VE320 – Summer 2021

Introduction to Semiconductor Devices

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Chapter 6 Non-Equilibrium Excess Carriers in Semiconductors

Outline

6.1 Carrier generation and recombination

6.2 Characteristics of excess carriers

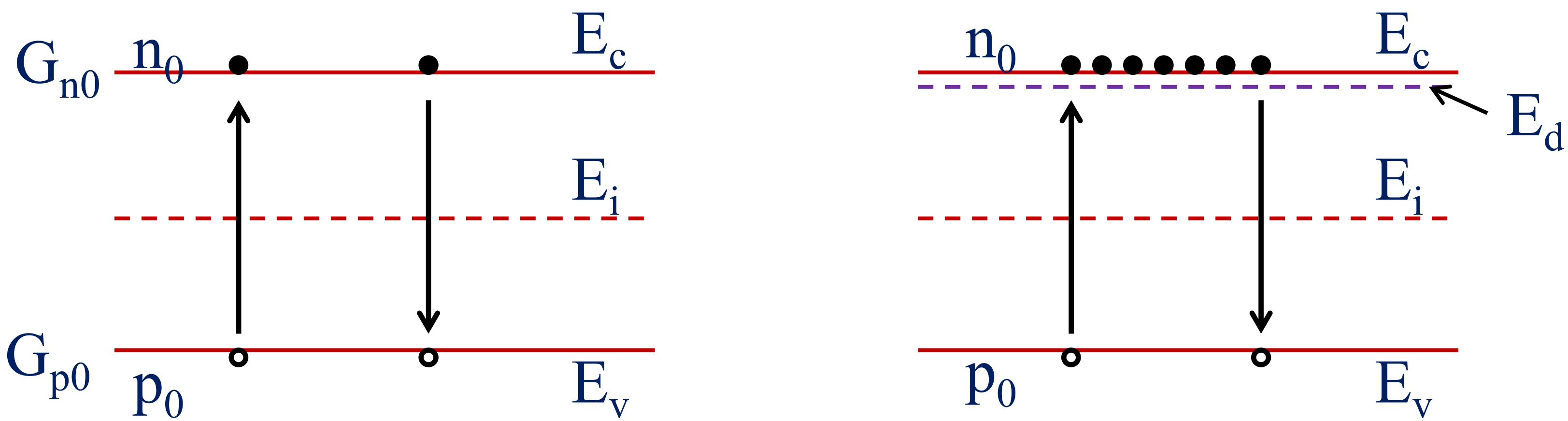
6.3 Quasi-Fermi levels

6.4 Excess carrier lifetime

6.5 Surface effects

6.1 Carrier generation and recombination

The semiconductor in equilibrium



Intrinsic: $n_0=p_0=n_i$

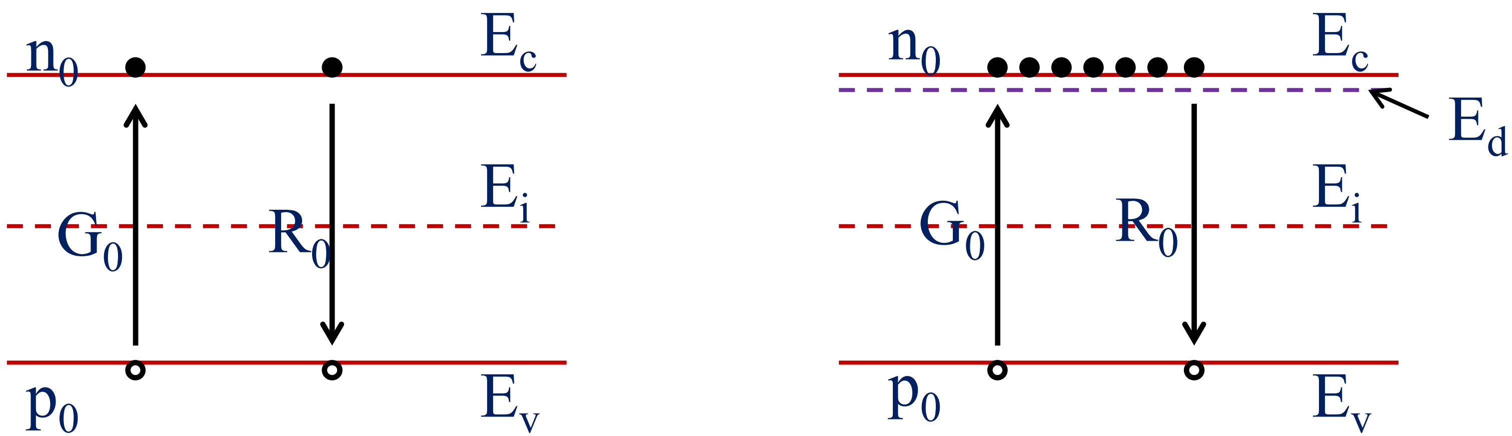
n type : $n_0>>n_i>>p_0$

G_{n0}: the thermal generation rate of electrons
G_{p0}: the thermal generation rate of holes
R_{n0}: the recombination rate of electrons
R_{p0}: the recombination rate of holes

$\boxed{G_{n0} = G_{p0} = R_{n0} = R_{p0}}$ (direct G and R from band to band)

6.1 Carrier generation and recombination

The semiconductor in equilibrium



Intrinsic: $n_0=p_0=n_i$

n type : $n_0>>n_i>>p_0$

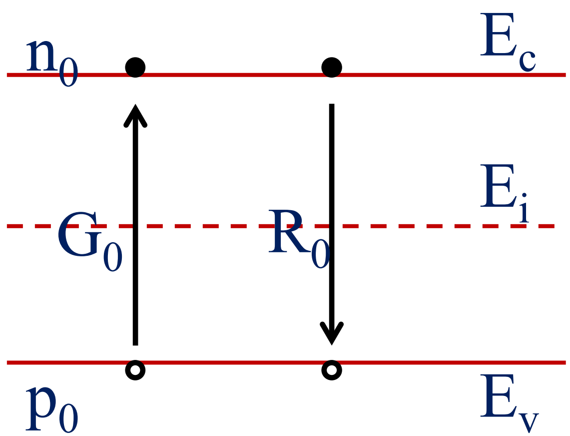
First, look at R_0

$\boxed{R_0 \sim n_0, \qquad R_0 \sim p_0}$ $\boxed{\Rightarrow R_0 \sim n_0 p_0}$ (limited by minority)

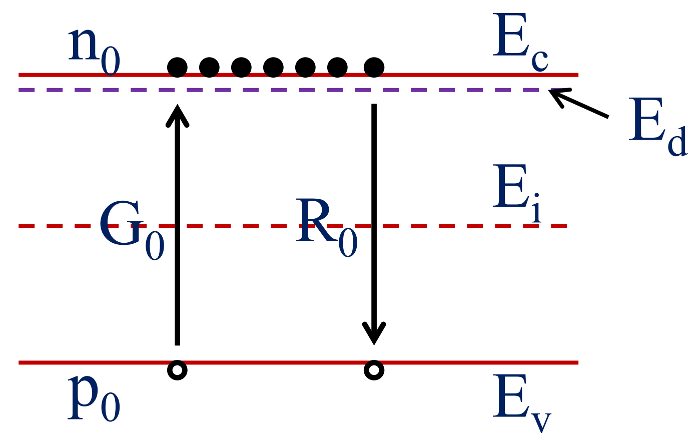
$$\boxed{\Rightarrow R_0 = \alpha_r n_0 p_0 \;\; where\ \alpha_r\ is \text{ recombination probability}}$$

6.1 Carrier generation and recombination

The semiconductor in equilibrium



Intrinsic: $n_0 = p_0 = n_i$



n type : $n_0 >> n_i >> p_0$

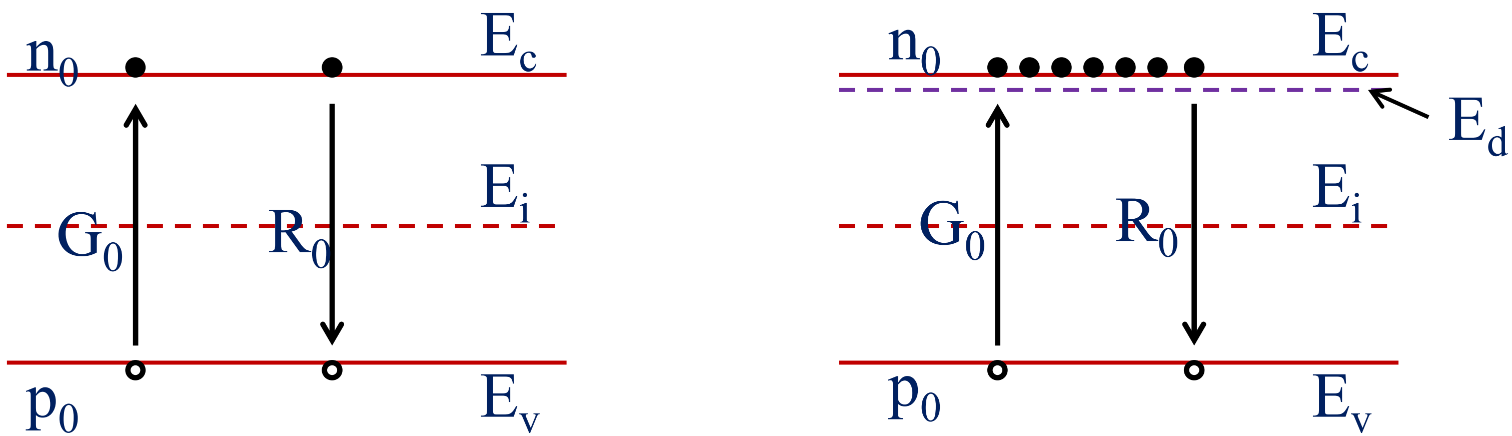
Then, look at G_0

Can we write similar equations? $G_0 \sim n$? $R_0 \sim p$?

