Derivation of BJT IV Equation

I-V Characteristic (I)

- **Assume $E_x = 0$**

$(n = n_o + \Delta n)$

$$J_n(\text{electron current in the base neutral region}) = q\mu_n n E_x + qD_n\frac{dn}{dx} = qD_n\frac{dn}{dx} = qD_n\frac{d\Delta n}{dx}$$

In steady − state $\Rightarrow \dfrac{\partial n}{\partial t} = \dfrac{1}{q}\dfrac{dJ_n}{dx} + G - R = 0$

$$\Rightarrow \frac{d^2\Delta n}{dx^2} = \frac{\Delta n}{D_n\tau_n} = \frac{\Delta n}{L_n{}^2}$$

Put Δn back into here

$$\Rightarrow \Delta n(x) = K_1 e^{\frac{-x}{L_n}} + K_2 e^{\frac{x}{L_n}}$$

$$\text{B.C.}\begin{cases}\Delta n(0) = \dfrac{n_i{}^2}{N_a}\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\\[2ex] \Delta n(W_B) = 0\end{cases}$$

$$\Rightarrow \Delta n(x) = \frac{n_i{}^2}{N_a}\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\frac{\sinh\left(\frac{W_B - x}{L_n}\right)}{\sinh\left(\frac{W_B}{L_n}\right)}$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

I-V Characteristic (II)

$$I_n(x=0) = I_E = \frac{AqD_n {n_i}^2}{L_n N_a}\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\coth\left(\frac{W_B}{L_n}\right) = \frac{AqD_n {n_i}^2}{W_B N_a}\left(e^{\frac{qV_{BE}}{kT}} - 1\right) \quad \text{if } L_n \gg W_B$$

$$I_n(x=W_B) = I_C = \frac{AqD_n {n_i}^2}{L_n N_a}\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\operatorname{csch}\left(\frac{W_B}{L_n}\right) = \frac{AqD_n {n_i}^2}{W_B N_a}\left(e^{\frac{qV_{BE}}{kT}} - 1\right) \quad \text{if } L_n \gg W_B$$

$$\boldsymbol{\alpha = \frac{I_C}{I_E} = \operatorname{sech}\left(\frac{W_B}{L_n}\right) \cong 1 - \frac{{W_B}^2}{2{L_n}^2}} \quad \text{if } L_n \gg W_B$$

$$\boldsymbol{\beta = \frac{I_C}{I_B} = \frac{\alpha I_E}{I_E - \alpha I_E} = \frac{\alpha}{1-\alpha}}$$

$$\coth(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}} \cong \frac{1}{x} \quad \text{if x small}$$

$$\operatorname{csch}(x) = \frac{2}{e^x - e^{-x}} \cong \frac{1}{x} \quad \text{if x small}$$

$$\operatorname{sech}(x) = \frac{2}{e^x + e^{-x}}$$