JOINT INSTITUTE
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FET Differential Pair

Ve311 Electronic Circuits (Fall 2020)

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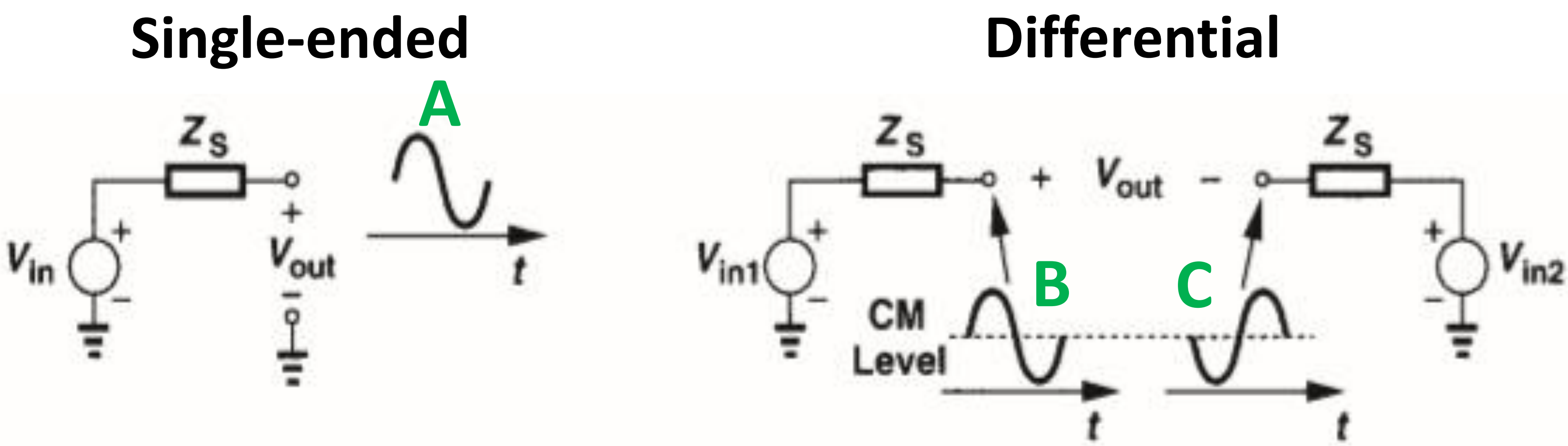
Single-Ended vs Differential Signals

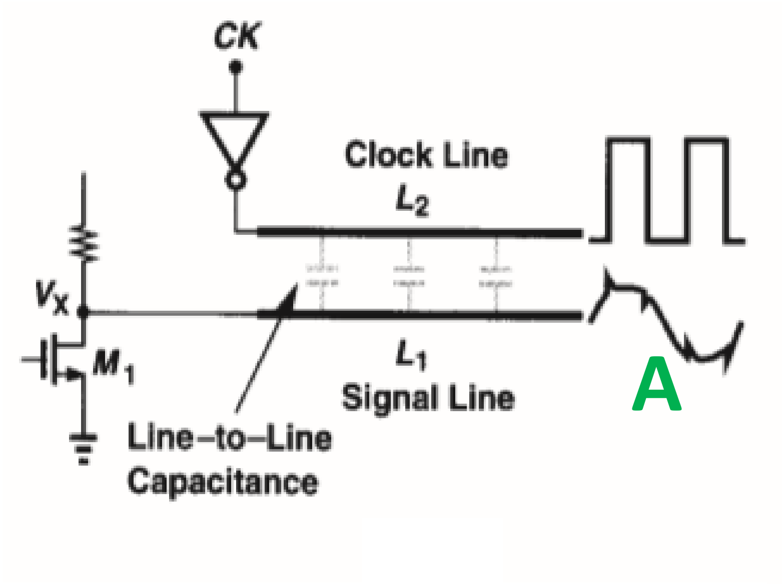


- B – C = A (matters)
- (B + C) / 2 = common-mode level (doesn't matter)
- Single-ended signal: a voltage signal measured with respect to ground
- Differential signal: a voltage signal measured between two nodes, each having equal amplitude and opposite phase around a common-mode (CM) level

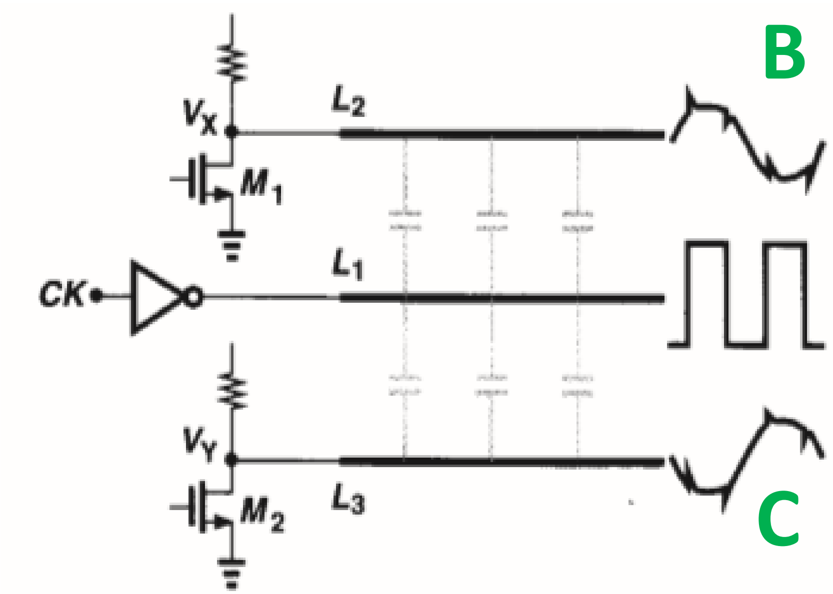
Advantages of Differential Operation

Common-Mode Noise Rejection

Single-ended



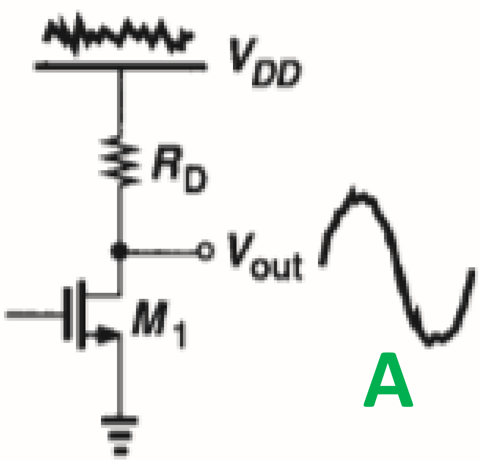
Differential

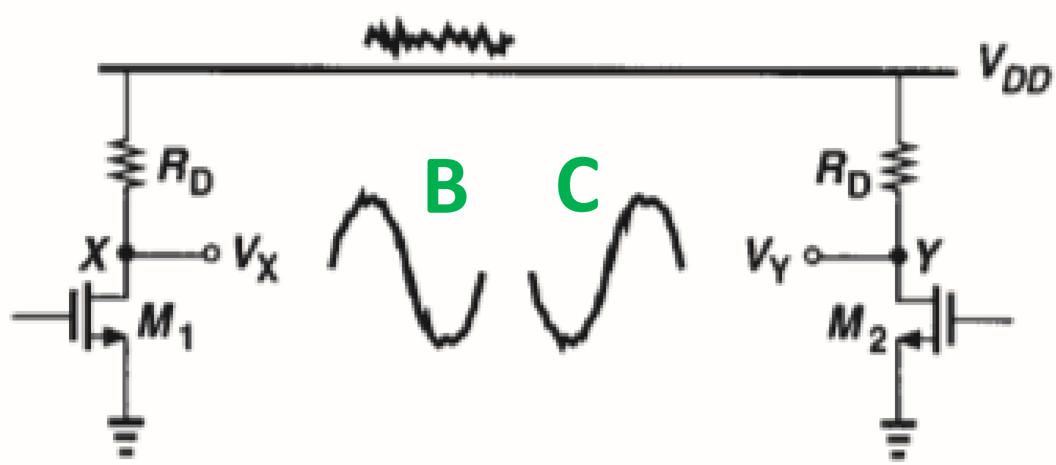


- A corrupted; B corrupted; C corrupted
- (B + C) / 2 = CM corrupted
- (B − C) not corrupted

Common-Mode Noise Rejection

Single-ended **Differential**

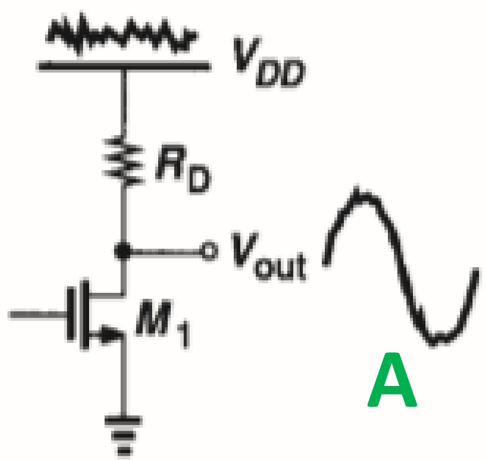




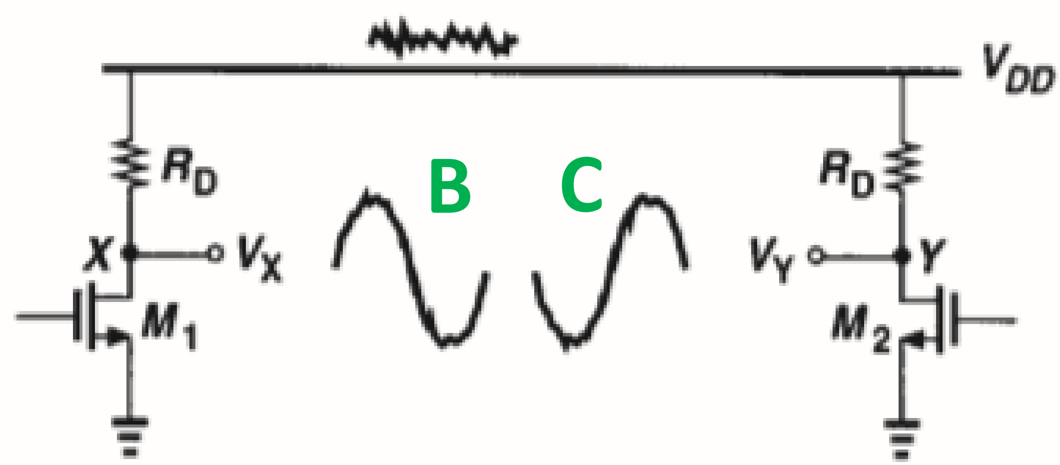
- A corrupted; B corrupted; C corrupted
- (B + C) / 2 = CM corrupted
- (B − C) not corrupted