No! G_0 is intrinsic and only a function of T

6.1 Carrier generation and recombination

The semiconductor in equilibrium



Intrinsic: $n_0=p_0=n_i$

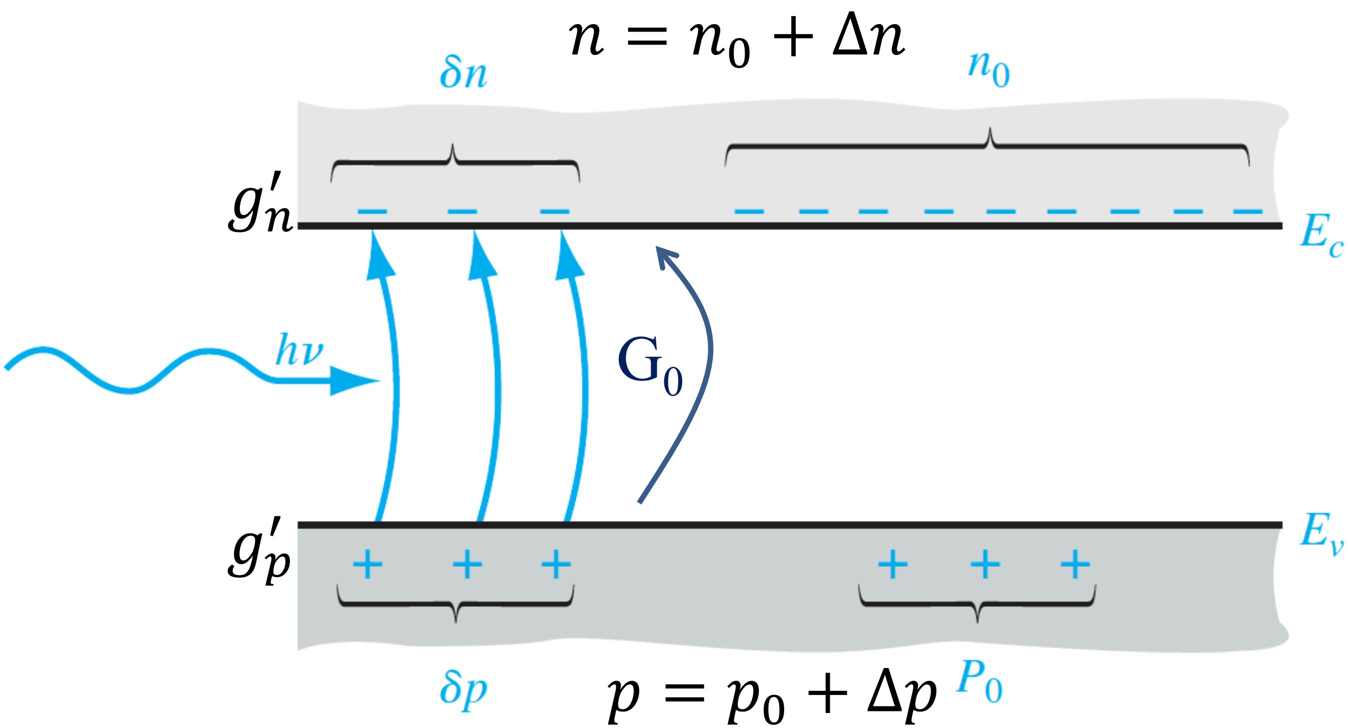
n type : $n_0>>n_i>>p_0$

At equilibrium, we must have

$$\boxed{G_0 = R_0 = \alpha_r \cdot n_0 \cdot p_0 = \alpha_r \cdot n_i^2}$$

6.1 Carrier generation and recombination

Excess carrier generation and recombination



g’ is not a function of n and p

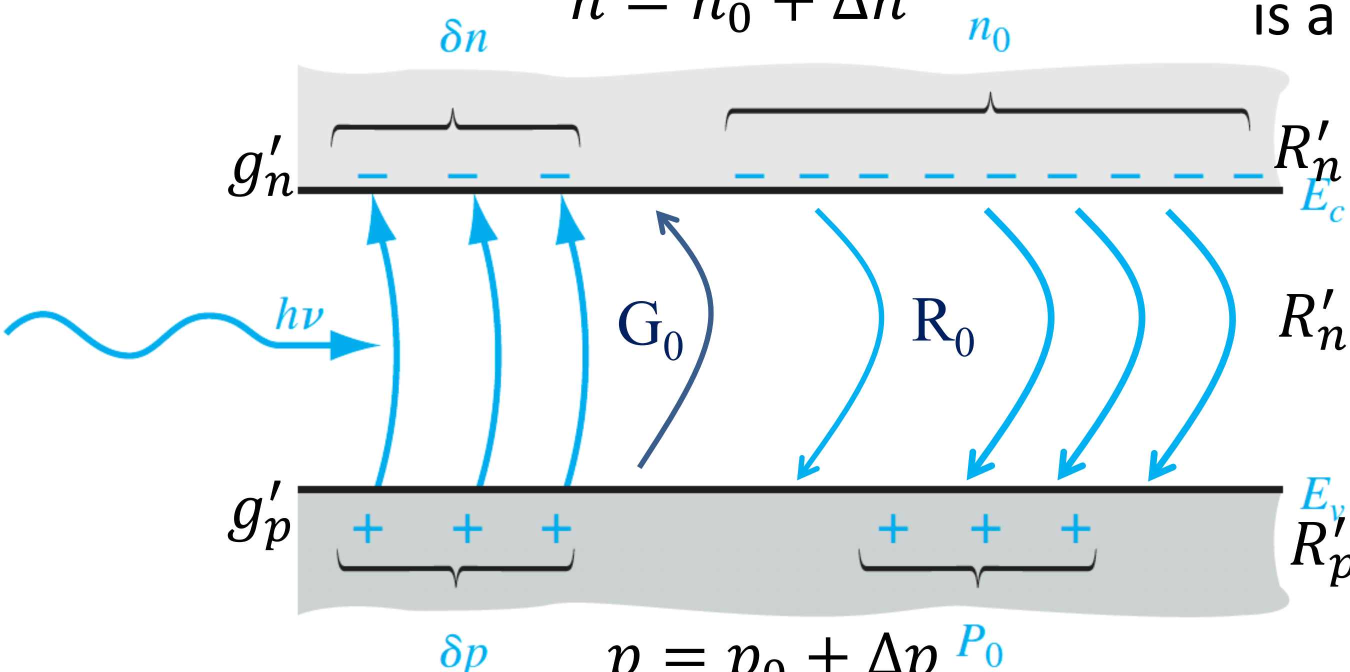
$$g_n' = g_p' = g', \qquad \Delta n = \Delta p$$

6.1 Carrier generation and recombination

Excess carrier generation and recombination

Recombination process is a function of n and p

$$n = n_0 + \Delta n$$



$$R'_n = R'_p = R'$$

$$p = p_0 + \Delta p$$

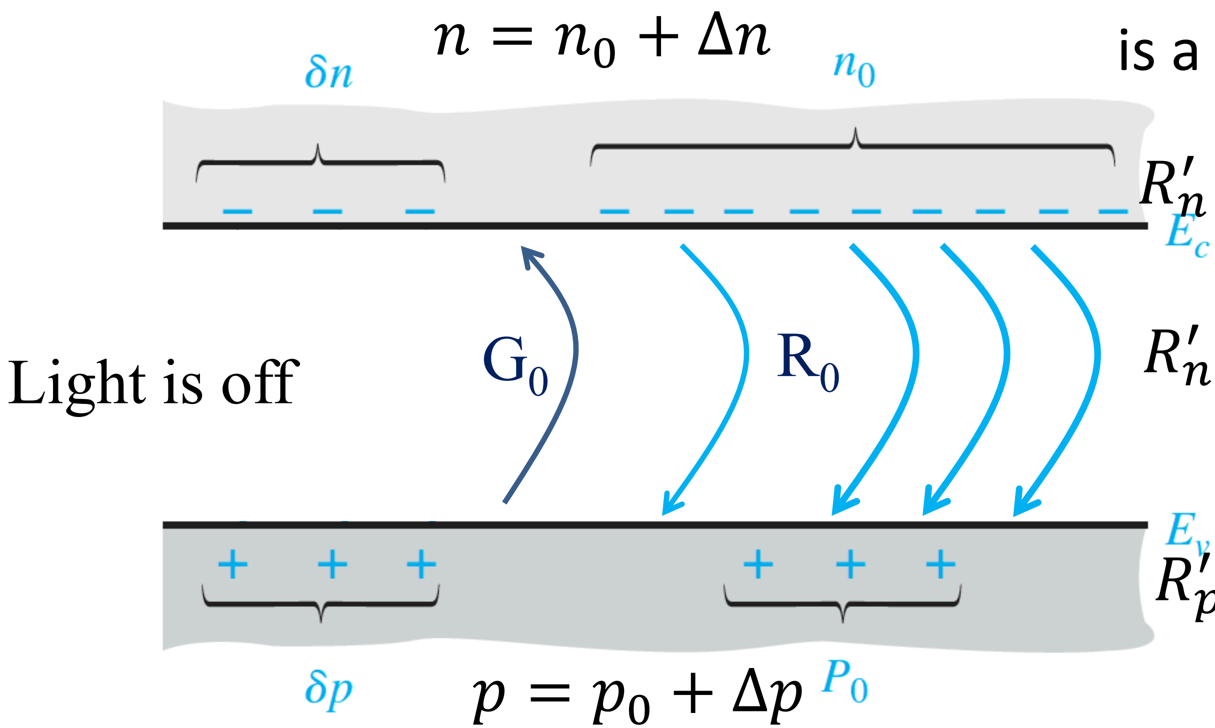
g’ is not a function of n and p

$$g'_n = g'_p = g', \qquad \Delta n = \Delta p \qquad R' + R_0 = \alpha_r (n_0 + \Delta n)(p_0 + \Delta p)$$

