I Chapter 6

Excess minority carrier lifetime:

$$\frac{\mathrm{d}(\delta n(t))}{\mathrm{d}t} = -\alpha_r(p_0+n_0)\delta n(t)$$

$$\tau = \frac{1}{\alpha_r(n_0+p_0)}$$

n-type:

$$R'_n = R'_p = \frac{\delta p(t)}{\tau_{p0}}$$

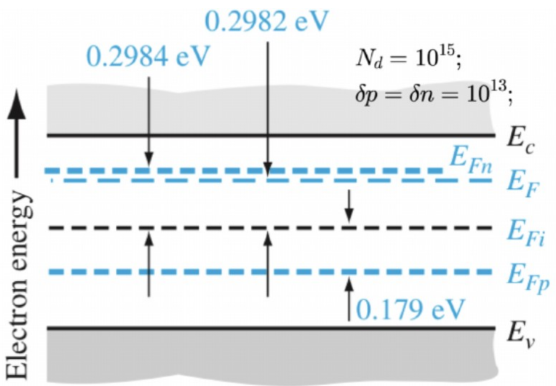
p-type:

$$R'_n = R'_p = \frac{\delta n(t)}{\tau_{n0}}$$

Quasi-Fermi Energy Level

$$n_0 + \delta n = n_i \exp\left(\frac{E_{Fn}-E_{Fi}}{kT}\right)$$

$$p_0 + \delta p = n_i \exp\left(\frac{E_{Fi}-E_{Fp}}{kT}\right)$$



Excess Carrier Lifetime

$$R_n = R_p = \frac{C_nC_pN_t(np-n_i^2)}{C_n(n+n')+C_p(p+p')} = \frac{(np-n_i^2)}{\tau_{p0}(n+n')+\tau_{n0}(p+p')}$$

where
$n' = N_c \exp\left[-\frac{E_c-E_t}{kT}\right], p' = N_v \exp\left[-\frac{E_t-E_v}{kT}\right]$, and $\tau_{n0} = \frac{1}{C_nN_t}$

Surface Effects

$$-D_p\left[\hat{n}\cdot\frac{\mathrm{d}(\delta p)}{\mathrm{d}x}\right]\Bigg|_{\text{surf}} = s\ \delta p|_{\text{surf}}$$

$$\delta p(x) = g'\tau_{p0}\left(1-\frac{sL_p\mathrm{e}^{-x/L_p}}{D_p+sL_p}\right)$$

Time-dependent Continuity Equation

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -\frac{F_p^+}{\mathrm{d}x} + g_p - \frac{p}{\tau_{pt}}$$

$$D_n\frac{\mathrm{d}^2n}{\mathrm{d}x^2} + \mu_n\left(E\frac{\mathrm{d}n}{\mathrm{d}x}+n\frac{\mathrm{d}E}{\mathrm{d}x}\right) + g_n - \frac{n}{\tau_{nt}} = \frac{\mathrm{d}n}{\mathrm{d}t}$$

$$D_p\frac{\mathrm{d}^2p}{\mathrm{d}x^2} - \mu_p\left(E\frac{\mathrm{d}p}{\mathrm{d}x}+p\frac{\mathrm{d}E}{\mathrm{d}x}\right) + g_p - \frac{p}{\tau_{pt}} = \frac{\mathrm{d}p}{\mathrm{d}t}$$

$$D_n\frac{\mathrm{d}^2(\delta n)}{\mathrm{d}x^2} + \mu_n\left(E\frac{\mathrm{d}(\delta n)}{\mathrm{d}x}+n\frac{\mathrm{d}E}{\mathrm{d}x}\right) + g_n - \frac{n}{\tau_{nt}} = \frac{\mathrm{d}(\delta n)}{\mathrm{d}t}$$

$$D_p\frac{\mathrm{d}^2(\delta p)}{\mathrm{d}x^2} - \mu_p\left(E\frac{\mathrm{d}(\delta p)}{\mathrm{d}x}+p\frac{\mathrm{d}E}{\mathrm{d}x}\right) + g_p - \frac{p}{\tau_{pt}} = \frac{\mathrm{d}(\delta p)}{\mathrm{d}t}$$

$$g_n - \frac{n}{\tau_{nt}} = g' - \frac{\delta n}{\tau_{nt}}$$

Table 6.2 | Common ambipolar transport equation simplifications

Specification	Effect
Steady state	$\frac{\partial(\delta n)}{\partial t} = 0, \quad \frac{\partial(\delta p)}{\partial t} = 0$
Uniform distribution of excess carriers (uniform generation rate)	$D_n\frac{\partial^2(\delta n)}{\partial x^2} = 0, \quad D_p\frac{\partial^2(\delta n)}{\partial x^2} = 0$
Zero electric field	$\mathrm{E}\frac{\partial(\delta n)}{\partial x} = 0, \quad \mathrm{E}\frac{\partial(\delta p)}{\partial x} = 0$
No excess carrier generation	$g' = 0$
No excess carrier recombination (infinite lifetime)	$\frac{\delta n}{\tau_{n0}} = 0, \quad \frac{\delta p}{\tau_{p0}} = 0$

General solutions:

