Recitation Class for Mid I
Chapter 5

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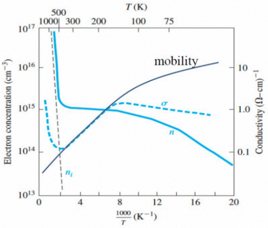
Summary

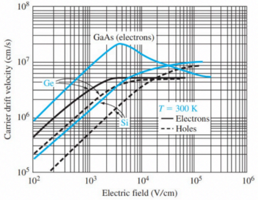
$$I_{drf} = \frac{\Delta Q}{\Delta t} = \frac{ep_0 A_c \Delta L}{\Delta t} = ep_0 A_c v_d$$

$$\frac{1}{\mu} = \frac{1}{\mu_L} + \frac{1}{\mu_I}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{e(\mu_n n + \mu_p p)}$$



$$\begin{aligned} J &= J_{drf} + J_{dif} \\ &= J_{nx|drf} + J_{px|drf} + J_{nx|dif} + J_{px|dif} \\ &= \boxed{en\mu_n E_x + ep\mu_p E_x + eD_n \frac{dn}{dx} - eD_p \frac{dp}{dx}} \end{aligned}$$



$$v_n = \frac{v_s}{\left[1 + \left(\frac{E_{on}}{E}\right)^2\right]^{1/2}}$$

$$v_p = \frac{v_s}{\left[1 + \left(\frac{E_{op}}{E}\right)^2\right]^{1/2}}$$

$$\boxed{\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e}}$$

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Drift

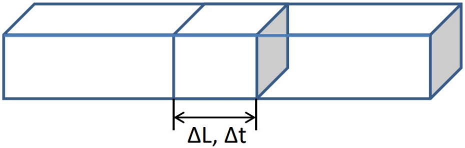


Figure: for p type semiconductor ($p_0 \gg n_0$)

$$I_{drf} = \frac{\Delta Q}{\Delta t} = \frac{ep_0 A_c \Delta L}{\Delta t} = ep_0 A_c v_d$$

How to derive v_d?

$$v = \frac{eEt}{m^*_{cp}},$$

where τ_{cp} - the mean time between collisions

$$v_d \approx \left(\frac{e\tau_{cp}}{m^*_{cp}}\right) E = \mu_p E$$

Drift

$$\boxed{J_{drf} = e(p_0\mu_p + n_0\mu_n)E}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{e(\mu_n n + \mu_p p)}$$

ρ : resistivity

σ : conductivity

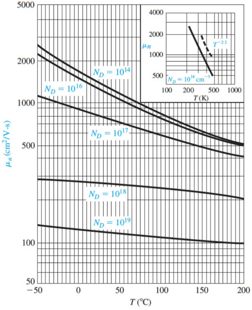
	μ_n (cm^2/V-s)	μ_p (cm^2/V-s)
Silicon	1350	480
Gallium arsenide	8500	400
Germanium	3900	1900

Figure: Typical mobility values at $T = 300K$ and low doping concentrations

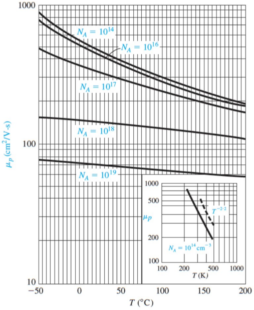
Mobility Effect - Scattering

- *Lattice scattering / phonon scattering*
  Lattice scatterings shorten $\tau_{cp} \implies \mu_L \propto T^{-3/2}$
- *Ionized impurity scattering*
  Impurity scatterings shorten $\tau_{cp} \implies \mu_I \propto \frac{T^{3/2}}{N_d^+ + N_a^-}$

$$\frac{1}{\mu} = \frac{1}{\mu_L} + \frac{1}{\mu_I}$$

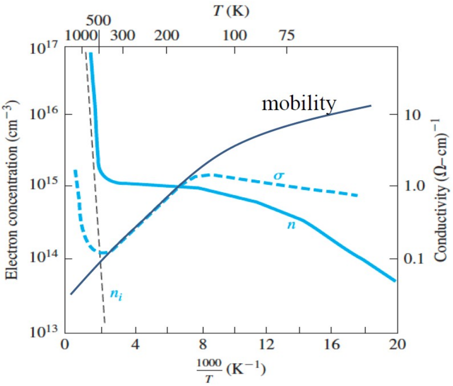


(a)



