VE320 – Summer 2021

Introduction to Semiconductor Devices

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Chapter 8 The pn Junction Diode

Outline

8.1 pn junction current

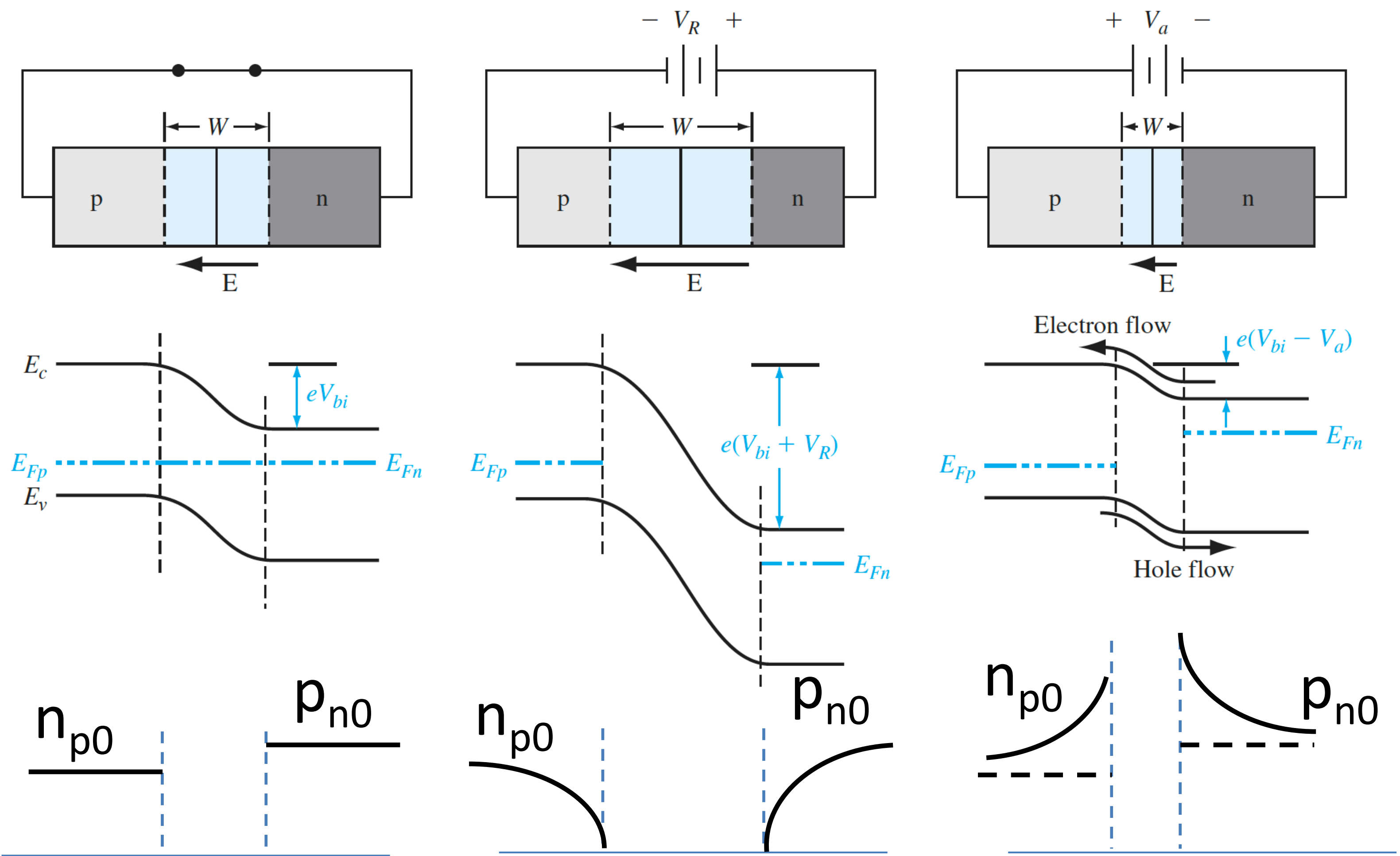
8.2 Generation-recombination currents

8.3 High-injection levels

8.4 A few more points on pn junctions (not in the textbook)

8.1 pn Junction Current

Qualitative Description of Charge Flow in a pn Junction



8.1 pn Junction Current

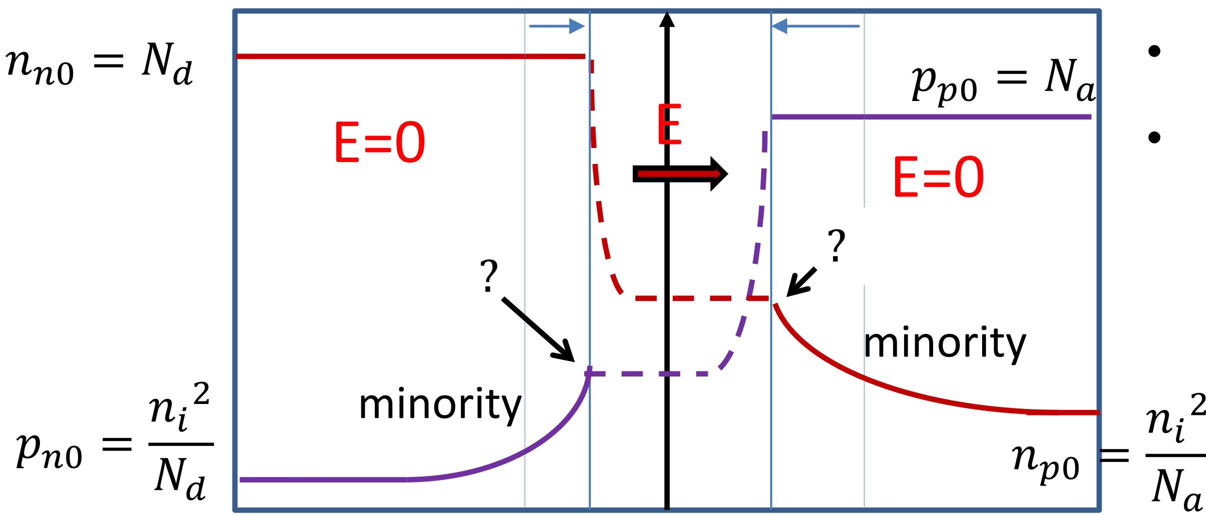
Goal: to find the analytical expression of current

$$\frac{\partial n}{\partial t} = D_n \frac{\partial^2 n}{\partial x^2} + \mu_n E \frac{\partial n}{\partial x} + n\mu_n \frac{\partial E}{\partial x} - \frac{\Delta n}{\tau} + G_{ex}$$

Electrons as minority

$$\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} - \mu_p E \frac{\partial p}{\partial x} - p\mu_p \frac{\partial E}{\partial x} - \frac{\Delta p}{\tau} + G_{ex}$$

holes as minority



- How to simplify?
- Boundary condition?
- how to get total current from minority currents?

8.1 pn Junction Current

Assumptions of an ideal PN junction

1. The abrupt depletion layer approximation applies. The space charge regions have abrupt boundaries, and the semiconductor is neutral outside of the depletion region.
2. The Maxwell–Boltzmann approximation applies to carrier statistics.
3. The concepts of low injection and complete ionization apply.

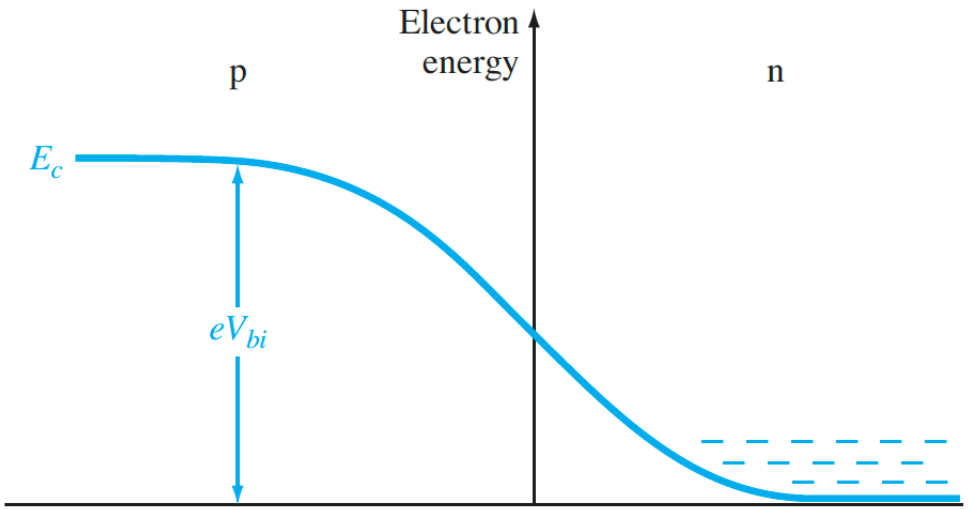
4a. The total current is a constant throughout the entire pn structure.

4b. The individual electron and hole currents are continuous functions through the pn structure.

4c. The individual electron and hole currents are constant throughout the depletion region.

8.1 pn Junction Current

Boundary condition



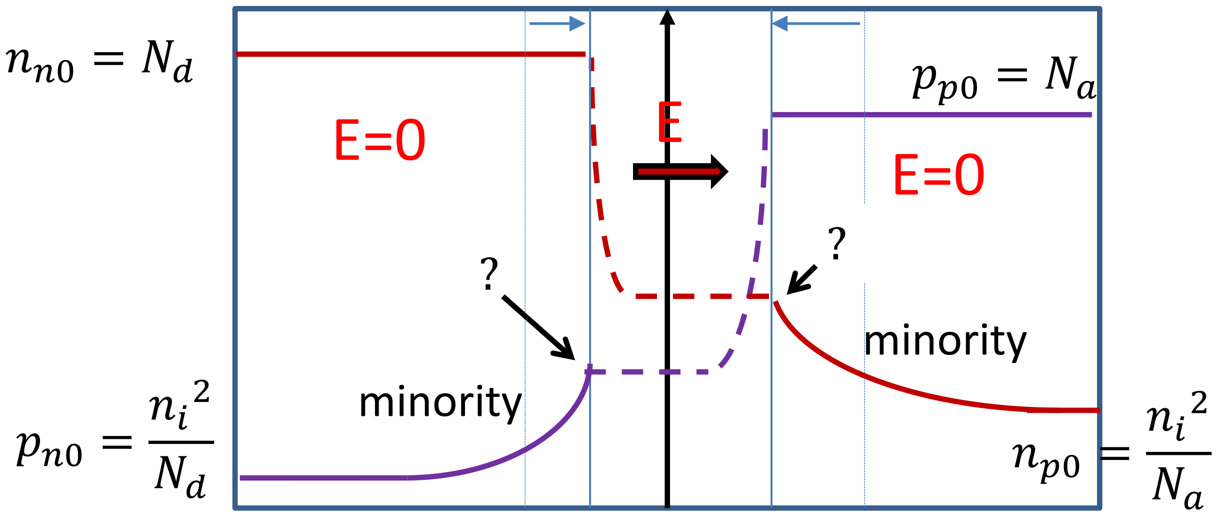
$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$n_{p0} = n_{n0} \exp\left(\frac{-eV_{bi}}{kT}\right)$$

8.1 pn Junction Current

Boundary condition

$$p_{n0} = p_{p0} exp\left(\frac{-eV_{bi}}{kT}\right) \qquad n_{p0} = n_{n0} \exp\left(\frac{-eV_{bi}}{kT}\right)$$