- $$\frac{\mathrm{d}(\delta p)}{\mathrm{d}t} = -\frac{\delta p}{\tau_{p0}}$$

  solution:

  $$\delta p(t) = \delta p(0)\mathrm{e}^{-t/\tau_{p0}}$$

- $$g' - \frac{\delta p}{\tau_{p0}} = \frac{\mathrm{d}(\delta p)}{\mathrm{d}t}$$

  solution:

  $$\delta p(t) = g'\tau_{p0}\left(1-\mathrm{e}^{-t/\tau_{p0}}\right)$$

- $$D_n\frac{\mathrm{d}^2(\delta n)}{\mathrm{d}x^2} - \frac{\delta n}{\tau_{n0}} = 0$$

  solution:

  $$\delta n(x) = A\mathrm{e}^{-x/L_n} + B\mathrm{e}^{x/L_n}, \quad L_n = \sqrt{D_n\tau_{n0}}$$

  special:

  $$\delta n(x) = \begin{cases}\delta n(0)\mathrm{e}^{-x/L_n}, x\geq 0\\ \delta n(0)\mathrm{e}^{+x/L_n}, x\leq 0\end{cases}$$

- $$D_p\frac{\mathrm{d}^2\delta p}{\mathrm{d}x^2} - \frac{\delta p}{\tau} + G_{ex} = 0$$

  solution:

  $$\delta p(x) = A\exp(\lambda x) + g\tau, \quad \lambda = \pm\frac{1}{\sqrt{D_p\tau}}$$

- $$D_p\frac{\mathrm{d}^2(\delta p)}{\mathrm{d}x^2} - \mu_pE_0\frac{\mathrm{d}(\delta p)}{\mathrm{d}x} - \frac{\delta p}{\tau_{p0}} = \frac{\mathrm{d}(\delta p)}{\mathrm{d}t}$$

  solution:

  $$\delta p(x,t) = \frac{\mathrm{e}^{-t/\tau_{p0}}}{(4\pi D_pt)^{1/2}}\exp\left[\frac{-(x-\mu_pE_0t)^2}{4D_pt}\right]$$

- $$D_p\frac{\mathrm{d}^2\delta p}{\mathrm{d}x^2} - \mu_pE\frac{\mathrm{d}\delta p}{\mathrm{d}x} - \frac{\delta p}{\tau} = 0$$

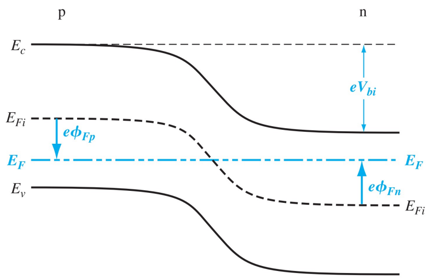
solution:
$\delta p(x) = A\exp(\lambda x) + C$

$$\lambda = \frac{L_p(E) \pm \sqrt{L_p^2(E) + 4L_p^2}}{2L_p^2}, \quad L_p = \sqrt{\tau D_p}, \quad L_p(E) = \tau\mu_p E$$

special:

$$\delta p(x) = \delta p(0)\exp\left[\frac{L_p(E) \pm \sqrt{L_p^2(E) + 4L_p^2}}{2L_p^2}x\right] = \begin{cases} \delta p(0)\exp\left(-\frac{x}{L_p}\right), & \text{if } L_p(E) \ll L_p \\ \delta p(0)\exp\left(-\frac{x}{L_p(E)}\right), & \text{if } L_p(E) \gg L_p \end{cases}$$

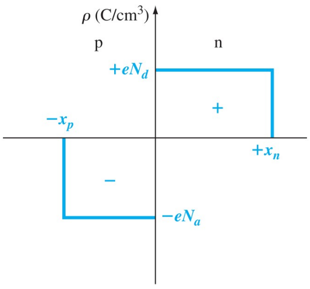
II Chapter 7

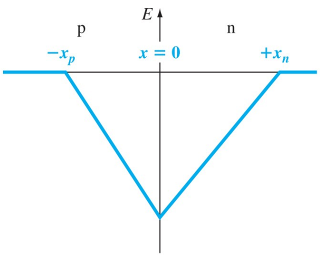


V_{bi}: built-in potential barrier.

$$\begin{aligned} V_{bi} &= |\phi_{Fn}| + |\phi_{Fp}| \\ &= \frac{kT}{e}\ln\left(\frac{N_a N_d}{n_i^2}\right) = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) \end{aligned}$$

$V_t = kT/e$ defined as the thermal voltage.





$$N_a x_p = N_d x_n \qquad |k| = \frac{eN_{a/d}}{\varepsilon_s}$$

$$N_a x_p = N_d x_n$$

$$x_n = \sqrt{\frac{2\varepsilon_s (V_{bi} + V_R)}{e}\left[\frac{N_a}{N_d}\right]\left[\frac{1}{N_a + N_d}\right]}$$

$$x_p = \sqrt{\frac{2\varepsilon_s (V_{bi} + V_R)}{e}\left[\frac{N_d}{N_a}\right]\left[\frac{1}{N_a + N_d}\right]}$$

$\varepsilon_s = \varepsilon_r\varepsilon_0$, where $\varepsilon_0 = 8.85 \times 10^{-14} F \cdot cm^{-1}$.
$\varepsilon_r = 11.7$ for Si.

