Derivation of

Si PN Junction Diode I-V Equation

Generation and Recombination

Generation and recombination constantly happens in the semiconductor (when $T > 0K$)

Recombination rate
unit: $1/(cm^3 \cdot sec)$

$$\boxed{\mathbf{R = Knp\ (K\ is\ a\ constant)}}$$

When in thermal equilibrium:

Generation rate
unit: $1/(cm^3 \cdot sec)$

$$G_o = R_o = Kn_op_o$$

Charge Carriers Flow in Semiconductor

Electric Field → Drift

Concentration Gradient → Diffusion

Current density equations in 1D:

$$\begin{cases} J_p = q\left(\mu_p p E_x - D_p \dfrac{dp}{dx}\right) \\ J_n = q\left(\mu_n n E_x + D_n \dfrac{dn}{dx}\right) \end{cases} \quad [A / cm^2]$$

μ: charge carrier mobility $[cm^2/(V \cdot sec)]$

D: charge carrier diffusion coefficient $[cm^2/sec]$

Electron Energy and Fermi Level

- **Electron energy level** (E_C, E_V, E_i) bending means there is **electric field.**

$$\text{electric field} \equiv -\frac{dV}{dx} = \frac{1}{q}\frac{dE_C}{dx} = \frac{1}{q}\frac{dE_V}{dx} = \frac{1}{q}\frac{dE_i}{dx}$$

- **Fermi level** (E_f) bending means there is **current.**

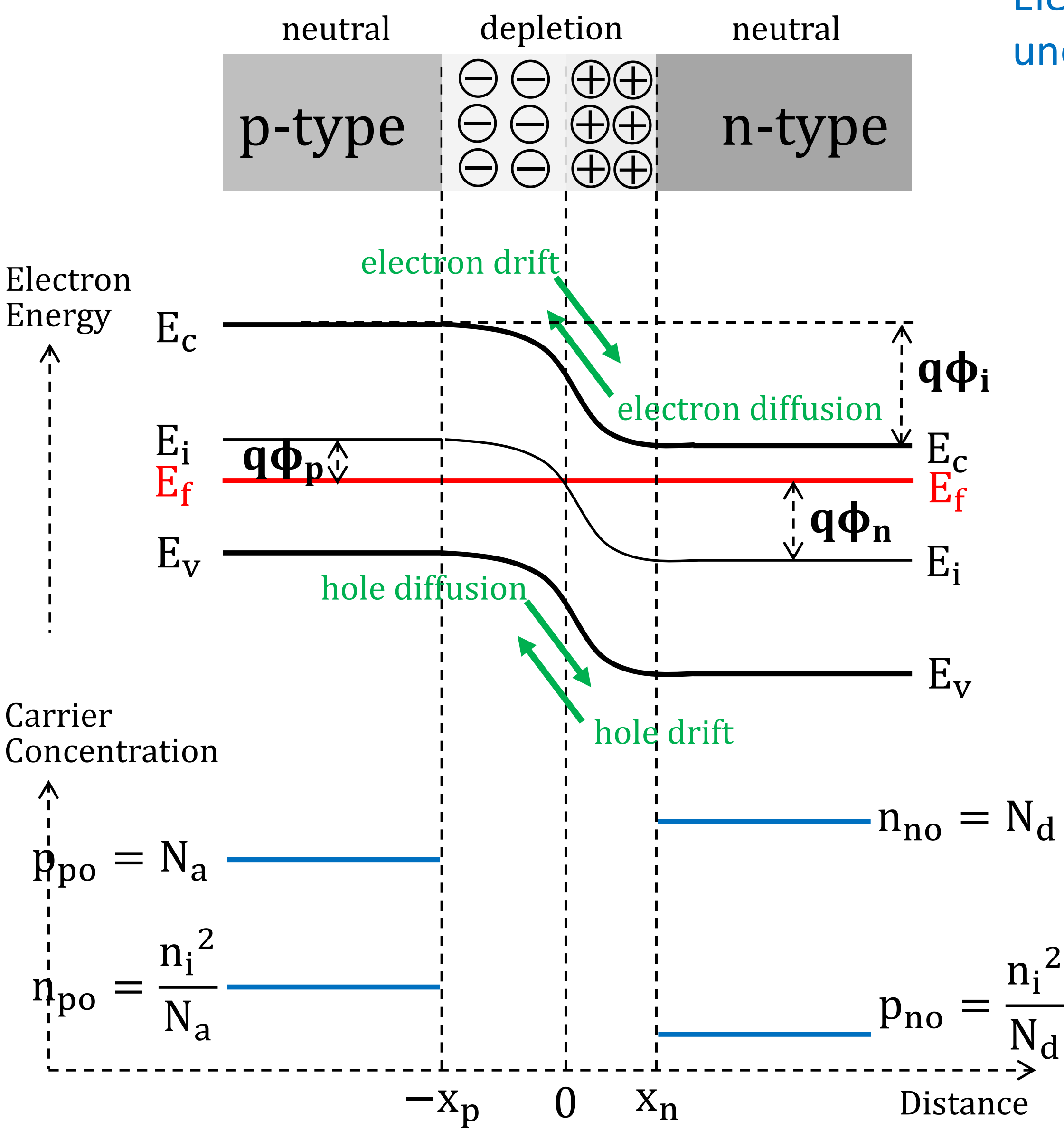
$$J_n = \mu_n n \frac{dE_{fn}}{dx} \qquad J_p = \mu_p p \frac{dE_{fp}}{dx}$$