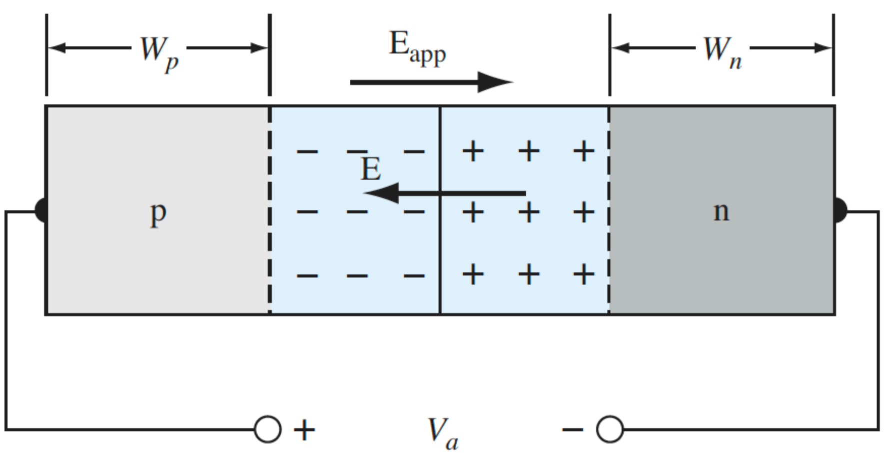


8.1 pn Junction Current

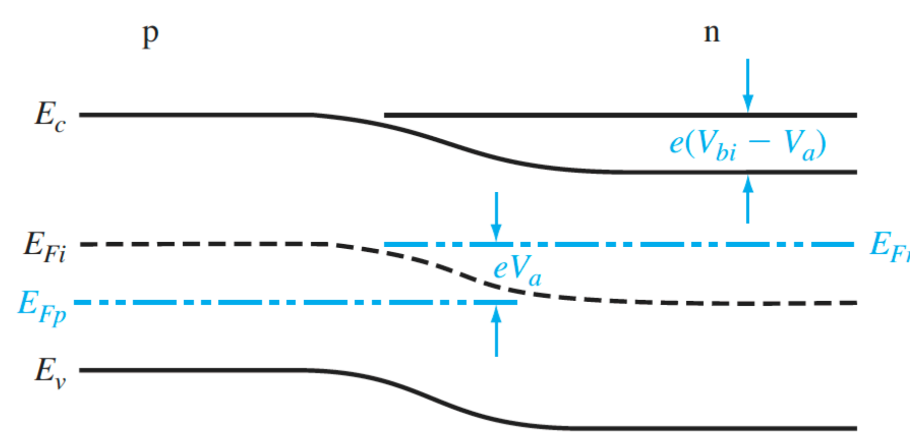
Boundary condition

$$p_{n0} = p_{p0} exp\left(\frac{-eV_{bi}}{kT}\right) \qquad n_{p0} = n_{n0} \exp\left(\frac{-eV_{bi}}{kT}\right)$$

$$p_n = p_{p0} exp\left(\frac{-e(V_{bi}-V_a)}{kT}\right) = p_{n0} exp\left(\frac{eV_a}{kT}\right) \qquad n_p = n_{p0} exp\left(\frac{eV_a}{kT}\right)$$



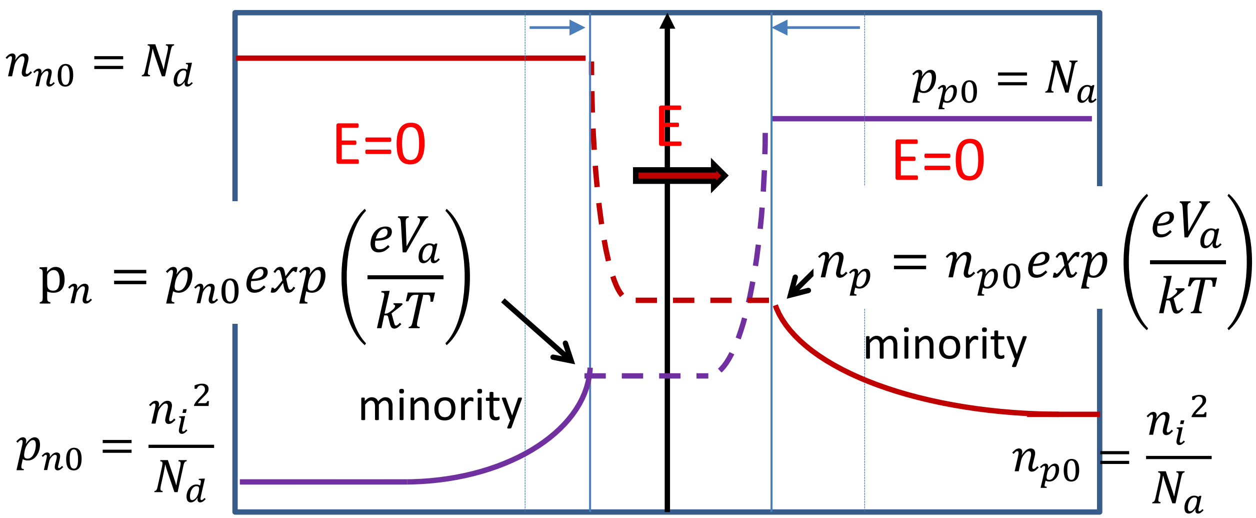
(a)



(b)

8.1 pn Junction Current

Boundary condition



8.1 pn Junction Current

Problem Example

Consider a silicon pn junction at T = 300K. Assume the doping concentration in the n region is $N_d = 10^{16}$ cm^{-3} and the doping concentration in the p region is $N_a = 6 \times 10^{15}$ cm^{-3}. Assume a forward bias of 0.6V is applied to the pn junction. Calculate the minority concentration at the edge of the depletion region.

8.1 pn Junction Current

<u>Minority carrier distribution</u>

$$\cancelto{0}{\frac{\partial p}{\partial t}} = D_p \frac{\partial^2 \cancelto{\Delta p}{p}}{\partial x^2} - \mu_p \cancelto{0}{E} \frac{\partial p}{\partial x} - p\mu_p \frac{\cancelto{0}{\partial E}}{\partial x} - \frac{\Delta p}{\tau} + \cancelto{0}{G_{ex}}$$

holes as minority

$$\boxed{D_p \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau} = 0}$$

- How to simplify?
- Boundary condition?
- how to get total current from minority currents?

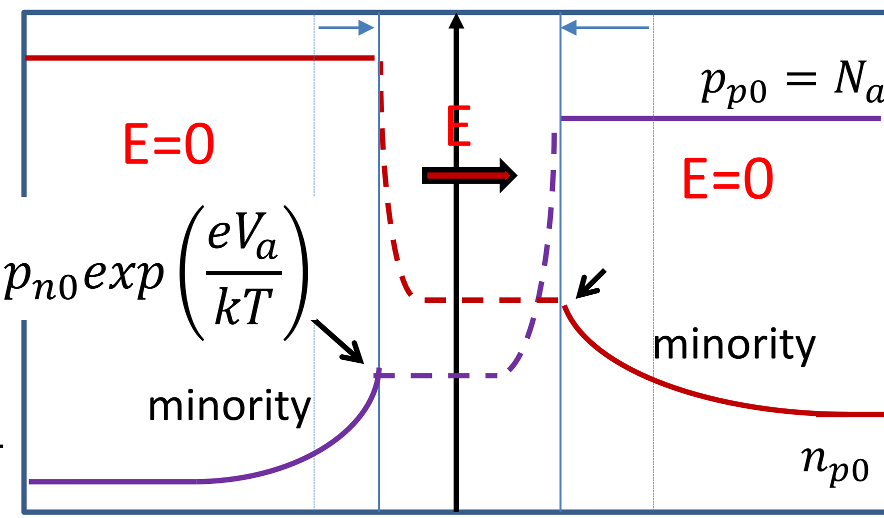
$n_{n0} = N_d$

$p_{p0} = N_a$

$p_n = p_{n0} exp\left(\frac{eV_a}{kT}\right)$

$p_{n0} = \frac{{n_i}^2}{N_d}$

$n_{p0} = \frac{{n_i}^2}{N_a}$



8.1 pn Junction Current

Minority carrier distribution

$$\delta p_n(x) = p_n(x) - p_{n0} = Ae^{x/L_p} + Be^{-x/L_p} \quad (x \le -x_p)$$

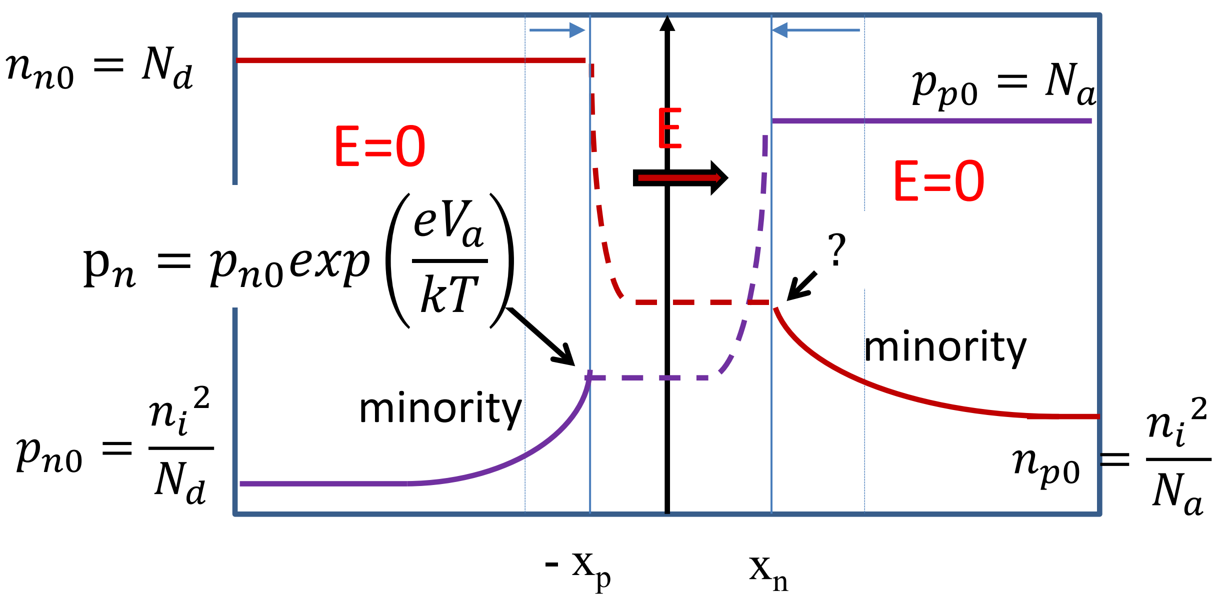
$$\delta n_p(x) = n_p(x) - n_{p0} = Ce^{x/L_n} + De^{-x/L_n} \quad (x \ge x_n)$$

$$p_n(x_n) = p_{n0}\exp\left(\frac{eV_a}{kT}\right)$$

$$n_p(-x_p) = n_{p0}\exp\left(\frac{eV_a}{kT}\right)$$

$$p_n(x \to +\infty) = p_{n0}$$

$$n_p(x \to -\infty) = n_{p0}$$

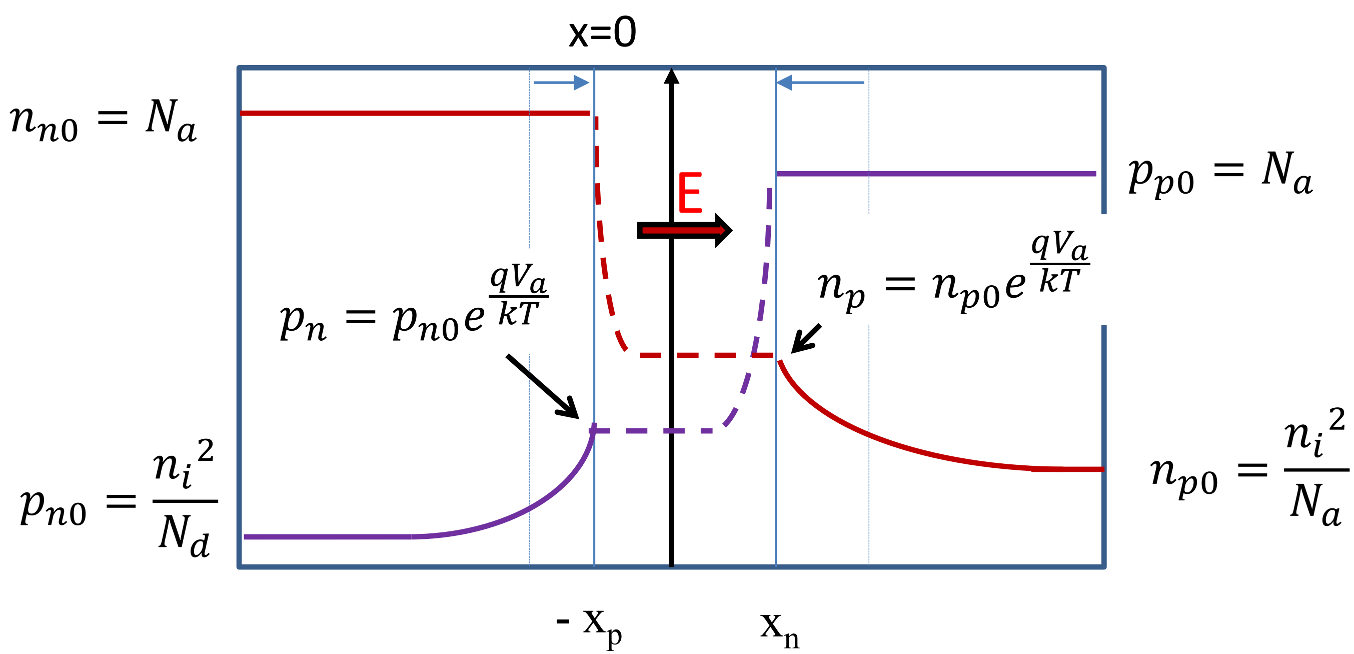


8.1 pn Junction Current

Minority carrier distribution

$$\Rightarrow \Delta p = p_n(x) - p_{n0} = p_{n0}(e^{\frac{qV_a}{kT}} - 1)e^{(x+\mathrm{x_p})/L_p}$$

