VE320 – Summer 2021

Introduction to Semiconductor Devices

Instructor: Yaping Dan （但亚平）
yaping.dan@sjtu.edu.cn

Chapter 5 Carrier Transport Phenomena

Outline

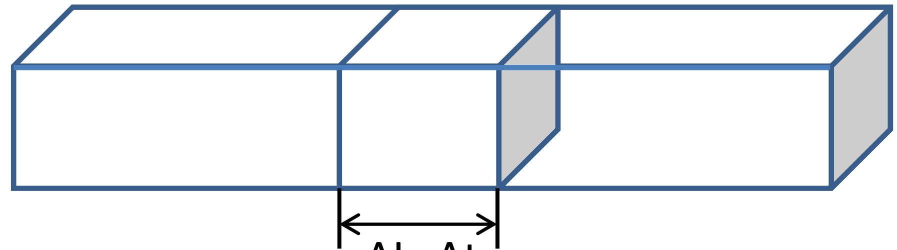
5.1 Carrier drift

5.2 Carrier diffusion

5.3 Graded impurity distribution

5.1 Carrier drift

Drift current density



$for\ p\ type\ semiconductor, p_0 \gg n_0$

Drift current

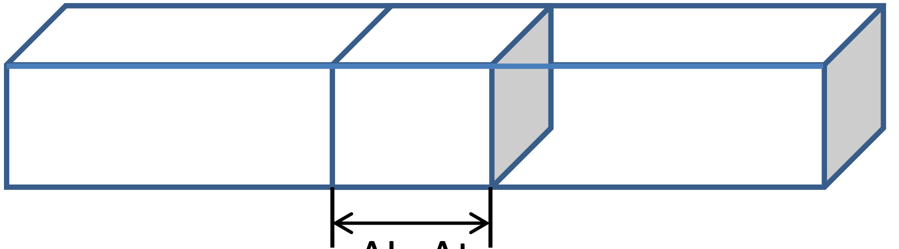
$$I_{drf} = \frac{\Delta Q}{\Delta t} = \frac{q p_0 A_c \Delta L}{\Delta t} = q p_0 A_c v_d$$

$\rho: charge\ density$

$$I_{drf} = \frac{\Delta Q}{\Delta t} = \frac{q p_0 \Delta L A_c}{\Delta t} = p_0 q v_d A_c$$

5.1 Carrier drift

<ins>Drift current density</ins>



$for\ p\ type\ semiconductor, p_0 \gg n_0$

Drift current

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$\rho: charge\ density$

$$I_{drf} = \frac{\Delta Q}{\Delta t} = \frac{q p_0 \Delta L A_c}{\Delta t} = p_0 q v_d A_c$$

$$\mathrm{L} = \frac{1}{2} a t^2 \quad \rightarrow t = \sqrt{2L/a}$$

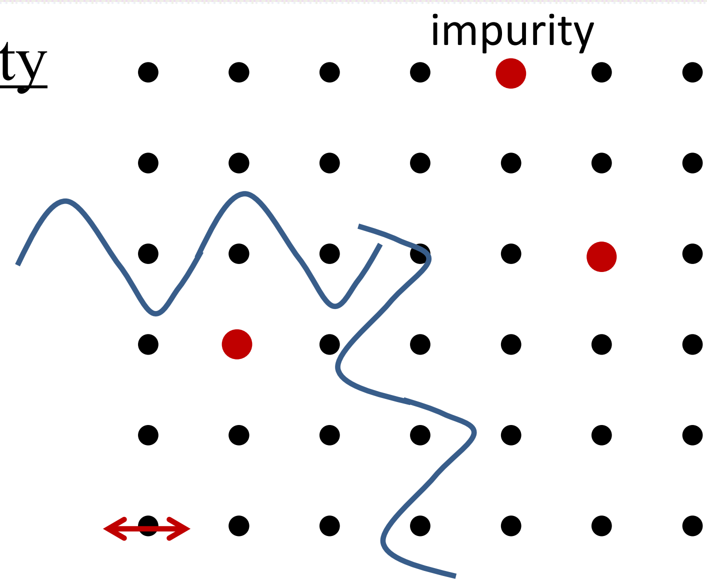
$$\left.\begin{array}{r} \rightarrow \quad vd = at = \sqrt{2La} = \sqrt{2LqE/m_{cp}^*} \\ E = V/L \end{array}\right\} \rightarrow vd = \sqrt{2qV/m_{cp}^*}$$

$$\therefore I_{drf} = q p_0 \sqrt{2qV/m_{cp}^*} A_c$$

$\boxed{However, Ohm's\ Law\ tells\ us: I = \sigma \cdot V}$

5.1 Carrier drift

Drift current density



Resistor heating up by current

- Thermal vibration of lattice ←→ phonon
- Impurity scattering

$$F = m_{cp}^{*}\frac{dv}{dt} = qE \quad \Rightarrow \quad v = \frac{qEt}{m_{cp}^{*}} \quad \text{if the initial drift velocity is zero}$$

$$\text{But the scattering is a random process}$$

$$\Rightarrow \text{the mean time between collisions: } \tau_{cp}$$

5.1 Carrier drift

Drift current density

$$v_d \approx \left(\frac{q\tau_{cp}}{m_{cp}^*}\right)E \quad \Rightarrow \quad \frac{v_d}{E} = \frac{q\tau_{cp}}{m_{cp}^*} = \mu_p \; (for\ holes)$$

$$\frac{v_d}{E} = \frac{q\tau_{cn}}{m_{cn}^*} = \mu_n \; (for\ electrons)$$

$$I_{drf} = \frac{\Delta Q}{\Delta t} = \frac{qp_0 A_c \Delta L}{\Delta t} = qp_0 A_c v = qp_0 A_c \mu_p E = qp_0 A_c \mu_p \frac{V}{L} = \sigma \cdot V$$

