VG441 PS1 周[illegible]凯 518021911039

1. We need to prove $\frac{\partial \theta^T X^T X \theta}{\partial \theta} = 2X^T X\theta$

Let $A = X^T X$, $A^T = (X^T X)^T = X^T (X^T)^T = X^T X = A$.

Therefore, $A = X^T X$ is symmetric.

Let A be a $n \times n$ matrix, and θ be a $n \times 1$ matrix.

Let $B = A\theta$, then $B_i = \sum_{m=1}^{n} A_{im}\theta_m$

Let $C = \theta^T B$, then $C = \sum_{p=1}^{n} \theta_p B_p = \sum_{p=1}^{n} \theta_p \left(\sum_{m=1}^{n} A_{pm}\theta_m\right)$

$$\begin{aligned} = & A_{11}\theta_1\theta_1 + A_{12}\theta_2\theta_1 + \cdots + A_{1n}\theta_n\theta_1 + \\ & A_{21}\theta_1\theta_2 + A_{22}\theta_2\theta_2 + \cdots + A_{2n}\theta_n\theta_2 + \\ & \vdots \\ & A_{n1}\theta_1\theta_n + A_{n2}\theta_2\theta_n + \cdots + A_{nn}\theta_n\theta_n \end{aligned}$$

$$\frac{\partial \theta^T X^T X \theta}{\partial \theta} = \frac{\partial C}{\partial \theta} = \begin{bmatrix} \frac{\partial C}{\partial \theta_1} \\ \frac{\partial C}{\partial \theta_2} \\ \vdots \\ \frac{\partial C}{\partial \theta_n} \end{bmatrix} = \begin{bmatrix} A_{11}\theta_1 + A_{12}\theta_2 + \cdots + A_{1n}\theta_n + A_{11}\theta_1 + A_{21}\theta_2 + \cdots + A_{n1}\theta_n \\ A_{21}\theta_1 + A_{22}\theta_2 + \cdots + A_{2n}\theta_n + A_{12}\theta_1 + A_{22}\theta_2 + \cdots + A_{n2}\theta_n \\ \vdots \\ A_{n1}\theta_1 + A_{n2}\theta_2 + \cdots + A_{nn}\theta_n + A_{1n}\theta_1 + A_{2n}\theta_2 + \cdots + A_{nn}\theta_n \end{bmatrix}$$

Since A is symmetric, we have $A_{ij} = A_{ji}$.

$$\frac{\partial \theta^T X^T X \theta}{\partial \theta} = \begin{bmatrix} 2(A_{11}\theta_1 + A_{12}\theta_2 + \cdots + A_{1n}\theta_n) \\ 2(A_{21}\theta_1 + A_{22}\theta_2 + \cdots + A_{2n}\theta_n) \\ \vdots \\ 2(A_{n1}\theta_1 + A_{n2}\theta_2 + \cdots + A_{nn}\theta_n) \end{bmatrix} = \begin{bmatrix} 2\sum_{m=1}^{n} A_{1m}\theta_m \\ 2\sum_{m=1}^{n} A_{2m}\theta_m \\ \vdots \\ 2\sum_{m=1}^{n} A_{nm}\theta_m \end{bmatrix} = \begin{bmatrix} 2B_1 \\ 2B_2 \\ \vdots \\ 2B_n \end{bmatrix} = 2B$$

$$= 2A\theta = 2X^T X\theta$$

Therefore, $\frac{\partial \theta^T X^T X \theta}{\partial \theta} = 2X^T X\theta$

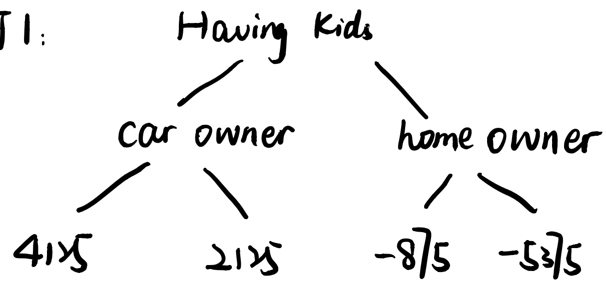
2. ①

F0	PR0	F1	PR1	F2	PR2
5875	4125	6287.5	3712.5	6658.75	3341.25
5875	-5375	5337.5	-4837.5	4853.75	-4353.75
5875	2125	6087.5	1912.5	6278.75	1721.25
5875	-875	5787.5	-787.5	5708.75	-708.75

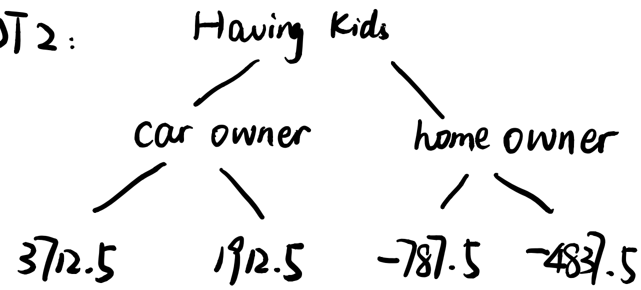
$F0 = \overline{salary} = 5875$

$PR0 = \text{True salary} - F0$

DT1:



DT2:



$F1 = F0 + \nu \times PR0$

$PR1 = \text{True salary} - F1$

$F_2 = F1 + \nu \times PR1$

$PR2 = \text{True salary} - F2$

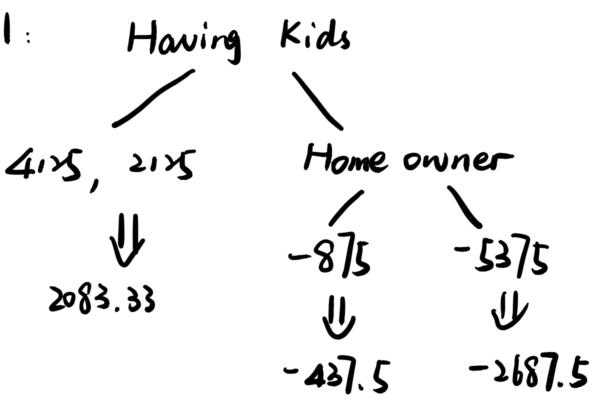
②

F0	PR0	F1	PR1	F2	PR2
5875	4125	6083.33	3916.67	6277.78	3722.22
5875	-5375	5606.25	-5106.25	5350.9375	-4850.9375
5875	2125	6083.33	1916.67	6277.78	1722.22
5875	-875	5831.25	-831.25	5789.6875	-789.6875

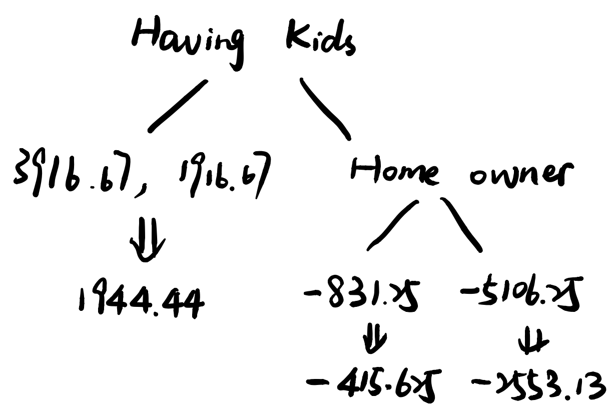
$F0 = \overline{salary} = 5875$

$PR0 = \text{True salary} - F0$

DT1:



DT2:



$F_1 = F_0 + \nu \cdot \text{output}$

$F_2 = F_1 + \nu \cdot \text{output}$