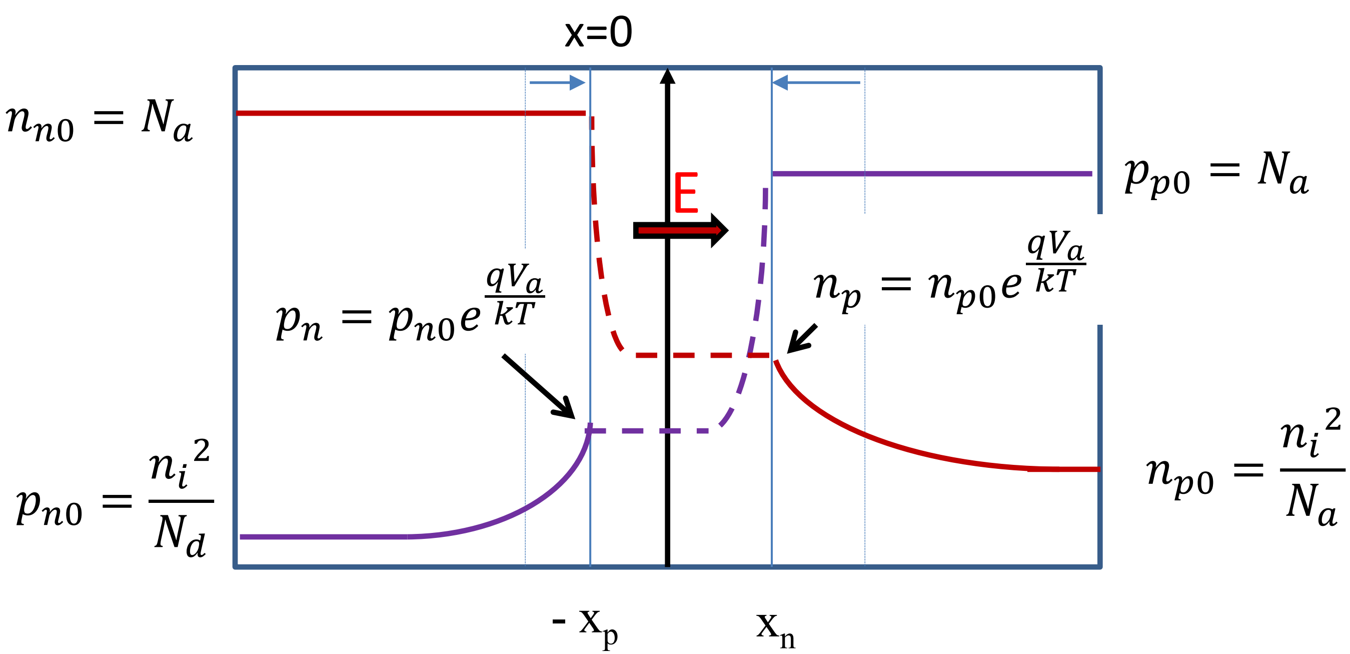
$$\Rightarrow \Delta n = n_p(x) - n_{p0} = n_{p0}(e^{\frac{qV_a}{kT}} - 1)e^{(x_n-\mathrm{x})/L_n}$$



8.1 pn Junction Current

Minority carrier distribution

$$p = p_o + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right) \qquad n = n_o + \delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

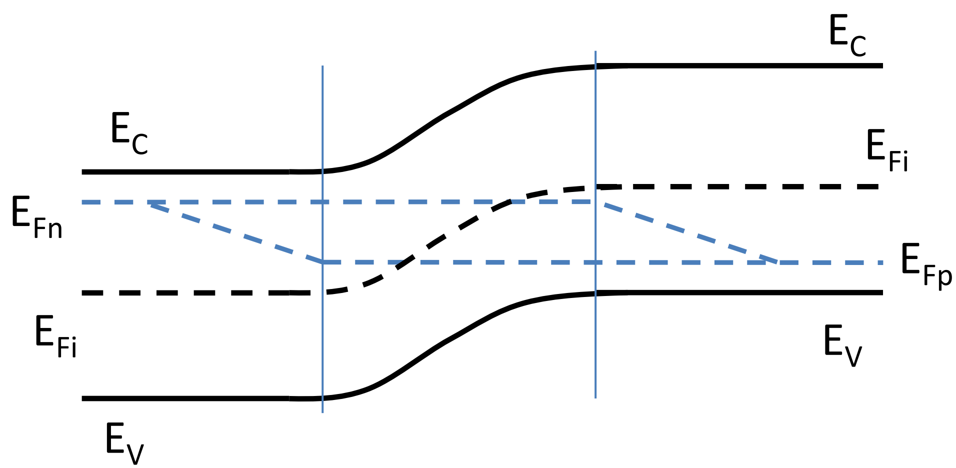


8.1 pn Junction Current

Minority carrier distribution

$$p = p_o + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right) \qquad n = n_o + \delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

$$np = n_i^2 \exp\left(\frac{E_{Fn} - E_{Fp}}{kT}\right)$$



8.1 pn Junction Current

<u>Minority carrier distribution</u>

$$p = p_o + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right) \qquad n = n_o + \delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

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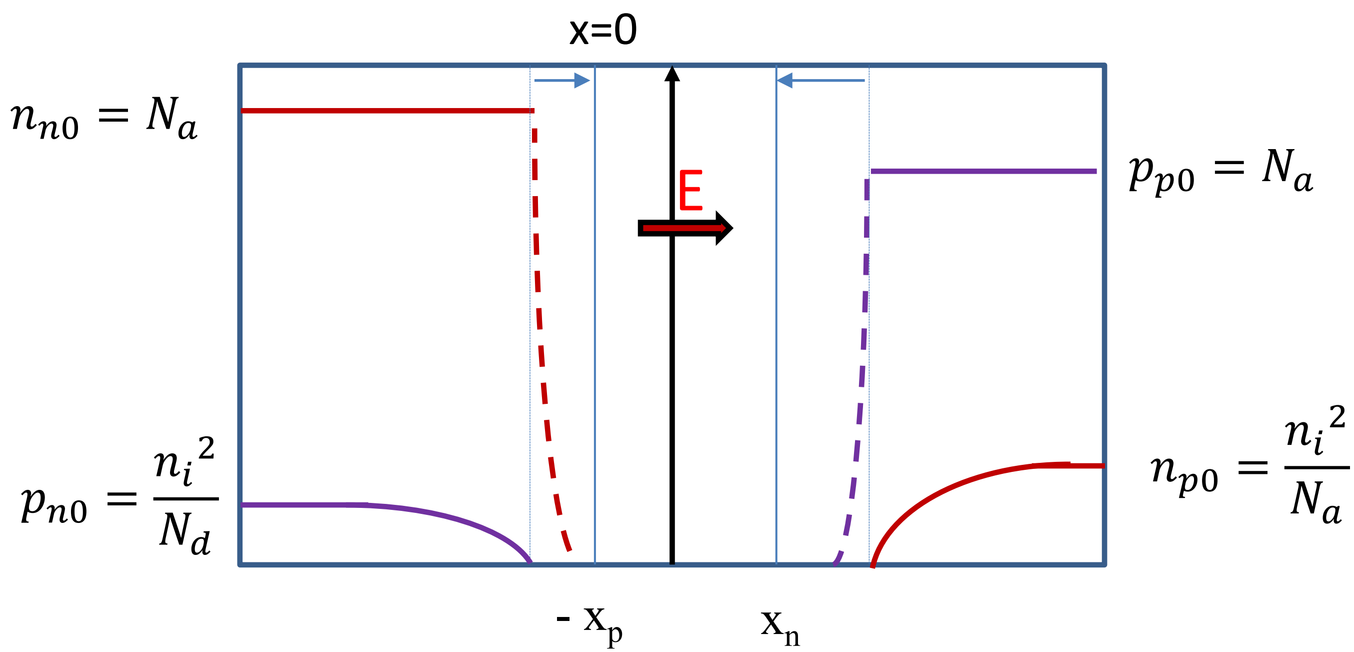
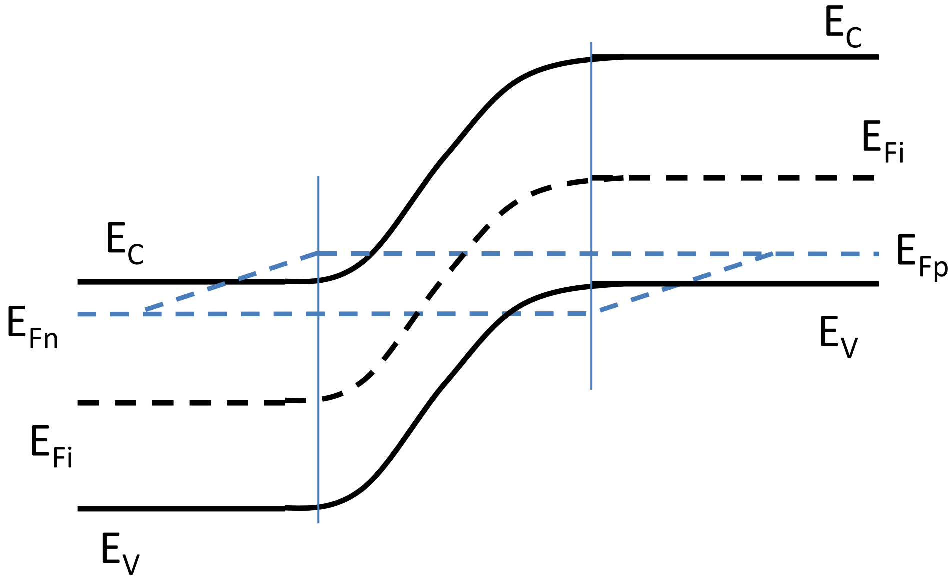