Proof:

$$n = n_i e^{\frac{E_{fn}-E_i}{kT}} \Rightarrow E_{fn} = E_i + kT\ln\left(\frac{n}{n_i}\right) \qquad \text{Einstein Relations: } \frac{\mu_n}{q} = \frac{D_n}{kT}$$

$$\mu_n n \frac{dE_{fn}}{dx} = \mu_n n\left(\frac{dE_i}{dx} + kT\frac{n_i}{n}\frac{1}{n_i}\frac{dn}{dx}\right) = q\mu_n n \underset{\text{electric field}}{\underset{\uparrow}{E_x}} + \mu_n kT\frac{dn}{dx} = q\mu_n nE_x + qD_n\frac{dn}{dx}$$

Si PN Junction in **Thermal Equilibrium**



Electron concentration in the n-type Si under thermal equilibrium condition

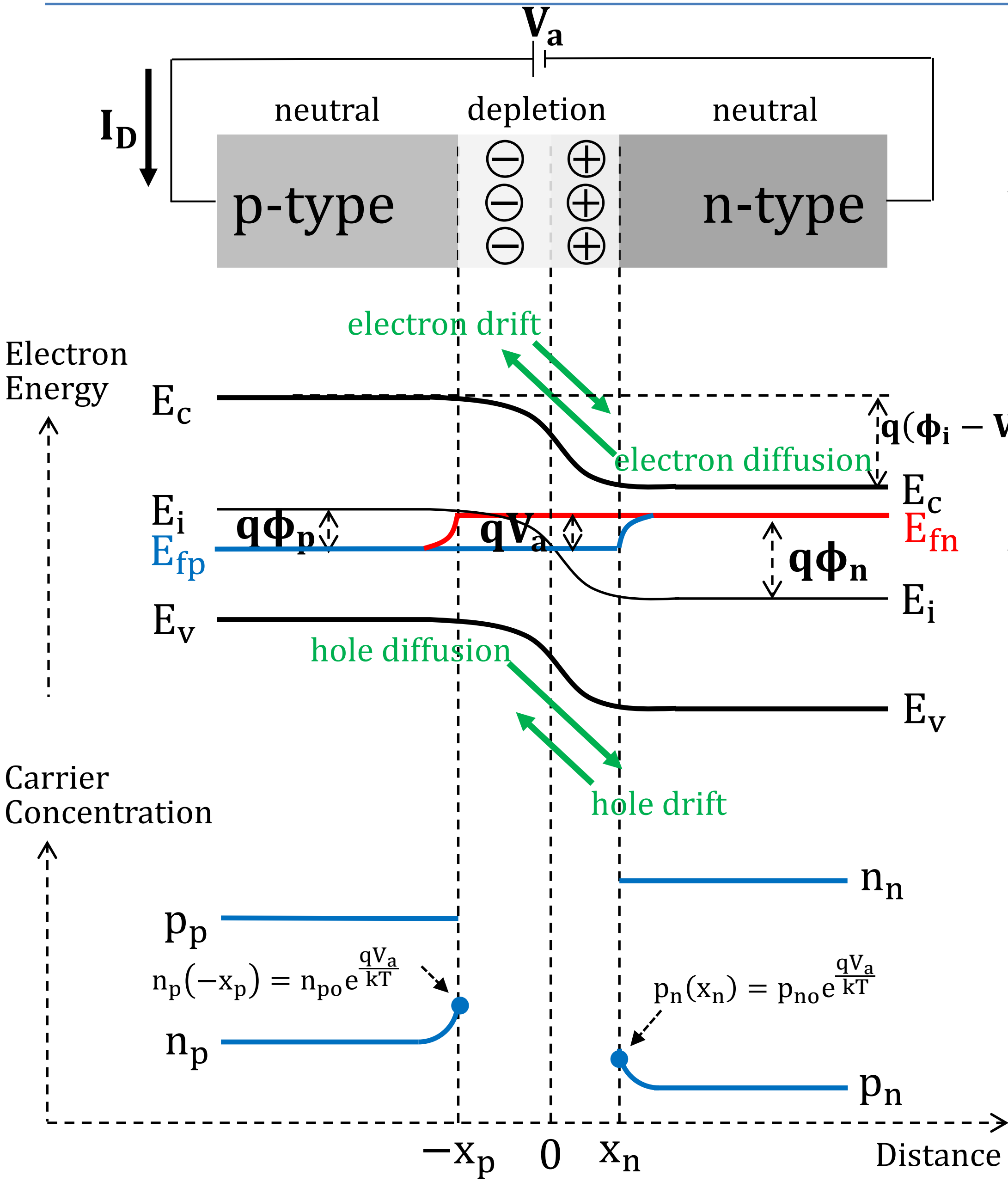
$$\begin{cases} n_{no} = N_d = n_i e^{\frac{E_f - E_i}{kT}} = n_i e^{\frac{q\phi_n}{kT}} \\ p_{po} = N_a = n_i e^{\frac{E_i - E_f}{kT}} = n_i e^{\frac{q\phi_p}{kT}} \end{cases}$$

