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2.(a). (i).

$$g_c(E) = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E-E_c}$$

$$N = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\int_{E_c}^{E_c+2kT}\sqrt{E-E_c}\,dE$$

$$= \frac{4\pi(2m_n^*)^{3/2}}{h^3}\cdot\frac{2}{3}\cdot(E-E_c)^{\frac{3}{2}}\Big|_{E_c}^{E_c+2kT}$$

$$= \frac{4\pi(2m_n^*)^{3/2}}{h^3}\cdot\frac{2}{3}\cdot(2kT)^{\frac{3}{2}}$$

$$= \frac{4\pi(2\times1.08\times(9.11\times10^{-31}))^{3/2}}{(6.625\times10^{-34})^3}\cdot\frac{2}{3}\cdot(2\times0.0259\times1.6\times10^{-19})^{3/2}$$

$$= 6.00\times10^{25}\ m^{-3} = 6.00\times10^{19}\ cm^{-3}$$

(ii).

$$N = \frac{4\pi(2\times1.08\times(9.11\times10^{-31}))^{3/2}}{(6.625\times10^{-34})^3}\cdot\frac{2}{3}\cdot(2\times1.38\times10^{-23}\times400)^{3/2}$$

$$= 9.23\times10^{25}\ m^{-3} = 9.23\times10^{19}\ cm^{-3}$$

(b).(i).

$$N = \frac{4\pi(2\times0.067\times(9.11\times10^{-31}))^{3/2}}{(6.625\times10^{-34})^3}\cdot\frac{2}{3}\cdot(2\times0.0259\times1.6\times10^{-19})^{3/2}$$

$$= 9.27\times10^{23}\ m^{-3} = 9.27\times10^{17}\ cm^{-3}$$

(ii).

$$N = \frac{4\pi(2\times0.067\times(9.11\times10^{-31}))^{3/2}}{(6.625\times10^{-34})^3}\cdot\frac{2}{3}\cdot(2\times1.38\times10^{-23}\times400)^{3/2}$$

$$= 1.43\times10^{24}\ m^{-3} = 1.43\times10^{18}\ cm^{-3}$$

3. (a). $$\frac{g_c(E)}{g_v(E)} = \frac{(2m^*_n)^{3/2}}{(2m^*_p)^{3/2}} = \left(\frac{m^*_n}{m^*_p}\right)^{3/2} = \left(\frac{1.08}{0.56}\right)^{3/2} = 2.68$$

(b). $$\frac{g_c(E)}{g_v(E)} = \left(\frac{m^*_n}{m^*_p}\right)^{3/2} = \left(\frac{0.067}{0.48}\right)^{3/2} = 0.052$$

4. (a). $$f_F(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)} = \frac{1}{1+\exp\left(\frac{kT}{kT}\right)} = 0.269$$

(b). $$f_F(E) = \frac{1}{1+\exp\left(\frac{5kT}{kT}\right)} = 6.69\times10^{-3}$$

(c). $$f_F(E) = \frac{1}{1+\exp\left(\frac{10kT}{kT}\right)} = 4.54\times10^{-5}$$

5. $$\frac{1}{1+\exp\left(\frac{E_c+kT-E_F}{kT}\right)} = 1 - \frac{1}{1+\exp\left(\frac{E_v-kT-E_F}{kT}\right)}$$

$$\frac{1}{1+\exp\left(\frac{E_c+kT-E_F}{kT}\right)} = \frac{1}{1+\exp\left(\frac{E_F-E_v+kT}{kT}\right)}$$

Therefore, $E_c + kT - E_F = E_F - E_v + kT$

$$E_F = \frac{1}{2}(E_c + E_v)$$

6. (a). $$f_F(E) = \frac{1}{1+\exp\left(\frac{5.8-5.5}{0.0259}\right)} = 9.32\times10^{-6}$$

(b). $$f_F(E) = \frac{1}{1+\exp\left(\frac{0.3}{8.62\times10^{-5}\times700}\right)} = 6.9\times10^{-3}$$

(c).

$$f_F(E) = 1 - \frac{1}{1+\exp\left(\frac{-0.25}{kT}\right)} = 0.02$$

$$\exp\left(\frac{-0.25}{kT}\right) = 0.0204$$

$$\frac{-0.25}{8.62\times10^{-5}\times T} = \ln(0.0204) \Rightarrow T = 745.21\ K$$

7. (a). $$f_F(E) = \frac{1}{1+\exp\left(\frac{0.6}{kT}\right)} = 10^{-8}$$

$$\exp\left(\frac{0.6}{kT}\right) = 10^{8}-1$$

$$\frac{0.6}{8.62\times10^{-5}\times T} = \ln(10^{8}-1) \Rightarrow T = 377.87\ K$$

(b). $$\exp\left(\frac{0.6}{kT}\right) = 10^{6}-1$$

$$\frac{0.6}{8.62\times10^{-5}\times T} = \ln(10^{6}-1) \Rightarrow T = 503.82\ K$$