5.1 Carrier drift

Drift current density

Hole drift current

$$J_{p|drf} = qp_0\mu_p E$$

Electron drift current

$$J_{n|drf} = qn_0\mu_n E$$

Both electrons and holes contribute to current:

$$J_{drf} = q(p_0\mu_p + n_0\mu_n)E$$

Table 5.1 | Typical mobility values at $T = 300$ K and low doping concentrations

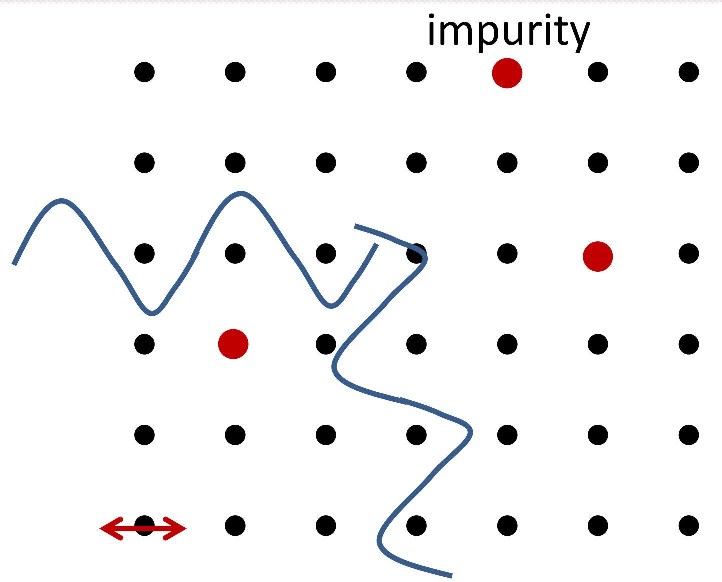
	μ_n (cm²/V-s)	μ_p (cm²/V-s)
Silicon	1350	480
Gallium arsenide	8500	400
Germanium	3900	1900

5.1 Carrier drift

Mobility effect

$$\frac{v_d}{E} = \frac{q\tau_{cp}}{m_{cp}^*} = \mu_p$$

$$\frac{v_d}{E} = \frac{q\tau_{cn}}{m_{cn}^*} = \mu_n$$



Resistor heating up by current

- Thermal vibration of lattice ←→ phonon

$$Lattice\ scatterings\ shorten\ \tau_{cp} \Rightarrow\ \mu_L \propto\ T^{-3/2}$$

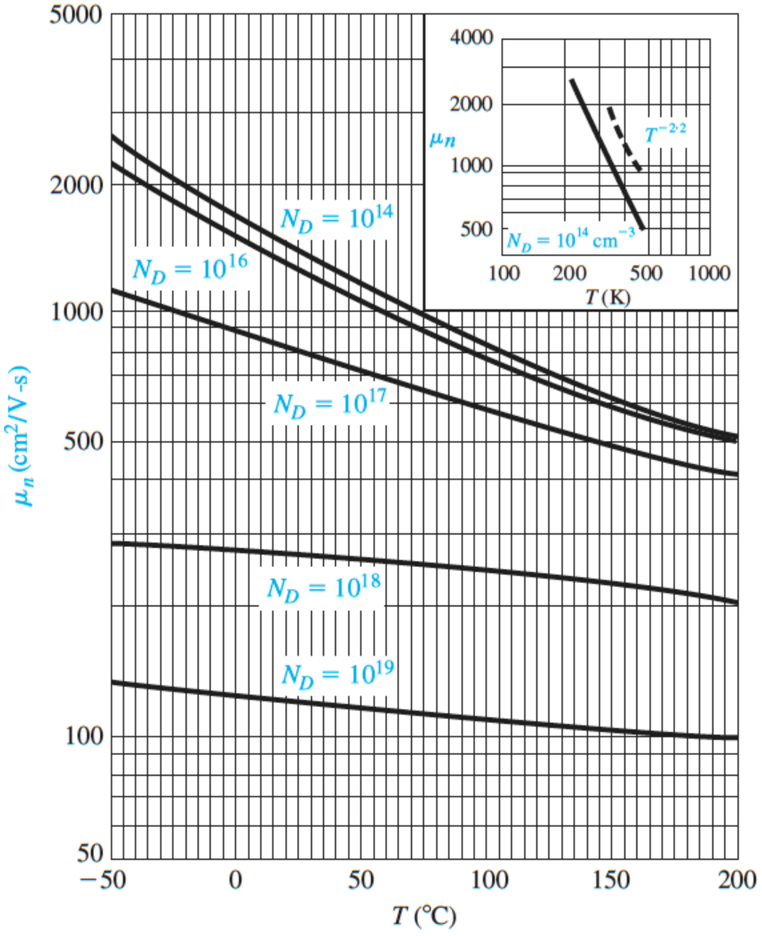
- Impurity scatterings

$$Impurity\ scatterings\ shorten\ \tau_{cp} \Rightarrow\ \mu_I \propto \frac{T^{3/2}}{N_d^+ + N_a^-}$$

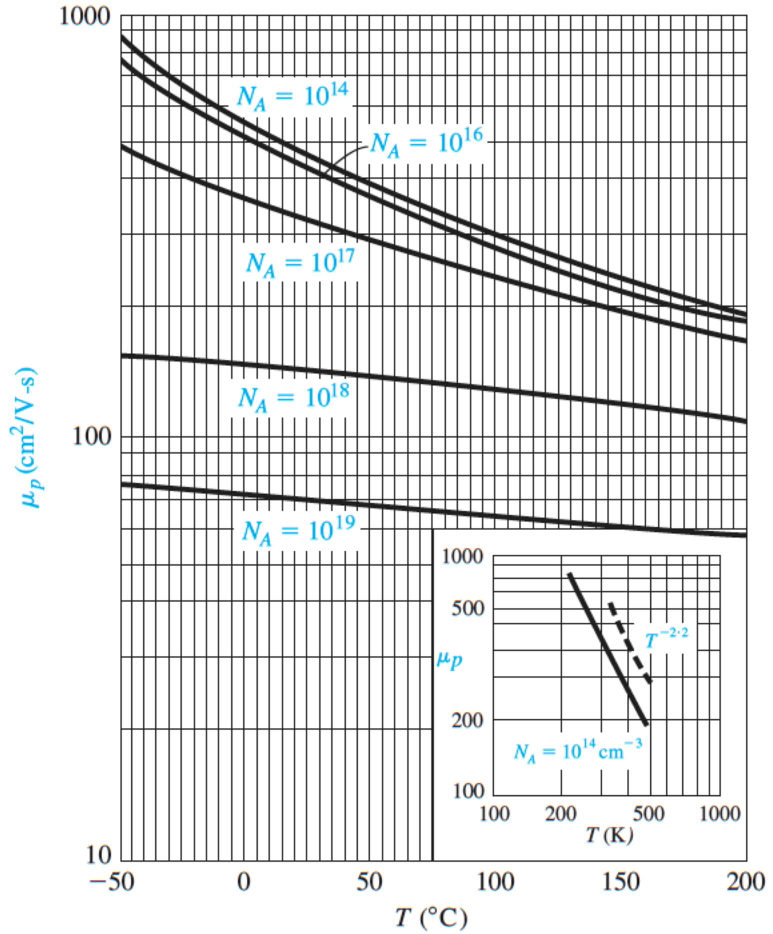
$$\frac{1}{\mu} = \frac{1}{\mu_L} + \frac{1}{\mu_I}$$

