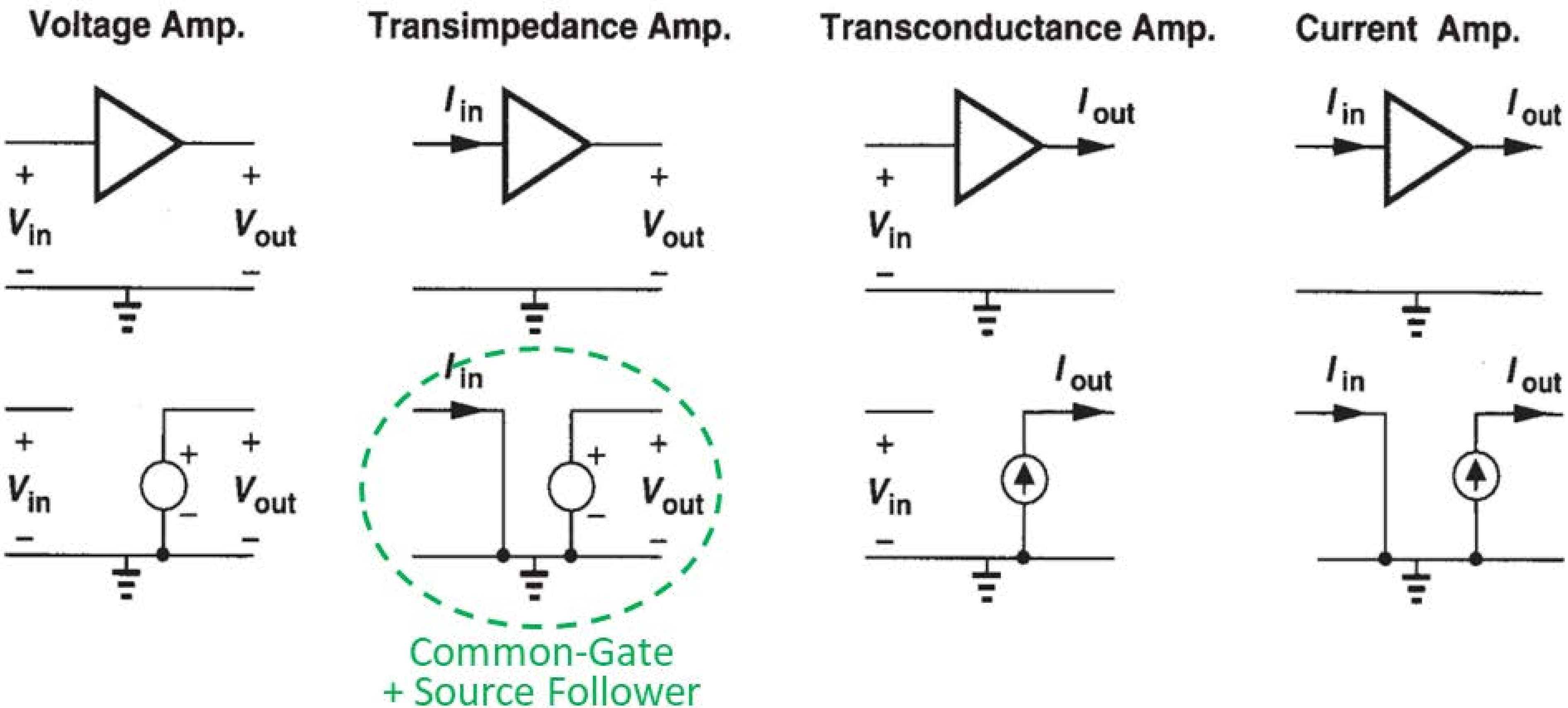
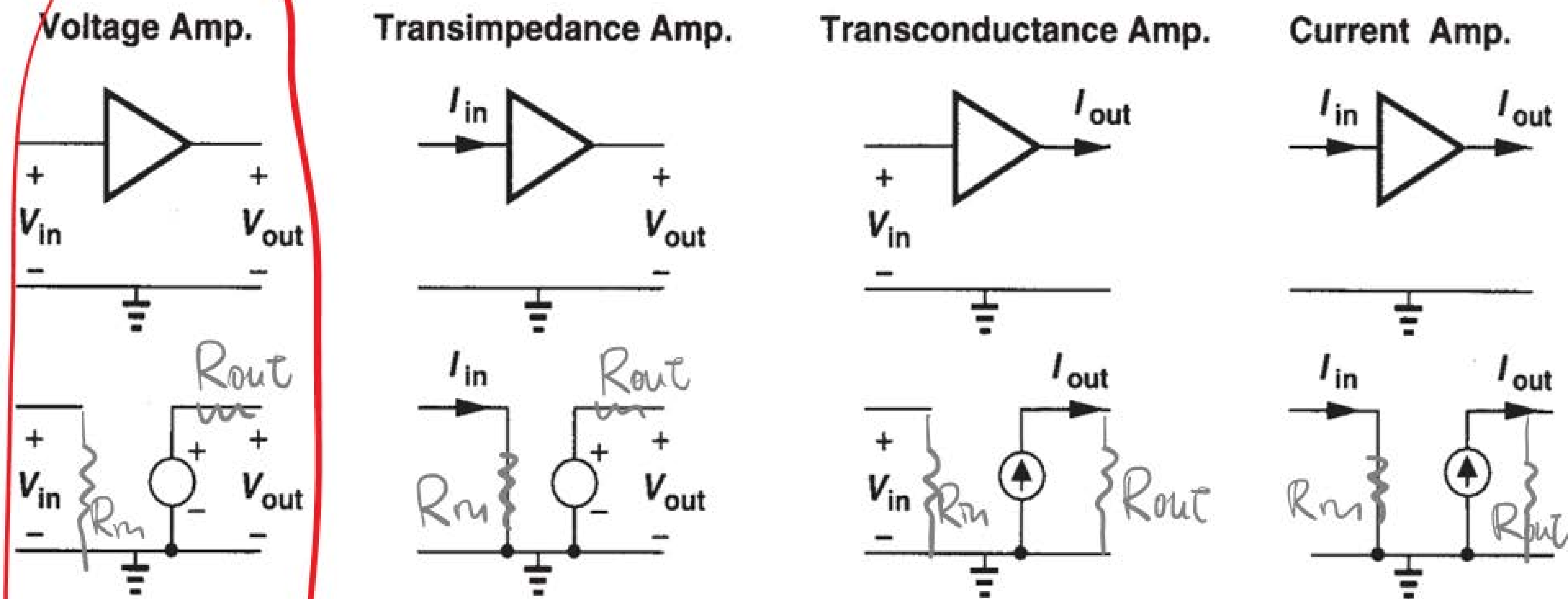
Ideal Amplifier



- For converting and amplifying small-signal current to voltages, common-gate provides **low input impedance** and **moderate gain**, but relatively **large output impedance**.

