VE320 – Summer 2021

Introduction to Semiconductor Devices

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Chapter 7 The pn Junction

Outline

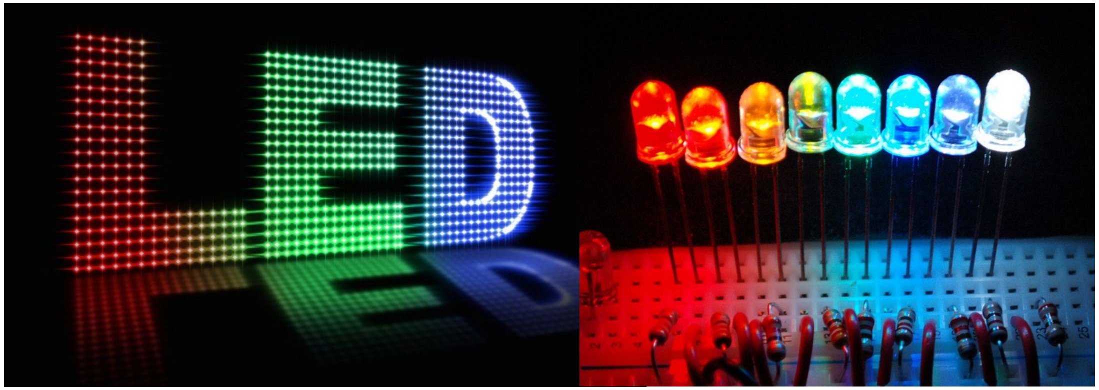
7.0 Introduction to semiconductor devices

7.1 Basic structure of the pn junction

7.2 Zero applied bias

7.3 Reverse applied bias

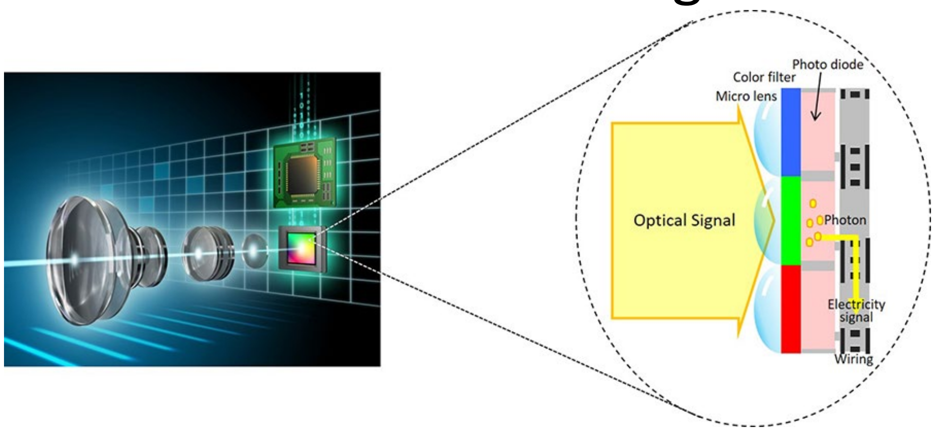
7.0 Introduction to semiconductor devices

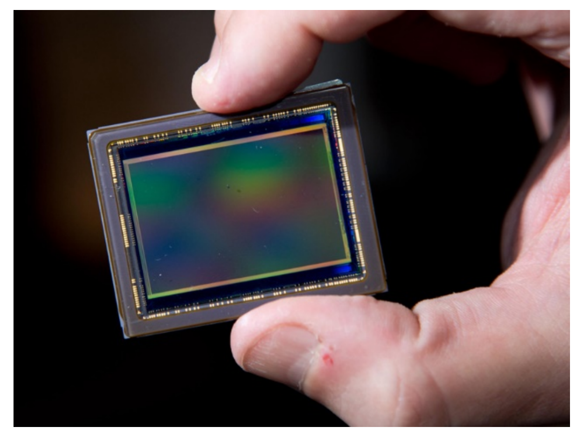


Light emitting diodes

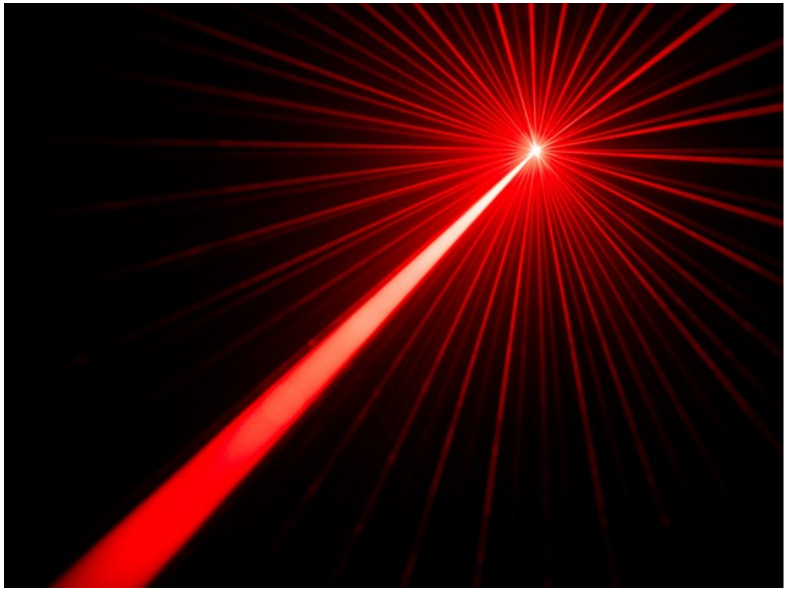
Cold light source

Photodetector: CMOS image sensor





7.0 Introduction to semiconductor devices



Semiconductor lasers

Solar cells

Outline

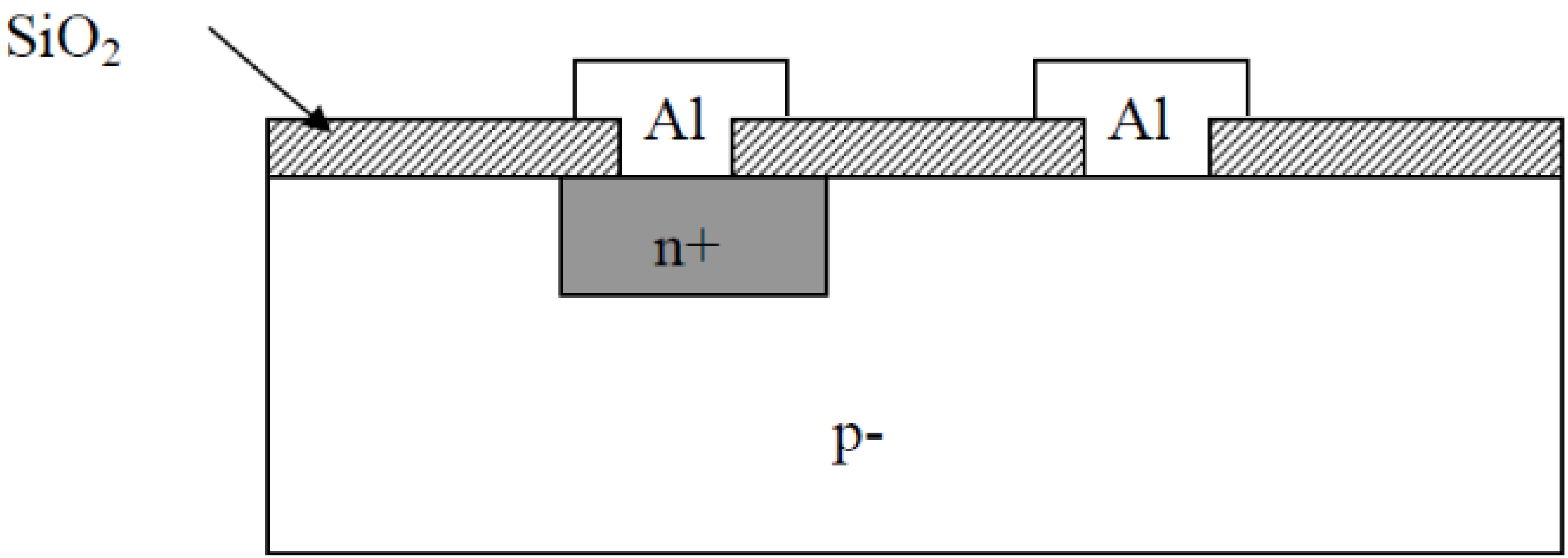
7.0 Introduction to semiconductor devices