6.1 Carrier generation and recombination

Excess carrier generation and recombination

Recombination process is a function of n and p



$R_n' = R_p' = R'$

$$\frac{d\Delta p}{dt} = -(R' + R_0 - G_0) = -[\alpha_r(n_0 + \Delta n)(p_0 + \Delta p) - \alpha_r n_0 p_0]$$

6.1 Carrier generation and recombination

Excess carrier generation and recombination

Net recombination rate

$$\frac{d\Delta p}{dt} = -(R' + R_0 - G_0) = -[\alpha_r(n_0 + \Delta n)(p_0 + \Delta p) - \alpha_r n_i^2]$$

$$= -\alpha_r \cdot \Delta p \cdot (p_0 + n_0) - \alpha_r \cdot (\Delta p)^2$$

$$if\ p_0 + n_0 \gg \Delta p \qquad \approx -\alpha_r \cdot \Delta p \cdot (p_0 + n_0)$$

(Small injection condition)

$$\Delta p(t) = \Delta p(0)\exp(-\frac{t}{\tau_{p0}}) \qquad \tau_{p0} = \frac{1}{\alpha_r(p_0 + n_0)}$$

6.1 Carrier generation and recombination

Excess carrier generation and recombination

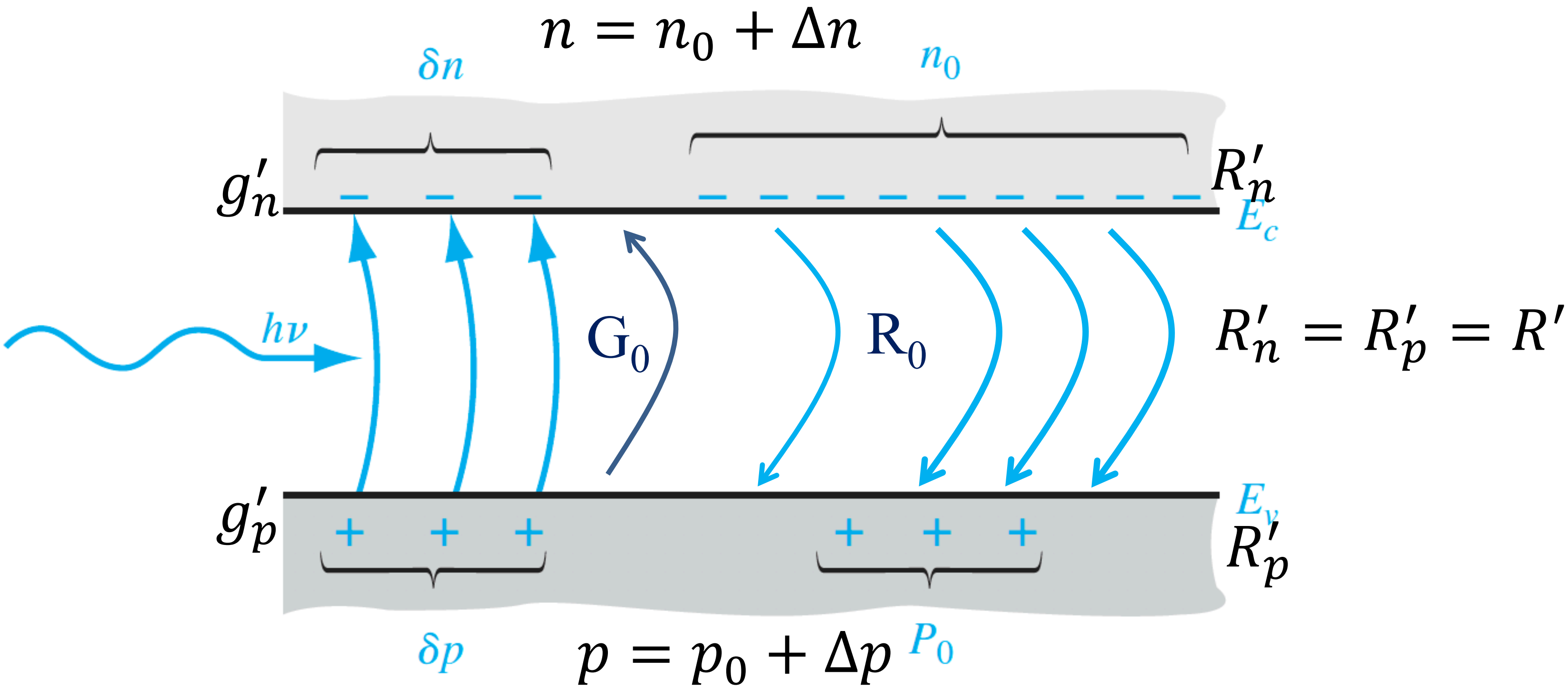
For n-type semiconductor, net recombination rate

$$R'_n = R'_p = \frac{\Delta p(t)}{\tau_{p0}}$$

For p-type semiconductor, net recombination rate

$$R'_n = R'_p = \frac{\Delta n(t)}{\tau_{n0}}$$

Excess carrier generation and recombination



$$\boxed{g' = R' \quad \Rightarrow \quad \Delta p(t \leq 0) = g'\tau_{p0} \; for \; n - type \; semiconductors}$$

$$\boxed{g' = R' \quad \Rightarrow \quad \Delta n(t \leq 0) = g'\tau_{n0} \; for \; p - type \; semiconductors}$$

6.1 Carrier generation and recombination

Problem Example

Assume that excess carriers have been generated uniformly in a semiconductor to a concentration of $\Delta n(0) = 10^{15}$ cm^{-3}. The generation of the excess carriers turns off at time t=0. Assuming the excess carrier lifetime is $\tau_{n0} = 10^{-6}$ s, calculate the recombination rate of excess carriers for t =4μs.