NOT Ideal Amplifier



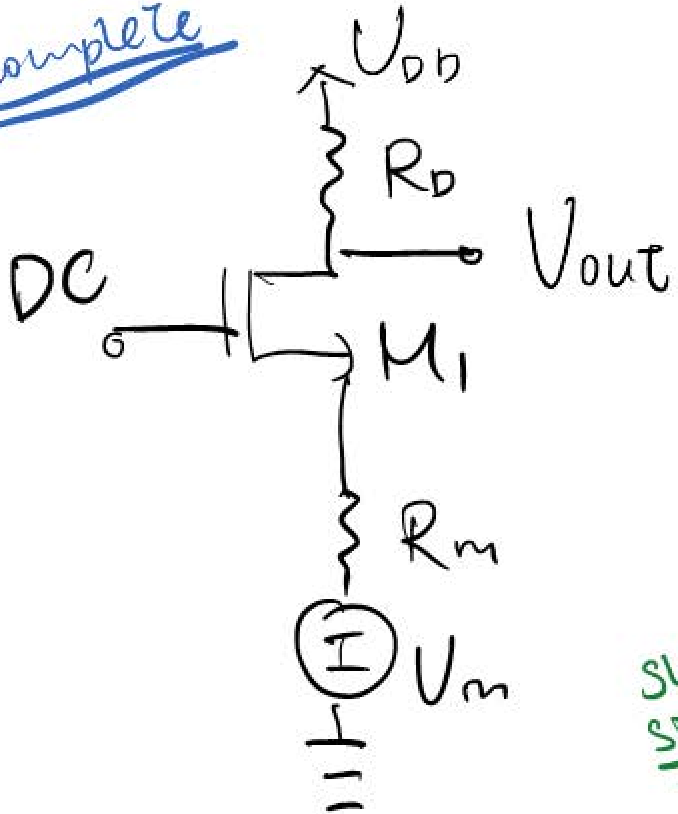
Voltage Amp.: $R_{in} \to \infty$, $R_{out} \to 0$

Transimpedance Amp.: $R_{in} \to 0$, $R_{out} \to 0$

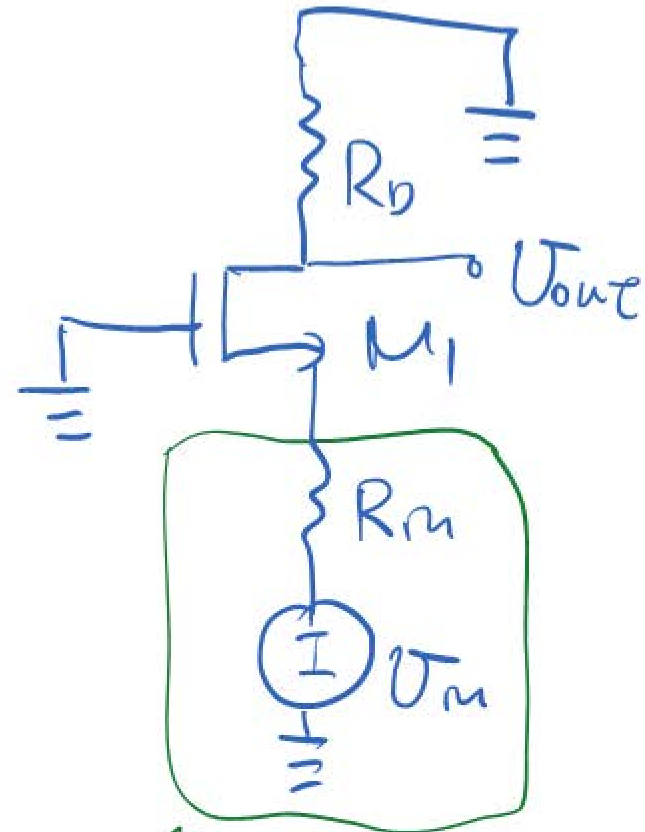
Transconductance Amp.: $R_{in} \to \infty$, $R_{out} \to \infty$

