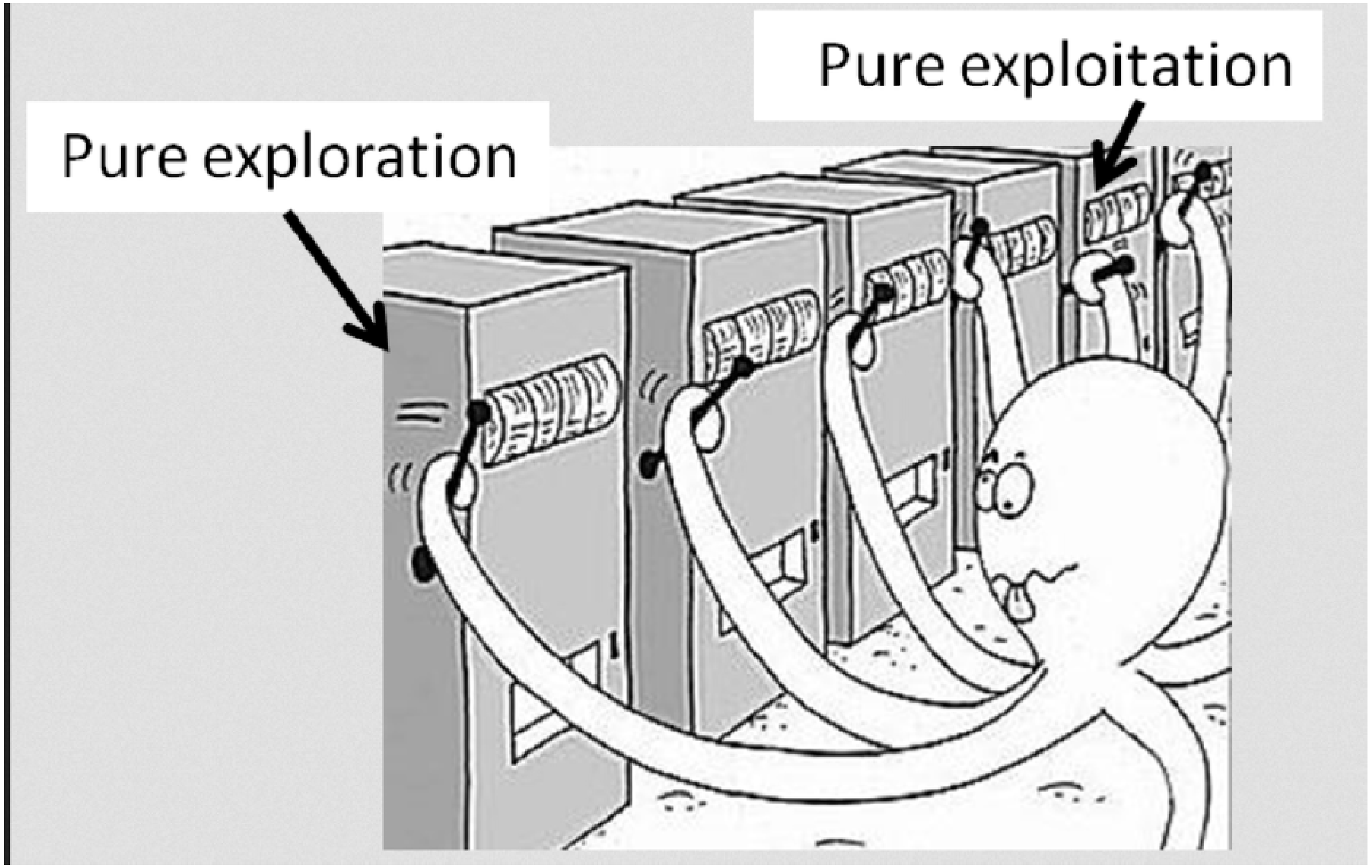
LEC019 MAB I

VG441 SS2021

Cong Shi
Industrial & Operations Engineering
University of Michigan

Basic RL for combining learning and decisions

- Multi-Armed Bandit Problem

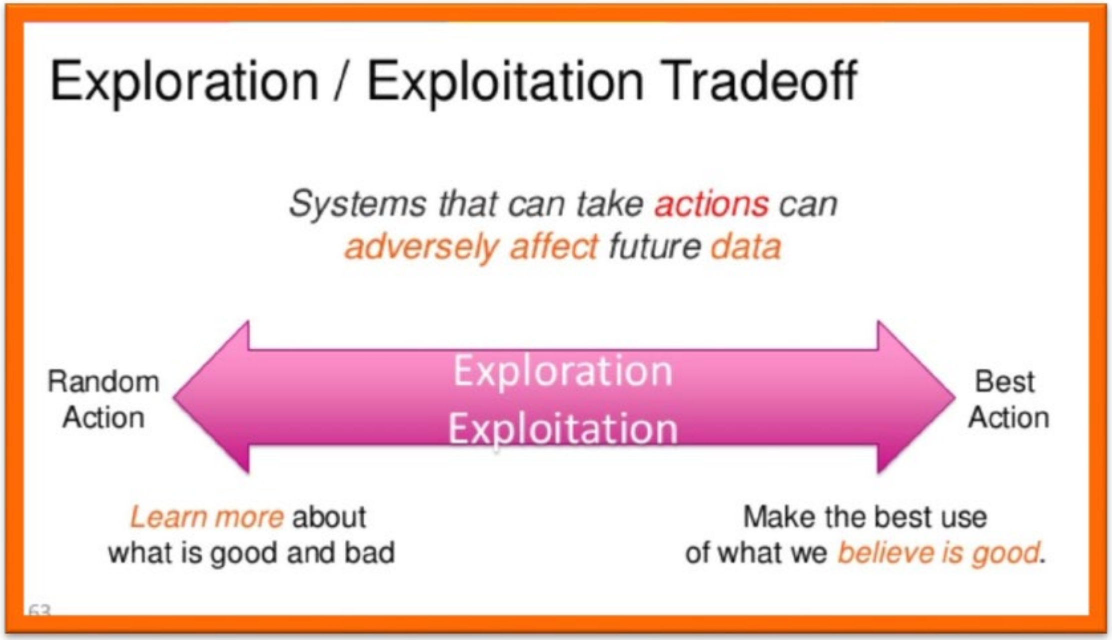


MAB

- Different machine generates different random rewards
- Gambler decides which slot machine to play with each token
- Maximize reward ($$)

Online decision-making: learning while doing

- Online decision-making involves a fundamental choice:
  - Exploration: Gather more information
  - Exploitation: Make the best decision given current information



- The best long-term strategy may involve short-term sacrifices

Example: Insufficient Exploration

1	2	3	4	5	6	7	8	

Example: Insufficient Exploration

1	2	3	4	5	6	7	8	
	$0		$0	$0				
$5		$5			$5	$5	$5	…

Example: Insufficient Exploration

	1	2	3	4	5	6	7	8	
		$0		$0	$0				
	$5	$5	$5	$5	$5	$5	$5	$5	…

It turns out  always pays $5/round

Example: Insufficient Exploration

	1	2	3	4	5	6	7	8	
		$0		$0	$0				
	$5	$5	$5	$5	$5	$5	$5	$5	…

It turns out  always pays **$5**/round

 pays **$100** a quarter of the time (**$25**/round on average)