(b)

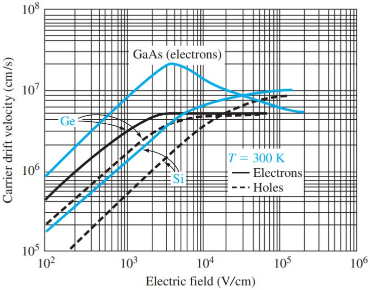
Conductivity



In the mid temperature range, we have complete ionization — the electron concentration remains essentially constant

$$\rho = \frac{1}{\sigma} = \frac{1}{e(\mu_n n + \mu_p p)}$$

Velocity Saturation

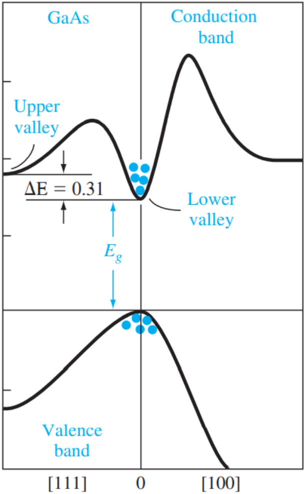


$$v_n = \frac{v_s}{\left[1 + \left(\frac{E_{on}}{E}\right)^2\right]^{1/2}}$$

$$v_p = \frac{v_s}{\left[1 + \left(\frac{E_{op}}{E}\right)^2\right]^{1/2}}$$

In silicon at $T = 300K$, $v_s = 10^7 cm/s$, $E_{on} = 7 \times 10^3 V/cm$, $E_{op} = 2 \times 10^4 V/cm$.

Special of Gallium Arsenide



Remember the effective mass is correlated with the second order derivative of the graph.

$$\left.\frac{d^2E}{dk^2}\right|_{k=0} = \frac{\hbar^2}{m^*}$$

Lower valley: $m_n^* = 0.067m_0$.
Upper valley: $m_n^* = 0.55m_0$.
Combining with equation (5.14):

$$\mu_n = \frac{e\tau_{cn}}{m_{cn}^*}$$

That's why the curve of GaAs looks like that.

Diffusion

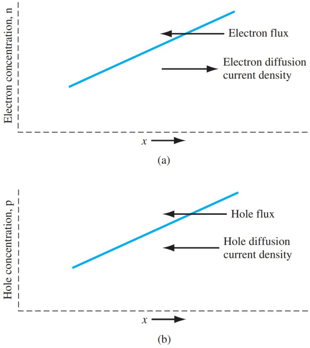


Figure: (a) Diffusion of electrons due to a density gradient. (b) Diffusion of holes due to a density gradient.

$$J_{nx|dif} = eD_n \frac{\mathrm{d}n}{\mathrm{d}x}$$

$$J_{px|dif} = -eD_p \frac{\mathrm{d}p}{\mathrm{d}x}$$

Total Current Density

$$
\begin{aligned}
J &= J_{drf} + J_{dif} \\
&= J_{nx|drf} + J_{px|drf} + J_{nx|dif} + J_{px|dif} \\
&= \boxed{en\mu_n E_x + ep\mu_p E_x + eD_n \frac{dn}{dx} - eD_p \frac{dp}{dx}}
\end{aligned}
$$