Outline

6.1 Carrier generation and recombination

6.2 Characteristics of excess carriers

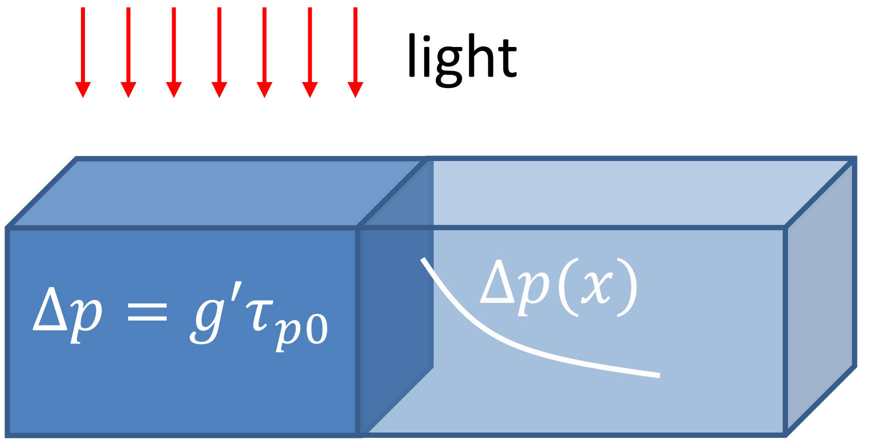
6.3 Quasi-Fermi levels

6.4 Excess carrier lifetime

6.5 Surface effects

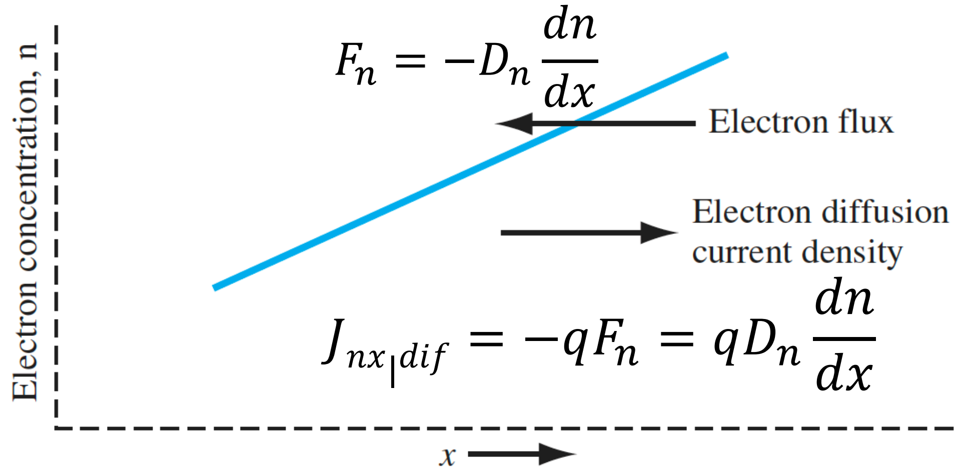
6.2 Characteristics of excess carriers

Continuity equation at steady state



6.2 Characteristics of excess carriers

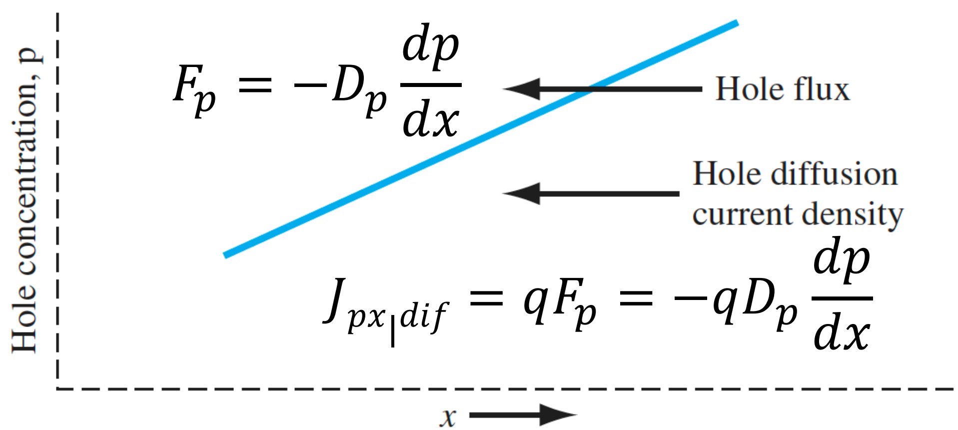
Previously…



$$F_n = -D_n \frac{dn}{dx}$$

$$J_{nx|dif} = -qF_n = qD_n \frac{dn}{dx}$$

(a)



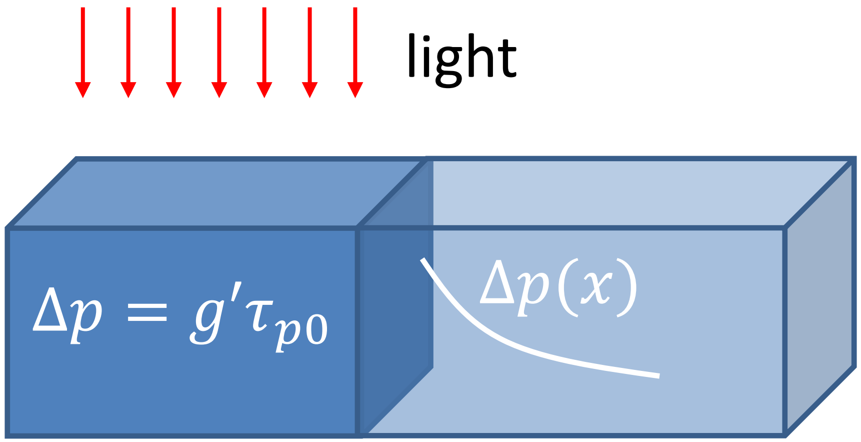
$$F_p = -D_p \frac{dp}{dx}$$

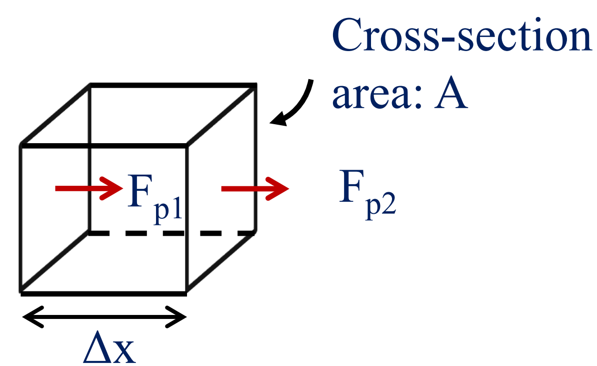
$$J_{px|dif} = qF_p = -qD_p \frac{dp}{dx}$$

(b)

6.2 Characteristics of excess carriers

Continuity equation at steady state





For diffusion current:

$\#\ of\ carriers\ passing\ (into)\ the\ area\ A\ at\ a\ unit\ time$: $\mathrm{F_{p1}} \cdot \mathrm{A}$

$\#\ of\ carriers\ passing\ (out)\ the\ area\ A\ at\ a\ unit\ time$: $\mathrm{F_{p2}} \cdot \mathrm{A}$

$\#\ of\ carriers\ recombined\ in\ that\ valume\ at\ a\ unit\ time$: $R_p' \cdot \mathrm{A} \cdot \Delta\mathrm{x}$

6.2 Characteristics of excess carriers

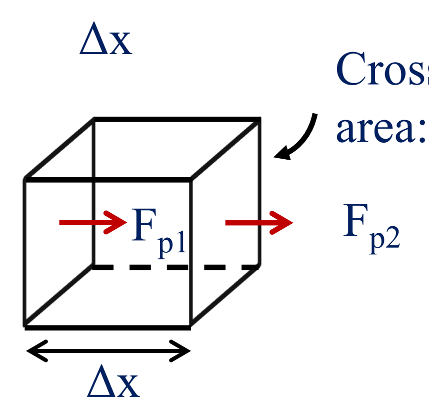
Continuity equation at steady state

For diffusion current:

$$F_{p2} \cdot A - F_{p1} \cdot A = -R_p' \cdot A \cdot \Delta x$$

$$\Rightarrow \frac{F_{p2} - F_{p1}}{\Delta x} = -R_p' = -\frac{\Delta p}{\tau_{p0}}$$ (small injection condition)

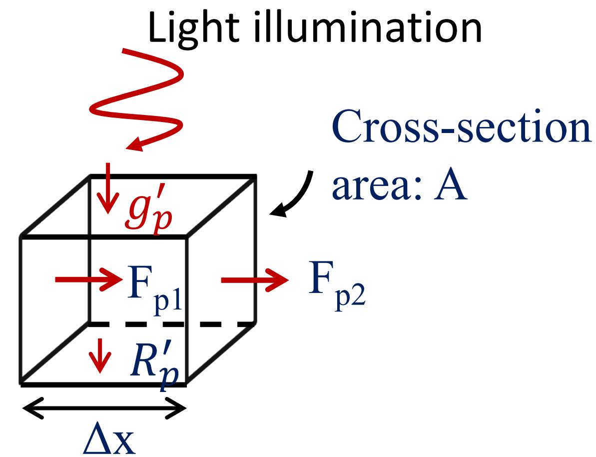
$$\Rightarrow \frac{dF_p}{dx} = -\frac{\Delta p}{\tau_{p0}} \Rightarrow D_p \frac{d^2 \Delta p}{dx^2} = \frac{\Delta p}{\tau_{p0}}$$



$$\Delta p(x) = Ae^{-x/L_p} + Be^{x/L_p} \qquad where\ L_p = \sqrt{D_p \tau}$$

6.2 Characteristics of excess carriers

Steady-state continuity equation



$$F_{p2} \cdot A - F_{p1} \cdot A = -R'_p \cdot A \cdot \Delta x + g'_p \cdot A \cdot \Delta x$$

$$\Rightarrow \frac{F_{p2} - F_{p1}}{\Delta x} = -R'_p + g'_p = -\frac{\Delta p}{\tau_{p0}} + g'_p$$

$$\Rightarrow \lim_{\Delta x \to 0} \left(\frac{F_{p2} - F_{p1}}{\Delta x}\right) = \frac{dF_p}{dx} = -\frac{\Delta p}{\tau_{p0}} + g'_p$$

$$\frac{dF_p}{dx} = \frac{1}{q}\frac{d}{dx}(J_p)_{dif} + \frac{1}{q}\frac{d}{dx}(J_p)_{drf} = -D_p\frac{d^2p}{dx^2} + \frac{d}{dx}(p\mu_p E) = -\frac{\Delta p}{\tau_{p0}} + g'_p$$

$$\boxed{D_p\frac{d^2p}{dx^2} - \mu_p E\frac{dp}{dx} - p\mu_p\frac{dE}{dx} - \frac{\Delta p}{\tau_{p0}} + g'_p = 0}$$

6.2 Characteristics of excess carriers

Steady-state continuity equation

Steady state:

$$\boxed{D_p \frac{d^2 p}{dx^2} - \mu_p E \frac{dp}{dx} - p\mu_p \frac{dE}{dx} - \frac{\Delta p}{\tau_{p0}} + g_p' = 0}$$

When the n-type semiconductor is uniformly doped,

$$p(x) = p_0 + \Delta p(x)$$

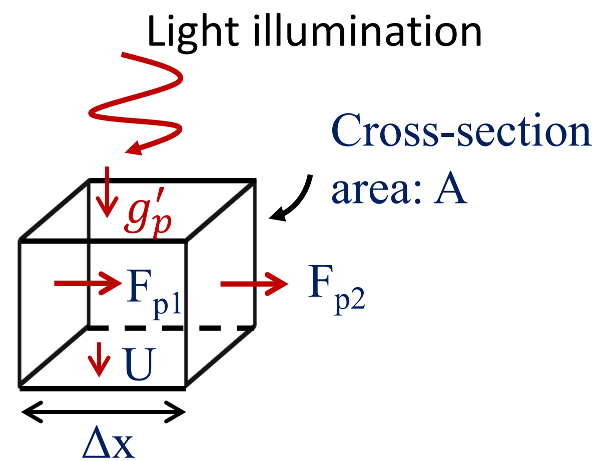
$$\boxed{D_p \frac{d^2 \Delta p}{dx^2} - \mu_p E \frac{d\Delta p}{dx} - \Delta p\mu_p \frac{dE}{dx} - \frac{\Delta p}{\tau_{p0}} + g_p' = 0}$$

