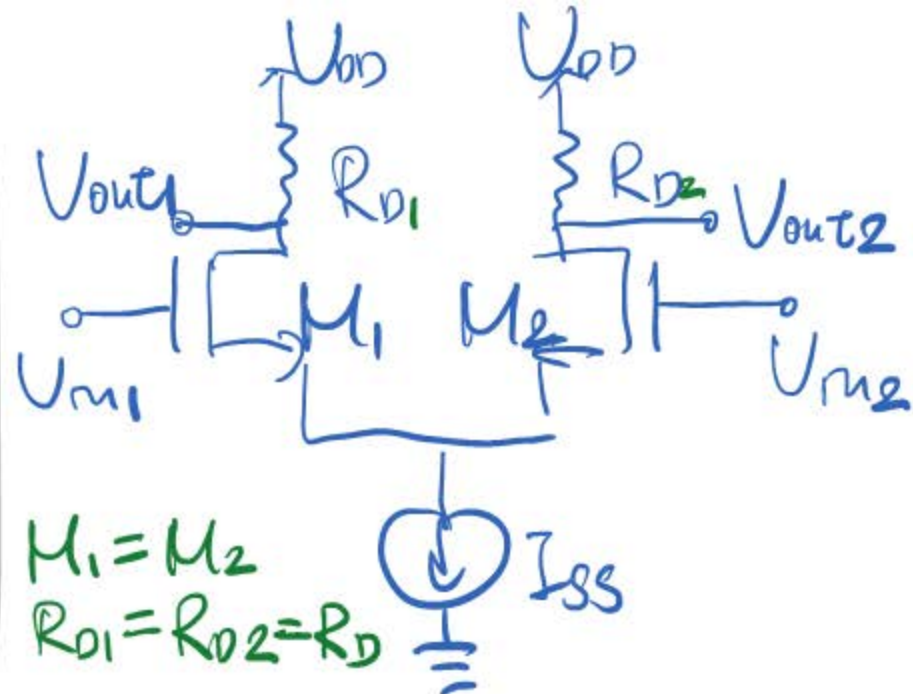
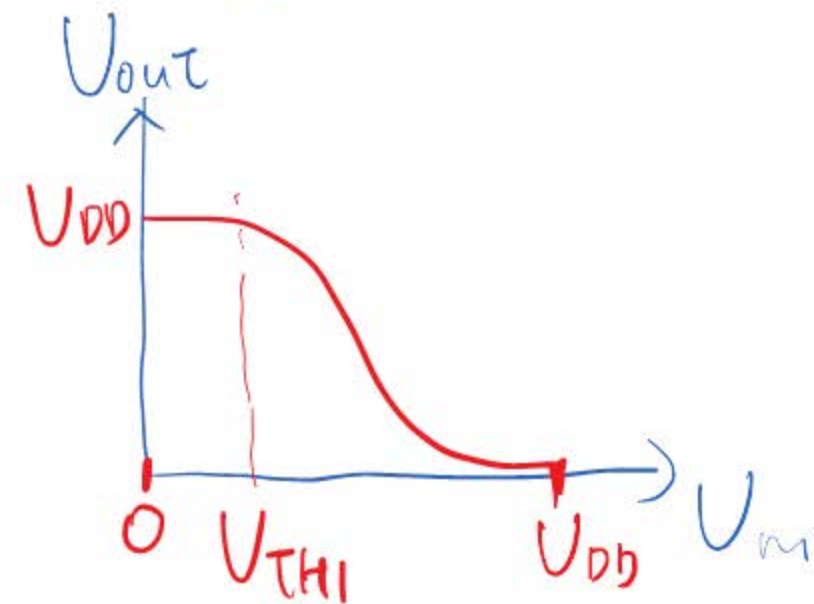
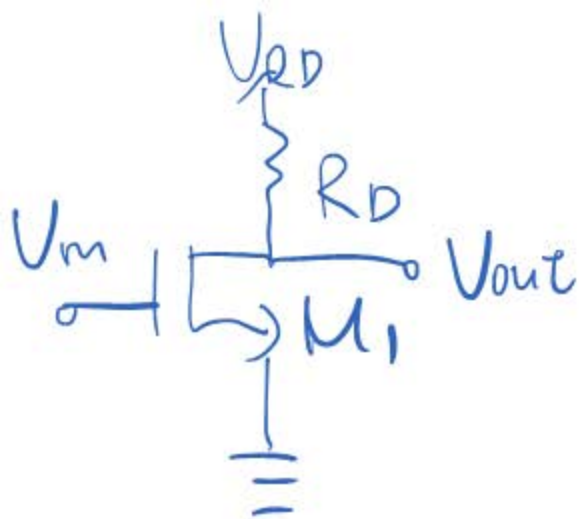


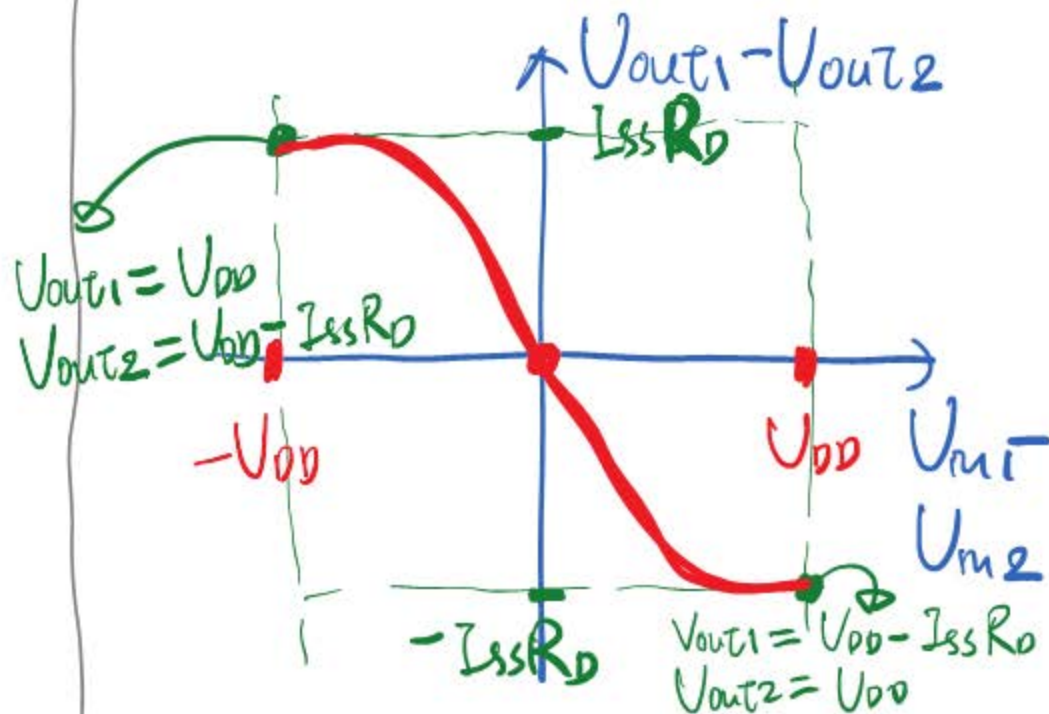
For the final exam:

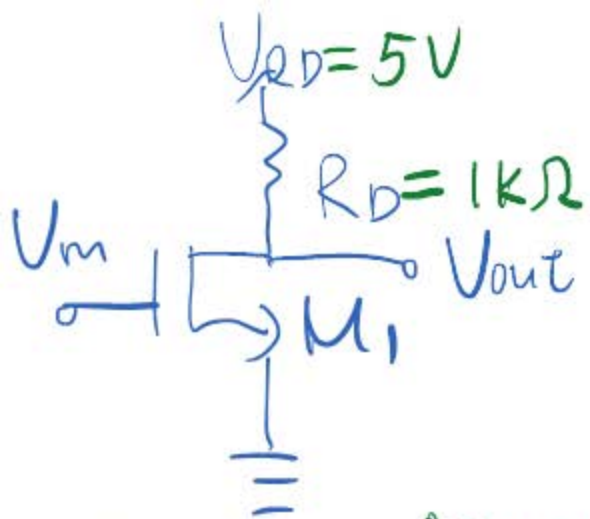
- $\sim 10\%$  upn BJT (CE, diode-connected load etc.)
- $\sim 80\%$  single stage amplifiers based on NMOS and PMOS
- $< 10\%$  differential pair



$$M_1 = M_2$$

$$R_{D1} = R_{D2} = R_D$$

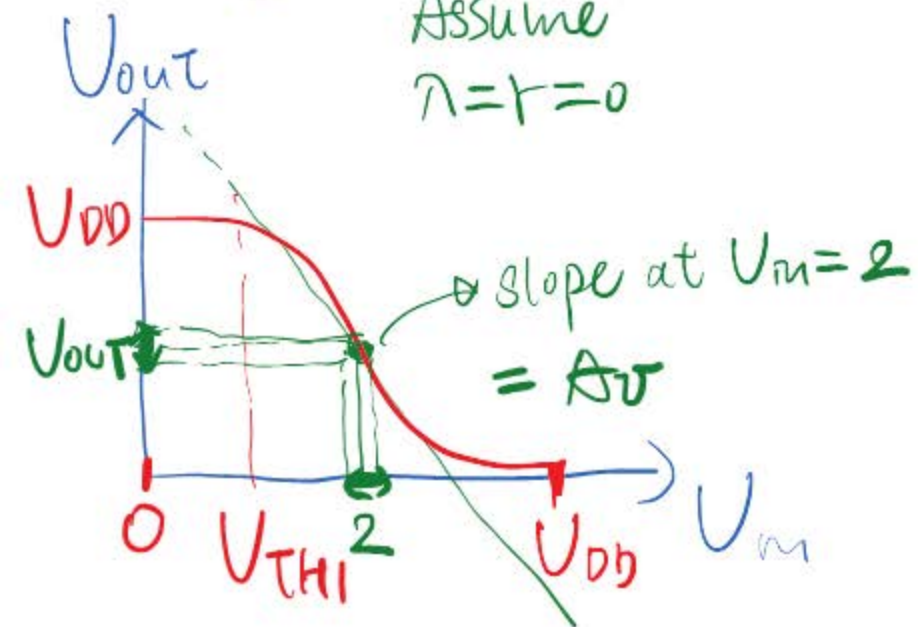
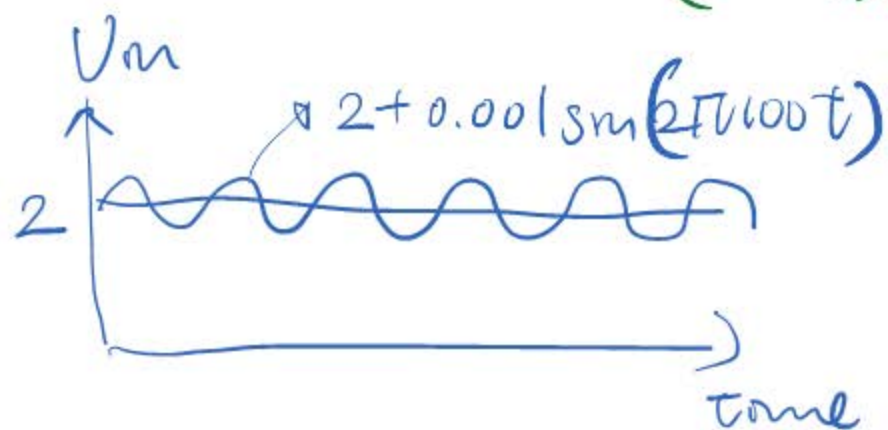




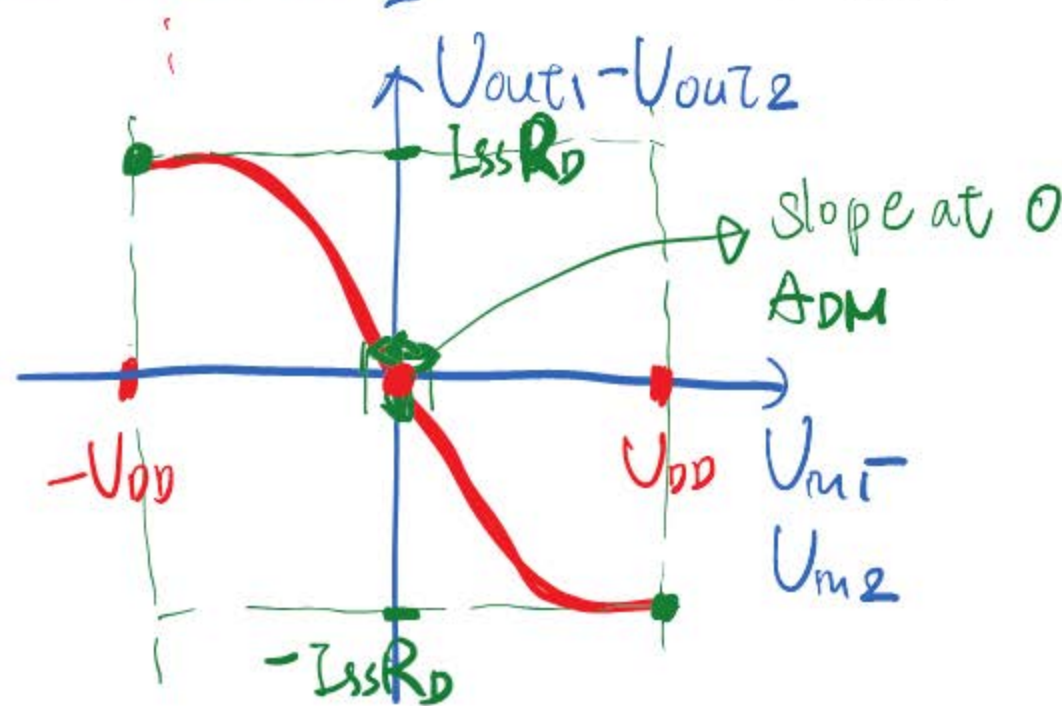
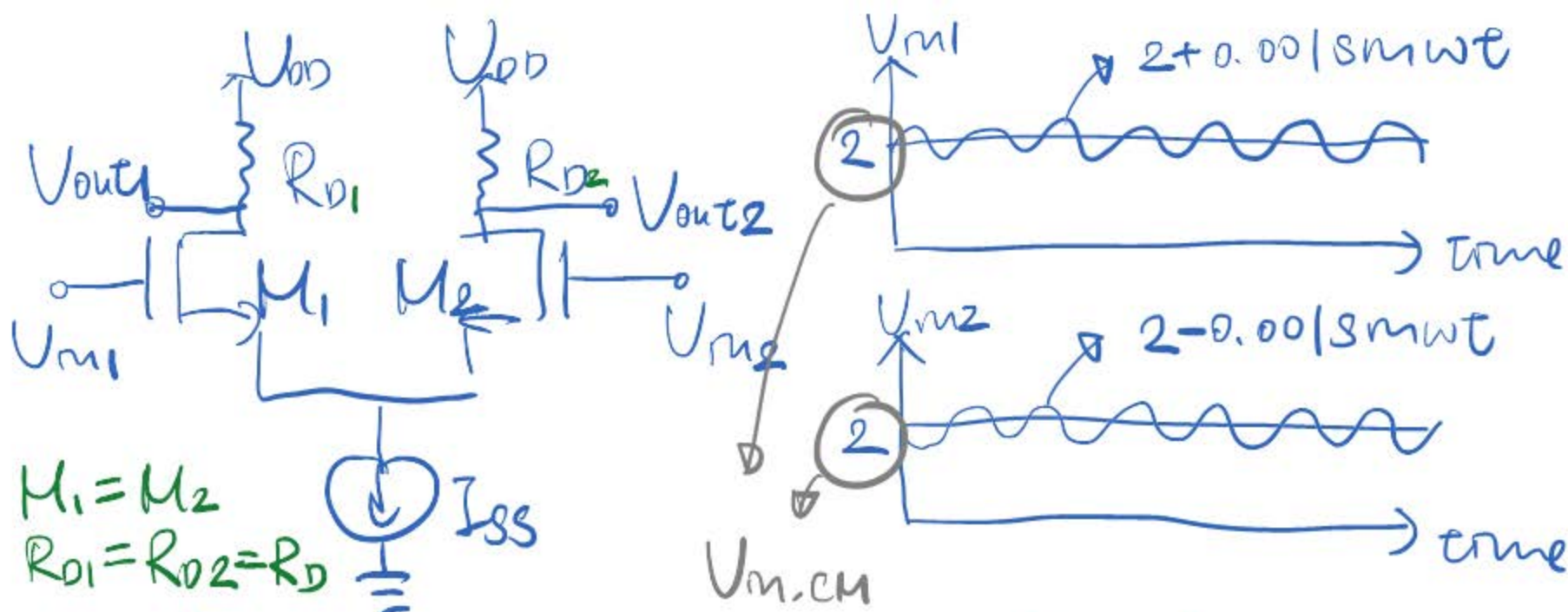
$$V_{in} = 2 + 0.001 \sin(2\pi 100t)$$