7.1 Basic structure of the pn junction

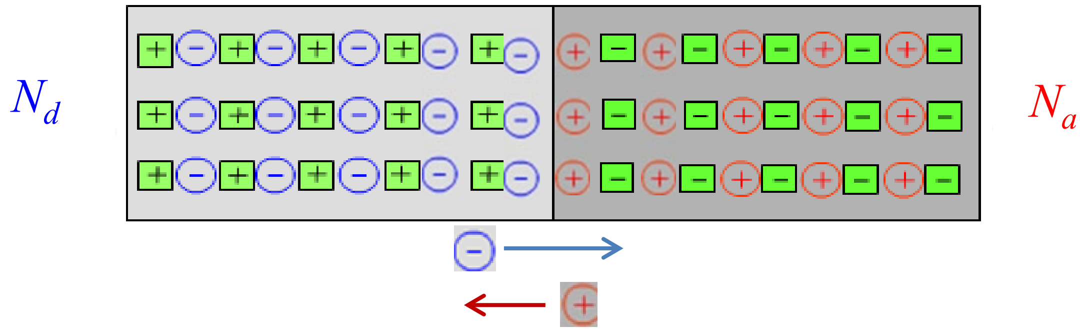
7.2 Zero applied bias

7.3 Reverse applied bias

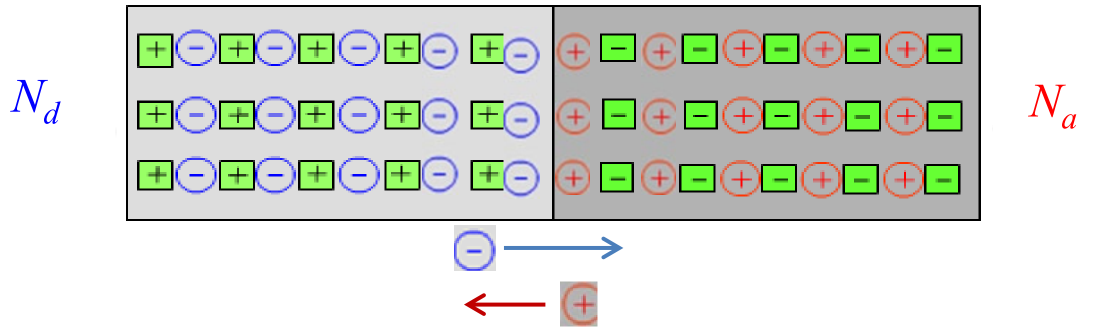
7.1 Basic structure of pn junction

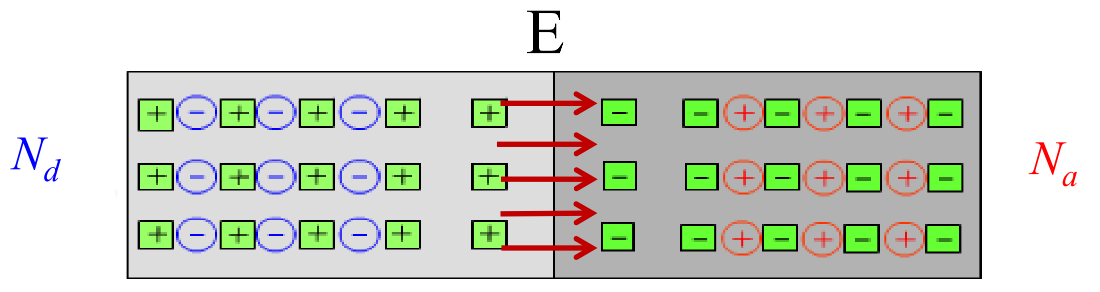


7.1 Basic structure of pn junction



7.1 Basic structure of pn junction





Outline

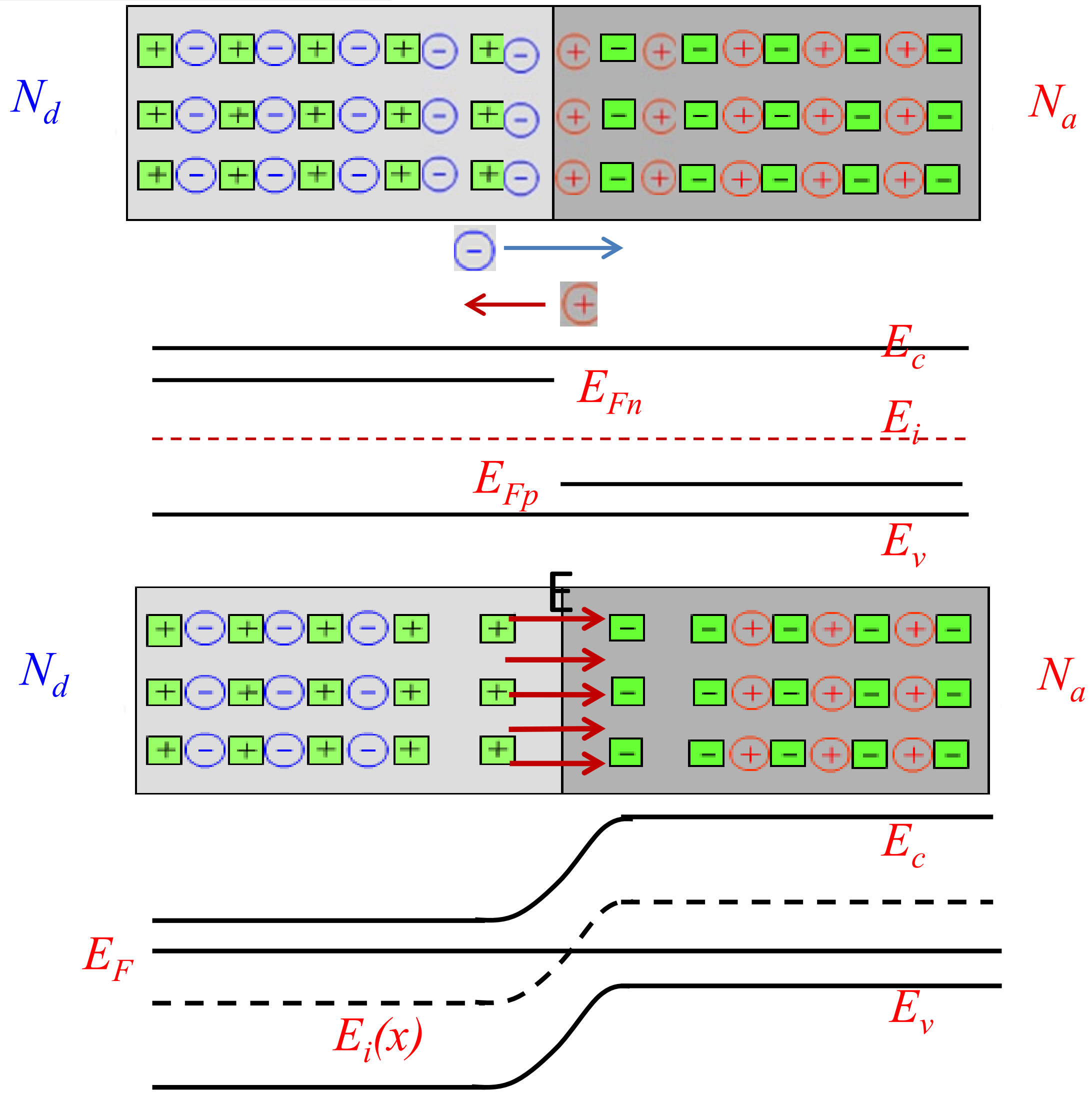
7.1 Basic structure of the pn junction

7.2 Zero applied bias

7.3 Reverse applied bias

7.2 Zero applied bias

Built-in potential barrier



7.2 Zero applied bias

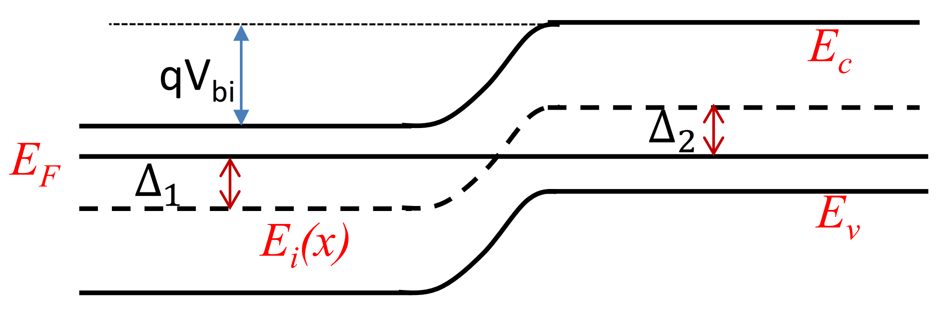
Built-in potential barrier

$$n_{n0} = n_i \exp\left(\frac{E_F - E_i}{kT}\right) = n_i \exp\left(\frac{q\Delta_1}{kT}\right)$$

$$p_{p0} = n_i \exp\left(\frac{E_i - E_F}{kT}\right) = n_i \exp\left(\frac{q\Delta_2}{kT}\right)$$

$$\Rightarrow V_{bi} = kTln\left(\frac{n_{n0}}{n_i}\right) + kTln\left(\frac{P_{p0}}{n_i}\right) = kTln\left(\frac{n_{n0}P_{p0}}{n_i^2}\right) = kTln\left(\frac{N_aN_d}{n_i^2}\right)$$