6.2 Characteristics of excess carriers

Time-dependent continuity equation

$$\frac{d\Delta p}{dt} \cdot A \cdot \Delta x = F_{p1} \cdot A - F_{p2} \cdot A + g'_p \cdot A \cdot \Delta x - R'_p \cdot A \cdot \Delta x$$

$$\frac{d\Delta p}{dt} = \lim_{\Delta x \to 0} \left(\frac{F_{p2} - F_{p1}}{\Delta x}\right) + g'_p - R'_p$$



$$\boxed{\frac{d\Delta p}{dt} = D_p \frac{d^2 p}{dx^2} - \mu_p E \frac{dp}{dx} - p\mu_p \frac{dE}{dx} - \frac{\Delta p}{\tau_{p0}} + g'_p}$$

6.2 Characteristics of excess carriers

Time-dependent continuity equation

For an n-type semiconductor,

$$\boxed{\frac{d\Delta p}{dt} = D_p \frac{d^2 p}{dx^2} - \mu_p E \frac{dp}{dx} - p\mu_p \frac{dE}{dx} - R'_p + g'_p}$$

$$R'_p = \frac{\Delta p}{\tau_{p0}}$$

(minority carriers)

$$\boxed{\frac{d\Delta n}{dt} = D_n \frac{d^2 n}{dx^2} + \mu_n E \frac{dn}{dx} + n\mu_n \frac{dE}{dx} - R'_n + g'_n}$$

$$R'_n = R'_p = \frac{\Delta p}{\tau_{p0}}$$

(majority carriers)

$$g'_n = g'_p$$

Summary

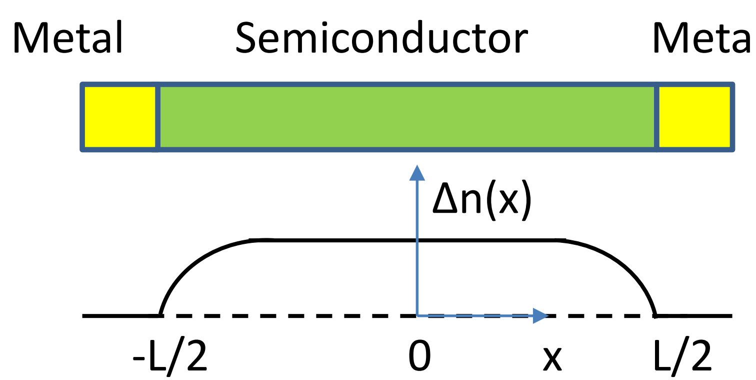
Table 6.2 |

Specification	Effect
Steady state	$\frac{\partial(\delta n)}{\partial t} = 0, \quad \frac{\partial(\delta p)}{\partial t} = 0$
Uniform distribution of excess carriers (uniform generation rate) + no boundary confinement	$D_n \frac{\partial^2(\delta n)}{\partial x^2} = 0, \quad D_p \frac{\partial^2(\delta n)}{\partial x^2} = 0$
Zero electric field	$\mathrm{E}\frac{\partial(\delta n)}{\partial x} = 0, \quad \mathrm{E}\frac{\partial(\delta p)}{\partial x} = 0$
No excess carrier generation	$g' = 0$
No excess carrier recombination (infinite lifetime)	$\frac{\delta n}{\tau_{n0}} = 0, \quad \frac{\delta p}{\tau_{p0}} = 0$

6.2 Characteristics of excess carriers

Problem Exmaple

Given a piece of p-type uniformly doped semiconductor in contact with two metal electrodes separated by a length of L, forming a photoconductor device. The light illumination will create electron-hole pairs at a generation rate of g. The minority carrier recombination lifetime is τ_0. Find the analytical distribution of the excess minority electrons at zero external bias. Note that light illumination will not create excess carriers in metals.



Outline

6.1 Carrier generation and recombination

6.2 Characteristics of excess carriers

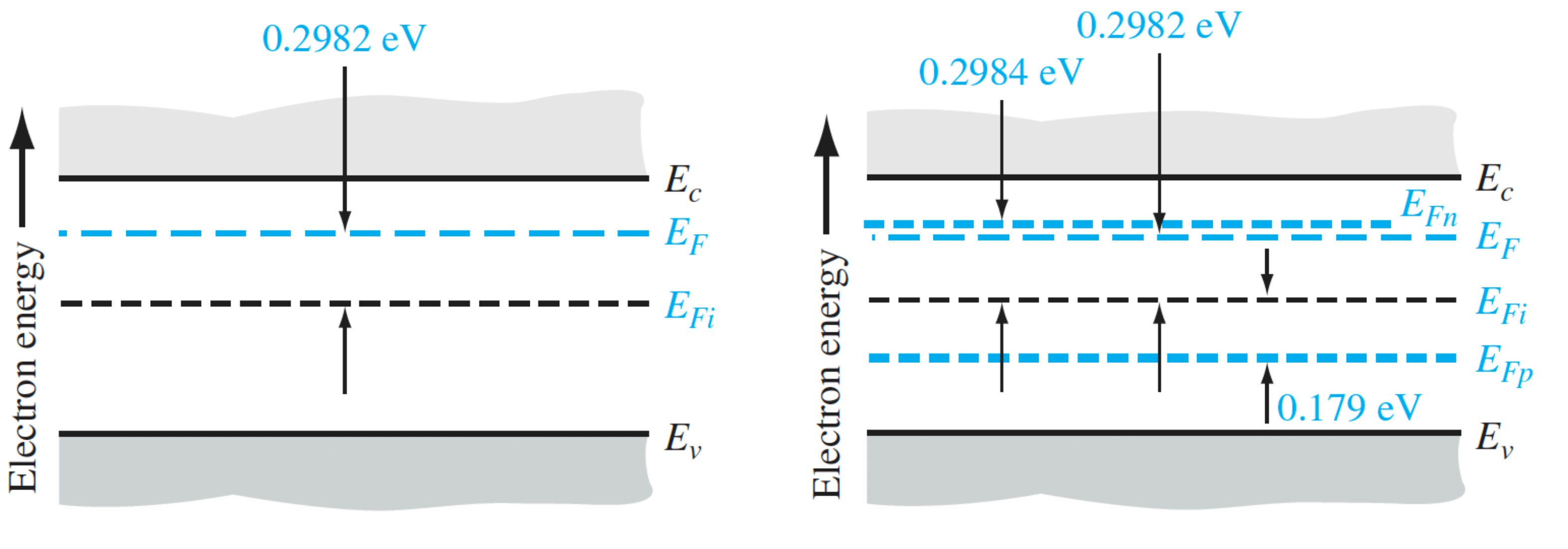
6.3 Quasi-Fermi levels

6.4 Excess carrier lifetime

6.5 Surface effects

6.3 Quasi-Fermi energy level

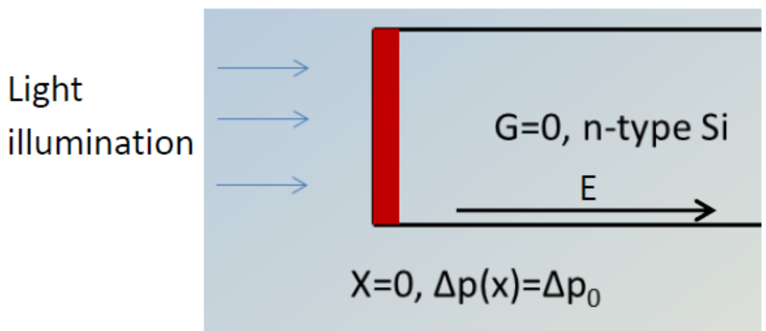
$$n_0 = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right) \longrightarrow n_0 + \Delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$



$$p_0 = n_i \exp\left(\frac{E_{Fi} - E_F}{kT}\right) \longrightarrow p_0 + \Delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$

Examples

A light beam is illuminated on the surface of a silicon wafer, generating excess carriers Δp_0 at the surface (x=0). The wafer is placed in a constant electric field with a known intensity E. We assume there is no external generation inside the wafer. The thickness of the wafer is infinite. Find the excess minority carriers at equilibrium as a function of the distance away from the surface (x=0). Small injection condition is always maintained and the wafer is uniformly doped as N_d.