$$V_{out} = V_{outT} + (0.001 A_v) \sin(2\pi 100t)$$

Assume  
 $\lambda = r = 0$



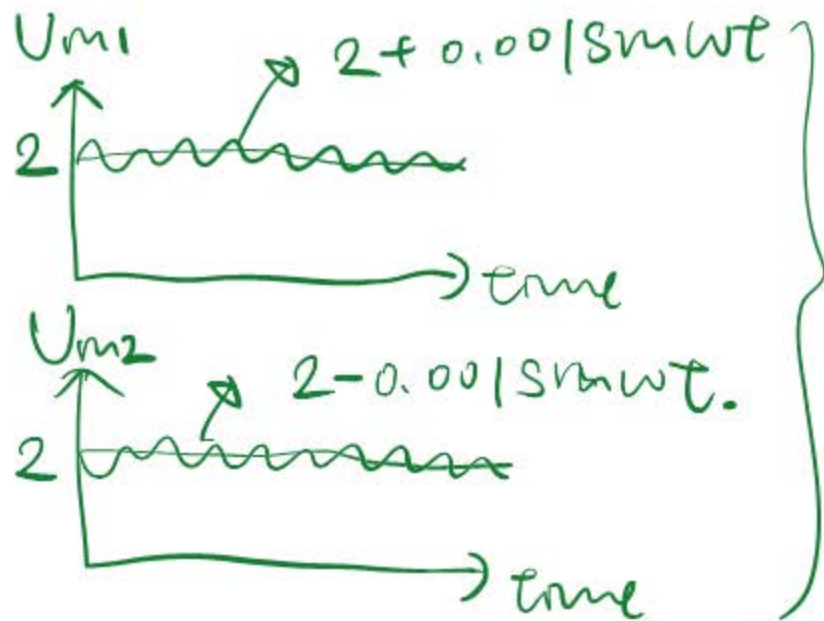
$$V_{outT} = 5 - (1k) \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L_{eff}} \right)_1 (2 - 0.7)^2$$



$$V_d = V_{m1} - V_{m2} = 0.002$$

$$V_{out} = V_{out1} - V_{out2} = 0.002 \text{ (ADM)}$$

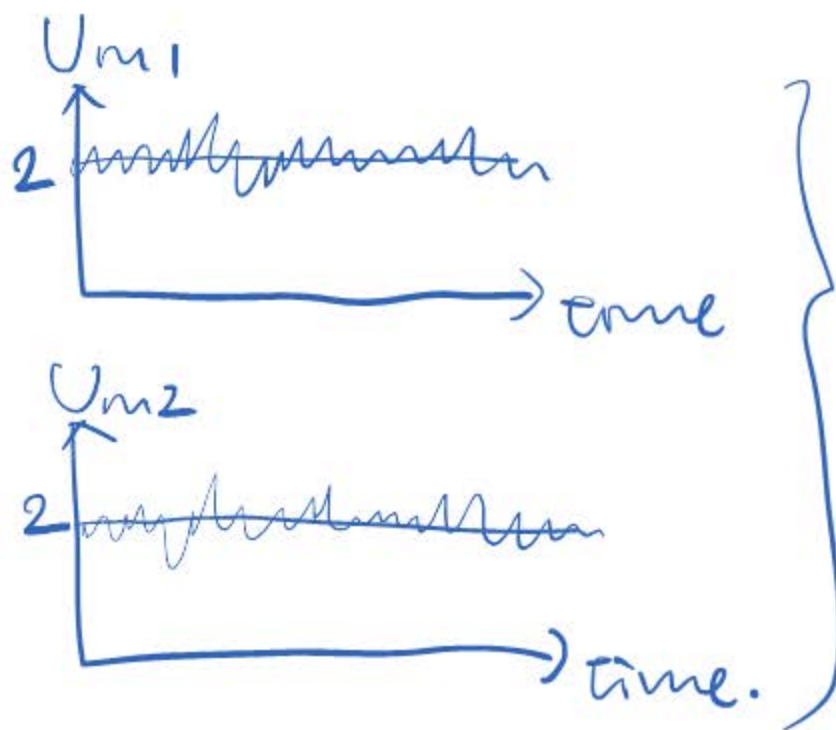




$$U_d = U_{m1} - U_{m2} = \overset{AC}{0.002 \sin \omega t}$$

$$U_{m,CM} = 2 = \frac{U_{m1} + U_{m2}}{2}$$

fully differential =  $\underbrace{U_{IN,CM}}_2 + \underbrace{U_{m,CM}}_0$



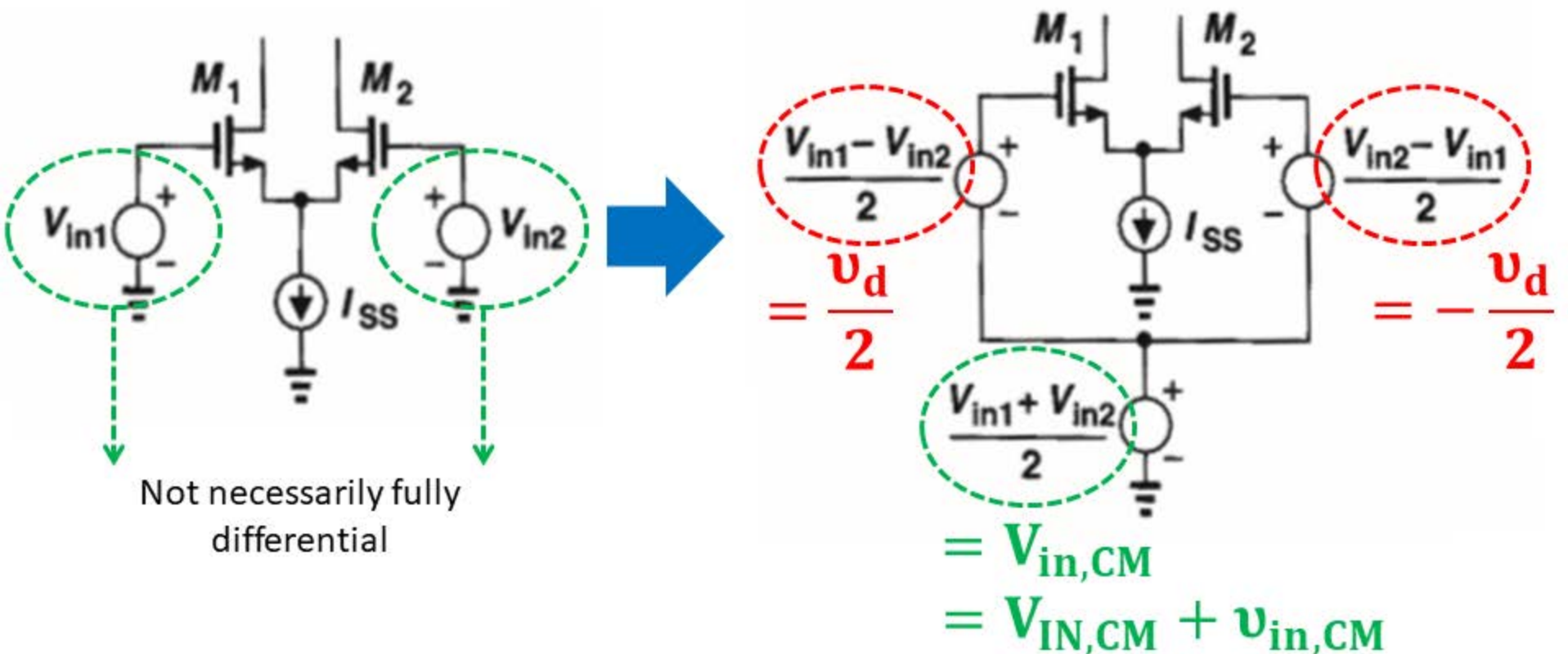
$$U_d = U_{m1} - U_{m2} = AC$$

$$U_{m,CM} = \frac{U_{m1} + U_{m2}}{2}$$

$$= \underbrace{U_{IN,CM}}_2 + \underbrace{U_{m,CM}}_0$$

Not fully differential

# Common-Mode + Differential-Mode



Not necessarily fully differential

If fully symmetric  
if  $I_{SS}$  ideal

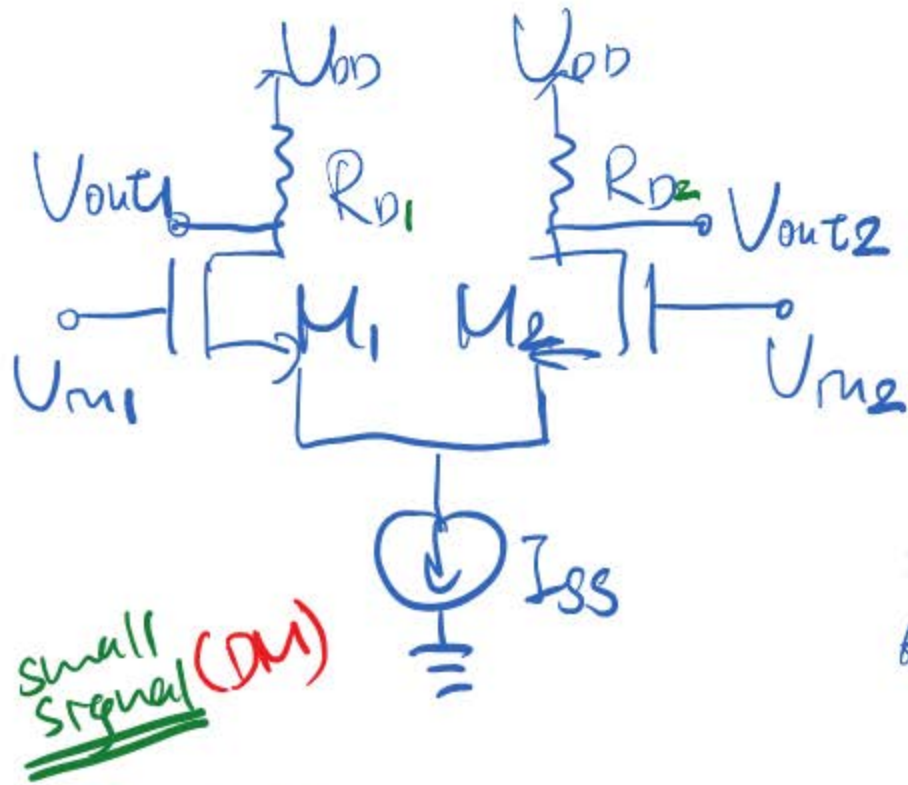
$$A_{DM} = \frac{v_{out1} - v_{out2}}{v_d}$$

$$A_{CM} = \frac{v_{out,CM}}{v_{in,CM}} = 0$$

$$A_{CM-DM} = \frac{v_{out1} - v_{out2}}{v_{in,CM}} = 0$$

$$CMRR = \left| \frac{A_{DM}}{A_{CM-DM}} \right|$$

If fully symmetric



$$\begin{aligned}
 1^{\circ} A_{DM} &= \frac{V_{out1} - V_{out2}}{V_{m1} - V_{m2}} = \frac{V_d}{V_d} \\
 &= \frac{V_{out1} - V_{out2}}{V_{m1} - V_{m2}} = \frac{V_d}{V_d} \\
 &= -g_{m1,2} (r_{o1,2} \parallel R_D)
 \end{aligned}$$

Assume all in sat.

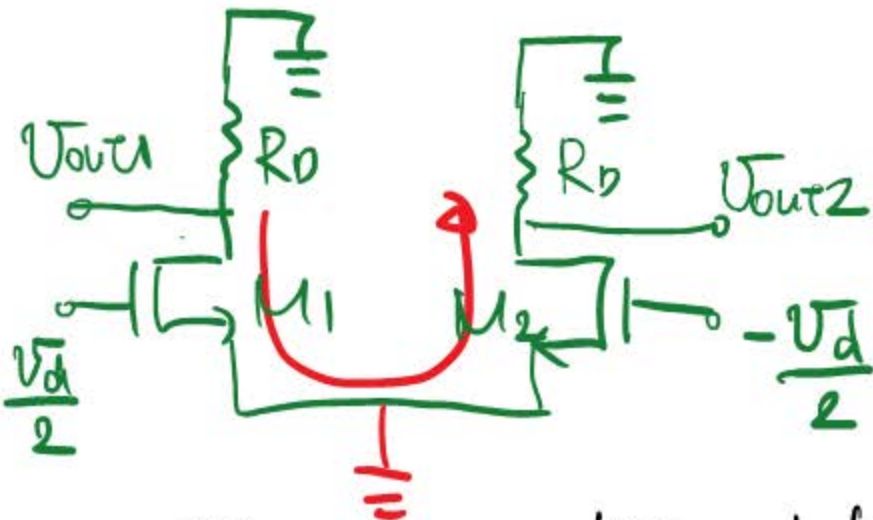
Assume fully symmetric circuit  
 $(R_{01} = R_{02} \text{ and } M_1 = M_2)$

$$g_{m1} = g_{m2} = g_{m1,2}$$

$$r_{o1} = r_{o2} = r_{o1,2}$$

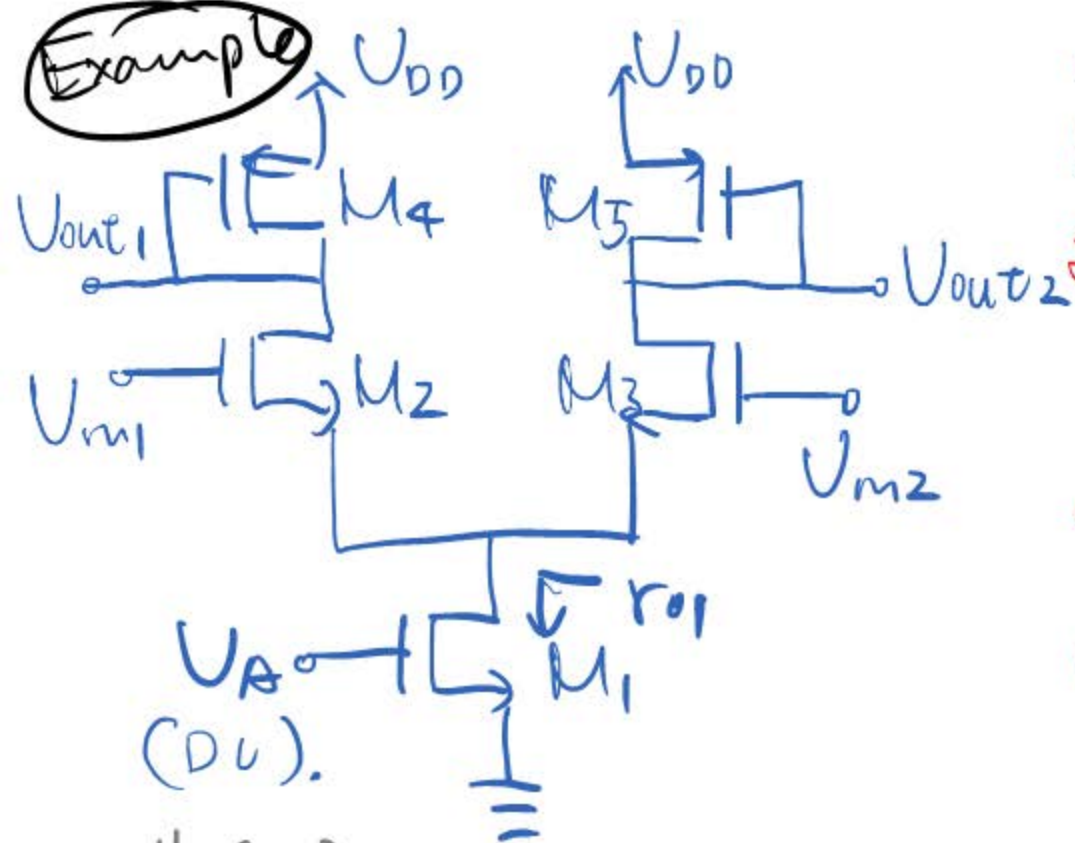
$$\begin{cases}
 V_{out1} = -g_{m1,2} (r_{o1,2} \parallel R_D) \frac{V_d}{2} \\
 V_{out2} = -g_{m1,2} (r_{o1,2} \parallel R_D) \left(-\frac{V_d}{2}\right)
 \end{cases}$$

$$V_{gs1} = V_{gs2} \quad \left| \frac{V_d}{2} g_{m1} \right| = \left| \left(-\frac{V_d}{2}\right) g_{m2} \right|$$

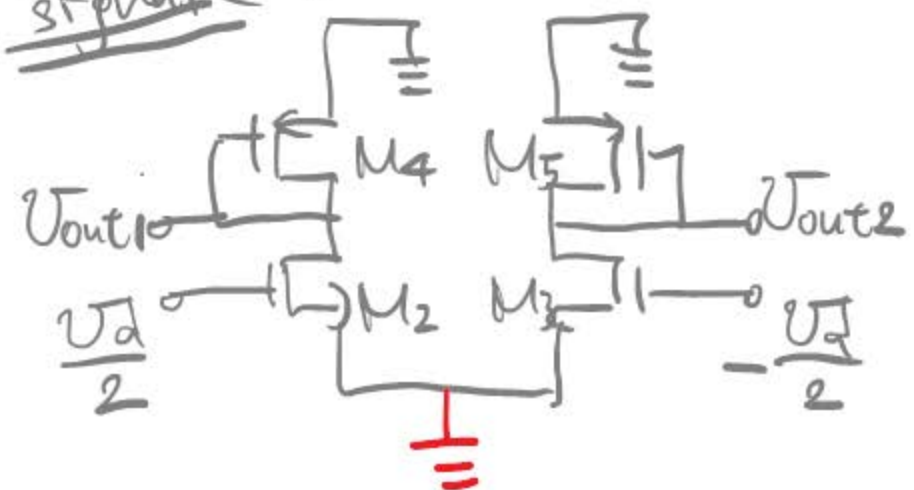




Example  $\uparrow V_{DD}$



Small signal (DM)



Assume  
1' All in sat.

2° Fully symmetric  
( $\mu_2 = \mu_3, \mu_4 = \mu_5$ )

3°  $\lambda = \tau = 0$ .