Increased Output Swing

Single-ended



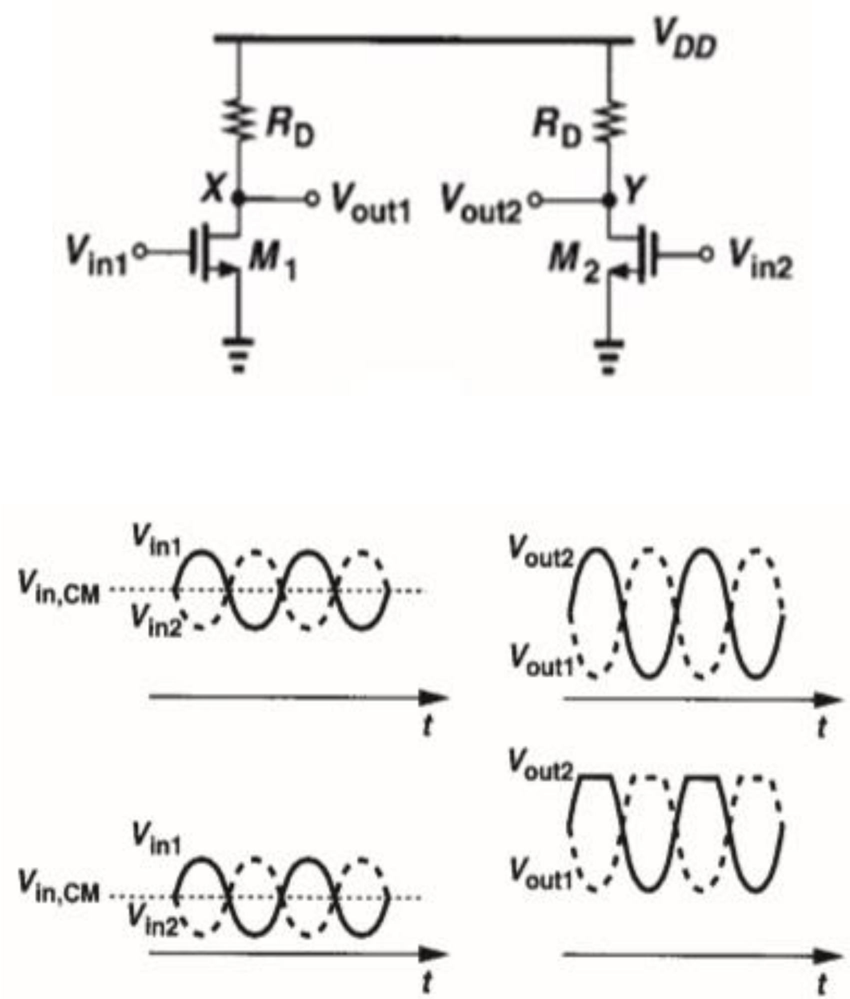
Differential

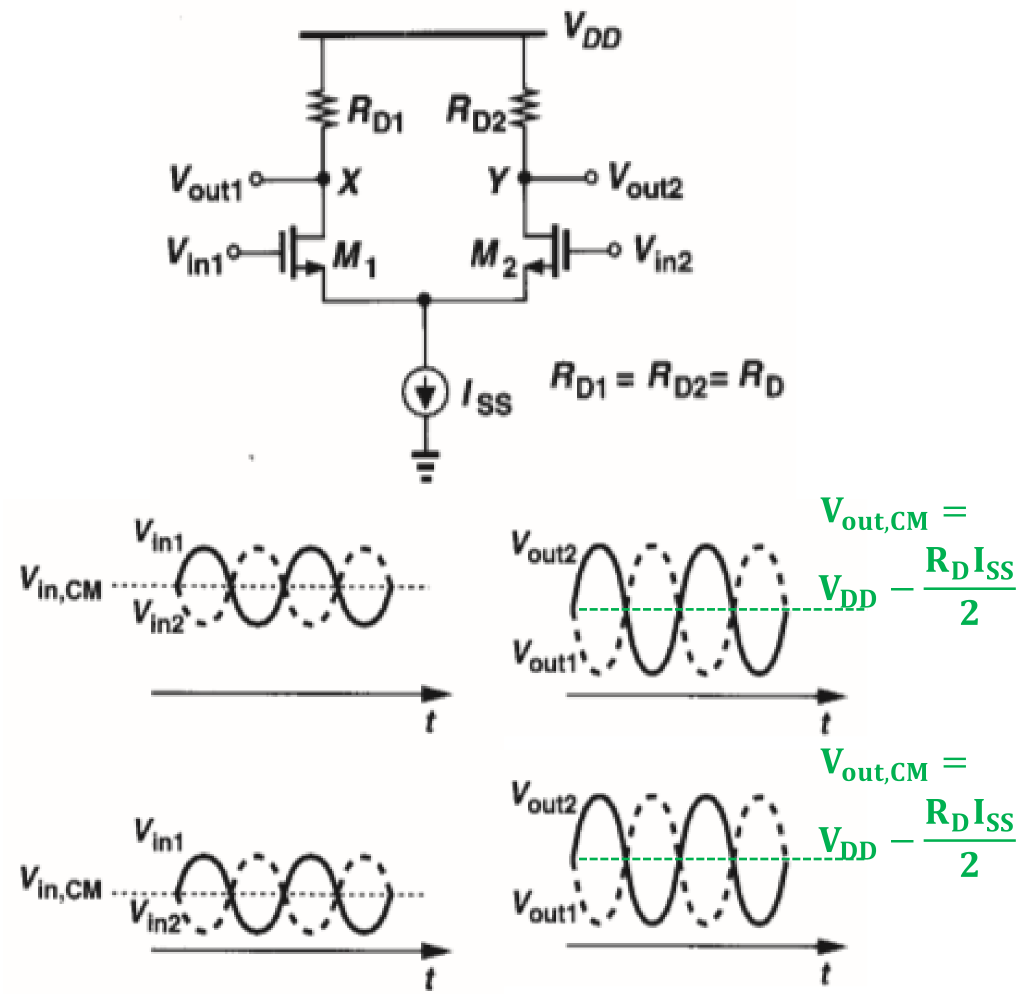


- $(V_{GS1} - V_{TH1}) \leq A \leq V_{DD}$
- $(V_{GS1,2} - V_{TH1,2}) - V_{DD} \leq (B - C) \leq V_{DD} - (V_{GS1,2} - V_{TH1,2})$