Example: Insufficient Exploration

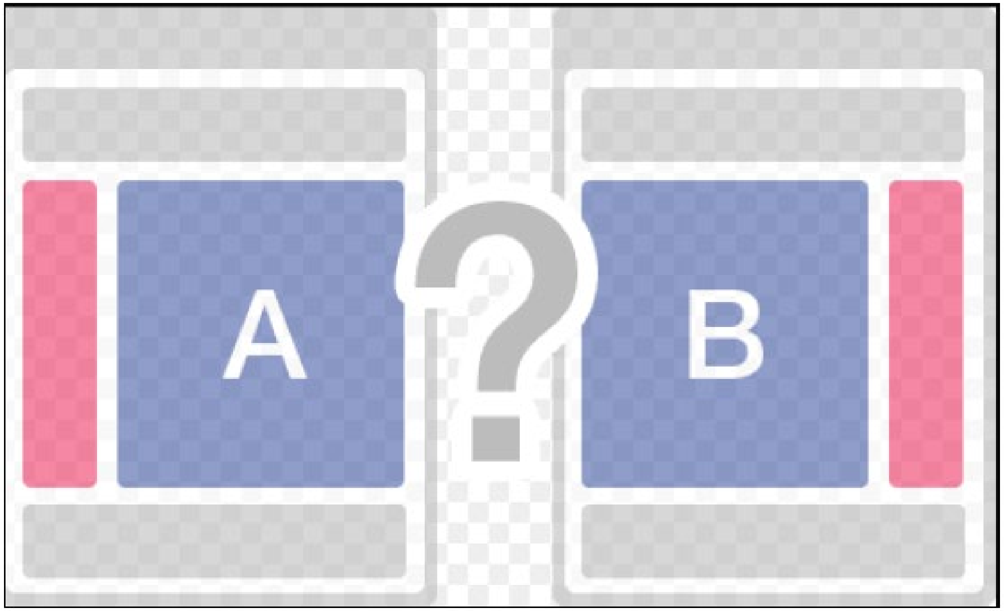
	1	2	3	4	5	6	7	8	
	$100	$0	$0	$0	$0	$100	$0	$100	
	$5	$5	$5	$5	$5	$5	$5	$5	…

It turns out  always pays **$5**/round

 pays **$100** a quarter of the time (**$25**/round on average)

A/B Testing

- Exploration: Gather more information about which design is better
- Exploration: Show the best design to the customer

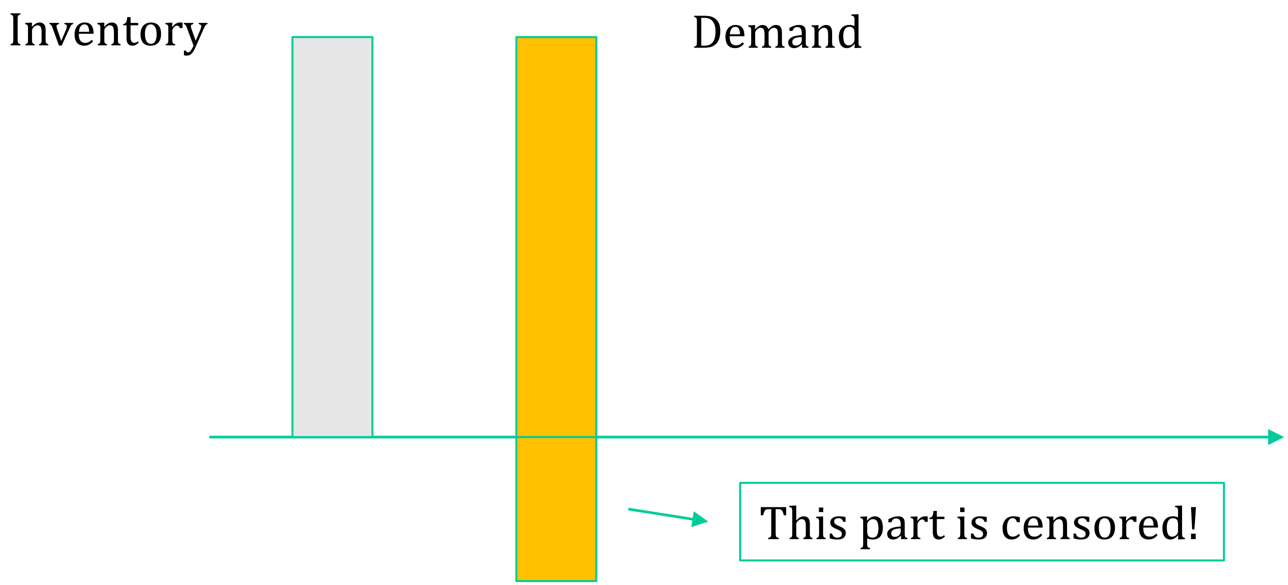


Revenue Management

- Retailers are interested in finding an optimal (pricing) policy to max revenue
- Unknown relationship between price and customer's purchasing decision (demand distribution)
  - Exploration: Gather more information about customers behavior using different prices
  - Exploitation: Make the best price based on the current information

