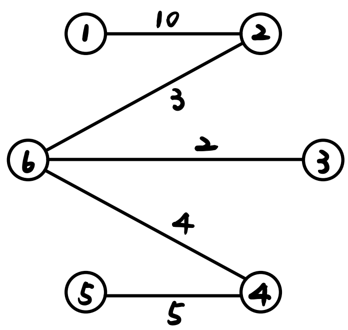
VG441 Final 周雅凯 518021911039

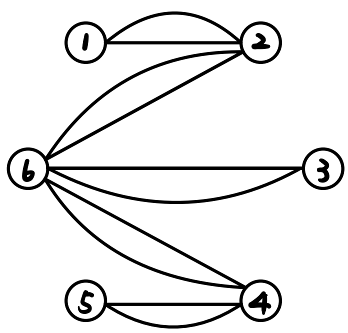
1. Task 1.

First, we need to find MST:

$2 < 3 < 4 < 5 < 6 < 9 < 10 < 22 < 33 < 50 < 66 < 86 \leq 86 < 100 < 952$



Then we double all edges



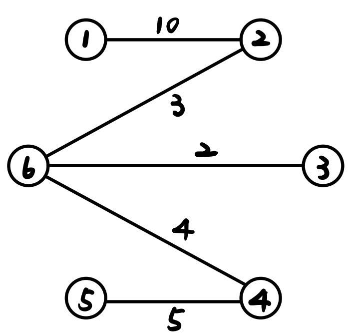
Then we find Eulerian path $1 \to 2 \to 6 \to 3 \to 6 \to 4 \to 5 \to 4 \to 6 \to 2 \to 1$

And shortcut: $1 \to 2 \to 6 \to 3 \to 4 \to 5 \to 1$

And the cost is $10 + 3 + 2 + 6 + 5 + 33 = 59$

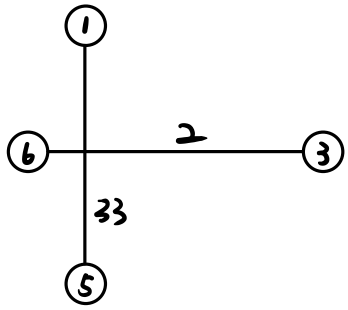
Task 2.

First, we need to find MST:

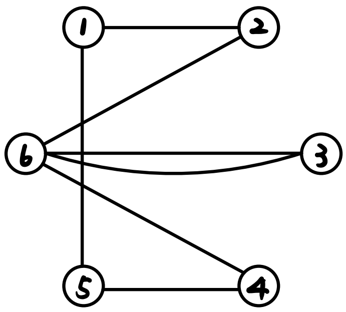


Then we find the set of odd degree vertices $U = \{1, 3, 5, 6\}$

And the minimum cost matching is on next page.



Add it back to MST, we get:



Then we find Eulerian path: $1 \to 2 \to 6 \to 3 \to 6 \to 4 \to 5 \to 1$

And shortcut: $1 \to 2 \to 6 \to 3 \to 4 \to 5 \to 1$

And the cost is 59.

2. Task 1.

$v_{max} = 6$

Let $T[i, w]$ be the min-size of subset $S \subseteq \{1, \dots, i\}$ such that $\sum_{k \in S} v_k = w$

$n v_{max} = 4 \times 6 = 24$

$T[1,0] = 0$, $T[1,1] = \infty$, $T[1,2] = \infty$, $T[1,3] = \infty$, $T[1,4] = 3$,

$T[1,w] = \infty$ for $5 \le w \le 24$;

$T[2,0] = 0$, $T[2,1] = \infty$, $T[2,2] = \infty$, $T[2,3] = \infty$, $T[2,4] = 3$,

$T[2,5] = \infty$, $T[2,6] = \infty$, $T[2,7] = \infty$, $T[2,8] = 6$,

$T[2,w] = \infty$ for $9 \le w \le 24$;

$T[3,0] = 0$, $T[3,1] = \infty$, $T[3,2] = \infty$, $T[3,3] = \infty$, $T[3,4] = 3$,

$T[3,5] = \infty$, $T[3,6] = 8$, $T[3,7] = \infty$, $T[3,8] = 6$, $T[3,9] = \infty$,

$T[3,10] = 11$, $T[3,11] = \infty$, $T[3,12] = \infty$, $T[3,13] = \infty$, $T[3,14] = 14$,

$T[3,w] = \infty$ for $15 \le w \le 24$;