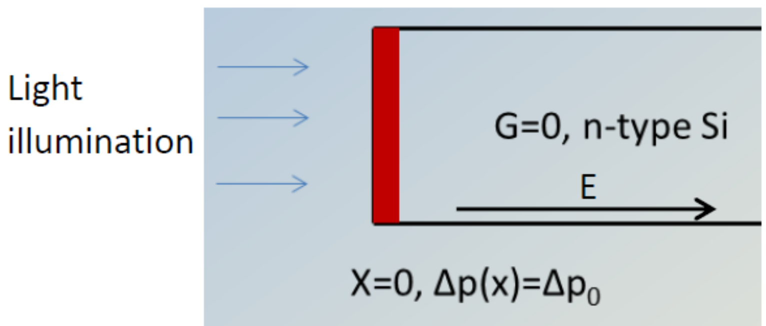


* Find the quasi Fermi level of holes.

$$\boxed{\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} - \mu_p E \frac{\partial p}{\partial x} - p\mu_p \frac{\partial E}{\partial x} + -\frac{\Delta p}{\tau} + G_{ex}}$$

Examples

A light beam is illuminated on the surface of a silicon wafer, generating excess carriers Δp_0 at the surface (x=0). The wafer is placed in a constant electric field with a known intensity E. We assume there is no external generation inside the wafer. The thickness of the wafer is infinite. Find the excess minority carriers at equilibrium as a function of the distance away from the surface (x=0). Small injection condition is always maintained and the wafer is uniformly doped as N_d.



* Find the quasi Fermi level of holes.

$$\boxed{\cancelto{0}{\frac{\partial p}{\partial t}} = D_p \frac{\partial^2 \cancelto{\Delta p}{p}}{\partial x^2} - \mu_p E \frac{\partial \cancelto{\Delta p}{p}}{\partial x} - p\mu_p \cancelto{0}{\frac{\partial E}{\partial x}} + -\frac{\Delta p}{\tau} + \cancelto{0}{G_{ex}}}$$

Examples

$$\boxed{D_p \frac{\partial^2 \Delta p}{\partial x^2} - \mu_p E \frac{\partial \Delta p}{\partial x} - \frac{\Delta p}{\tau} = 0}$$

$The\ solution\ likely\ looks\ like\ this:$

$$\Delta p = Aexp(\lambda x) + C$$

$$\frac{\partial \Delta p}{\partial x} = A\lambda exp(\lambda x) \qquad \frac{\partial^2 \Delta p}{\partial x^2} = A\lambda^2 \exp(\lambda x)$$

$$D_p[A\lambda^2 \exp(\lambda x)] - \mu_p E[A\lambda exp(\lambda x)] - \frac{Aexp(\lambda x)}{\tau} - \frac{C}{\tau} = 0$$

$$\text{Aexp}(\lambda x)\,(D_p\lambda^2 - \mu_p E\lambda - \frac{1}{\tau}) - \frac{C}{\tau} = 0$$

$$D_p\lambda^2 - \mu_p E\lambda - \frac{1}{\tau} = 0, C = 0$$

Examples

$$\boxed{D_p \frac{\partial^2 \Delta p}{\partial x^2} - \mu_p E \frac{\partial \Delta p}{\partial x} - \frac{\Delta p}{\tau} = 0}$$

$$\tau D_p \lambda^2 - \tau \mu_p E \lambda - 1 = 0 \qquad L_p = \sqrt{\tau D_p} \qquad L_p(E) = \tau \mu_p E$$

$$\lambda = \frac{L_p(E) \pm \sqrt{L_p^2(E) + 4L_p^2}}{2L_p^2}$$

$$\Delta p = (\Delta p)_0 exp\left(\frac{L_p(E) - \sqrt{L_p^2(E) + 4L_p^2}}{2L_p^2} x\right)$$

Examples

$$\boxed{D_p \frac{\partial^2 \Delta p}{\partial x^2} - \mu_p E \frac{\partial \Delta p}{\partial x} - \frac{\Delta p}{\tau} = 0}$$

$$\Delta p = (\Delta p)_0 exp\left(\frac{L_p(E) - \sqrt{L_p^2(E) + 4L_p^2}}{2L_p^2} x\right) \qquad L_p = \sqrt{\tau D_p} \qquad L_p(E) = \tau \mu_p E$$

Case #1: E is small so that $L_p(E) << L_p$

$$\Delta p = (\Delta p)_0 exp\left(-\frac{x}{L_p}\right)$$

Case #2: E is big so that $L_p(E) >> L_p$

$$\Delta p = (\Delta p)_0 exp\left(-\frac{x}{L_p(E)}\right)$$

Examples

$$\boxed{D_p \frac{\partial^2 \Delta p}{\partial x^2} - \mu_p E \frac{\partial \Delta p}{\partial x} - \frac{\Delta p}{\tau} = 0}$$

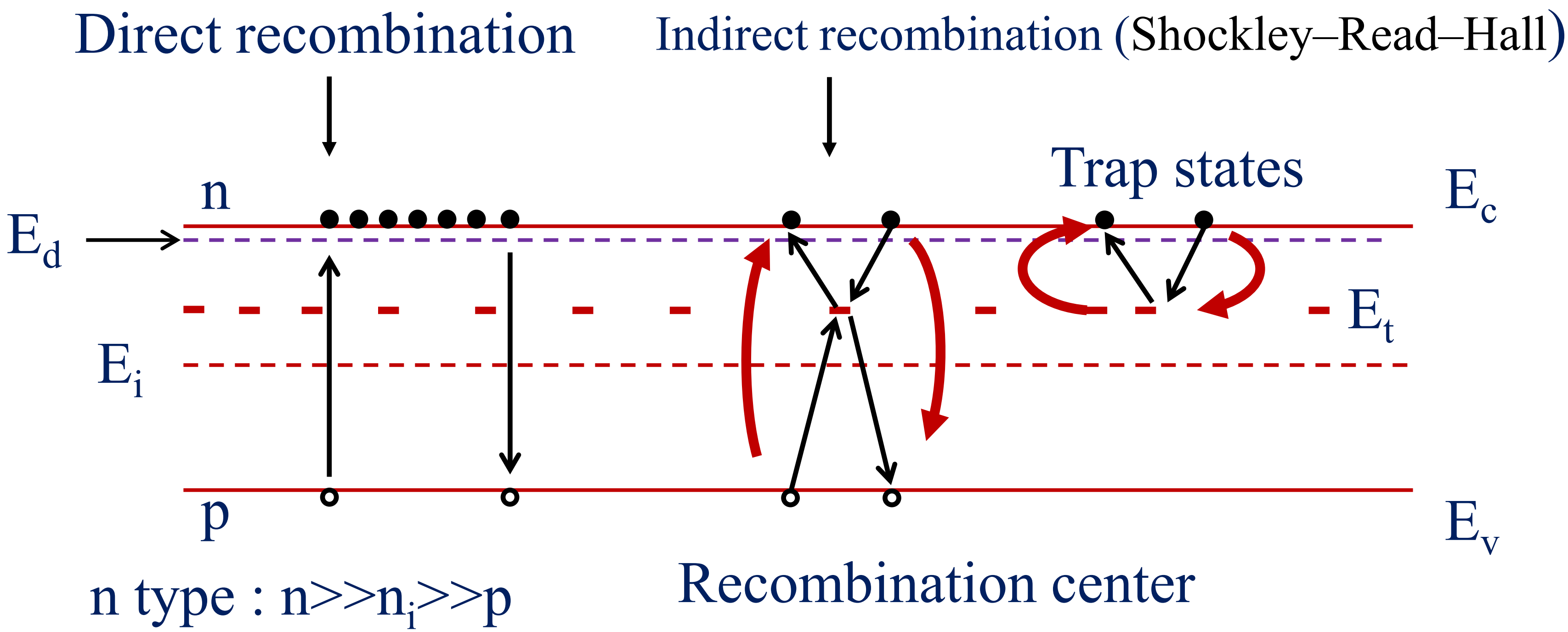
$$\Delta p = (\Delta p)_0 exp\left(\frac{L_p(E) - \sqrt{L_p^2(E) + 4L_p^2}}{2L_p^2} x\right) \qquad L_p = \sqrt{\tau D_p} \qquad L_p(E) = \tau \mu_p E$$

$$p = p_0 + \Delta p = p_0 + (\Delta p)_0 exp\left(\frac{L_p(E) - \sqrt{L_p^2(E) + 4L_p^2}}{2L_p^2} x\right)$$

$$p = N_V \exp\left(\frac{E_V - E_F^p}{kT}\right) \Rightarrow E_F^p = E_V - kTln(\frac{p}{N_V})$$

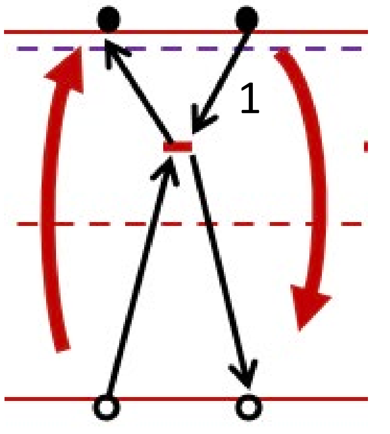
Outline

6.4 Excess carrier lifetime



6.4 Excess carrier lifetime

1. Capture of an electron from conductance band by an initially neutral empty trap



$$R_{cn} = C_n N_t [1 - f_F(E_t)] n$$

R_{cn} = capture rate (#/cm^3-s)

C_n = constant proportional to electron-capture cross section

N_t = total concentration of trapping centers

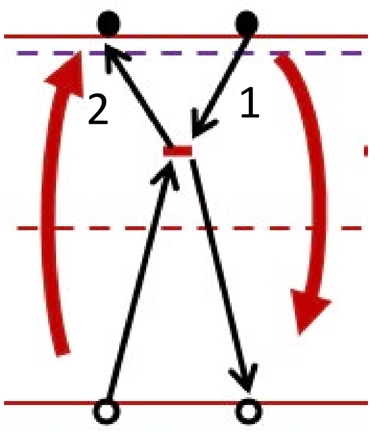
n = electron concentration in the conduction band

$f_F(E_t)$ = Fermi function at the trap energy

$$f_F(E_t) = \frac{1}{1 + \exp\left(\frac{E_t - E_F}{kT}\right)}$$

6.4 Excess carrier lifetime

2. Inverse of process 1—the emission of an electron that is initially occupying a trap level back into the conduction band



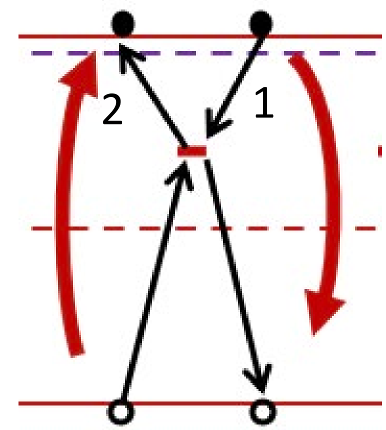
$$R_{en} = E_n N_t f_F(E_t)$$

R_{en} = emission rate (#/cm^3-s)

E_n = constant

$f_F(E_t)$ = probability that the trap is occupied

6.4 Excess carrier lifetime



3. Capture of an hole from valence band by a trap containing an electron (Or we may consider the process to be the emission of an electron from the trap into the valence band.)

