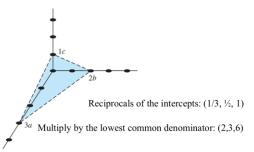
Resistivity:

Conductors	Semiconductors	Insulators
< 10 ⁻³ Ω•cm	$10^{-3} - 10^9 \Omega$ cm	> 10 ⁹ Ω•cm
Metals (Au, Al, Cu, Hg)	Si, Ge, GaAs, InP	SiO ₂ , HfO ₂
Solids, liquids (Hg)	Solids	Solids, liquids gases

Unit cell: any small volume of crystal to reproduce the entire crystal. Primitive cell: smallest unit cell

Crytalline Plane and Miller Index



$$\frac{\partial^2 y}{\partial x^2} = k^2 y$$
 General solution: $y = Ae^{bx}$

Plug into the equation: $b^2Ae^{bx}=k^2Ae^{bx}$

$$\Rightarrow b = \pm k$$

$$\Rightarrow y = A_1 e^{kx} + A_2 e^{-kx}$$

$$\frac{\partial^2 y}{\partial x^2} = -k^2 y$$
 General solution: $y = Ae^{bx}$

Plug into the equation: $b^2Ae^{bx} = -k^2Ae^{bx}$

$$\Rightarrow b = \pm ki$$

$$\Rightarrow y = A_1 e^{ikx} + A_2 e^{-ikx}$$

$$K = \frac{2\pi}{\lambda}, E = mc^2 = hv = \frac{hc}{\lambda}, p = \frac{h}{\lambda} = mv$$

$$\frac{\partial^2 \psi(x)}{\partial x^2} + \frac{2m}{\hbar^2} (E - V(x))\psi(x) = 0$$

$$E = \frac{k^2 \hbar^2}{2m} = \frac{n^2 \pi^2 \hbar^2}{2ma^2}$$

$$k = \frac{n\pi}{a}$$
 $n = 0, \pm 1, \pm 2, ...$

$$p = \hbar k = mv$$

$$\frac{dE}{dk} = \frac{\hbar^2 k}{m} \xrightarrow{mv = \hbar k} \frac{\hbar mv}{m} = \hbar v$$

$$\mathsf{E} = \frac{v^2}{2m} = \frac{\hbar^2 k^2}{2m} \qquad \qquad v = \frac{1}{\hbar} \frac{dE}{dk}$$

$$J = qNv_d = q\sum_{i}^{N} v_i$$

Conduction Band:

$$E = E(k) = E_c + \frac{\hbar^2}{2m_n^*} (k - k_1)^2$$

Valence Band:

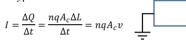
$$E = E(k) = E_c - \frac{\hbar^2}{2m_p^*} (k - k_2)^2$$

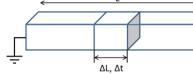
$$E - E_c = C_1(k)^2$$

$$\frac{1}{\hbar^2} \frac{d^2 E}{dk^2} = \frac{2C_1}{\hbar^2}$$

$$\frac{1}{\hbar^2} \frac{d^2 E}{dk^2} = \frac{2C_1}{\hbar^2} = \frac{1}{m^*}$$

n type semiconductor





$$v = \mu E = \mu V/L$$

$$I = \frac{\Delta Q}{\Delta t} = \frac{nqA_c\Delta L}{\Delta t} = nqA_c\mu V/L \qquad \Rightarrow \quad \sigma = \frac{I}{V} = \frac{N_D qA_c\mu}{L}$$

$$f_F(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})} \quad f_F(E) \approx \exp(-\frac{E - E_F}{kT})$$

$$E = E(k) = E_c + \frac{\hbar^2}{2m_n^*} k^2$$

$$k = \mp \frac{\sqrt{2m_n^*(E - E_c)}}{\hbar}$$

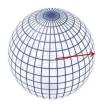


Within ΔE , we have the number of k is $\frac{d(k/\pi)}{dE}\Delta E$

$$g(E) = \frac{1}{2} \frac{d(k/\pi)}{dE}$$

$$E = E(k) = E_c + \frac{\hbar^2}{2m_n^*}k^2$$

$$k = \mp \frac{\sqrt{2m_n^*(E - E_c)}}{\hbar}$$



Within ΔE , we have the number of k is $\frac{d(4\pi \left(\frac{k}{\pi}\right)^2/3)}{dE} \Delta E$

$$g(E) = \frac{1}{8} \frac{d(4\pi \left(\frac{k}{\pi}\right)^2 / 3)}{dE}$$

$$g_v(E) = \frac{4\pi (2m_p^*)^{3/2}}{h^3} \sqrt{E_v - E}$$

$$n_0 = \int_{E_c}^{\infty} g_c(E) f_F(E) dE$$

$$p_0 = \int_{-\infty}^{E_v} g_v(E) [1 - f_F(E)] dE$$

$$if \exp(x - \mathcal{E}) > 10 \Leftrightarrow \frac{E - E_F}{kT} > 3 \Leftrightarrow E - E_F > 3kT$$

