VP240 Homework 7 周[illegible] 518021911039

1. $B_1 = \frac{\mu_0}{4\cancel{\pi}} \times \frac{\cancel{\pi R\times\frac{1}{4}}}{R^2} \times I_1 = \frac{\mu_0 I_1}{4R}$ The direction is into the paper

$B_2 = \frac{\mu_0}{4\cancel{\pi}} \times \frac{\cancel{\pi R\times\frac{1}{4}}}{R^2} \times I_2 = \frac{\mu_0 I_2}{4R}$ The direction is out of the paper

$B = \frac{\mu_0}{4R}|I_1 - I_2|$

If $I_1 = I_2$, $B=0$

2. 

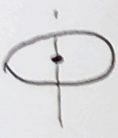
For ①: $F_1 = \frac{\mu_0 I}{2\pi s}\times a \times I = \frac{a\mu_0 I^2}{2\pi s}$

For ②: $F_2 = \int_0^a \frac{\mu_0 I}{2\pi(s+\frac{\sqrt{3}}{2}a)} I\,da = \frac{I^2\mu_0\sqrt{3}}{3\pi}\left(\ln\left(\frac{\sqrt{3}a+2s}{2}\right) - \ln s\right)$

For ③: $F_3 = \int_0^a \frac{\mu_0 I}{2\pi(s+\frac{\sqrt{3}}{2}a)} I\,da = \frac{I^2\mu_0\sqrt{3}}{3\pi}\left(\ln\left(\frac{\sqrt{3}a+2s}{2}\right) - \ln s\right)$

$\vec{F} = \left(F_1 - \frac{1}{2}F_2 - \frac{1}{2}F_3\right)\hat{n}_{up} = \left[\frac{a\mu_0 I^2}{2\pi s} - \frac{I^2\mu_0\sqrt{3}}{3\pi}\left(\ln\left(\frac{\sqrt{3}a+2s}{2}\right) - \ln s\right)\right]\hat{n}_{up}$

The direction of $\hat{n}_{up}$ is upward.

3. 

$\rho = \frac{Q}{\pi a^2}$ $T = \frac{1}{n}$

$dI(r) = \frac{\frac{Q}{\cancel{\pi} a^2}\cdot 2\cancel{\pi} r\,dr}{T} = \frac{2nQr}{a^2}dr$

$B = \int \frac{\mu_0\, dI}{2\cancel{\pi r}} \times \frac{\cancel{2\pi r}}{r^2}$

$= \int_0^a \frac{\mu_0}{\cancel{2}\cancel{r}}\cdot\frac{\cancel{2}nQ\cancel{r}}{a^2}dr$

$= \frac{\mu_0 nQ}{a^2}\cdot a = \frac{\mu_0 nQ}{a}$

If $Q>0$, rotates counterclockwise, the direction is upward.

4.(a). $\rho = \frac{I}{L}$

$B = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\mu_0 \frac{I}{L}}{2\pi\sqrt{x^2+y^2}}\cdot\frac{y}{\sqrt{x^2+y^2}}dx = \frac{I\mu_0}{\pi L}\arctan\left(\frac{L}{2y}\right)$

(b). When $y \gg L$, $\arctan \approx \frac{L}{2y}$

$B = \frac{I\mu_0}{\pi L}\times\frac{L}{2y} = \frac{I\mu_0}{2\pi y}$, which is the result of an infinitely long line em 5. a: $\oint \vec{B} \cdot d\vec{l} = 0$

b: $\oint \vec{B} \cdot d\vec{l} = -\mu_0 I_1$

c: $\oint \vec{B} \cdot d\vec{l} = \mu_0 (I_2 - I_1)$

d: $\oint \vec{B} \cdot d\vec{l} = \mu_0 (I_2 + I_3 - I_1)$

Problem 6.

The unit of a and δ is m, the unit of b is A/m.

(a). $I_0 = \int_0^a \frac{b}{r} e^{\frac{r-a}{\delta}} 2\pi r \, dr$

$= 2\pi b (\delta - \delta e^{-\frac{a}{\delta}})$

(b). Let the loop traversed in the counterclockwise direction

And the loop is the circule with radius r to the z-axis.

$\oint \vec{B} \cdot d\vec{r} = \mu_0 I_0$

$B \cdot 2\pi r = \mu_0 I_0$

$B = \frac{\mu_0 I_0}{2\pi r}$

If the current is upward, then B is counterclocle wise.

(c). $I = \int_0^r \frac{b}{r} e^{\frac{r-a}{\delta}} 2\pi r \, dr = 2\pi b \delta e^{-\frac{a}{\delta}} (e^{\frac{r}{\delta}} - 1)$

From (a), we can know that $2\pi b \delta = \frac{I_0}{1 - e^{-\frac{a}{\delta}}}$

Then $I = \frac{I_0 e^{-\frac{a}{\delta}}}{1 - e^{-\frac{a}{\delta}}} (e^{\frac{r}{\delta}} - 1)$

(d). Let the loop traversed in the counterclockwise direction

And the loop is the circule with radius r to the z-axis

$\oint \vec{B} \cdot d\vec{r} = \mu_0 I$

$B = \frac{\mu_0}{2\pi r} I = \frac{\mu_0}{2\pi r} \cdot \frac{I_0 e^{-\frac{a}{\delta}}}{1 - e^{-\frac{a}{\delta}}} (e^{\frac{r}{\delta}} - 1)$

If the current is upward, the B is counterclockwise.