Current Amp.: $R_{in} \to 0$, $R_{out} \to \infty$

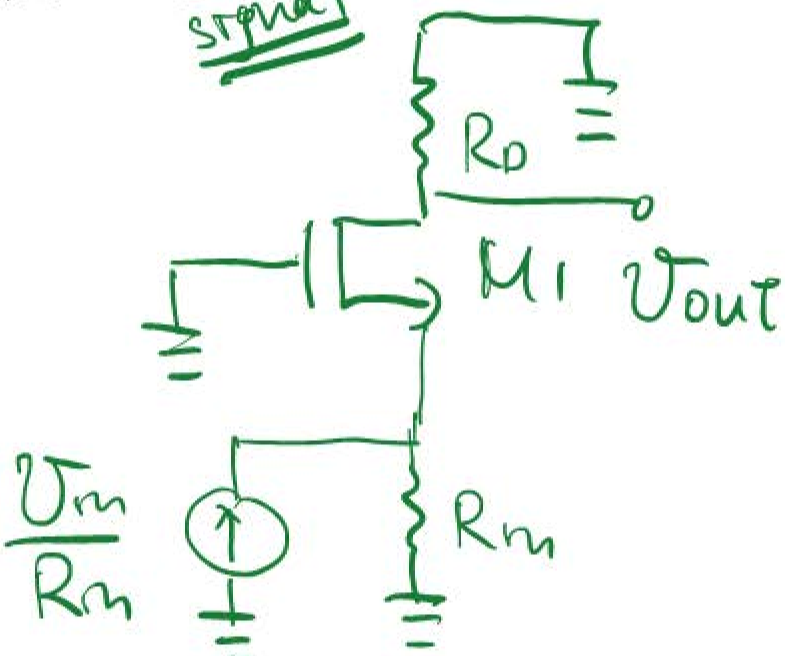
complete



small signal

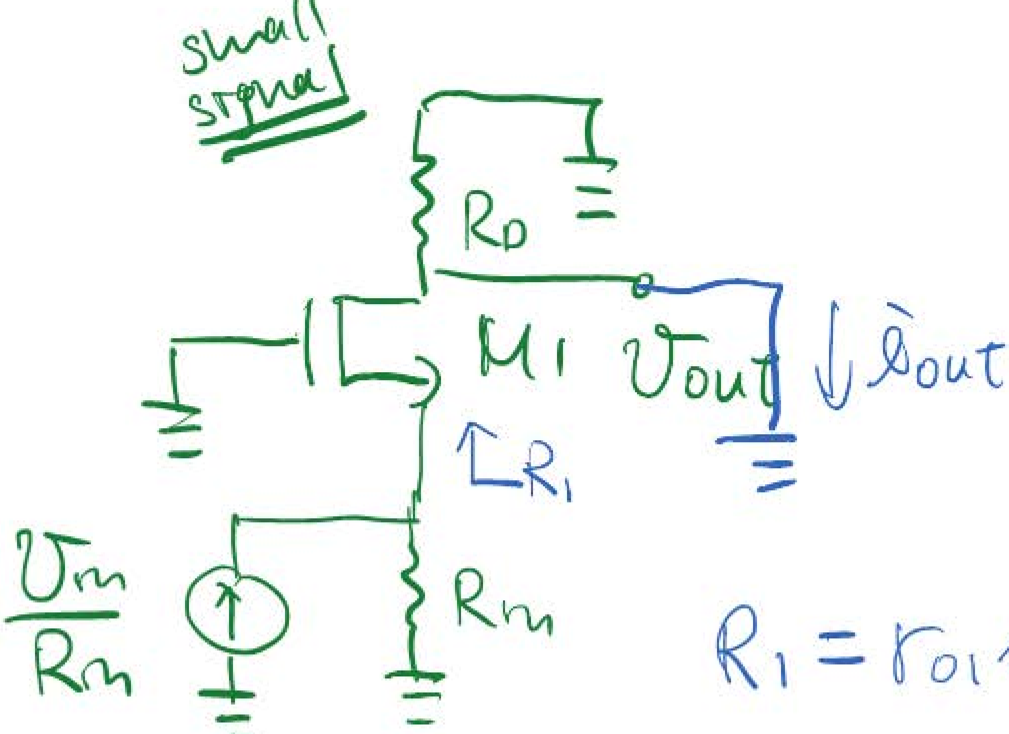


small signal



When calculating $G_m = ?$

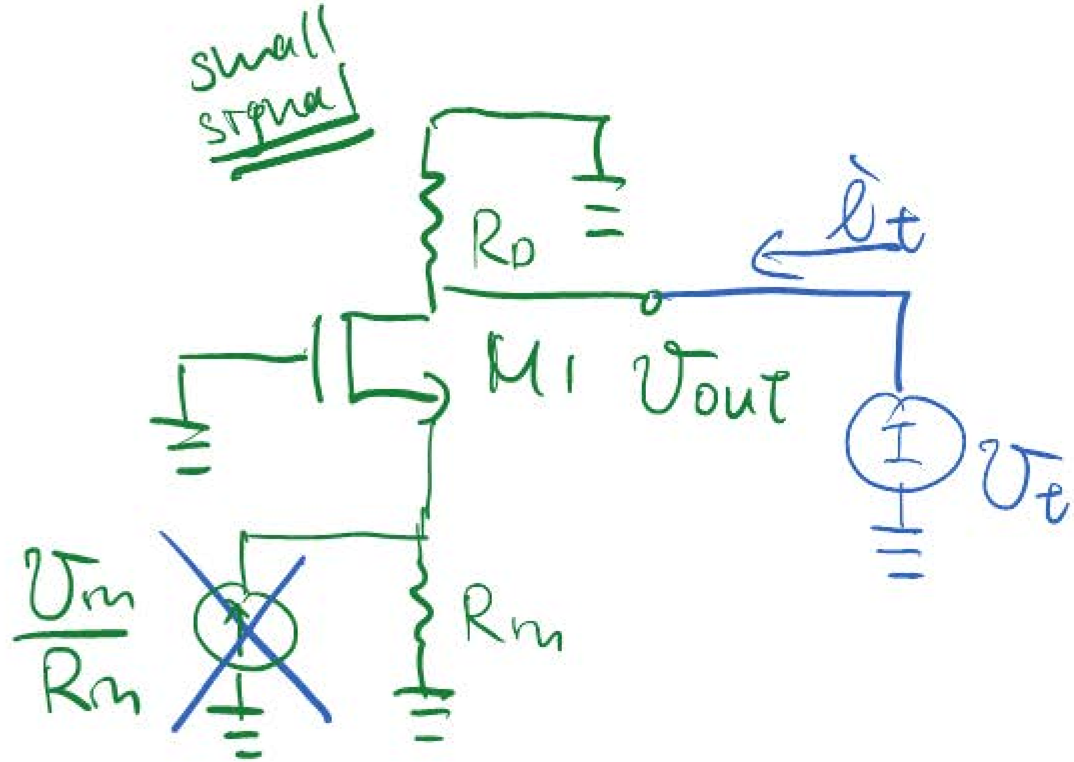
$$i_{out} = \frac{V_{in}}{\cancel{R_{in}}} \cdot \frac{\cancel{R_{in}}}{R_1 + R_{in}} = V_{in} \frac{1}{\left(r_{o1} \parallel \frac{1}{g_{m1}+g_{mb1}}\right) + R_{in}}$$



$$R_1 = r_{o1} \parallel \frac{1}{g_{m1}+g_{mb1}}$$

When calculating Rout = ?

$$R_{out} = \frac{v_t}{i_t} = R_D \,//\, \left[r_{o1} + R_{in} + (g_{m1} + g_{mb1}) r_{o1} R_{in} \right]$$