5.1 Carrier drift

Mobility effect



(a)



(b)

5.1 Carrier drift

Conductivity

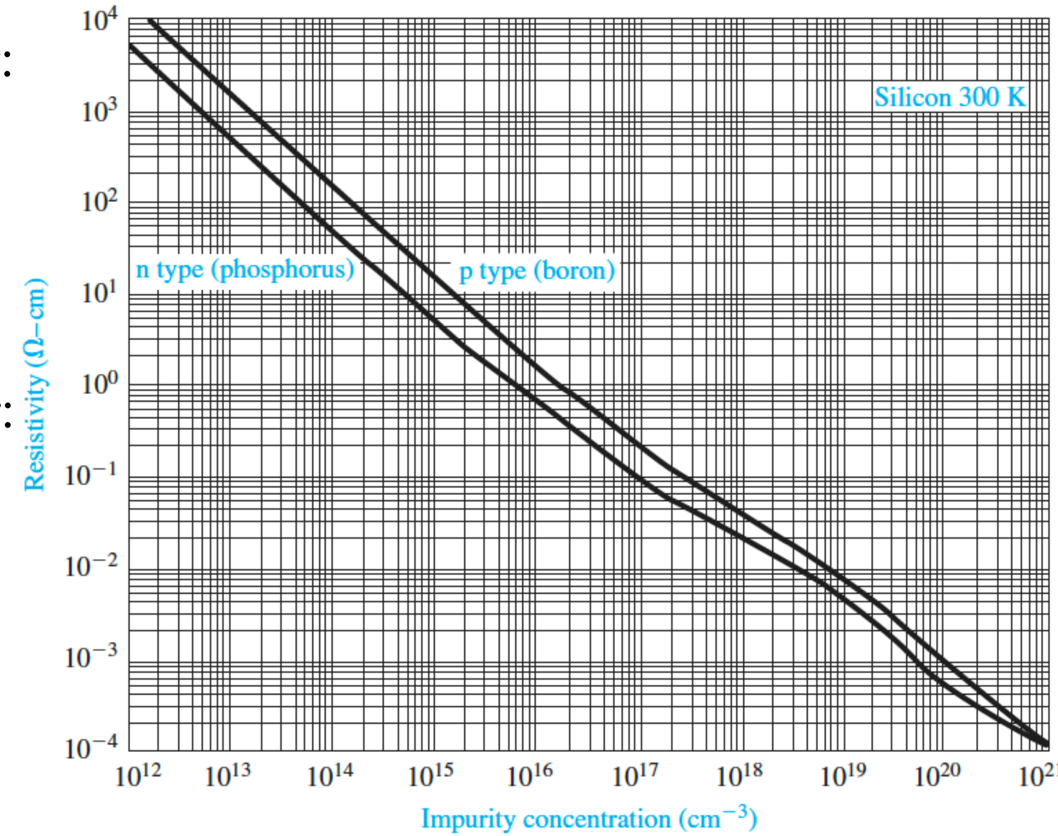
$$J_{drf} = q(p_0\mu_p + n_0\mu_n)E \;\Rightarrow\; \rho = \frac{1}{\sigma} = \frac{1}{q(\mu_n n + \mu_p p)}$$

For n-type doped semiconductor:

$$\rho = \frac{1}{\sigma} = \frac{1}{q\mu_n n} = \frac{1}{q\mu_n N_d}$$

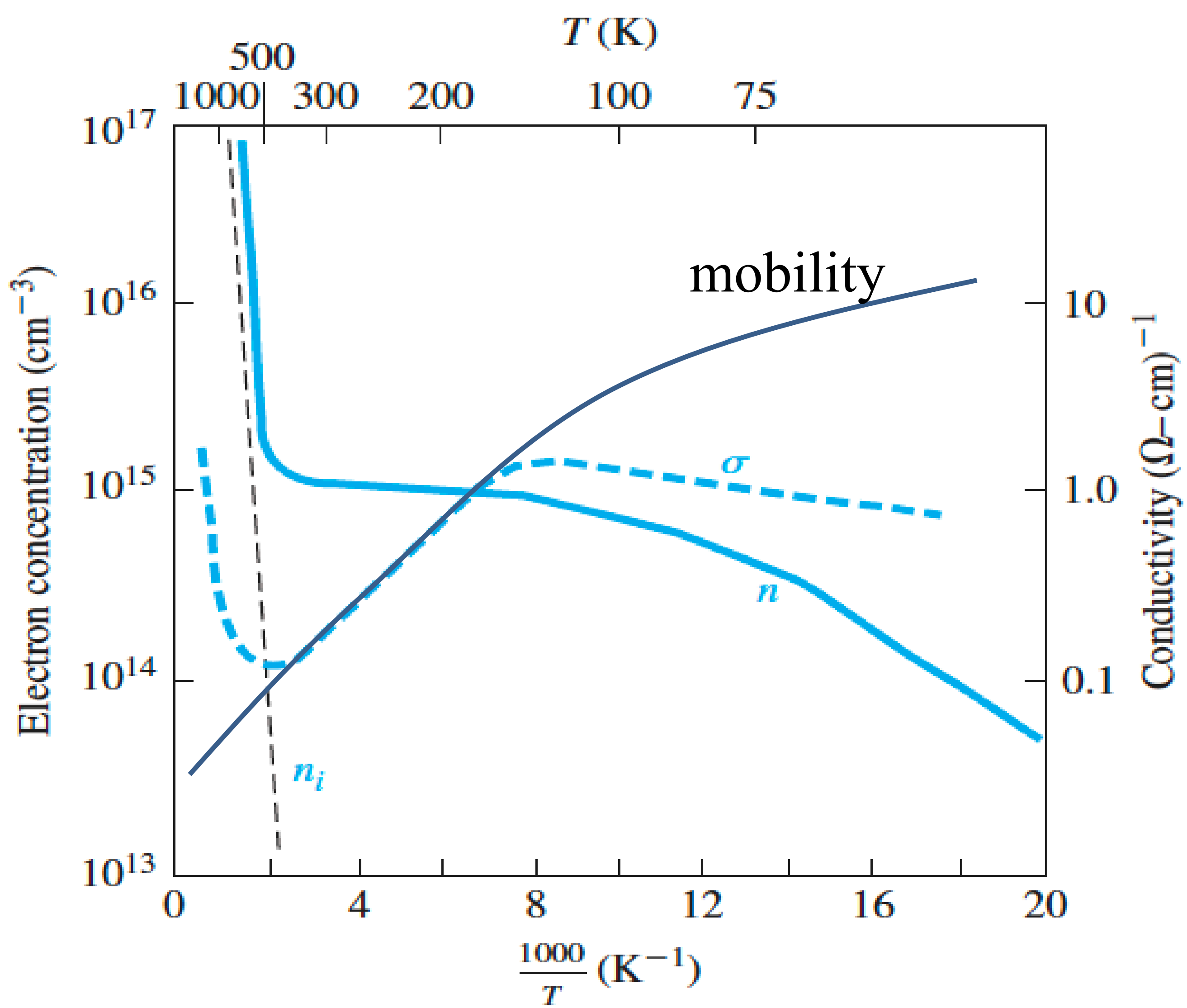
For p-type doped semiconductor:

$$\rho = \frac{1}{\sigma} = \frac{1}{q\mu_n p} = \frac{1}{q\mu_n N_a}$$

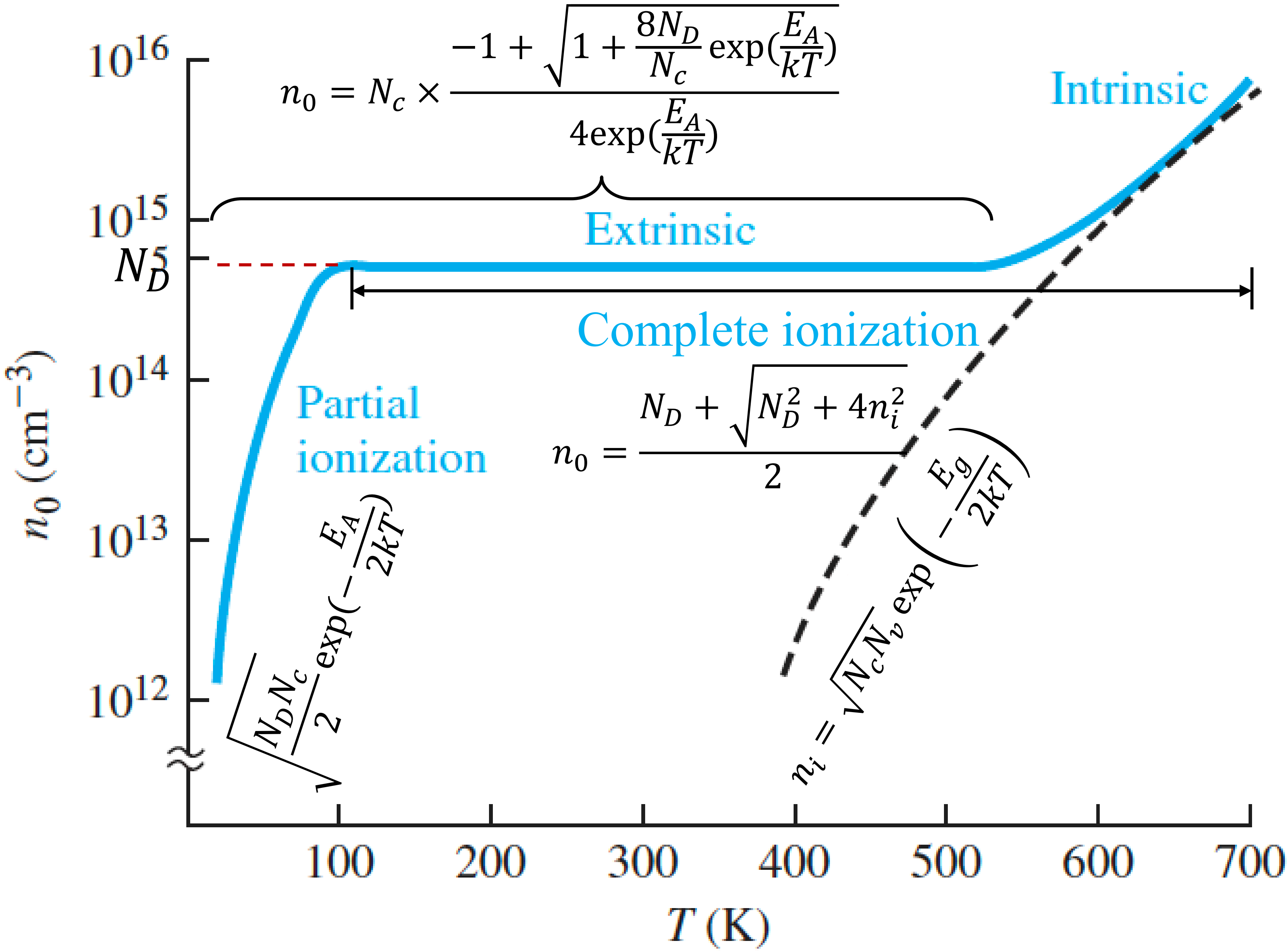


5.1 Carrier drift

<u>Conductivity</u>



Previously… Ionization of dopants



5.1 Carrier drift

Velocity saturation

$$\frac{1}{2}mv_{th}^2 = \frac{3}{2}kT = 0.03885eV\ (300K)$$

$$\Rightarrow thermal\ velocity\ v_{th} \approx 10^7\ cm/s$$

$$Drift\ velocity\ v_d = \mu_n E$$

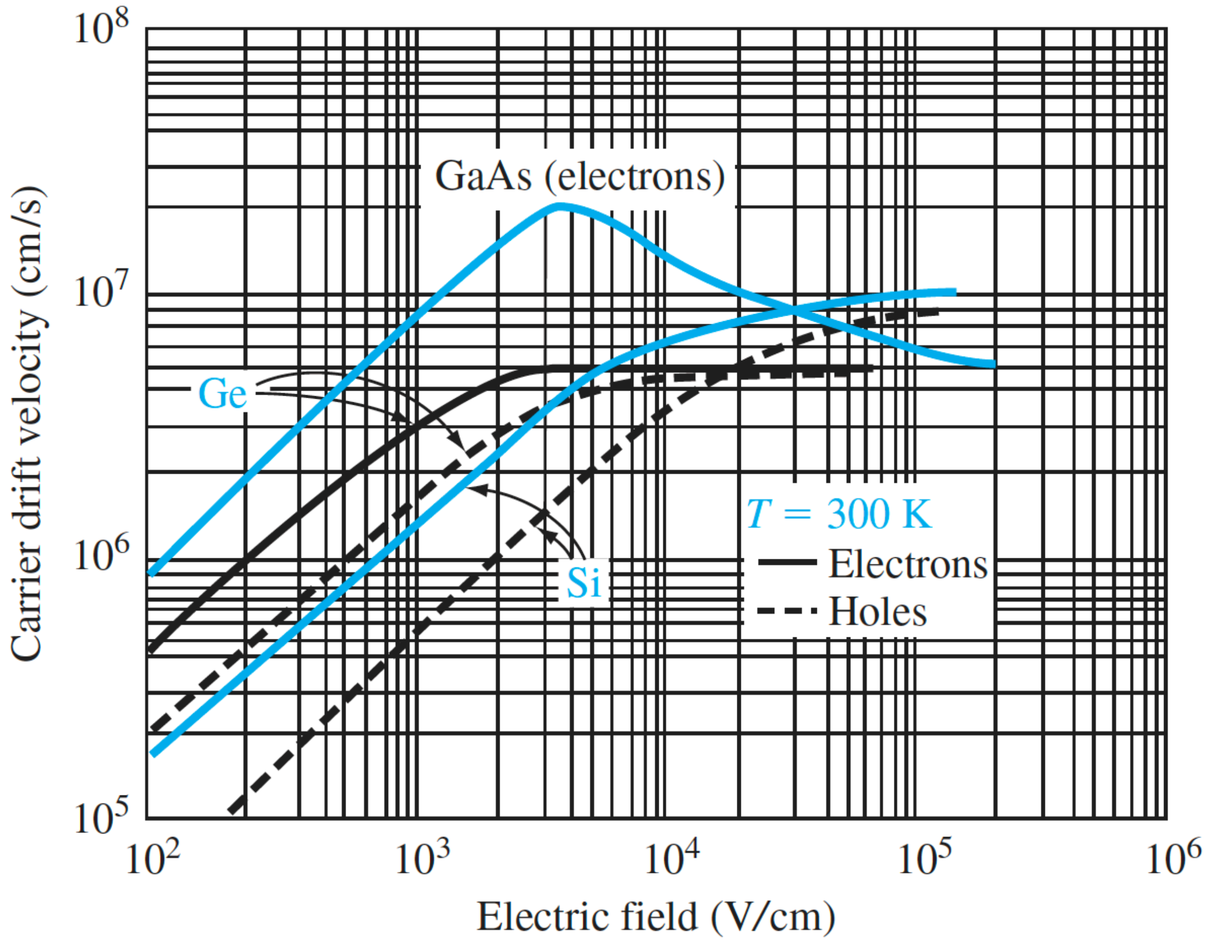