$$W = x_n + x_p = \sqrt{\frac{2\varepsilon_s (V_{bi} + V_R)}{e}\left[\frac{N_a + N_d}{N_a N_d}\right]}$$

$$E = \begin{cases} -\frac{eN_a}{\varepsilon_s}(x + x_p), & -x_p \le x \le 0 \\ -\frac{eN_d}{\varepsilon_s}(x_n - x), & 0 \le x \le x_n \end{cases}$$

$$\begin{aligned} |E_{max}| &= -\frac{eN_d x_n}{\varepsilon_s} = -\frac{eN_a x_p}{\varepsilon_s} \\ &= -\frac{2(V_{bi} + V_R)}{W} \end{aligned}$$

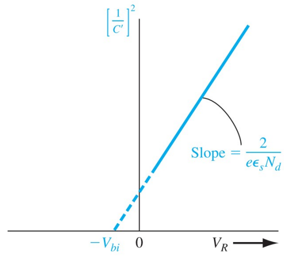
$$\phi(x) = -\int E(x)\,\mathrm{d}x$$

Junction Capacitance

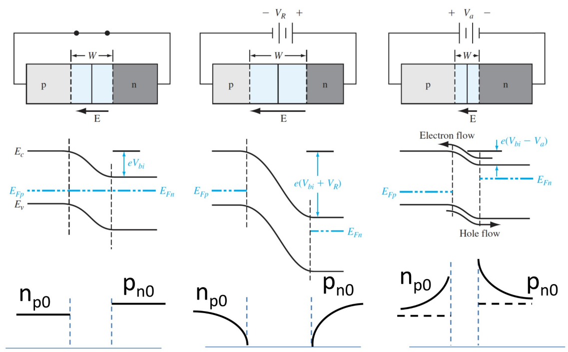
$$C' = \frac{\mathrm{d}Q'}{\mathrm{d}V} = \sqrt{\frac{e\varepsilon_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)}} = \frac{\varepsilon_s}{W}$$

One-sided Junction - p$^+$n junction:

$$\begin{aligned} &x_p \ll x_n \\ &W \approx x_n \\ &C' \approx \sqrt{\frac{e\varepsilon_s N_d}{2(V_{bi} + V_R)}} \\ &\left(\frac{1}{C'}\right)^2 = \frac{2(V_{bi} + V_R)}{e\varepsilon_s N_d} \end{aligned}$$



III Chapter 8



$$n_{p0} = n_{n0}\exp\left(-\frac{eV_{bi}}{kT}\right)$$

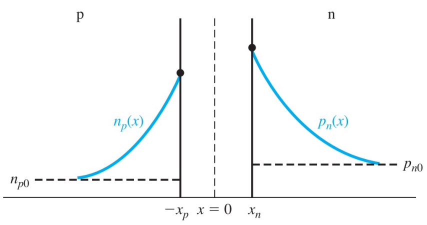
$$p_{n0} = p_{p0}\exp\left(-\frac{eV_{bi}}{kT}\right)$$

Forward biased

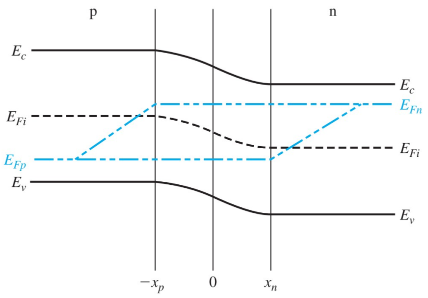
$$n_p = n_{p0}\exp\left(\frac{eV_a}{kT}\right)$$

$$p_n = p_{n0}\exp\left(\frac{eV_a}{kT}\right)$$

$$\begin{aligned} \delta p_n(x) &= p_n(x) - p_{n0} \\ &= p_{n0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_n - x}{L_p}\right), \quad x \ge x_n \\ \delta n_p(x) &= n_p(x) - n_{p0} \\ &= n_{p0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_p + x}{L_n}\right), \quad x \le -x_p \end{aligned}$$



Quasi-Fermi Level



$$p = p_0 + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$

$$np = n_i^2 \exp\left(\frac{E_{Fn} - E_{Fp}}{kT}\right)$$

Durrent density

$$J_n(x) = \frac{eD_n n_{p0}}{L_n}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_p + x}{L_n}\right)$$

$$J_p(x) = \frac{eD_p p_{n0}}{L_p}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_n - x}{L_p}\right)$$

Ideal pn junciton current

$$J = J_s\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

$$\text{where } J_s = \left[\frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n}\right] = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\right]$$

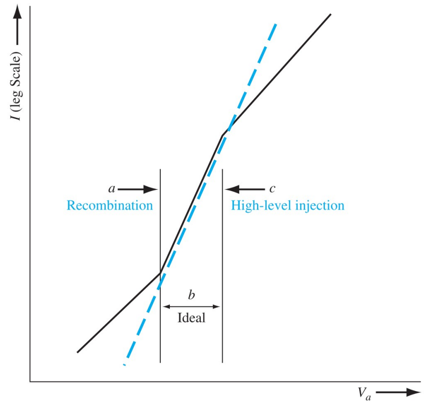
Non-ideal I - Generation-recombination currents

$$R_n = \frac{(np - n_i^2)}{\tau_p\left[n + n_i \exp\left(\frac{E_t - E_i}{kT}\right)\right] + \tau_n\left[p + n_i \exp\left(\frac{E_i - E_t}{kT}\right)\right]} = -\frac{n_i}{2\tau} = -G_0 \qquad \text{assume } E_t = E_i, \tau_n = \tau_p = \tau$$