$$n_0 = \frac{2(2\pi m_n^* kT)^{\frac{3}{2}}}{h^3} \exp\left(\frac{E_F - E_c}{kT}\right) = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$$

$$p_0 = \frac{2(2\pi m_p^* kT)^{\frac{3}{2}}}{h^3} \exp\left(\frac{E_v - E_F}{kT}\right) = N_v \exp(\frac{E_v - E_F}{kT})$$

$$n \times p = n_i^2 = N_c N_v \exp\left(\frac{E_v - E_c}{kT}\right) \Rightarrow n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2kT}\right)$$

$$n = N_c \exp(\frac{E_F - E_c}{kT}) \qquad p = N_v \exp(\frac{E_v - E_F}{kT})$$

$$n = N_c \exp(\frac{E_F - E_c}{kT}) \qquad p = N_v \exp(\frac{E_v - E_F}{kT}) \qquad N_c \approx 10^{19} cm^{-3}$$

$$N_v \approx 10^{19} cm^{-3}$$

$$n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right] \qquad p_0 = n_i \exp\left[\frac{-(E_F - E_{Fi})}{kT}\right] \qquad n_i \approx 10^{10} cm^{-3}$$

$$n = N_c \exp\left(\frac{E_{Fi} - E_c}{kT}\right) = p = N_v \exp\left(\frac{E_v - E_{Fi}}{kT}\right)$$

$$E_{Fi} = \frac{1}{2}(E_c + E_v) + \frac{1}{2}kTln(\frac{N_v}{N_c})$$

$$E_{midgap} = \frac{1}{2} (E_c + E_v)$$

$$E_{Fi} = E_{midgap} + \frac{3}{4}kTln(\frac{m_p^*}{m_n^*})$$

$$n_{d} = N_{d} - N_{d}^{+}$$

$$= \frac{N_{d}}{1 + \frac{1}{2} \exp(\frac{E_{d} - E_{F}}{kT})}$$

$$p_{a} = \frac{N_{a}}{1 + \frac{1}{8} \exp(\frac{E_{F} - E_{a}}{kT})} = N_{a} - N_{a}^{-}$$

$$n_{0} + (N_{a} - p_{a}) = p_{0} + (N_{d} - n_{d})$$

$$n_{0} = \frac{(N_{d} - N_{a})}{2} + \sqrt{\left(\frac{N_{d} - N_{a}}{2}\right)^{2} + n_{i}^{2}}$$

$$n_{0} = \frac{N_{d}^{+} + \sqrt{(N_{d}^{+})^{2} + 4n_{i}^{2}}}{2} \quad (but \ N_{d}^{+} \text{ unknown})$$

$$n_{0} = N_{c} \times \frac{-1 + \sqrt{1 + \frac{8N_{D}}{N_{c}} \exp(\frac{E_{A}}{kT})}}{4 \exp(\frac{E_{A}}{kT})} = \begin{cases} \sqrt{\frac{N_{D}N_{c}}{2} \exp(-\frac{E_{A}}{2kT})} & partial \ ionization, \\ N_{D} & complete \ ionization \end{cases}$$

$$n_{0} = \frac{N_{D} + \sqrt{N_{D}^{2} + 4n_{i}^{2}}}{2} \quad Complete \ ionization \ at \ high \ T$$

$$E_{C} = E_{C} + kT \ln(\sqrt{1 + \frac{8N_{D}}{N_{c}}} \exp(\frac{E_{A}}{kT}) - 1) - \frac{E_{C} + E_{D}}{2} + \frac{kT}{2} \ln \frac{N_{D}}{2N_{c}} \quad T \ small \ T$$

$$E_F = E_c + kT ln(\frac{\sqrt{1 + \frac{8N_D}{N_c} \exp(\frac{E_A}{kT})} - 1}{4 \exp(\frac{E_A}{kT})}) = \begin{cases} \frac{E_c + E_D}{2} + \frac{kT}{2} ln \frac{N_D}{2N_c} & T \text{ small} \\ E_c - kT ln \frac{N_c}{N_D} & T \text{ big} \end{cases}$$

$$v_d pprox \left(rac{q au_{cp}}{m_{cp}^*}
ight)E \ \Rightarrow \ rac{v_d}{E} = rac{q au_{cp}}{m_{cp}^*} = \mu_p \ (for \ holes)$$