$$\Rightarrow E = \frac{v_d}{\mu_n} = \frac{10^7 cm/s}{1350cm^2/(Vs)} = 7 \times 10^3\ V/cm$$

5.1 Carrier drift

Velocity saturation

$$\boxed{v_d \to v_{th}}$$

- Electric field is heating up electrons
- Electrons transfer energy to lattice to reach thermal equilibrium



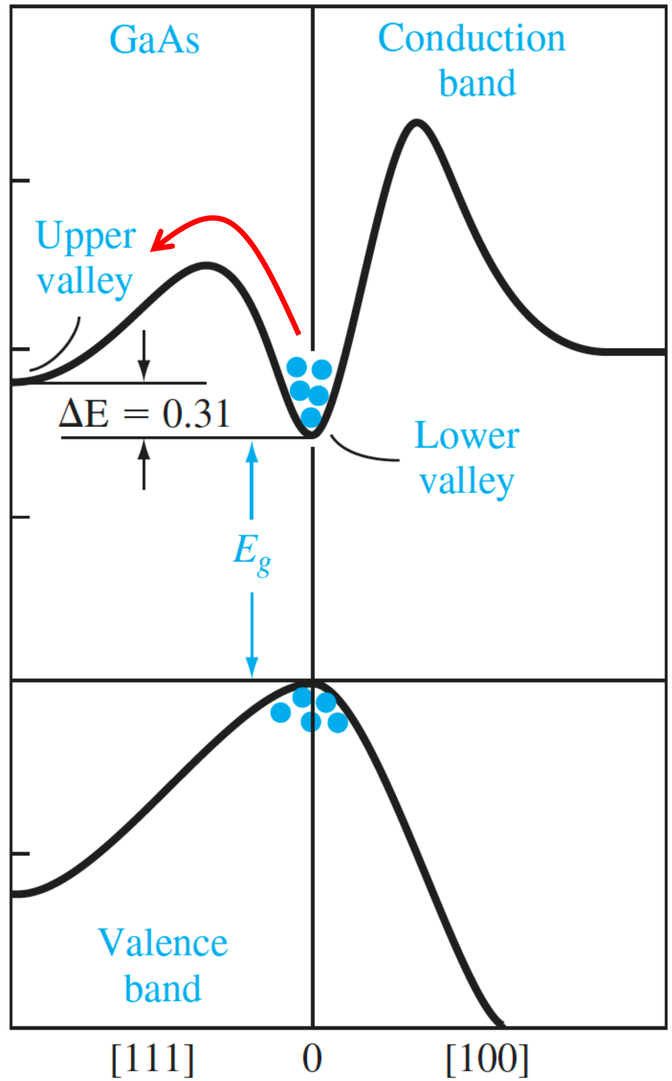
$$v_n = \frac{v_s}{\left[1 + \left(\frac{E_{on}}{E}\right)^2\right]^{1/2}}$$

$$v_p = \frac{v_s}{\left[1 + \left(\frac{E_{op}}{E}\right)\right]}\ {}^{2}\ {}^{1/2}$$