4. Inverse of process 3—the emission of a hole from a neutral trap into the valence band. (Or we may consider this process to be the capture of an electron from the valence band.)

$$\boxed{R_n = R_p = \frac{C_n C_p N_t\left(np - n_i^2\right)}{C_n(n + n') + C_p(p + p')} \equiv R}$$

$$n' = N_c \exp\left[\frac{-(E_c - E_t)}{kT}\right] \qquad p' = N_v \exp\left[\frac{-(E_t - E_v)}{kT}\right]$$

6.4 Excess carrier lifetime

$$R_n = R_p = \frac{C_n C_p N_t (np - n_i^2)}{C_n(n + n') + C_p(p + p')} \equiv R$$

$$n' = N_c \exp\left[\frac{-(E_c - E_t)}{kT}\right] \qquad p' = N_v \exp\left[\frac{-(E_t - E_v)}{kT}\right]$$

$$n' = n_i \exp(\frac{E_t - E_i}{kT}) \qquad p' = n_i \exp(\frac{E_i - E_t}{kT})$$

$$R_n = \frac{np - n_i^2}{\tau_{p0}\left[n + n_i \exp\left(\frac{E_t - E_i}{kT}\right)\right] + \tau_{n0}\left[p + n_i \exp\left(\frac{E_i - E_t}{kT}\right)\right]}$$

$$where\ \tau_{p0} = \frac{1}{N_t C_p}, \tau_{n0} = \frac{1}{N_t C_n}$$

6.4 Excess carrier lifetime

Problem Example

A PN junction consisting an n-type semiconductor in contact with another p-type semicondcutor (to be covered later) has a depletion region in which n_0 and p_0 are nearly zero. Suppose a silicon PN junction has defects located at the middle of the semiconductor. The defect concentration is 10^{16} cm^{-3} and the capture rate C_n and C_p for electrons and holes are 10^{-10} cm^{-3}/s. Find the recombination rate of charge carriers in the depletion region of the Si PN junction.