$$Example: N_a = 10^{17}cm^{-3}, N_d = 10^{17}cm^{-3}, \Rightarrow V_{bi} = 0.026/q * \ln\left(\frac{10^{17}10^{17}}{10^{20}}\right) = 0.84V$$



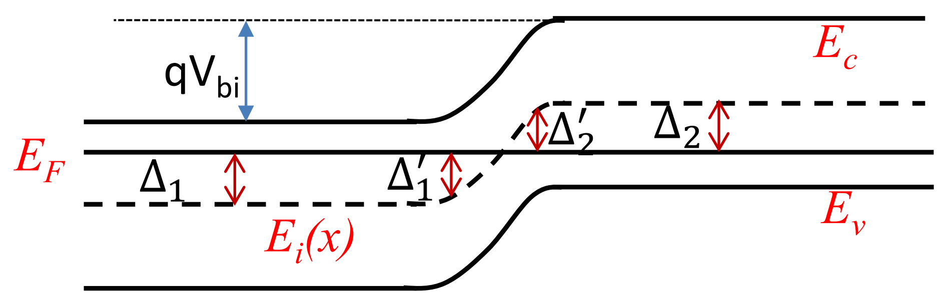
7.2 Zero applied bias

Charge carrier distribution

$$n = n_i \exp\left(\frac{E_F - E_i(x)}{kT}\right) = N_d exp\left(-\frac{0.1eV}{0.026eV}\right) \approx \frac{N_d}{50}$$

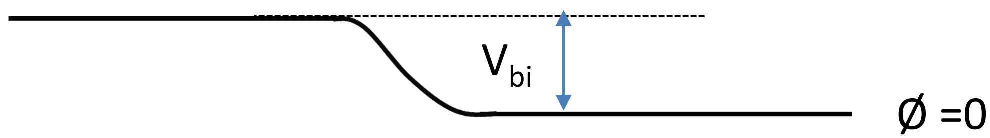
$$p = n_i \exp\left(\frac{E_i(x) - E_F}{kT}\right) = N_a exp\left(-\frac{0.1eV}{0.026eV}\right) \approx \frac{N_a}{50}$$

$\Delta_1 - \Delta_1' = 0.1eV$ $\Delta_2 - \Delta_2' = 0.1eV$

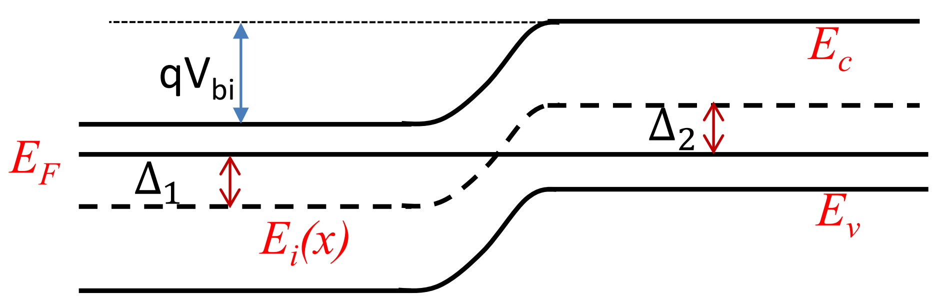


7.2 Zero applied bias

Potential profile



Potential profile

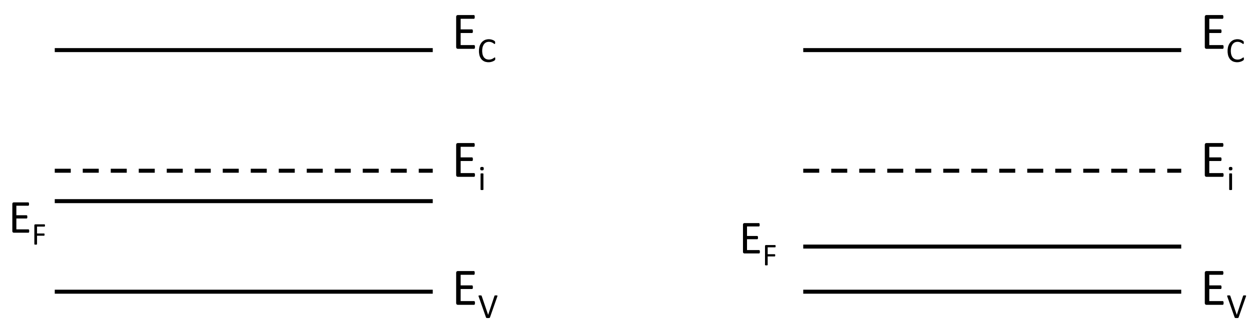


Energy band diagram

7.2 Zero applied bias

Problem Example #1

Two pieces of p-type silicon are in contact. The doping concentrations are 10^{16} cm^{-3} and 10^{18} cm^{-3}. Calculate the built-in potential between these two pieces of silicon and plot the energy band bending diagram.