$$\boxed{\phi_i = \phi_n + \phi_p = \frac{kT}{q} \ln \frac{N_d N_a}{n_i^2}}$$

$$\begin{cases} N_d = \frac{n_i^2}{N_a} e^{\frac{q\phi_i}{kT}} \\ N_a = \frac{n_i^2}{N_d} e^{\frac{q\phi_i}{kT}} \end{cases}$$

$$\Rightarrow \boxed{\begin{cases} n_{no} = n_{po} e^{\frac{q\phi_i}{kT}} \\ p_{po} = p_{no} e^{\frac{q\phi_i}{kT}} \end{cases}}$$

Si PN Junction in **Forward Bias (I)**



$$\begin{cases} n_n(x_n) = n_p(-x_p)e^{\frac{q(\phi_i - V_a)}{kT}} \\ p_p(-x_p) = p_n(x_n)e^{\frac{q(\phi_i - V_a)}{kT}} \end{cases}$$

Assume low-level injection $\Rightarrow$

$$\begin{cases} n_p(-x_p)e^{\frac{q(\phi_i - V_a)}{kT}} = N_d = n_{po}e^{\frac{q\phi_i}{kT}} \\ p_n(x_n)e^{\frac{q(\phi_i - V_a)}{kT}} = N_a = p_{no}e^{\frac{q\phi_i}{kT}} \end{cases}$$

$$\boxed{\begin{cases} n_p(-x_p) = n_{po}e^{\frac{qV_a}{kT}} \\ p_n(x_n) = p_{no}e^{\frac{qV_a}{kT}} \end{cases}}$$ B.C. #1

$$\boxed{\begin{cases} n_p(-\infty) = n_{po} \\ p_n(\infty) = p_{no} \end{cases}}$$ B.C. #2

Si PN Junction in **Forward Bias** (II)

Current density equations in 1D:

$$\begin{cases} J_p = q\left(\mu_p p E_x - D_p \dfrac{dp}{dx}\right) \\ J_n = q\left(\mu_n n E_x + D_n \dfrac{dn}{dx}\right) \end{cases}$$

Continuity equations (steady-state) in 1D:

Put J_p or J_n into here

$$\begin{cases} \dfrac{\partial p}{\partial t} = \dfrac{-1}{q}\dfrac{dJ_p}{dx} + G - R = 0 \\ \dfrac{\partial n}{\partial t} = \dfrac{1}{q}\dfrac{dJ_n}{dx} + G - R = 0 \end{cases}$$

Genergation rate: G

Recombination rate: R

(see next page)

Excess carrier lifetime: τ_p or τ_n

By depletion approximation
(i.e. $E = 0$ outside the depletion region)

$$\Rightarrow \boxed{\begin{cases} J_p = -qD_p \dfrac{d\Delta p_n}{dx} \\ J_n = qD_n \dfrac{d\Delta n_p}{dx} \end{cases}}$$

$$\Rightarrow \begin{cases} \dfrac{d^2\Delta p_n}{dx^2} = \dfrac{\Delta p_n}{D_p\tau_p} = \dfrac{\Delta p_n}{{L_p}^2} \\ \dfrac{d^2\Delta n_p}{dx^2} = \dfrac{\Delta n_p}{D_n\tau_n} = \dfrac{\Delta n_p}{{L_n}^2} \end{cases}$$

