Midterm Problem 2

(a). $\lambda = 50$, $K = 50$, $iC_j = h = \frac{200}{12}$

$$Q_3^* = \sqrt{\frac{2 \times 50 \times 50}{200/12}} = 17.32$$

$$Q_2^* = \sqrt{\frac{2 \times 50 \times 50}{200/12}} = 17.32$$

$$Q_1^* = \sqrt{\frac{2 \times 50 \times 50}{200/12}} = 17.32 \approx 17$$

$$Q_0^* = \sqrt{\frac{2 \times 50 \times 50}{200/12}} = 17.32$$

Only Q_1^* is realizable, and it has cost

$$g(17) = 510 \times 50 + \frac{50 \times 50}{17} + \frac{\frac{200}{12} \times 17}{2} = 25788.7$$

Next, we calculate the cost of the breakpoints to the right of Q_1^*

$$g_2(65) = 495 \times 50 + \frac{50 \times 50}{65} + \frac{\frac{200}{12} \times 65}{2} = 25330.1$$

$$g_3(129) = 485 \times 50 + \frac{50 \times 50}{129} + \frac{\frac{200}{12} \times 129}{2} = 25344.4$$

Therefore, the optimal order quantity is $Q = 65$, and the cost per month is \$25330.1

(b). $$g(65) = 520 \times 50 + \frac{0 \cdot 50}{65} + \frac{0 \cdot 65}{2} = 2600 > 25330.1$$

Therefore, Zeus should not accept the offer.

Midterm Problem 3

(a). $K = 1000 \quad h = 1.2$

$$\theta_8 = 0$$

$$\theta_7 = K + h(0 \cdot d_4) + \theta_5$$
$$= 1000 \quad [S(7)=8]$$

$$\theta_6 = \min\{K + h(0 \cdot d_6) + \theta_7,\ K + h(0 \cdot d_6 + 1 \cdot d_7) + \theta_8\}$$
$$= \min\{2000, 1348\}$$
$$= 1348 \quad [S(6)=8]$$

$$\theta_5 = \min\{K + h(0 \cdot d_5) + \theta_6,\ K + h(0 \cdot d_5 + 1 \cdot d_6) + \theta_7,\ K + h(0 \cdot d_5 + 1 \cdot d_6 + 2 \cdot d_7) + \theta_8\}$$
$$= \min\{2348, 2252, 1948\}$$
$$= 1948 \quad [S(5)=8]$$

$$\theta_4 = \min\{K + h(0 \cdot d_4) + \theta_5,\ K + h(0 \cdot d_4 + 1 \cdot d_5) + \theta_6,\ K + h(0 \cdot d_4 + 1 \cdot d_5 + 2 \cdot d_6) + \theta_7,$$
$$K + h(0 \cdot d_4 + 1 \cdot d_5 + 2 \cdot d_6 + 3 \cdot d_7) + \theta_8\}$$
$$= \min\{2948, 2552, 2708, 2752\}$$
$$= 2552 \quad [S(4)=6]$$