7.2 Zero applied bias

<u>Poisson's equation</u>

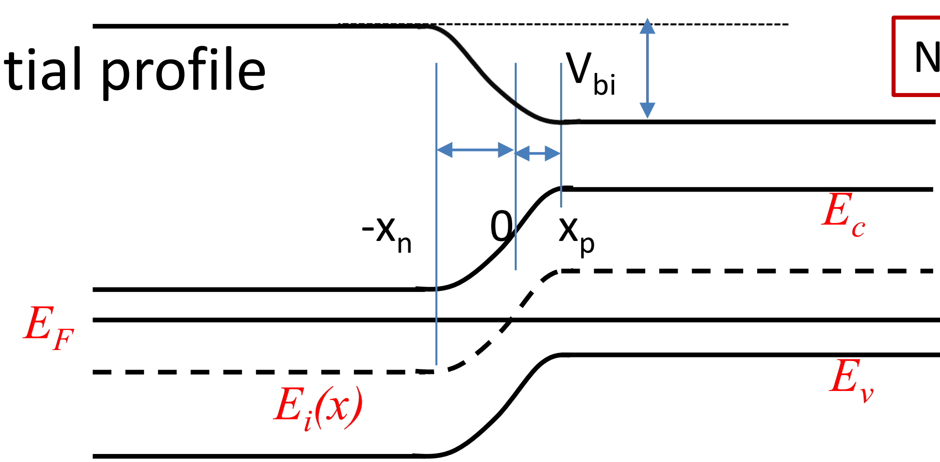
$$\frac{d^2V(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon}$$

$$= -\frac{q}{\varepsilon}[N_d\ (x) - N_a\ (x) + p(x) - n(x)]$$

$$= -\frac{q}{\varepsilon}[N_d\ (x) - N_a\ (x) + n_i \exp(\frac{E_i(x) - E_F}{kT}) - n_i \exp(\frac{E_F - E_i(x)}{kT})]$$

$$= -\frac{q}{\varepsilon}[N_d\ (x) - N_a\ (x) + n_i \exp(\frac{-qV(x) - E_F}{kT}) - n_i \exp(\frac{E_F + qV(x)}{kT})]$$

Potential profile



NO analytical solution

Ø =0

7.2 Zero applied bias

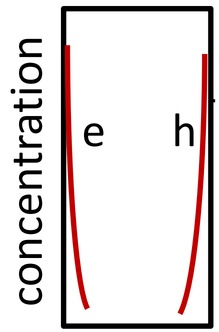
Poisson's equation

$$\frac{d^2V(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon}$$

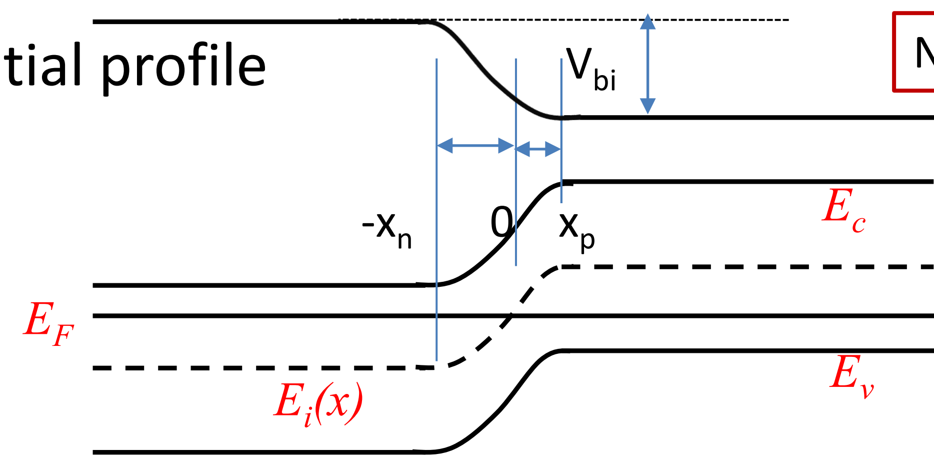
$$= -\frac{q}{\varepsilon}[N_d\ (x) - N_a\ (x) + p(x) - n(x)]$$

$$= -\frac{q}{\varepsilon}[N_d\ (x) \quad n_i \exp(\frac{E_i(x) - E_F}{kT}) \ - n_i \exp(\frac{E_F - E_i(x)}{kT})]$$

$$= -\frac{q}{\varepsilon}[N_d\ (x \quad + n_i \exp(\frac{-qV(x) - E_F}{kT}) \ - n_i \exp(\frac{E_F + qV(x)}{kT})]$$



Potential profile



NO analytical solution

Ø =0

7.2 Zero applied bias

Poisson's equation

Third time approximation

$$\frac{d^2V(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon}$$

$$= -\frac{q}{\varepsilon}[N_d\ (x) - N_a\ (x) + p(x) - n(x)]$$

$$= -\frac{q}{\varepsilon}[N_d\ (x) - N_a\ (x) + n_i\exp(\frac{E_i(x) - E_F}{kT})\ - n_i\exp(\frac{E_F - E_i(x)}{kT})]$$

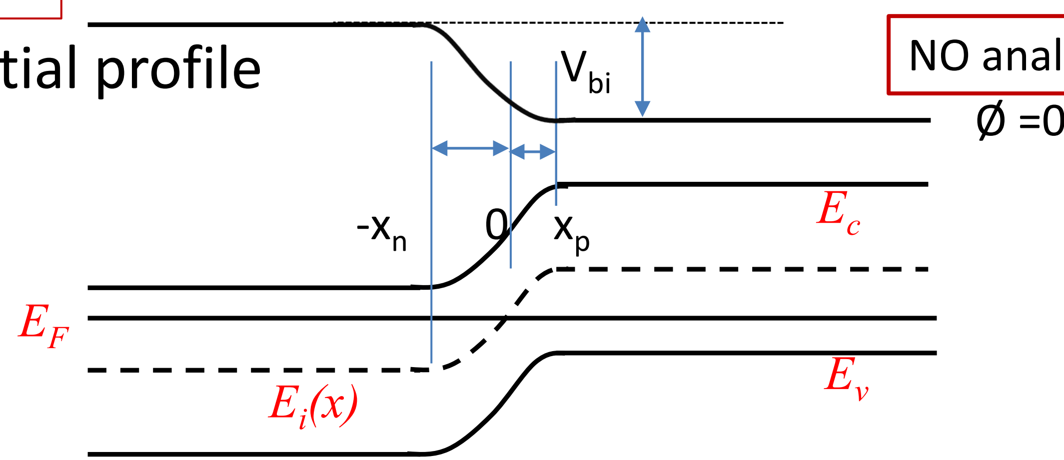
$-x_n \le x \le x_p$

Depletion region

$$= -\frac{q}{\varepsilon}[N_d\ (x) - N_a\ (x) + \cancelto{0}{n_i\exp(\frac{-qV(x) - E_F}{kT})}\ - \cancelto{0}{n_i\exp(\frac{E_F + qV(x)}{kT})}]$$