Diffusion legnth: $L_n = \sqrt{D_n\tau_n}$

Apply B.C. #1 and B.C. #2 $\Rightarrow$

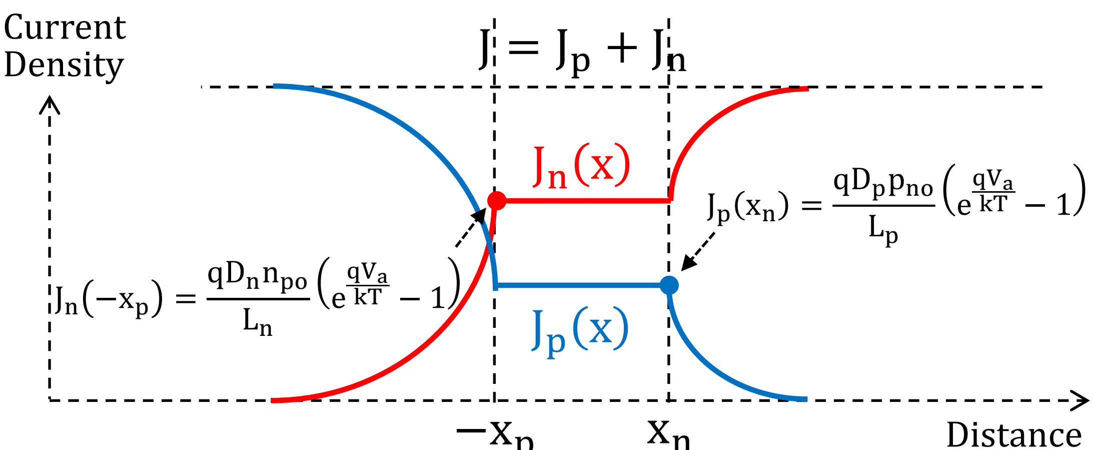
$$\boxed{\begin{cases} \Delta p_n(x) = p_{no}\left(e^{\frac{qV_a}{kT}} - 1\right)e^{-\left(\frac{x-x_n}{L_p}\right)} & (x \geq x_n) \\ \Delta n_p(x) = n_{po}\left(e^{\frac{qV_a}{kT}} - 1\right)e^{\left(\frac{x+x_p}{L_n}\right)} & (x \leq -x_p) \end{cases}}$$

Put Δp_n or Δn_p back into here

Si PN Junction in **Forward Bias** (III)

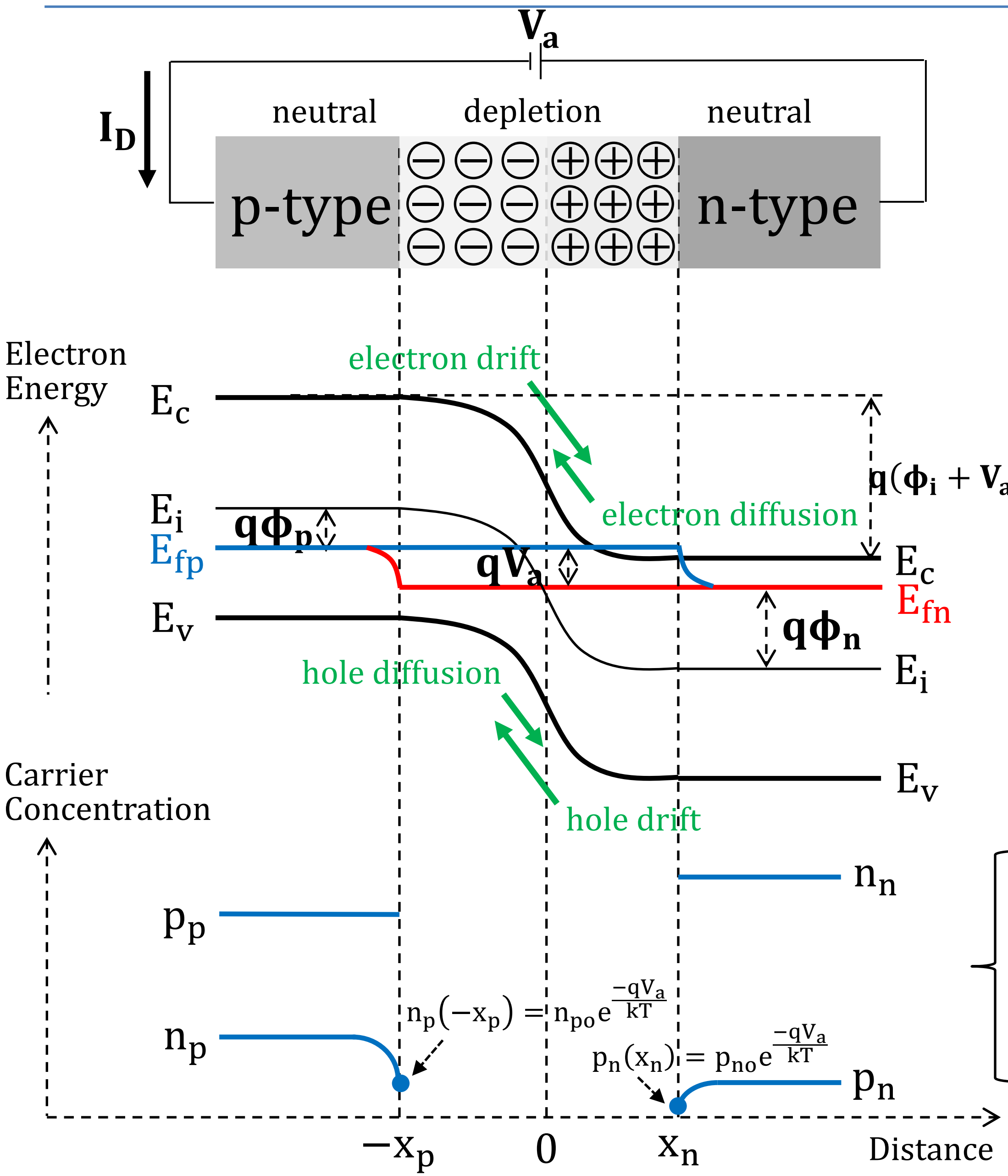
$$\begin{cases} J_p(x) = \dfrac{qD_p p_{no}}{L_p}\left(e^{\frac{qV_a}{kT}} - 1\right)e^{-\left(\frac{x-x_n}{L_p}\right)} & (x \geq x_n) \\ J_n(x) = \dfrac{qD_n n_{po}}{L_n}\left(e^{\frac{qV_a}{kT}} - 1\right)e^{\left(\frac{x+x_p}{L_n}\right)} & (x \leq -x_p) \end{cases}$$