Inventory Management

- Retailers are interested in finding an optimal (ordering) policy to min cost
- Unknown demand distribution (can only observe sales – censored demand)
  - Exploration: Order more to find out about true demand distribution
  - Exploitation: Order just right to minimize the cost

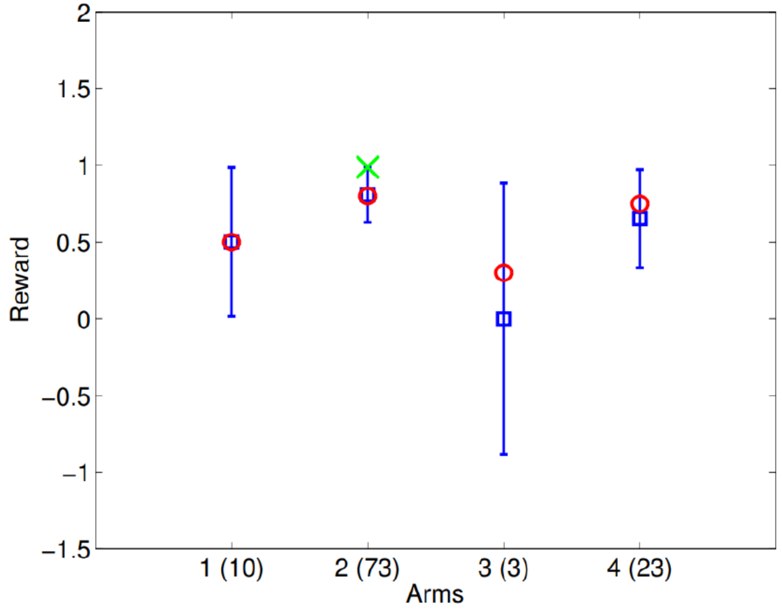


Other Applications

- Clinical trials
- Recommender systems
- Advertising: what ad to put on a web-page?
- Auctions
- Financial portfolio design
- Crowdsourcing

Many algorithms for MAB

- ϵ-greedy algorithm
- Upper confidence bound (UCB)
  - Add confidence bonus to the estimated mean
  - If the estimator is reliable, add less; if not, add more



$$i_t = \arg\max \left[\hat{\mu}_i + \underbrace{\sqrt{\frac{c \log t}{n_i}}}_{\mathrm{ucb}_i} \right]$$

- Thompson sampling
  - Bayesian setup with a prior distribution over reward parameters
  - Choose the auction that maximizes the expected reward under posterior

Online Network RM using TS

- ~$300B industry with ~10% annual growth over the last 5 years
  - IBISWorldUS Industry Report; excludes online sales of traditionally brick & mortar stores

- Online retailers have additional information as compared to brick & mortar retailers, e.g. real-time customer purchase decisions (buy / no buy)
  - How can we use this information to develop a more effective revenue management strategy?

Setting

- Finite selling horizon of T periods
  - One customer arrives per period
  - Sequentially observe customer purchase decisions

- Finite set of prices; i-th price denoted by p_i

- Unknown mean demand per price (“purchase probability”) d_i

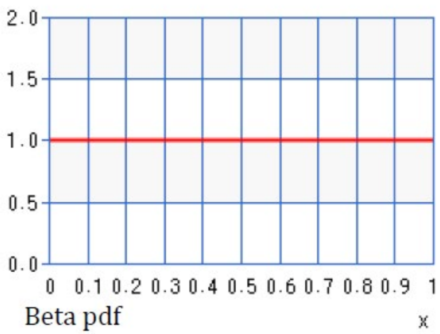
- Given unlimited inventory and known demand, select price with highest revenue $= p_i \times d_i$

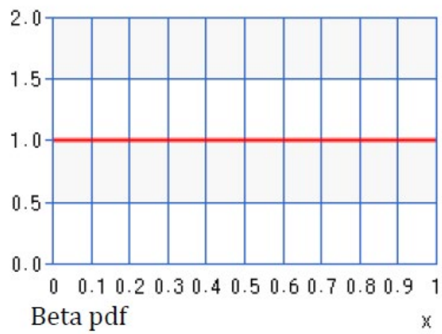
- Challenges: unknown demand
- Exploration vs. Exploitation Tradeoff