DC and Small-Signal Analysis

$V_{in,CM}$ and $V_{out,CM}$

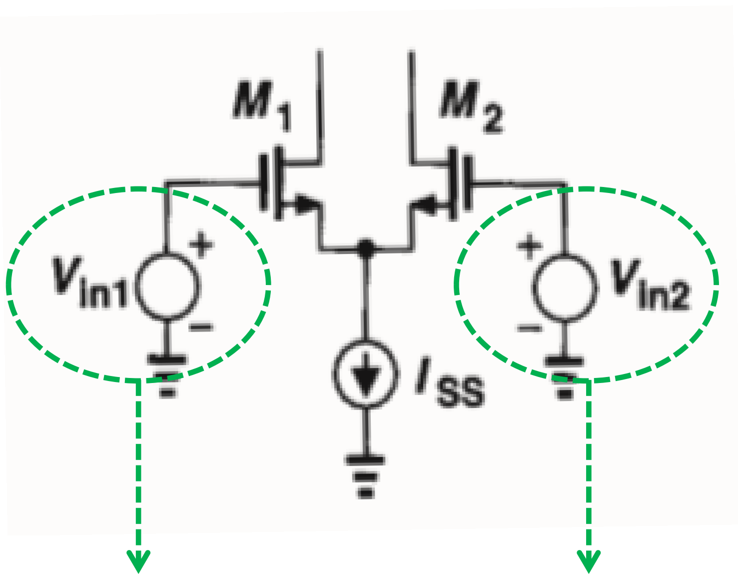




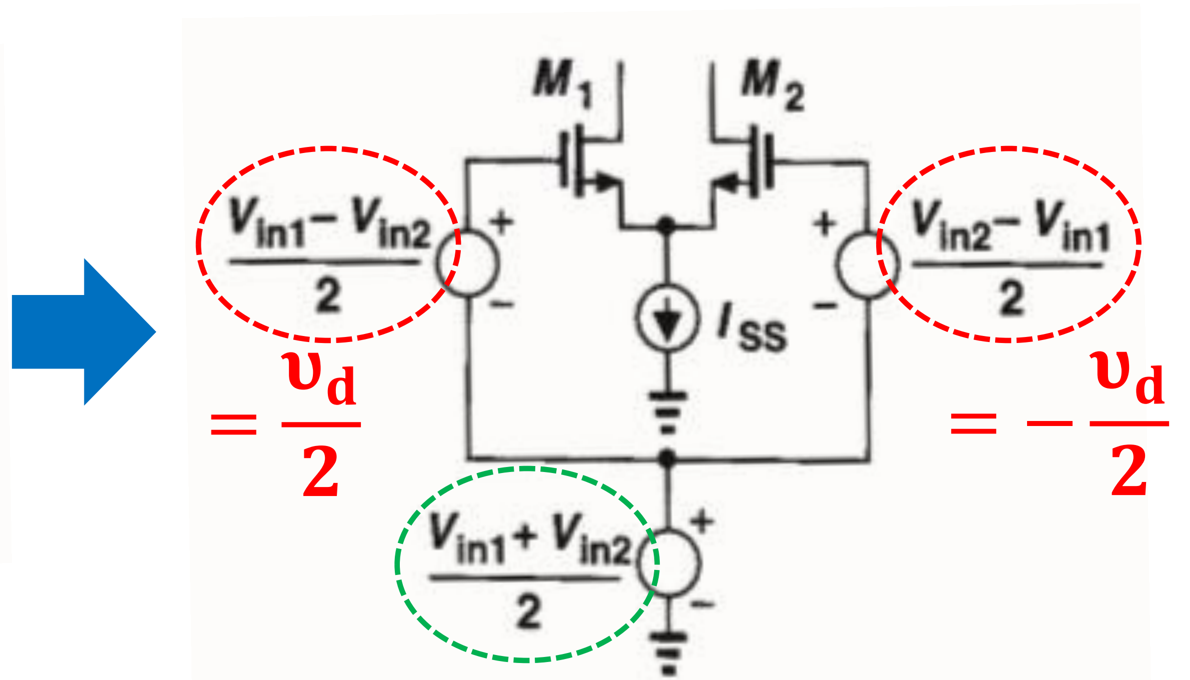
- $V_{out,CM}$ dependent on $V_{in,CM}$

- $V_{out,CM}$ independent from $V_{in,CM}$
- Better design

Common-Mode + Differential-Mode



Not necessarily fully differential



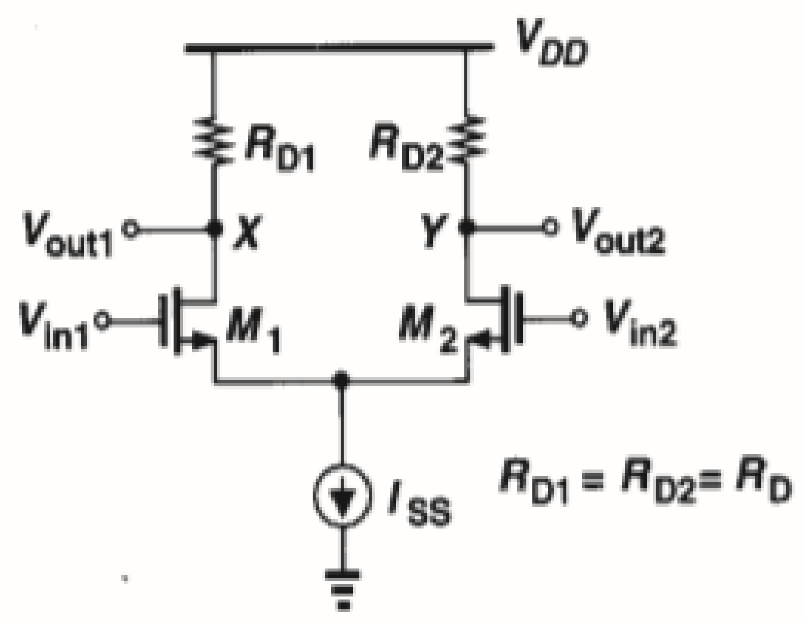
$$= V_{in,CM}$$
$$= V_{IN,CM} + \upsilon_{in,CM}$$

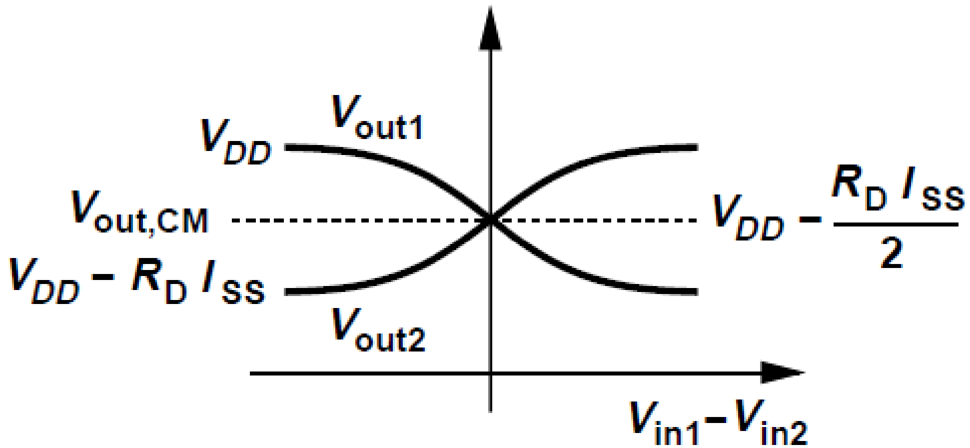
$$A_{DM} = \frac{\upsilon_{out1} - \upsilon_{out2}}{\upsilon_d} \qquad A_{CM-DM} = \frac{\upsilon_{out1} - \upsilon_{out2}}{\upsilon_{in,CM}}$$

$$A_{CM} = \frac{\upsilon_{out,CM}}{\upsilon_{in,CM}} \qquad CMRR = \left|\frac{A_{DM}}{A_{CM-DM}}\right|$$

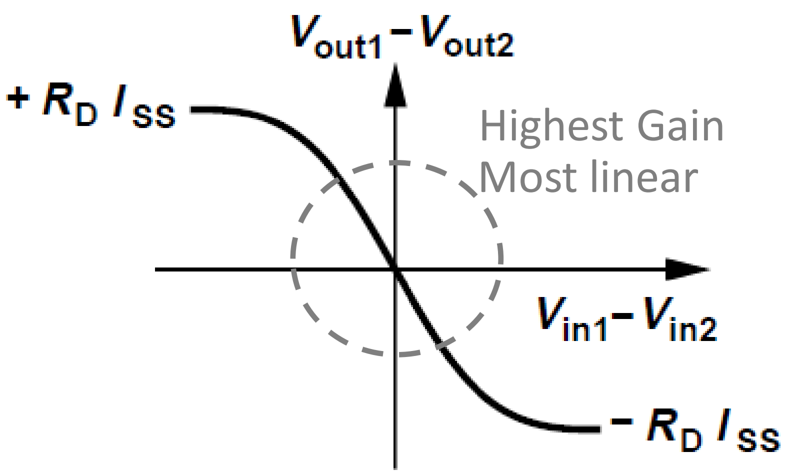
Differential-Mode (Qualitative Analysis)

Qualitative Analysis





(a)

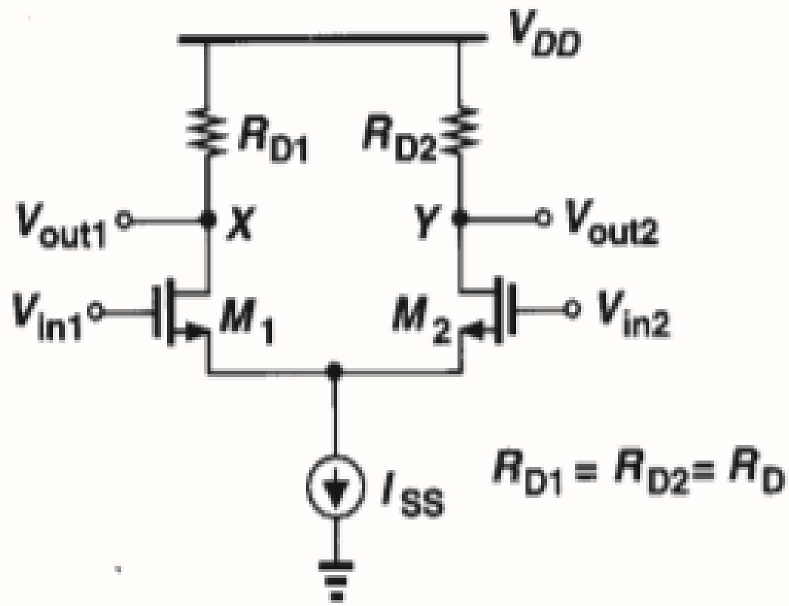


(b)

Differential-Mode (DC Analysis)

DC Analysis

$\lambda = 0 \quad \gamma = 0$



$$V_{in1} - V_{GS1} = V_{in2} - V_{GS2}$$

$$V_{in1} - V_{in2} = V_{GS1} - V_{GS2} = (V_{GS1} - V_{TH}) - (V_{GS2} - V_{TH}) = \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}\frac{W}{L}}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}\frac{W}{L}}}$$

$$(V_{in1} - V_{in2})^2 = \frac{2I_{D1}}{\mu_n C_{ox}\frac{W}{L}} + \frac{2I_{D2}}{\mu_n C_{ox}\frac{W}{L}} - 2\frac{\sqrt{4I_{D1}I_{D2}}}{\mu_n C_{ox}\frac{W}{L}} = \frac{2}{\mu_n C_{ox}\frac{W}{L}}\left(I_{SS} - 2\sqrt{I_{D1}I_{D2}}\right)$$

