

University of Michigan – Shanghai Jiao Tong University Joint Institute (UM-SJTU JI)

**Maxwell's Equations** 

# Maxwell's Equations

Gauss's law for  $\vec{E}$ :

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$$
 Charge enclosed by surface Electric constant

(29.18)

Gauss's law for  $\vec{B}$ :

Flux of magnetic field through any closed surface ...

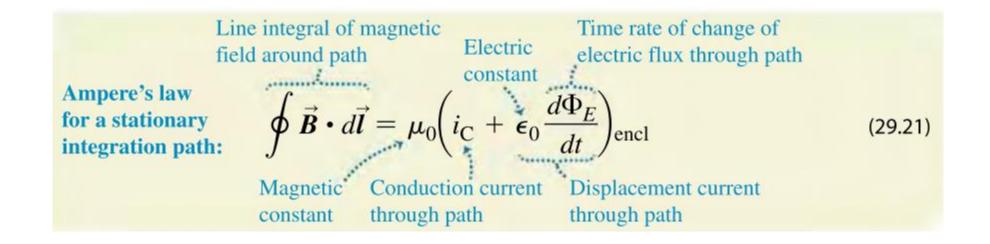
$$\oint \vec{B} \cdot d\vec{A} = 0 \quad \dots \text{ equals zero.}$$
 (29.19)

Faraday's law for a stationary integration path: Line integral of electric field around path

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$
Negative of the time rate of change of magnetic flux through path (29.20)



# Maxwell's Equations



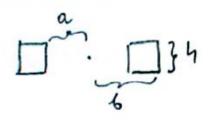


# Inductance

## Toroidal solenoid

(inductance of a toroidal coillittle rectangular cross-section)





Magnetic field inside  $B = \frac{M_0 I N}{2\pi r}$  (see rec. chass)

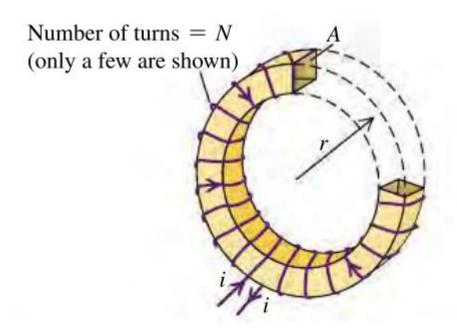
Flux through a single turn

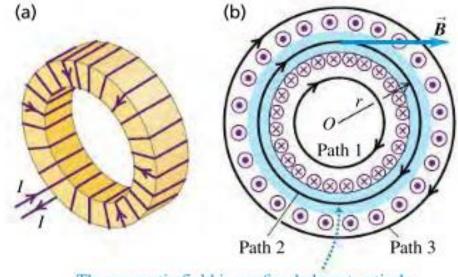
$$\overline{\Phi}_{B}^{(i)} = \int \overline{B} \cdot d\overline{S} = \frac{\mu_{0} I N}{2\pi} h \int \frac{dr}{r} = \frac{\mu_{0} I N h}{2\pi} ln(\frac{b}{a})$$

$$\Phi_{B} = N \cdot \Phi_{B}^{(1)} = \frac{\mu_{o} N^{2} h}{2\pi} \ln(\frac{6}{a}) I ; \qquad L = \frac{\mu_{o} N^{2} h}{2\pi} \ln(\frac{6}{a})$$
# of turns



## Toroidal solenoid





The magnetic field is confined almost entirely to the space enclosed by the windings (in blue).



#### Toroidal solenoid

Example. Energy Horeol in a toroidal rolenoid. (Assume 6-2 x 2)

$$L = \frac{M_0 N^2 A}{2 \sqrt{R}}$$

$$U = \frac{1}{2}LI^2 = \frac{1}{2}\frac{\mu_0 N^2 A}{2\pi R}I^2$$

Volume of the torroidal coil: V = 211RA

$$R = \frac{2\pi}{2}$$

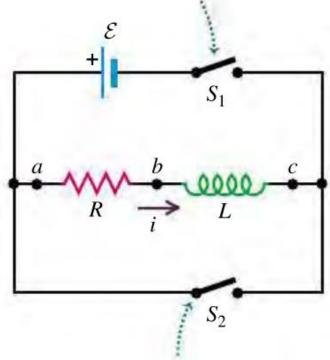
$$2\pi R \times A \otimes$$





# **RL** circuits

Closing switch  $S_1$  connects the R-L combination in series with a source of emf  $\mathcal{E}$ .

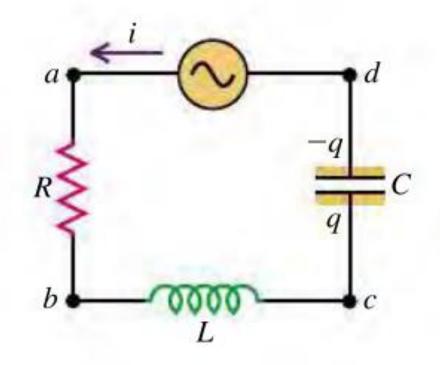


Closing switch  $S_2$  while opening switch  $S_1$  disconnects the combination from the source.



**RLC** circuits and AC circuits

# RLC & AC circuits





# Good luck for your final exam!