Depletion region

$N_t = 10^{16}$ cm^{-3}

$C_n = C_p = 10^{-10}$ cm^{-3}/s

Outline

6.1 Carrier generation and recombination

6.2 Characteristics of excess carriers

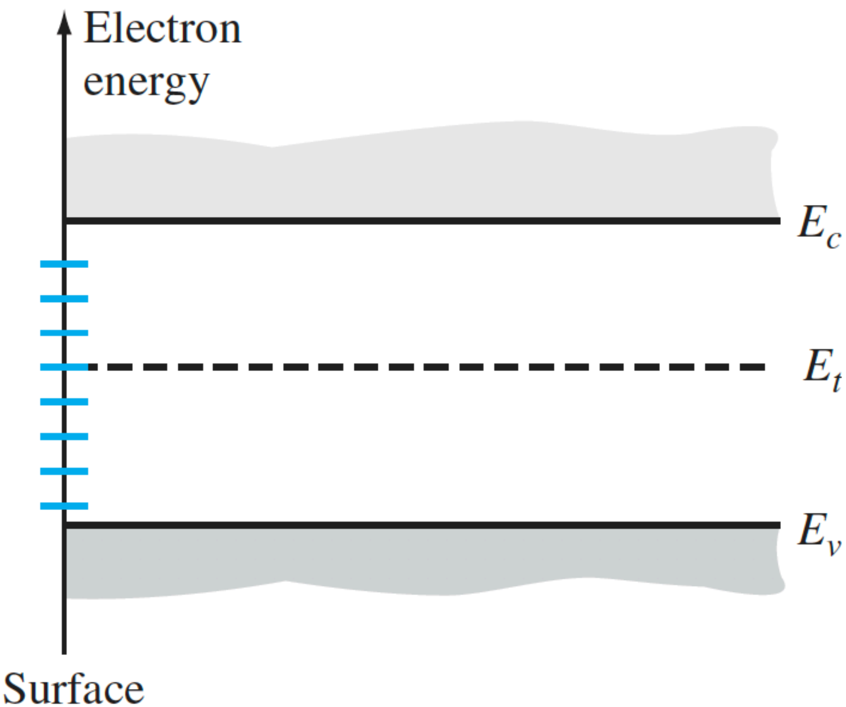
6.3 Quasi-Fermi levels

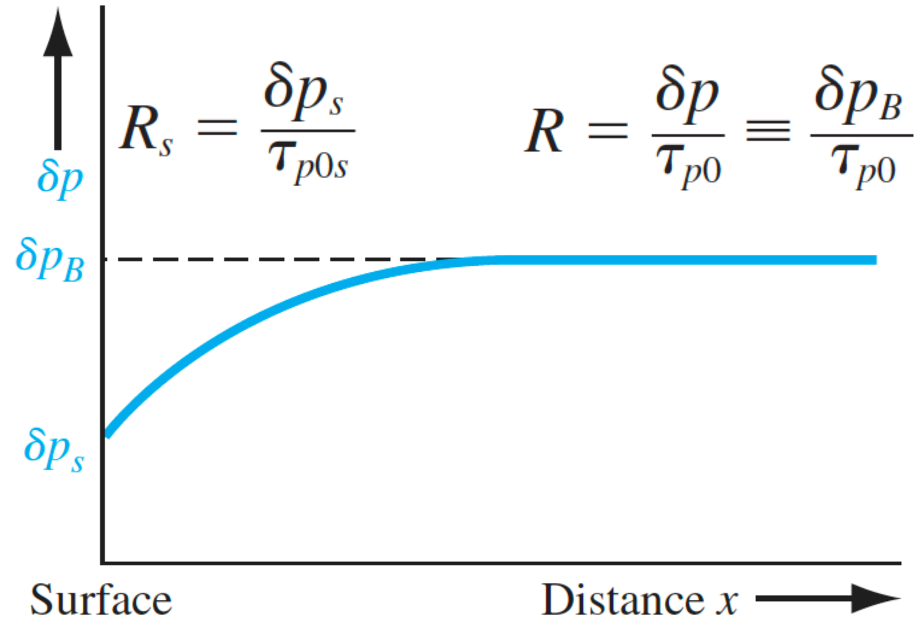
6.4 Excess carrier lifetime

6.5 Surface effects

6.5 Surface effects

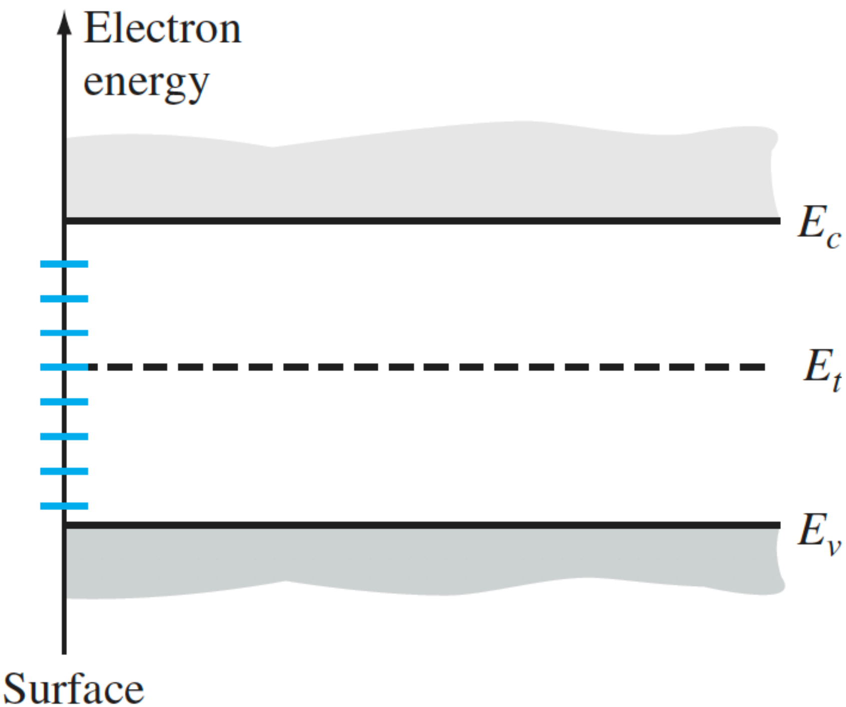
Surface States

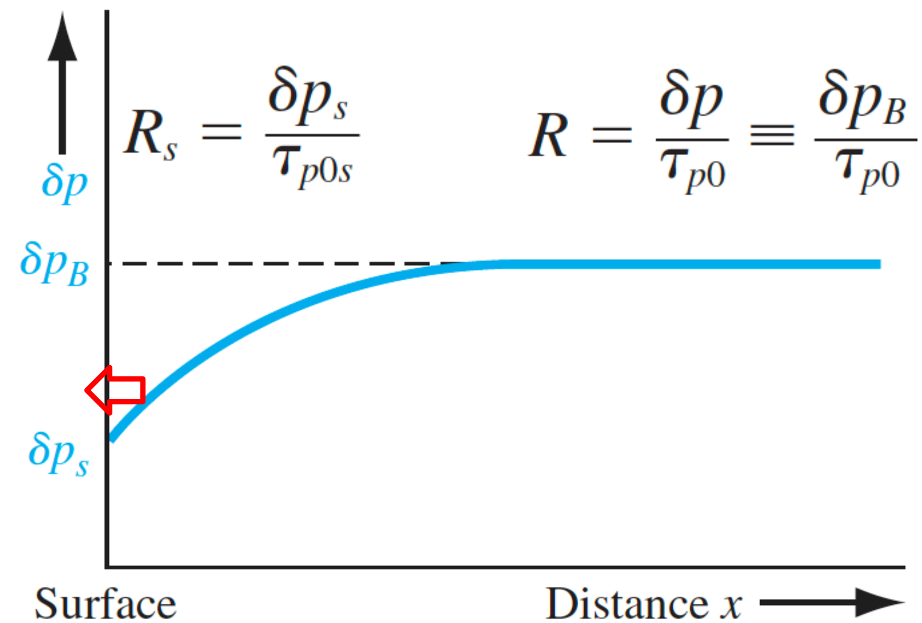




6.5 Surface effects

Surface recombination velocity





Surface recombination rate:
number of recombined carriers in a unit surface area at a give unit time

$$-D_p\left[\hat{n}\cdot\frac{d(\delta p)}{dx}\right]\bigg|_{\text{surf}} = s\delta p|_{\text{surf}}$$

s: surface recombination velocity

Examples

A n-type semiconductor wafer is uniformly doped and uniformly illuminated by light. There is no electric field. At equilibrium, the excess minority carrier concentration is $(\Delta p)_0$. Start from zero time t=0, the illumination is cut off. Find how does the concentration of the excess minority carriers change over time. Small injection condition is always maintained.

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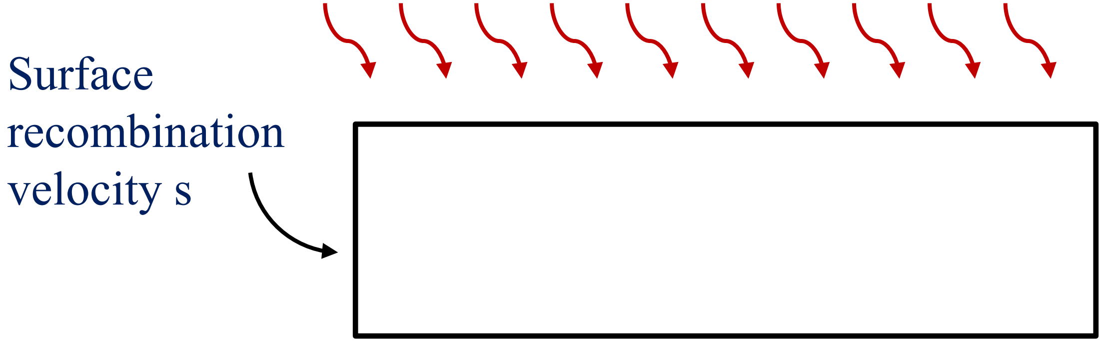
$$\boxed{\frac{\partial p}{\partial t} = D_p \cancelto{0}{\frac{\partial^2 p}{\partial x^2}} - \mu_p E \cancelto{0}{\frac{\partial p}{\partial x}} - p\mu_p \cancelto{0}{\frac{\partial E}{\partial x}} - \frac{\Delta p}{\tau} + \cancelto{0}{G_{ex}}}$$

Examples

A n-type semiconductor wafer is uniformly doped and uniformly illuminated by light. There is no electric field. The illumination generation rate is g and the minority carrier lifetime is τ. The semiconductor is long enough on the right side. On the left side edge, the surface recombination velocity is s. Find how does the concentration of the excess minority carriers change along x coordinate at equilibrium. Small injection condition is always maintained.



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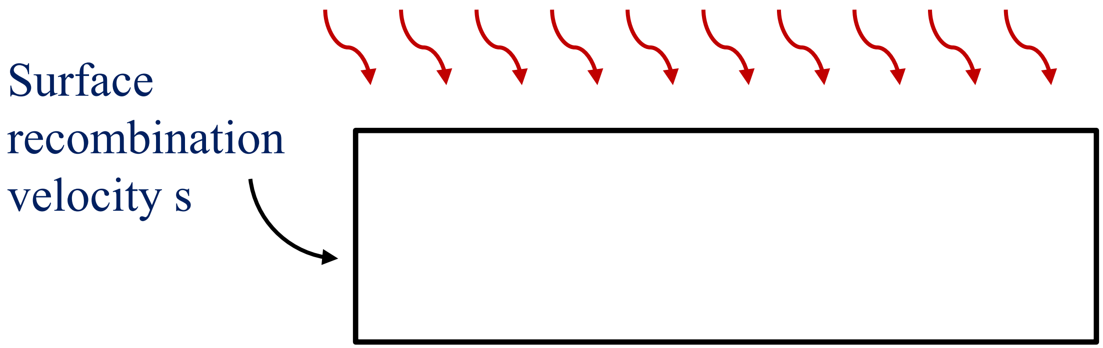

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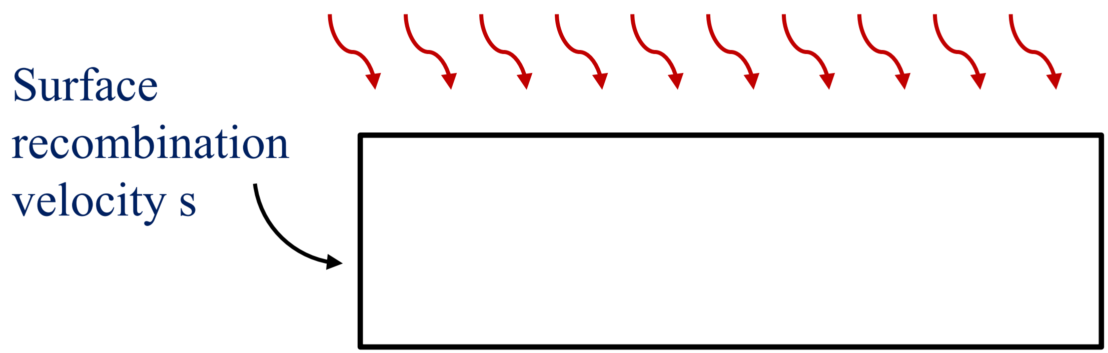

$$\boxed{0 = D_p \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau} + G_{ex}}$$

The solution likely looks like this:

$$\Delta p = A exp(\lambda x) + C$$

Examples

$$\boxed{0 = D_p \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau} + G_{ex}}$$

$The\ solution\ likely\ looks\ like\ this:$

$$\Delta p = Aexp(\lambda x) + C$$

$$\frac{\partial \Delta p}{\partial x} = A\lambda exp(\lambda x) \qquad \frac{\partial^2 \Delta p}{\partial x^2} = A\lambda^2 \exp(\lambda x)$$

$$D_p[A\lambda^2 \exp(\lambda x)] - \frac{Aexp(\lambda x)}{\tau} - \frac{C}{\tau} + G_{ex} = 0$$

$$\mathrm{Aexp}(\lambda x)\,(D_p\lambda^2 - \frac{1}{\tau}) - \frac{C}{\tau} + G_{ex} = 0$$

$$D_p\lambda^2 - \frac{1}{\tau} = 0; \quad -\frac{C}{\tau} + G_{ex} = 0 \Rightarrow \lambda = \pm\sqrt{D_p\tau}; \quad C = G_{ex}\tau$$

Examples

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Boundary conditions:

① $x \to \infty, \qquad \Delta p\ limited \Rightarrow \Delta p = Aexp\left(-\frac{x}{\sqrt{D_p \tau}}\right) + g\tau$

② $x = 0, \qquad -\mathrm{F}_p = s \cdot \Delta p(x = 0) \Rightarrow D_p \frac{\partial \Delta p(x)}{\partial x}|_{x=0} = s \cdot \Delta p(x = 0)$

Examples

$$\boxed{0 = D_p \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau} + G_{ex}}$$

The solution likely looks like this:

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$$\Rightarrow -D_p \frac{Aexp\left(-\frac{x}{\sqrt{D_p \tau}}\right)}{\sqrt{D_p \tau}}|_{x=0} = s \cdot (A + g\tau)$$

$$\Rightarrow -D_p \frac{A}{\sqrt{D_p \tau}} = s \cdot (A + g\tau) \Rightarrow \ A = -\frac{g\tau s}{s + D_p/L_p}$$