$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1} - V_{in2})^2 = I_{SS} - 2\sqrt{I_{D1}I_{D2}}$$

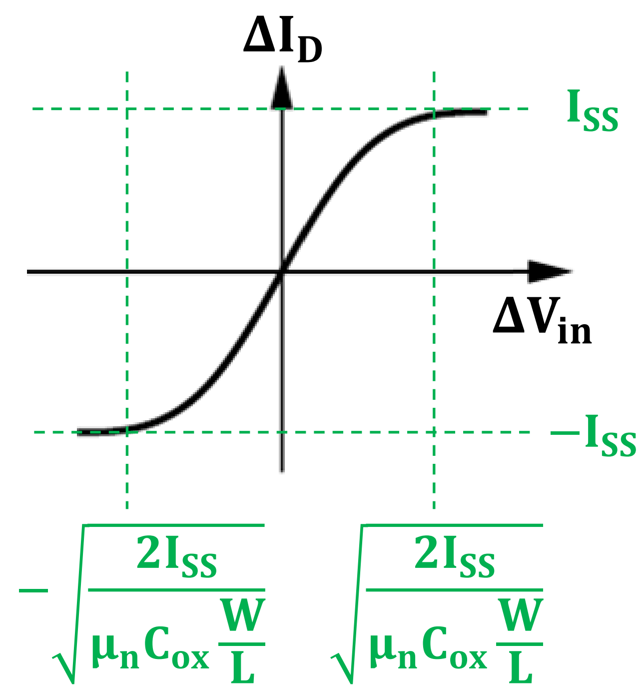
Differential-Mode (DC Analysis)

$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1}-V_{in2})^2 - I_{SS} = -2\sqrt{I_{D1}I_{D2}}$$

$$\frac{1}{4}\left(\mu_n C_{ox}\frac{W}{L}\right)^2(V_{in1}-V_{in2})^4 + I_{SS}{}^2 - \mu_n C_{ox}\frac{W}{L}(V_{in1}-V_{in2})^2 I_{SS} = 4I_{D1}I_{D2}$$

$$\frac{1}{4}\left(\mu_n C_{ox}\frac{W}{L}\right)^2(\underbrace{\boxed{V_{in1}-V_{in2}}}_{=\Delta V_{in}})^4 + \cancel{I_{SS}}{}^2 - \mu_n C_{ox}\frac{W}{L}(\underbrace{\boxed{V_{in1}-V_{in2}}}_{=\Delta V_{in}})^2 I_{SS} = \cancel{I_{SS}}{}^2 - (\underbrace{\boxed{I_{D1}-I_{D2}}}_{=\Delta I_D})^2$$

$$\boxed{\Delta I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\Delta V_{in}\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - \Delta V_{in}{}^2}}$$

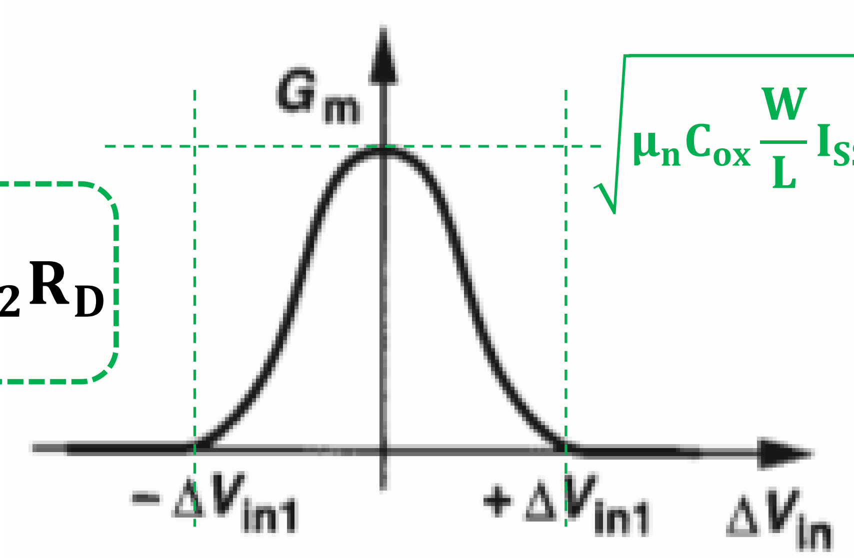


Differential-Mode (DC Analysis)

$$G_m = \frac{\partial \Delta I_D}{\partial \Delta V_{in}} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\frac{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - 2\Delta V_{in}{}^2}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - \Delta V_{in}{}^2}}$$

At $\Delta V_{in} = 0$,

$$A_{DM} = \frac{V_{out1} - V_{out2}}{\Delta V_{in}} = -gm_{1,2}R_D$$



$$\sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}} = gm_{1,2}$$

$$-\sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \qquad \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}$$

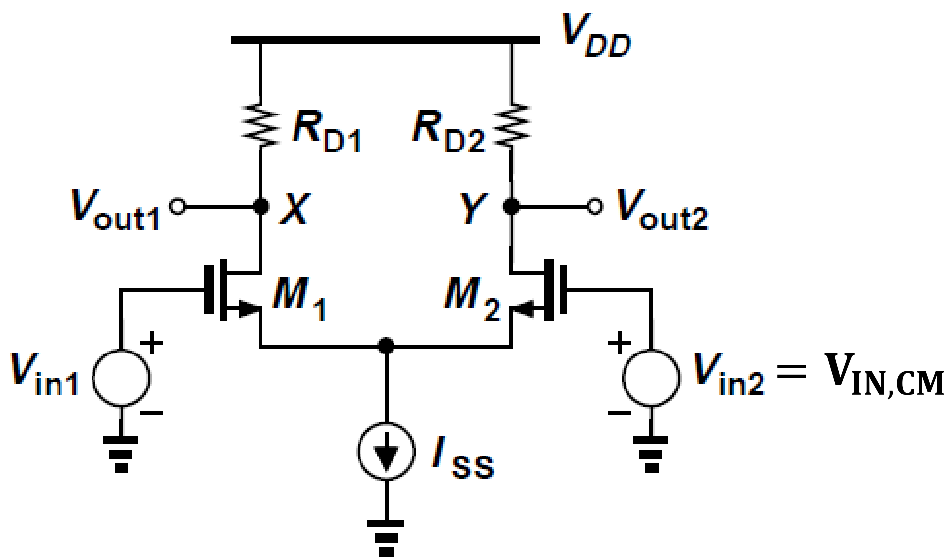
- Larger I_{SS} leads to higher G_m and wider input range.
- Smaller W/L leads to lower G_m but wider input range.

Differential-Mode (Small-Signal, Superposition)

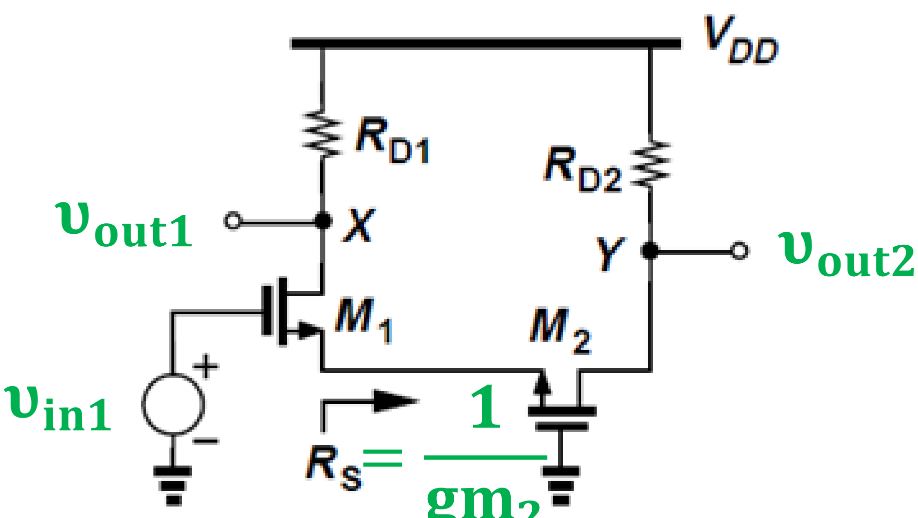
Small-signal Analysis

$\lambda = 0 \ \gamma = 0$

Complete circuit



Small-signal circuit



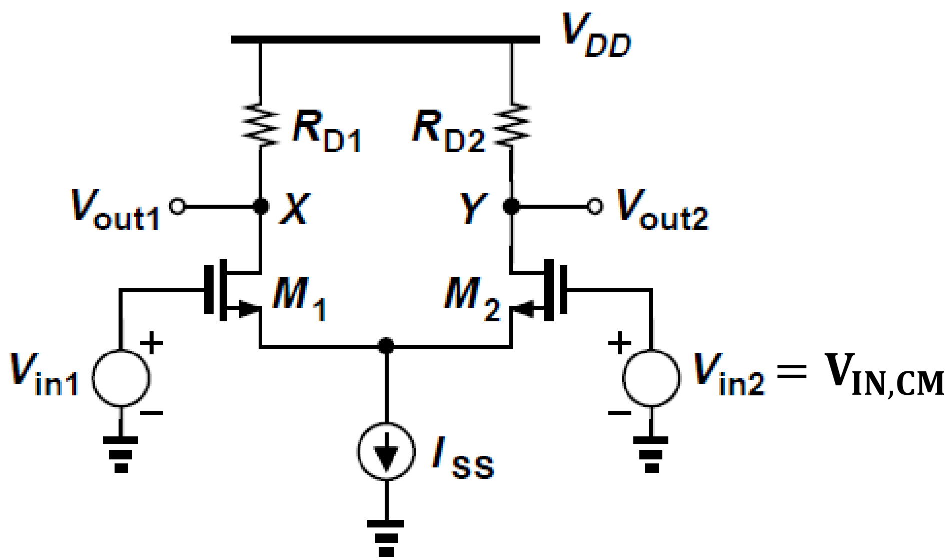
$$\upsilon_{out1} = -\frac{R_D}{\frac{1}{gm_1} + \frac{1}{gm_2}}\upsilon_{in1}$$