$T[4,0] = 0$, $T[4,1] = \infty$, $T[4,2] = \infty$, $T[4,3] = \infty$, $T[4,4] = 3$,

$T[4,5] = 5$, $T[4,6] = 8$, $T[4,7] = \infty$, $T[4,8] = 6$, $T[4,9] = 8$,

$T[4,10] = 11$, $T[4,11] = 13$, $T[4,12] = \infty$, $T[4,13] = 11$, $T[4,14] = 14$,

$T[4,15] = 16$, $T[4,16] = \infty$, $T[4,17] = \infty$, $T[4,18] = \infty$, $T[4,19] = 19$,

$T[4,w] = \infty$ for $20 \le w \le 24$.

$\max\{w : T[n,w] \le 8\} = 9$

Therefore, maximum value is 9.

Task 2.

#1	#2	#3	#4
$s_1 = 3$	$s_2 = 3$	$s_3 = 8$	$s_4 = 5$
$v_1 = 4$	$v_2 = 4$	$v_3 = 6$	$v_4 = 5$
$\frac{v_1}{s_1} = \frac{4}{3}$	$\frac{v_2}{s_2} = \frac{4}{3}$	$\frac{v_3}{s_3} = \frac{3}{4}$	$\frac{v_4}{s_4} = 1$

By ranking, $\frac{v_1}{s_1} \ge \frac{v_2}{s_2} > \frac{v_4}{s_4} > \frac{v_3}{s_3}$

By greedy algorithm, we first choose item 1 and item 2, get value = 8 and size = 6. Now we have 2 more size, which can not fit any other item.

But if we choose item 1 and item 4 or item 2 and item 4, we get value = 9 and size = 8, which is a better solution for greedy algorithm.

Therefore, it is not optimal.

3. Task 1.

① $R_1 = \frac{6}{5}$

$R_2 = \frac{15}{5} = 3$

$R_3 = \frac{7}{7} = 1$

$R_2 > R_1 > R_3$. Therefore, we choose set 3.

② $R_1 = \frac{6}{3} = 2$

$R_2 = \frac{15}{5} = 3$

$R_2 > R_1$. Therefore, we choose set 1.

③ Only S_2 is left and V is not fully covered. $R_2 = \frac{15}{2}$

And after choosing S_2, V is fully covered.

After 3 iterations, the total cost is $7+6+15=28$.

Task 2.

It is not optimal.

Picking S_2 and S_3 covers V fully and the total cost is $15+7=22$.

It is better than greedy solution.

Bonus: Task 1.

Let $i = |F|$, $j = |D|$

(P):

$$\min(f_1 y_1 + f_2 y_2 + \cdots + f_i y_i + d_{11} x_{11} + d_{12} x_{12} + \cdots + d_{ij} x_{ij})$$

$$\text{s.t.} \begin{cases} x_{11} + x_{21} + \cdots + x_{i1} \geq 1 \\ x_{12} + x_{22} + \cdots + x_{i2} \geq 1 \\ \vdots \\ x_{1j} + x_{2j} + \cdots + x_{ij} \geq 1 \end{cases}$$

$$\begin{cases} x_{11} \leq y_1 \\ x_{12} \leq y_1 \\ \vdots \\ x_{1j} \leq y_1 \\ \vdots \\ x_{ij} \leq y_i \end{cases}$$

$$x_{ij}, y_i \geq 0$$

$\Rightarrow$

(D):

$$\max \sum_{j \in D} \alpha_j$$

$$\text{s.t.} \sum_{j \in D} \beta_{ij} \leq f_i \quad \forall i \in F$$

$$\alpha_j - \beta_{ij} \leq d_{ij} \quad \forall i \in F, j \in D$$

$$\alpha_j, \beta_{ij} \geq 0.$$

Task 2.

$\sum_{j \in D} \beta_{ij} \leq f_i \;\; \forall i \in F$ can be interpreted as: the total money needed to open facility i is at least the total contribution from every demand j towards opening facility i.

$\alpha_j - \beta_{ij} \leq d_{ij} \;\; \forall i \in F, j \in D$ can be written as $\alpha_j \leq \beta_{ij} + d_{ij} \;\; \forall i \in F, j \in D$.

It can be interpreted as: the total money demand j is willing to contribute is at most the sum of the money demand j contributes towards opening facility plus the cost of distance it travels to i.

$\max \sum_{j \in D} \alpha_j$ can be interpreted as: we want to maximize the total amount of money demands are willing to contribute to the solution.