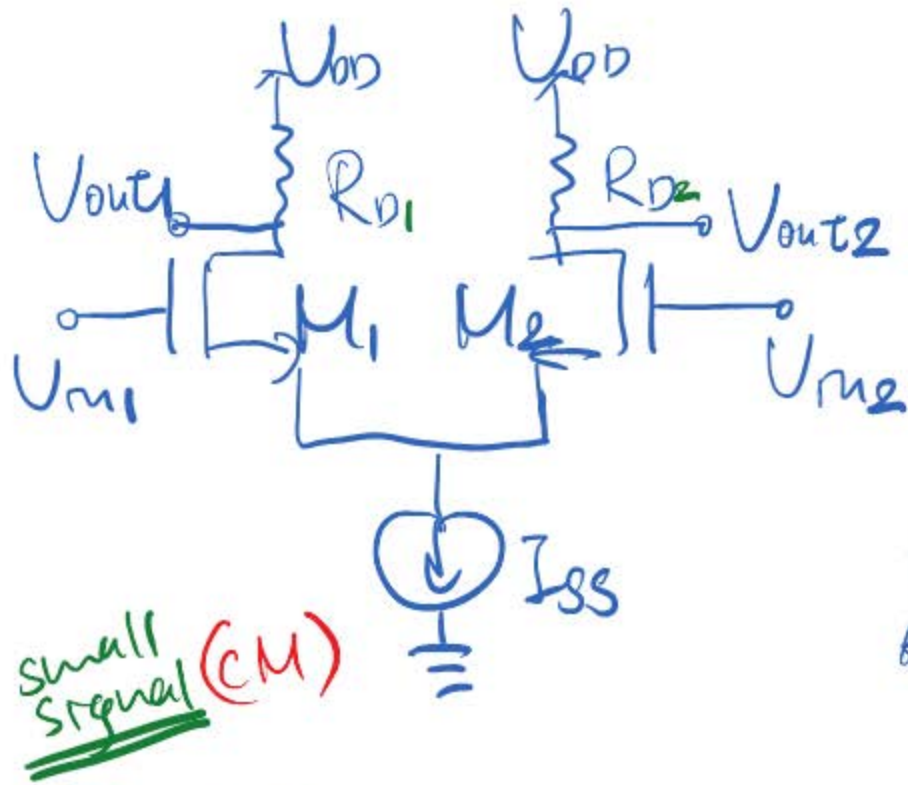
$$A_{OM} = \frac{V_{out1} - V_{out2}}{V_d}$$

$$= -g_{m2,3} \left( \frac{1}{g_{m4,5}} \right)$$

$$g_{m2} = g_{m3} = g_{m2,3}$$

$$g_{m4} = g_{m5} = g_{m4,5}$$



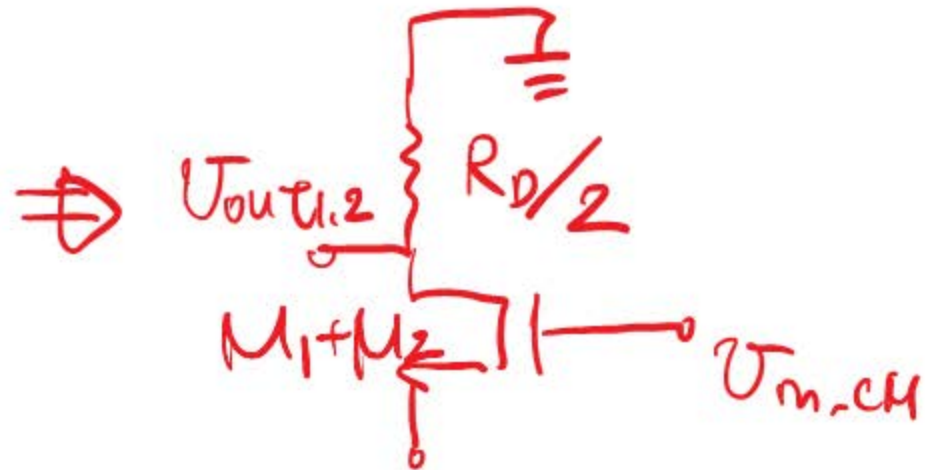
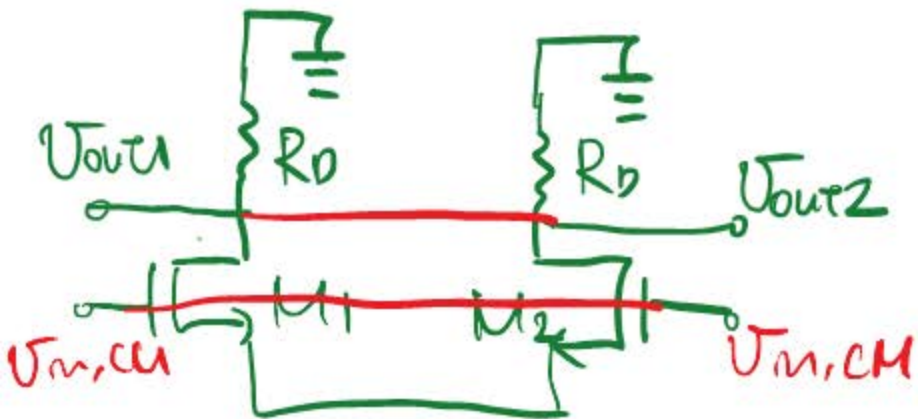


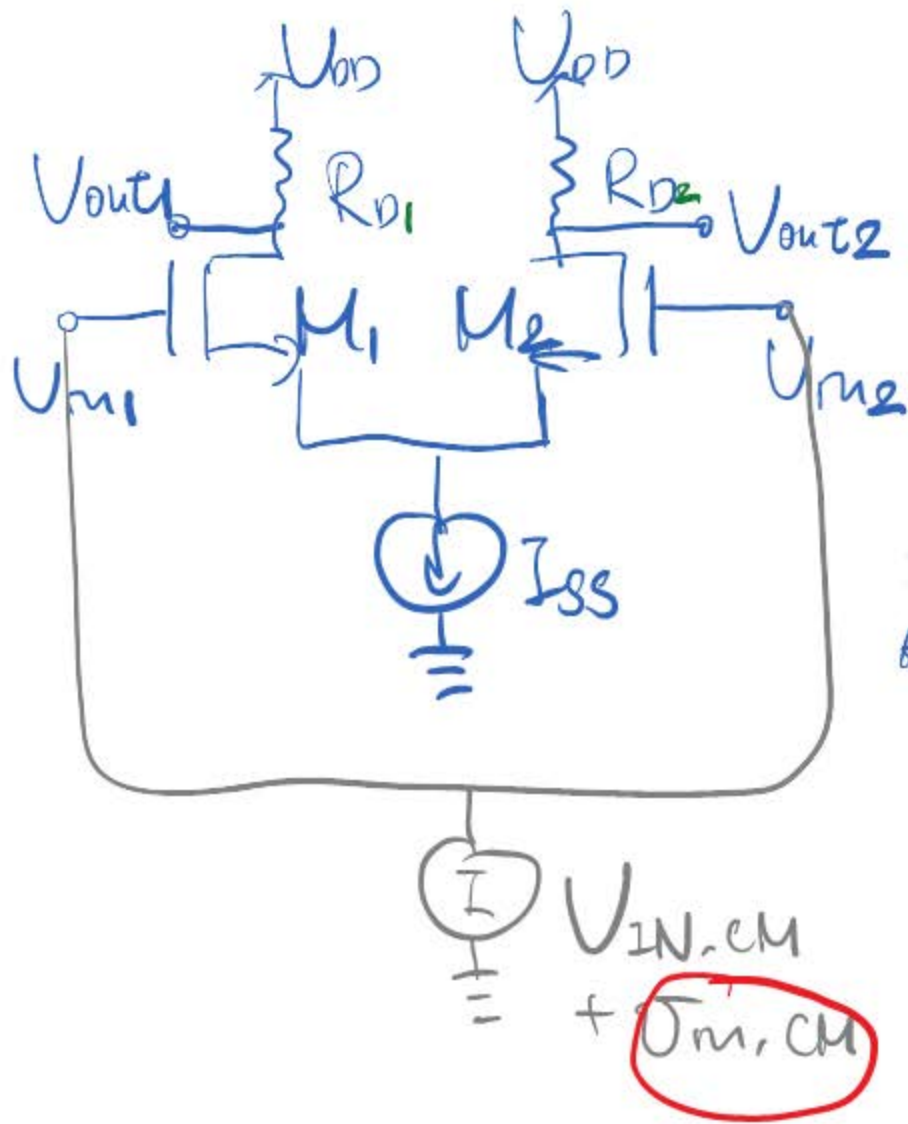
$$2^{\circ} A_{CM} = \frac{V_{out1,2}}{V_{in,CM}} = 0$$

$$V_{out1} = V_{out2} = V_{out1,2}$$

Assume all in sat.

Assume fully symmetric  
( $R_{D1} = R_{D2}$ ,  $\mu_1 = \mu_2$ )



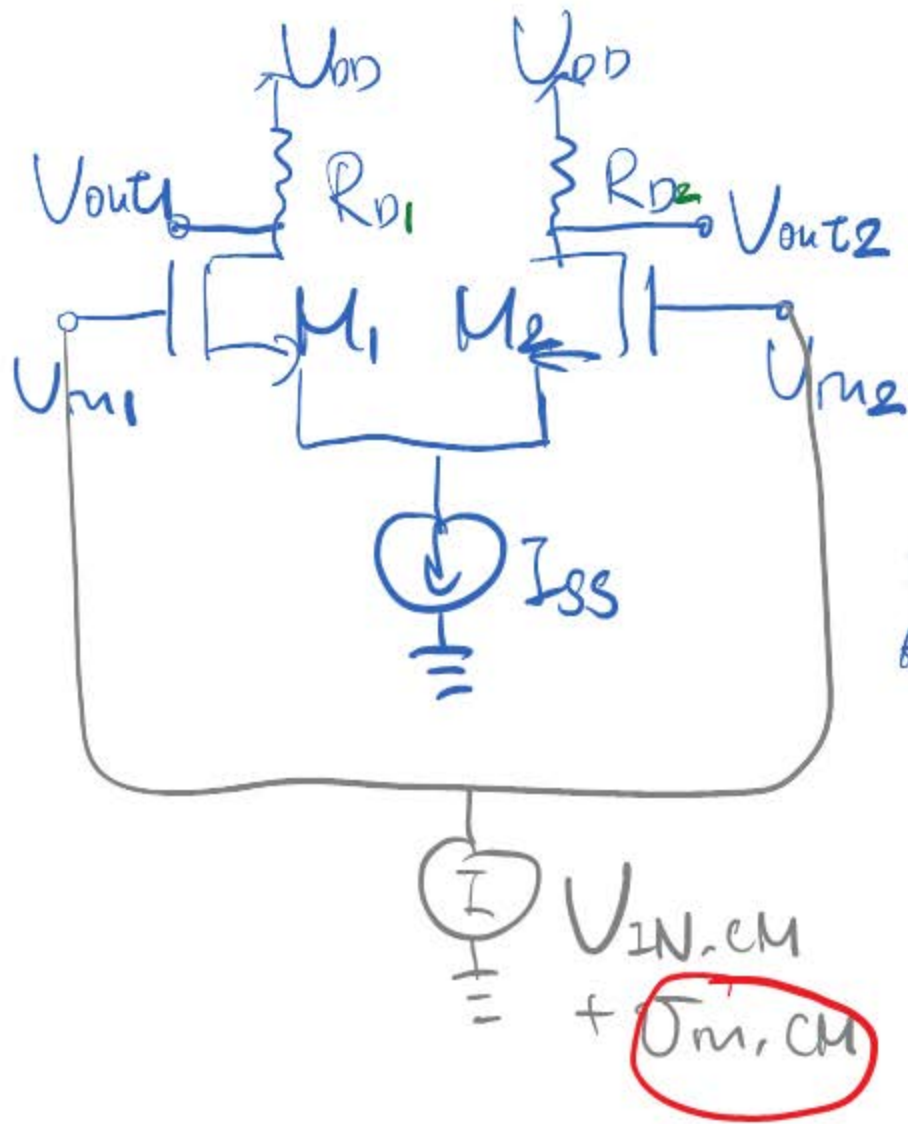


$$2^{\circ} A_{CM} = \frac{V_{out1,2}}{V_{in,CM}} = 0$$

Assume all in sat.

Assume fully symmetric  
( $R_{D1} = R_{D2}$ ,  $M_1 = M_2$ )

$$V_{out1} = V_{out2} = V_{DD} - R_D \frac{I_{SS}}{2}$$



$$3^{\circ} A_{CM-DM} = \frac{V_{out1} - V_{out2}}{V_{m,CM}}$$

$$= 0$$

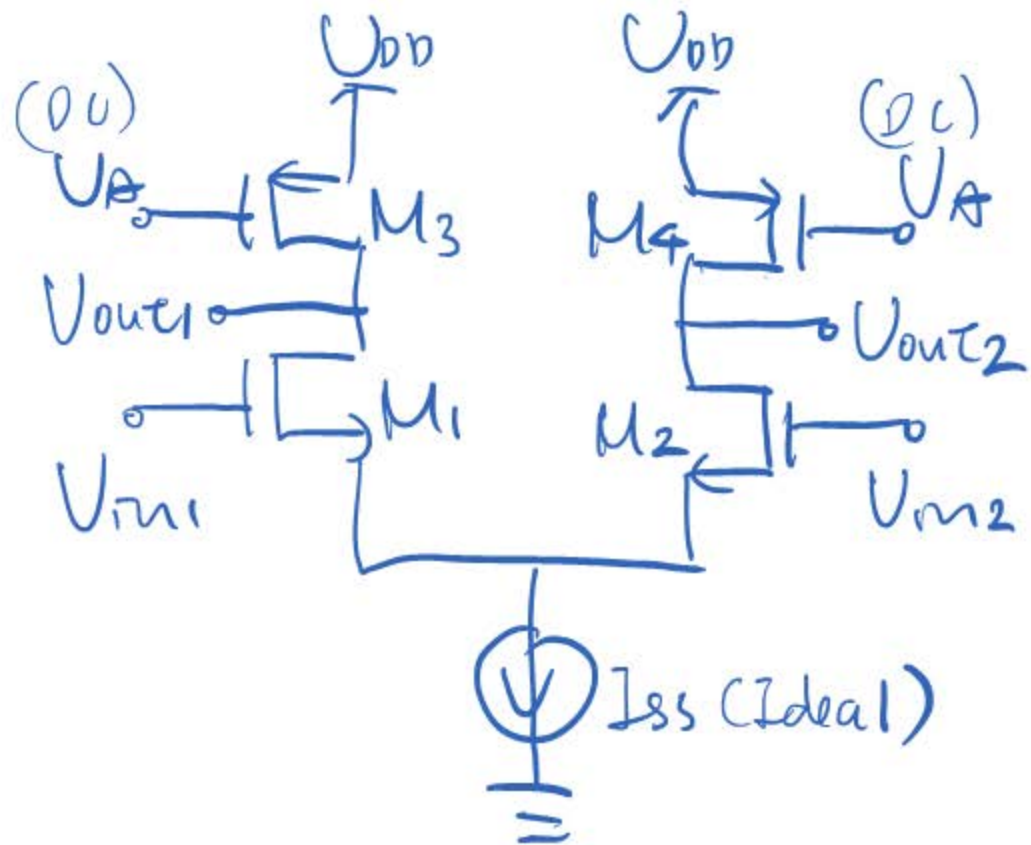
Assume all in sat.

Assume fully symmetric  
( $R_{D1} = R_{D2}$ ,  $M_1 = M_2$ )

$$V_{out1} = V_{out2} = V_{DD} - R_D \frac{I_{SS}}{2}$$



# Example



Assume all  $m \text{ sat}$

Assume fully Symmetric.