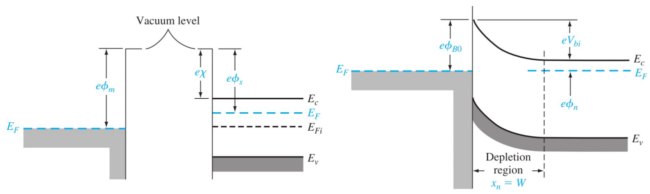
$$J_r = \int_0^W qG_0 \, \mathrm{d}x = \frac{qWn_i}{2\tau}$$

$$J = J_s\left[\exp\left(\frac{qV_a}{kT}\right) - 1\right] + \frac{qWn_i}{2\tau}\left[\exp\left(\frac{qV_a}{2kT}\right) - 1\right]$$

Non-ideal II - High Level Injection

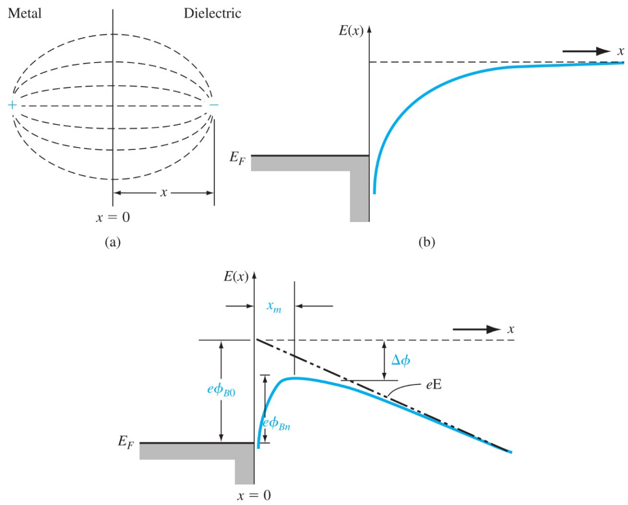


IV Chapter 9



Work function: ϕ
Electron affinity: χ
Schottky barrier: $\phi_{B0} = \phi_m - \chi$
Built-in potential barrier: $V_{bi} = \phi_{B0} - \phi_n$
$\phi_n = kT \ln\left(\frac{N_c}{N_d}\right)$

Schottky Barrier Lowering



(a) (b) (c)

$$F = \frac{-e^2}{4\pi\varepsilon_s(2x)^2} = -eE$$

$$-\phi(x) = +\int_x^\infty E \, \mathrm{d}x' = \frac{-e}{16\pi\varepsilon_s x}$$

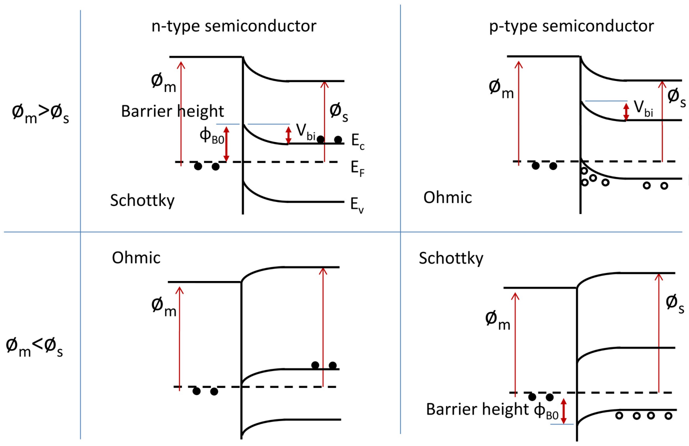
$$\text{With electric field: } -\phi(x) = \frac{-e}{16\pi\varepsilon_s x} - Ex$$

$$\frac{\mathrm{d}(e\phi(x))}{\mathrm{d}x} = 0 \quad \Rightarrow \quad \Delta\phi = \sqrt{\frac{eE}{4\pi\varepsilon_s}}$$

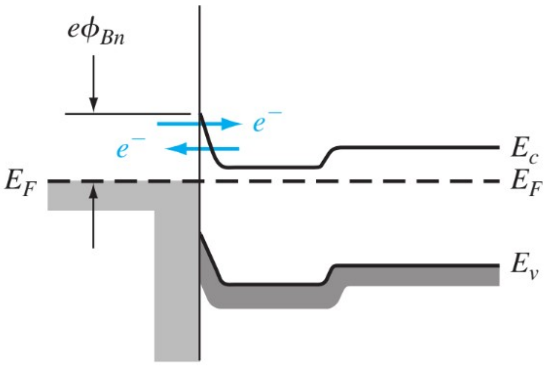
Current-Voltage Relationship

$$
\begin{aligned}
J &= J_{sT}\left[\exp\left(\frac{eV_a}{kT}\right)-1\right] \\
J_{sT} &= A^*T^2\exp\left(\frac{-e\phi_{Bn}}{kT}\right) \\
A^* &= \frac{4\pi e m_n^* k^2}{h^3}
\end{aligned}
$$

A^*: effective Richardson constant for thermionic emission.