Potential profile

NO analytical solution



7.2 Zero applied bias

Poisson's equation

$$\frac{d^2V(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon}$$

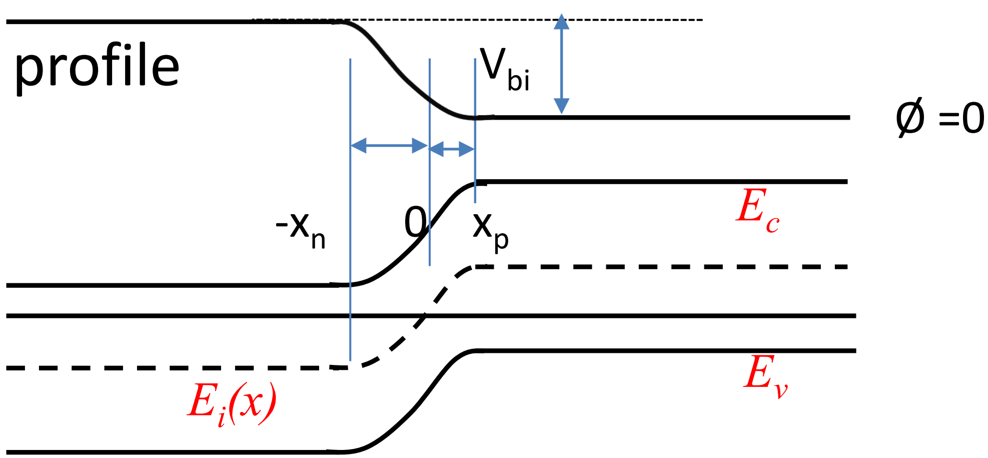
$$= -\frac{q}{\varepsilon}[N_d\ (x) - N_a\ (x) + p(x) - n(x)]$$

$$= -\frac{q}{\varepsilon}[N_d\ (x) - N_a\ (x) + n_i \exp(\frac{E_i(x) - E_F}{kT}) - n_i \exp(\frac{E_F - E_i(x)}{kT})]$$

$$\boxed{-x_n \le x \le x_p} = -\frac{q}{\varepsilon}[N_d\ (x) - N_a\ (x)] = \begin{cases} \frac{q}{\varepsilon} N_a & 0 \le x \le x_p \\ -\frac{q}{\varepsilon} N_d & -x_n \le x < 0 \end{cases}$$

Depletion region

Potential profile



7.2 Zero applied bias

$$\frac{d^2V(x)}{dx^2} = \begin{cases} \frac{q}{\varepsilon} N_a & 0 \le x \le x_p \\ -\frac{q}{\varepsilon} N_d & -x_p \le x < 0 \end{cases}$$

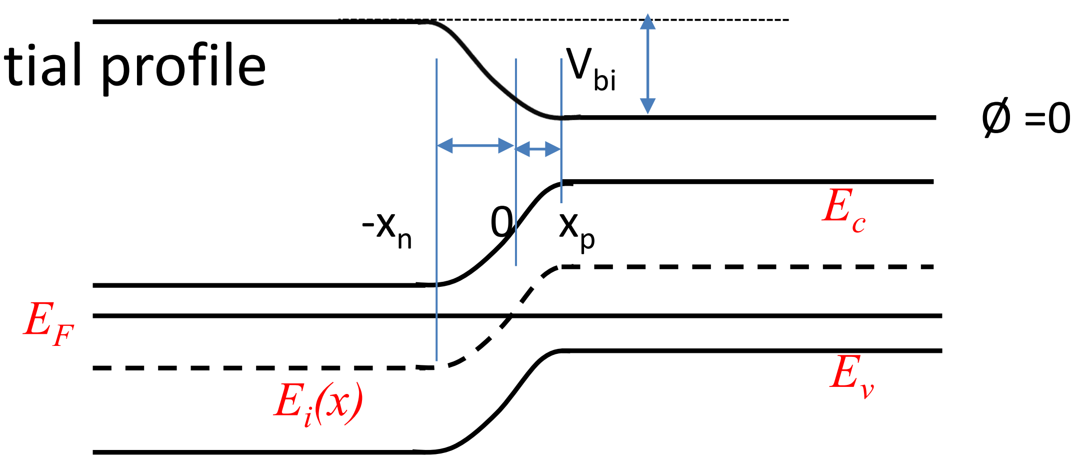
$$E = -\frac{dV(x)}{dx} = \begin{cases} -\frac{q}{\varepsilon} N_a \; x + A_1 & 0 \le x \le x_p \\ \frac{q}{\varepsilon} N_d \; x + A_2 & -x_n \le x < 0 \end{cases}$$

Boundary condition:

$$\boxed{E(x = x_p) = 0}$$

$$\boxed{E(x = -x_n) = 0}$$

Potential profile



7.2 Zero applied bias

$$\frac{d^2V(x)}{dx^2} = \begin{cases} \frac{q}{\varepsilon} N_a & 0 \le x \le x_p \\ -\frac{q}{\varepsilon} N_d & -x_n \le x < 0 \end{cases}$$

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Boundary condition:

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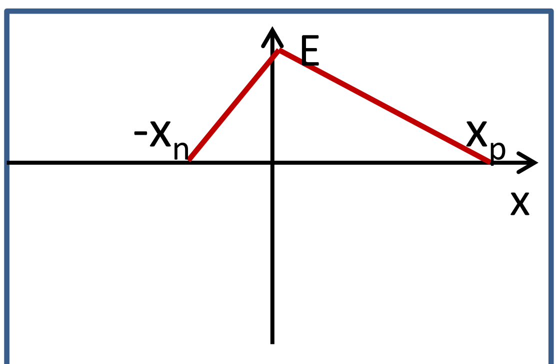
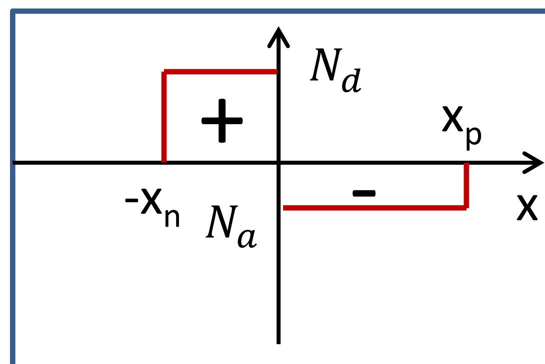
$$E = -\frac{dV(x)}{dx} = \begin{cases} -\frac{q}{\varepsilon} N_a\ x + \frac{q}{\varepsilon} N_a\ x_p & 0 \le x \le x_p \\ \frac{q}{\varepsilon} N_d\ x + \frac{q}{\varepsilon} N_d\ x_n & -x_n \le x < 0 \end{cases}$$

$$\boxed{x = 0 \Rightarrow N_a\ x_p = N_d\ x_n}$$





7.2 Zero applied bias

Space charge width

$$E = -\frac{dV(x)}{dx} = \begin{cases} -\frac{q}{\varepsilon}N_a\ x + \frac{q}{\varepsilon}N_a\ x_p & 0 \le x \le x_p \\ \frac{q}{\varepsilon}N_d\ x + \frac{q}{\varepsilon}N_d\ x_n & -\mathrm{x}_n \le x < 0 \end{cases}$$

$$V(x) = \begin{cases} \frac{q}{\varepsilon}N_a\ (\frac{1}{2}x^2 - x_p x + C_1) & 0 \le x \le x_p \\ -\frac{q}{\varepsilon}N_d\ (\frac{1}{2}x^2 + x_n x + C_2) & -x_n \le x < 0 \end{cases}$$