Differential-Mode (Small-Signal, Superposition)

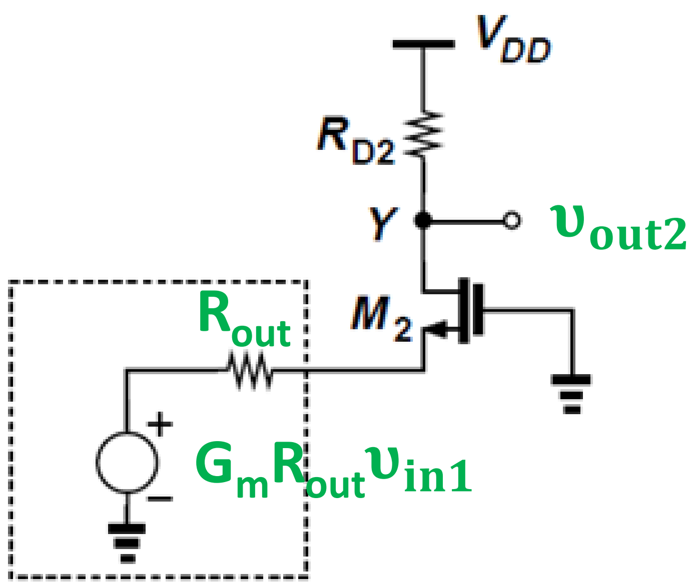
Small-signal Analysis

$\lambda = 0 \quad \gamma = 0$

Complete circuit



Small-signal circuit



$$G_m = gm_1$$

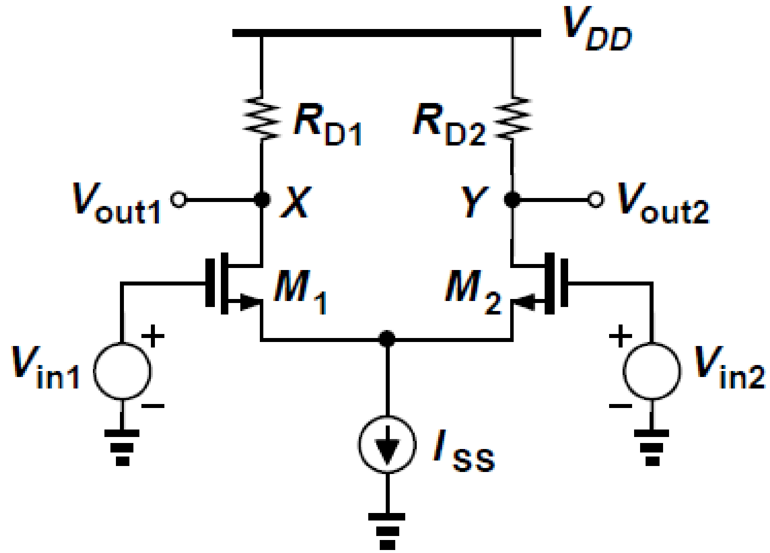
$$R_{out} = \frac{1}{gm_1}$$

$$\upsilon_{out2} = \frac{R_D}{\frac{1}{gm_1} + \frac{1}{gm_2}} \upsilon_{in1}$$

Differential-Mode (Small-Signal, Superposition)

Small-signal Analysis

λ = 0 γ = 0



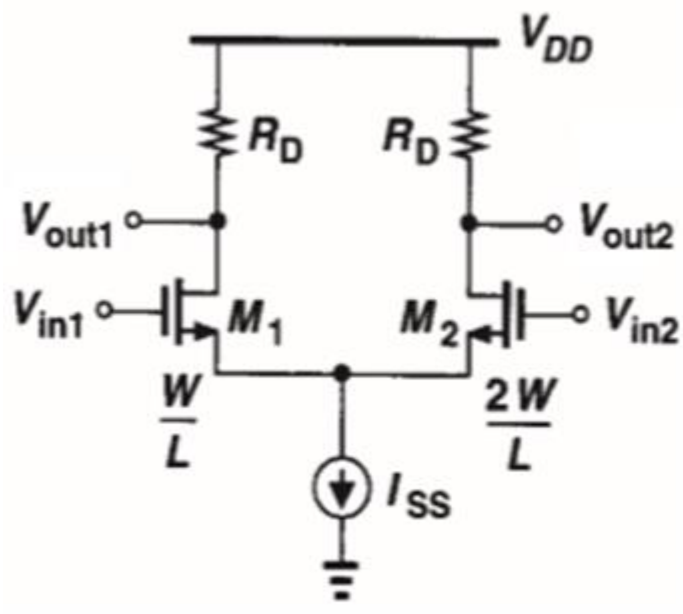
$$\upsilon_{out1} - \upsilon_{out2} = -\frac{2R_D}{\frac{1}{gm_1} + \frac{1}{gm_2}} \upsilon_{in1} = -gmR_D\upsilon_{in1} \qquad (1)$$

$$\upsilon_{out1} - \upsilon_{out2} = \frac{2R_D}{\frac{1}{gm_1} + \frac{1}{gm_2}} \upsilon_{in2} = gmR_D\upsilon_{in2} \qquad (2)$$

$$\boxed{A_{DM} = \frac{\upsilon_{out1} - \upsilon_{out2}}{\upsilon_{in1} - \upsilon_{in2}} = -gmR_D} \qquad (1) + (2)$$

Example

Calculate the A_{DM} of the differential pair below if the biasing conditions of M_1 and M_2 are the same.



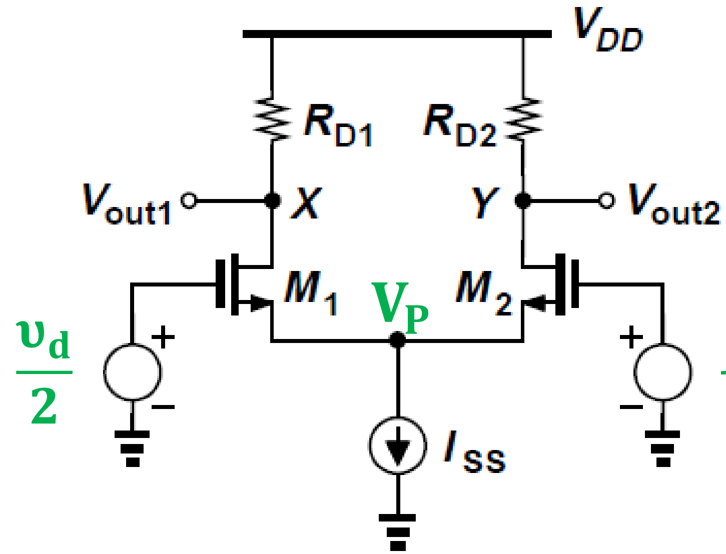
$$\begin{cases} \upsilon_{out1}-\upsilon_{out2}=-\dfrac{2R_D}{\dfrac{1}{gm_1}+\dfrac{1}{2gm_1}}\upsilon_{in1}=-\dfrac{4}{3}gmR_D\upsilon_{in1} \\ \upsilon_{out1}-\upsilon_{out2}=\dfrac{2R_D}{\dfrac{1}{gm_1}+\dfrac{1}{2gm_1}}\upsilon_{in2}=\dfrac{4}{3}gmR_D\upsilon_{in2} \end{cases}$$

$$\mathbf{A_{DM}=\frac{\upsilon_{out1}-\upsilon_{out2}}{\upsilon_{in1}-\upsilon_{in2}}=-\frac{4}{3}gmR_D}$$

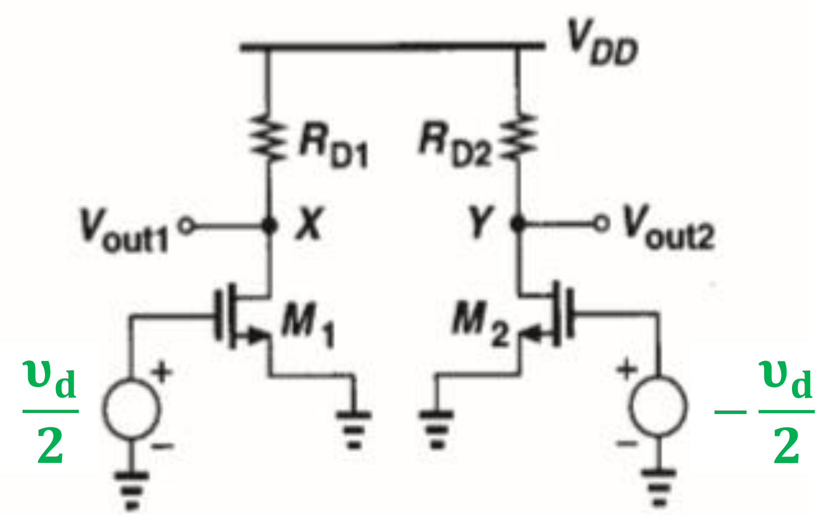
Differential-Mode (Small-Signal, Half-circuit)