This equation may be generalized to three dimensions as

$$
J = en\mu_n E + ep\mu_p E + eD_n \nabla \mathrm{n} - eD_p \nabla p
$$

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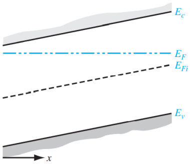
Induced Electric Field

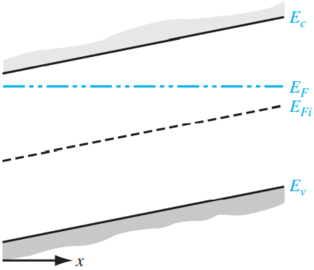
This part is telling how to derive the Einstein Relation

$$\boxed{\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e}}$$



Electrons diffuse from high concentration to low concentration while left the positive charge behind.
Finally reach an equilibrium in this semiconductor where the diffusion current and the drift current caused by the induced electric field cancel with each other.

Induced Electric Field



Diffusion goes from left to right.
The induced electric field also points from left to right. That is to say, the drift current goes from right to left.
The we can try to write an equation:

$$J_{n|drf} = en(x)\mu_n|E| = eD_n \frac{dn(x)}{dx} = J_{n|dif}$$

Induced Electric Field

$$J_{n|drf} = en(x)\mu_n|E| = eD_n\frac{dn(x)}{dx} = J_{n|dif}$$

How to find E ?

$$\phi = +\frac{1}{e}\left(E_F - E_{Fi}\right)$$

We know that E_F is flat when at equilibrium, therefore

$$\mathrm{E}_x = -\frac{d\phi}{dx} = \frac{1}{e}\frac{dE_{Fi}}{dx}$$

from $E_F - E_{Fi} = kT\ln\left(\frac{n(x)}{n_i}\right)$, we get

$$-\frac{dE_{Fi}}{dx} = \frac{kT}{n(x)}\frac{dn(x)}{dx}$$

plugging in $\frac{dE_{Fi}}{dx}$, we finally get (there is a minus sign missing on the slide ch.3 p28)

$$\mathrm{E}_x = -\left(\frac{kT}{e}\right)\frac{1}{n(x)}\frac{dn(x)}{dx}$$

The Einstein Relation

After plug in $\mathrm{E}_x = -\left(\frac{kT}{e}\right)\frac{1}{n(x)}\frac{dn(x)}{dx}$ into

$$J_{n|drf} = en(x)\mu_n|E| = eD_n\frac{dn(x)}{dx} = J_{n|dif}$$

We get

$$en(x)\mu_n\left(\frac{kT}{e}\right)\frac{1}{n(x)}\frac{dn(x)}{dx} = eD_n\frac{dn(x)}{dx}$$

$$\frac{D_n}{\mu_n} = \frac{kT}{e}$$

Finally we get the Einstein Relation:

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e}$$

Summary

$$I_{drf} = \frac{\Delta Q}{\Delta t} = \frac{ep_0 A_c \Delta L}{\Delta t} = ep_0 A_c v_d \qquad \frac{1}{\mu} = \frac{1}{\mu_L} + \frac{1}{\mu_I}$$

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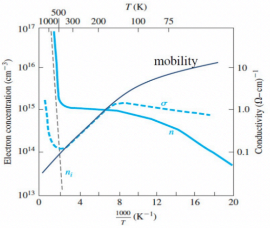

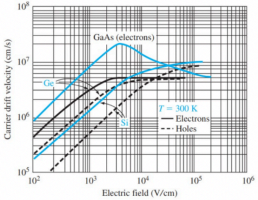

$$v_n = \frac{v_s}{\left[1 + \left(\frac{E_{on}}{E}\right)^2\right]^{1/2}}$$

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$$\boxed{\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e}}$$

Some tips for the exam

1. Look at the questions carefully. Plot? Coordinate? Label? In units of what?
2. We have three exams. Don't be to worried. You can still be a 320 TA even if you do not do well in the first mid term.
3. Exercises on the textbook will help you learn this course.

Good luck to your midterm exam!