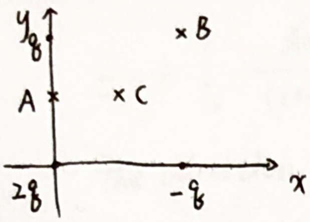
VP 240 Homework - 1 周维凯 518021911039

Problem 1.



Set the coordinate like shown on the left, suppose $k = \frac{1}{4\pi\varepsilon_0}$

For A:

$$\vec{E_A} = k\frac{2q}{(\frac{a}{2})^2}\hat{n_y} - k\frac{q}{(\frac{a}{2})^2}\hat{n_y} + \frac{2}{\sqrt{5}}k\frac{q}{(\frac{\sqrt{5}a}{2})^2}\hat{n_x} - \frac{1}{\sqrt{5}}k\frac{q}{(\frac{\sqrt{5}a}{2})^2}\hat{n_y}$$

$$= 6.43\times10^{20}\ N/C\ \hat{n_x} + 3.27\times10^{21}\ N/C\ \hat{n_y}$$

For B:

$$\vec{E_B} = k\frac{q}{a^2}\hat{n_x} + \frac{\sqrt{2}}{2}k\frac{2q}{(\sqrt{2}a)^2}\hat{n_x} + \frac{\sqrt{2}}{2}k\frac{2q}{(\sqrt{2}a)^2}\hat{n_y} - k\frac{q}{a^2}\hat{n_y}$$

$$= 1.53\times10^{21}\ N/C\ \hat{n_x} - 2.63\times10^{20}\ N/C\ \hat{n_y}$$

For C:

$$\vec{E_C} = \frac{\sqrt{2}}{2}k\frac{q}{(\frac{\sqrt{2}}{2}a)^2}\hat{n_x} - \frac{\sqrt{2}}{2}k\frac{q}{(\frac{\sqrt{2}}{2}a)^2}\hat{n_y} + \frac{\sqrt{2}}{2}k\frac{2q}{(\frac{\sqrt{2}}{2}a)^2}\hat{n_x} + \frac{\sqrt{2}}{2}k\frac{2q}{(\frac{\sqrt{2}}{2}a)^2}\hat{n_y}$$

$$+ \frac{\sqrt{2}}{2}k\frac{q}{(\frac{\sqrt{2}}{2}a)^2}\hat{n_x} - \frac{\sqrt{2}}{2}k\frac{q}{(\frac{\sqrt{2}}{2}a)^2}\hat{n_y}$$

$$= 5.08\times10^{21}\ N/C\ \hat{n_x}$$

Problem 2.

According to symmetry, $E_D = 0$ and for E_A, we just need to consider the part where $x>0$ and $y<0$. Since the charge of this part in A is $\frac{Q}{3}$ and the charge of the similiar part in C is Q, we can get $E_A = \frac{1}{3}E_C$

And for B $E_B = \frac{\sqrt{2}}{2}E_C$

Therefore $E_C > E_B > E_A > E_D$

Problem 3.

a). $dq = \lambda dx$, and $q = \lambda l$.

Suppose $k = \frac{1}{4\pi\varepsilon_0}$

$$E = k\int_0^l \frac{\lambda\, dx}{(l+a-x)^2} = k\cdot\frac{\lambda l}{a(a+l)} = \frac{\lambda l}{4\pi\varepsilon_0 a(a+l)}$$

The direction is horizontally to the left.

b). $dq = Ax\,dx$, and $q = \frac{1}{2}AL^2$

$$E = k\int_0^L \frac{Ax\,dx}{(L+a-x)^2} = \frac{A}{4\pi\varepsilon_0}\left[\ln\left(\frac{a}{a+L}\right) + \frac{L}{a}\right]$$

The direction is horizontally to the left.

Problem 4.

a). $dq = \lambda\,dx$, and $Q = \lambda l$

$$E = \frac{1}{4\pi\varepsilon_0}\int_{-\frac{a}{2}-L}^{-\frac{a}{2}} \frac{\lambda\,dx}{(x_0-x)^2}\,dx = \frac{Q}{4\pi\varepsilon_0}\cdot\frac{1}{(x_0+\frac{a}{2})(x_0+\frac{a}{2}+L)}$$

$$F = \frac{Q}{4\pi\varepsilon_0}\int_{\frac{a}{2}}^{\frac{a}{2}+L} \frac{\lambda\,dx_0}{(x_0+\frac{a}{2})(x_0+\frac{a}{2}+L)} = \frac{Q}{4\pi\varepsilon_0}\,\frac{\lambda\ln\left[\frac{(2L+2a)^2}{2a(2a+4L)}\right]}{L}$$

$$= \frac{Q^2}{4\pi\varepsilon_0 L^2}\ln\left[\frac{(L+a)^2}{a(a+2L)}\right]$$

b). $\ln\left[\frac{(L+a)^2}{a(a+2L)}\right] = \ln\left(1+\frac{L^2}{a^2+2aL}\right) = \ln\left(1+\frac{1}{(\frac{a}{L})^2+2(\frac{a}{L})}\right)$

Since $a \gg L$, $\left|\frac{1}{(\frac{a}{L})^2+2(\frac{a}{L})}\right| \ll 1$, then

$$\ln\left[\frac{(L+a)^2}{a(a+2L)}\right] \approx \frac{L^2}{a^2+2aL} - \frac{\left(\frac{L^2}{a^2+2aL}\right)^2}{2} + \frac{\left(\frac{L^2}{a^2+2aL}\right)^3}{3}\cdots \approx \frac{L^2}{a^2+2aL}$$

Therefore $F \approx \frac{Q^2}{4\pi\varepsilon_0 L^2}\cdot\frac{L^2}{a^2+2aL} = \frac{Q^2}{4\pi\varepsilon_0 a^2(1+2\frac{L}{a})}$

Since $a \gg L$, $\frac{L}{a} \to 0$, $1+2\frac{L}{a} \approx 1$

Therefore $F \approx \frac{Q^2}{4\pi\varepsilon_0 a^2}$

It means that when the length of rods is small enough compared with the distance, the rods can be regarded as one point or the point charge.

Problem 5.

a). From class, we know that the disk with radius R,

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{1+\frac{R^2}{x^2}}}\right)\hat{n}_x$$

Then, in this problem

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{1+\frac{R_2^2}{x^2}}}\right)\hat{n}_x - \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{1+\frac{R_1^2}{x^2}}}\right)\hat{n}_x$$

$$= \frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{\sqrt{x^2+R_1^2}} - \frac{1}{\sqrt{x^2+R_2^2}}\right)\hat{n}_x$$

b). When $x \to 0$, $\frac{1}{\sqrt{x^2+R_1^2}} - \frac{1}{\sqrt{x^2+R_2^2}} \to \frac{1}{R_1} - \frac{1}{R_2}$

Then $\vec{E} \approx \frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\hat{n}_x$

When it is sufficiently close to the geometric center of the disk, the magnitude of the force exerted by the electric field is approximately propotional to the distance from the center.