8.1 pn Junction Current

Minority carrier distribution

$$p = p_o + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right) \qquad n = n_o + \delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

$$np = n_i^2 \exp\left(\frac{E_{Fn} - E_{Fp}}{kT}\right)$$

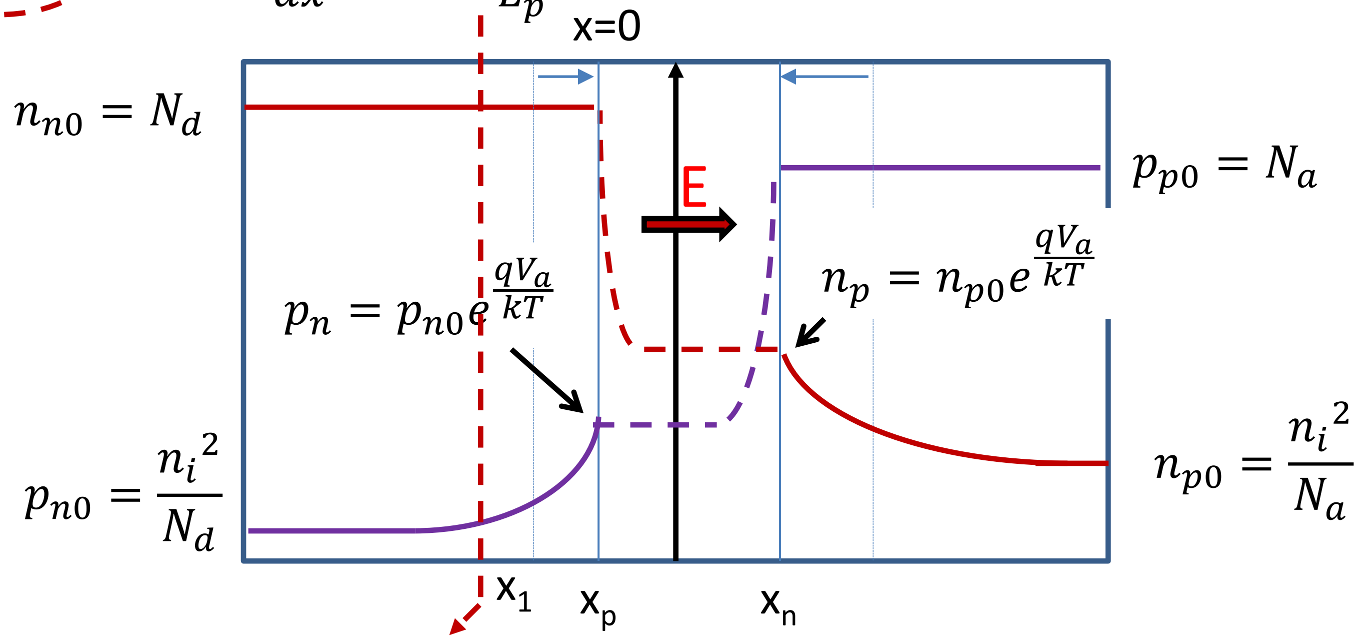


8.1 pn Junction Current

- charge carrier transport: forward bias

$$J_{n,diff} = qD_n\frac{dn_p}{dx} = -\frac{qD_n n_{p0}}{L_n}(e^{\frac{qV_a}{kT}} - 1)e^{\frac{x_n - x}{L_n}}$$

$$J_{p,diff} = -qD_p\frac{dp_n}{dx} = -\frac{qD_p p_{n0}}{L_p}(e^{\frac{qV_a}{kT}} - 1)e^{\frac{x + x_p}{L_p}}$$



8.1 pn Junction Current

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$$J_{n,diff} = qD_n \frac{dn_p}{dx} = -\frac{qD_n n_{p0}}{L_n}(e^{\frac{qV_a}{kT}} - 1)e^{\frac{x_n - x}{L_n}}$$

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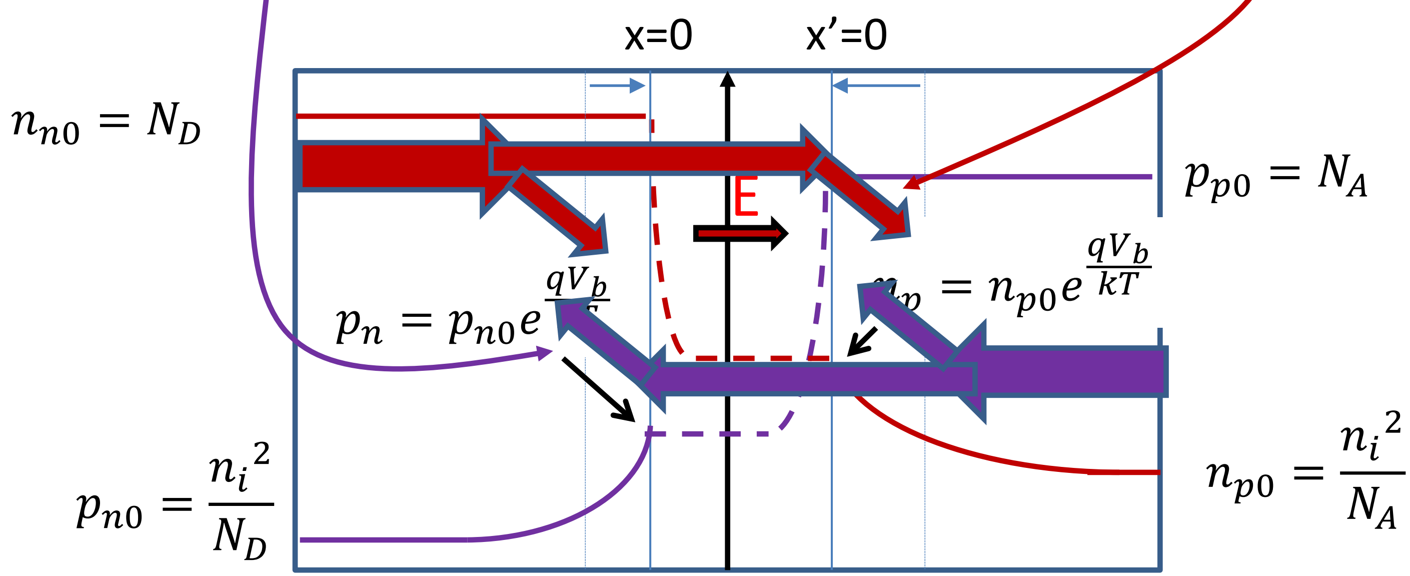
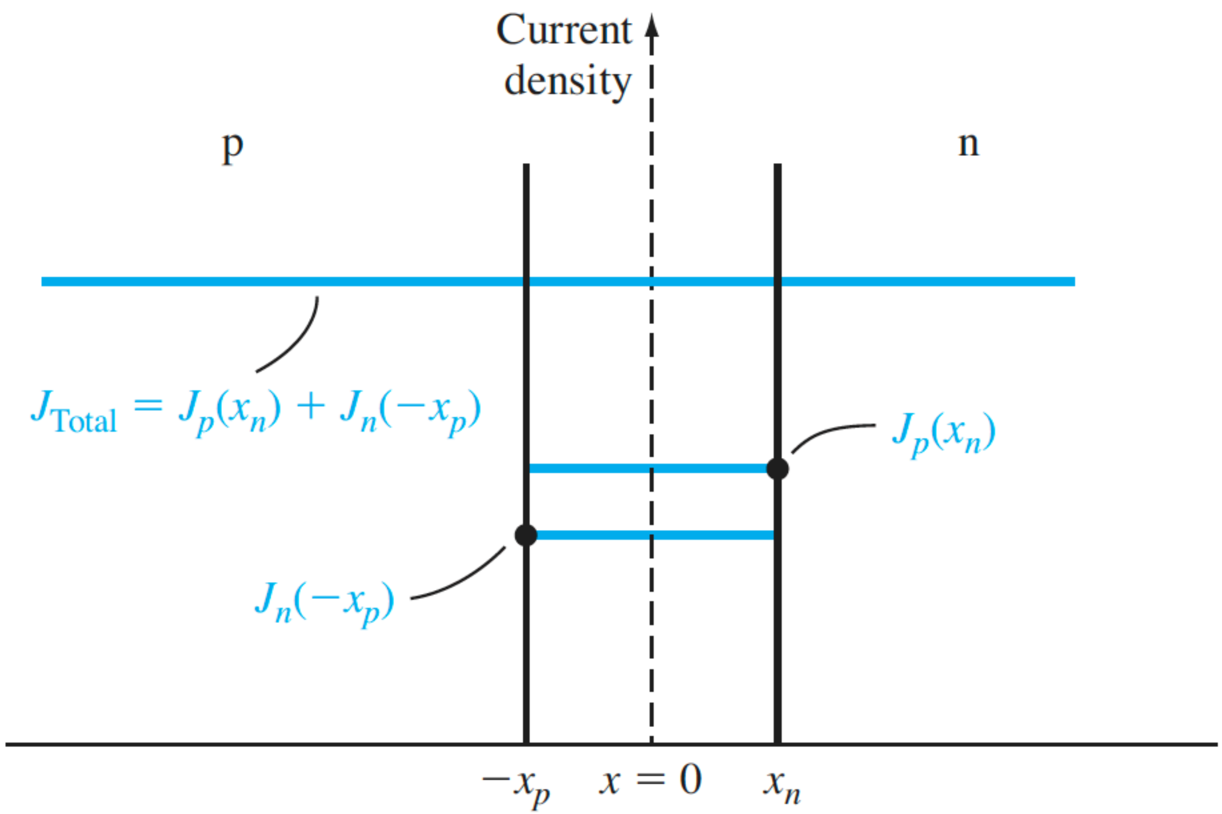
$$\boxed{J = J_n|_{x'=0} + J_n|_{x=0}}$$



Assumption: No recombination-generation in depletion region.

8.1 pn Junction Current

- Ideal pn junction current



Assumption: No recombination-generation in depletion region.

8.1 pn Junction Current

- Ideal pn junction current

$$\boxed{J = J_n|_{x'=0} + J_n|_{x=0}}$$

$$\boxed{J = J_s\left(e^{\frac{qV_b}{kT}} - 1\right)}$$

$$\boxed{J_s = \frac{qD_n n_{p0}}{L_n} + \frac{qD_p p_{n0}}{L_p}}$$

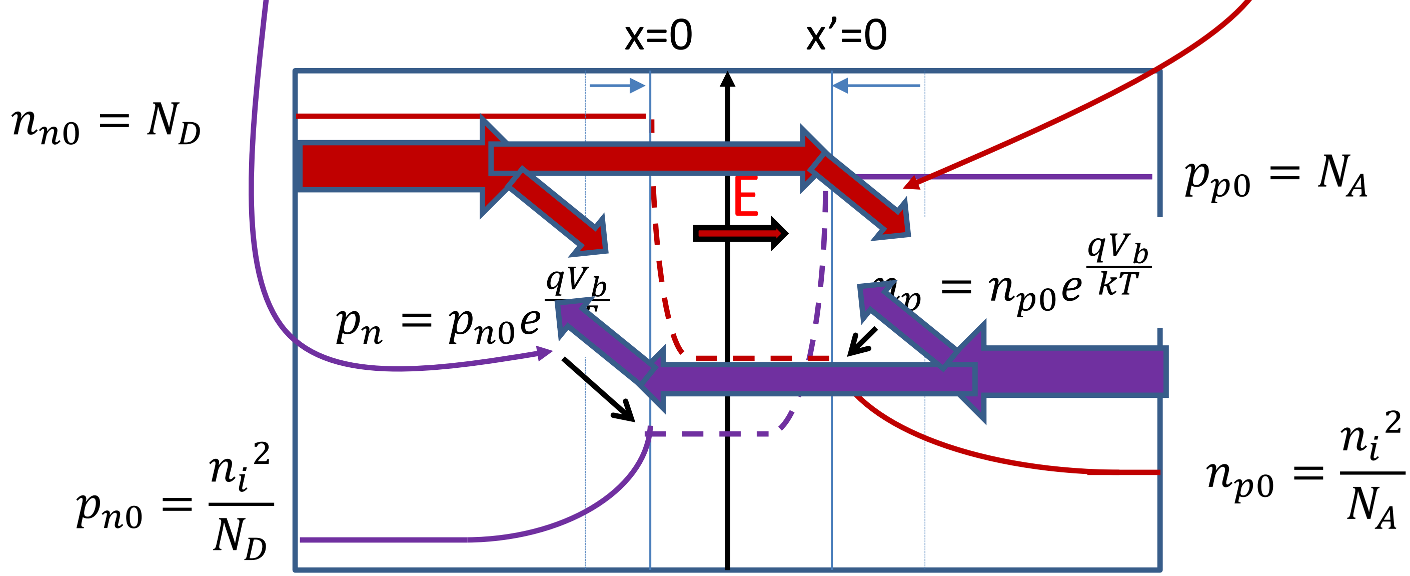
8.1 pn Junction Current

- charge carrier transport: forward bias: current ratio

$$J_n = qD_n \frac{dn_p}{dx} = -\frac{qD_n n_{p0}}{L_n}(e^{\frac{qV_b}{kT}} - 1)$$

$$J_p = -qD_p \frac{dp_n}{dx} = -\frac{qD_p p_{n0}}{L_p}(e^{\frac{qV_b}{kT}} - 1)$$

$$\boxed{\frac{J_n}{J_p} = \frac{D_n n_{p0}/L_n}{D_p p_{n0}/L_p}}$$



Assumption: No recombination-generation in depletion region.