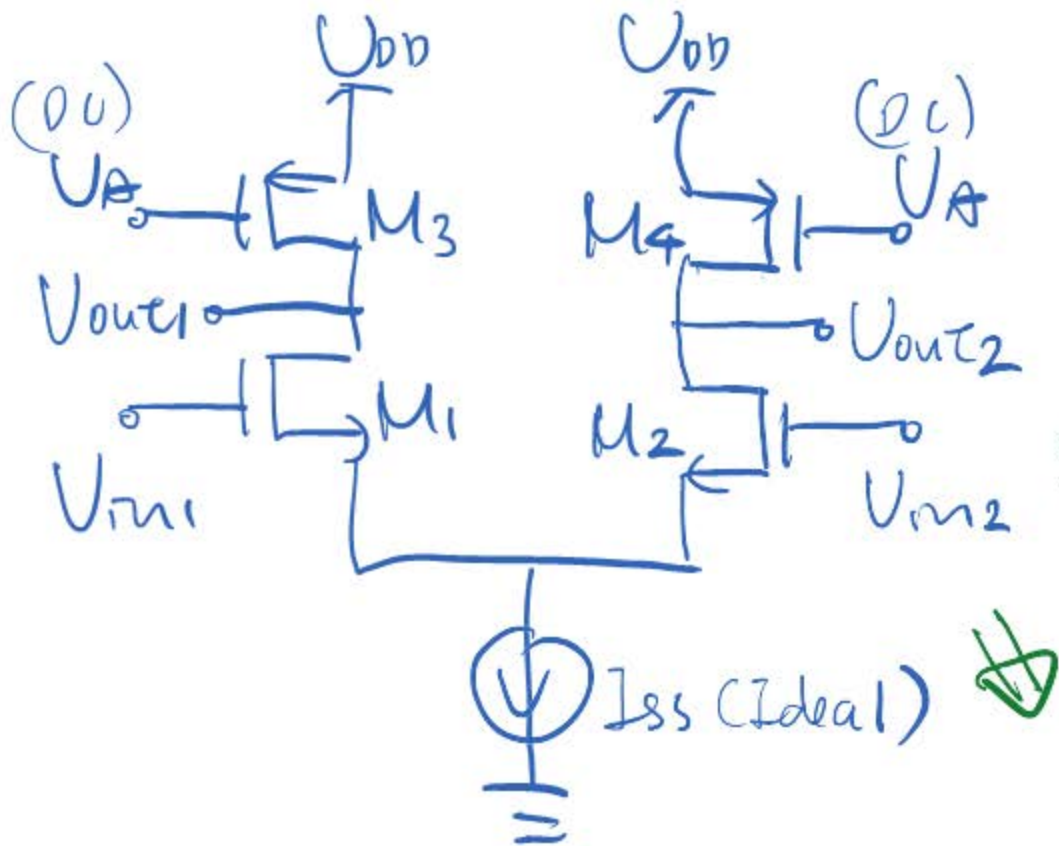
$$(M_1 = M_2, M_3 = M_4)$$

$$A_{DM} = ?$$

$$A_{CM} = ?$$

$$A_{CM-DM} = ?$$

# Example



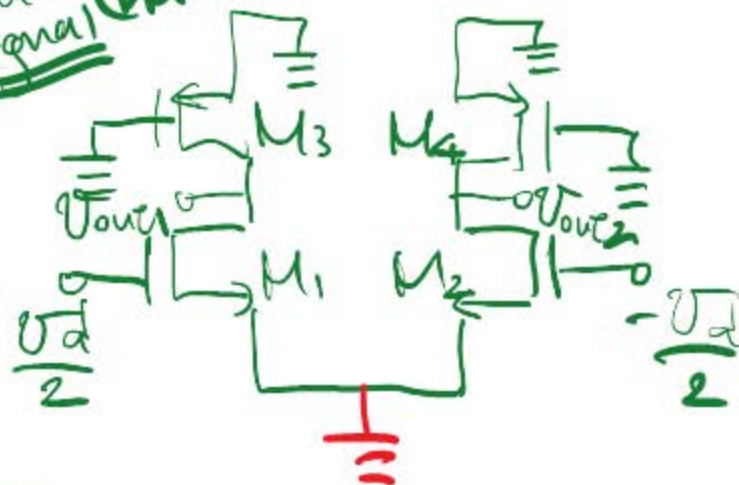
Assume  $r \neq 0, \lambda \neq 0$   
Assume all in sat

Assume fully symmetric.

$$(M_1 = M_2, M_3 = M_4)$$

$$A_{DM} = \frac{V_{out1} - V_{out2}}{V_d}$$

Small signal (AC)



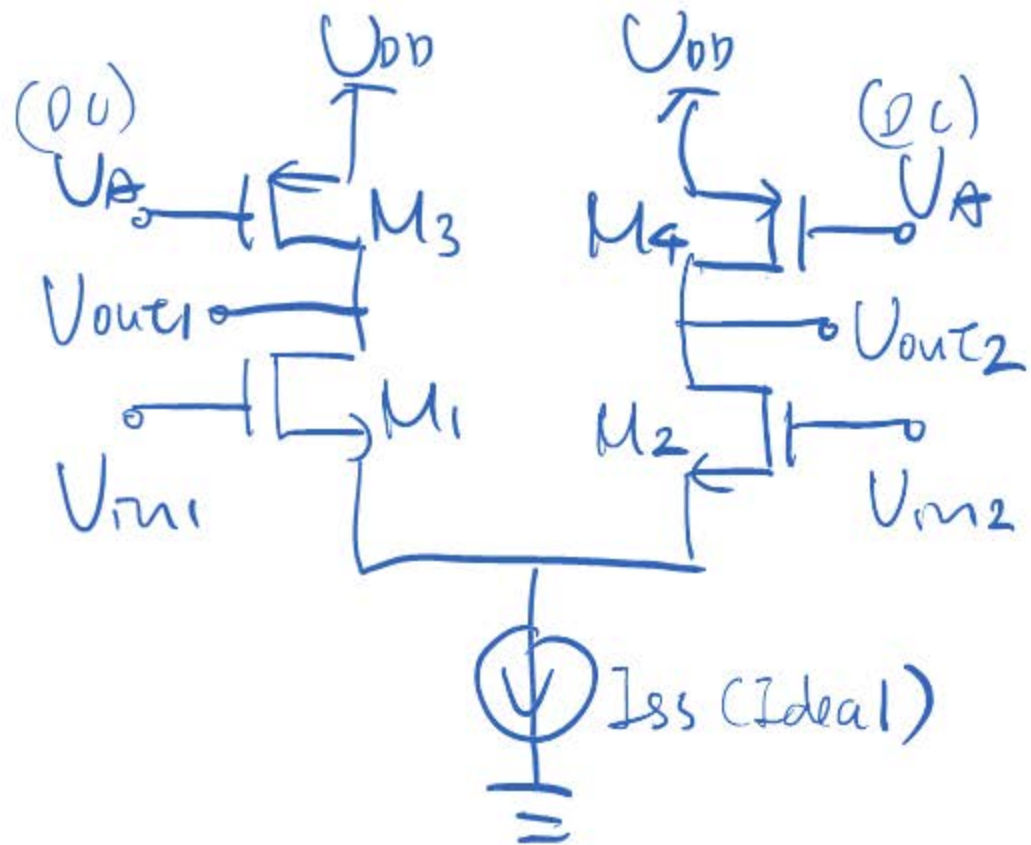
Apply half circuit method

$$g_{m1} = g_{m2} = g_{m1,2}$$

$$r_{o1} = r_{o2} = r_{o1,2}, r_{o3} = r_{o4} = r_{o3,4}$$

$$\begin{cases} V_{out1} = -g_{m1,2} (r_{o1,2} // r_{o3,4}) \frac{V_d}{2} \\ V_{out2} = -g_{m1,2} (r_{o1,2} // r_{o3,4}) (-\frac{V_d}{2}) \end{cases}$$

# Example



Assume all  $m \text{ sat}$

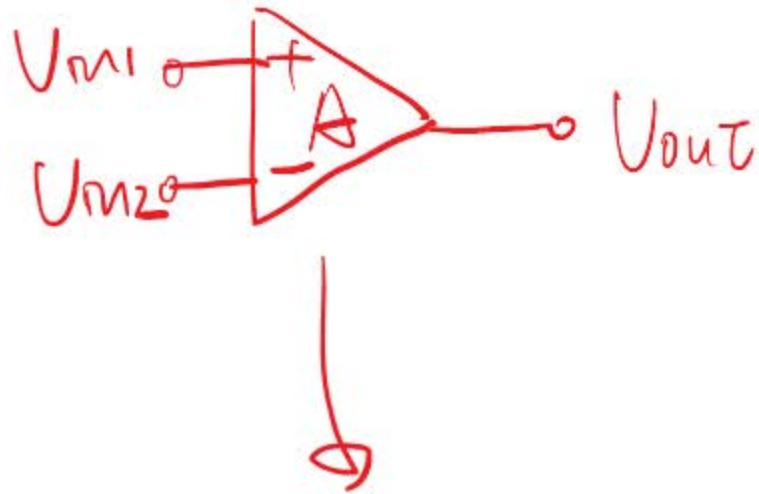
Assume fully Symmetric.

$$(M_1 = M_2, M_3 = M_4)$$

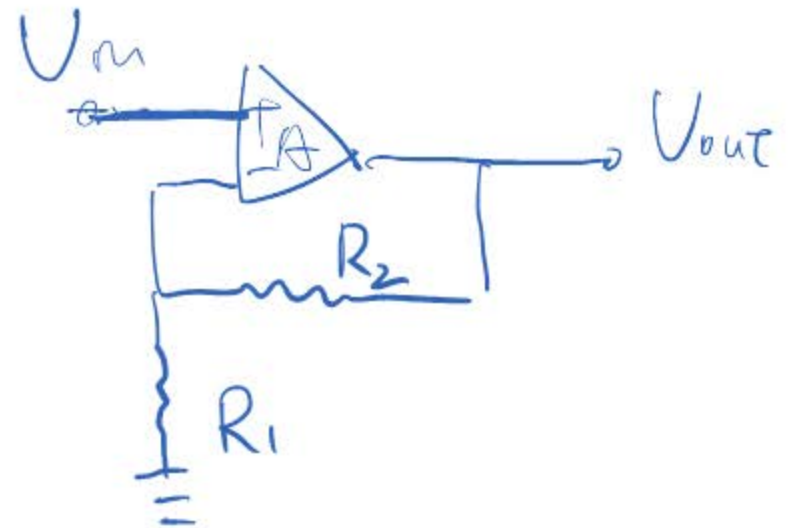
$$A_{CM} = 0$$

$$A_{CM-DM} = 0$$





"Differential  
Pair"  
+  
as the input stage



"Current  
Mirror"  
to convert differential  
output to single-ended  
output.



# JOINT INSTITUTE

---

## 交大密西根学院

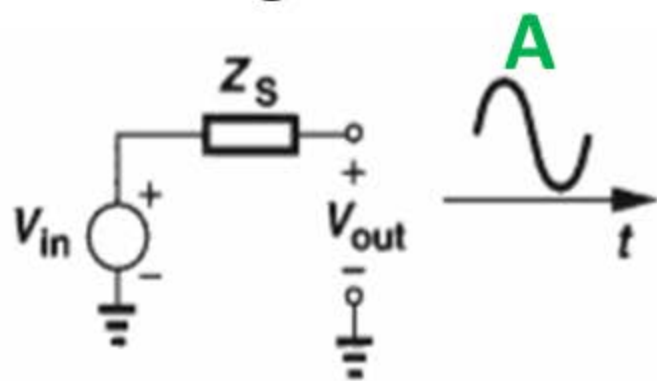
### **FET Differential Pair**

Ve311 Electronic Circuits (Fall 2020)

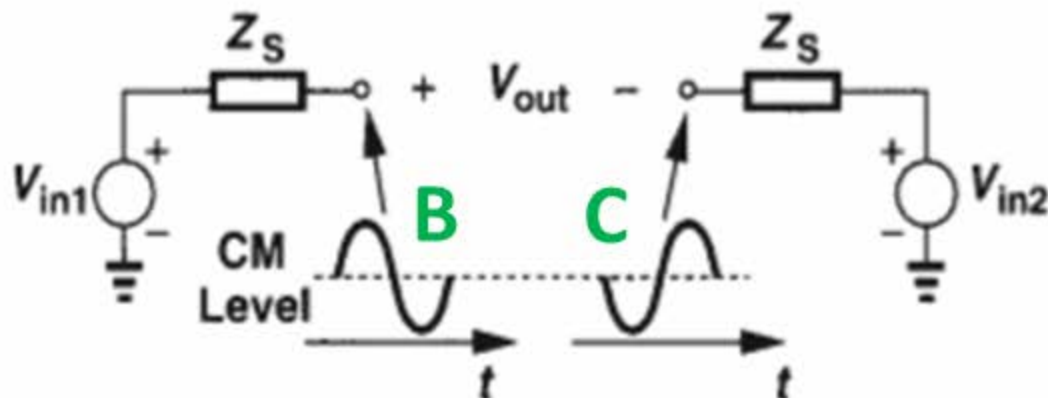
Dr. Chang-Ching Tu

# Single-Ended vs Differential Signals

## Single-ended



## Differential



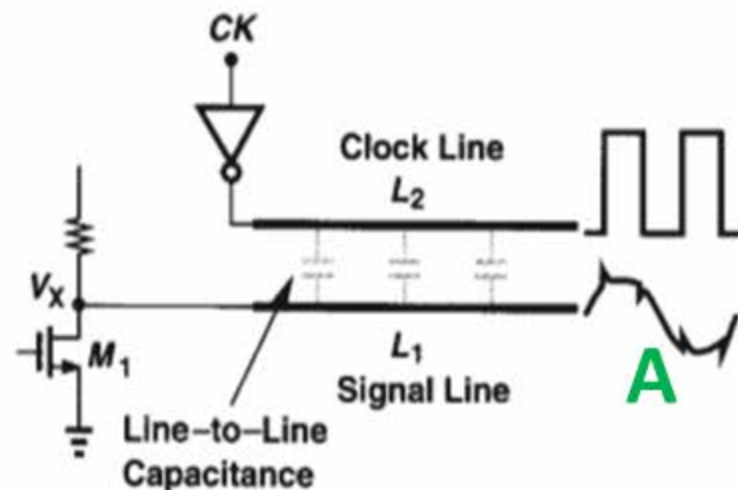
- $B - C = A$  (matters)
- $(B + C) / 2 = \text{common-mode level}$  (doesn't matter)
- Single-ended signal: a voltage signal measured with respect to ground
- Differential signal: a voltage signal measured between two nodes, each having equal amplitude and opposite phase around a common-mode (CM) level



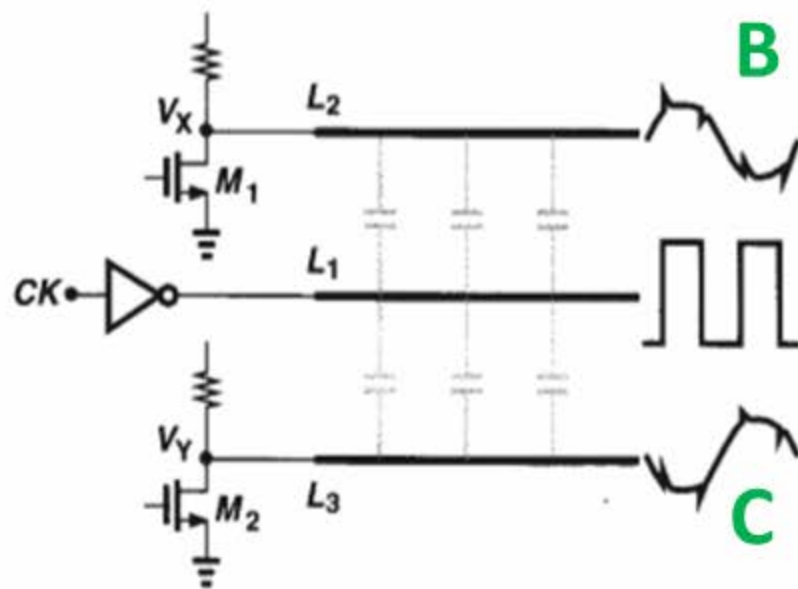
## Advantages of Differential Operation

# Common-Mode Noise Rejection

## Single-ended



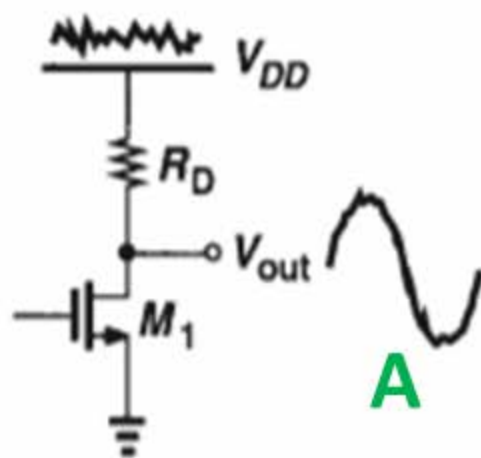
## Differential



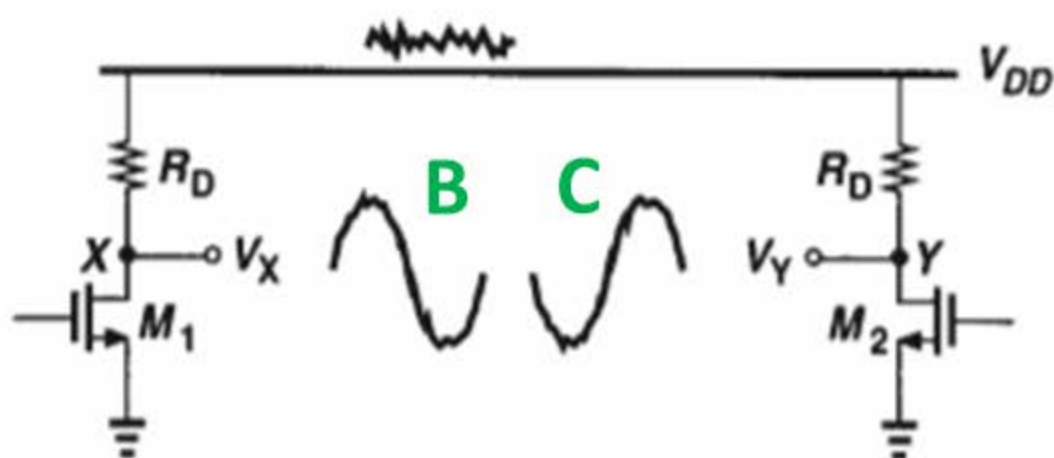
- A corrupted; B corrupted; C corrupted
- $(B + C) / 2 = \text{CM corrupted}$
- $(B - C)$  not corrupted

# Common-Mode Noise Rejection

## Single-ended



## Differential

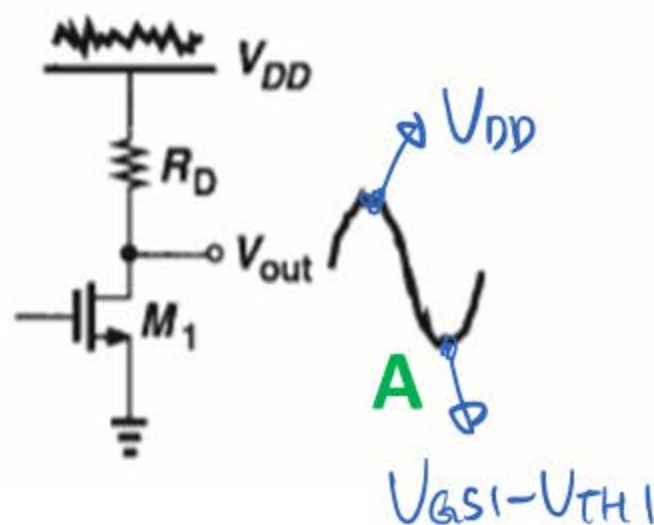


- A corrupted; B corrupted; C corrupted
- $(B + C) / 2 = \text{CM corrupted}$
- $(B - C)$  not corrupted