Common-Gate

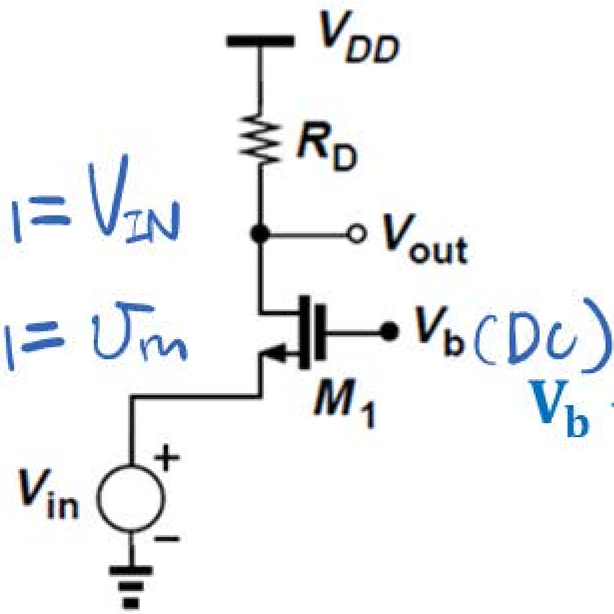
DC Analysis

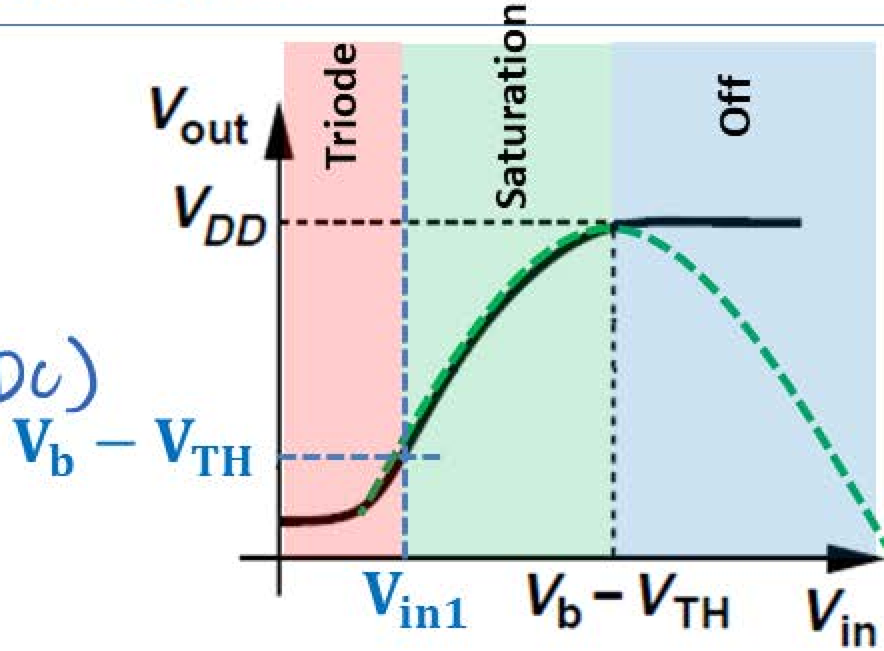
$\lambda = 0$

$\gamma \neq 0$

$V_{SB1} = V_{IN}$

$v_{sb1} = v_{in}$





$$V_{out} = V_b - V_{TH}$$
$$= V_{DD} - R_D \frac{1}{2}\mu_n C_{ox}\frac{W}{L_{eff}}(V_b - V_{in1} - V_{TH})^2$$

- **$V_{in} > V_b - V_{TH}$ → M_1 Off**

  $$V_{out} = V_{DD}$$

- **$V_b - V_{TH} > V_{in} > V_{in1}$ → M_1 in Saturation**

  $$V_{out} = V_{DD} - R_D \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_b - V_{in} - V_{TH})^2$$

- **$V_{in} < V_{in1}$ → M_1 in Triode**

  $$V_{out} = V_{DD} - R_D \mu_n C_{ox}\frac{W}{L}\left[(V_b - V_{in} - V_{TH})(V_{out} - V_{in}) - \frac{1}{2}(V_{out} - V_{in})^2\right]$$

Common-Gate

DC Analysis

$\lambda = 0$

$\gamma \neq 0$

- **$V_b - V_{TH} > V_{in} > V_{in1} \rightarrow M_1$ in Saturation**

$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_b - V_{in} - V_{TH})^2$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} 2 (V_b - V_{in} - V_{TH}) \left(-1 - \frac{\partial V_{TH}}{\partial V_{in}}\right)$$

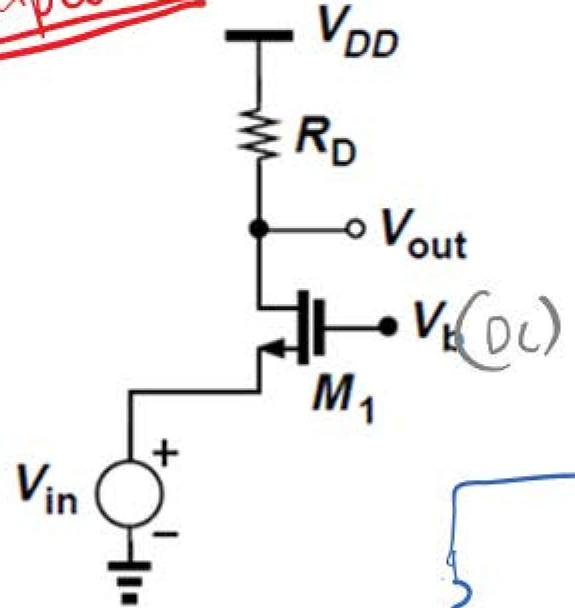
$$= R_D \underbrace{\boxed{\mu_n C_{ox} \frac{W}{L} (V_b - V_{in} - V_{TH})}}_{= gm} \left(1 + \underbrace{\frac{\partial V_{TH}}{\partial V_{in}}}_{= \eta = \frac{\gamma}{2\sqrt{2\Phi_F + V_{SB}}}}\right)$$

$V_{in} = V_{SB}$

$$\boxed{A_v = \frac{\partial V_{out}}{\partial V_{in}} = R_D gm (1 + \eta)} = R_D (g_m + g_{mb})$$

- gm is a function of I_D and η is a function of V_{SB}.
- A_v is not quite linear.

complete



Assume $\lambda = 0$, $\gamma \neq 0$

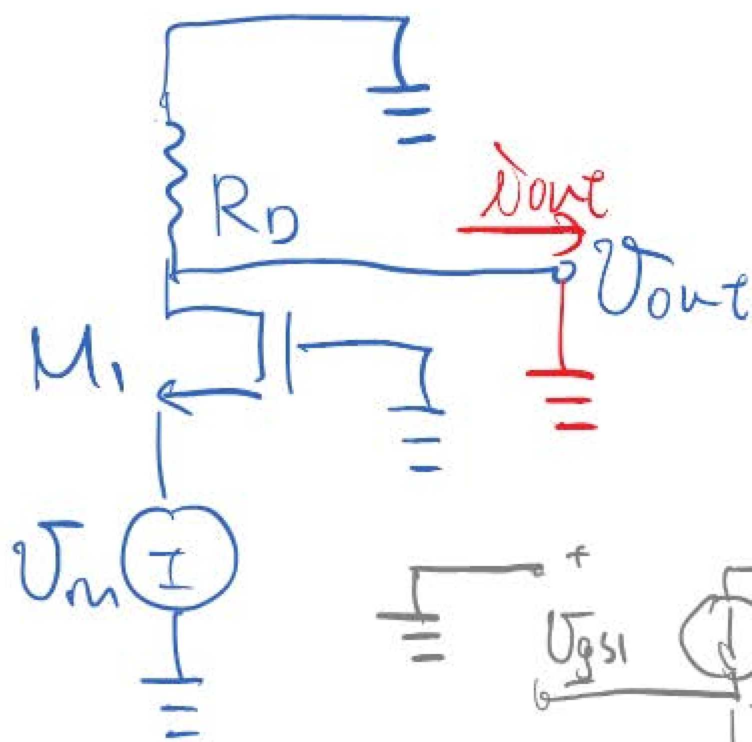
Assume M_1 in sat.