8.1 pn Junction Current

- charge carrier transport: reverse bias

$$J = J_n|_{x'=0} + J_n|_{x=0}$$

$$J_n = qD_n\frac{dn_p}{dx} = \frac{qD_n n_{p0}}{L_n}$$

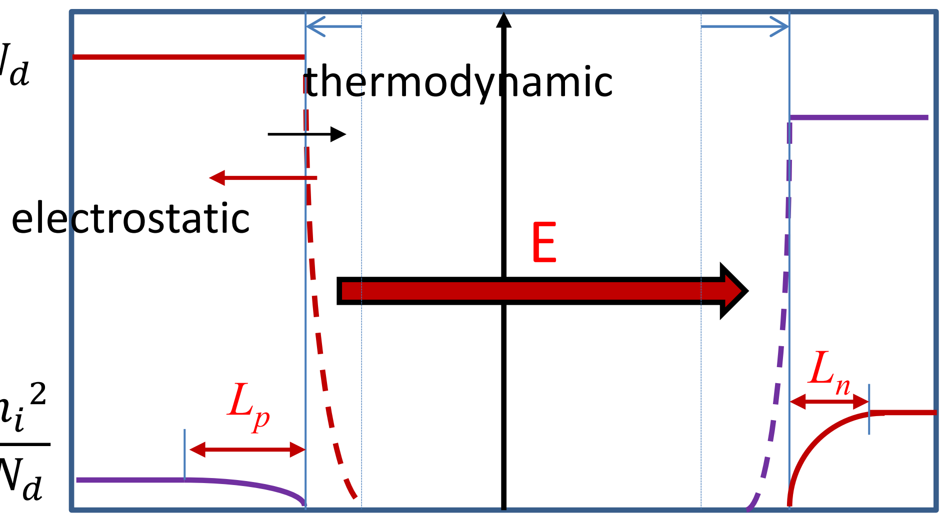
$$J = J_s\left(e^{\frac{qV_b}{kT}} - 1\right) = -J_s$$

$$J_p = -qD_p\frac{dp_n}{dx} = \frac{qD_p p_{n0}}{L_p}$$

$$J_s = \frac{qD_n n_{p0}}{L_n} + \frac{qD_p p_{n0}}{L_p}$$

$n_{n0} = N_d$

$p_{p0} = N_a$



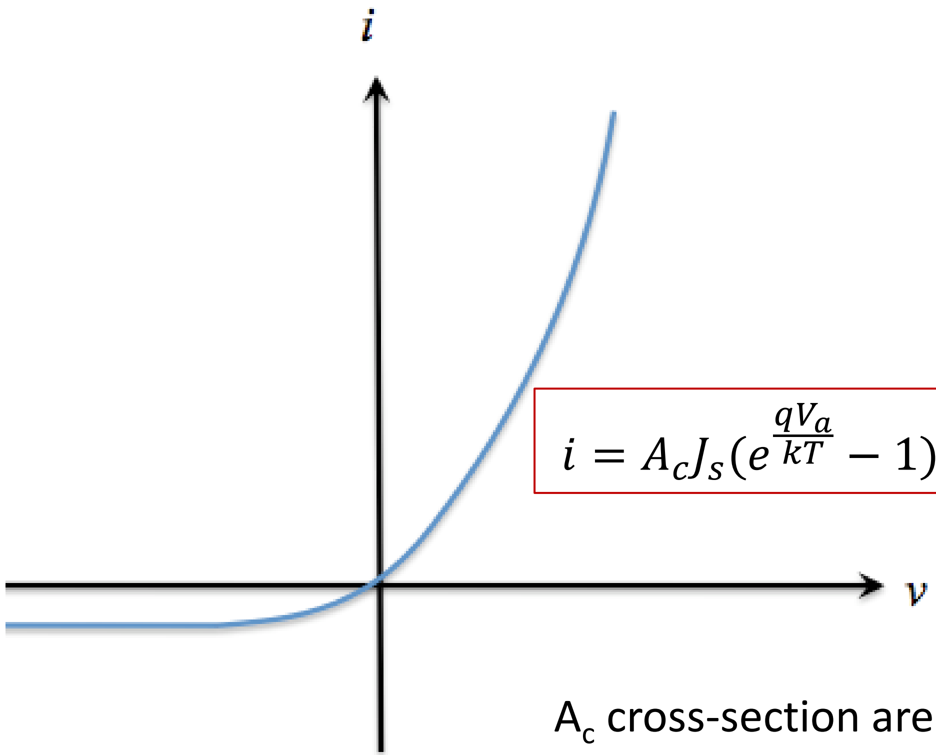
$p_{n0} = \frac{{n_i}^2}{N_d}$

$n_{p0} = \frac{{n_i}^2}{N_a}$

Assumption: No recombination-generation in depletion region.

8.1 pn Junction Current

- charge carrier transport: forward bias



$$J = J_n|_{x'=0} + J_n|_{x=0}$$

$$J = J_s(e^{\frac{qV_b}{kT}} - 1)$$

$$J_s = \frac{qD_n n_{p0}}{L_n} + \frac{qD_p p_{n0}}{L_p}$$

$$i = A_c J_s(e^{\frac{qV_a}{kT}} - 1)$$

A_c cross-section area

8.1 pn Junction Current

Problem Example #2

Given the following parameters in a silicon pn junction, determine the ideal reverse-saturation current density of this pn junction at 300K.

$$N_a = N_d = 10^{16}\ \text{cm}^{-3} \qquad n_i = 1.5 \times 10^{10}\ \text{cm}^{-3}$$

$$D_n = 25\ \text{cm}^2/\text{s} \qquad \tau_{p0} = \tau_{n0} = 5 \times 10^{-7}\ \text{s}$$

$$D_p = 10\ \text{cm}^2/\text{s} \qquad \epsilon_r = 11.7$$

Outline

8.1 pn junction current

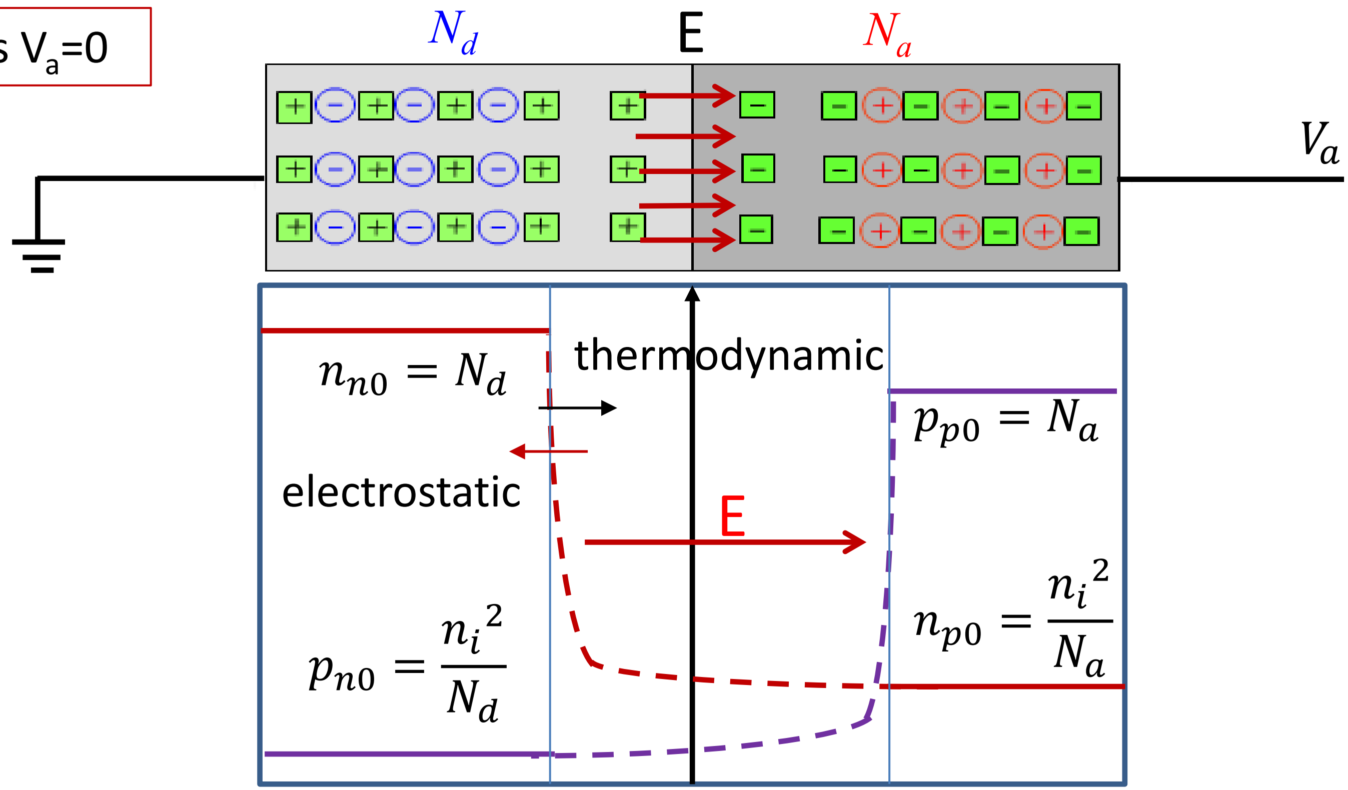
8.2 Generation-recombination currents

8.3 High-injection levels

8.4 A few more points on pn junctions (not in the textbook)

8.2 Generation-recombination currents

Zero Bias V_a=0

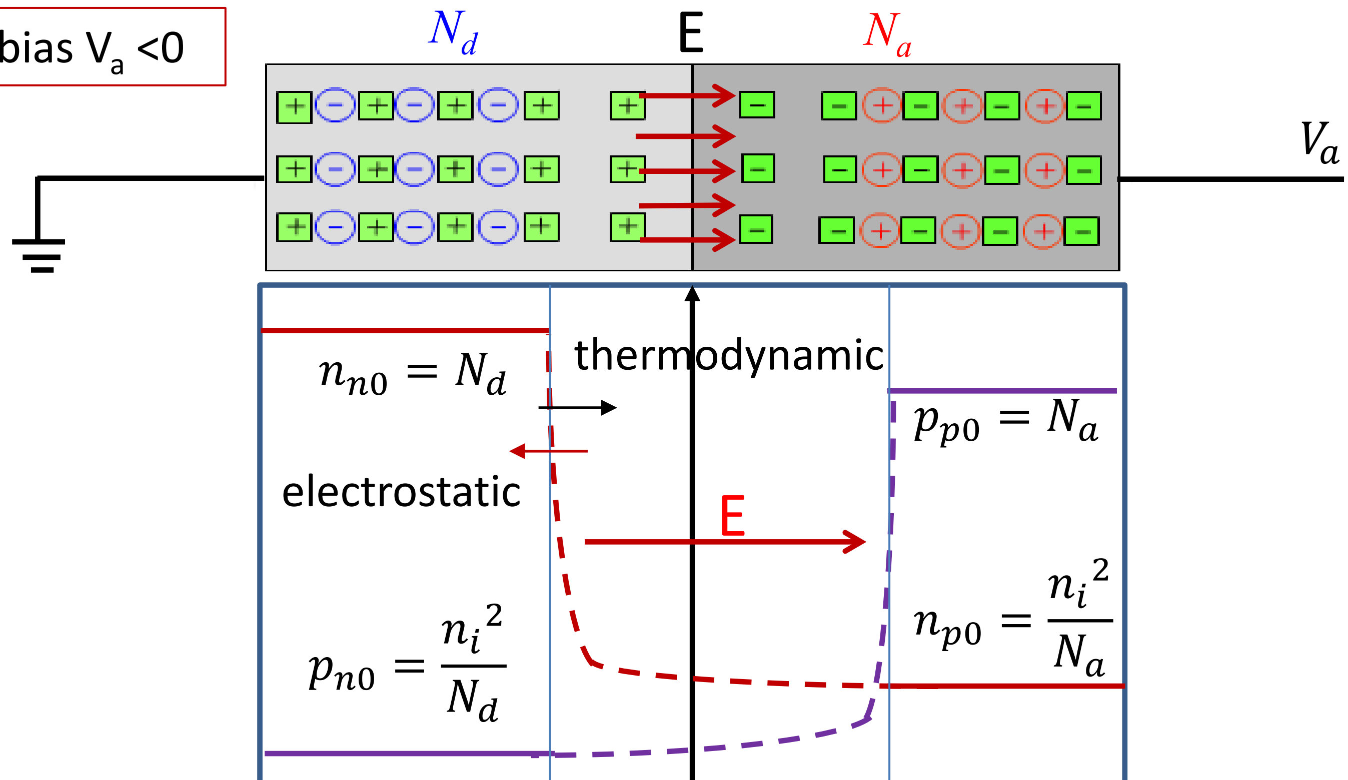


$$R_n = \frac{(np - n_i^2)}{\tau_p\left[n + n_i \exp\left(\frac{E_t - E_i}{kT}\right)\right] + \tau_n\left[p + n_i \exp\left(\frac{E_i - E_t}{kT}\right)\right]}$$

$$In\ depletion\ region{:}\ np = n_i^2 \exp\left(\frac{qV_a}{kT}\right)$$

8.2 Generation-recombination currents

Reverse bias V_a <0



$$R_n = \frac{(\cancelto{0}{np} - n_i^2)}{\tau_p\left[\cancelto{0}{n} + n_i \exp\left(\frac{E_t - E_i}{kT}\right)\right] + \tau_n\left[\cancelto{0}{p} + n_i \exp\left(\frac{E_i - E_t}{kT}\right)\right]}$$