Probably a typo in textbook

5.1 Carrier drift

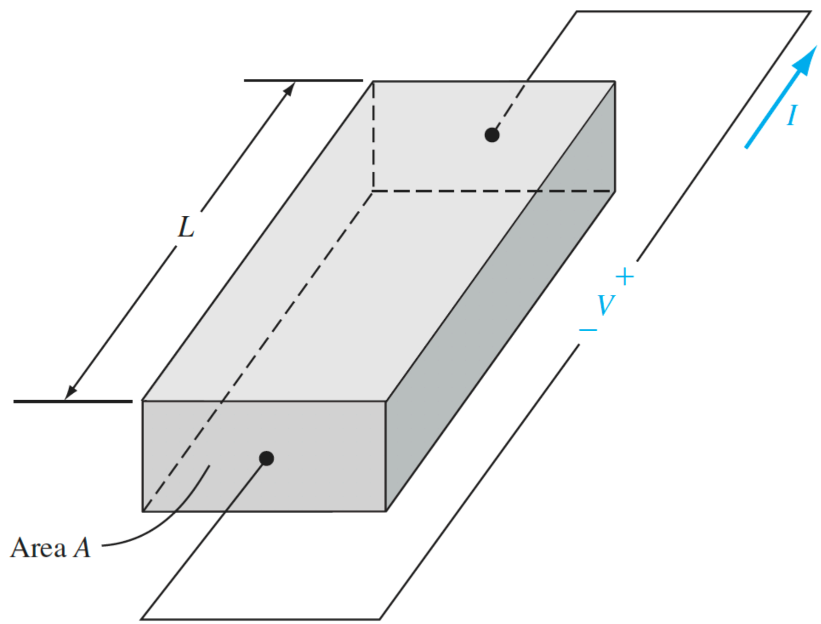
Velocity saturation



5.1 Carrier drift

Problem Example

A bar of p-type silicon at 300K in the figure below has a cross-sectional area A = 10^{-6} cm^2 and a length L = 1.2×10^{-3} cm. For an applied voltage of 5V, a current of 2mA is required. What is the required (a) resistance, (b) resistivity, and (c) impurity doping concentration? (d) What is the resulting hole mobility?



Outline

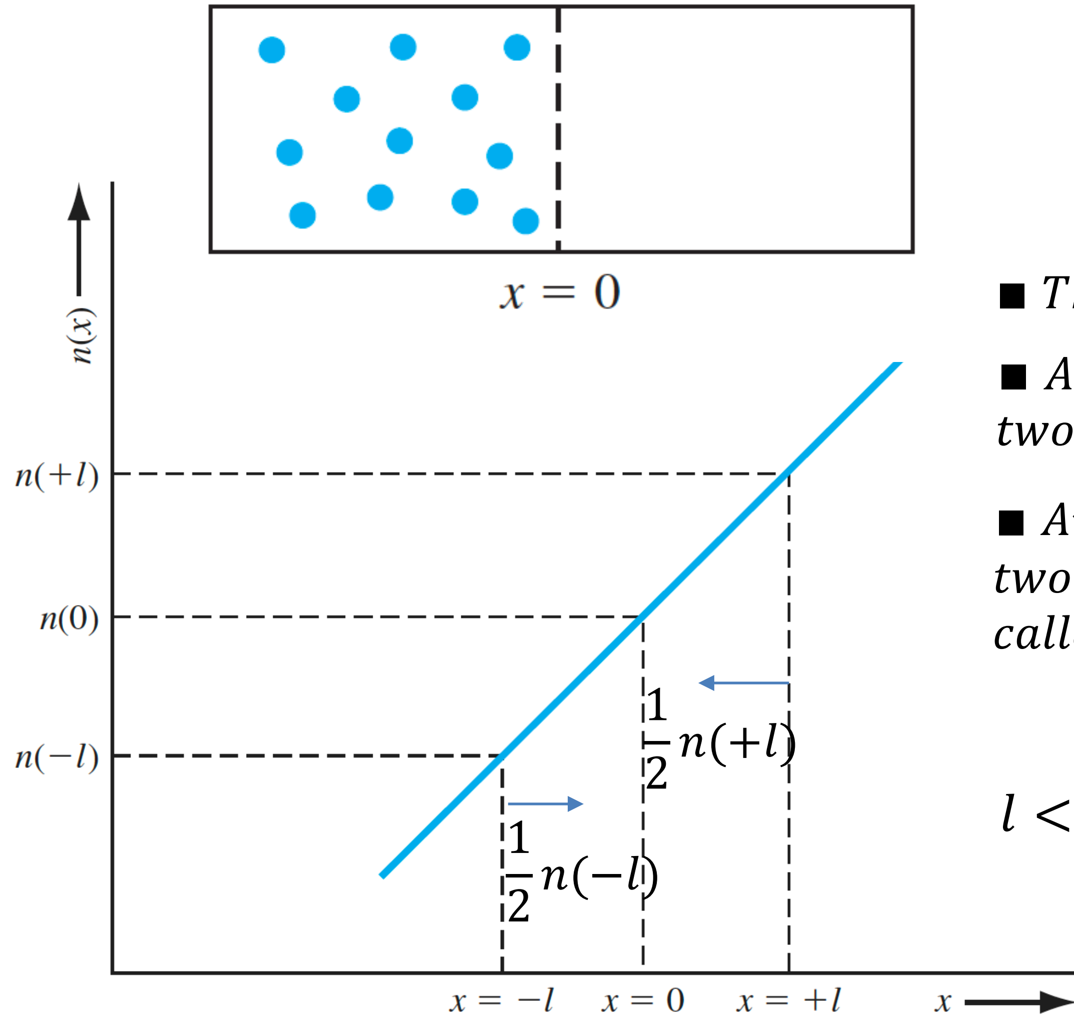
5.1 Carrier drift

5.2 Carrier diffusion

5.3 Graded impurity distribution

5.2 Carrier diffusion

Diffusion current density



- $Thermal\ velocity$: υ_{th}
- $Average\ time\ between\ two\ collisions$: τ_{cn}
- $Average\ distance\ between\ two\ collisions, which\ is\ called\ mean\ free\ path$: $\upsilon_{th}\tau_{cn}$

$l < \upsilon_{th}\tau_{cn}$

5.2 Carrier diffusion

Diffusion current density

Net rate of electron flow in the +x direction at x=0:

$$F_n = \frac{1}{2}n(-l)v_{th} - \frac{1}{2}n(+l)v_{th} = \frac{1}{2}v_{th}[n(-l) - n(+l)]$$