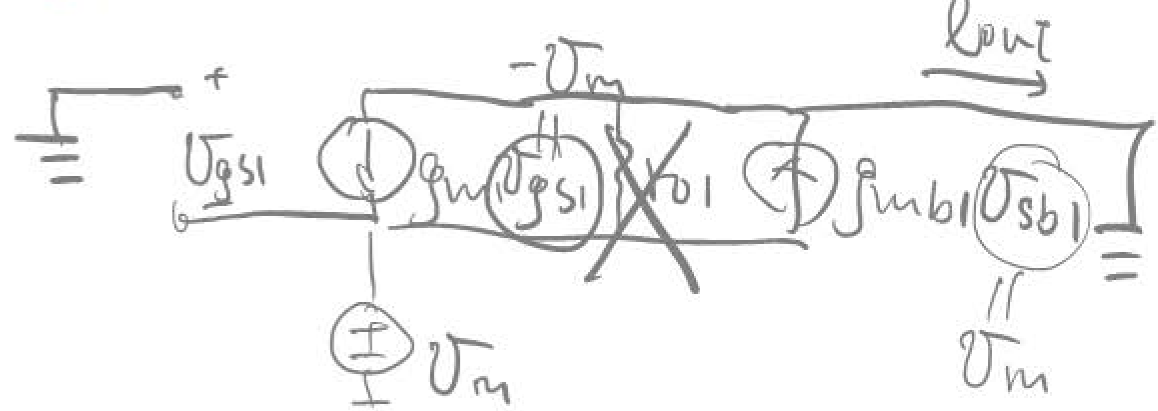
1° $G_m = ?$

$$G_m = \frac{i_{out}}{v_{in}}$$

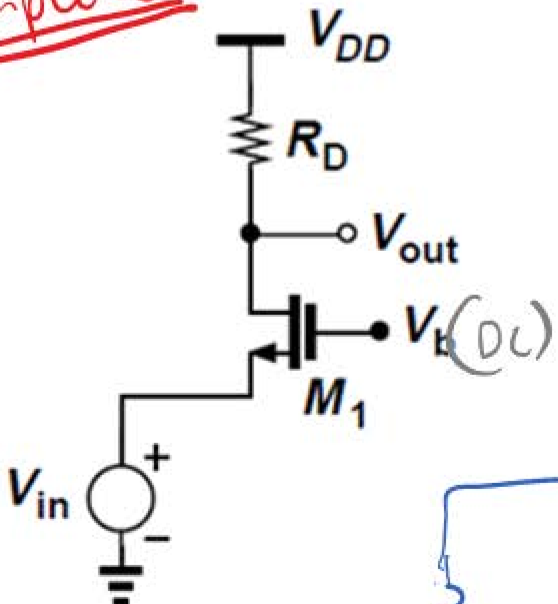
$$= g_{m1} + g_{mb1}$$

small signal





complete



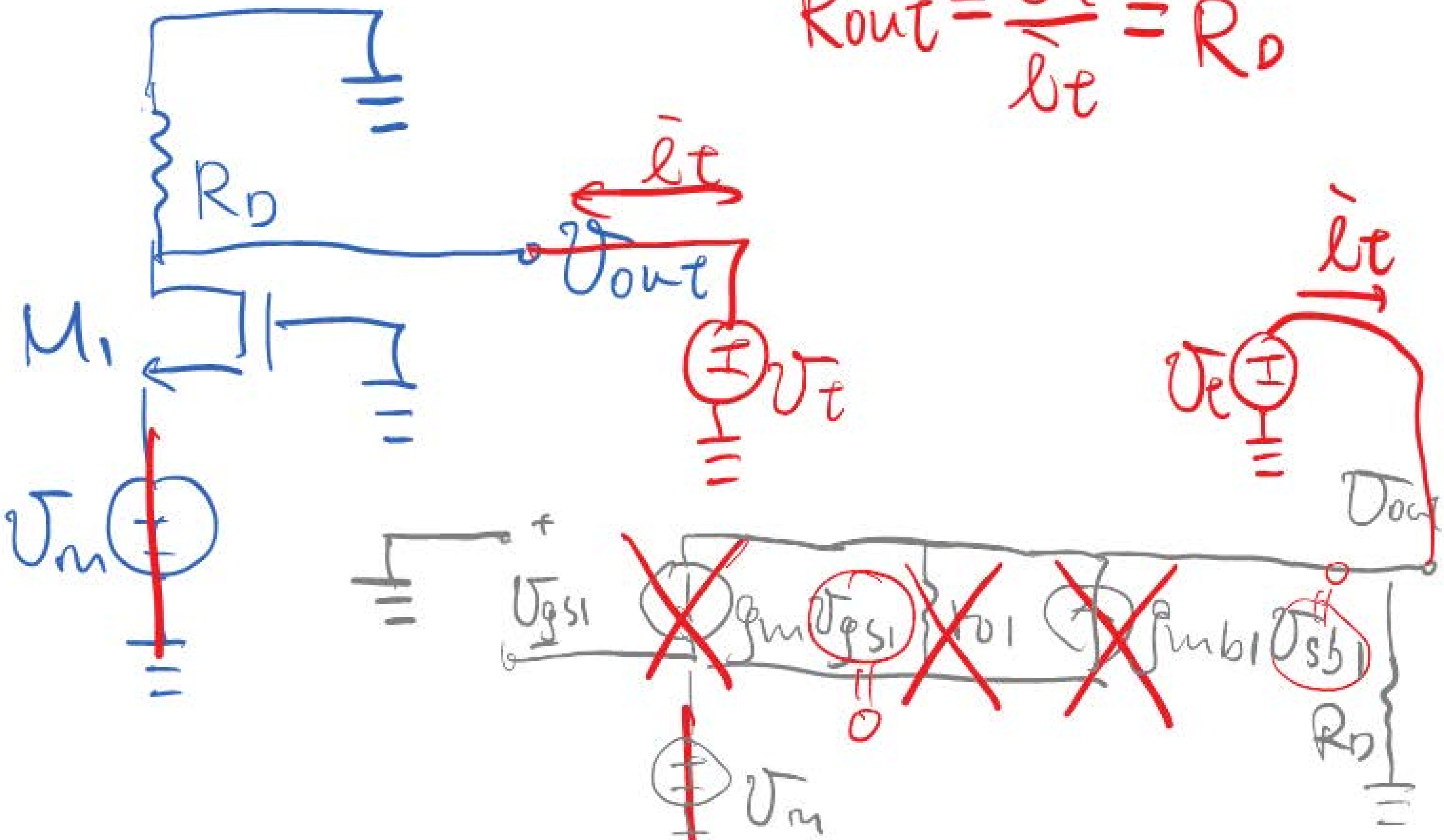
Assume $\lambda = 0$, $\gamma \neq 0$

Assume M_1 in sat.

