VP240-1 Recitation class
Week #7

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Brief introduction

- Do the exercise and check your answers.
- Don't leave the questions to the last minutes.

1 Short Review

1. Electrostatics and Magnetostatics
   ELECTROSTATICS: stationary charges $\Rightarrow$ constant (in time) electric field
   MAGNETOSTATICS: steady currents $\Rightarrow$ constant (in time) magnetic field
2. Law of Biot and Savart
   Magnetic constant: $\epsilon_0 = 4\pi \times 10^{-7} N/A^2$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2} \tag{1}$$

Info:

- Principle of superposition of magnetic fields: the total magnetic field caused by several moving charges is the vector sum of the fields caused by the individual charges.
- The volume of the segment is A $d\vec{l}$, where A is the cross-sectional area of the conductor. If there are n moving charged particles per unit volume, each of charge q, the total moving charge dQ in the segment is:

  $$dQ = nqAdl \tag{2}$$

  Thus, the magnitude of the resulting field d$\vec{B}$ at any field point P is:

  $$dB = \frac{\mu_0}{4\pi}\frac{|dQ|v_\mathrm{d}\sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{n|q|v_\mathrm{d}Adl\sin\phi}{r^2} \tag{3}$$

  But from the equation above, we have $n|q|v_\mathrm{d}A$ equals the current I in the element. So:

  $$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2} \tag{4}$$

- $$\vec{B} = \frac{\mu_0}{4\pi}\int\frac{Id\vec{l} \times \hat{r}}{r^2} \tag{5}$$

3. magnetic field of a straight current carrying conductor

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{x dy}{(x^2 + y^2)^{3/2}} \Rightarrow B = \frac{\mu_0 I}{2\pi x} \tag{6}$$

> **Info:**
>
> - Magnetic field around a long, straight, current-carrying conductor. The field lines are circles, with directions determined by the right-hand rule.

4. Force between parallel wires

From above equation, the force that this field exerts on a length L of the upper conductor is $\overrightarrow{\boldsymbol{F}} = I' \overrightarrow{\boldsymbol{L}} \times \overrightarrow{\boldsymbol{B}}$, where the vector $\overrightarrow{\boldsymbol{L}}$ is in the direction of the current I' and has magnitude L. since $\overrightarrow{\boldsymbol{B}}$ is perpendicular to the length of the conductor and hence to $\overrightarrow{\boldsymbol{L}}$, the magnitude of this force is:

$$F = I'LB = \frac{\mu_0 I I' L}{2\pi r} \tag{7}$$

and the force *per unit length F/L* is:

$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r} \tag{8}$$

> **Info:**
>
> - Magnetic force per unit length between two long, parallel, current-carrying conductors
> - Thus two parallel conductors carrying current in the same direction attract each other. If the direction of either current is reversed, the forces also reverse. Parallel conductors carrying currents in opposite directions repel each other.

5. Ampere's law

The definition of the SI definition of the **ampere**: One ampere is that unvarying current that, if present in each of two parallel conductors of infinite length and one meter apart in empty space, causes each conductor to experience a force of exactly 2×10^{-7} newtons per meter of length.

$$\oint \overrightarrow{\boldsymbol{B}} \cdot d\vec{l} = \mu_0 I_{\text{encl}} \tag{9}$$

If $\oint \overrightarrow{\boldsymbol{B}} \cdot d\vec{l} = 0$, it does not necessarily mean that $\overrightarrow{\boldsymbol{B}} = \mathbf{0}$ everywhere along the path, only that the total current through an area bounded by the path is zero.

- the line integrals of electric and magnetic fields–Page 960.
- the field of a **toroidal solenoid or solenoid**.

2 Discussion

- Q28.1 A topic of current interest in physics research is the search (thus far unsuccessful) for an isolated magnetic pole, or magnetic monopole. If such an entity were found, how could it be recognized? What would its properties be?

- Q28.2 Streams of charged particles emitted from the sun during periods of solar activity create a disturbance in the earth's magnetic field. How does this happen?

- Q28.5 Pairs of conductors carrying current into or out of the power-supply components of electronic equipment are sometimes twisted together to reduce magnetic-field effects. Why does this help?

- Q28.6 Suppose you have three long, parallel wires arranged so that in cross section they are at the corners of an equilateral triangle. Is there any way to arrange the currents so that all three wires attract each other? So that all three wires repel each other? Explain.
- Q28.9 A current was sent through a helical coil spring. The spring contracted, as though it had been compressed. Why?
- Q28.11 Magnetic field lines never have a beginning or an end. Use this to explain why it is reasonable for the field of an ideal toroidal solenoid to be confined entirely to its interior, while a straight solenoid must have some field outside.

3 Problems and exercises

Question 1

A conductor shaped as shown in the figure carries constant current I. Find the magnetic field in the center of the circle.

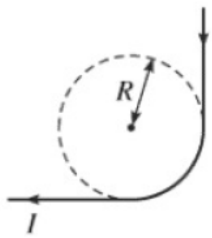

Figure 1: Rehash table using double hashing

Question 2

A conical surface $(x^2 + y^2 = z^2, 0 \leq z \leq h)$ charged uniformly with surface density of charge σ rotates about it axis of symmetry with constant angular velocity ω. Find the magnetic field at the origin.

Question 3

Exercises: 28.15, 28.24, 28.28, 28.37, 28.42, 28.45, 28.48.

Question 4

Problems: 28.70, 28.71, 28.77, 28.83, 28.88 (challenge problem)

Question 5

(optional) Charge Q is distributed uniformly over the volume of a ball with radius R. One half of the ball rotates about its symmetry axis with constant angular velocity $\omega 1$ and the other one with constant angular velocity $\omega 2$ but in the direction opposite to $\omega 1$. (a) Find the magnetic field at the origin. (b) What should be the ratio of the charge (distributed uniformly) in one half to the charge in the other half, so that the field in the center of the ball is zero.