$$n(-l) = n(0) - l\frac{dn}{dx}$$

$$n(+l) = n(0) + l\frac{dn}{dx}$$

$$F_n = -v_{th}l\frac{dn}{dx}$$

Electron current density: $J = -qF_n = qv_{th}l\frac{dn}{dx}$

5.2 Carrier diffusion

Diffusion current density

In the end, l is limited to be the mean free path $\upsilon_{th}\tau_{cn}$,

$\upsilon_{th}l\ will\ become\ a\ constant\ (D_n)\ at\ a\ given\ temperature$

$for\ specific\ material$

Electron diffusion current density: $J_{nx|dif} = -qF_n = qD_n\dfrac{dn}{dx}$

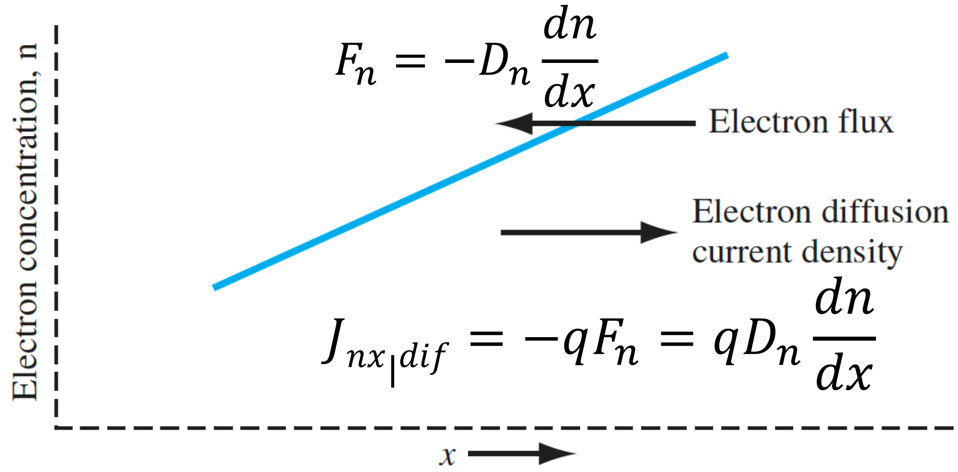
D_n is called the electron diffusion coefficient

Hole diffusion current density: $J_{px|dif} = qF_p = -qD_p\dfrac{dp}{dx}$

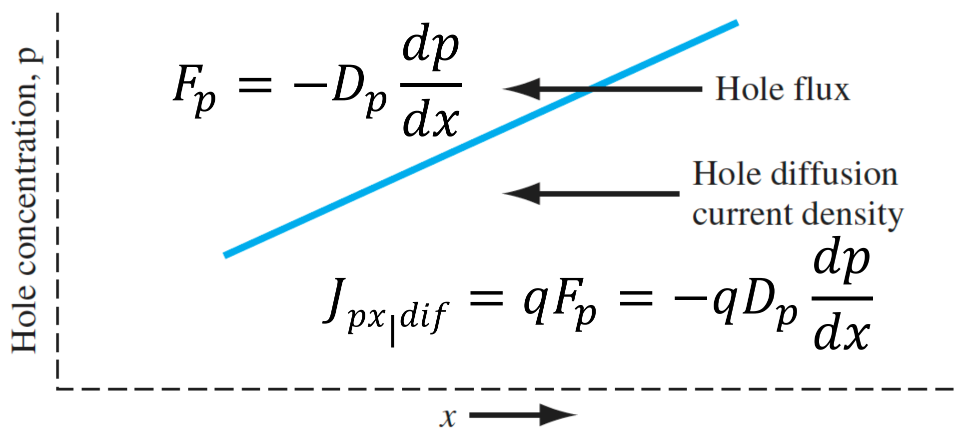
D_p is called the hole diffusion coefficient

5.2 Carrier diffusion

Diffusion current density



(a)



(b)

5.2 Carrier diffusion

Total current density

$$J = J_{drf} + J_{dif} = J_{n|drf} + J_{p|drf} + J_{n|dif} + J_{p|dif}$$

$$= qn\mu_n E_x + qp\mu_p E_x + qD_n\frac{dn}{dx} - qD_p\frac{dp}{dx}$$

$$\mathrm{J} = qn\mu_n E_x + qp\mu_p E_x + qD_n\nabla \mathrm{n} - qD_p\nabla p$$

5.2 Carrier diffusion

Problem Example

The hole density in silicon is given by $p(x) = 10^{16} \exp(-x/L_p)$ $(x \geq 0)$ where L_p = 2×10^{-4} cm. Assume the hole diffusion coefficient is $D_p = 8 cm^2/s$. Determine the hole current density at $x = 2 \times 10^{-4}$ cm.

$$J_{p|diff} = -qD_p \frac{dp}{dx}$$

Outline

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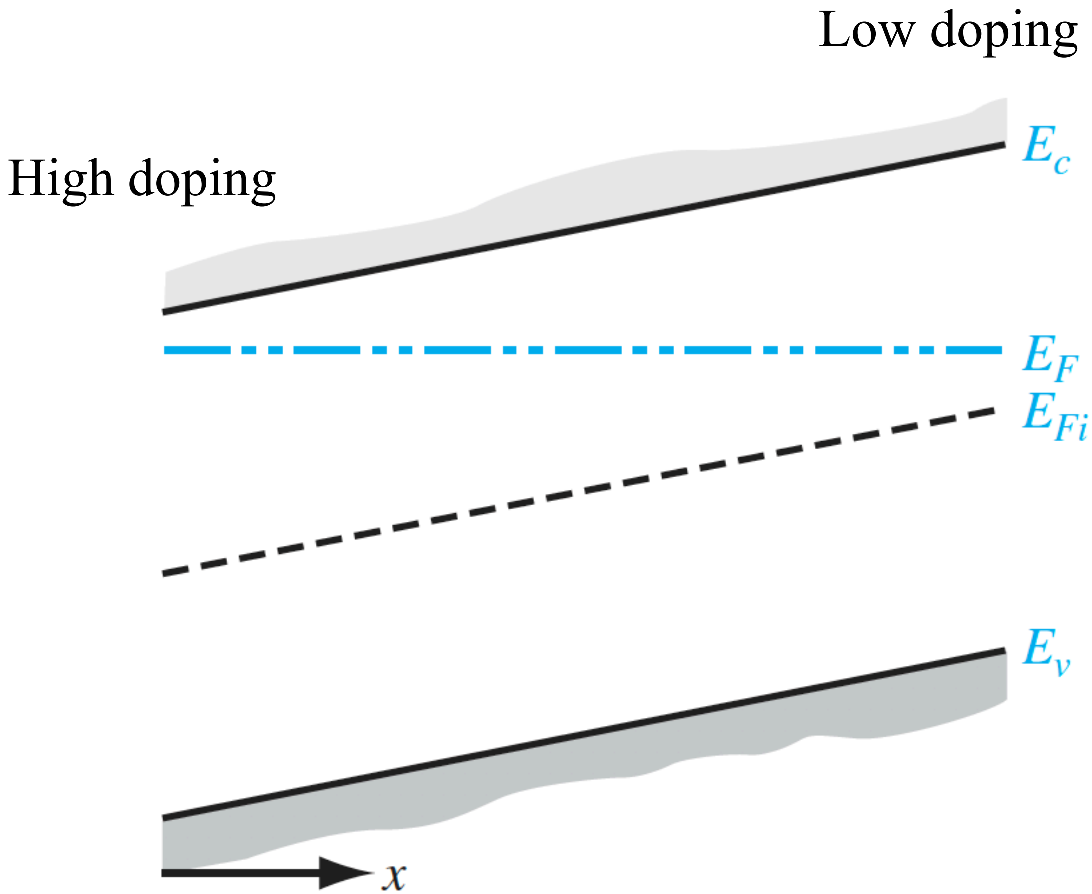
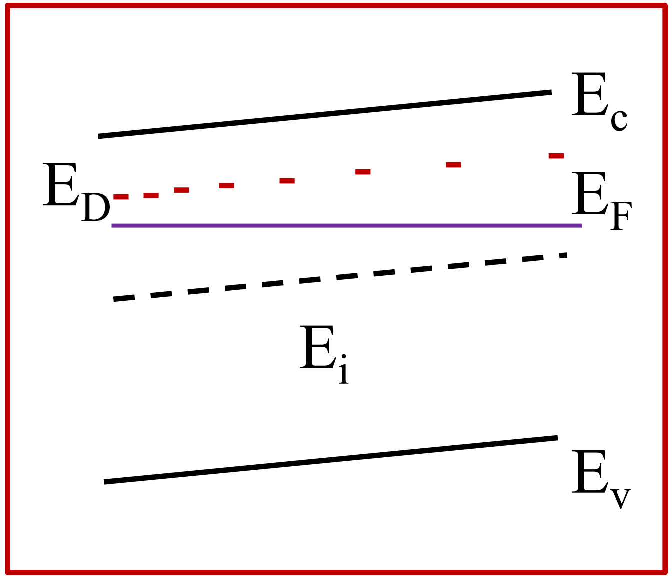
5.3 Graded impurity distribution