$$\boxed{\mathbf{I_D = qAn_i{}^2\left(\frac{D_p}{L_pN_d} + \frac{D_n}{L_nN_a}\right)\left(e^{\frac{qV_a}{kT}} - 1\right) = I_S\left(e^{\frac{qV_a}{kT}} - 1\right)}}$$



- Minority current gradually transformed into majority current
- Total current across PN junction is a constant

Si PN Junction in **Reverse Bias** (I)



Replace V_a with $-V_a$

$$\begin{cases} n_p(-x_p) = n_{po}e^{\frac{-qV_a}{kT}} \\ p_n(x_n) = p_{no}e^{\frac{-qV_a}{kT}} \end{cases}$$

$$\begin{cases} n_p(-\infty) = n_{po} \\ p_n(\infty) = p_{no} \end{cases}$$

$$\begin{cases} \Delta p_n(x) = p_{no}\left(e^{\frac{-qV_a}{kT}} - 1\right)e^{-\left(\frac{x-x_n}{L_p}\right)} < 0 \\ \Delta n_p(x) = n_{po}\left(e^{\frac{-qV_a}{kT}} - 1\right)e^{\left(\frac{x+x_p}{L_n}\right)} < 0 \end{cases}$$

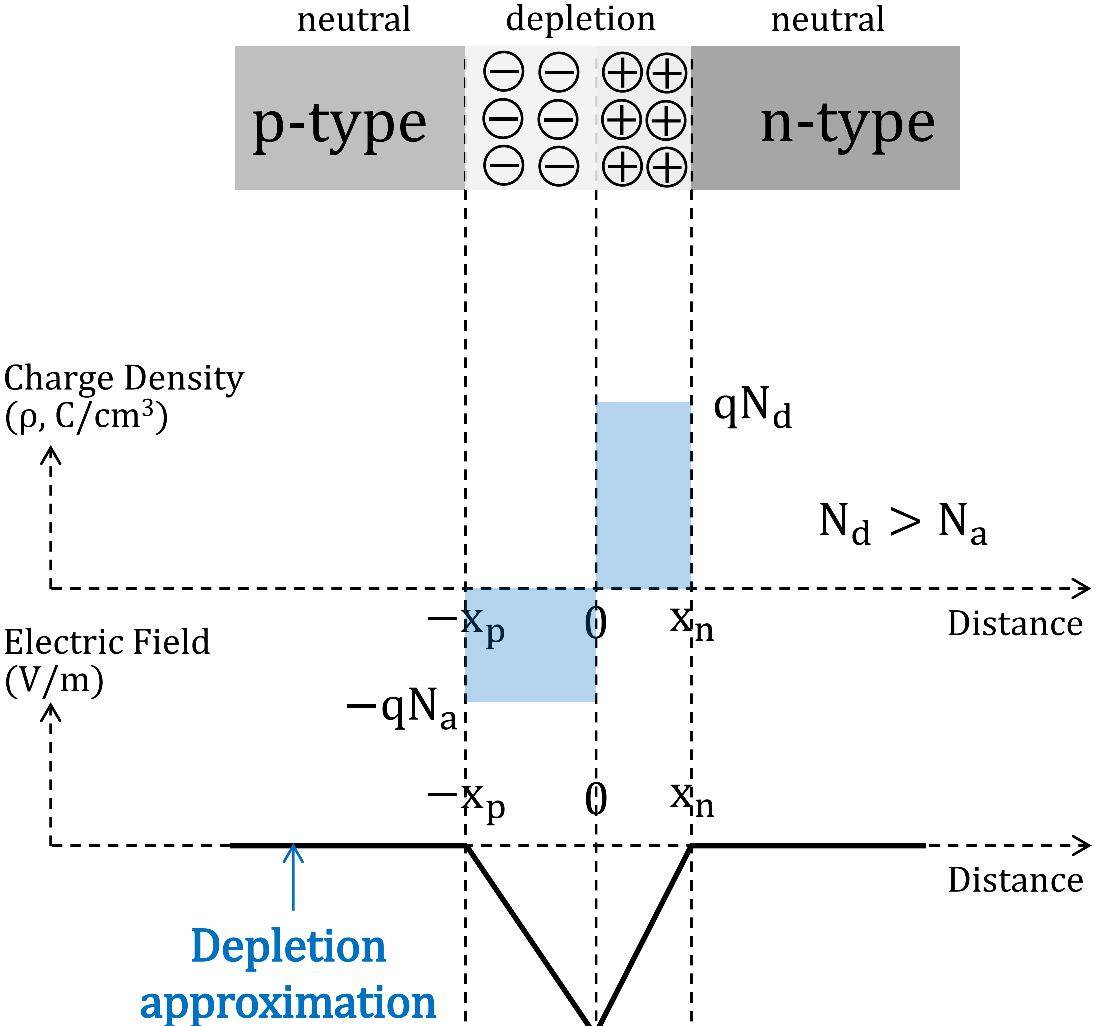
Si PN Junction in **Reverse Bias** (II)

