Recitation Class 3

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2021.06.01

Outline

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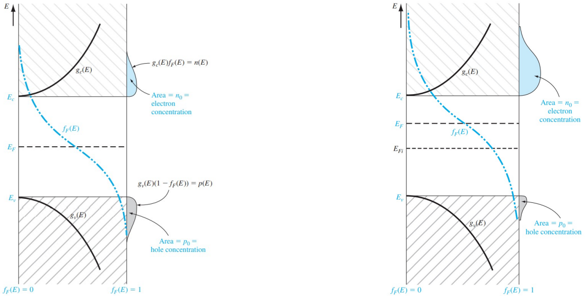
One More Equation

$$\boxed{E_{Fi} - E_{midgap} = \frac{1}{2} kT \ln\left(\frac{N_v}{N_c}\right) = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)}$$

The Extrinsic Semiconductor



(a) Intrinsic

(b) n-type semiconductor

Figure: Density of states functions, Fermi-Dirac probability function, and areas representing electron and hole concentrations

The Extrinsic Semiconductor

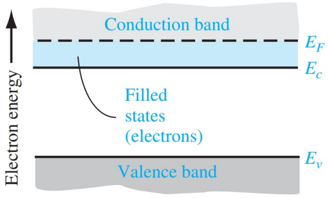
$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$$
$$p_0 = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

$$\boxed{n_0 p_0 = N_c N_v \exp\left(-\frac{E_g}{kT}\right) = n_i^2}$$

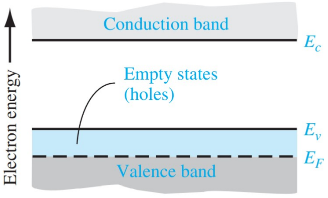
Degenerate Semiconductors

Impurity concentration increases $\Rightarrow$ distance between impurity atoms decreases $\Rightarrow$ donor electrons start to interact with each other $\Rightarrow$ single discrete donor energy level splits into a band $\Rightarrow$ overlaps with conduction band.

Degenerate Semiconductors



(a) n-type



(b) p-type

Figure: Simplified energy-band diagrams for degenerately doped semiconductors

Statistics of Donors and Acceptors

$$f_d(E) = \frac{1}{1 + \frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)}$$

$$n_d = f_d(E)N_d = N_d - N_d^+$$

where N_d^+ is the concentration of ionized donors.

$$f_a(E) = \frac{1}{1 + \frac{1}{g}\exp\left(\frac{E_F - E_a}{kT}\right)}$$

$1/g$ is the degeneracy factor, normally taken as 4 for acceptor level in silicon and gallium arsenide (because of detailed band structure).

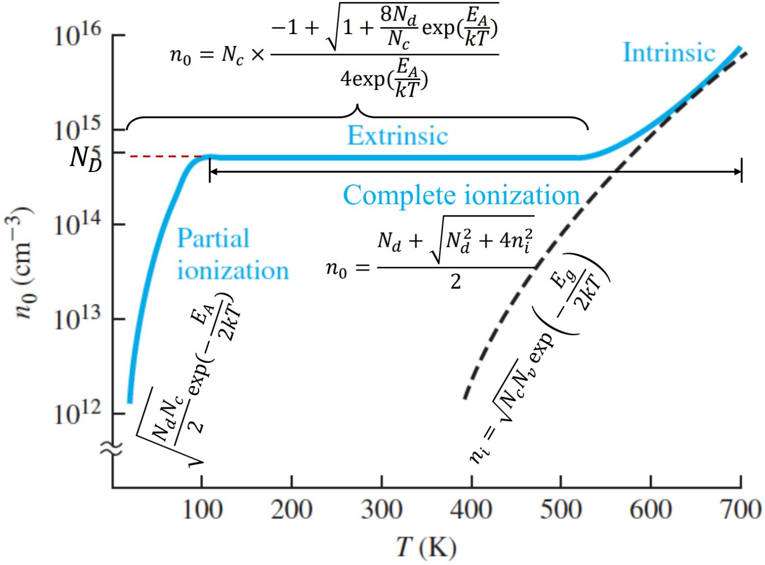
$$n_a = f_a(E)N_a = N_a - N_a^+$$

Statistics of Donors and Acceptors

We calculate the relative number of electrons in the donor state compared with the total number of electrons: (assuming $(E_d - E_F) \gg kT$)

$$\frac{n_d}{n_d + n_0} = \frac{1}{1 + \frac{N_c}{2N_d} \exp\left[\frac{-(E_c - E_d)}{kT}\right]}$$

Example: Determine the fraction of total electrons still in the donor states at $T = 300K$. Consider phosphorus doping in silicon, for $T = 300K$, at a concentration of $N_d = 10^{16} cm^{-3}$.
Answer: 0.41%. Very few electrons remains in the donor states (completely ionized).