Boundary condition:

$$V(x = x_p) = 0 \ \Rightarrow \ C_1 = \frac{{x_p}^2}{2}$$

$$V(x = 0)\ is\ continuous$$

$$\Rightarrow \ C_2 = -\frac{N_a}{2N_d}{x_p}^2$$

$$V(x) = \begin{cases} \frac{q}{\varepsilon}N_a\ (\frac{1}{2}x^2 - x_p x + \frac{{x_p}^2}{2}) & 0 \le x \le x_p \\ -\frac{q}{\varepsilon}N_d\ (\frac{1}{2}x^2 + x_n x - \frac{N_a}{N_d}\frac{{x_p}^2}{2}) & -x_n \le x < 0 \end{cases}$$

7.2 Zero applied bias

Space charge width

$$\boxed{x = 0 \Rightarrow N_d\ x_n = N_a\ x_p}$$

$$\boxed{V(x = x_n) = V_{bi} = \frac{kT}{q} ln(\frac{N_d\ N_a}{n_i^2})}$$

$$V(x) = \begin{cases} \frac{q}{\varepsilon} N_a\ (\frac{1}{2}x^2 - x_p x + \frac{{x_p}^2}{2}) & 0 \le x \le x_p \\ -\frac{q}{\varepsilon} N_d\ (\frac{1}{2}x^2 + x_n x - \frac{N_a}{N_d}\frac{{x_p}^2}{2}) & -x_n \le x < 0 \end{cases}$$

$$\frac{q}{\varepsilon} N_d \left(\frac{1}{2}{x_n}^2 + \frac{N_a}{2N_d}{x_p}^2\right) = V_{bi}$$

$$x_p = \sqrt{\frac{2\varepsilon V_{bi}}{q}\frac{N_d}{N_a}\frac{1}{N_a + N_d}}$$

$$\boxed{N_d\ x_n = N_a\ x_p \Rightarrow x_n = \sqrt{\frac{2\varepsilon V_{bi}}{q}\frac{N_a}{N_d}\frac{1}{N_a + N_d}}}$$

7.2 Zero applied bias

Space charge width

$$x_p = \sqrt{\frac{2\varepsilon V_{bi}}{q}\frac{N_d}{N_a}\frac{1}{N_a + N_d}}$$

$$N_d\ x_n = N_a\ x_p \Rightarrow x_n = \sqrt{\frac{2\varepsilon V_{bi}}{q}\frac{N_a}{N_d}\frac{1}{N_a + N_d}}$$

$$W = x_p + x_n = \sqrt{\frac{2\varepsilon V_{bi}}{q}\frac{N_d + N_a}{N_a N_d}}$$

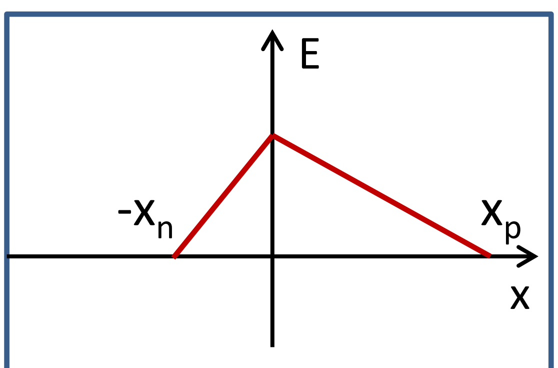
7.2 Zero applied bias

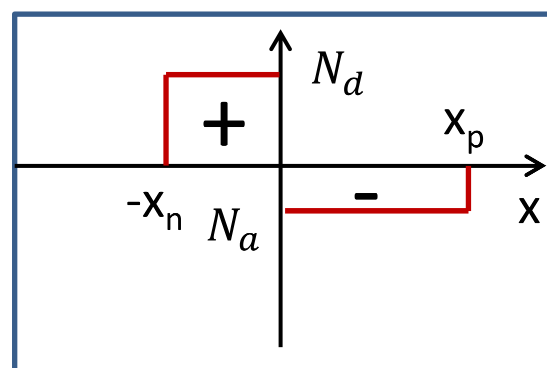
Space charge width

$$\boxed{x_p = \sqrt{\frac{2\varepsilon V_{bi}}{q}\frac{N_d}{N_a}\frac{1}{N_a+N_d}}}$$

$$\boxed{N_d\ x_n = N_a\ x_p \Rightarrow x_n = \sqrt{\frac{2\varepsilon V_{bi}}{q}\frac{N_a}{N_d}\frac{1}{N_a+N_d}}}$$

$$E = -\frac{dV(x)}{dx} = \begin{cases} -\frac{q}{\varepsilon}N_a\ x + \frac{q}{\varepsilon}N_a\ x_p & 0 \le x \le x_p \\ \frac{q}{\varepsilon}N_d\ x + \frac{q}{\varepsilon}N_d\ x_n & -x_n \le x < 0 \end{cases}$$





7.2 Zero applied bias

Problem Example #2

A silicon pn junction at T=300K with zero applied bias has doping concentration of $N_d = 5 \times 10^{16}$ cm^{-3} and $N_a = 5 \times 10^{15}$ cm^{-3}. Determine x_n, x_p, W and $|E_{max}|$.

Outline

7.1 Basic structure of the pn junction

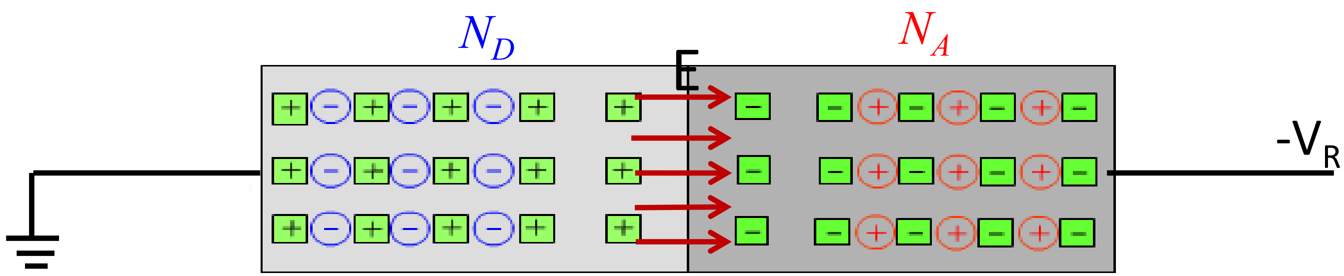
7.2 Zero applied bias

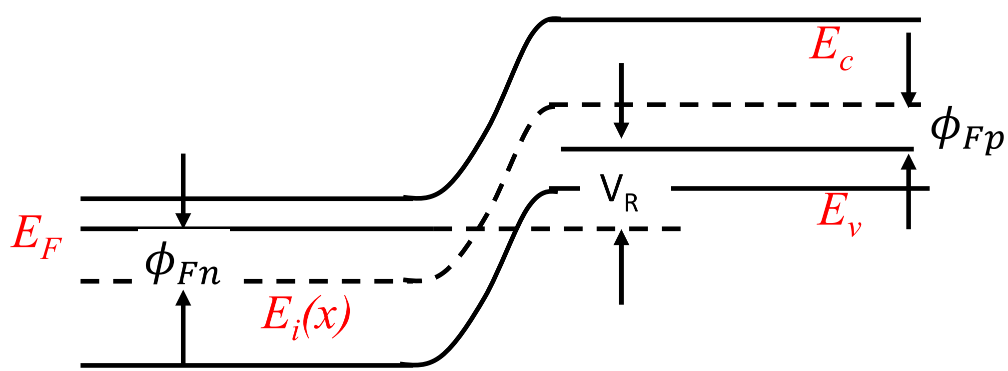
7.3 Reverse applied bias

7.3 Reverse applied bias

Space charge width and electric field

$$V_{\text{total}} = |\phi_{Fn}| + |\phi_{Fp}| + V_R$$





7.3 Reverse applied bias

Space charge width and electric field

$$x_p = \sqrt{\frac{2\varepsilon(V_{bi}+V_R)}{q}\frac{N_d}{N_a}\frac{1}{N_a+N_d}}$$