2° $R_{out} = ?$

$$R_{out} = \frac{v_t}{i_t} = R_D$$

small signal

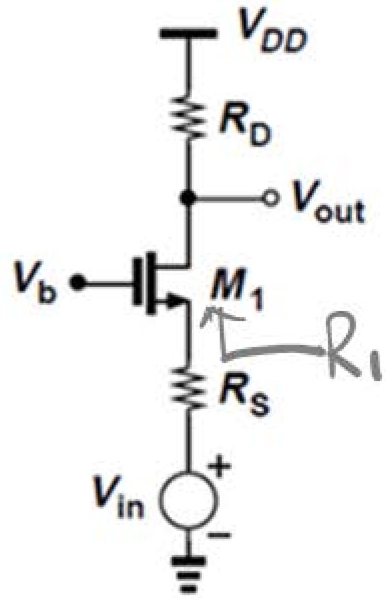


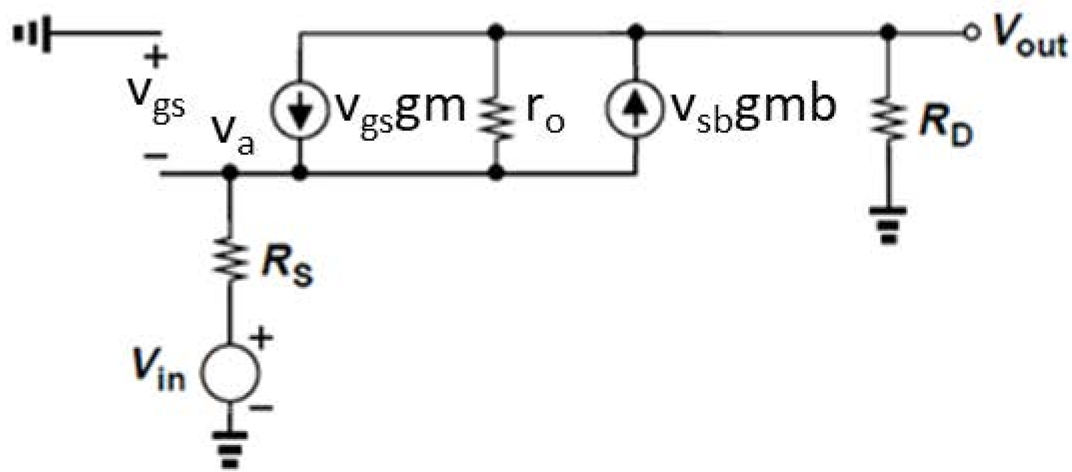
Common-Gate

Small-signal Analysis

$\lambda \neq 0$

$\gamma \neq 0$





$$G_m = \frac{(gm + gmb)r_o + 1}{r_o + R_S + (gm + gmb)r_o R_S}$$

$$R_{out} = R_D \parallel [r_o + R_S + (gm + gmb)r_o R_S]$$

$$\boxed{A_v = \frac{(gm + gmb)r_o + 1}{r_o + R_S + (gm + gmb)r_o R_S + R_D} R_D} \approx R_D gm(1 + \eta)$$

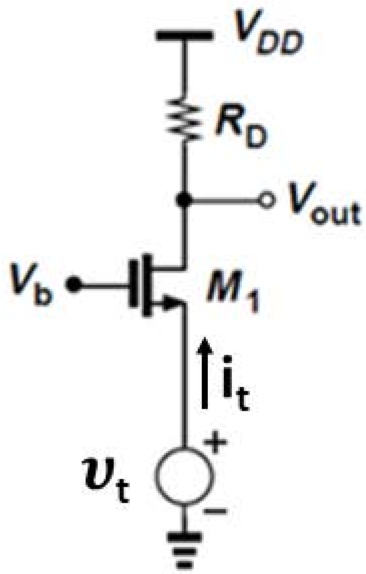
If $R_S = 0$ and $r_o = \infty$

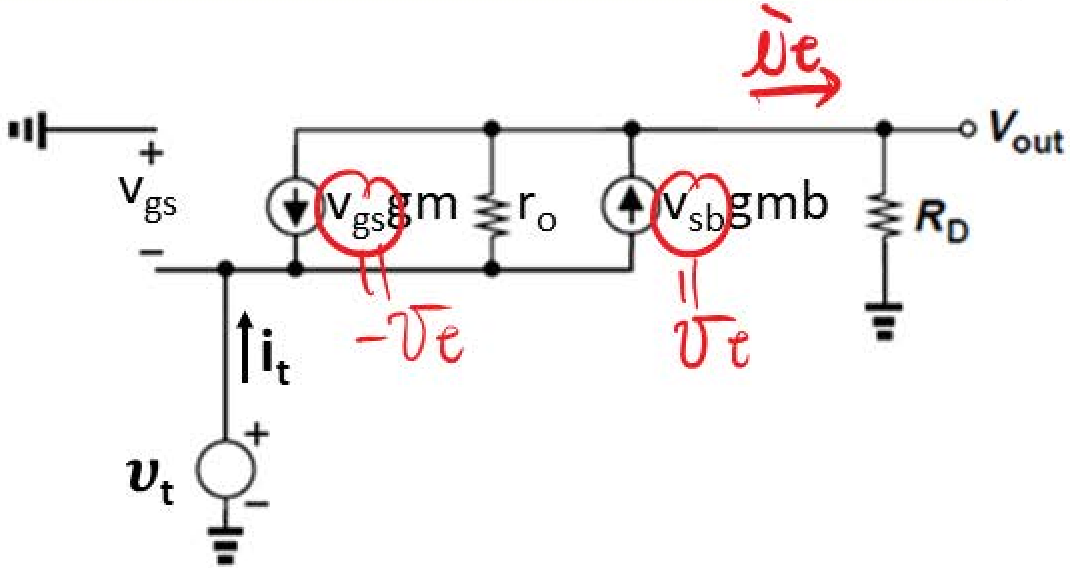
Common-Gate (Input Impedance)

Small-signal Analysis

$\lambda \neq 0$

$\gamma \neq 0$





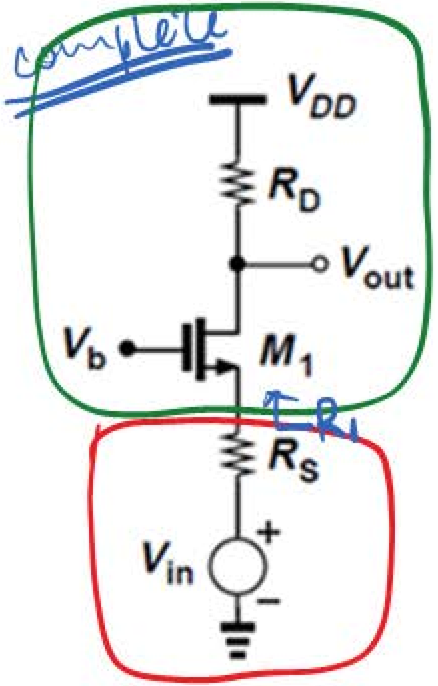
$$\begin{cases} i_t = \upsilon_t(gm + gmb) + \dfrac{\upsilon_t - \upsilon_{out}}{r_o} \\ \upsilon_{out} = R_D i_t \end{cases}$$

