

# Midterm Problem 4.

$$(a) \quad C = \begin{pmatrix} 6 \\ 14 \\ 13 \end{pmatrix} \quad A = \begin{pmatrix} 3 & 2 & 1 \\ 1 & 2 & 4 \end{pmatrix} \quad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad b = \begin{pmatrix} 24 \\ 60 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 3 & 1 \\ 2 & 2 \\ 1 & 4 \end{pmatrix} \quad b^T = (24 \quad 60)$$

$$\min \quad 24y_1 + 60y_2$$

$$\text{s.t.} \quad 3y_1 + y_2 \geq 6$$

$$2y_1 + 2y_2 \leq 14$$

$$y_1 + 4y_2 = 13$$

$$y_1 \geq 0$$

$$y_2 \leq 0$$

$$(b) \quad C = \left( \begin{array}{c} | \\ \vdots \\ | \end{array} \right\} |I| \times |J| \right) \quad b = \left( \begin{array}{c} | \\ \vdots \\ | \end{array} \right\} |I| + |J| \right)$$

$$A = \begin{pmatrix} 111 \dots 100 \dots 0000 \\ 000 \dots 111 \dots 100 \dots 0 \\ \vdots \\ 000 \dots 00111 \dots 1 \\ 1000 \dots 1000 \dots 1000 \dots \\ 0100 \dots 0100 \dots 0100 \dots \\ 0010 \dots 0010 \dots 0010 \dots \\ \vdots \\ 0000 \dots 0001 \dots 0001 \end{pmatrix}$$

$$b^T = \left( \underbrace{1 \dots 1}_{|I| + |J|} \right)$$

$$A^T = \left( \begin{array}{cc} \overbrace{\begin{matrix} 000 \dots 000 & 100 \dots 0 \\ 000 \dots 000 & 010 \dots 0 \\ 000 \dots 000 & 001 \dots 0 \end{matrix}}^{|I|} & \overbrace{\begin{matrix} 000 \dots 000 \\ 000 \dots 000 \\ 000 \dots 000 \end{matrix}}^{|J|} \end{array} \right\} |I| \times |J| \right)$$

we notice that for  $A^T$ , there are two '1' in each row, one in  $|I|$  part and one in  $|J|$  part.

$$\text{Therefore:} \quad \min \sum_{i \in I} u_i + \sum_{j \in J} v_j$$

$$\text{s.t.} \quad u_i + v_j \geq 1 \quad \text{for all } i \in I, j \in J$$

$$u_i \geq 0 \quad \text{for } i \in I$$

$$v_j \geq 0 \quad \text{for } j \in J$$