Small-signal Analysis

$\lambda \neq 0$ $\gamma \neq 0$



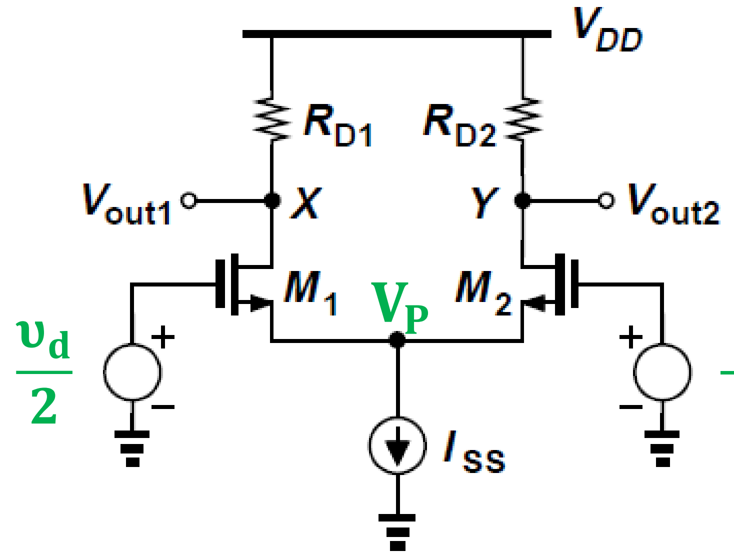
- Assume the circuit is fully symmetric.
- For $i_{d1}+i_{d2} = 0$and $gm_1 \frac{\upsilon_d}{2} + gm_2\left(-\frac{\upsilon_d}{2}\right) = 0$, V_P must be a constant voltage in DC analysis and a virtual ground in small-signal analysis.

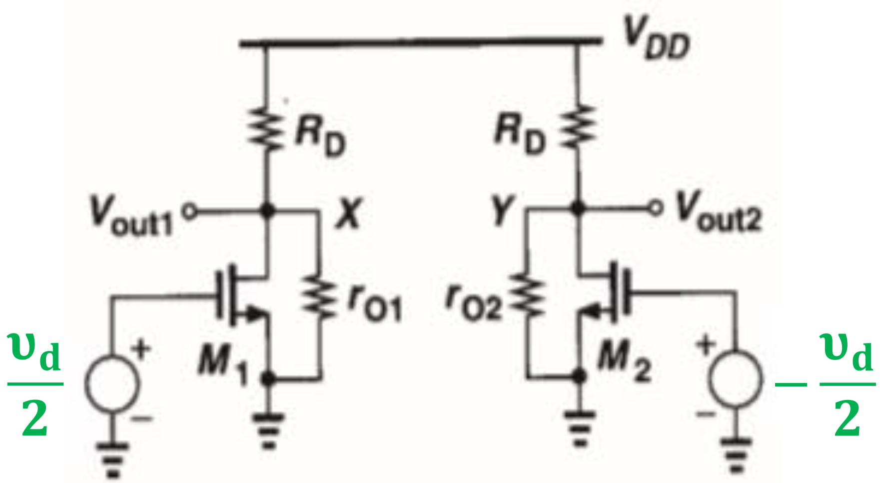


Differential-Mode (Small-Signal, Half-circuit)

Small-signal Analysis

$\lambda \neq 0 \ \gamma \neq 0$





$$\upsilon_{out1} = -gm(R_D \parallel r_o)\frac{\upsilon_d}{2}$$

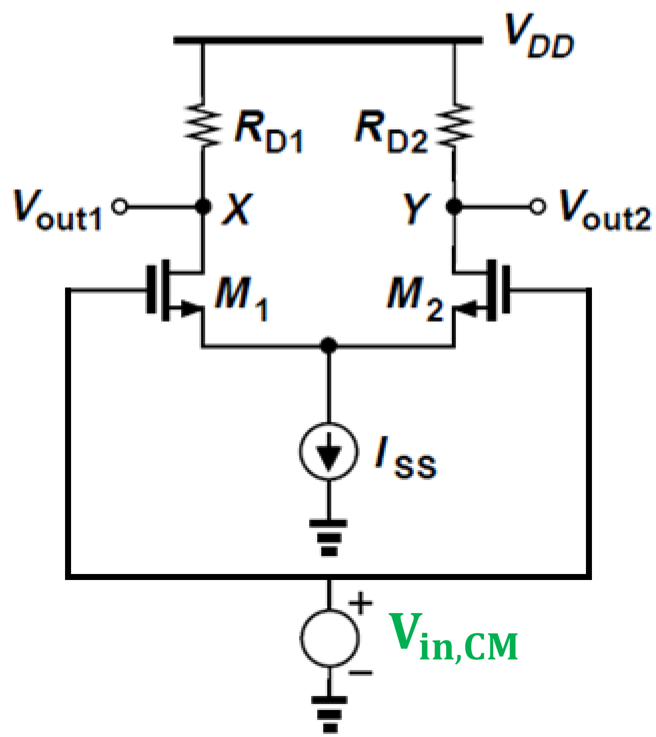
$$\upsilon_{out2} = -gm(R_D \parallel r_o)\left(-\frac{\upsilon_d}{2}\right)$$

$$\boxed{A_{DM} = \frac{\upsilon_{out1} - \upsilon_{out2}}{\upsilon_d} = -gm(R_D \parallel r_o)}$$

Common-Mode Response

Small-signal Analysis

$\lambda \neq 0$ $\gamma \neq 0$



If the circuit is fully symmetric,

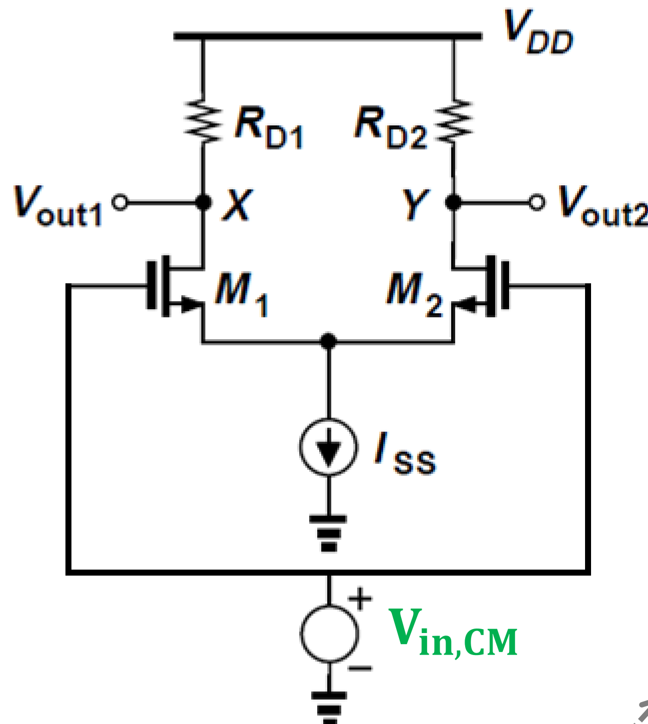
$$\mathbf{A_{CM-DM}} = \frac{\boldsymbol{\upsilon_{out1} - \upsilon_{out2}}}{\boldsymbol{\upsilon_{in,CM}}} = \mathbf{0}$$

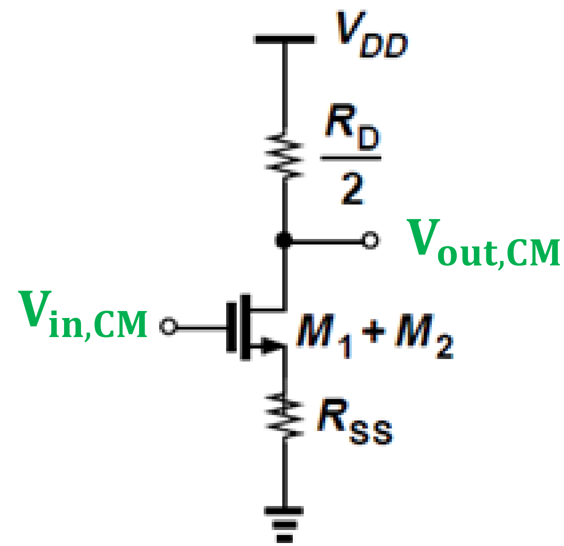
$$\mathbf{CMRR} = \left|\frac{\mathbf{A_{DM}}}{\mathbf{A_{CM-DM}}}\right| = \infty$$

Common-Mode Response

Small-signal Analysis

$\lambda \neq 0 \ \gamma \neq 0$





- Perturbing biasing condition → Altering transconductance (gm)

If the circuit is fully symmetric, $A_{CM} = \frac{\upsilon_{out,CM}}{\upsilon_{in,CM}}$

$$= \frac{-2gm\frac{r_o}{2}}{\cancel{\left[R_{SS} + \frac{r_o}{2} + (2gm + 2gmb)\frac{r_o}{2}R_{SS}\right]}} \cdot \frac{\cancel{\left[R_{SS} + \frac{r_o}{2} + (2gm + 2gmb)\frac{r_o}{2}R_{SS}\right]}\frac{R_D}{2}}{\left[R_{SS} + \frac{r_o}{2} + (2gm + 2gmb)\frac{r_o}{2}R_{SS}\right] + \frac{R_D}{2}}$$