$$\begin{cases} J_p(x) = \dfrac{qD_p p_{no}}{L_p}\left(e^{\frac{-qV_a}{kT}} - 1\right)e^{-\left(\frac{x-x_n}{L_p}\right)} < 0 & (x \geq x_n) \\ J_n(x) = \dfrac{qD_n n_{po}}{L_n}\left(e^{\frac{-qV_a}{kT}} - 1\right)e^{\left(\frac{x+x_p}{L_n}\right)} < 0 & (x \leq -x_p) \end{cases}$$

$$\boxed{\mathbf{I_D = qAn_i{}^2\left(\frac{D_p}{L_pN_d} + \frac{D_n}{L_nN_a}\right)\left(e^{\frac{-qV_a}{kT}} - 1\right) = I_S\left(e^{\frac{-qV_a}{kT}} - 1\right)} < 0}$$

Si PN Junction Diode Depletion Width

Depletion Width for **Thermal Equilibrium**



$$\boxed{E_{max} = \frac{-qN_d x_n}{\epsilon_{Si}} = \frac{-qN_a x_p}{\epsilon_{Si}}}$$

Gauss's law:

Electric field / Charge density

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon} \xrightarrow{\text{1D}} \frac{dE}{dx} = \frac{\rho}{\epsilon}$$