Tunneling Barrier



$$
\begin{aligned}
J_t &\propto \exp\left(\frac{-e\phi_{Bn}}{E_{oo}}\right) \\
E_{oo} &= \frac{e\hbar}{2}\sqrt{\frac{N_d}{\varepsilon_s M_n^*}}
\end{aligned}
$$

Specific Contact Resistance

$$
\begin{aligned}
R_c &= \left(\frac{\partial J}{\partial V}\right)^{-1}\Bigg|_{V=0} \quad \Omega - cm^2 \\
J_n &= A^*T^2\exp\left(\frac{-e\phi_{Bn}}{kT}\right)\left[\exp\left(\frac{eV}{kT}\right)-1\right] \\
R_c &= \frac{\left(\frac{kT}{e}\right)\exp\left(\frac{+e\phi_{Bn}}{kT}\right)}{A^*T^2}
\end{aligned}
$$

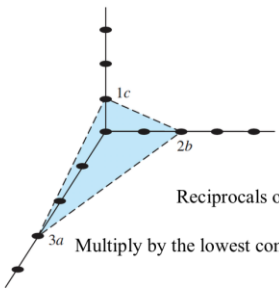
R_c: the reciprocal of the derivative of current density with respect to voltage evaluated at zero bias.

Resistivity:

Conductors	Semiconductors	Insulators
< 10^{-3} Ω•cm	10^{-3} – 10^{9} Ω•cm	> 10^{9} Ω•cm
Metals (Au, Al, Cu, Hg…)	Si, Ge, GaAs, InP…	SiO_2, HfO_2 …
Solids, liquids (Hg)	Solids	Solids, liquids gases

Unit cell: any small volume of crystal to reproduce the entire crystal.
Primitive cell: smallest unit cell

Crytalline Plane and Miller Index



Reciprocals of the intercepts: (1/3, ½, 1)

Multiply by the lowest common denominator: (2,3,6)

$\frac{\partial^2 y}{\partial x^2} = k^2 y$ General solution: $y = Ae^{bx}$

Plug into the equation: $b^2Ae^{bx} = k^2Ae^{bx}$

$\Rightarrow b = \pm k$

$\Rightarrow y = A_1e^{kx} + A_2e^{-kx}$

$\frac{\partial^2 y}{\partial x^2} = -k^2 y$ General solution: $y = Ae^{bx}$

Plug into the equation: $b^2Ae^{bx} = -k^2Ae^{bx}$

$\Rightarrow b = \pm ki$

$\Rightarrow y = A_1e^{ikx} + A_2e^{-ikx}$

$K = \frac{2\pi}{\lambda}, E = mc^2 = hv = \frac{hc}{\lambda}, p = \frac{h}{\lambda} = mv$

$$\frac{\partial^2\psi(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V(x))\psi(x) = 0$$

$$\mathrm{E} = \frac{k^2\hbar^2}{2m} = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

$$k = \frac{n\pi}{a} \quad n = 0, \pm 1, \pm 2, \ldots$$

$p = \hbar k = mv$

$$\frac{dE}{dk} = \frac{\hbar^2 k}{m} \xRightarrow{mv=\hbar k} \frac{\hbar m v}{m} = \hbar v$$

$\mathrm{E} = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$

$$v = \frac{1}{\hbar}\frac{dE}{dk}$$

$$J = qNv_d = q\sum_i^N v_i$$

Conduction Band:

$$\boxed{E = E(k) = E_c + \frac{\hbar^2}{2m_n^*}(k - k_1)^2}$$

Valence Band:

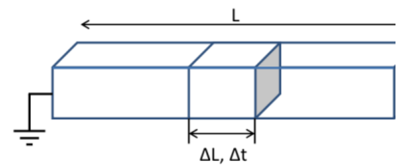
$$\boxed{E = E(k) = E_c - \frac{\hbar^2}{2m_p^*}(k - k_2)^2}$$

$$E - E_c = C_1(k)^2$$

$$\frac{1}{\hbar^2}\frac{d^2E}{dk^2} = \frac{2C_1}{\hbar^2}$$

$$\frac{1}{\hbar^2}\frac{d^2E}{dk^2} = \frac{2C_1}{\hbar^2} = \frac{1}{m^*}$$

n type semiconductor



$I = \frac{\Delta Q}{\Delta t} = \frac{nqA_c\Delta L}{\Delta t} = nqA_c v$

$v = \mu E = \mu V/L$

$I = \frac{\Delta Q}{\Delta t} = \frac{nqA_c\Delta L}{\Delta t} = nqA_c\mu V/L \quad \Rightarrow \quad \sigma = \frac{I}{V} = \frac{N_D qA_c\mu}{L}$

$$f_F(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}$$

$$\boxed{f_F(E) \approx \exp(-\frac{E - E_F}{kT})}$$

$$\boxed{E = E(k) = E_c + \frac{\hbar^2}{2m_n^*}k^2}$$

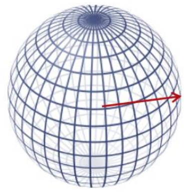
$$\boxed{k = \mp\frac{\sqrt{2m_n^*(E - E_c)}}{\hbar}}$$