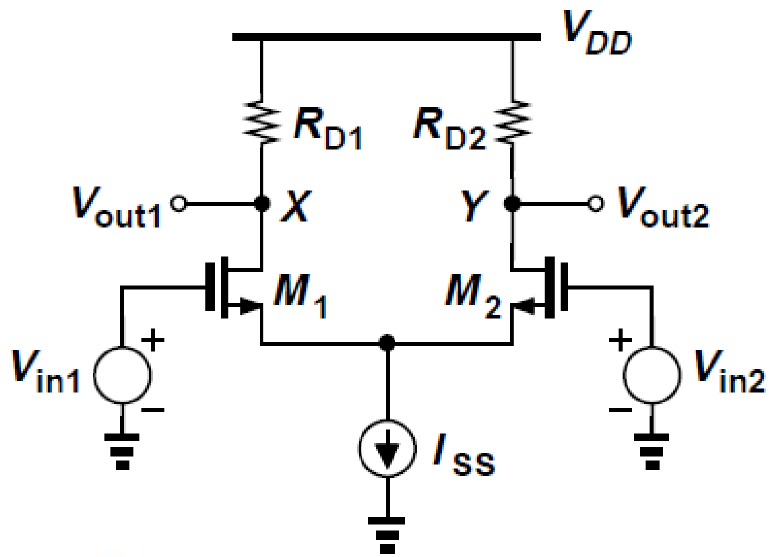
$= 0$ if $R_{SS} = \infty$

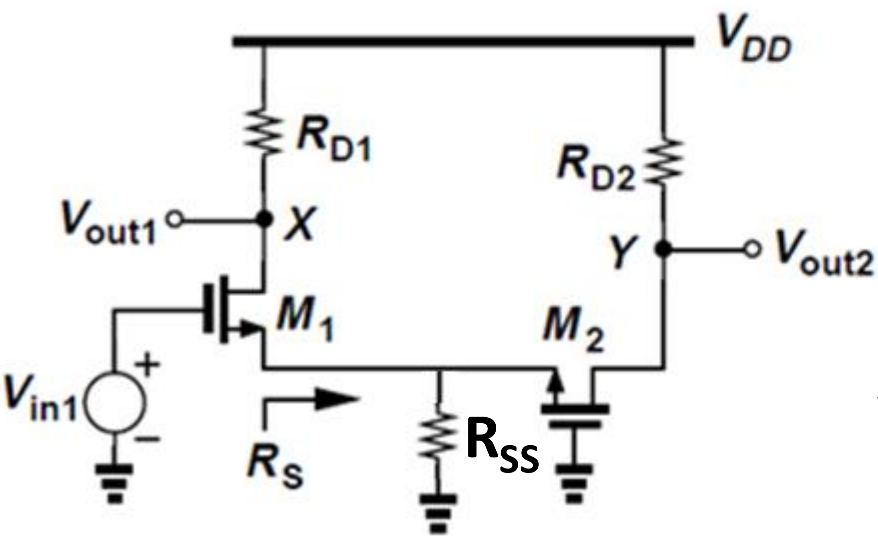
A_{DM} for Finite R_{SS}

Differential-Mode (Small-Signal, Superposition)

Small-signal Analysis

$\lambda = 0 \quad \gamma = 0$





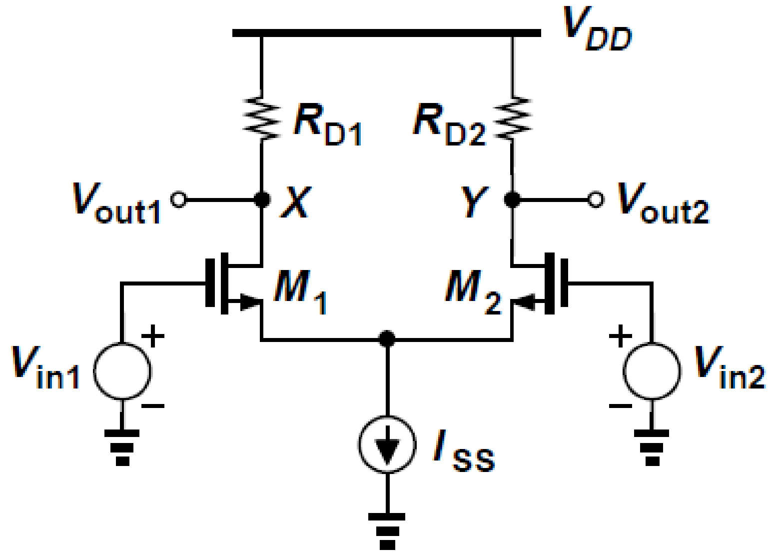
$$R_S = \frac{1}{gm_2} \parallel R_{SS}$$

$$\upsilon_{out1} = -\frac{R_D}{\frac{1}{gm_1} + \left(\frac{1}{gm_2} \parallel R_{SS}\right)}\upsilon_{in1}$$

Differential-Mode (Small-Signal, Superposition)

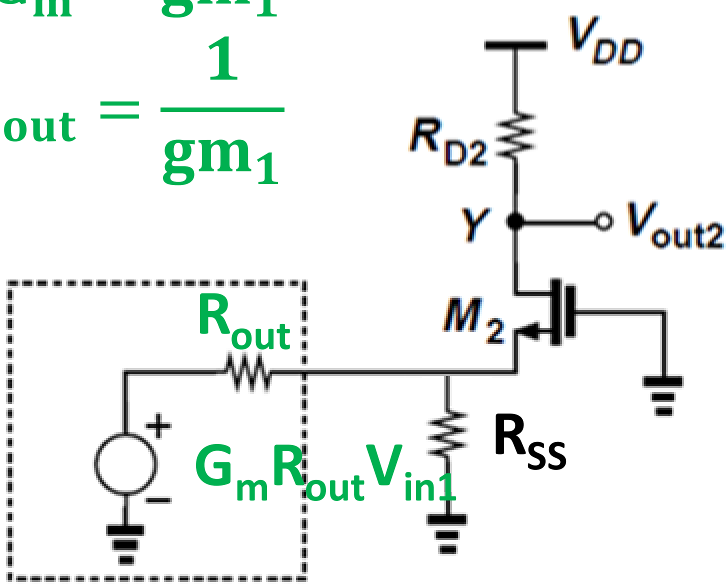
Small-signal Analysis

$\lambda = 0 \quad \gamma = 0$



$$G_m = gm_1$$

$$R_{out} = \frac{1}{gm_1}$$

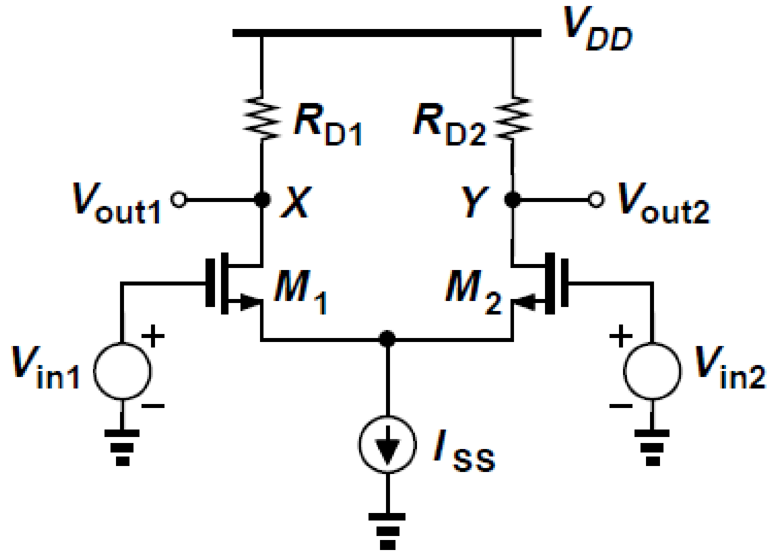


$$\upsilon_{out2} = \frac{\dfrac{R_{SS}}{R_{SS} + \dfrac{1}{gm_2}} R_D}{\dfrac{1}{gm_1} + \left(\dfrac{1}{gm_2} \parallel R_{SS}\right)} \upsilon_{in1}$$

Differential-Mode (Small-Signal, Superposition)

Small-signal Analysis

$\lambda = 0$ $\gamma = 0$



$$\upsilon_{out1} - \upsilon_{out2} = -\frac{(gm_1 + 2R_{SS}gm_1gm_2)R_D}{1 + (gm_1 + gm_2)R_{SS}}\upsilon_{in1} = -gmR_D\upsilon_{in1} \quad (1)$$

$$\upsilon_{out1} - \upsilon_{out2} = \frac{(gm_2 + 2R_{SS}gm_1gm_2)R_D}{1 + (gm_1 + gm_2)R_{SS}}\upsilon_{in2} = gmR_D\upsilon_{in2} \quad (2)$$

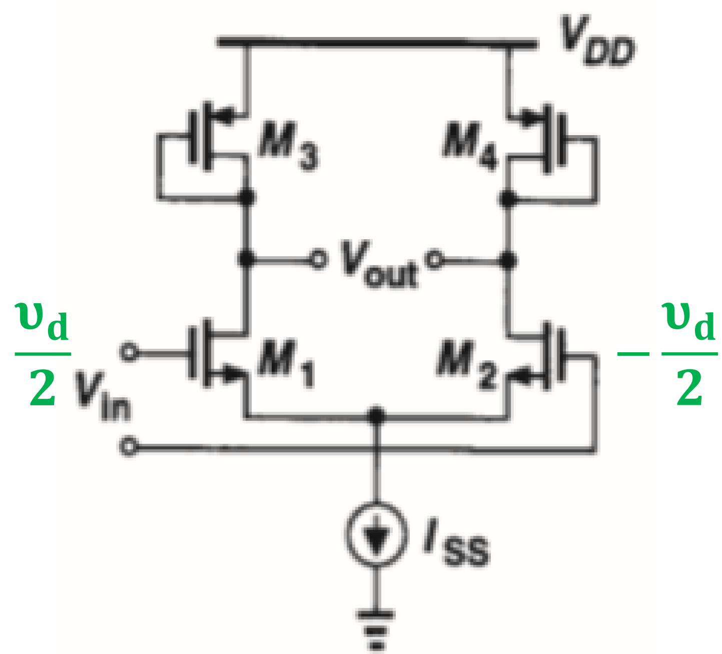
$$A_{DM} = \frac{\upsilon_{out1} - \upsilon_{out2}}{\upsilon_{in1} - \upsilon_{in2}} = -gmR_D \quad (1) + (2)$$

