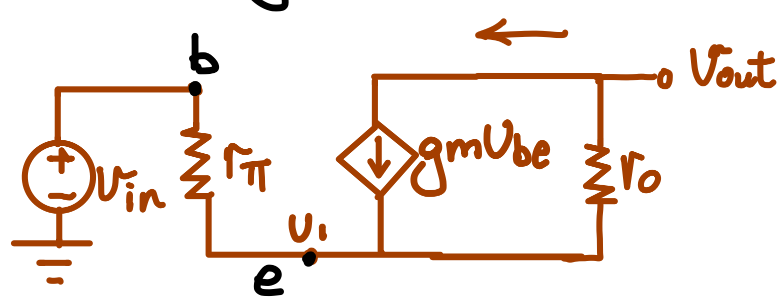
$$\begin{cases} \frac{V_1}{R_E} = \frac{V_{in}-V_1}{r_\pi} + g_m V_{be} + \frac{V_{out}-V_1}{r_o} \\ V_{be} = V_{in} - V_1 \\ g_m V_{be} + \frac{V_{out}-V_1}{r_o} = 0 \end{cases}$$

$\Rightarrow$ Calculate $A_V = \frac{V_{out}}{V_{in}}$

Complete circuit:



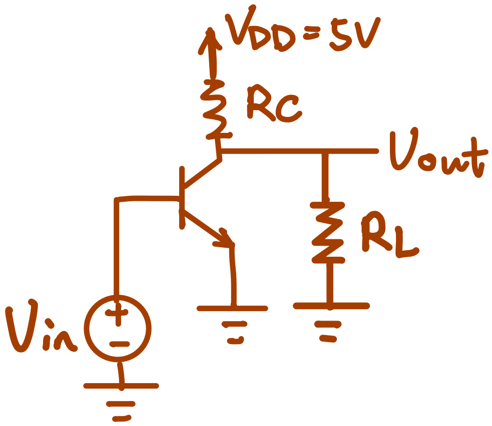
Small-signal:



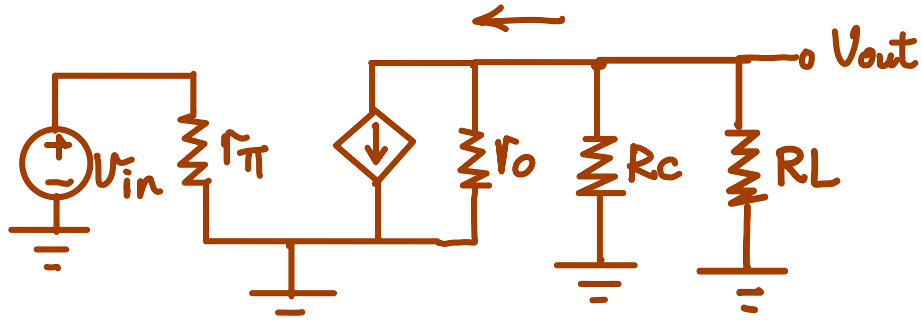
open circuit here

$$A_V = \frac{V_{out}}{V_{in}} = \frac{V_{in}}{V_{in}} = 1$$

Complete circuit:

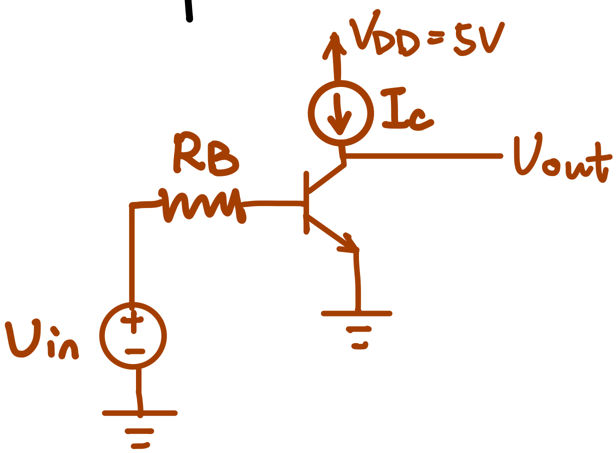


Small-signal:

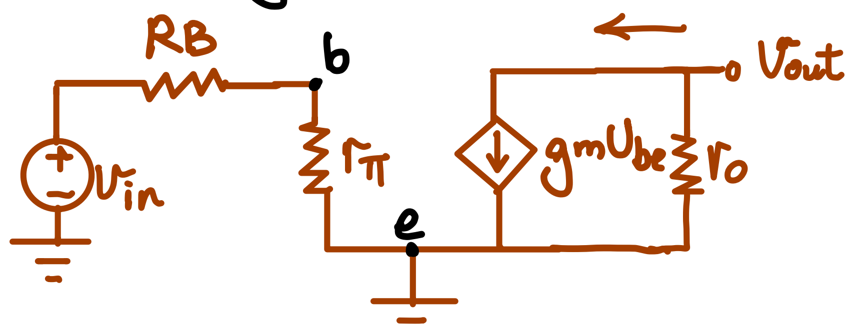


$$A_v = \frac{v_{out}}{v_{in}} = -g_m(r_o // R_C // R_L)$$

Complete circuit:



Small-signal:

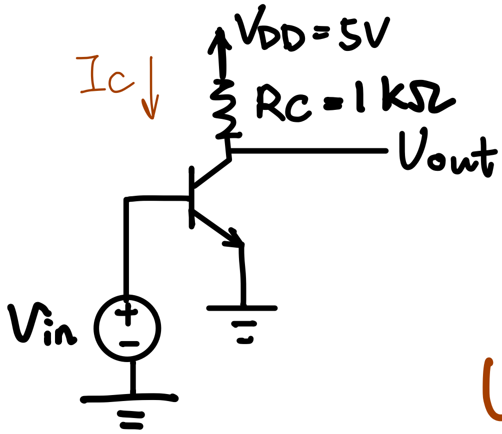


KCL: $v_{be} = v_{in} \frac{r_\pi}{r_\pi + R_B}$

$$A_v = -g_m r_o \cdot \frac{r_\pi}{r_\pi + R_B}$$

* What if there's a diode connected ?