8.2 Generation-recombination currents

Reverse bias $V_a < 0$

To simplify the calculation, we assume

$$\boxed{E_t = E_i, \tau_n = \tau_p = \tau}$$

$$\boxed{R_n = \frac{-n_i}{2\tau} = -G_0}$$

$$R_n = \frac{(\cancel{np}^{\,0} - n_i^2)}{\tau_p \left[\cancel{n}_{0} + n_i \exp\left(\frac{E_t - E_i}{kT}\right)\right] + \tau_n \left[\cancel{p}_{0} + n_i \exp\left(\frac{E_i - E_t}{kT}\right)\right]}$$

8.2 Generation-recombination currents

Reverse bias V_a <0

To simplify the calculation, we assume

$$\boxed{E_t = E_i, \tau_n = \tau_p = \tau}$$

$$\boxed{R_n = \frac{-n_i}{2\tau} = -G_0}$$

Current density from G-R in the depletion region:

$$J_r = \int_0^W qG_0 dx = \frac{qWn_i}{2\tau}$$

$$\boxed{R_n = \frac{(\underbrace{np}_{0} - n_i^2)}{\tau_p\left[\underbrace{n}_{0} + n_i \exp\left(\frac{E_t - E_i}{kT}\right)\right] + \tau_n\left[\underbrace{p}_{0} + n_i \exp\left(\frac{E_i - E_t}{kT}\right)\right]}}$$

8.2 Generation-recombination currents

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Current density from G-R in the depletion region:

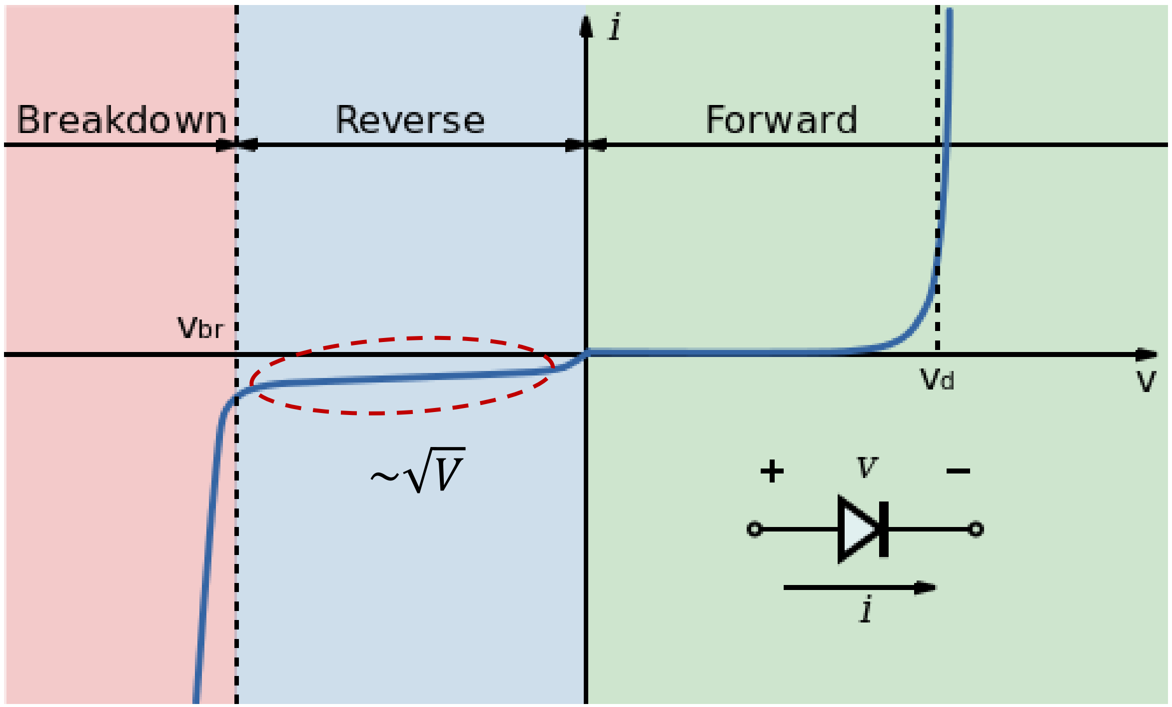
$$J_r = \int_0^W qG_0 dx = \frac{qWn_i}{2\tau}$$

$$\boxed{W = a + b = \sqrt{\frac{2\varepsilon(V_{bi}-V_a)}{q}\frac{N_d+N_a}{N_aN_d}}}$$

$$\boxed{R_n = \frac{(\cancel{np}^{\,0} - n_i^2)}{\tau_p\left[\cancel{n}^{\,0} + n_i \exp\left(\frac{E_t - E_i}{kT}\right)\right] + \tau_n\left[\cancel{p}^{\,0} + n_i \exp\left(\frac{E_i - E_t}{kT}\right)\right]}}$$

8.2 8.2 Generation-recombination currents

Reverse bias $V_a < 0$

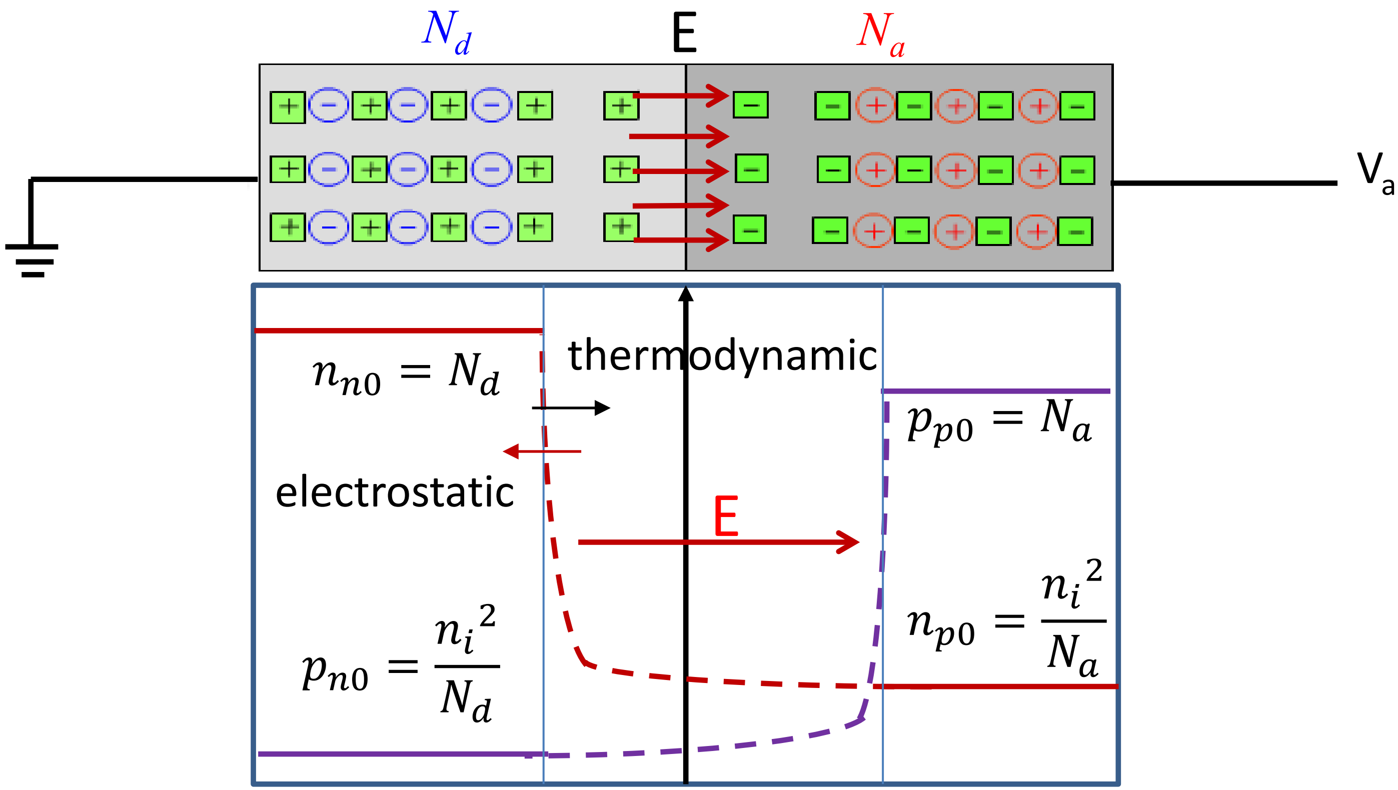


Current density from G-R in the depletion region:

$$J_r = \int_0^W qG dx = \frac{qWn_i}{2\tau}$$

$$\boxed{W = a + b = \sqrt{\frac{2\varepsilon(V_{bi} - V_a)}{q} \frac{N_d + N_a}{N_a N_d}}}$$

8.2 Generation-recombination currents



$$In\ depletion\ region:\ np = n_i^2 \exp(\frac{qV_a}{kT})$$

8.2 Generation-recombination currents

To simplify the calculation, we assume

$$E_t = E_i, \tau_n = \tau_p = \tau$$

$$R_n = \frac{np - n_i^2}{\tau(n + p + 2n_i)}$$

When n=p, U reaches its max value.

$$R_{n,max} = \frac{np - n_i^2}{\tau\left[n_i \exp(\frac{qV_a}{2kT}) + n_i \exp(\frac{qV_a}{2kT}) + 2n_i\right]} = \frac{n_i\left[\exp\left(\frac{qV_a}{kT}\right) - 1\right]}{2\tau\left[\exp\left(\frac{qV_a}{2kT}\right) + 1\right]}$$

$$= \frac{n_i\left[\exp\left(\frac{qV_a}{2kT}\right) - 1\right]}{2\tau}$$

$$In\ depletion\ region:\ np = n_i^2 \exp(\frac{qV_a}{kT})$$

8.2 Generation-recombination currents

$$\boxed{R_{n,max} = \frac{n_i}{2\tau}\left[\exp\left(\frac{qV_a}{2kT}\right) - 1\right]}$$

Current density from G-R in the depletion region:

$$J_r = \int_0^W qR_{n,max}dx = \frac{qWn_i}{2\tau}\left[\exp\left(\frac{qV_a}{2kT}\right) - 1\right]$$

For a non-ideal pn junction, the total current density:

$$J = J_F + J_r = J_s\left[\exp\left(\frac{qV_a}{kT}\right) - 1\right] + \frac{qWn_i}{2\tau}\left[\exp\left(\frac{qV_a}{2kT}\right) - 1\right]$$