A_{DM} with MOS Loads

Differential-Mode (Small-Signal, Half-circuit)

Small-signal Analysis

$\lambda \neq 0 \ \gamma \neq 0$



- Higher A_{DM}
  → Smaller $(W/L)_P$
  → Larger $(V_{SGP} - V_{THP})$
  → Smaller $V_{in,CM}$ headroom

$$\upsilon_{out1} = -gm_N\left(r_{oN} \parallel r_{oP} \parallel \frac{1}{gm_P}\right)\frac{\upsilon_d}{2}$$

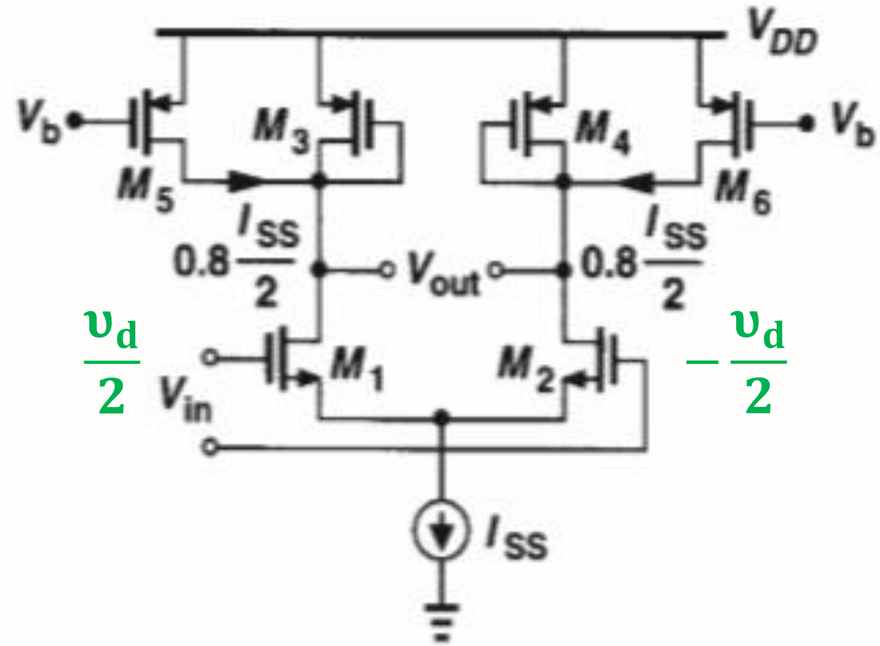
$$\upsilon_{out2} = -gm_N\left(r_{oN} \parallel r_{oP} \parallel \frac{1}{gm_P}\right)\left(-\frac{\upsilon_d}{2}\right)$$

$$A_{DM} = \frac{\upsilon_{out1} - \upsilon_{out2}}{\upsilon_d} = -gm_N\left(r_{oN} \parallel r_{oP} \parallel \frac{1}{gm_P}\right) \approx -\frac{gm_N}{gm_P} \approx -\sqrt{\frac{\mu_n (W/L)_N}{\mu_p (W/L)_P}}$$

Differential-Mode (Small-Signal, Half-circuit)

Small-signal Analysis

$\lambda \neq 0 \quad \gamma \neq 0$



$$\upsilon_{out1} = -gm_{1,2}\left(r_{o1,2} \parallel r_{o3,4} \parallel \frac{1}{gm_{3,4}} \parallel r_{o5,6}\right)\frac{\upsilon_d}{2}$$

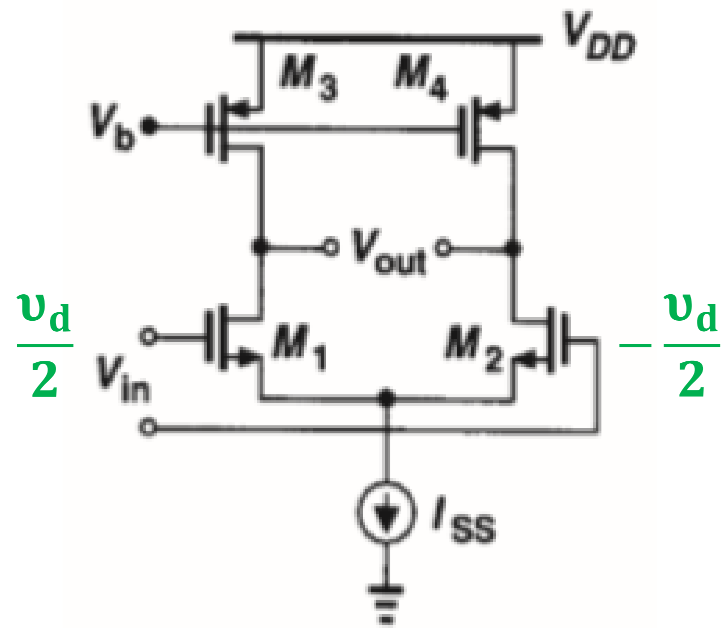
$$\upsilon_{out2} = -gm_{1,2}\left(r_{o1,2} \parallel r_{o3,4} \parallel \frac{1}{gm_{3,4}} \parallel r_{o5,6}\right)\left(-\frac{\upsilon_d}{2}\right)$$

$$A_{DM} = \frac{\upsilon_{out1} - \upsilon_{out2}}{\upsilon_d} \approx -\frac{gm_{1,2}}{gm_{3,4}} \approx -\sqrt{\frac{5\mu_n (W/L)_{1,2}}{\mu_p (W/L)_{3,4}}}$$

Differential-Mode (Small-Signal, Half-circuit)

Small-signal Analysis

$\lambda \neq 0 \quad \gamma \neq 0$



$$\upsilon_{out1} = -g m_{1,2}\left(r_{o1,2} \parallel r_{o3,4}\right)\frac{\upsilon_d}{2}$$

$$\upsilon_{out2} = -g m_{1,2}\left(r_{o1,2} \parallel r_{o3,4}\right)\left(-\frac{\upsilon_d}{2}\right)$$

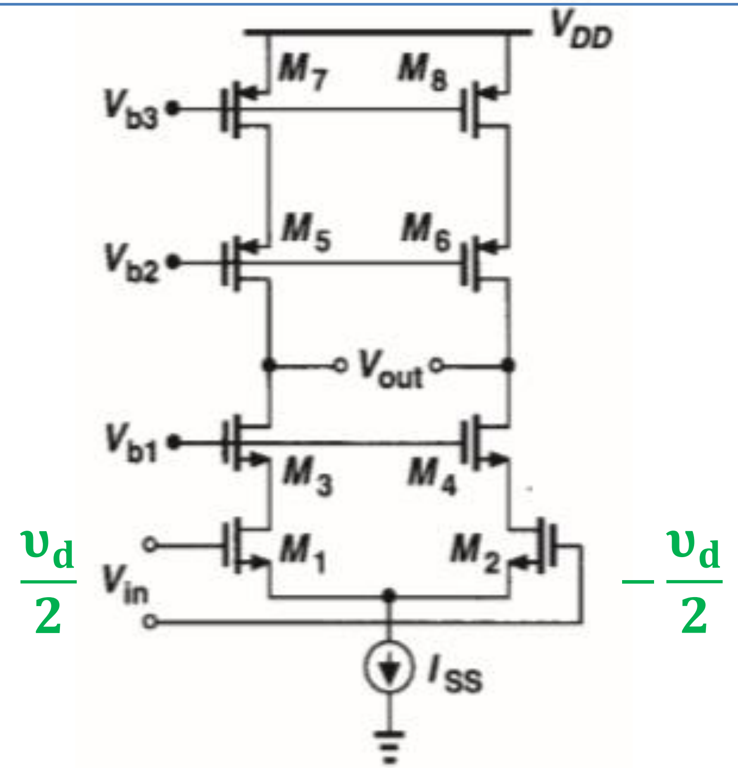
$$\boxed{A_{DM} = \frac{\upsilon_{out1} - \upsilon_{out2}}{\upsilon_d} = -g m_{1,2}\left(r_{o1,2} \parallel r_{o3,4}\right)}$$

Differential-Mode (Small-Signal, Half-circuit)

Small-signal Analysis

$\lambda \neq 0 \ \gamma \neq 0$

- High R_{out}
  → High A_{DM}
  → Small $V_{in,CM}$ headroom



$$\upsilon_{out1} \cong -gm_{1,2}\{[r_{o1,2} + r_{o3,4} + (gm_{3,4} + gmb_{3,4})r_{o3,4}r_{o1,2}] \parallel [r_{o7,8} + r_{o5,6} + (gm_{5,6} + gmb_{5,6})r_{o5,6}r_{o7,8}]\}\frac{\upsilon_d}{2}$$

$$\upsilon_{out2} \cong -gm_{1,2}\{[r_{o1,2} + r_{o3,4} + (gm_{3,4} + gmb_{3,4})r_{o3,4}r_{o1,2}] \parallel [r_{o7,8} + r_{o5,6} + (gm_{5,6} + gmb_{5,6})r_{o5,6}r_{o7,8}]\}\left(-\frac{\upsilon_d}{2}\right)$$

$$A_{DM} = \frac{\upsilon_{out1} - \upsilon_{out2}}{\upsilon_d} \cong -gm_{1,2}[(gm_{3,4} + gmb_{3,4})r_{o3,4}r_{o1,2} \parallel (gm_{5,6} + gmb_{5,6})r_{o5,6}r_{o7,8}]$$