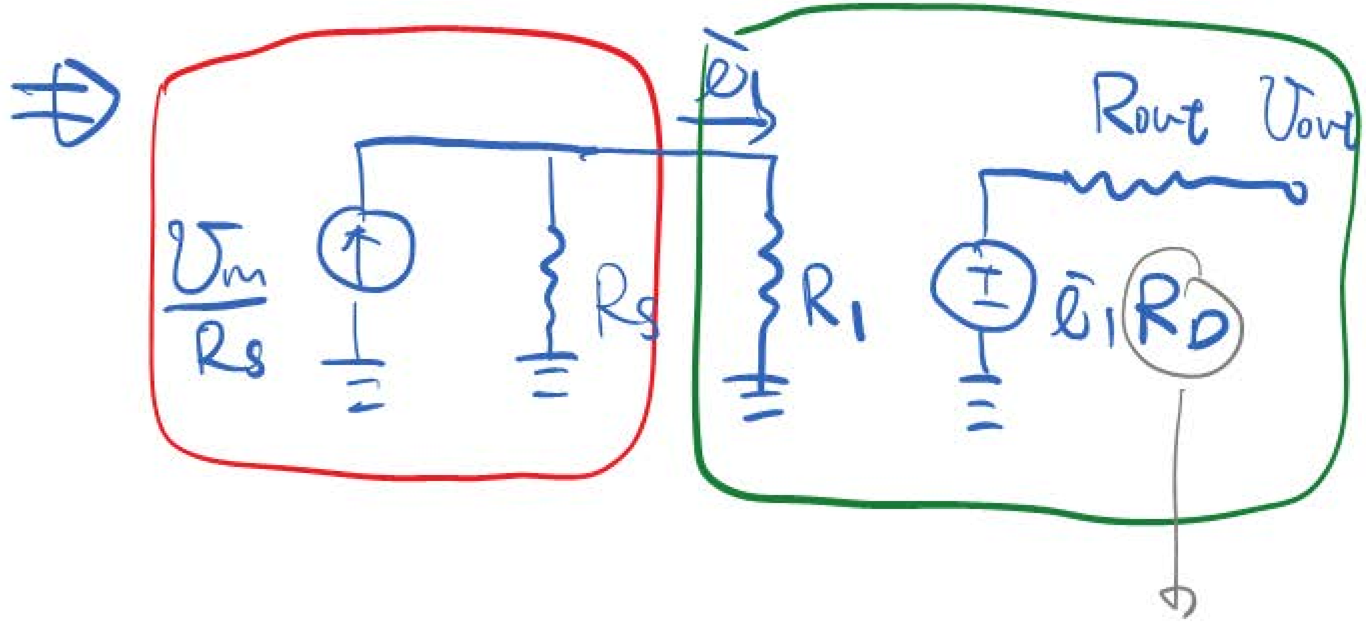
$$\boxed{R_{in} = \frac{R_D + r_o}{1 + (gm + gmb)r_o}}$$

If $R_D = 0$: $R_{in} = r_o \parallel \dfrac{1}{gm} \parallel \dfrac{1}{gmb}$

If $R_D = \infty$: $R_{in} = \infty$

Equivalent small-signal





$$R_1 = \frac{R_D + r_{o1}}{1 + (gm_1 + gmb_1) r_{o1}}$$

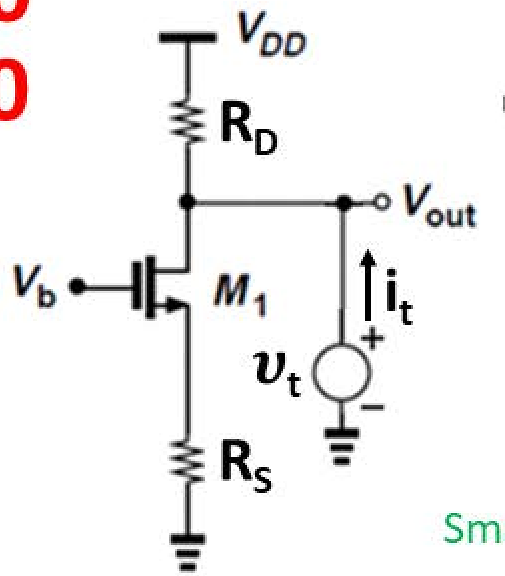
trans-impedance gain.

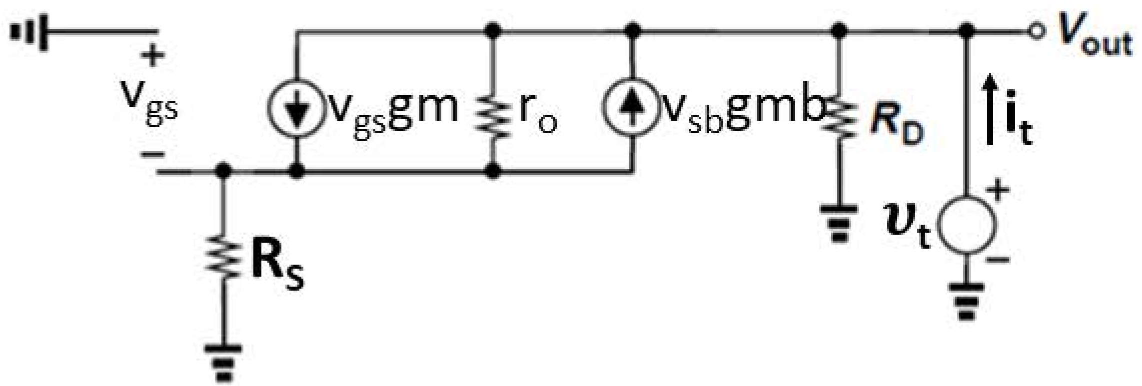
$$i_1 = \frac{R_S}{R_S + R_1}$$

Common-Gate (Output Impedance)

Small-signal Analysis

$\lambda \neq 0$
$\gamma \neq 0$





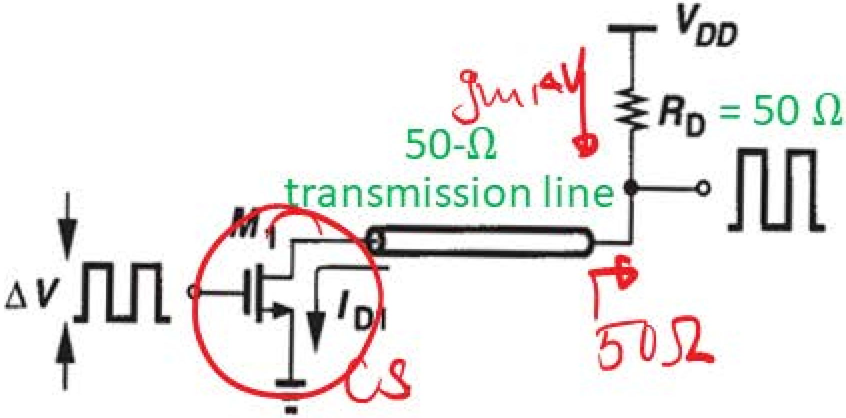
Small-signal model the same as CS with source degradation