RM-MAB

- Retailer decides…
  - Which price to offer to a customer
  - How many times to offer each price
  - In what order to offer prices to customers
- Learns demand at each price to max revenue

RM-MAB



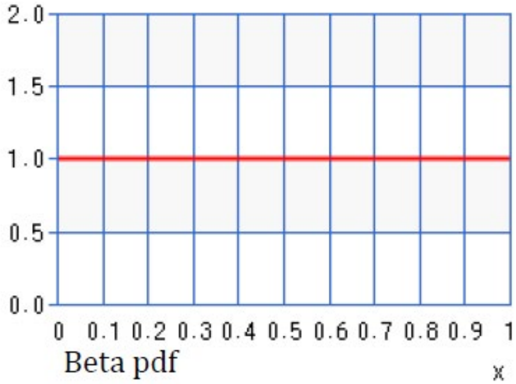


$\hat{d}_1 \sim Beta(1,1)$
True (unknown) $d_1 = 0.6$

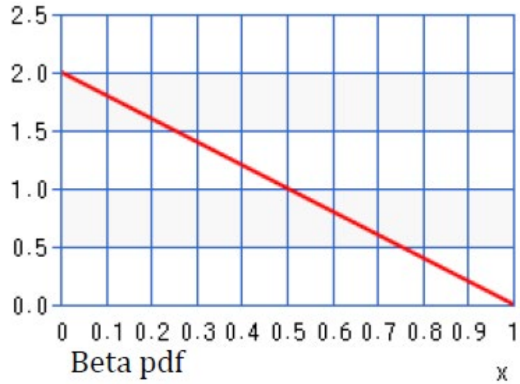
$\hat{d}_2 \sim Beta(1,1)$
True (unknown) $d_2 = 0.3$

1. Customer arrives
2. Retailer samples θ_1 and θ_2 from current distributional estimation of d_1 and d_2
3. Retailer offers price that maximizes $p_i \theta_i$
4. Customer makes purchase decision (according to d_i)
5. Retailer observes purchase decision and updates demand estimation

RM-MAB



$\hat{d}_1 \sim Beta(1,1)$
True (unknown) $d_1 = 0.6$



$\hat{d}_2 \sim Beta(1,1)$
True (unknown) $d_2 = 0.3$

update

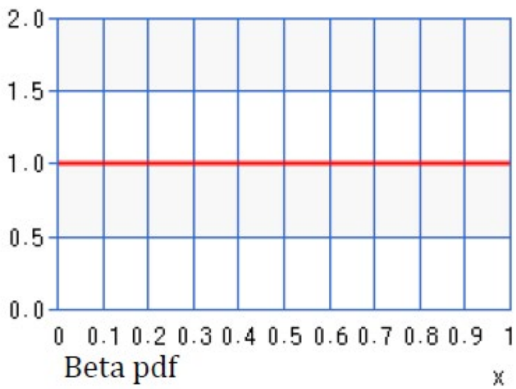
$\hat{d}_2 \sim Beta(1,1+1)$
True (unknown) $d_2 = 0.3$