Within ΔE, we have the number of k is $\frac{d(k/\pi)}{dE}\Delta E$

$$\boxed{g(E) = \frac{1}{2}\frac{d(k/\pi)}{dE}}$$

$$E = E(k) = E_c + \frac{\hbar^2}{2m_n^*}k^2$$

$$k = \mp\frac{\sqrt{2m_n^*(E-E_c)}}{\hbar}$$



Within ΔE, we have the number of k is $\frac{d(4\pi\left(\frac{k}{\pi}\right)^2/3)}{dE}\Delta E$

$$g(E) = \frac{1}{8}\frac{d(4\pi\left(\frac{k}{\pi}\right)^2/3)}{dE}$$

$$g_v(E) = \frac{4\pi(2m_p^*)^{3/2}}{h^3}\sqrt{E_v - E}$$

$$n_0 = \int_{E_c}^{\infty} g_c(E) f_F(E) dE$$

$$p_0 = \int_{-\infty}^{E_v} g_v(E)[1 - f_F(E)]dE$$

$$if\ \exp(x-\mathcal{E}) > 10 \Leftrightarrow \frac{E-E_F}{kT} > 3 \Leftrightarrow E - E_F > 3kT$$

$$n_0 = \frac{2(2\pi m_n{}^*kT)^{\frac{3}{2}}}{h^3}\exp\left(\frac{E_F-E_c}{kT}\right) = N_c\exp(\frac{E_F-E_c}{kT})$$

$$p_0 = \frac{2(2\pi m_p{}^*kT)^{\frac{3}{2}}}{h^3}\exp\left(\frac{E_v-E_F}{kT}\right) = N_v\exp(\frac{E_v-E_F}{kT})$$

$$n\times p = n_i{}^2 = N_c N_v \exp\left(\frac{E_v-E_c}{kT}\right) \Rightarrow n_i = \sqrt{N_c N_v}\exp(-\frac{E_g}{2kT})$$

$$n = N_c\exp(\frac{E_F-E_c}{kT}) \qquad p = N_v\exp(\frac{E_v-E_F}{kT})$$

$N_c \approx 10^{19}cm^{-3}$

$N_v \approx 10^{19}cm^{-3}$

$$n_0 = n_i\exp\left[\frac{E_F-E_{Fi}}{kT}\right] \qquad p_0 = n_i\exp\left[\frac{-(E_F-E_{Fi})}{kT}\right]$$

$n_i \approx 10^{10}cm^{-3}$

$$n = N_c\exp\left(\frac{E_{Fi}-E_c}{kT}\right) = p = N_v\exp(\frac{E_v-E_{Fi}}{kT})$$

$$E_{Fi} = \frac{1}{2}(E_c+E_v) + \frac{1}{2}kTln(\frac{N_v}{N_c})$$

$$E_{midgap} = \frac{1}{2}(E_c+E_v)$$

$$E_{Fi} = E_{midgap} + \frac{3}{4}kTln(\frac{m_p^*}{m_n^*})$$

$$n_d = N_d - N_d^+ = \frac{N_d}{1+\frac{1}{2}\exp(\frac{E_d-E_F}{kT})}$$

$$p_a = \frac{N_a}{1+\frac{1}{g}\exp\left(\frac{E_F-E_a}{kT}\right)} = N_a - N_a^-$$

$$n_0 + (N_a - p_a) = p_0 + (N_d - n_d)$$

$$n_0 = \frac{(N_d-N_a)}{2} + \sqrt{\left(\frac{N_d-N_a}{2}\right)^2 + n_i^2}$$

$$n_0 = \frac{N_d^+ + \sqrt{(N_d^+)^2 + 4n_i^2}}{2}\ (but\ N_d^+\ \text{unknown})$$

$$n_0 = N_c\times\frac{-1+\sqrt{1+\frac{8N_D}{N_c}\exp(\frac{E_A}{kT})}}{4\exp(\frac{E_A}{kT})} = \begin{cases}\sqrt{\frac{N_D N_c}{2}}\exp(-\frac{E_A}{2kT}) & partial\ ionization, \\ N_D & complete\ ionization\end{cases}$$

$$n_0 = \frac{N_D + \sqrt{N_D^2 + 4n_i^2}}{2} \quad \text{Complete ionization at high T}$$