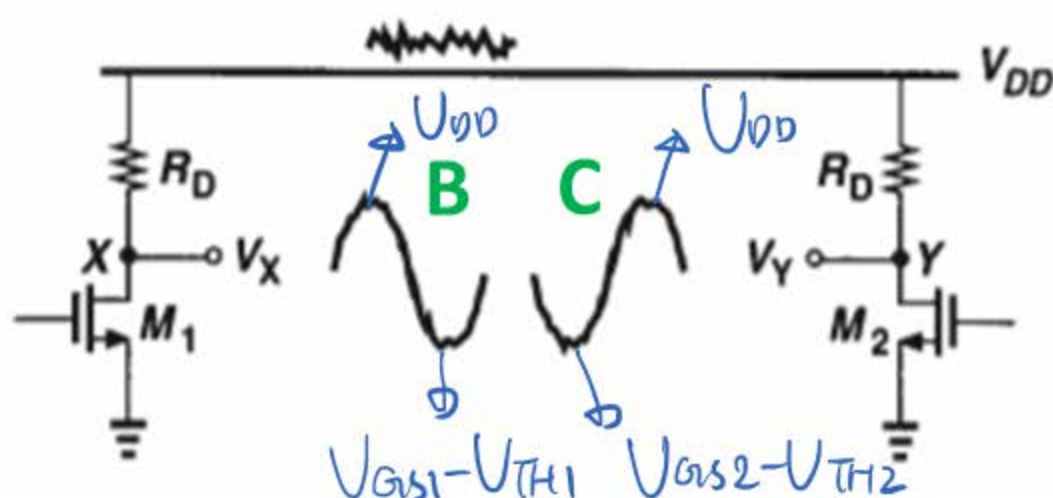


# Increased Output Swing

## Single-ended



## Differential



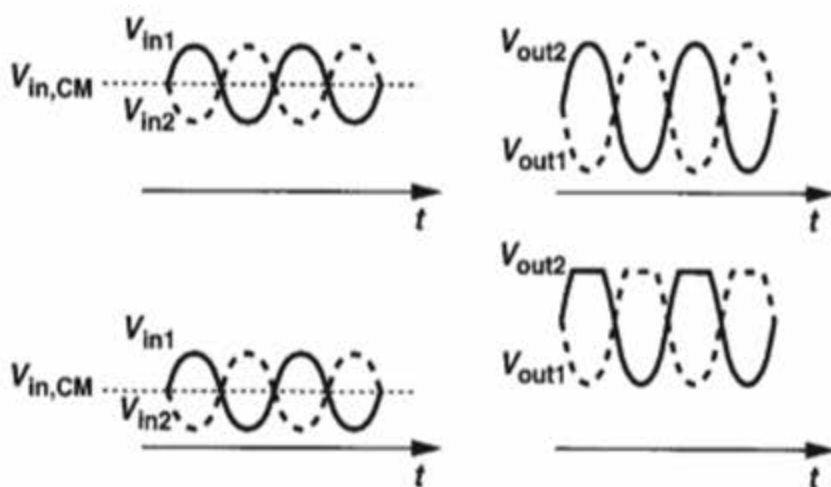
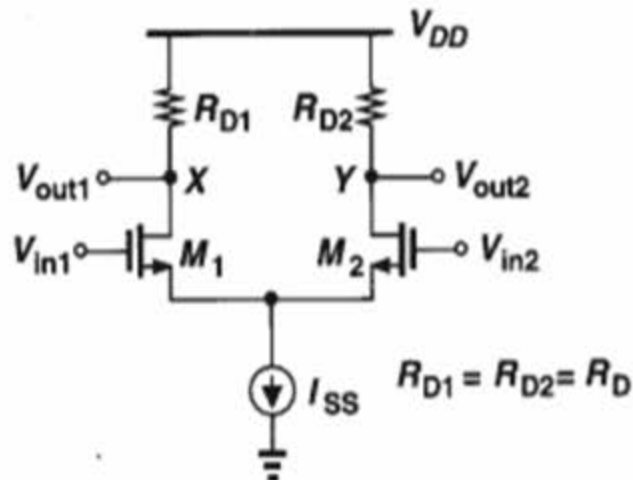
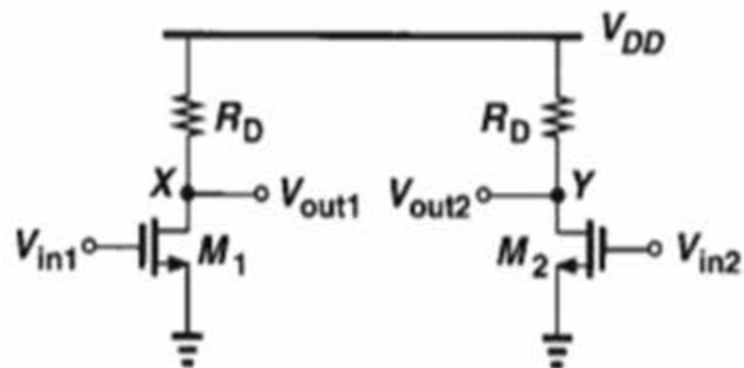
- $(V_{GS1} - V_{TH1}) \leq A \leq V_{DD}$
- $(V_{GS1,2} - V_{TH1,2}) - V_{DD} \leq (B - C) \leq V_{DD} - (V_{GS1,2} - V_{TH1,2})$

$$\text{Range} = V_{DD} - (V_{GS} - V_{TH})$$

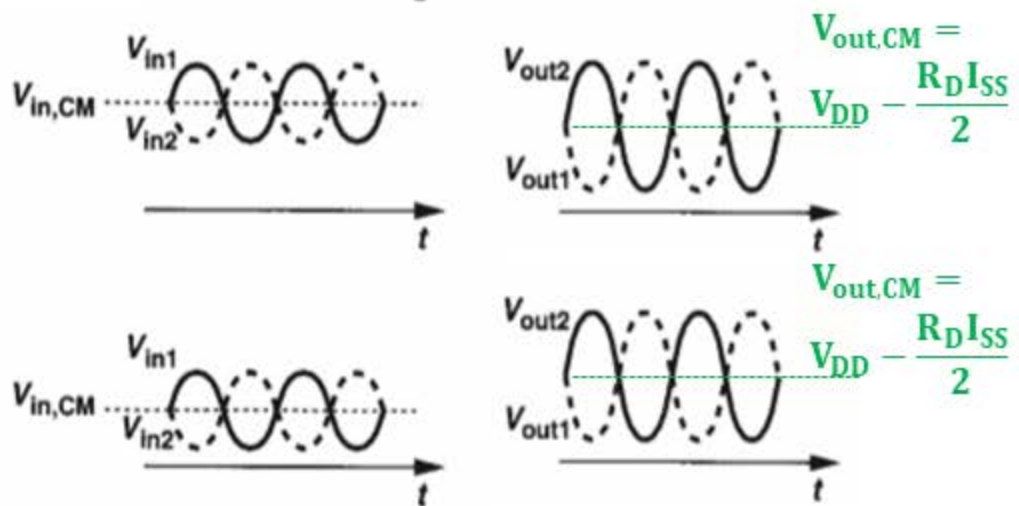
$$\text{Range} = 2[V_{DD} - (V_{GS} - V_{TH})]$$

# DC and Small-Signal Analysis

# $V_{in,CM}$ and $V_{out,CM}$

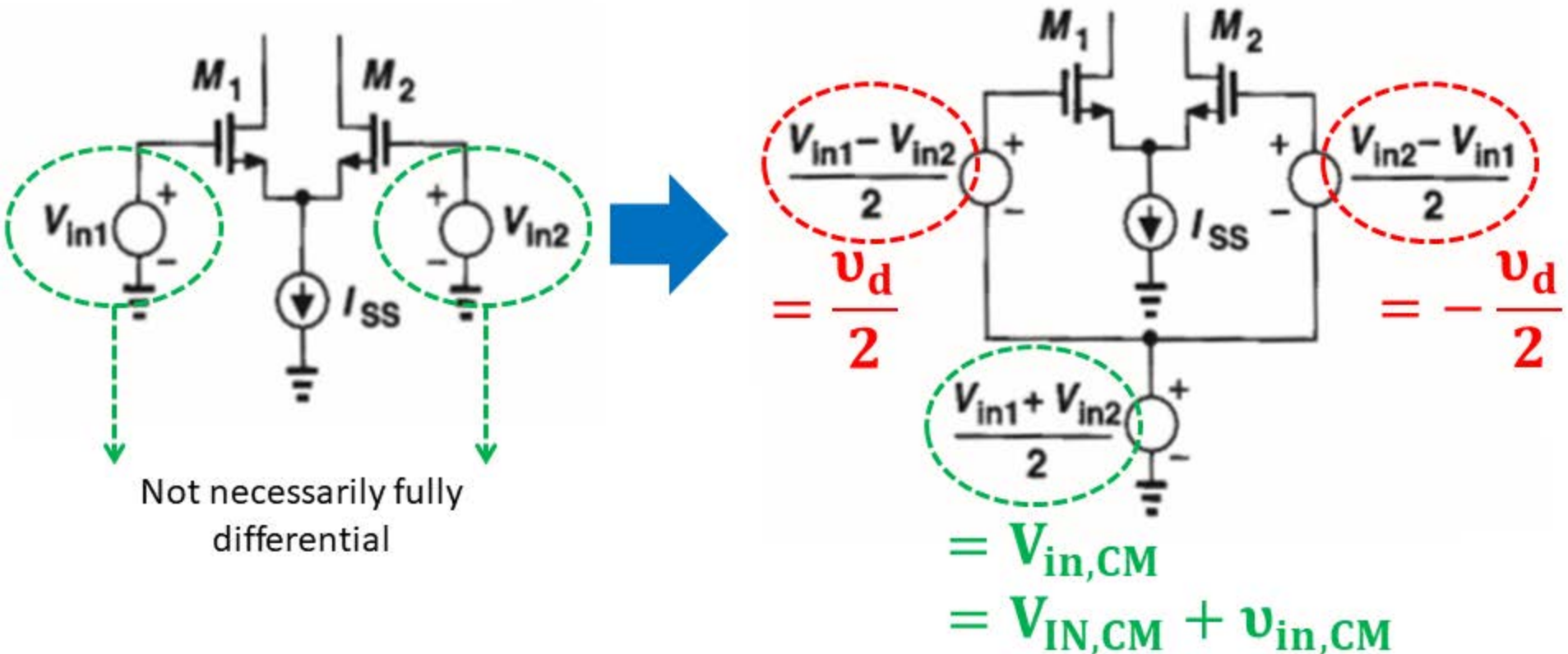


- $V_{out,CM}$  dependent on  $V_{in,CM}$



- $V_{out,CM}$  independent from  $V_{in,CM}$
- Better design

# Common-Mode + Differential-Mode



$$A_{DM} = \frac{v_{out1} - v_{out2}}{v_d}$$

$$A_{CM-DM} = \frac{v_{out1} - v_{out2}}{v_{in,CM}}$$

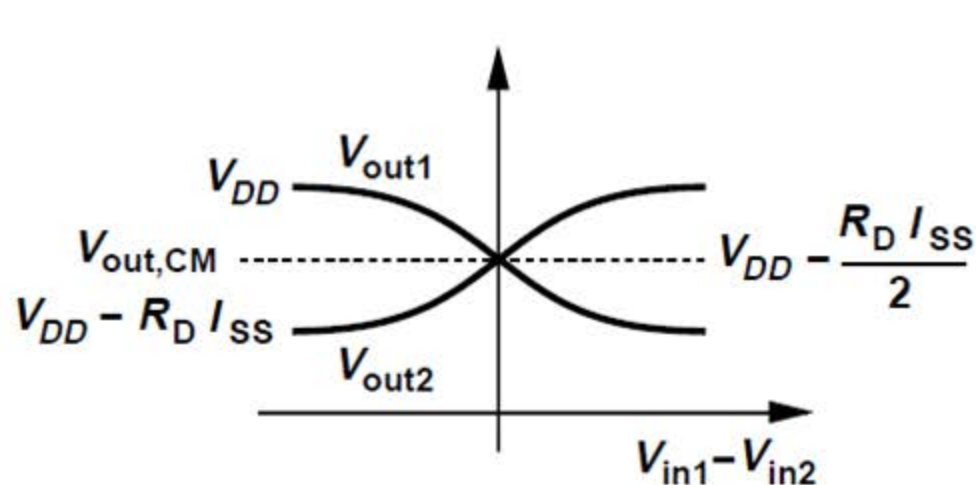
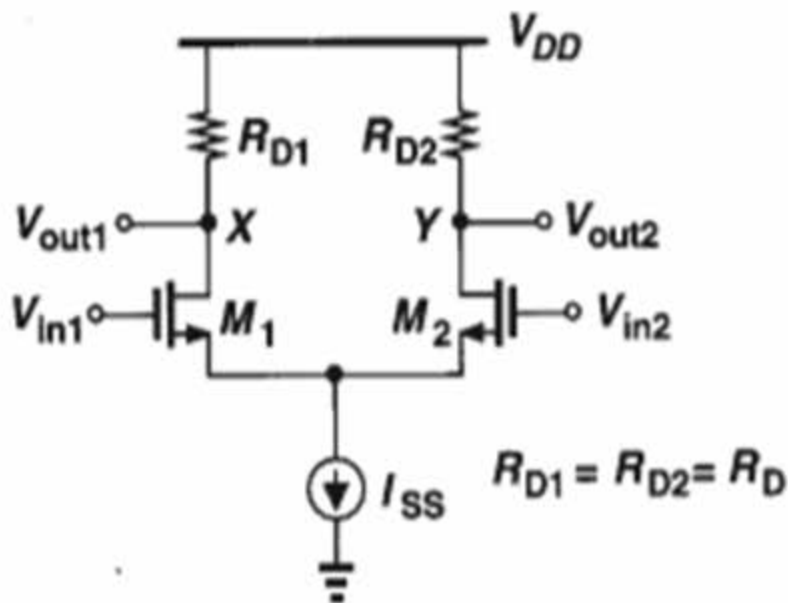
$$A_{CM} = \frac{v_{out,CM}}{v_{in,CM}}$$

$$CMRR = \left| \frac{A_{DM}}{A_{CM-DM}} \right|$$

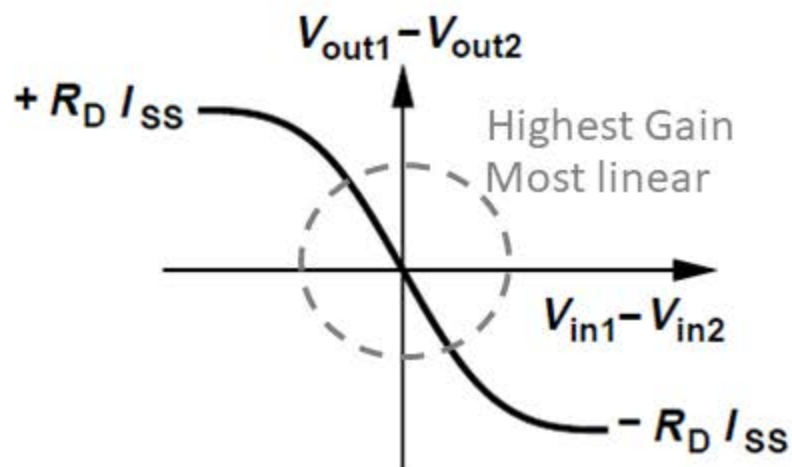


# Differential-Mode (Qualitative Analysis)

## Qualitative Analysis



(a)

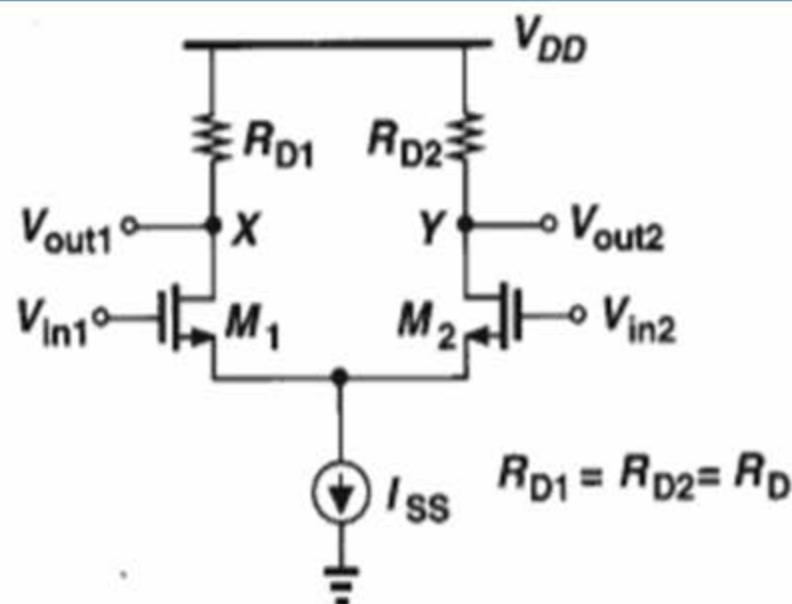


(b)

# Differential-Mode (DC Analysis)

DC  
Analysis

$\lambda = 0 \quad \gamma = 0$



$$V_{in1} - V_{GS1} = V_{in2} - V_{GS2}$$

$$V_{in1} - V_{in2} = V_{GS1} - V_{GS2} = (V_{GS1} - V_{TH}) - (V_{GS2} - V_{TH}) = \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} \frac{W}{L}}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox} \frac{W}{L}}}$$

$$(V_{in1} - V_{in2})^2 = \frac{2I_{D1}}{\mu_n C_{ox} \frac{W}{L}} + \frac{2I_{D2}}{\mu_n C_{ox} \frac{W}{L}} - 2 \frac{\sqrt{4I_{D1}I_{D2}}}{\mu_n C_{ox} \frac{W}{L}} = \frac{2}{\mu_n C_{ox} \frac{W}{L}} (I_{SS} - 2\sqrt{I_{D1}I_{D2}})$$

$$\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2})^2 = I_{SS} - 2\sqrt{I_{D1}I_{D2}}$$