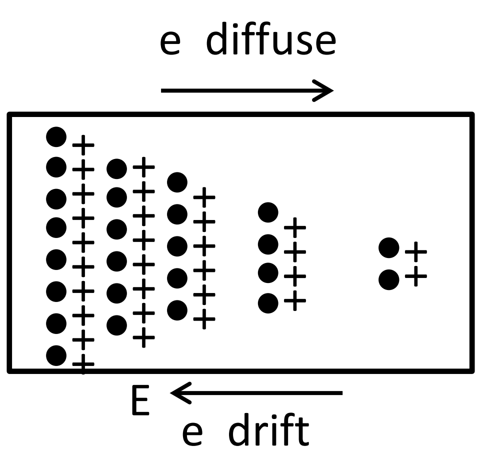
Induced electric field



5.3 Graded impurity distribution

- Induced electric field



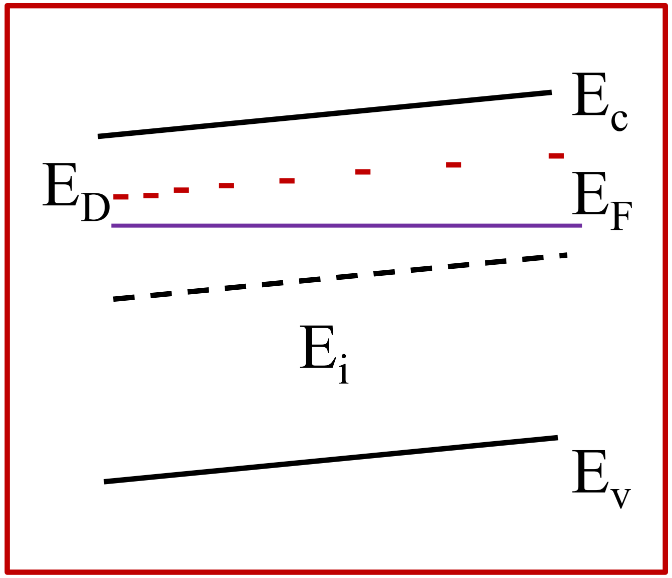


5.3 Graded impurity distribution

- The Einstein relation

$$E_x = -\frac{d\phi}{dx} = \frac{1}{q}\frac{dE_i}{dx}$$

$$= \frac{1}{q}\frac{kT}{n(x)}\frac{dn(x)}{dx}$$



$$\phi = \frac{1}{q}(E_F - E_i)$$

$$E_x = -\frac{d\phi}{dx} = \frac{1}{q}\frac{dE_i}{dx}$$

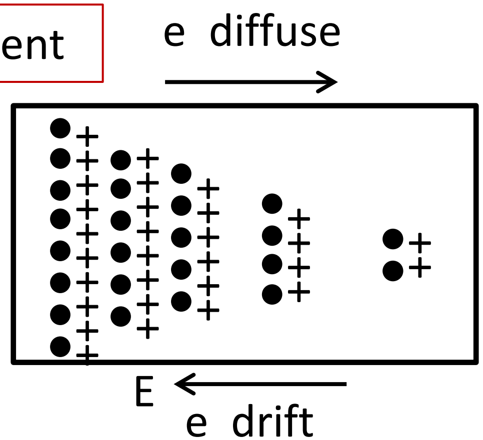
$$n = n_i \exp(\frac{E_F - E_i}{kT})$$

$$E_F - E_i = kTln(n/n_i)$$

Drift current = diffusion current

$$J_{n,drift} = qn(x)\mu_n|E|$$

$$J_{n,diff} = qD_n\frac{dn(x)}{dx}$$



5.3 Graded impurity distribution

- The Einstein relation

$$J_{n,drift} = qn(x)\mu_n|E| = qD_n\frac{dn(x)}{dx} = J_{n,diff}$$

$$qn(x)\mu_n\left(\frac{1}{q}\frac{kT}{n(x)}\frac{dn(x)}{dx}\right) = qD_n\frac{dn(x)}{dx}$$

$$\cancel{q}\cancel{n(x)}\mu_n\left(\frac{1}{q}\frac{kT}{\cancel{n(x)}}\cancel{\frac{dn(x)}{dx}}\right) = \cancel{q}D_n\cancel{\frac{dn(x)}{dx}}$$

$$E_x = -\frac{d\phi}{dx} = \frac{1}{e}\frac{dE_i}{dx}$$

$$= \frac{1}{q}\frac{kT}{n(x)}\frac{dn(x)}{dx}$$

$$\boxed{D_n = \frac{\mu_n kT}{q}}$$

5.3 Graded impurity distribution

Problem Example

Assume the donor concentration in an n-type semiconductor at T =300K is given by $N_d(x) = 10^{16}\exp(-x/L)$ where L = 2 x 10^{-2} cm. Determine the induced electric field and drift current density in the semiconductor at x = 2 x 10^{-2} cm. Note $\mu_n \approx$ 1350 cm^2/Vs and 1200 cm^2/Vs near the doping concentration of 3.68 x 10^{15} cm^{-3} and 10^{16} cm^{-3}, respectively.

$$E_x = \frac{1}{q}\frac{kT}{n(x)}\frac{dn(x)}{dx}$$