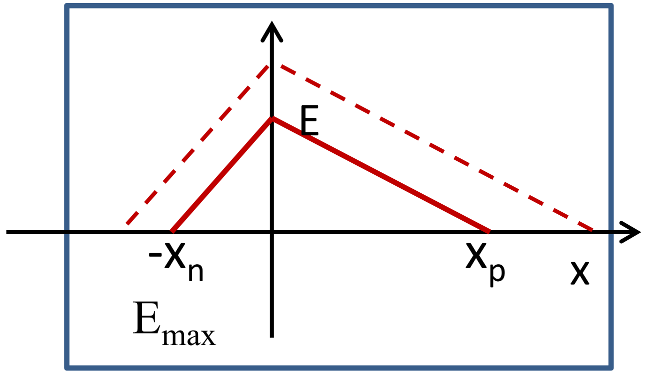
$$N_a^- x_n = N_d^+ x_p \Rightarrow x_n = \sqrt{\frac{2\varepsilon(V_{bi}+V_R)}{q}\frac{N_a}{N_d}\frac{1}{N_a+N_d}}$$

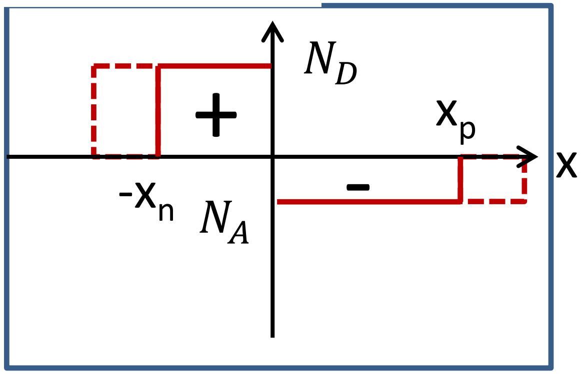
$$W = \sqrt{\frac{2\varepsilon(V_{bi}+V_R)}{q}\frac{N_d+N_a}{N_aN_d}}$$

$$E = -\frac{dV(x)}{dx} = \begin{cases} -\frac{q}{\varepsilon}N_a\ x + \frac{q}{\varepsilon}N_a\ x_p & 0 \le x \le x_p \\ \frac{q}{\varepsilon}N_d\ x + \frac{q}{\varepsilon}N_d\ x_n & -x_n \le x < 0 \end{cases}$$

$$E_{max} = \left\{\frac{2e(V_{bi}+V_R)}{\epsilon_s}\left(\frac{N_aN_d}{N_a+N_d}\right)\right\}^{1/2}$$

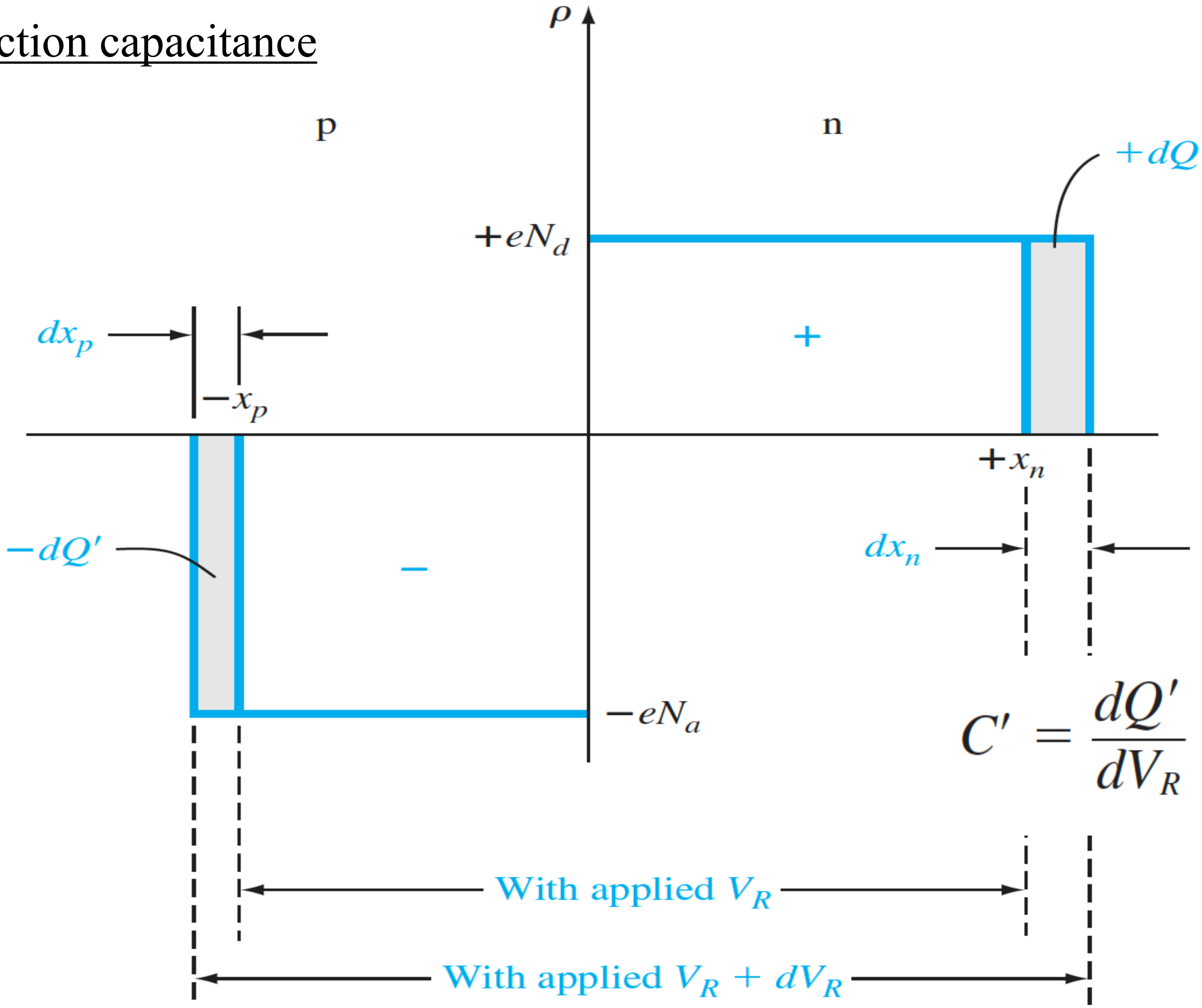
$$E_{max} = \frac{2(V_{bi}+V_R)}{W}$$





7.3 Reverse applied bias

Junction capacitance



$$C' = \frac{dQ'}{dV_R}$$

7.3 Reverse applied bias

Junction capacitance

$$C' = \frac{dQ'}{dV_R}$$

$$dQ' = eN_d\,dx_n = eN_a\,dx_p$$

$$x_n = \left\{\frac{2\epsilon_s(V_{bi} + V_R)}{e}\left[\frac{N_a}{N_d}\right]\left[\frac{1}{N_a + N_d}\right]\right\}^{1/2}$$

$$\boxed{C' = \frac{dQ}{dV_\mathrm{R}}\Big|_{V_R=V_{R_0}} = qN_d\frac{db}{dV_R}\Big|_{V_R=V_{R_0}} = \sqrt{\frac{q\varepsilon}{2(V_{bi}+V_{R_0})}\frac{N_dN_a}{N_a+N_d}} = \frac{\varepsilon}{W}}$$

