Ve401 Probabilistic Methods in Engineering

Spring 2020 — Assigment 1

Date Due: 11:00 PM, Friday, the 13th of March 2020

This assignment has a total of **(23 Marks)**.

Exercise 1.1 Elementary Probability

To get the opportunity to enter the McNeill River Brown (Grizzly) Bear Sanctuary in Alaska, one must enter a lottery. For a given year there are 2000 individuals entered, and of these a set of 120 names will be randomly selected. Assume that you and a friend are both entered into the lottery. What is the probability that you and your friend will both be chosen?
(2 Marks)

Exercise 1.2 Some Routine Calculations

Let A and B be events in a sample space S. Use the axioms of probability to show the following statements:

i) Let $A \subset B$. Show that $P[A] \le P[B]$.
(1 Mark)

ii) Assume that A and B are independent and that $P[A]P[B] > 0$. Show that A and B are not mutually exclusive.
(2 Marks)

iii) Show that $P[A \cup B] = P[A] + P[B] - P[A \cap B]$
(1 Mark)

Exercise 1.3 D'Alembert's Coins

The French mathematician Jean D'Alembert claimed that in tossing a coin twice (or two coins at once), we have only three possible outcomes: "two heads," "one head," and "no heads." This is a legitimate sample space, of course. However, D'Alembert also claimed that each outcome in this space has the same probability 1/3.

i) Is it possible to have a coin biased in such a way so as to make D'Alembert's claim true? If so, how? If not, why not?
(2 Marks)

ii) Is it possible to make two coins with different probabilities of heads so as to make D'Alembert's claim true if both coins are tossed at once? If so, how? If not, why not?
(2 Marks)

Exercise 1.4 Independence

The ablity to observe and recall details is important in science. Unfortunately, the power of suggestion can distort memory. A study of recall is conducted as follows: Subjects are shown a film in which a car is moving along a country road. There is no barn in the film. The subjects are then asked a series of questions concerning the film. Half the subjects are asked, "How fast was the car moving when it passed the barn?" The other half is not asked the question. Later each subject is asked, "Is there a barn in the film?" Of those asked the first question concerning the barn, 17% answer "yes"; only 3% of the others answer "yes."

i) What is the probability that a randomly selected participant in this study claims to have seen the non-existent barn?
(2 Marks)

ii) Is claiming to see the barn independent of being asked the first question about the barn?
(2 Marks)

Exercise 1.5 This one may need a little thinking about...

It is reported that 50% of all computer chips produced are defective. Inspection assures that only 5% of the chips legally marketed are defective. Unfortunately, some chips are stolen before inspection. If 1% of all chips on the market are stolen, find the probability that a given chip is stolen given that it is defective.
(3 Marks)

Exercise 1.6 Monty Hall in Prison?

One[1] of three prisoners, A, B, and C, is to be executed the next morning. They all know about it, but they do not know who is going to die. The warden knows, but he is not allowed to tell them until just before the execution.

In the evening, one of the prisoners, say A, goes to the warden and asks him: "Please, tell me the name of one of the two prisoners, B and C, who is not going to die. If both are not to die, tell me one of their names at random. Since I know anyway that one of them is not going to die, you will not be giving me any information."

The warden thought about it for a while, and replied: "I cannot tell you who is not going to die. The reason is that now you think you have only 1/3 chance of dying. Suppose I told you that B is not to be executed. You would then think that you have a 1/2 chance of dying, so, in effect, I would have given you some information."

Was the warden right or was the prisoner right?
(3 Marks)

Exercise 1.7 Two Children Paradox - Birthday Party!

At a meeting of boy scouts in 2010 (at the time, girls were not allowed to become boy scouts but could join the girl scouts instead), you meet a friendly lady, mother of one of the boys. She tells you that her son actually has a sibling, but neglects to mention whether he or she is younger or older. Being familiar with basic probability, you conclude that the sibling is most likely (with probability 2/3) a girl.

But then you recall that the meeting is actually a communal birthday party only for those boy scouts born in July, so that must be the month of birth of the boy. You assume that the probability for any given child of being born in July is 1/12, ignoring the variable lengths of the months as well as all sociological, biological or other distortions. What is the probability that the lady's other child is a girl?
(3 Marks)

[1] Exercise 4.4.9 of R. Bartoszynski and M. Niewiadomska-Buga *Probability and Statistical Inference*, Second Edition, Wiley 2008