8.2 Generation-recombination currents

$$J = J_F + J_r = J_s \left[\exp\left(\frac{qV_a}{kT}\right) - 1\right] + \frac{qWn_i}{2\tau}\left[\exp\left(\frac{qV_a}{2kT}\right) - 1\right]$$

Forward bias V> 3kT/q=0.078V:

$$J = J_F + J_r = J_s \exp\left(\frac{qV_a}{kT}\right) + \frac{qWn_i}{2\tau}\exp\left(\frac{qV_a}{2kT}\right) = J_0 \exp\left(\frac{qV_a}{nkT}\right)$$

the ideality factor

8.2 Generation-recombination currents

$$J = J_F + J_r = J_s\left[\exp\left(\frac{qV_a}{kT}\right) - 1\right] + \frac{qWn_i}{2\tau}\left[\exp\left(\frac{qV_a}{2kT}\right) - 1\right]$$

Forward bias V> 3kT/q=0.078V:

$$J = J_F + J_r = J_s \exp\left(\frac{qV_a}{kT}\right) + \frac{qWn_i}{2\tau}\exp\left(\frac{qV_a}{2kT}\right) = J_0 \exp\left(\frac{qV_a}{nkT}\right)$$

the ideality factor

Reverse bias:

$$J_0 = -J_s - \frac{qWn_i}{2\tau} = -\left(\frac{qD_n n_{p0}}{L_n} + \frac{qD_p p_{n0}}{L_p}\right) - \frac{qWn_i}{2\tau}$$

Outline

8.1 pn junction current

8.2 Generation-recombination currents

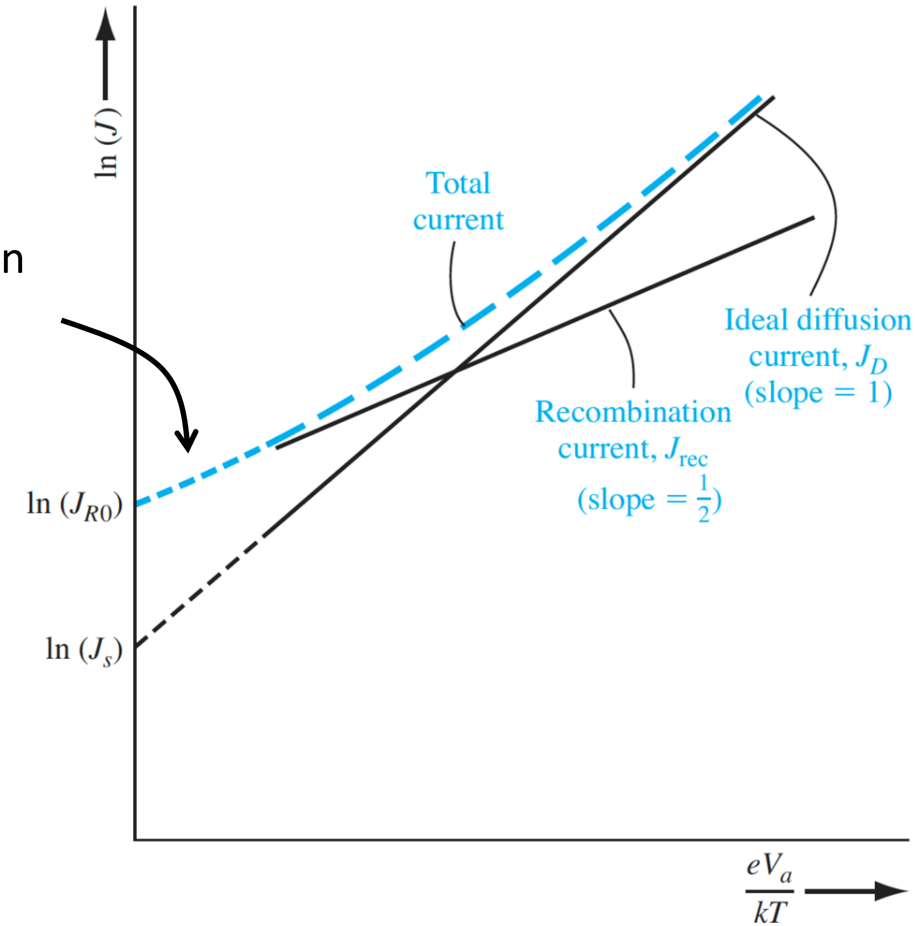
8.3 High-injection levels

8.4 A few more points on pn junctions (not in the textbook)

8.3 High inject level

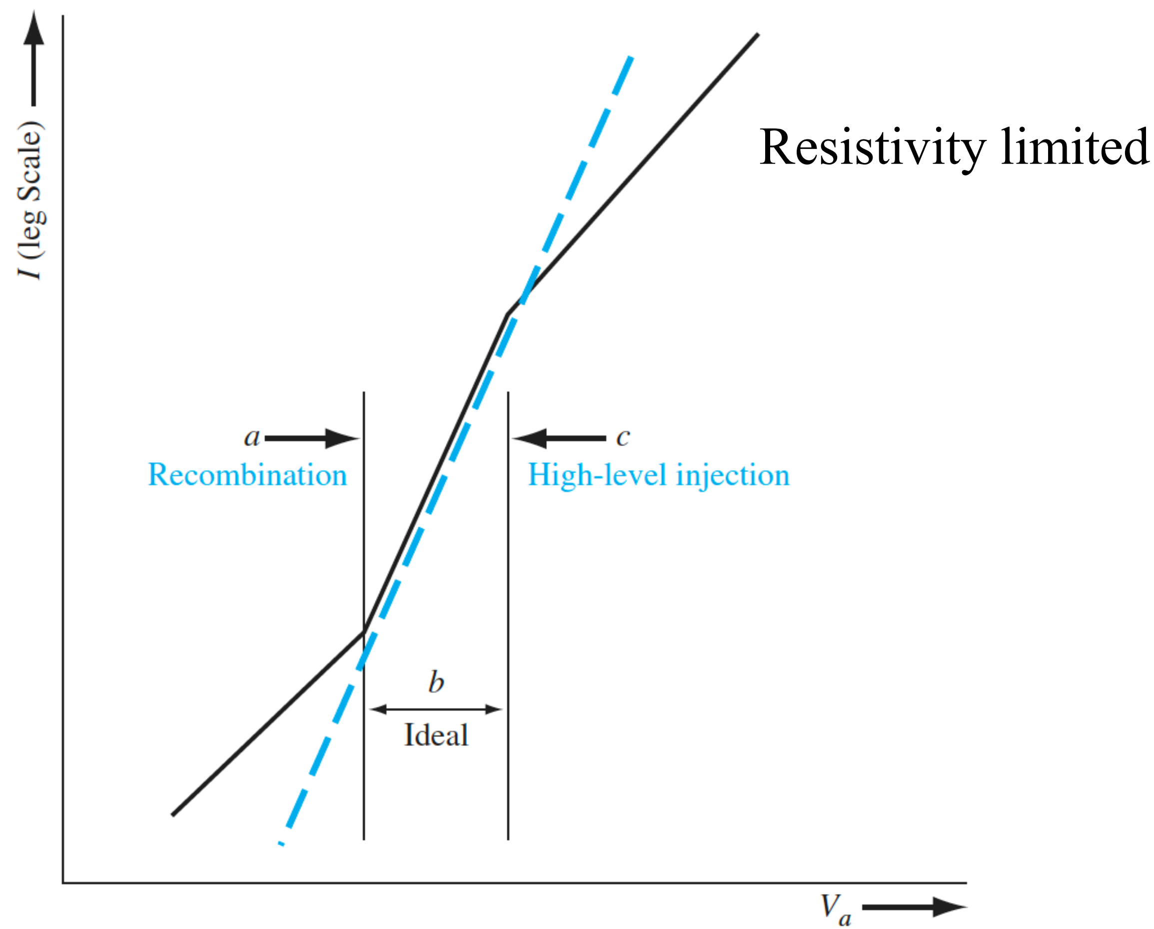
$$J = J_F + J_r = J_s \exp\left(\frac{qV_a}{kT}\right) + \frac{qWn_i}{2\tau}\exp\left(\frac{qV_a}{2kT}\right) = J_0 \exp\left(\frac{qV_a}{nkT}\right)$$

e-h pairs recombine in the depletion region



8.3 High inject level

$$J = J_F + J_r = J_s \exp\left(\frac{qV_a}{kT}\right) + \frac{qWn_i}{2\tau}\exp\left(\frac{qV_a}{2kT}\right) = J_0 \exp(\frac{qV_a}{nkT})$$



Outline

8.1 pn junction current

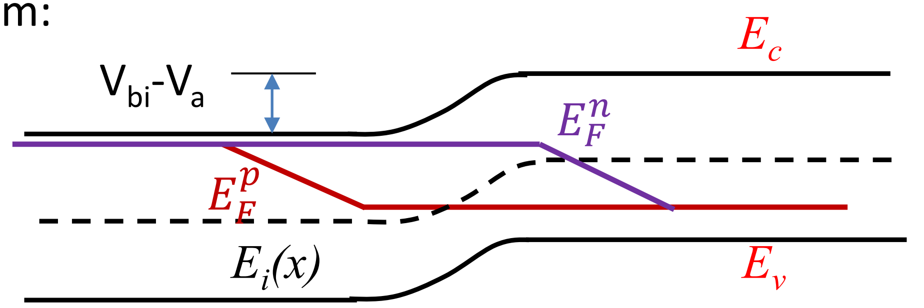
8.2 Generation-recombination currents

8.3 High-injection levels

8.4 A few more points on pn junctions (not in the textbook)

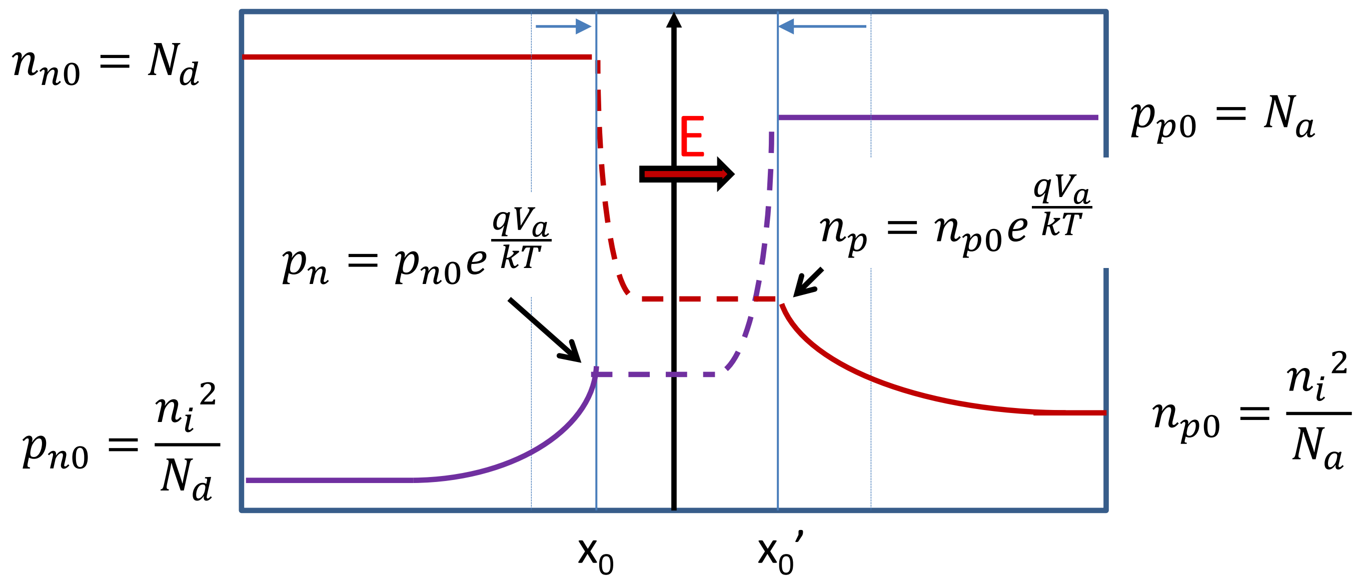
8.4 A few points about pn junction

- Energy diagram:



Forward bias

To understand this, see the next page.