Electric permittivity

- For $-x_p \leq x \leq 0$

$$E(x) = \frac{-qN_a}{\epsilon_{Si}}(x + x_p)$$

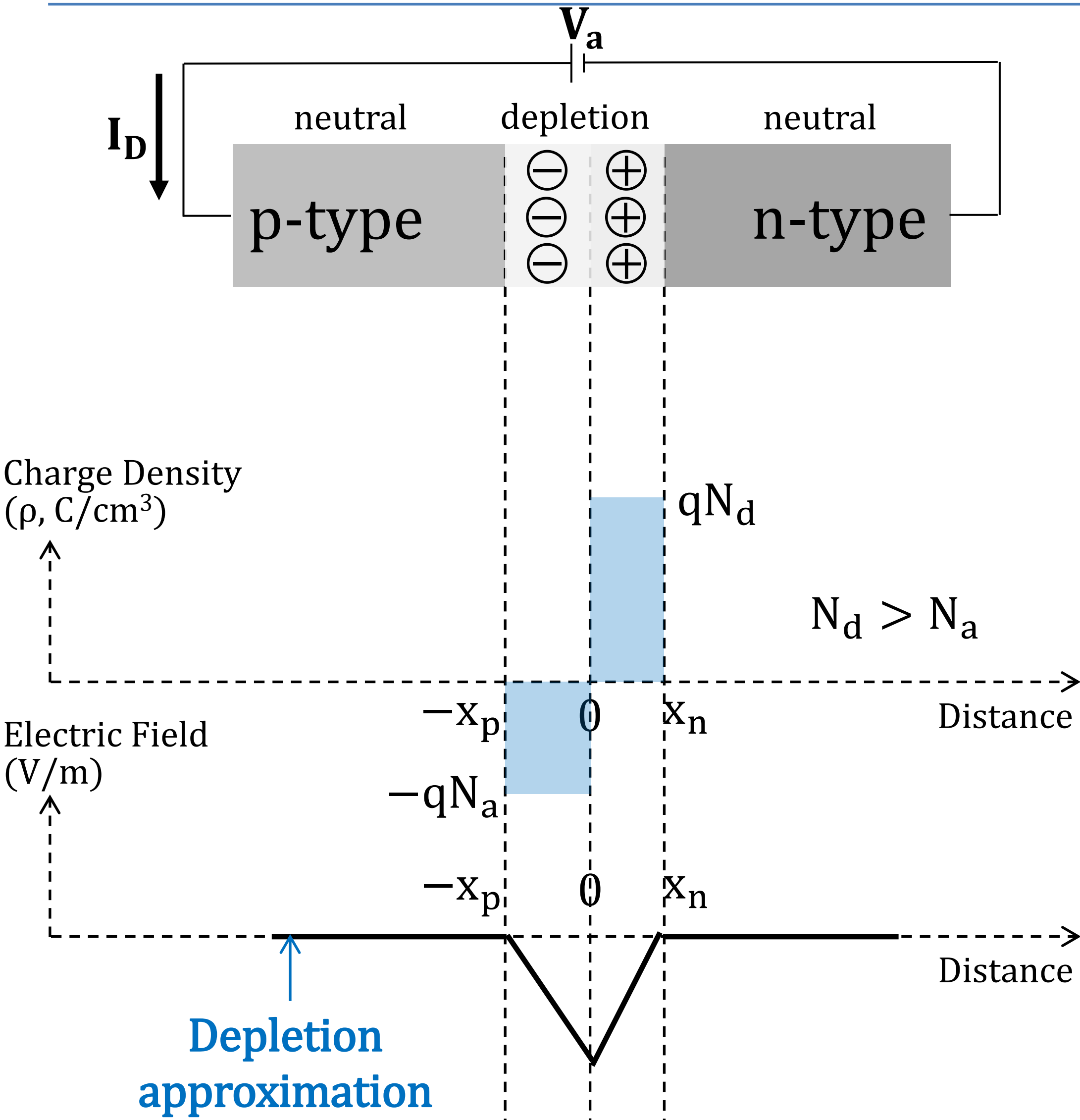
- For $0 \leq x \leq x_n$

$$E(x) = \frac{qN_d}{\epsilon_{Si}}x + \frac{-qN_a x_p}{\epsilon_{Si}}$$

$$\begin{cases} \phi_i = \dfrac{1}{2}E_{max}(x_n + x_p) \\ qN_a x_p = qN_d x_n \end{cases}$$

$$\boxed{\begin{aligned} x_d &= x_n + x_p \\ &= \left[\frac{2\epsilon_{Si}}{q}\phi_i\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{1/2} \end{aligned}}$$

Depletion Width for **Forward Bias**



$$E_{max} = \frac{-qN_d x_n}{\epsilon_{Si}} = \frac{-qN_a x_p}{\epsilon_{Si}}$$

Gauss's law:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon} \xrightarrow{\text{1D}} \frac{dE}{dx} = \frac{\rho}{\epsilon}$$