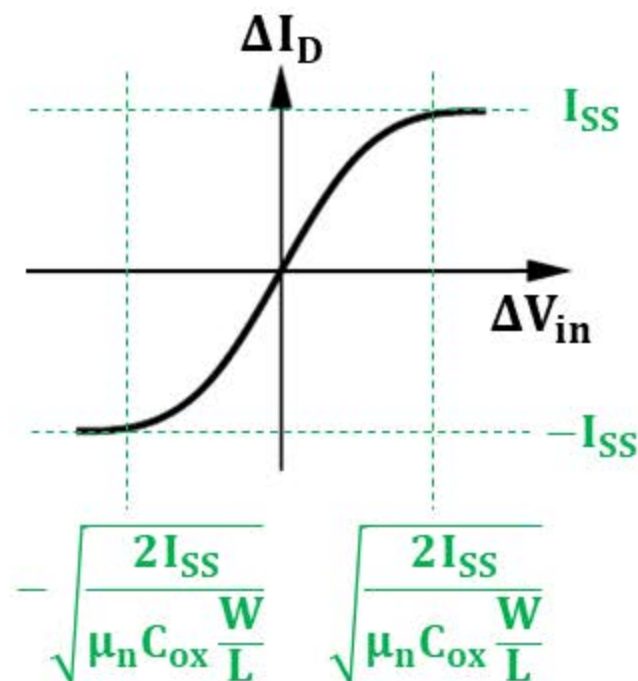
# Differential-Mode (DC Analysis)

$$\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2})^2 - I_{SS} = -2\sqrt{I_{D1}I_{D2}}$$

$$\frac{1}{4} \left( \mu_n C_{ox} \frac{W}{L} \right)^2 (V_{in1} - V_{in2})^4 + I_{SS}^2 - \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2})^2 I_{SS} = 4I_{D1}I_{D2}$$

$$\frac{1}{4} \left( \mu_n C_{ox} \frac{W}{L} \right)^2 \underbrace{(V_{in1} - V_{in2})^4}_{= \Delta V_{in}^4} + \cancel{I_{SS}^2} - \mu_n C_{ox} \frac{W}{L} \underbrace{(V_{in1} - V_{in2})^2}_{= \Delta V_{in}^2} I_{SS} = \cancel{I_{SS}^2} - \underbrace{(I_{D1} - I_{D2})^2}_{= \Delta I_D^2}$$

$$\Delta I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \Delta V_{in} \sqrt{\frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - \Delta V_{in}^2}$$



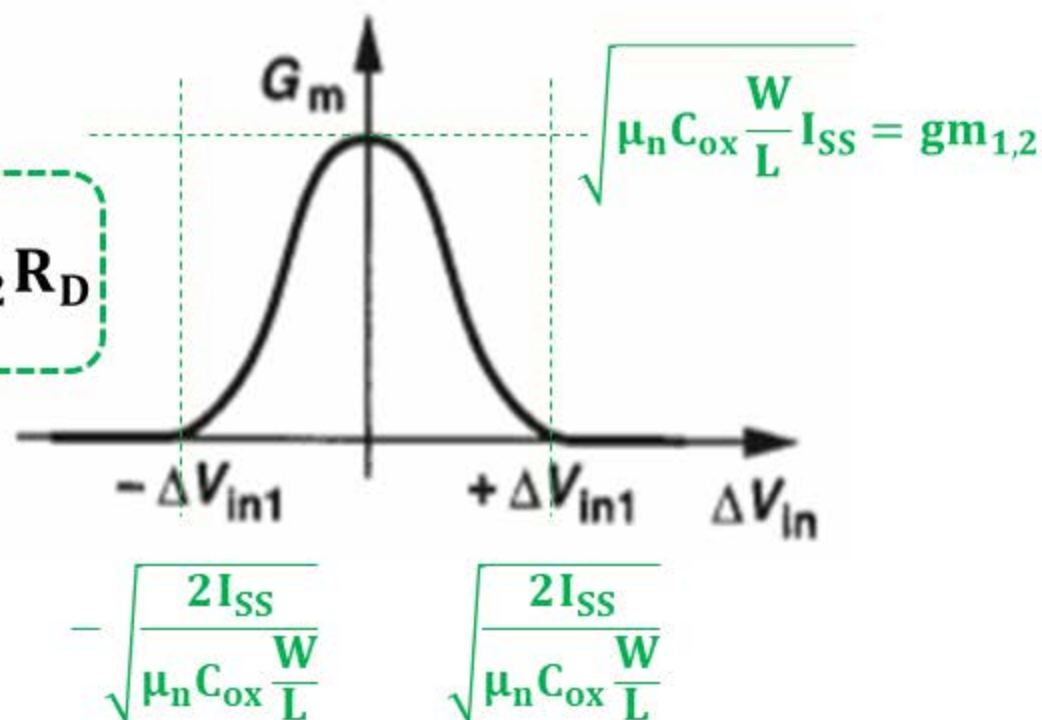
# Differential-Mode (DC Analysis)

$$G_m = \frac{\partial \Delta I_D}{\partial \Delta V_{in}} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \frac{\frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - 2\Delta V_{in}^2}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - \Delta V_{in}^2}}$$

At  $\Delta V_{in} = 0$ ,

$$A_{DM} = \frac{V_{out1} - V_{out2}}{\Delta V_{in}} = -gm_{1,2} R_D$$

- Larger  $I_{SS}$  leads to higher  $G_m$  and wider input range.
- Smaller  $W/L$  leads to lower  $G_m$  but wider input range.



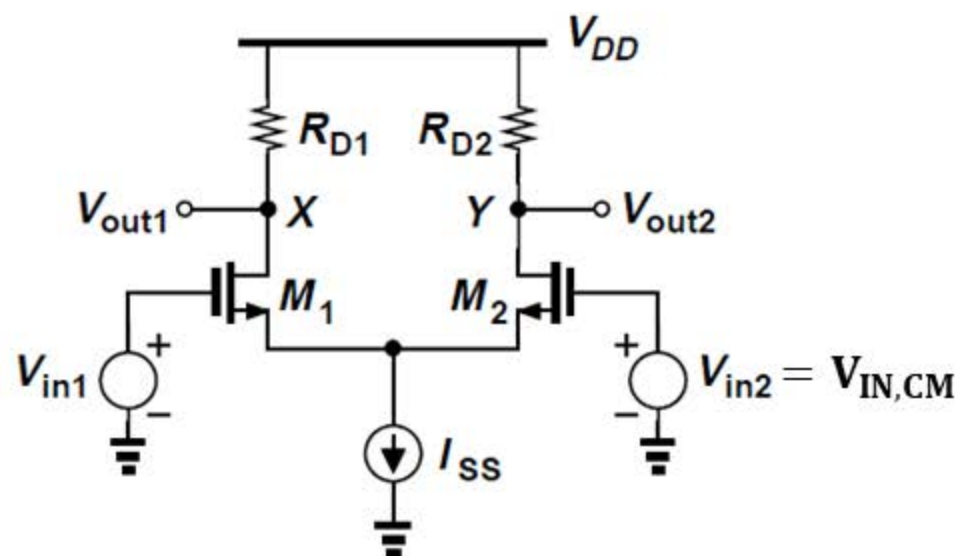


# Differential-Mode (Small-Signal, Superposition)<sup>29</sup>

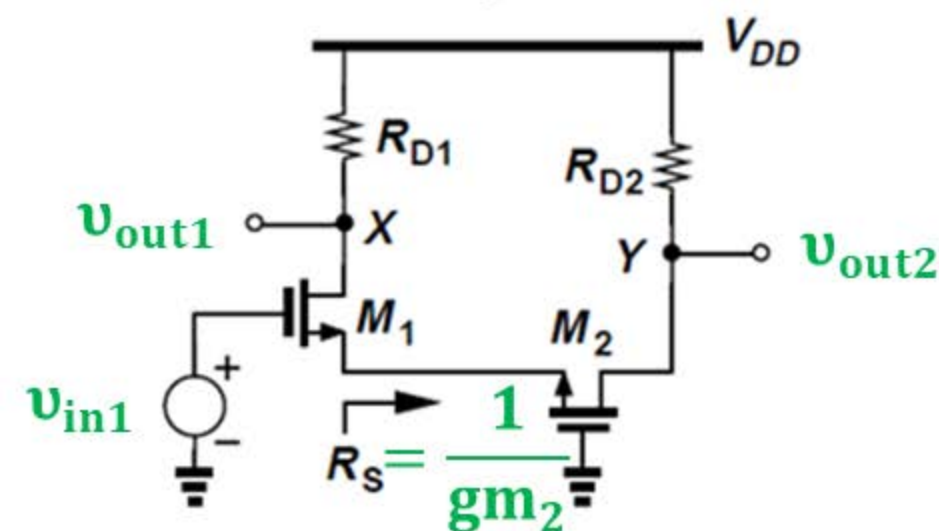
Small-signal  
Analysis

$$\lambda = 0 \quad \gamma = 0$$

Complete circuit



Small-signal circuit



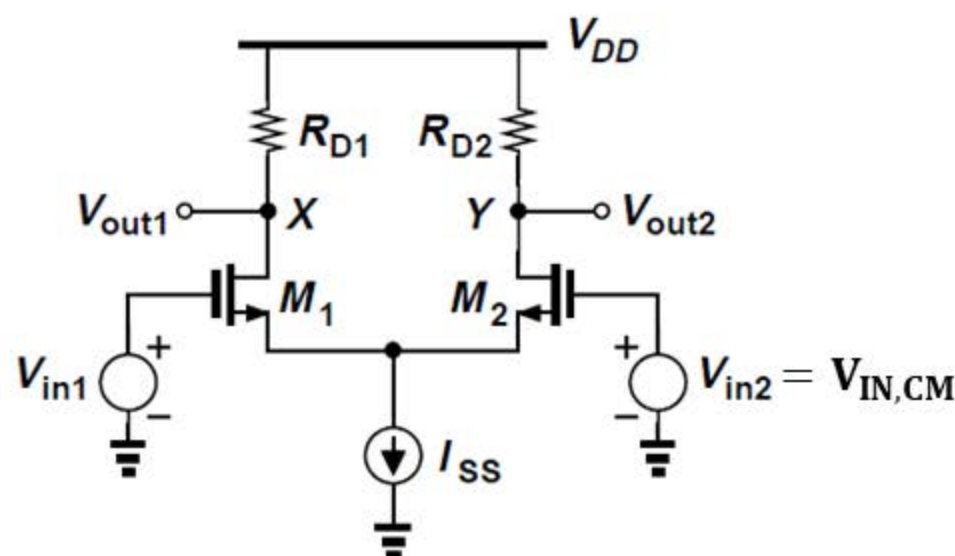
$$v_{out1} = - \frac{R_D}{\frac{1}{gm_1} + \frac{1}{gm_2}} v_{in1}$$

# Differential-Mode (Small-Signal, Superposition)<sup>30</sup>

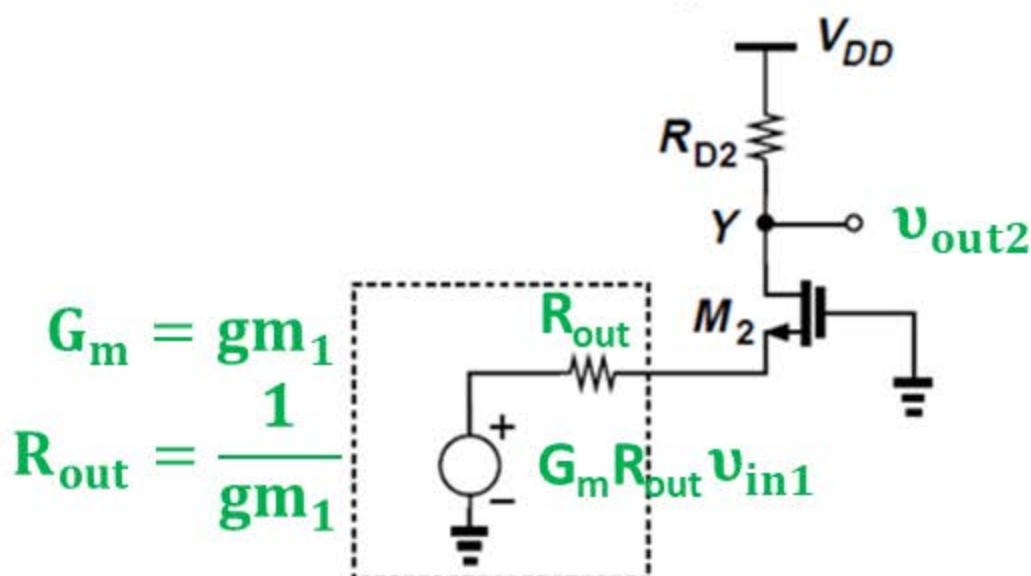
Small-signal  
Analysis

$$\lambda = 0 \quad \gamma = 0$$

Complete circuit



Small-signal circuit



$$v_{out2} = \frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} v_{in1}$$

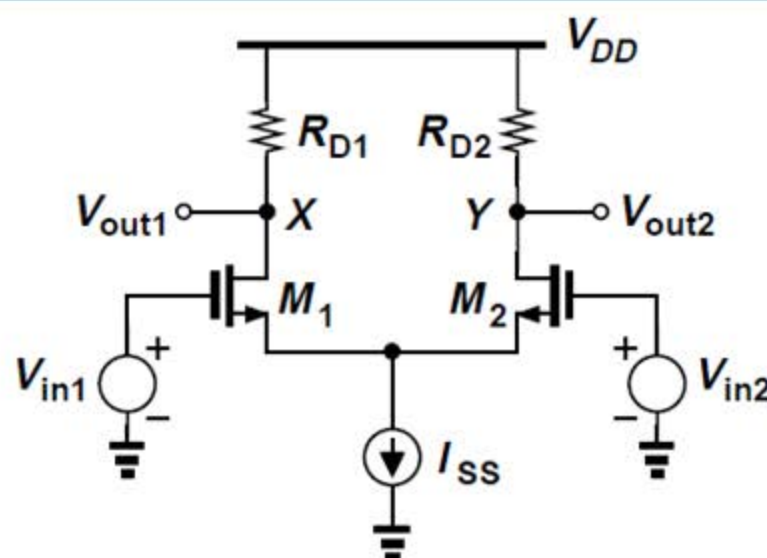
$$G_m = g_{m1}$$

$$R_{out} = \frac{1}{g_{m1}}$$

# Differential-Mode (Small-Signal, Superposition)<sup>31</sup>

Small-signal  
Analysis

$$\lambda = 0 \quad \gamma = 0$$



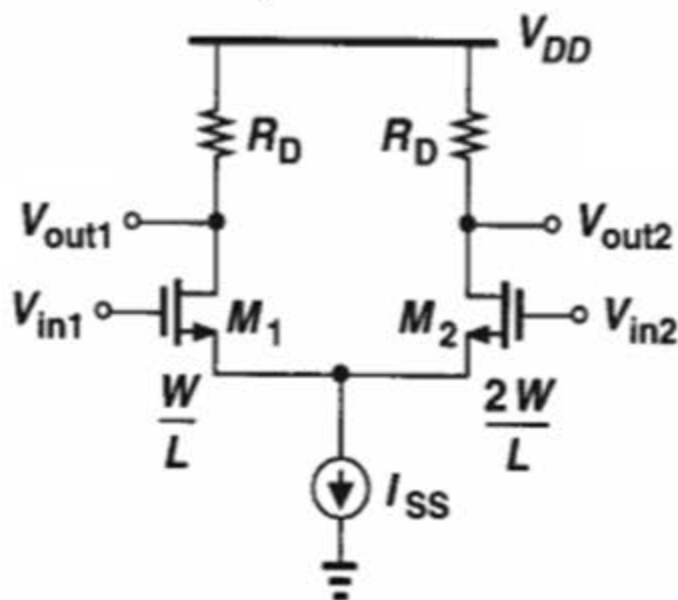
$$v_{out1} - v_{out2} = - \frac{2R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} v_{in1} = -g_m R_D v_{in1} \quad (1)$$

$$v_{out1} - v_{out2} = \frac{2R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} v_{in2} = g_m R_D v_{in2} \quad (2)$$

$$A_{DM} = \frac{v_{out1} - v_{out2}}{v_{in1} - v_{in2}} = -g_m R_D \quad (1) + (2)$$

# Example

Calculate the  $A_{DM}$  of the differential pair below if the biasing conditions of  $M_1$  and  $M_2$  are the same.



$$\left\{ \begin{array}{l} v_{out1} - v_{out2} = -\frac{2R_D}{\frac{1}{gm_1} + \frac{1}{2gm_1}} v_{in1} = -\frac{4}{3} gm_1 R_D v_{in1} \\ v_{out1} - v_{out2} = \frac{2R_D}{\frac{1}{gm_1} + \frac{1}{2gm_1}} v_{in2} = \frac{4}{3} gm_1 R_D v_{in2} \end{array} \right.$$

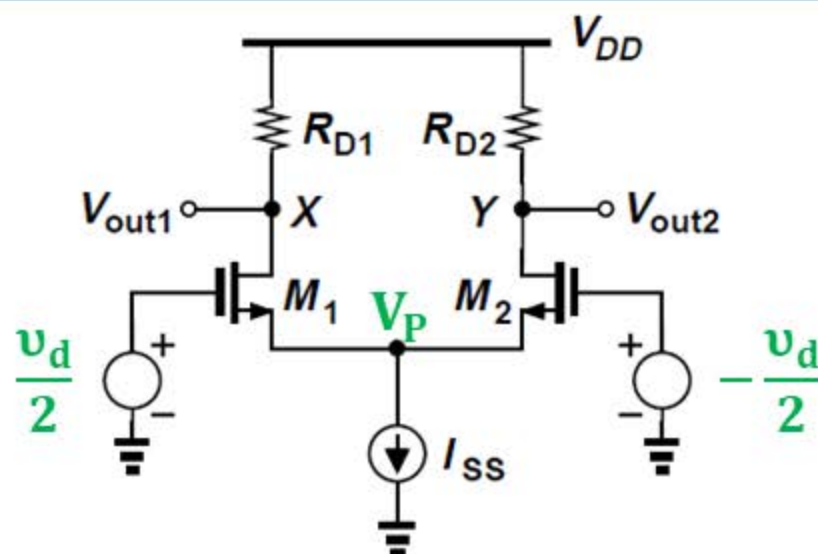
$$A_{DM} = \frac{v_{out1} - v_{out2}}{v_{in1} - v_{in2}} = -\frac{4}{3} gm_1 R_D$$



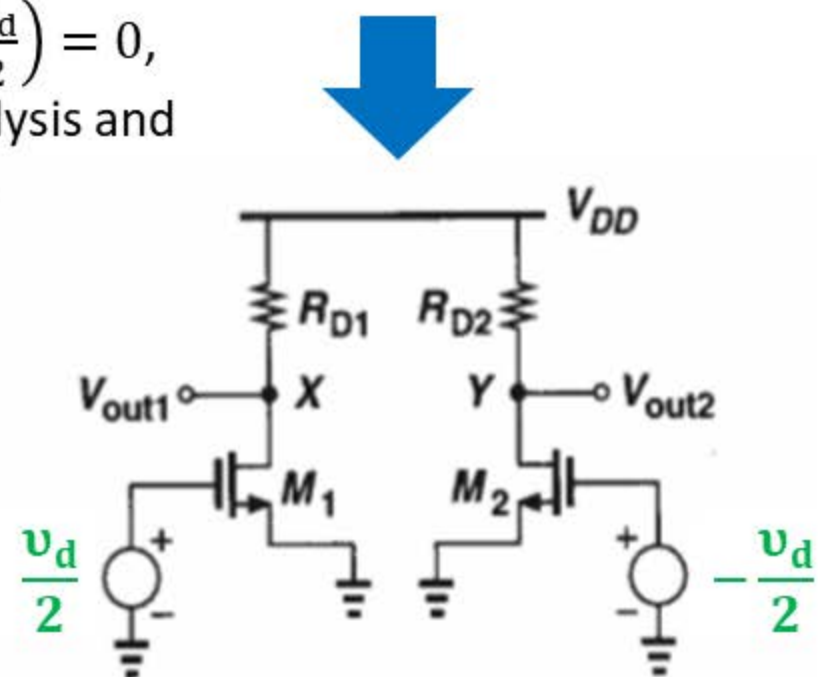
# Differential-Mode (Small-Signal, Half-circuit)

## Small-signal Analysis

$$\lambda \neq 0 \quad \gamma \neq 0$$



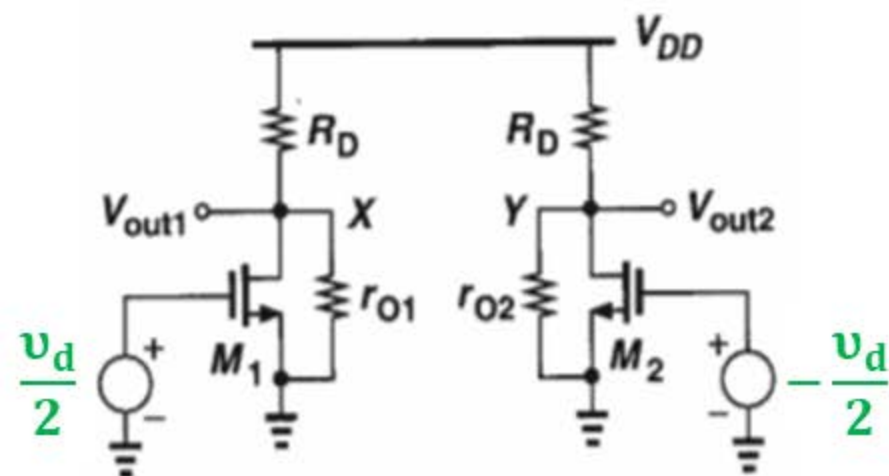
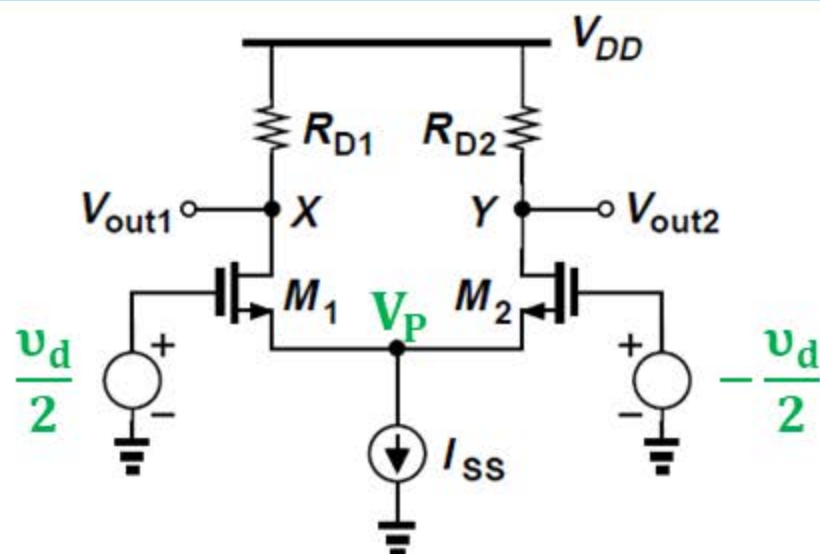
- Assume the circuit is fully symmetric.
- For  $i_{d1} + i_{d2} = 0$  and  $g_{m1} \frac{v_d}{2} + g_{m2} \left(-\frac{v_d}{2}\right) = 0$ ,  $V_P$  must be a constant voltage in DC analysis and a virtual ground in small-signal analysis.



# Differential-Mode (Small-Signal, Half-circuit)

Small-signal  
Analysis

$\lambda \neq 0 \quad \gamma \neq 0$



$$v_{out1} = -gm(R_D \parallel r_o) \frac{v_d}{2}$$

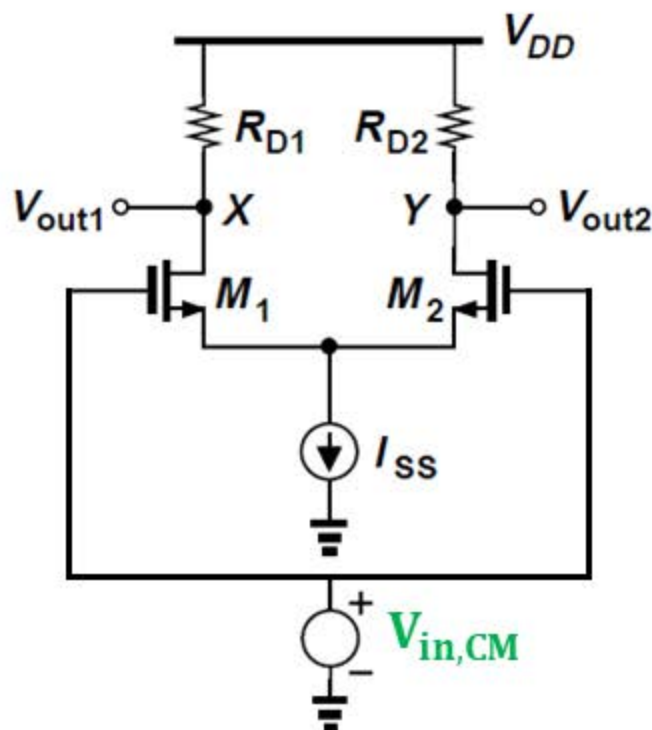
$$v_{out2} = -gm(R_D \parallel r_o) \left(-\frac{v_d}{2}\right)$$

$$A_{DM} = \frac{v_{out1} - v_{out2}}{v_d} = -gm(R_D \parallel r_o)$$

# Common-Mode Response

Small-signal  
Analysis

$\lambda \neq 0 \quad \gamma \neq 0$



If the circuit is fully symmetric,

$$A_{\text{CM-DM}} = \frac{v_{\text{out1}} - v_{\text{out2}}}{v_{\text{in,CM}}} = 0$$

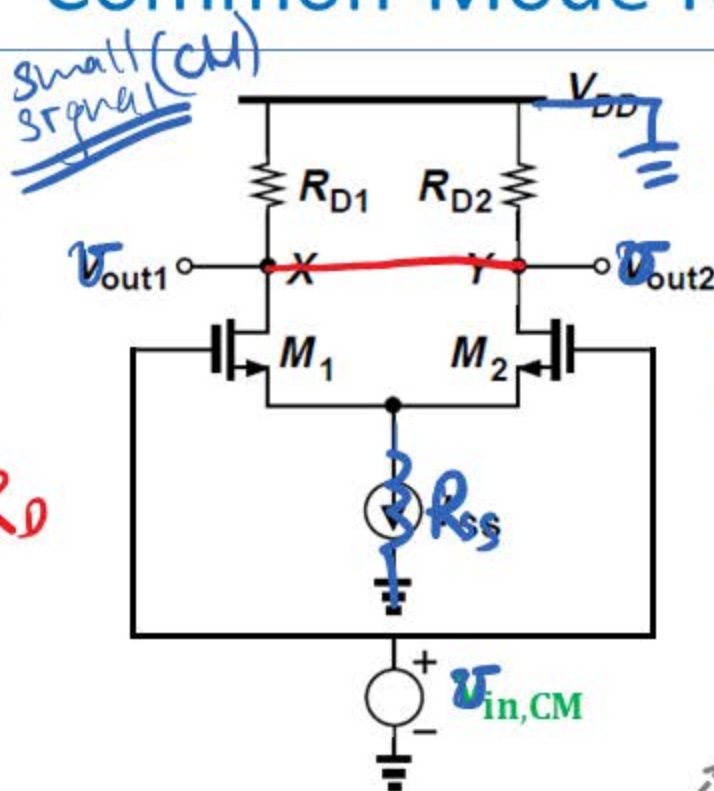
$$\text{CMRR} = \left| \frac{A_{\text{DM}}}{A_{\text{CM-DM}}} \right| = \infty$$

# Common-Mode Response

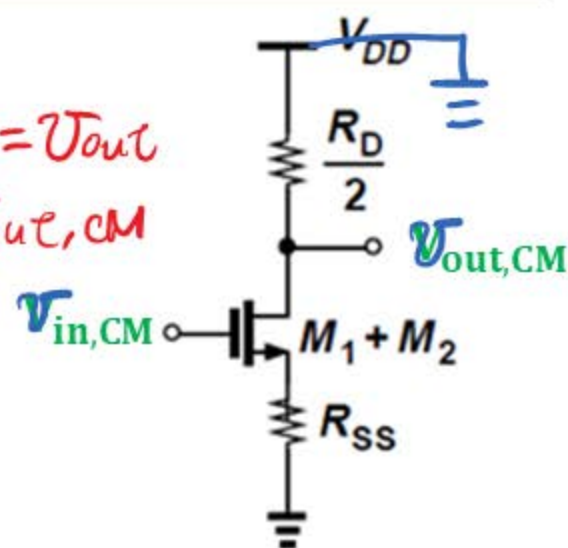
## Small-signal Analysis

$$\lambda \neq 0 \quad \gamma \neq 0$$

Assume  
 $R_{D1} = R_{D2} = R_D$   
 $\mu_1 = \mu_2$



$$v_{out1} = v_{out2} \\ = v_{out,CM}$$



- Perturbing biasing condition  $\rightarrow$  Altering transconductance ( $g_m$ )

If the circuit is fully symmetric,

$$A_{CM} = \frac{v_{out,CM}}{v_{in,CM}}$$

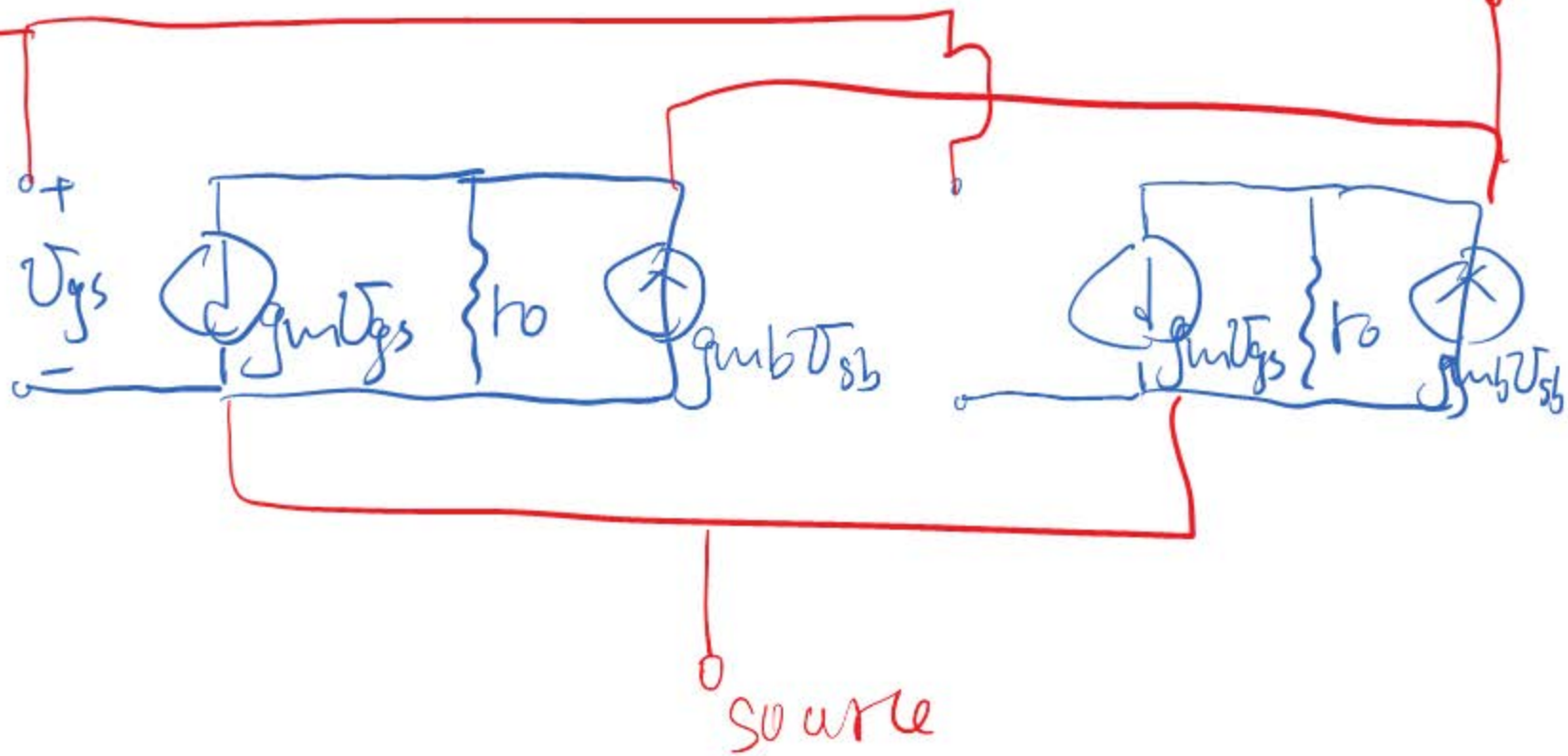
$$= \frac{-2g_m \frac{r_o}{2}}{\left[ R_{SS} + \frac{r_o}{2} + (2g_m + 2g_{mb}) \frac{r_o}{2} R_{SS} \right]} \cdot \frac{\left[ R_{SS} + \frac{r_o}{2} + (2g_m + 2g_{mb}) \frac{r_o}{2} R_{SS} \right] \frac{R_D}{2}}{\left[ R_{SS} + \frac{r_o}{2} + (2g_m + 2g_{mb}) \frac{r_o}{2} R_{SS} \right] + \frac{R_D}{2}}$$

$$= 0 \text{ if } R_{SS} = \infty$$



gate

drain



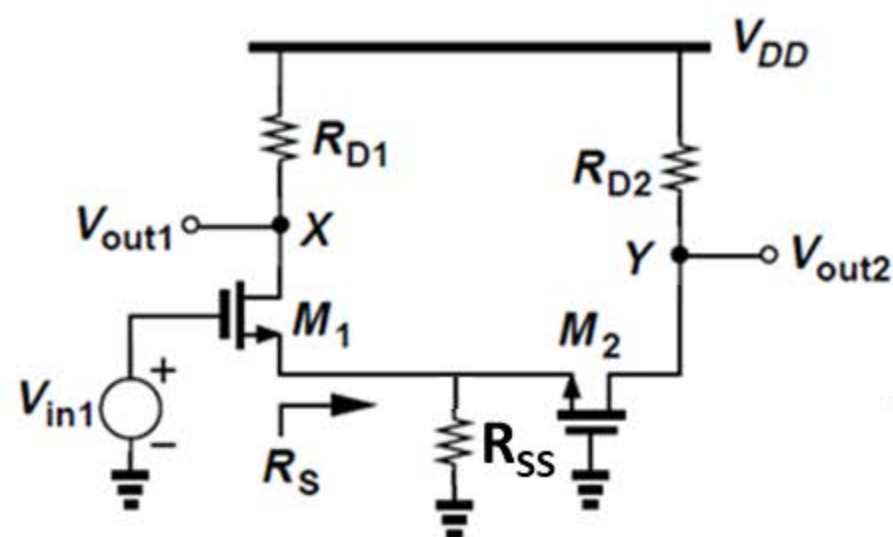
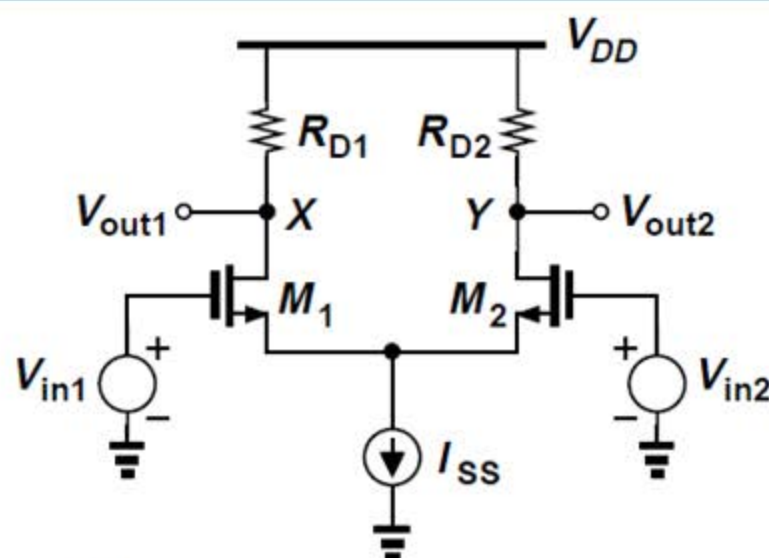
## $A_{DM}$ for Finite $R_{SS}$