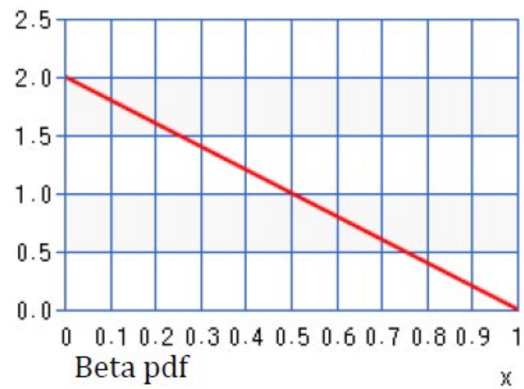
$\theta_1 = 0.41, \theta_2 = 0.83$

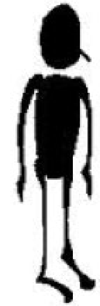
$p_2\theta_2 > p_1\theta_1$

Customer does not buy item 2

RM-MAB







$\theta_1 = 0.93, \theta_2 = 0.12$

$p_1\theta_1 > p_2\theta_2$

$\hat{d}_1 \sim Beta(1,1)$

True (unknown) $d_1 = 0.6$

$\hat{d}_2 \sim Beta(1,2)$

True (unknown) $d_2 = 0.3$

Customer buys item 1

update

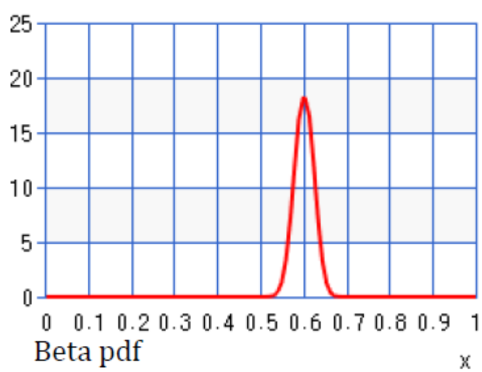
$\hat{d}_1 \sim Beta(1+1,1)$

True (unknown) $d_1 = 0.6$

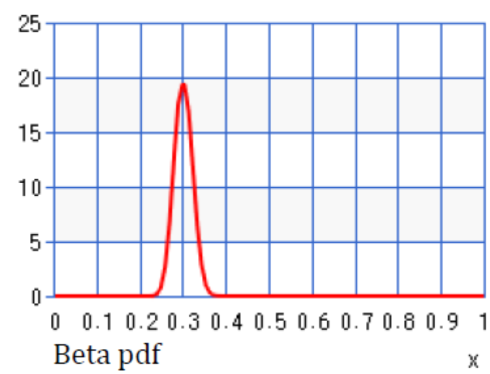
RM-MAB: 2 Price Example

As each price is offered more times…

- Beta pdf converges to reflect true mean demand
- Will choose optimal price with high probability



$\hat{d}_1$ ~ Beta(1 + # "buy", 1 + # "no buy")
True (unknown) d_1=0.6



$\hat{d}_2$ ~ Beta(1 + # "buy", 1 + # "no buy")
True (unknown) d_2=0.3

Advantages of Thompson Sampling

- Empirical and theoretical results show it's a highly competitive algorithm for unlimited inventory
- Easy to implement and understand
- Non-parametric
- Continuous exploration & exploitation

How do we incorporate inventory constraints?

Key Tradeoffs:

- Exploration vs. Exploitation
- Explore at the cost of running out of inventory

RM-with inventory constraint

1. Customer arrives
2. Retailer samples θ_1 and θ_2
3. Retailer solves a deterministic LP to identify the optimal fraction of remaining customers to offer p_1 and p_2, using
   - θ_1 and θ_2
   - Remaining unsold inventory & customers
4. Retailer offers price p_i with probability based on fraction found in Step 3
5. Customer makes purchase decision
6. Retailer observes decision and updates $\hat{d}_i$

RM-with inventory constraint

$x_i =$ fraction of remaining customers (T-t) to offer price p_i

$\max_{x_1,x_2} \sum_{T-t} p_1\theta_1 x_1 + p_2\theta_2 x_2$	maximize revenue over remaining customers
$s.t.\ x_1 + x_2 \leq 1$	fraction of remaining customers ≤ 1
$(T-t)(\theta_1 x_1 + \theta_2 x_2) \leq Inv(t)$	expected inventory sold is upper-bounded by remaining inventory
$x_1, x_2 \geq 0$	

RM-with inventory constraint

Regret = E[Revenue of Optimal Policy with Known Demand] –
E[Revenue of Algorithm]

≤ Upper Bound on Optimal Policy –
E[Revenue of Algorithm]

Theorem

Suppose the LP of the underlying true demand (i.e. benchmark) is nondegenerate. Then, for the modified Thompson Sampling with Inventory Algorithm,

$$Regret(T) \leq O\left(\sqrt{T} \log T \log \log T\right) = \tilde{O}\left(\sqrt{T}\right)$$