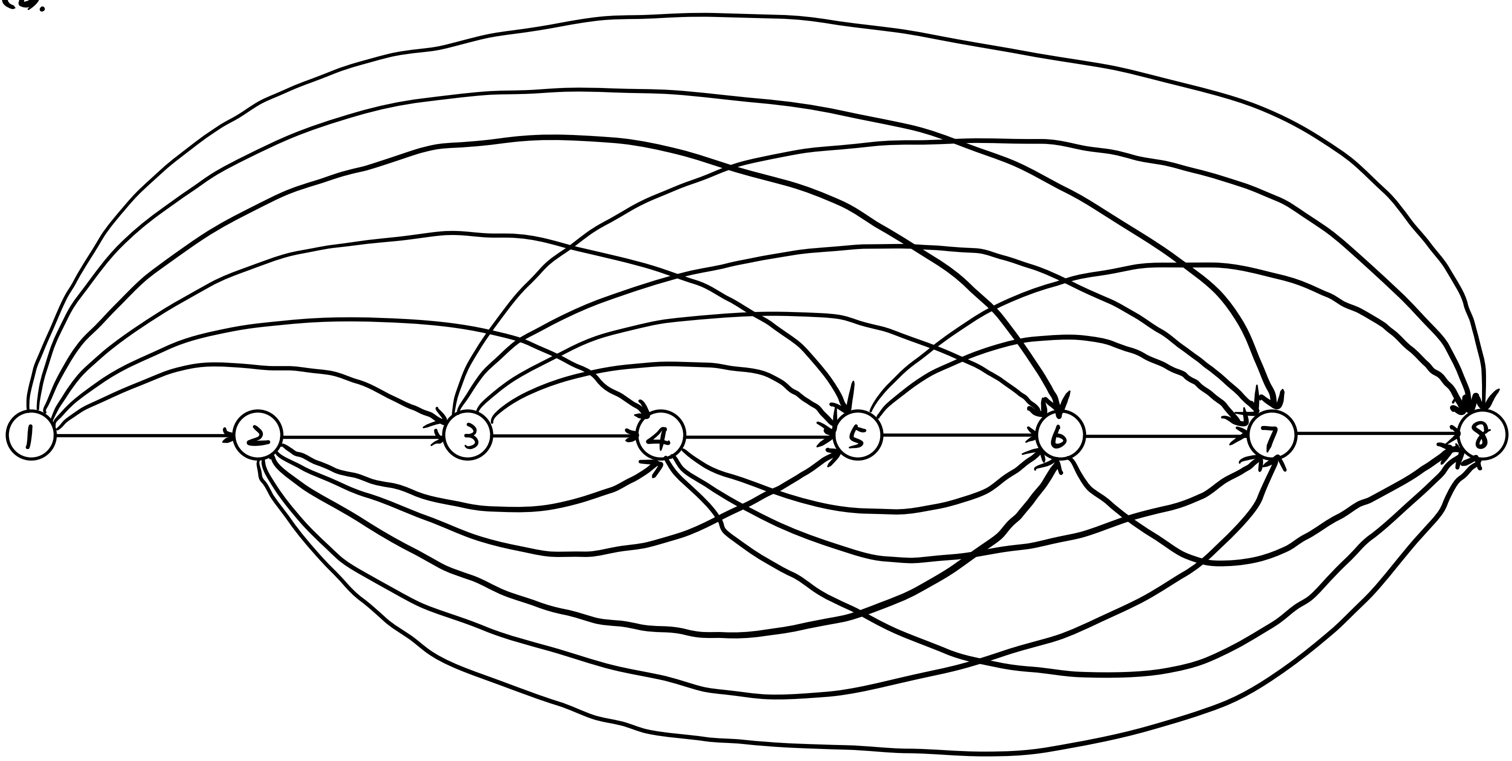
$$\theta_3 = \min\{K + h(0 \cdot d_3) + \theta_4,\ K + h(0 \cdot d_3 + 1 \cdot d_4) + \theta_5,\ K + h(0 \cdot d_3 + 1 \cdot d_4 + 2 \cdot d_5) + \theta_6,$$
$$K + h(0 \cdot d_3 + 1 \cdot d_4 + 2 \cdot d_5 + 3 \cdot d_6) + \theta_7,\ K + h(0 \cdot d_3 + 1 \cdot d_4 + 2 \cdot d_5 + 3 \cdot d_6 + 4 \cdot d_7) + \theta_8\}$$
$$= \min\{3552, 3056, 2864, 3272, 3664\}$$
$$= 2864 \quad [S(3)=6]$$

$$\theta_2 = \min\{K + h(0 \cdot d_2) + \theta_3,\ K + h(0 \cdot d_2 + 1 \cdot d_3) + \theta_4,\ K + h(0 \cdot d_2 + 1 \cdot d_3 + 2 \cdot d_4) + \theta_5,$$
$$K + h(0 \cdot d_2 + 1 \cdot d_3 + 2 \cdot d_4 + 3 \cdot d_5) + \theta_6,\ K + h(0 \cdot d_2 + 1 \cdot d_3 + 2 \cdot d_4 + 3 \cdot d_5 + 4 \cdot d_6) + \theta_7,$$
$$K + h(0 \cdot d_2 + 1 \cdot d_3 + 2 \cdot d_4 + 3 \cdot d_5 + 4 \cdot d_6 + 5 \cdot d_7) + \theta_8\}$$
$$= \min\{3864, 3678, 3290, 3302, 3962, 4702\}$$
$$= 3290 \quad [S(2)=5]$$