- For $-x_p \le x \le 0$

$$E(x) = \frac{-qN_a}{\epsilon_{Si}}(x + x_p)$$

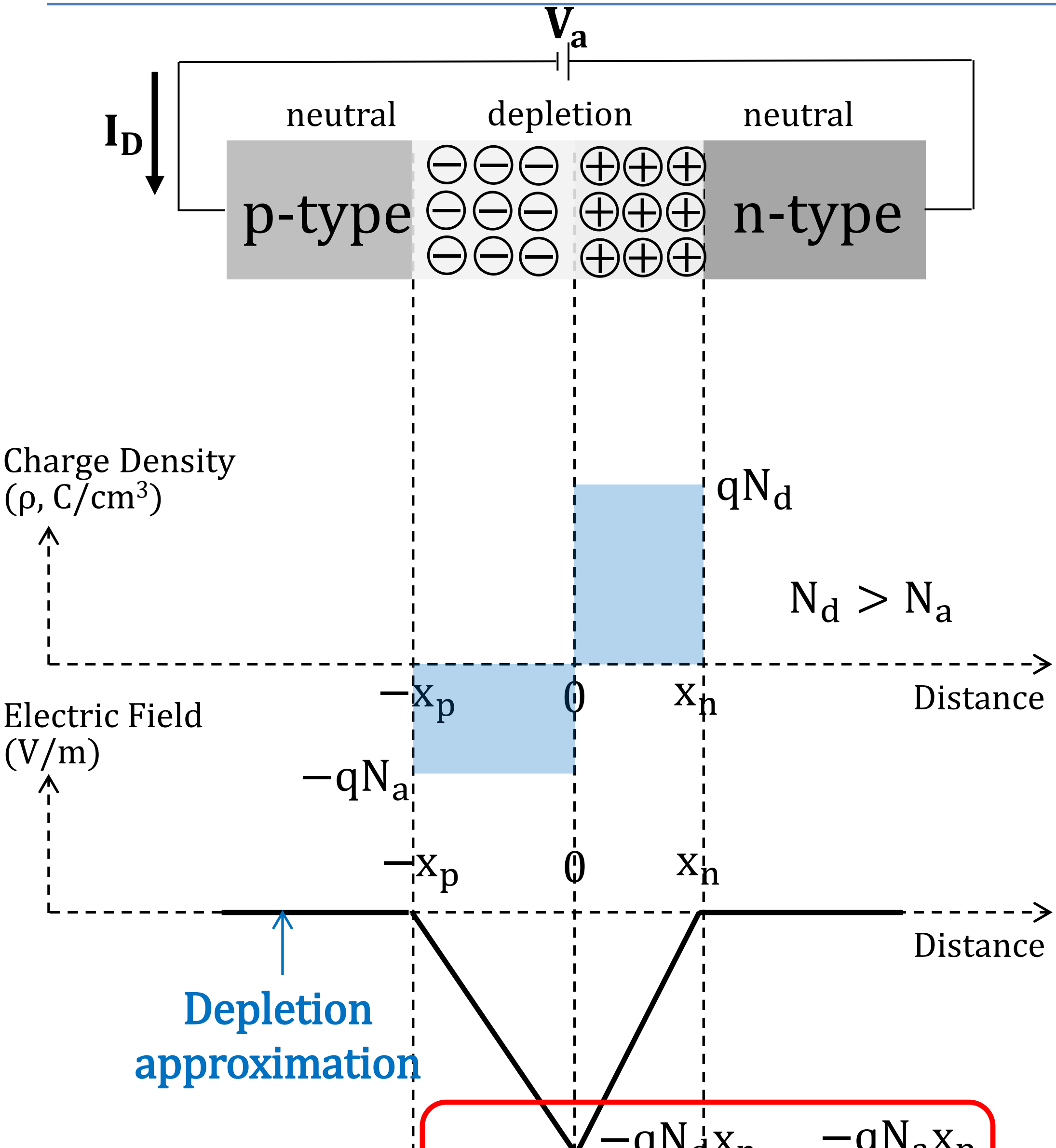
- For $0 \le x \le x_n$

$$E(x) = \frac{qN_d}{\epsilon_{Si}}x + \frac{-qN_a x_p}{\epsilon_{Si}}$$

$$\begin{cases} (\phi_i - V_a) = \dfrac{1}{2}E_{max}(x_n + x_p) \\ qN_a x_p = qN_d x_n \end{cases}$$

$$x_d = x_n + x_p = \left[\frac{2\epsilon_{Si}}{q}(\phi_i - V_a)\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{1/2}$$

Depletion Width for **Reverse Bias**



$$\boxed{E_{max} = \frac{-qN_d x_n}{\epsilon_{Si}} = \frac{-qN_a x_p}{\epsilon_{Si}}}$$

Gauss's law:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon} \xrightarrow{\text{1D}} \frac{dE}{dx} = \frac{\rho}{\epsilon}$$

- For $-x_p \le x \le 0$

$$E(x) = \frac{-qN_a}{\epsilon_{Si}}(x + x_p)$$

- For $0 \le x \le x_n$

$$E(x) = \frac{qN_d}{\epsilon_{Si}} x + \frac{-qN_a x_p}{\epsilon_{Si}}$$

$$\begin{cases} (\phi_i + V_a) = \dfrac{1}{2} E_{max}(x_n + x_p) \\ qN_a x_p = qN_d x_n \end{cases}$$

$$\boxed{\begin{aligned} x_d &= x_n + x_p \\ &= \left[\frac{2\epsilon_{Si}}{q}(\phi_i + V_a)\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{1/2} \end{aligned}}$$

Example

For a silicon diode with $N_a = 10^{17}$ $1/cm^3$, $N_d = 10^{20}$ $1/cm^3$ and $V_a = 0$, calculate the ϕ_i, x_n, x_p and E_{max}. ($k = 1.38 \times 10^{-23}$ J/K, $n_i = 10^{10}$ $1/cm^3$ at 300 K, $q = 1.6 \times 10^{-19}$ C, $\epsilon_{Si} = 11.7 \times 8.85 \times 10^{-14}$ F/cm)

$$\phi_i = \phi_n + \phi_p = \frac{kT}{q} \ln \frac{N_d N_a}{{n_i}^2} = \frac{(1.38 \times 10^{-23})(300)}{(1.6 \times 10^{-19})} \ln \frac{10^{20} \times 10^{17}}{(10^{10})^2} = 1.01\ (\mathrm{V})$$

$$x_d = x_n + x_p = \left[\frac{2\epsilon_{Si}}{q}\phi_i\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{1/2} = \sqrt{\frac{2 \times 11.7 \times 8.85 \times 10^{-14}}{1.6 \times 10^{-19}} 1.01 \left(\frac{1}{10^{17}} + \frac{1}{10^{20}}\right)}$$

$$= 1.144 \times 10^{-5}\ (\mathrm{cm}) = 0.1144\ (\mu\mathrm{m})$$

$$\begin{cases} x_p = \dfrac{10^{20}}{10^{17} + 10^{20}} \times 0.1144 = 0.1143\ (\mu\mathrm{m}) \\ x_n = \dfrac{10^{17}}{10^{17} + 10^{20}} \times 0.1144 = 0.0001\ (\mu\mathrm{m}) \end{cases}$$

Almost all depletion region on the p-side

$$E_{max} = \frac{-qN_a x_p}{\epsilon_{Si}} = \frac{-(1.6 \times 10^{-19}) \times 10^{17} \times (1.143 \times 10^{-5})}{11.7 \times 8.85 \times 10^{-14}} = -1.77 \times 10^5\ (\mathrm{V/cm})$$