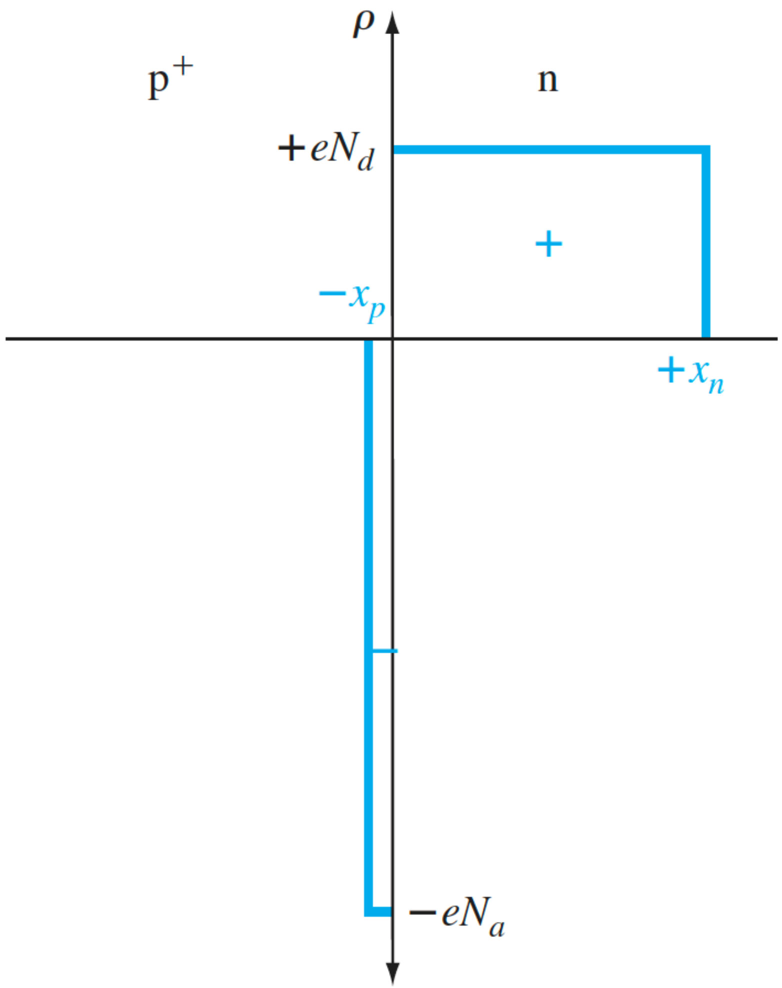
7.3 Reverse applied bias

Problem Example #3

Consider a GaAs pn junction at T = 300K doped to $N_a = 5 \times 10^{15}$ cm^{-3} and $N_d = 2 \times 10^{16}$ cm^{-3}. (a) Calculate V_{bi}. (b) Determine the junction capacitance C' for V_R =4V.

7.3 Reverse applied bias

One-sided junction



$$W = \sqrt{\frac{2\varepsilon(V_{bi}+V_R)}{q}\frac{N_d+N_a}{N_aN_d}}$$

$$\Big\downarrow \; N_a \to \infty$$

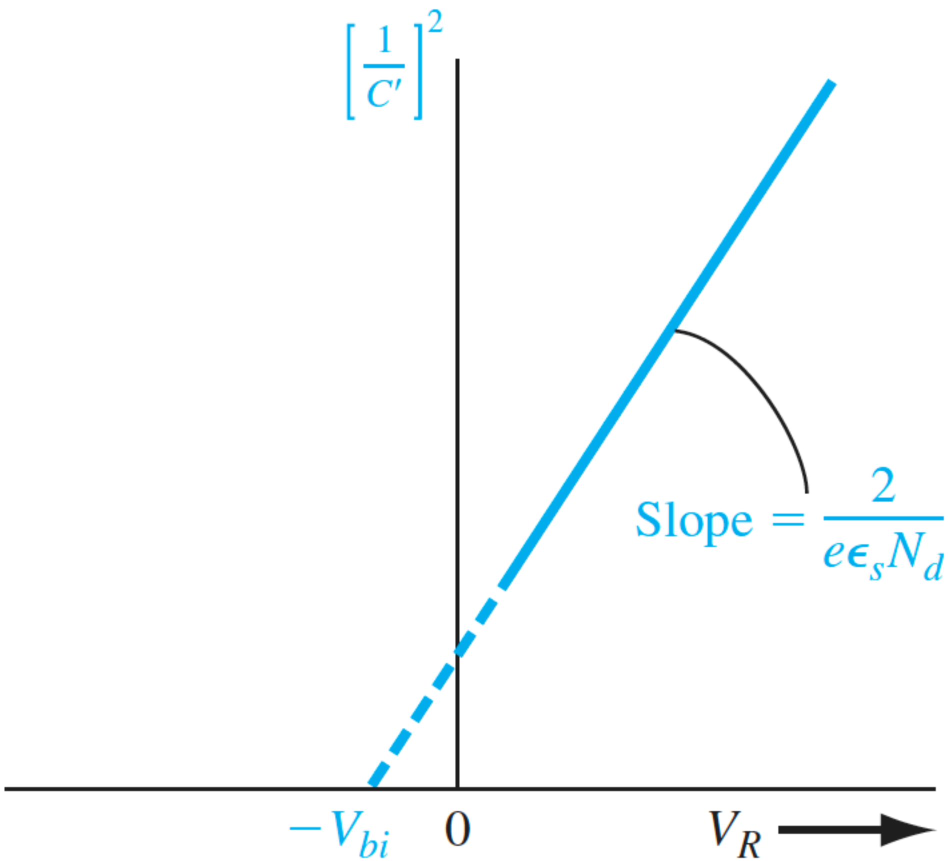
$$W = \sqrt{\frac{2\varepsilon(V_{bi}+V_R)}{q}\frac{1}{N_d}} \approx x_n$$

$$\Big\downarrow$$

$$C' = \frac{\varepsilon}{W} = \sqrt{\frac{q\varepsilon N_d}{2\,(V_{bi}+V_R)}}$$

7.3 Reverse applied bias

One-sided junction



$$\boxed{C' = \frac{\varepsilon}{W} = \sqrt{\frac{q\varepsilon N_d}{2\,(V_{bi} + V_R)}}}$$

$$\downarrow$$

$$\boxed{\frac{1}{C'^2} = \frac{2\,(V_{bi} + V_R)}{q\varepsilon N_d}}$$