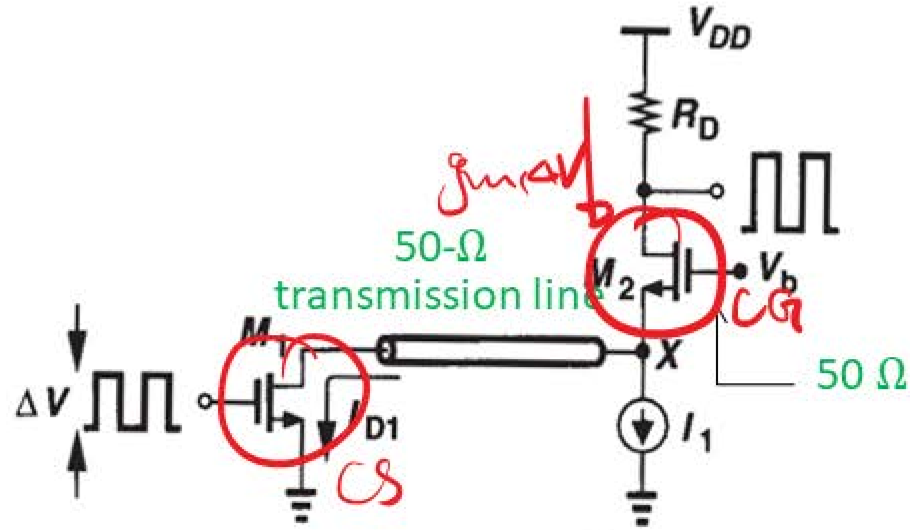
$$R_{out} = [R_S + r_{o1} + (gm_1 + gmb_1) r_{o1} R_S] \parallel R_D$$

Example

Calculate the small-signal voltage gain at low frequencies of the circuits below. To minimize wave reflection at point X, the input impedance must be equal to 50 Ω.

Assume $\lambda = 0$



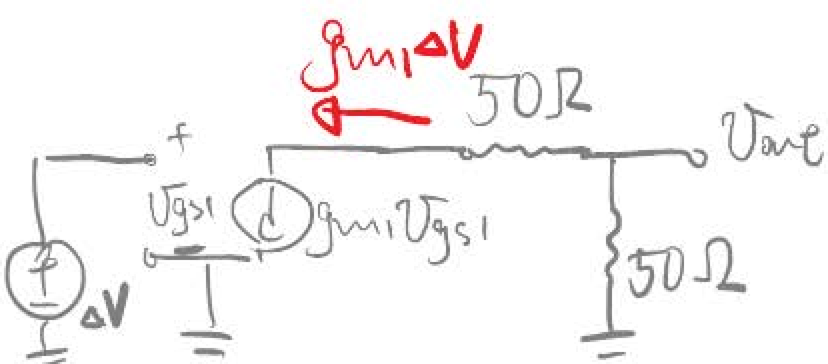


$$A_v = -gm_1 R_D$$

= 50 Ω

$$A_v = -gm_1 R_D$$

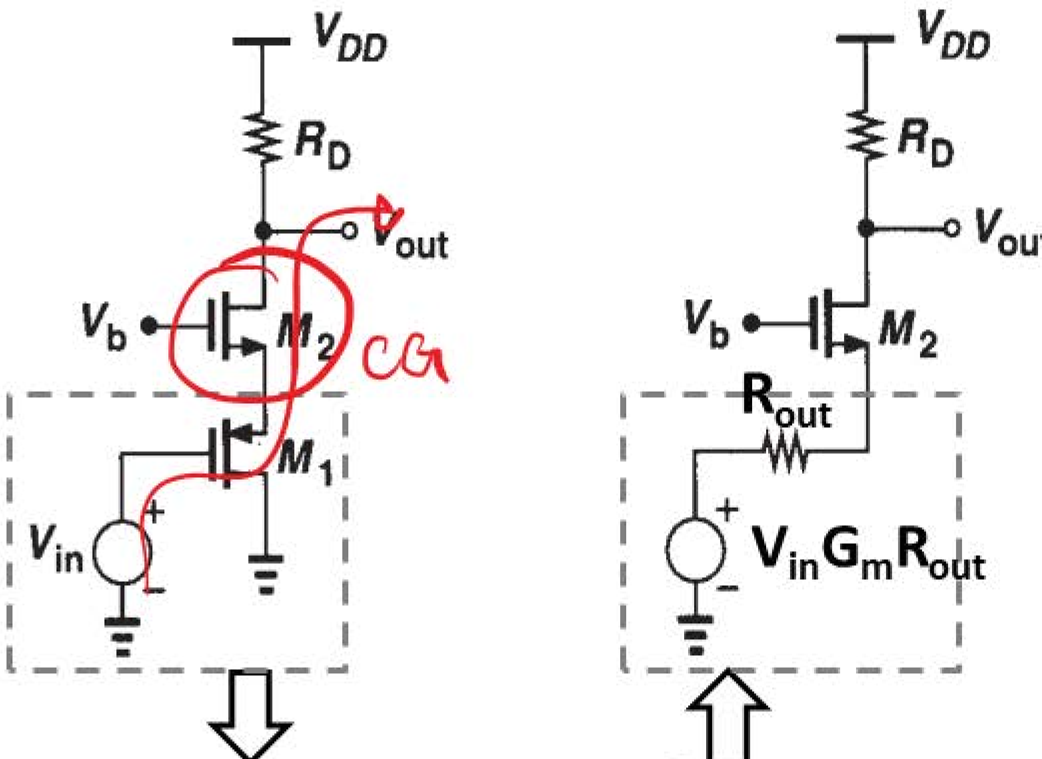
can be much larger than 50 Ω, so as to achieve a much higher gain.

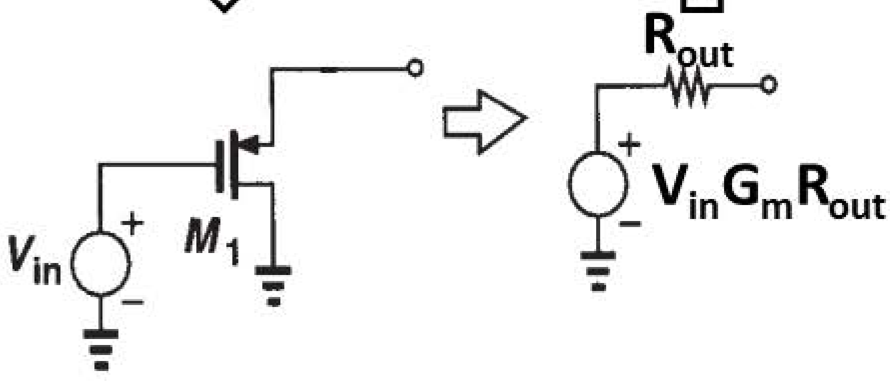


$$R_{in} = \frac{R_D + r_{o2}}{1 + (gm_2 + gmb_2) r_{o2}} = 50\ \Omega$$

Example

Calculate the small-signal voltage gain of the circuit below. ($\lambda \neq 0$, $\gamma \neq 0$)





$$\begin{cases} G_m = gm_1 \\ R_{out} = r_{o1} \parallel \dfrac{1}{gm_1} \parallel \dfrac{1}{gmb_1} \end{cases}$$