8.4 A few points about pn junction

- Current and Fermi level

$$\boxed{J_n = J_{diff,n} + J_{drift,n}}$$

$$J_n = qD_n\frac{dn}{dx} + nq\mu_n|E|$$

$$\boxed{\therefore D_n = kT\mu_n/q}$$

$$= kT\mu_n\frac{dn}{dx} + nq\mu_n|E|$$

$$= nq\mu_n(\frac{kT}{nq}\frac{dn}{dx} + |E|)$$

$$= nq\mu_n\left[\frac{kT}{q}\frac{dln(n)}{dx} + |E|\right]$$

$$\boxed{\therefore n = n_i\exp\left(\frac{E_F - E_i}{kT}\right) \Rightarrow \ln(n) = \ln(n_i) + \frac{E_F - E_i}{kT}}$$

$$= nq\mu_n\left[\frac{dE_F}{qdx} - \frac{dE_i}{qdx} + |E|\right]$$

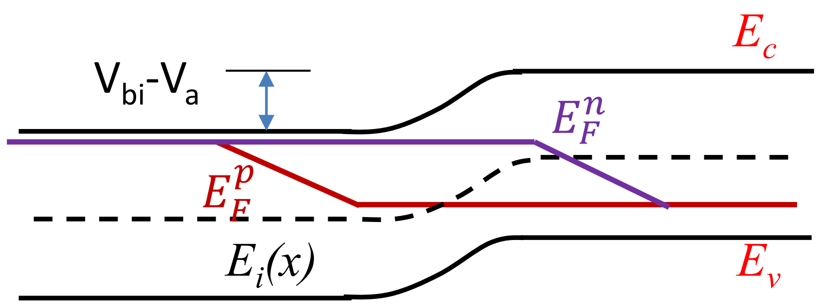
$$\boxed{\Rightarrow \frac{d}{dx}\ln(n) = \frac{d}{dx}\left(\frac{E_F - E_i}{kT}\right) = \frac{1}{kT}(\frac{dE_F}{dx} - \frac{dE_i}{dx})}$$

$$\boxed{J_n = n\mu_n\frac{dE_F}{dx}}$$

$$\boxed{\therefore \frac{dE_i}{dx} = -q\frac{dV(x)}{dx} = |E|}$$

8.4 A few points about pn junction

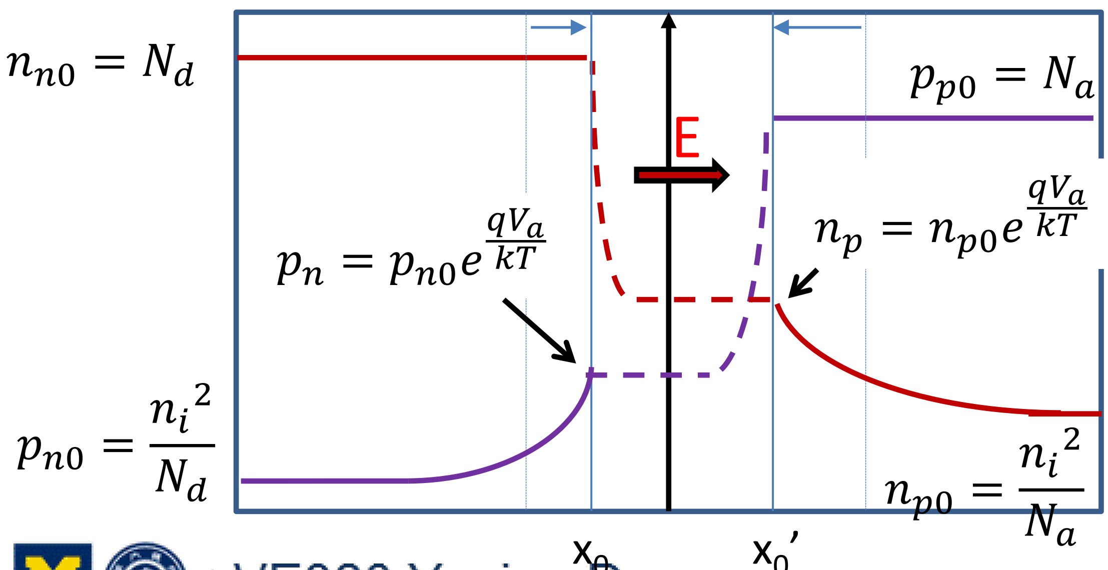
- Energy diagram:



Forward bias

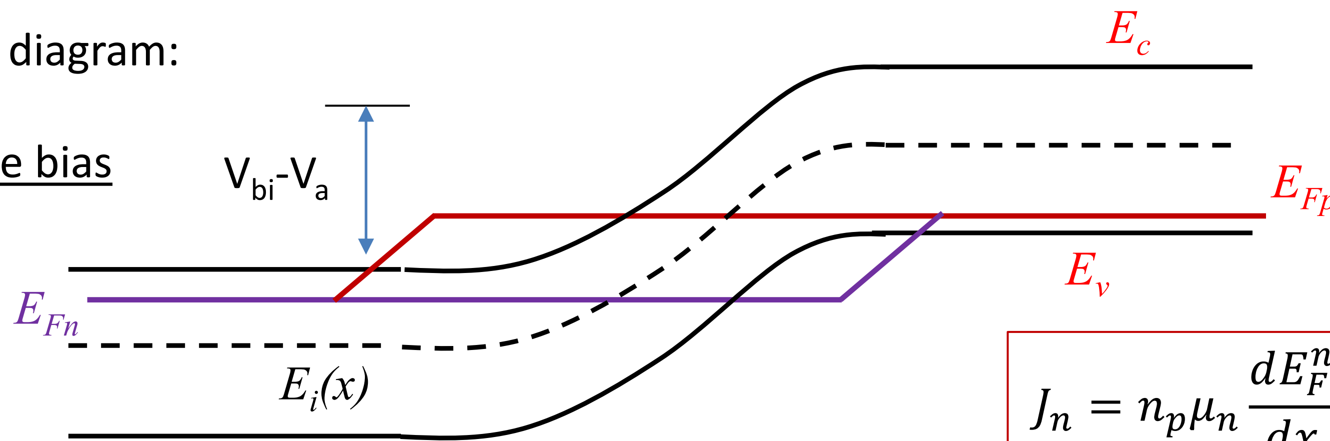
$$n_p = n_{p0} + \Delta n \approx \Delta n$$

$$\boxed{J_n = n_p \mu_n \frac{dE_F^n}{dx}}$$



8.4 A few points about pn junction

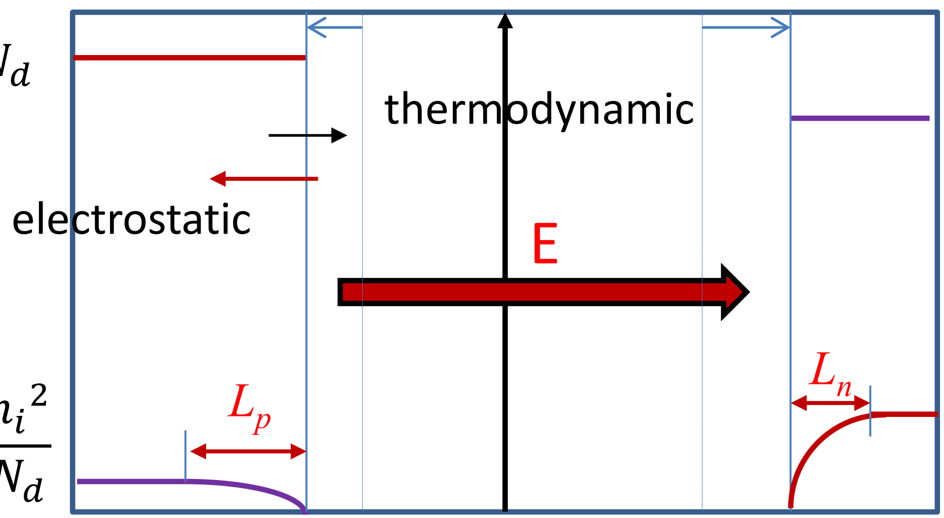
- Energy diagram:



$$J_n = n_p \mu_n \frac{dE_F^n}{dx}$$

$n_{n0} = N_d$

$p_{p0} = N_a$

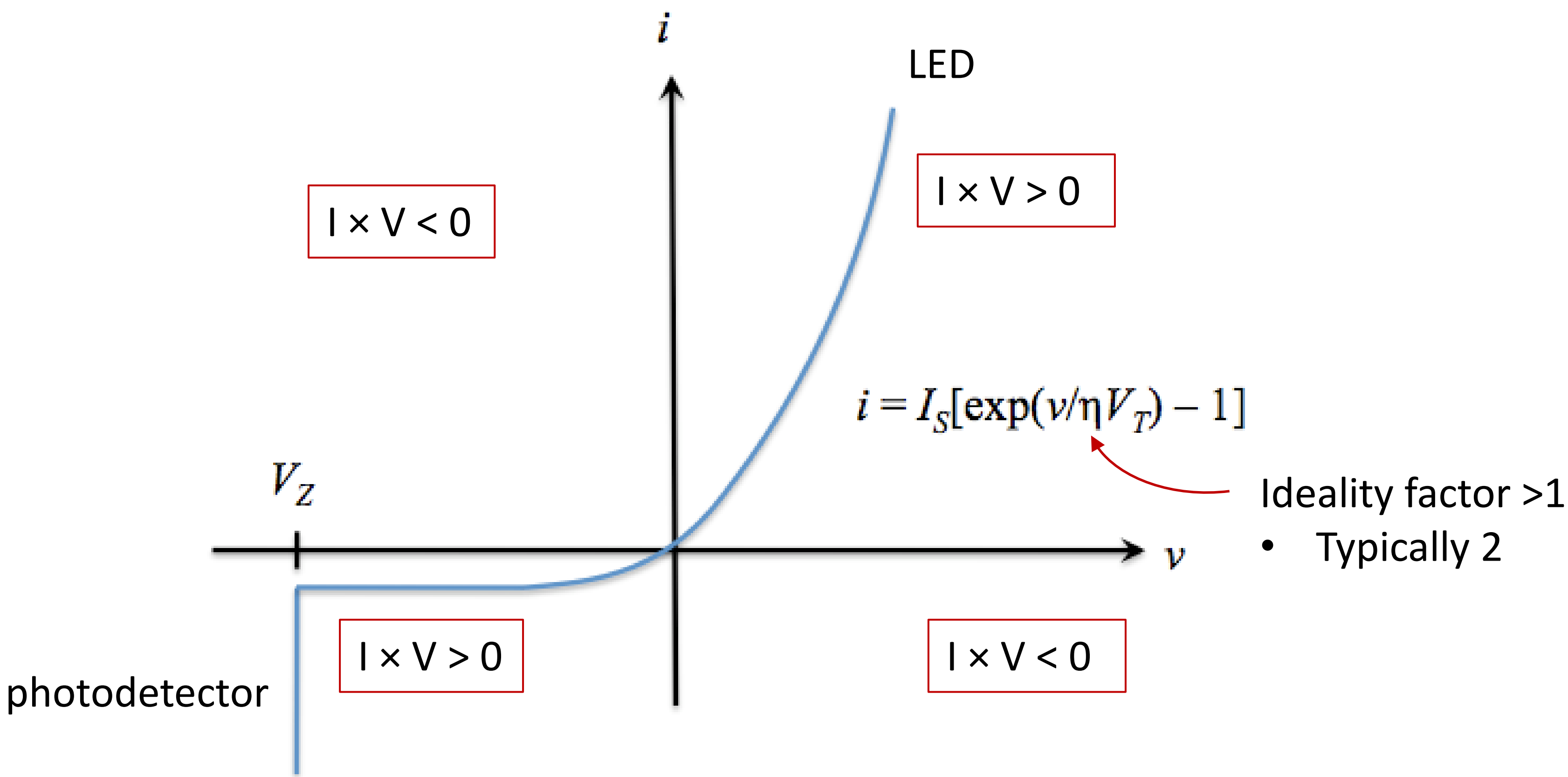


$p_{n0} = \frac{{n_i}^2}{N_d}$

$n_{p0} = \frac{{n_i}^2}{N_a}$

8.4 A few points about pn junction

- Energy consumption:



8.4 A few points about pn junction

- Energy consumption: