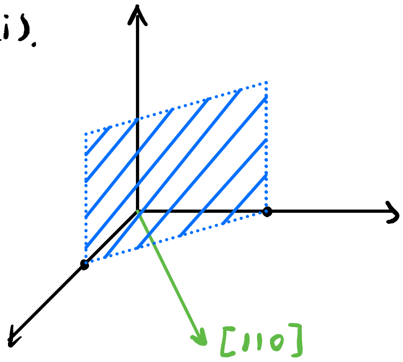
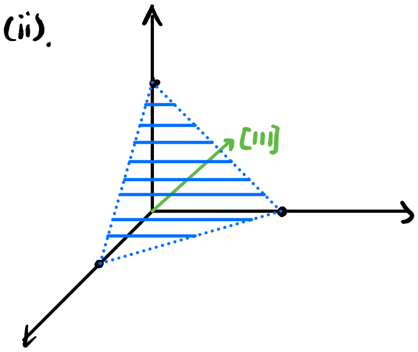
VE320 HW1 周维凯 518021911039

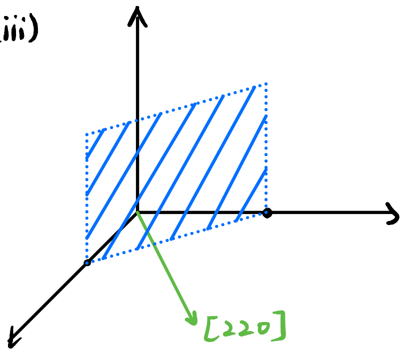
1. (i).



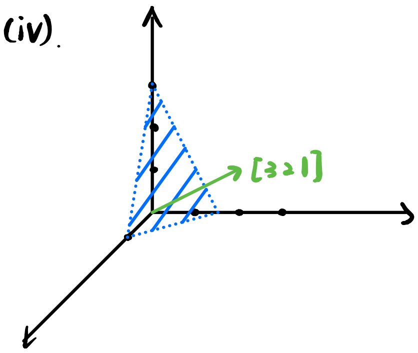
(ii).



(iii)



(iv).



a) The planes are in blue.

b). The directions are in green.

2. (a). simple cubic

(i). $\frac{1}{(4.73\times10^{-8})^2} = 4.47\times10^{14}\ cm^{-2}$

(ii). $\frac{1}{\sqrt{2}\,(4.73\times10^{-8})^2} = 3.16\times10^{14}\ cm^{-2}$

(iii) $\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}(4.73\times10^{-8})^2} = 2.58\times10^{14}\ cm^{-2}$

(b). body centered

(i). $\frac{1}{(4.73\times10^{-8})^2} = 4.47\times10^{14}\ cm^{-2}$

(ii). $\frac{2}{\sqrt{2}\,(4.73\times10^{-8})^2} = 6.32\times10^{14}\ cm^{-2}$

(iii). $\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}(4.73\times10^{-8})^2} = 2.58\times10^{14}\ cm^{-2}$

(c). face centered

(i). $\frac{2}{(4.73\times10^{-8})^2} = 8.94\times10^{14}\ cm^{-2}$

(ii). $\frac{2}{\sqrt{2}\,(4.73\times10^{-8})^2} = 6.32\times10^{14}\ cm^{-2}$

(iii) $\frac{2}{\frac{\sqrt{3}}{2}(4.73\times10^{-8})^2} = 1.03\times10^{15}\ cm^{-2}$

3. For gold:

$$E = h\nu = h \cdot \frac{c}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = \frac{6.625 \times 10^{-34} \times 3 \times 10^{8}}{4.9 \times 1.6 \times 10^{-19}} = 2.54 \times 10^{-7}\ m$$

For cesium:

$$\lambda = \frac{hc}{E} = 6.54 \times 10^{-7}\ m$$

4. $E = \frac{3kT}{2} = 0.03885\ eV$

$$\left.\begin{array}{l} E = \frac{1}{2} m v^2 \\ p = mv \end{array}\right\} \Rightarrow p = m\sqrt{\frac{2E}{m}} = \sqrt{2 \times 9.11 \times 10^{-31} \times 0.03885 \times 1.6 \times 10^{-19}} = 1.06 \times 10^{-25}\ kg \cdot m/s$$

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}}{1.06 \times 10^{-25}} = 6.23 \times 10^{-9}\ m$$

5. (a).

$$\left.\begin{array}{l} -\frac{\hbar^2}{2m} \frac{\partial^2 \psi_1}{\partial x^2} + V(x)\psi_1 = E\psi_1 \quad ① \\ -\frac{\hbar^2}{2m} \frac{\partial^2 \psi_2}{\partial x^2} + V(x)\psi_2 = E\psi_2 \quad ② \end{array}\right\} \xRightarrow{①+②} -\frac{\hbar^2}{2m} \frac{\partial^2 (\psi_1 + \psi_2)}{\partial x^2} + V(x)(\psi_1 + \psi_2) = E(\psi_1 + \psi_2)$$

Therefore, $\psi_1 + \psi_2$ is a solution

(b). No, it is not a solution.

If it is, then we have

$$-\frac{\hbar^2}{2m} \frac{\partial^2 (\psi_1 \cdot \psi_2)}{\partial x^2} + V(x)(\psi_1 \cdot \psi_2) = E\psi_1\psi_2$$

$$-\frac{\hbar^2}{2m}\left(\psi_1 \frac{\partial^2 \psi_2}{\partial x^2} + \psi_2 \frac{\partial^2 \psi_1}{\partial x^2} + 2\frac{\partial \psi_1}{\partial x} \cdot \frac{\partial \psi_2}{\partial x}\right) + V(x)(\psi_1 \psi_2) = E\psi_1 \cdot \psi_2$$

$$-\frac{\hbar^2}{2m}\left(\frac{1}{\psi_2} \frac{\partial^2 \psi_2}{\partial x^2} + \frac{1}{\psi_1} \frac{\partial^2 \psi_1}{\partial x^2} + \frac{2}{\psi_1\psi_2} \frac{\partial \psi_1}{\partial x} \frac{\partial \psi_2}{\partial x}\right) + V(x) = E \quad ③$$

From ①, we know

$$-\frac{\hbar^2}{2m} \frac{1}{\psi_1} \frac{\partial^2 \psi_1}{\partial x^2} + V(x) = E \quad ④$$

③ - ④:

$$-\frac{\hbar^2}{2m}\left(\frac{1}{\psi_2} \frac{\partial^2 \psi_2}{\partial x^2} + \frac{2}{\psi_1\psi_2} \frac{\partial \psi_1}{\partial x} \frac{\partial \psi_2}{\partial x}\right) = 0.$$

However, it may not true for all ψ_1 and ψ_2.

Therefore, $\psi_1 \cdot \psi_2$ is not a solution.

6. a). for $x<0$, $V(x)=V_0$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+V_0\psi=E\psi$$

$$\psi_1=Ae^{-|k_1|x}+Be^{|k_1|x}, \quad k_1=i\sqrt{\frac{2m(V_0-E)}{\hbar^2}}$$

Since ψ_1 should be finite, $\psi_1=Be^{kx}$, $k=\sqrt{\frac{2m(V_0-E)}{\hbar^2}}$

for $0<x<a$, $V(x)=0$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}=E\psi$$

$$\psi_2=C\sin k_2x+D\cos k_2x$$

$$k_2=\sqrt{\frac{2mE}{\hbar^2}}$$

for $x>a$, $V(x)=\infty$

$$\psi_3=0$$

(b). At $x=0$

$$\psi_1=\psi_2 \Rightarrow B=D$$

$$\frac{\partial\psi_1}{\partial x}=\frac{\partial\psi_2}{\partial x} \Rightarrow Bk=Ck_2$$

At $x=a$

$$\psi_2=\psi_3 \Rightarrow C\sin k_2a+D\cos k_2a=0$$

(c).

$$\begin{cases} B=D \\ Bk=Ck_2 \Rightarrow C=\frac{k}{k_2}B \\ C\sin k_2a+D\cos k_2a=0 \end{cases}$$

Therefore, $\frac{k}{k_2}B\sin k_2a+B\cos k_2a=0$

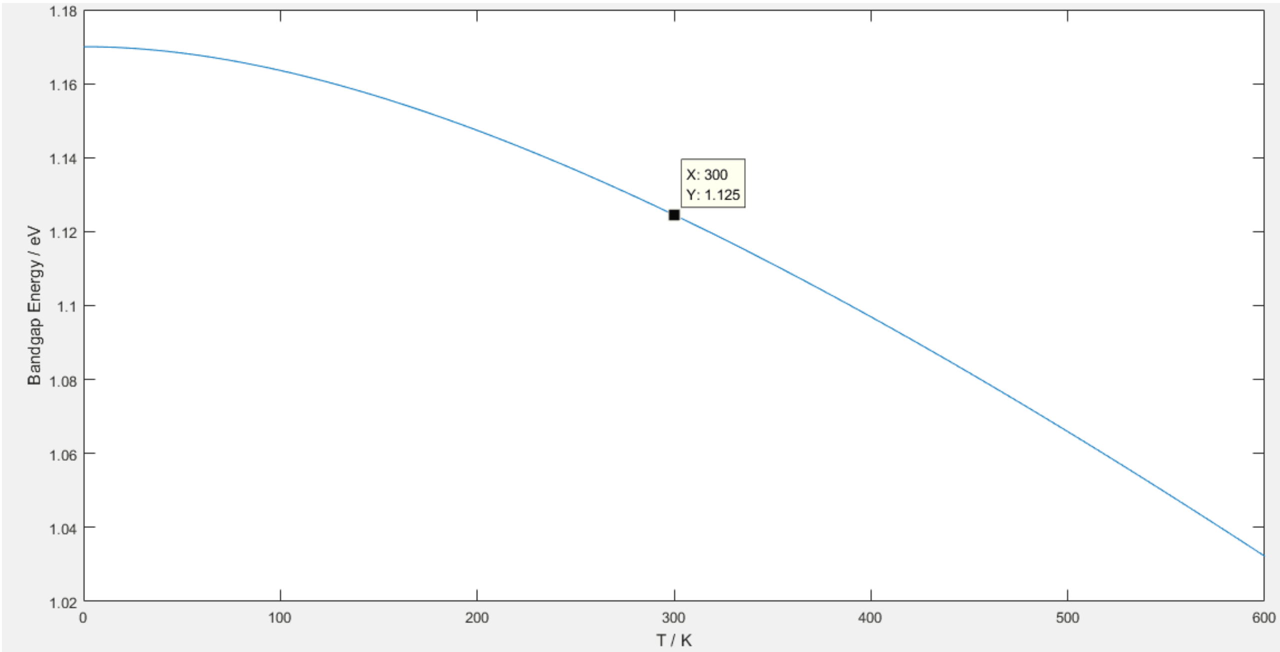
$$\frac{\sqrt{\frac{2m(V_0-E)}{\hbar^2}}}{\sqrt{\frac{2mE}{\hbar^2}}}\sin k_2a+\cos k_2a=0$$

$$\tan k_2a=-\sqrt{\frac{E}{V_0-E}}$$

$$\tan\sqrt{\frac{2mE}{\hbar^2}}\,a=-\sqrt{\frac{E}{V_0-E}}$$

It is only for specific value of E. Therefore, it is quantized.

7.



8. $E_c - E = Ck^2 = \frac{\hbar^2}{2m} \cdot k^2$

For A: $0.05\text{eV} = \frac{k^2 \hbar^2}{2m}$

$$\Rightarrow m = \frac{k^2 \hbar^2}{0.05\text{eV}} = \frac{[0.08 \times (0.1 \times 10^{-9})^{-1}]^2 \cdot (1.054 \times 10^{-34})^2}{0.05 \times 1.6 \times 10^{-19}} = 8.89 \times 10^{-31} \text{kg} \approx 0.98\, m_e$$

For B: $0.3\text{ eV} = \frac{k^2 \hbar^2}{2m}$

$$\Rightarrow m = \frac{k^2 \hbar^2}{0.6\text{eV}} = \frac{[0.08 \times (0.1 \times 10^{-9})^{-1}]^2 \cdot (1.054 \times 10^{-34})^2}{0.6 \times 1.6 \times 10^{-19}} = 7.41 \times 10^{-32} \text{kg} \approx 0.08\, m_e$$