$$\theta_1 = \min\{K + h(0 \cdot d_1) + \theta_2,\ K + h(0 \cdot d_1 + 1 \cdot d_2) + \theta_3,\ K + h(0 \cdot d_1 + 1 \cdot d_2 + 2 \cdot d_3) + \theta_4,$$
$$K + h(0 \cdot d_1 + 1 \cdot d_2 + 2 \cdot d_3 + 3 \cdot d_4) + \theta_5,\ K + h(0 \cdot d_1 + 1 \cdot d_2 + 2 \cdot d_3 + 3 \cdot d_4 + 4 \cdot d_5) + \theta_6,$$
$$K + h(0 \cdot d_1 + 1 \cdot d_2 + 2 \cdot d_3 + 3 \cdot d_4 + 4 \cdot d_5 + 5 \cdot d_6) + \theta_7,$$
$$K + h(0 \cdot d_1 + 1 \cdot d_2 + 2 \cdot d_3 + 3 \cdot d_4 + 4 \cdot d_5 + 5 \cdot d_6 + 6 \cdot d_7) + \theta_8\}$$
$$= \min\{4290, 4050, 3990, 3710, 3926, 4838, 5926\}$$
$$= 3710 \quad [S(1)=5]$$

Therefore, on Day # 1, we should order 570, and then on Day # 5, we should order 670. Total cost is \$ 3710.

(b).



Edge cost:

$1\to2$: $K = 1000$

$1\to3$: $K+hD_2 = 1186$

$1\to4$: $K+hD_2+2hD_3 = 1438$

$1-5$: $K+hD_2+2hD_3+3hD_4 = 1762$

$1\to6$: $K+hD_2+2hD_3+3hD_4+4hD_5 = 2578$

$1\to7$: $K+hD_2+2hD_3+3hD_4+\cdots+5hD_6 = 3838$

$1\to8$: $K+hD_2+2hD_3+3hD_4+\cdots+6hD_7 = 5926$

$2\to3$: $K = 1000$

$2\to4$: $K+hD_3 = 1126$

$2\to5$: $K+hD_3+2hD_4 = 1342$

$2\to6$: $K+hD_3+2hD_4+3hD_5 = 1954$

$2\to7$: $K+hD_3+2hD_4+3hD_5+4hD_6 = 2962$

$2\to8$: $K+hD_3+2hD_4+\cdots+5hD_7 = 4702$

$3\to4$: $K = 1000$

$3\to5$: $K+hD_4 = 1108$

$3\to6$: $K+hD_4+2hD_5 = 1516$

$3\to7$: $K+hD_4+2hD_5+3hD_6 = 2272$

$3\to8$: $K+hD_4+2hD_5+3hD_6+4hD_7 = 3664$

$4\to5$: $K = 1000$

$4\to6$: $K+hD_5 = 1204$

$4\to7$: $K+hD_5+2hD_6 = 1708$

$4\to8$: $K+hD_5+2hD_6+3hD_7 = 2752$

$5\to6$: $K = 1000$

$5\to7$: $K+hD_6 = 1252$

$5\to8$: $K+hD_6+2hD_7 = 1948$

$6\to7$: $K = 1000$

$6\to8$: $K+hD_7 = 1348$

$7\to8$: $K = 1000$

i $A[1] = 0$
$B[1] = \emptyset$

ii $A[2] = 1000$
$B[2] = \{1\to2\}$

iii $A[3] = 1186$
$B[3] = \{1\to3\}$

iv. $A[4] = 1438$
$B[4] = \{1\to4\}$

v. $A[5] = 1762$
$B[5] = \{1\to5\}$

vi. $A[6] = 2578$
$B[6] = \{1\to6\}$

vii. $A[7] = 3014$
$B[7] = \{1\to5, 5\to7\}$

viii. $A[8] = 3710$
$B[8] = \{1\to5, 5\to8\}$

Therefore, on Day #1, we should order 570, and then on Day #5, we should order 670. Total cost is \$3710.

(c). Let q_t = the number of units ordered in period

$y_t = 1$ if we order in period t, 0 otherwise

x_t = the inventory level at the end of period, with $x_0 \equiv 0$.

$$\text{minimize} \sum_{t=1}^{T} (1000 y_t + 1.2 x_t)$$

$$\begin{aligned}
\text{subject to } \quad & x_t = x_{t-1} + q_t - d_t && \forall t = 1, \dots, T \\
& q_t \le M y_t && \forall t = 1, \dots, T \\
& x_t \ge 0 && \forall t = 1, \dots, T \\
& q_t \ge 0 && \forall t = 1, \dots, T \\
& y_t \in \{0, 1\} && \forall t = 1, \dots, T
\end{aligned}$$

where $T = 7$, $M = 100000$

The code is on next page.