Intrinsic

$$E_F = E_c + kTln(\frac{\sqrt{1+\frac{8N_D}{N_c}\exp(\frac{E_A}{kT})}-1}{4\exp(\frac{E_A}{kT})}) = \begin{cases}\frac{E_c+E_D}{2}+\frac{kT}{2}ln\frac{N_D}{2N_c} & T\ small \\ E_c - kTln\frac{N_c}{N_D} & T\ big\end{cases}$$

$$v_d \approx \left(\frac{q\tau_{cp}}{m_{cp}^*}\right)E \ \Rightarrow\ \frac{v_d}{E} = \frac{q\tau_{cp}}{m_{cp}^*} = \mu_p\ (for\ holes)$$

$$\frac{v_d}{E} = \frac{q\tau_{cn}}{m_{cn}^*} = \mu_n\ (for\ electrons)$$

$$I_{drf} = \frac{\Delta Q}{\Delta t} = \frac{qp_0A_c\Delta L}{\Delta t} = qp_0A_cv = qp_0A_c\mu_pE = qp_0A_c\mu_p\frac{V}{L} = \sigma\cdot V$$

$$J_{drf} = q(p_0\mu_p + n_0\mu_n)E \qquad \frac{1}{\mu} = \frac{1}{\mu_L} + \frac{1}{\mu_I}$$

$$v_n = \frac{v_s}{\left[1+\left(\frac{E_{on}}{E}\right)^2\right]^{1/2}} \qquad v_p = \frac{v_s}{\left[1+\left(\frac{E_{op}}{E}\right)^2\right]^{1/2}}$$

$$J_{nx|dif} = -qF_n = qD_n\frac{dn}{dx}$$

$$J_{px|dif} = qF_p = -qD_p\frac{dp}{dx}$$

$$J = qn\mu_nE_x + qp\mu_pE_x + qD_n\nabla n - qD_p\nabla p$$

$$E_x = -\frac{\mathrm{d}\Phi}{\mathrm{d}x} = \frac{1}{e}\frac{\mathrm{d}E_{Fi}}{\mathrm{d}x} = -\frac{1}{e}\frac{kT}{n(x)}\frac{\mathrm{d}n(x)}{\mathrm{d}x}$$

$$\boxed{D_n = \frac{\mu_n kT}{q}}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{q\mu_n n} = \frac{1}{q\mu_n N_d} \qquad \rho = \frac{1}{\sigma} = \frac{1}{q\mu_n p} = \frac{1}{q\mu_n N_a}$$

Infinite quantum well

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi = E\Psi, \quad \begin{cases} V(x) = +\infty, & x \le 0 \text{ or } x \ge a \\ V(x) = 0, & 0 < x < a \end{cases}$$

General solution:

$$\Psi(x) = Ae^{-ikx} + Be^{ikx}$$

Boundary condition:

$$\Psi(x)|_{x=a,0} = 0$$

$$\int_0^a \Psi(x)\Psi^*(x)\,\mathrm{d}x = 1$$

conclusion:

$$k = \frac{n\pi}{a}, n = 0, \pm 1, \pm 2, \dots$$

$$E = \frac{k^2\hbar^2}{2m} = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

Finite quantum well

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi = E\Psi, \quad \begin{cases} V(x) = V_0, & x \le 0 \text{ or } x \ge a \\ V(x) = 0, & 0 < x < a \end{cases}$$

General solution:

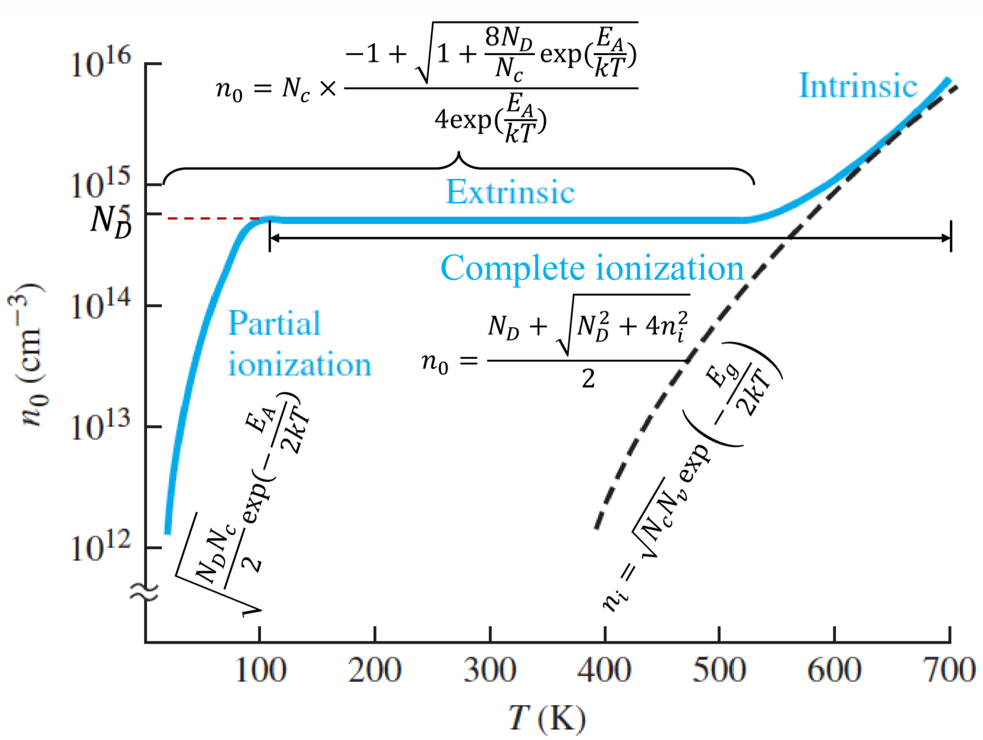
$$\Psi(x) = \begin{cases} Ae^{-ik_1x} + Be^{ik_1x}, & k_1 = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}, & x \le 0 \text{ or } x \ge a \\ Ce^{-ik_2x} + De^{ik_2x}, & k_2 = \sqrt{\frac{2mE}{\hbar^2}}, & 0 < x < a \end{cases}$$