$$rac{v_d}{E} = rac{q au_{cn}}{m_{cp}^*} = \mu_n \ (for \ electrons)$$

$$I_{drf} = \frac{\Delta Q}{\Delta t} = \frac{q p_0 A_c \Delta L}{\Delta t} = q p_0 A_c v = q p_0 A_c \mu_p E = q p_0 A_c \mu_p \frac{V}{L} = \sigma \cdot V$$

$$J_{drf} = q(p_0\mu_p + n_0\mu_n)E$$
 $\frac{1}{\mu} = \frac{1}{\mu_L} + \frac{1}{\mu_L}$

$$v_n = \frac{v_s}{\left[1 + \left(\frac{E_{on}}{E}\right)^2\right]^{1/2}} \quad v_p = \frac{v_s}{\left[1 + \left(\frac{E_{op}}{E}\right)^2\right]^{1/2}}$$

$$J_{nx|dif} = -qF_n = qD_n \frac{dn}{dx}$$

$$J_{px|dif} = qF_p = -qD_p \frac{dp}{dx}$$

$$J = qn\mu_n E_x + qp\mu_p E_x + qD_n \nabla n - qD_p \nabla p$$

$$E_{x} = -\frac{\mathrm{d}\Phi}{\mathrm{d}x} = \frac{1}{e} \frac{\mathrm{d}E_{Fi}}{\mathrm{d}x}$$

$$= -\frac{1}{e} \frac{kT}{n(x)} \frac{\mathrm{d}n(x)}{\mathrm{d}x}$$

$$D_{n} = \frac{\mu_{n}kT}{q}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{q\mu_n n} = \frac{1}{q\mu_n N_d} \quad \rho = \frac{1}{\sigma} = \frac{1}{q\mu_n p} = \frac{1}{q\mu_n N_a}$$

Infinite quantum well

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi = E\Psi, \quad \begin{cases} V(x) = +\infty, & x \le 0 \text{ or } x \ge \\ V(x) = 0, & 0 < x < a \end{cases}$$

General solution:

$$\Psi(x) = Ae^{-ikx} + Be^{ikx}$$

Boundary condition:

$$\Psi(x)|_{x=a,0} = 0$$
$$\int_0^a \Psi(x)\Psi^*(x) \, \mathrm{d}x = 1$$

conclusion:

$$k = \frac{n\pi}{a}, n = 0, \pm 1, \pm 2, \dots$$

$$E = \frac{k^2 \hbar^2}{2m} = \frac{n^2 \pi^2 \hbar^2}{2ma^2}$$

Finite quantum well

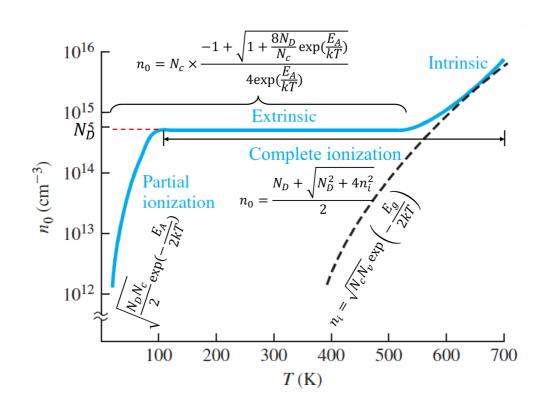
$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V(x)\Psi = E\Psi, \quad \left\{ \begin{array}{ll} V(x) = V_0, & x \leq 0 \text{ or } x \geq a \\ V(x) = 0, & 0 < x < a \end{array} \right.$$

General solution:

$$\Psi(x) = \left\{ \begin{array}{ll} A \mathrm{e}^{-ik_1 x} + B \mathrm{e}^{ik_1 x}, & k_1 = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}, & x \leq 0 \text{ or } x \geq a \\ C \mathrm{e}^{-ik_2 x} + D \mathrm{e}^{ik_2 x}, & k_2 = \sqrt{\frac{2mE}{\hbar^2}}, & 0 < x < a \end{array} \right.$$

Boundary condition:

$$\Psi(x)|_{x=0}$$
 continuous $\Psi(x)|_{x=a}$ continuous
$$\int_{-\infty}^{\infty} \Psi(x) \Psi^*(x) \, \mathrm{d}x = 1$$



From textbook Semiconductor Physics and Devices: Basic Principles 4th edition. P716-718 (Appendix B)

Table B.2 | Conversion factors

	Prefixes		
$1 \text{ Å (angstrom)} = 10^{-8} \text{ cm} = 10^{-10} \text{ m}$	10^{-15}	femto-	= f
$1 \mu\mathrm{m} (\mathrm{micrometer}) = 10^{-4} \mathrm{cm}$	10^{-12}	pico-	= p
$1 \text{ mil} = 10^{-3} \text{ in.} = 25.4 \ \mu\text{m}$	10^{-9}	nano-	= n
2.54 cm = 1 in.	10^{-6}	micro-	$=\mu$
$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$	10^{-3}	milli-	= m
$1 \mathrm{J} = 10^7 \mathrm{erg}$	10^{+3}	kilo-	= k
	10^{+6}	mega-	= M
	10^{+9}	giga-	= G
	10^{+12}	tera	= T

Table B.3 | Physical constants

Avogadro's number	$N_A = 6.02 \times 10^{+23}$
	atoms per gram
	molecular weight
Boltzmann's constant	$k = 1.38 \times 10^{-23} \text{J/K}$
	$= 8.62 \times 10^{-5} \mathrm{eV/K}$
Electronic charge	$e = 1.60 \times 10^{-19} \mathrm{C}$
(magnitude)	
Free electron rest mass	$m_0 = 9.11 \times 10^{-31} \mathrm{kg}$
Permeability of free space	$\mu_0=4\pi imes 10^{-7}$ H/m
Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-14} \mathrm{F/cm}$
	$= 8.85 \times 10^{-12} \text{F/m}$
Planck's constant	$h = 6.625 \times 10^{-34} \text{J-s}$
	$= 4.135 \times 10^{-15} \mathrm{eV}$ -s
	$\frac{h}{2\pi} = \hbar = 1.054 \times 10^{-34} \text{J-s}$
	2π
Proton rest mass	$M = 1.67 \times 10^{-27} \mathrm{kg}$
Speed of light in vacuum	$c = 2.998 \times 10^{10} \mathrm{cm/s}$
Thermal voltage ($T = 300 \text{ K}$)	$V_t = \frac{kT}{e} = 0.0259 \text{ V}$
	kT = 0.0259 eV

Table B.4 | Silicon, gallium arsenide, and germanium properties (T = 300 K)

Property	Si	GaAs	Ge
Atoms (cm ⁻³)	5.0×10^{22}	4.42×10^{22}	4.42×10^{22}
Atomic weight	28.09	144.63	72.60
Crystal structure	Diamond	Zincblende	Diamond
Density (g/cm ³)	2.33	5.32	5.33
Lattice constant (Å)	5.43	5.65	5.65
Melting point (°C)	1415	1238	937
Dielectric constant	11.7	13.1	16.0
Bandgap energy (eV)	1.12	1.42	0.66
Electron affinity, χ (V)	4.01	4.07	4.13
Effective density of states in conduction band, N_c (cm ⁻³)	2.8×10^{19}	4.7×10^{17}	1.04×10^{19}
Effective density of states in valence band, N_{ν} (cm ⁻³)	1.04×10^{19}	7.0×10^{18}	6.0×10^{18}
Intrinsic carrier concentration (cm ⁻³)	1.5×10^{10}	1.8×10^{6}	2.4×10^{13}
Mobility (cm²/V-s)			
Electron, μ_n	1350	8500	3900
Hole, μ_p	480	400	1900
Effective mass $\left(\frac{m^*}{m_0}\right)$			
Electrons	$m_I^* = 0.98$	0.067	1.64
	$m_i^* = 0.19$		0.082
Holes	$m_{th}^* = 0.16$	0.082	0.044
	$m_{bb}^* = 0.49$	0.45	0.28
Density of states effective mass			
Electrons $\left(\frac{m_{dn}^*}{m_o}\right)$	1.08	0.067	0.55
Holes $\left(\frac{m_{dp}^*}{m_o}\right)$	0.56	0.48	0.37
Conductivity effective mass			
Electrons $\left(\frac{m_{ci}^*}{m_o}\right)$	0.26	0.067	0.12
$ ext{Holes}\left(rac{m_{cp}^*}{m_o} ight)$	0.37	0.34	0.21

Table B.5 | Other semiconductor parameters

Material	$E_g(\mathrm{eV})$	a (Å)	ϵ_r	χ	\overline{n}
Aluminum arsenide	2.16	5.66	12.0	3.5	2.97
Gallium phosphide	2.26	5.45	10	4.3	3.37
Aluminum phosphide	2.43	5.46	9.8		3.0
Indium phosphide	1.35	5.87	12.1	4.35	3.37

Table B.6 | Properties of SiO_2 and Si_3N_4 (T = 300 K)

Property	SiO ₂	Si ₃ N ₄	
Crystal structure	[Amorphous for most integrated circuit applications]		
Atomic or molecular density (cm ⁻³)	2.2×10^{22}	1.48×10^{22}	
Density (g/cm ³)	2.2	3.4	
Energy gap	$\approx 9 \text{ eV}$	4.7 eV	
Dielectric constant	3.9	7.5	
Melting point (°C)	≈1700	≈1900	