A typical complete question for BJT common-emitter :



Given $V_{in} = 0.5 + 0.001 \sin(2\pi 60 t)$

plot the waveform of V_{out}

(.model VAF = 100)

$$v_{out} = V_{OUT} + A_v \cdot 0.001 \sin(2\pi 60 t)$$

Step 1: Find V_{OUT} (DC part of v_{out})

$$V_{OUT} = 5 - I_C \cdot (1K)$$

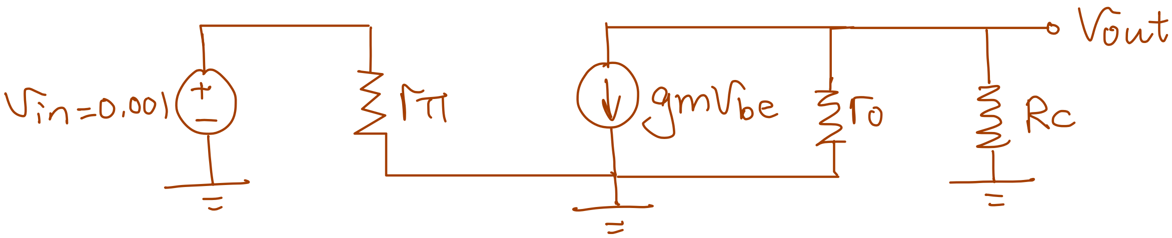
Plug in $I_C = I_S \cdot \left(e^{q\frac{V_{BE}}{KT}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right)$ $\quad * \frac{KT}{q} \simeq 0.026$

$$\Rightarrow V_{OUT} = 5 - (1 \times 10^{-18}) \times \left(e^{\frac{0.5}{0.026}} - 1\right)\left(1 + \frac{V_{OUT}}{100}\right) \cdot (1K)$$

Solve the Eq. $\Rightarrow V_{OUT} \simeq 5V$

$*$ Check if $V_{out} > V_{BE}$

Step 2: Use small-signal circuit to find A_v



$$A_v = \frac{v_{out}}{v_{in}} = -g_m (r_o // R_C)$$

(If $V_A \to \infty$, no this term. $A_v = -g_m R_C$)

Step 3: find I_c, g_m, r_o, r_π if needed in A_v

$$I_c = I_S \cdot \left(e^{\frac{qV_{BE}}{KT}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right)$$

plug in V_{OUT} calculated in Step 1

$$\Rightarrow I_c = 2.36 \times 10^{-10}\ A$$

$$\Rightarrow g_m = \frac{I_c}{KT/q} = 9.079 \times 10^{-9}$$

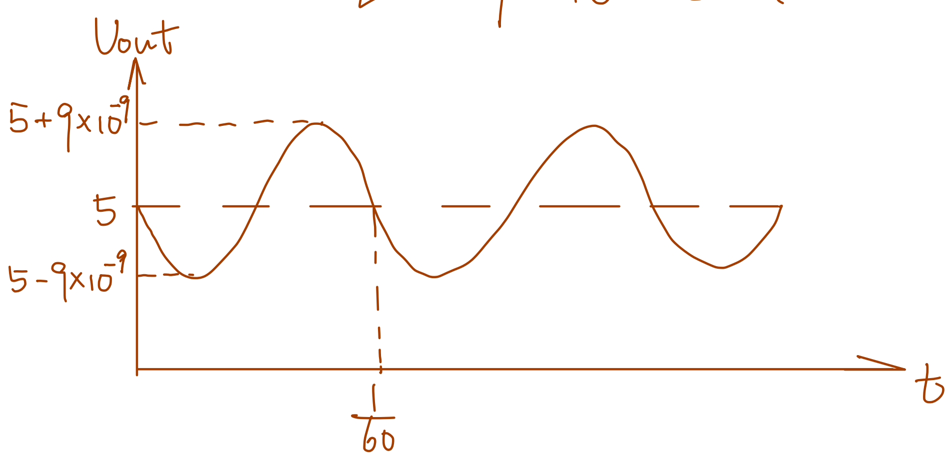
$$\Rightarrow r_o = \frac{V_A}{I_c} = \frac{100}{I_c} = 4.24 \times 10^{11}\ \Omega$$

$$A_v = -g_m(r_o // R_c) \cong -9 \times 10^{-6}$$

Step 4: Combine all to obtain V_{out}

$$V_{out} = V_{OUT} + A_v \cdot 0.001 \sin(2\pi 60 t)$$

$$= 5 - 9 \times 10^{-9} \sin(2\pi 60 t)$$



npn BJT Pspice Model

.model Qbreakn NPN IS=1e-18 BF=100 VAF=100

$$I_C = \mathbf{IS}\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\left(1 + \frac{V_{CE}}{\mathbf{VAF}}\right)$$

$$\mathbf{BF} = \frac{I_C}{I_B} \qquad I_E = I_C + I_B$$

$$gm \cong \frac{I_C}{kT/q} \qquad r_\pi = \frac{\mathbf{BF}}{gm} \qquad r_o \cong \frac{\mathbf{VAF}}{I_C}$$