Boundary condition:

$$\Psi(x)|_{x=0} \text{ continuous}$$

$$\Psi(x)|_{x=a} \text{ continuous}$$

$$\int_{-\infty}^{\infty} \Psi(x)\Psi^*(x)\,\mathrm{d}x = 1$$



From textbook
Semiconductor Physics and Devices: Basic Principles 4th edition. P716-718 (Appendix B)

Table B.2 | Conversion factors

	Prefixes		
1 Å (angstrom) $= 10^{-8}$ cm $= 10^{-10}$ m	10^{-15}	femto-	= f
1 μm (micrometer) $= 10^{-4}$ cm	10^{-12}	pico-	= p
1 mil $= 10^{-3}$ in. $= 25.4\ \mu$m	10^{-9}	nano-	= n
2.54 cm = 1 in.	10^{-6}	micro-	$= \mu$
1 eV $= 1.6 \times 10^{-19}$ J	10^{-3}	milli-	= m
1 J $= 10^7$ erg	10^{+3}	kilo-	= k
	10^{+6}	mega-	= M
	10^{+9}	giga-	= G
	10^{+12}	tera	= T

Table B.3 | Physical constants

Avogadro's number	$N_A = 6.02 \times 10^{+23}$ atoms per gram molecular weight
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ J/K $= 8.62 \times 10^{-5}$ eV/K
Electronic charge (magnitude)	$e = 1.60 \times 10^{-19}$ C
Free electron rest mass	$m_0 = 9.11 \times 10^{-31}$ kg
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7}$ H/m
Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-14}$ F/cm $= 8.85 \times 10^{-12}$ F/m
Planck's constant	$h = 6.625 \times 10^{-34}$ J-s $= 4.135 \times 10^{-15}$ eV-s $\frac{h}{2\pi} = \hbar = 1.054 \times 10^{-34}$ J-s
Proton rest mass	$M = 1.67 \times 10^{-27}$ kg
Speed of light in vacuum	$c = 2.998 \times 10^{10}$ cm/s
Thermal voltage ($T = 300$ K)	$V_t = \frac{kT}{e} = 0.0259$ V $kT = 0.0259$ eV

Table B.4 | Silicon, gallium arsenide, and germanium properties ($T = 300$ K)

Property	Si	GaAs	Ge
Atoms (cm^{-3})	5.0×10^{22}	4.42×10^{22}	4.42×10^{22}
Atomic weight	28.09	144.63	72.60
Crystal structure	Diamond	Zincblende	Diamond
Density (g/cm^3)	2.33	5.32	5.33
Lattice constant (Å)	5.43	5.65	5.65
Melting point (°C)	1415	1238	937
Dielectric constant	11.7	13.1	16.0
Bandgap energy (eV)	1.12	1.42	0.66
Electron affinity, χ (V)	4.01	4.07	4.13
Effective density of states in conduction band, N_c (cm^{-3})	2.8×10^{19}	4.7×10^{17}	1.04×10^{19}
Effective density of states in valence band, N_v (cm^{-3})	1.04×10^{19}	7.0×10^{18}	6.0×10^{18}
Intrinsic carrier concentration (cm^{-3})	1.5×10^{10}	1.8×10^{6}	2.4×10^{13}
Mobility (cm^2/V-s)			
Electron, μ_n	1350	8500	3900
Hole, μ_p	480	400	1900
Effective mass $\left(\frac{m^*}{m_0}\right)$			
Electrons	$m_l^* = 0.98$ $m_t^* = 0.19$	0.067	1.64 0.082
Holes	$m_{lh}^* = 0.16$ $m_{hh}^* = 0.49$	0.082 0.45	0.044 0.28
Density of states effective mass			
Electrons $\left(\frac{m_{dn}^*}{m_o}\right)$	1.08	0.067	0.55
Holes $\left(\frac{m_{dp}^*}{m_o}\right)$	0.56	0.48	0.37
Conductivity effective mass			
Electrons $\left(\frac{m_{cn}^*}{m_o}\right)$	0.26	0.067	0.12
Holes $\left(\frac{m_{cp}^*}{m_o}\right)$	0.37	0.34	0.21

Table B.5 | Other semiconductor parameters

Material	E_g (eV)	a (Å)	ϵ_r	χ	$\bar{n}$
Aluminum arsenide	2.16	5.66	12.0	3.5	2.97
Gallium phosphide	2.26	5.45	10	4.3	3.37
Aluminum phosphide	2.43	5.46	9.8		3.0
Indium phosphide	1.35	5.87	12.1	4.35	3.37

Table B.6 | Properties of SiO_2 and Si_3N_4 ($T = 300$ K)

Property	SiO_2	Si_3N_4
Crystal structure	[Amorphous for most integrated circuit applications]	
Atomic or molecular density (cm^{-3})	2.2×10^{22}	1.48×10^{22}
Density (g/cm^3)	2.2	3.4
Energy gap	≈ 9 eV	4.7 eV
Dielectric constant	3.9	7.5
Melting point (°C)	≈ 1700	≈ 1900