$n_0 = N_c \times \frac{-1 + \sqrt{1 + \frac{8N_d}{N_c}\exp(\frac{E_A}{kT})}}{4\exp(\frac{E_A}{kT})}$
Intrinsic
Extrinsic
N_D
Complete ionization
Partial ionization
$n_0 = \frac{N_d + \sqrt{N_d^2 + 4n_i^2}}{2}$
$\sqrt{\frac{N_d N_c}{2}}\exp(-\frac{E_A}{2kT})$
$n_i = \sqrt{N_c N_v}\exp\left(-\frac{E_g}{2kT}\right)$
10^{16}
10^{15}
10^{14}
10^{13}
10^{12}
n_0 (cm^{-3})
100
200
300
400
500
600
700
T (K)

Two Important Equations

$$\boxed{n_0 = \frac{N_d}{2} + \sqrt{\frac{N_d}{2}^2 + n_i^2}}$$

Charge neutrality:

$$n_0 = p_0 + N_d^+$$

Complete ionization:

$$n_0 = \frac{n_i^2}{n_0} + N_d$$

$$\Rightarrow \quad n_0^2 - N_d n_0 - n_i^2 = 0$$

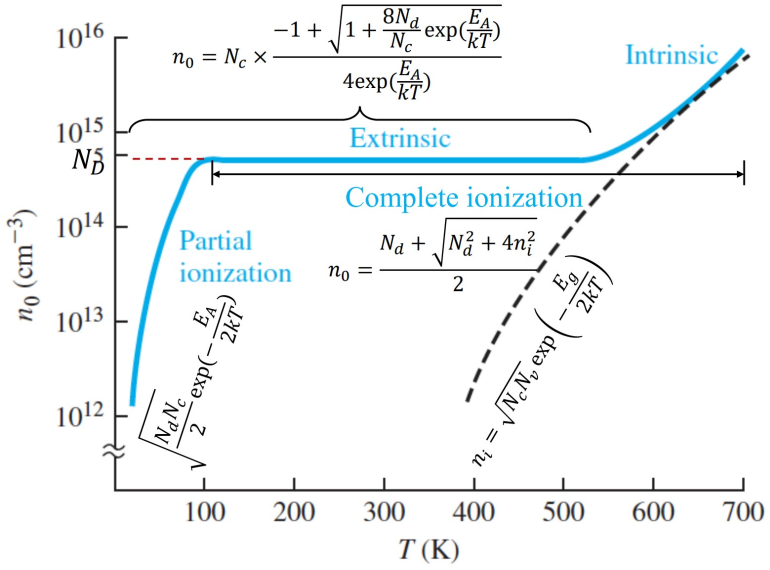
Two Important Equations

$$\boxed{n_0 = N_c \times \frac{-1 + \sqrt{1 + \frac{8N_d}{N_c}\exp\left(\frac{E_A}{kT}\right)}}{4\exp\left(\frac{E_A}{kT}\right)}}$$

$$n_0 = N_d^+ \quad \text{when } T \text{ is not high}$$

$$\begin{aligned}
n_0 &= \frac{N_d}{1 + 2\exp\left(\frac{E_F - E_d}{kT}\right)} \\
&= \frac{N_d}{1 + 2\exp\left(\frac{E_c - E_d}{kT}\right)\exp\left(\frac{E_F - E_c}{kT}\right)} \\
&= \frac{N_d}{1 + 2\exp\left(\frac{E_A}{kT}\right)\frac{n_0}{N_c}}
\end{aligned}$$

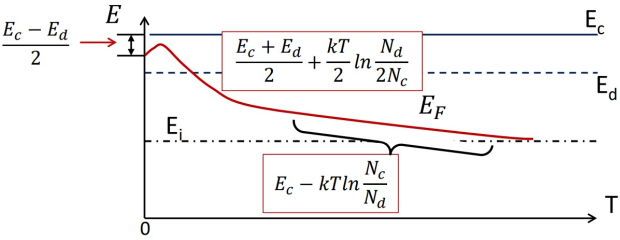
$$\Rightarrow \quad 2\exp\left(\frac{E_A}{kT}\right)n_0^2 + N_c n_0 - N_d N_c = 0$$


$n_0 = N_c \times \frac{-1 + \sqrt{1 + \frac{8N_d}{N_c}\exp(\frac{E_A}{kT})}}{4\exp(\frac{E_A}{kT})}$
Extrinsic
Intrinsic
Complete ionization
Partial ionization
$n_0 = \frac{N_d + \sqrt{N_d^2 + 4n_i^2}}{2}$
$\sqrt{\frac{N_d N_c}{2}}\exp(-\frac{E_A}{2kT})$
$n_i = \sqrt{N_c N_v}\exp\left(-\frac{E_g}{2kT}\right)$
N_D
10^{16}
10^{15}
10^{14}
10^{13}
10^{12}
$n_0\ (\mathrm{cm}^{-3})$
100
200
300
400
500
600
700
T (K)

Fermi Level Position

$$E_F = E_c + kT \ln \left(\frac{\sqrt{1 + \frac{8N_d}{N_c} \exp\left(\frac{E_A}{kT}\right)} - 1}{4 \exp\left(\frac{E_A}{kT}\right)} \right)$$

$$= \begin{cases} \dfrac{E_c + E_D}{2} + \dfrac{kT}{2} \ln \dfrac{N_d}{2N_c}, & T \text{ small} \\ E_c - kT \ln \dfrac{N_c}{N_d}, & T \text{ big} \end{cases}$$



