

2. For n items, each item I_i has a value v_i and size S_i . The capacity is B . First, we can re-index them such that $\frac{v_1}{S_1} \geq \frac{v_2}{S_2} \geq \dots \geq \frac{v_n}{S_n}$

We can write the greedy solution as $G = (x_1, x_2, x_3, \dots, x_n)$, where x_i indicates fraction of item I_i taken.

We can write any optimal solution as $O = (y_1, y_2, y_3, \dots, y_n)$, where y_i indicates fraction of item I_i taken.

We can get $\sum_{i=1}^n x_i S_i = \sum_{i=1}^n y_i S_i = B$

For the first item I_a where the two solutions differ from each other, we can have $x_a > y_a$ since greedy always takes as much as it can.

Then we consider a new solution $O' = (y'_1, y'_2, y'_3, \dots, y'_n)$.

For $j < a$, we let $y'_j = y_j$.

For $j = a$, we let $y'_j = x_j$.

For $j > a$, we remove items of total size $(x_i - y_i) S_i$ and reset y'_j .

The total value of solution O' is no less than O .

Since O is an optimal solution, the total value of O' and O must equal to each other.

Therefore, O' is also an optimal solution.

If we continue this process, we can finally convert O into G without changing the total value.

Therefore, G is an optimal solution.