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1. Let F = finish dinner and P = play video games.
   Then logician says $\neg F \Rightarrow \neg P$, which can be interpret as $P \Rightarrow F$.
   Hower, being able to play video games after dinner should be $F \Rightarrow P$.
   $P \Rightarrow F$ and $F \Rightarrow P$ are not logically equivalent. However, he treats them equivalent.

2. a. $\neg(p \vee (q \wedge r)) \equiv \neg p \wedge \neg(q \wedge r) \equiv \neg p \wedge (\neg q \vee \neg r)$

   b. $(\neg p \Rightarrow q) \vee \neg(q \wedge r) \equiv p \vee q \vee \neg q \vee \neg r \equiv \text{True}$

   c. $(p \Rightarrow \neg q) \Leftrightarrow ((q \wedge \neg r) \Rightarrow (\neg p))$

   $\equiv (\neg p \vee \neg q) \Leftrightarrow (\neg(q \wedge \neg r) \vee \neg p) \equiv (\neg p \vee \neg q) \Leftrightarrow (\neg q \vee r \vee \neg p)$

   $\equiv ((\neg p \vee \neg q) \Rightarrow (\neg q \vee r \vee \neg p)) \wedge ((\neg q \vee r \vee \neg p) \Rightarrow (\neg p \vee \neg q))$

   $\equiv ((p \wedge q) \vee (\neg q \vee r \vee \neg p)) \wedge ((q \wedge \neg r \wedge p) \vee (\neg p \vee \neg q))$

   $\equiv (p \vee \neg q \vee r \vee \neg p) \wedge (q \vee \neg q \vee r \vee \neg p) \wedge (q \vee \neg p \vee \neg q) \wedge (\neg r \vee \neg p \vee \neg q) \wedge$
   $(p \vee \neg p \vee \neg q)$

3. a. 12
   b. 15
   c. 0

4. a. Yes. e.g. XOR
   b. $4 \times 4 = 16$
   c. Some of them can be expressed by other connectives.
      e.g. $\neg A \vee B$ is equivalent to $A \Rightarrow B$

5. ① Modus Tollens $\Rightarrow$ Modus Ponens
   Modus Tollens is equivalent to $(\neg q, p \Rightarrow q) \Rightarrow \neg p \equiv (\neg q \wedge (\neg p \vee q)) \Rightarrow \neg p$
   Let $a = \neg q$, $b = \neg p$, we have $(a \wedge (b \vee \neg a)) \Rightarrow b \equiv \dfrac{a,\ a \Rightarrow b}{b}$,
   which is the Modus Ponens

   ② Modus Ponens $\Rightarrow$ Modus Tollens
   Modus Ponens is equivalent to $(a \Rightarrow b, a) \Rightarrow b \equiv ((\neg a \vee b) \wedge a) \Rightarrow b$
   Let $q = \neg a$, $p = \neg b$, we have $((q \vee \neg p) \wedge \neg q) \Rightarrow \neg p \equiv \dfrac{\neg q,\ p \Rightarrow q}{\neg p}$,
   which is the Modus Tollens

6. a. $\forall x \; \neg like(Bob, x) \Rightarrow like(Alice, x)$

b. $\exists x \; \neg like(Bob, x) \land like(Alice, x)$

c. $\forall x \; like(Alice, x) \Rightarrow \neg like(Charles, x)$

d. $\forall x, p \; (Person(p) \land \neg(p = David) \land \neg like(p, x)) \Rightarrow like(David, x)$

e. $\forall s, \; WriteFirstLogicSentence(s) \Rightarrow Like(I, s)$

f. $\forall p, \; Parent(p, my\ sibling) \Rightarrow Parent(p, I)$

g. $\forall c, (Child(c, my\ parent) \land \neg \; (c = I)) \Rightarrow Sibling(c, I)$

7. For anypeople speak the same langnage, they can understand each other.

8. a. $Wrote(Gershwin, TheManILove)$

b. $\neg Wrote(Gershwin, EleanorRigby)$

c. $Wrote(Gershwin, TheManILove) \lor Wrote(McCartney, TheManILove)$

d. $\exists s \; Wrote(Joe, s)$

e. $\exists d \; CopyOf(d, Revolver) \land Owns(Joe, d)$

f. $\forall s \; Sings(McCartney, s, Revolver) \Rightarrow Wrote(McCartney, s)$

g. $\forall s, p \; Sings(p, s, Revolver) \Rightarrow \neg Wrote(Gershwin, s)$

h. $\forall s \; Wrote(Gershwin, s) \Rightarrow \exists a, p \; Sings(p, s, a)$

i. $\exists a \; \forall s, p \; Wrote(Joe, s) \land Sings(p, s, a)$

j. $\exists d, a \; CopyOf(d, a) \land Owns(Jeo, d) \land Sings(BillieHoliday, TheManILove, a)$

k. $\forall a \; \exists s \; Sings(McCartney, s, a) \Rightarrow \exists d \; CopyOf(d, a) \land Owns(Joe, d)$

l. $\forall a, s \; Sings(BillieHoliday, s, a) \Rightarrow \exists d \; CopyOf(d, a) \land Owns(Joe, d)$