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Drift

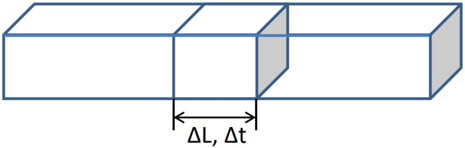


Figure: for p type semiconductor ($p_0 \gg n_0$)

$$I_{drf} = \frac{\Delta Q}{\Delta t} = \frac{q p_0 A_c \Delta L}{\Delta t} = q p_0 A_c v_d$$

How to derive v_d?

$$v = \frac{qEt}{m^*_{cp}},$$

where τ_{cp} - the mean time between collisions

$$v_d \approx \left(\frac{q\tau_{cp}}{m^*_{cp}}\right) E = \mu_p E$$

Drift

$$\boxed{J_{drf} = q(p_0\mu_p + n_0\mu_n)E}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{q(\mu_n n + \mu_p p)}$$

ρ : resistivity

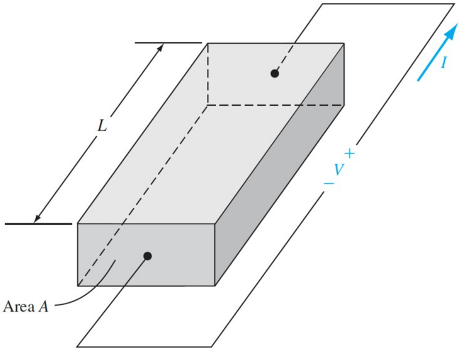
σ : conductivity

	μ_n (cm²/V-s)	μ_p (cm²/V-s)
Silicon	1350	480
Gallium arsenide	8500	400
Germanium	3900	1900

Figure: Typical mobility values at $T = 300K$ and low doping concentrations

Example

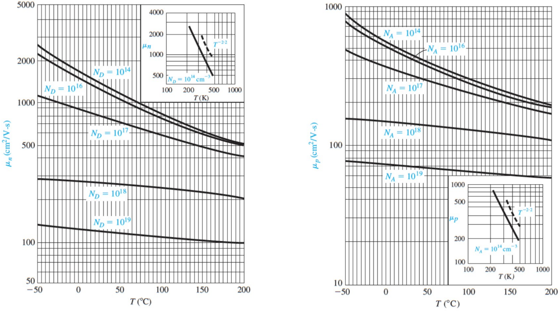
A bar of p-type silicon at $300K$ in the figure below has a cross-sectional area $A = 10^{-6} cm^2$ and a length $L = 1.2 \times 10^{-3} cm$. For an applied voltage of $5V$, a current of $2mA$ is required. What is the required (a) resistance, (b) resistivity, and (c) impurity doping concentration? (d) What is the resulting hole mobility? $(p = 7 \times 10^{15} cm^{-3})$



Mobility Effect - Scattering

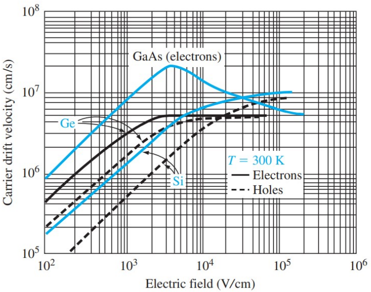
- *Lattice scattering / phonon scattering*
  Lattice scatterings shorten $\tau_{cp} \implies \mu_L \propto T^{-3/2}$
- *Ionized impurity scattering*
  Impurity scatterings shorten $\tau_{cp} \implies \mu_I \propto \frac{T^{3/2}}{N_d^+ + N_a^-}$

$$\frac{1}{\mu} = \frac{1}{\mu_L} + \frac{1}{\mu_I}$$



(a) (b)

Velocity Saturation



$$v_n = \frac{v_s}{\left[1 + \left(\frac{E_{on}}{E}\right)^2\right]^{1/2}}$$

$$v_p = \frac{v_s}{\left[1 + \left(\frac{E_{op}}{E}\right)^2\right]^{1/2}}$$

In silicon at $T = 300K$, $v_s = 10^7 cm/s$, $E_{on} = 7 \times 10^3 V/cm$, $E_{op} = 2 \times 10^4 V/cm$.

Diffusion

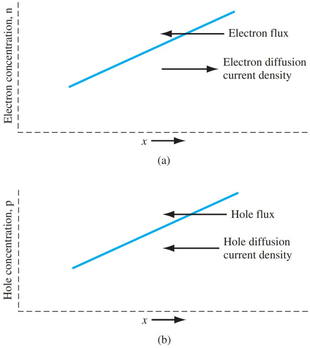


Figure: (a) Diffusion of electrons due to a density gradient. (b) Diffusion of holes due to a density gradient.

$$J_{nx|dif} = eD_n \frac{\mathrm{d}n}{\mathrm{d}x}$$

$$J_{px|dif} = -eD_p \frac{\mathrm{d}p}{\mathrm{d}x}$$

End