To prove that even when the   $I_{SS}$  is non-ideal

# Differential-Mode (Small-Signal, Superposition)<sup>39</sup>

Small-signal  
Analysis

$$\lambda = 0 \quad \gamma = 0$$



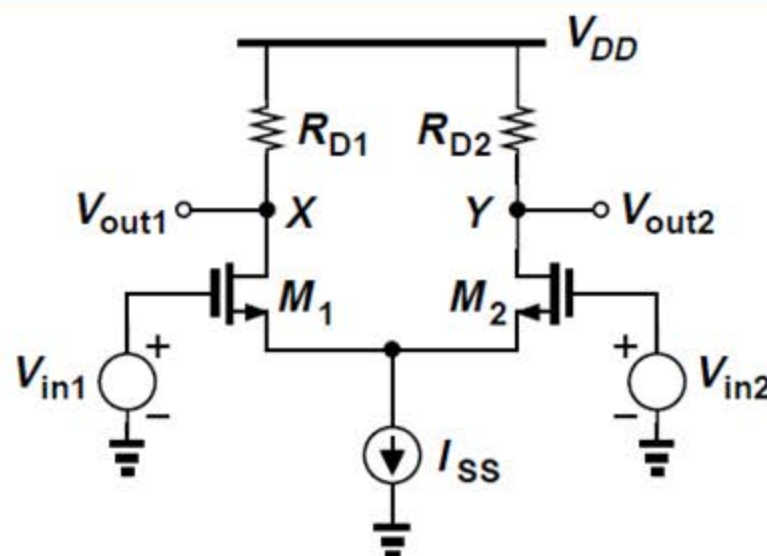
$$R_S = \frac{1}{g_{m2}} \parallel R_{SS}$$

$$v_{out1} = - \frac{R_D}{\frac{1}{g_{m1}} + \left( \frac{1}{g_{m2}} \parallel R_{SS} \right)} v_{in1}$$

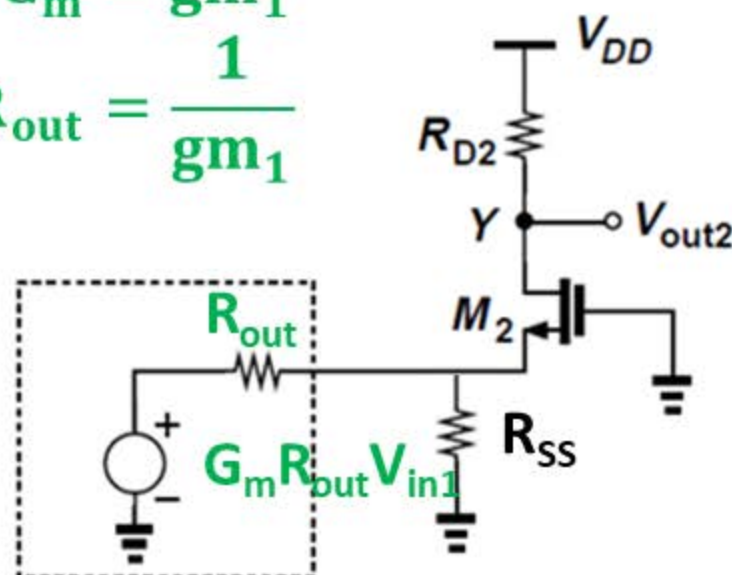
# Differential-Mode (Small-Signal, Superposition)<sup>40</sup>

Small-signal  
Analysis

$$\lambda = 0 \quad \gamma = 0$$



$$G_m = gm_1$$
$$R_{out} = \frac{1}{gm_1}$$

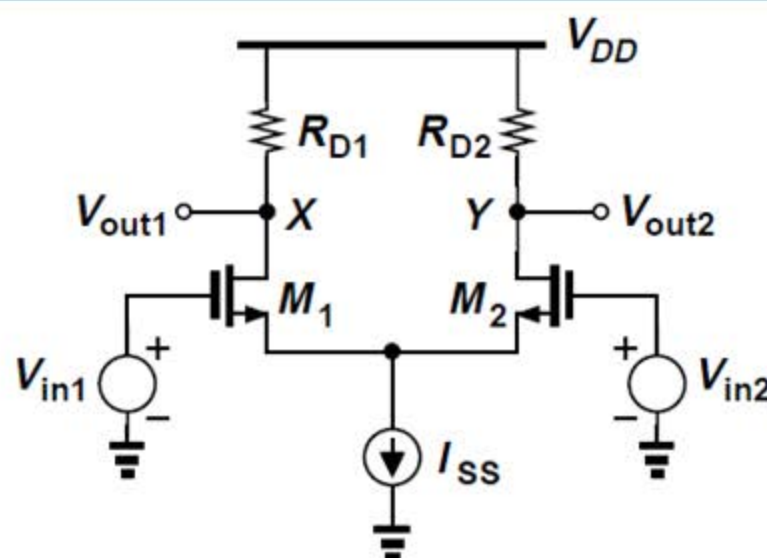


$$v_{out2} = \frac{\frac{R_{SS}}{R_{SS} + \frac{1}{gm_2}} R_D}{\frac{1}{gm_1} + \left( \frac{1}{gm_2} \parallel R_{SS} \right)} v_{in1}$$

# Differential-Mode (Small-Signal, Superposition)<sup>41</sup>

Small-signal  
Analysis

$$\lambda = 0 \quad \gamma = 0$$



$$v_{out1} - v_{out2} = -\frac{(gm_1 + 2R_{SS}gm_1gm_2)R_D}{1 + (gm_1 + gm_2)R_{SS}} v_{in1} = -gmR_D v_{in1} \quad (1)$$

$$v_{out1} - v_{out2} = \frac{(gm_2 + 2R_{SS}gm_1gm_2)R_D}{1 + (gm_1 + gm_2)R_{SS}} v_{in2} = gmR_D v_{in2} \quad (2)$$

$$A_{DM} = \frac{v_{out1} - v_{out2}}{v_{in1} - v_{in2}} = -gmR_D$$

(1) + (2)



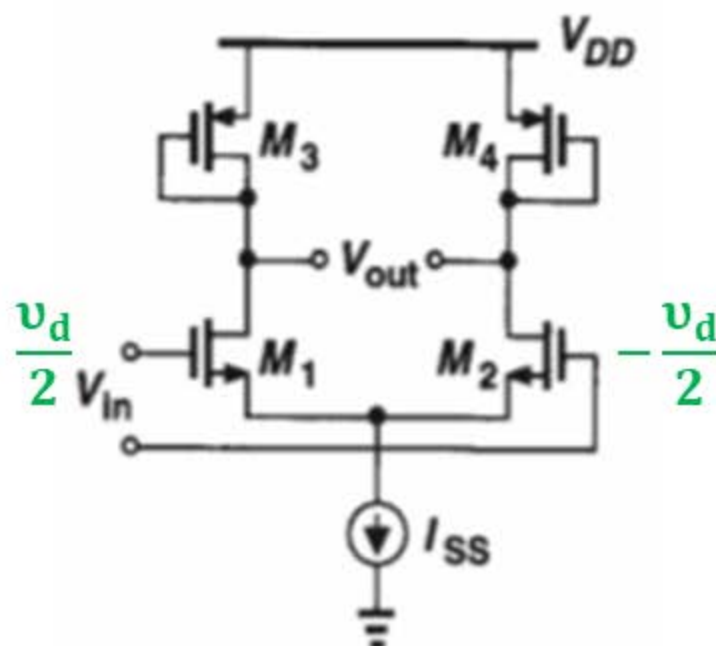
$A_{DM}$  with MOS Loads

# Differential-Mode (Small-Signal, Half-circuit)

## Small-signal Analysis

$$\lambda \neq 0 \quad \gamma \neq 0$$

- Higher  $A_{DM}$ 
  - Smaller  $(W/L)_p$
  - Larger  $(V_{SGP} - V_{THP})$
  - Smaller  $V_{in,CM}$  headroom



$$v_{out1} = -g_{mN} \left( r_{oN} \parallel r_{oP} \parallel \frac{1}{g_{mP}} \right) \frac{v_d}{2}$$

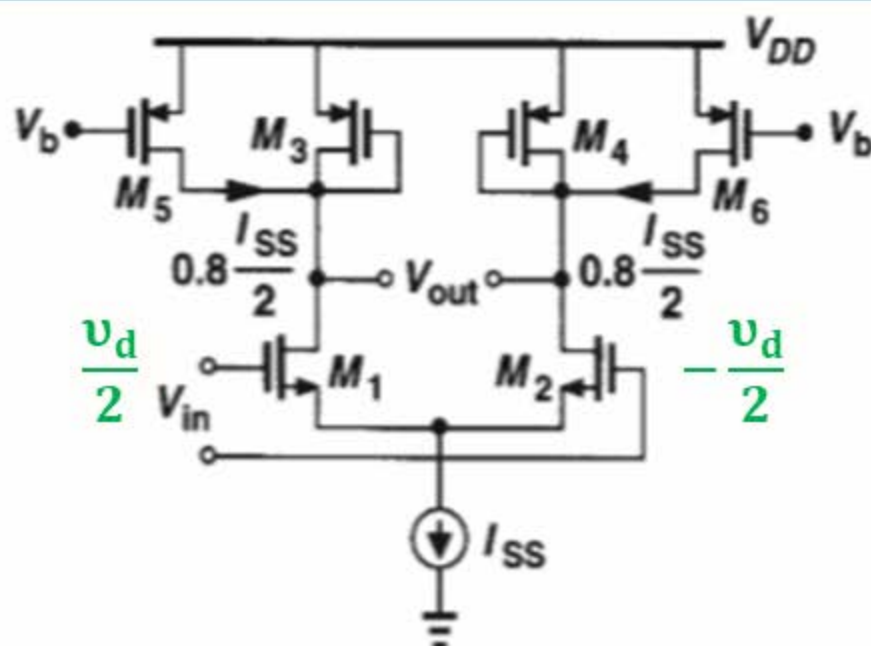
$$v_{out2} = -g_{mN} \left( r_{oN} \parallel r_{oP} \parallel \frac{1}{g_{mP}} \right) \left( -\frac{v_d}{2} \right)$$

$$A_{DM} = \frac{v_{out1} - v_{out2}}{v_d} = -g_{mN} \left( r_{oN} \parallel r_{oP} \parallel \frac{1}{g_{mP}} \right) \approx -\frac{g_{mN}}{g_{mP}} \approx -\sqrt{\frac{\mu_n (W/L)_N}{\mu_p (W/L)_P}}$$

# Differential-Mode (Small-Signal, Half-circuit)

Small-signal  
Analysis

$\lambda \neq 0 \quad \gamma \neq 0$



$$v_{out1} = -gm_{1,2} \left( r_{o1,2} \parallel r_{o3,4} \parallel \frac{1}{gm_{3,4}} \parallel r_{o5,6} \right) \frac{v_d}{2}$$

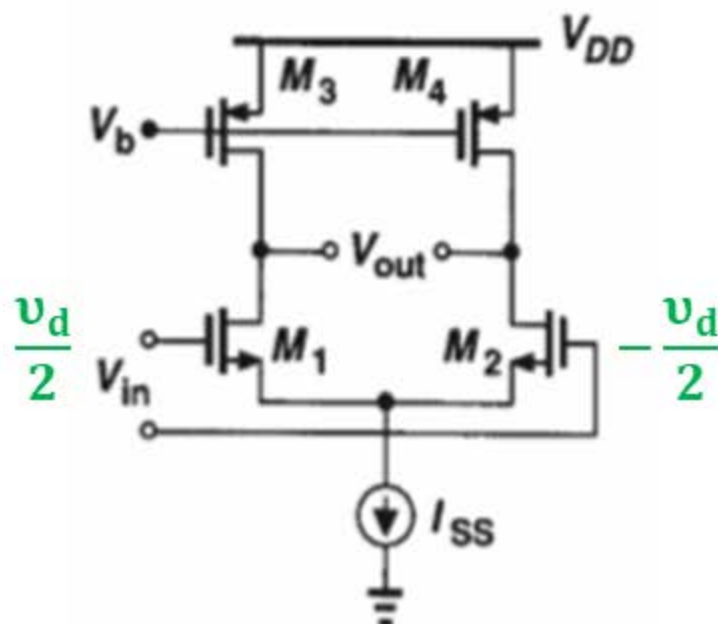
$$v_{out2} = -gm_{1,2} \left( r_{o1,2} \parallel r_{o3,4} \parallel \frac{1}{gm_{3,4}} \parallel r_{o5,6} \right) \left( -\frac{v_d}{2} \right)$$

$$A_{DM} = \frac{v_{out1} - v_{out2}}{v_d} \approx -\frac{gm_{1,2}}{gm_{3,4}} \approx -\sqrt{\frac{5\mu_n(W/L)_{1,2}}{\mu_p(W/L)_{3,4}}}$$

# Differential-Mode (Small-Signal, Half-circuit)

Small-signal  
Analysis

$\lambda \neq 0 \quad \gamma \neq 0$



$$v_{out1} = -g_{m1,2}(r_{o1,2} \parallel r_{o3,4}) \frac{v_d}{2}$$

$$v_{out2} = -g_{m1,2}(r_{o1,2} \parallel r_{o3,4}) \left(-\frac{v_d}{2}\right)$$

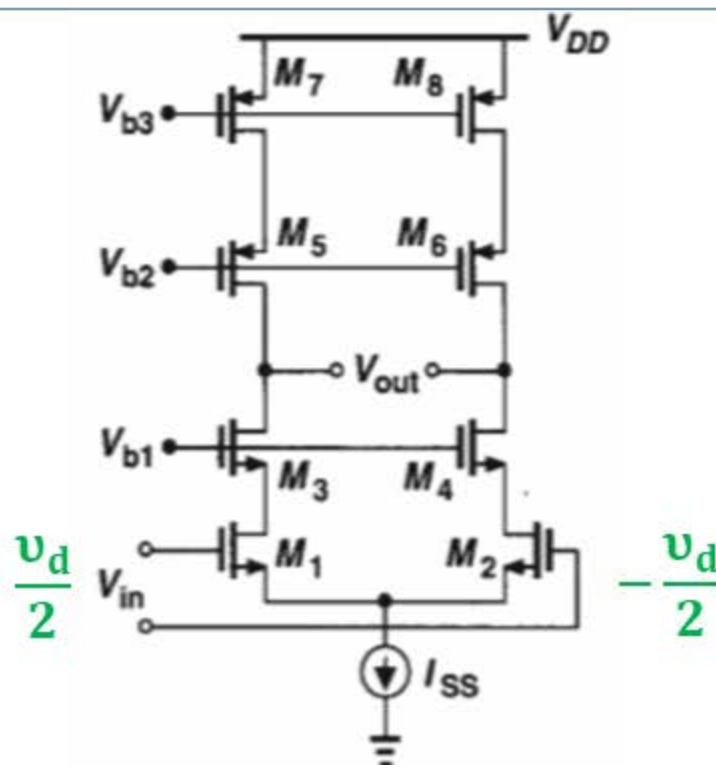
$$A_{DM} = \frac{v_{out1} - v_{out2}}{v_d} = -g_{m1,2}(r_{o1,2} \parallel r_{o3,4})$$

# Differential-Mode (Small-Signal, Half-circuit)

## Small-signal Analysis

$$\lambda \neq 0 \quad \gamma \neq 0$$

- High  $R_{out}$   
 $\rightarrow$  High  $A_{DM}$   
 $\rightarrow$  Small  $V_{in,CM}$  headroom



$$v_{out1} \cong -g_{m1,2} \left\{ [r_{o1,2} + r_{o3,4} + (g_{m3,4} + g_{mb3,4})r_{o3,4}r_{o1,2}] \parallel [r_{o7,8} + r_{o5,6} + (g_{m5,6} + g_{mb5,6})r_{o5,6}r_{o7,8}] \right\} \frac{v_d}{2}$$

$$v_{out2} \cong -g_{m1,2} \left\{ [r_{o1,2} + r_{o3,4} + (g_{m3,4} + g_{mb3,4})r_{o3,4}r_{o1,2}] \parallel [r_{o7,8} + r_{o5,6} + (g_{m5,6} + g_{mb5,6})r_{o5,6}r_{o7,8}] \right\} \left( -\frac{v_d}{2} \right)$$

$$A_{DM} = \frac{v_{out1} - v_{out2}}{v_d} \cong -g_{m1,2} \left[ (g_{m3,4} + g_{mb3,4})r_{o3,4}r_{o1,2} \parallel (g_{m5,6} + g_{mb5,6})r_{o5,6}r_{o7,8} \right]$$