JOINT INSTITUTE
交大密西根学院

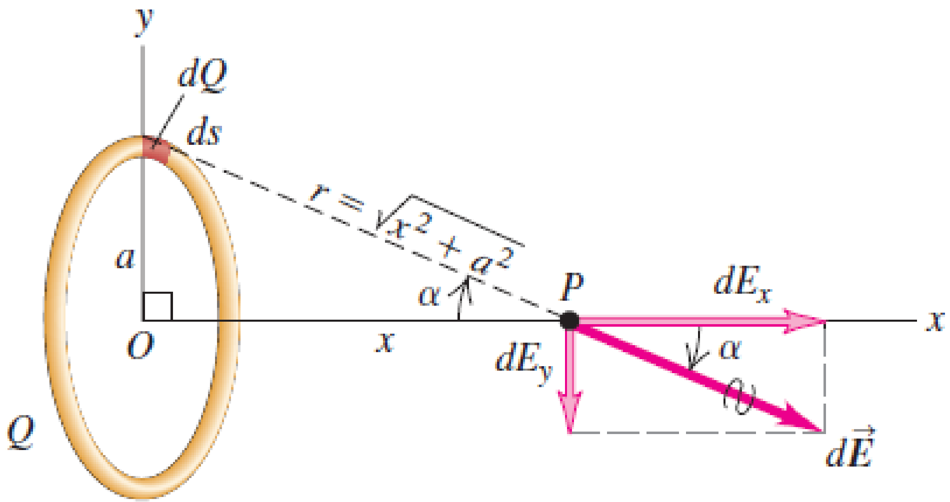
Recitation Class
Week 1
(Examples)

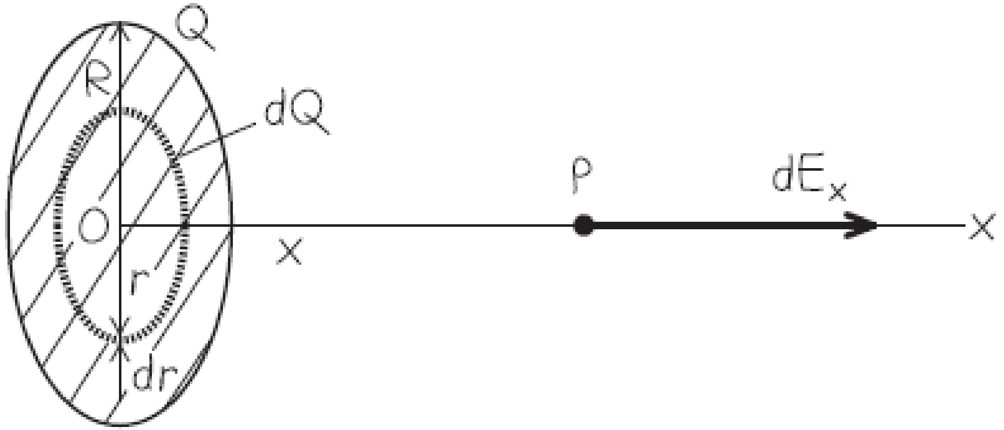
Teaching Assistant
Xie Jinglei

1

Review

Superposition (different methods)





Superposition (polar)

Polar coordinates

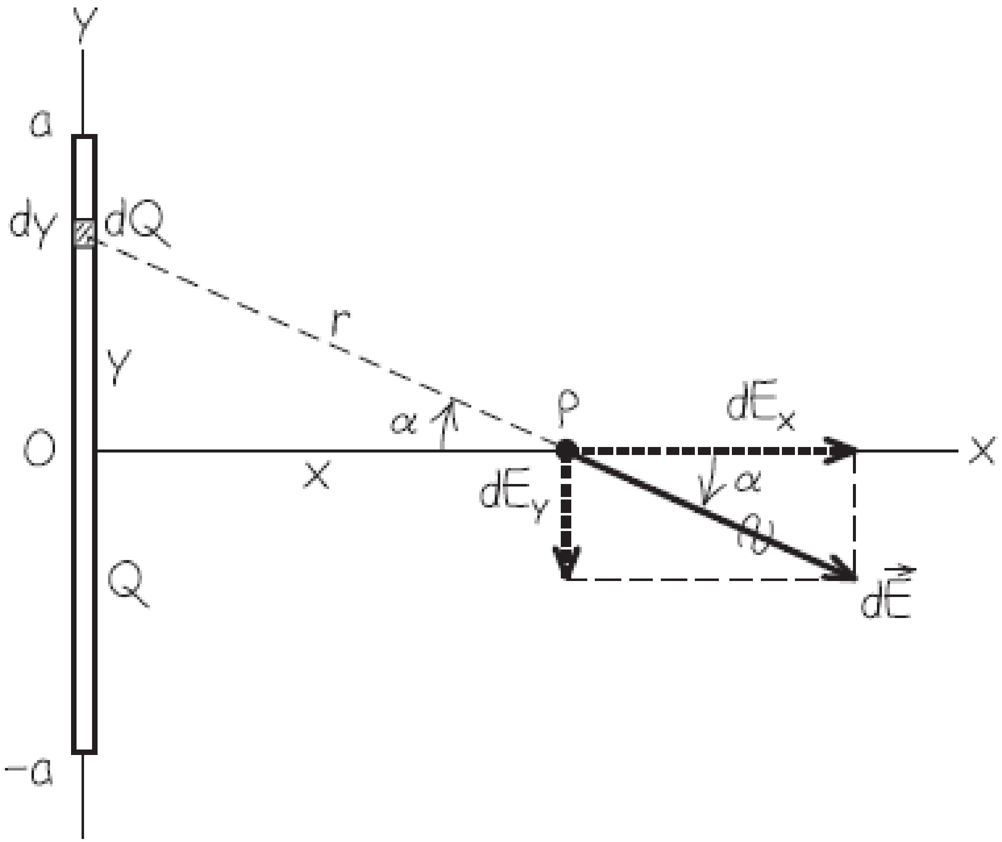
Cylindrical:

$$\int_{z_1}^{z_2}\int_{y_1}^{y_2}\int_{x_1}^{x_2} f(x,y,z)\,dx\,dy\,dz = \int_{z_1}^{z_2}\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2} F(r,\theta,z)r\,dr\,d\theta\,dz$$

Spherical:

$$\iiint_S f(x,y,z)\,dV = \int_{\rho_1}^{\rho_2}\int_{\theta_1}^{\theta_2}\int_{\phi_1}^{\phi_2} F(\rho,\theta,\phi)\rho^2\sin\phi\,d\phi\,d\theta\,d\rho$$

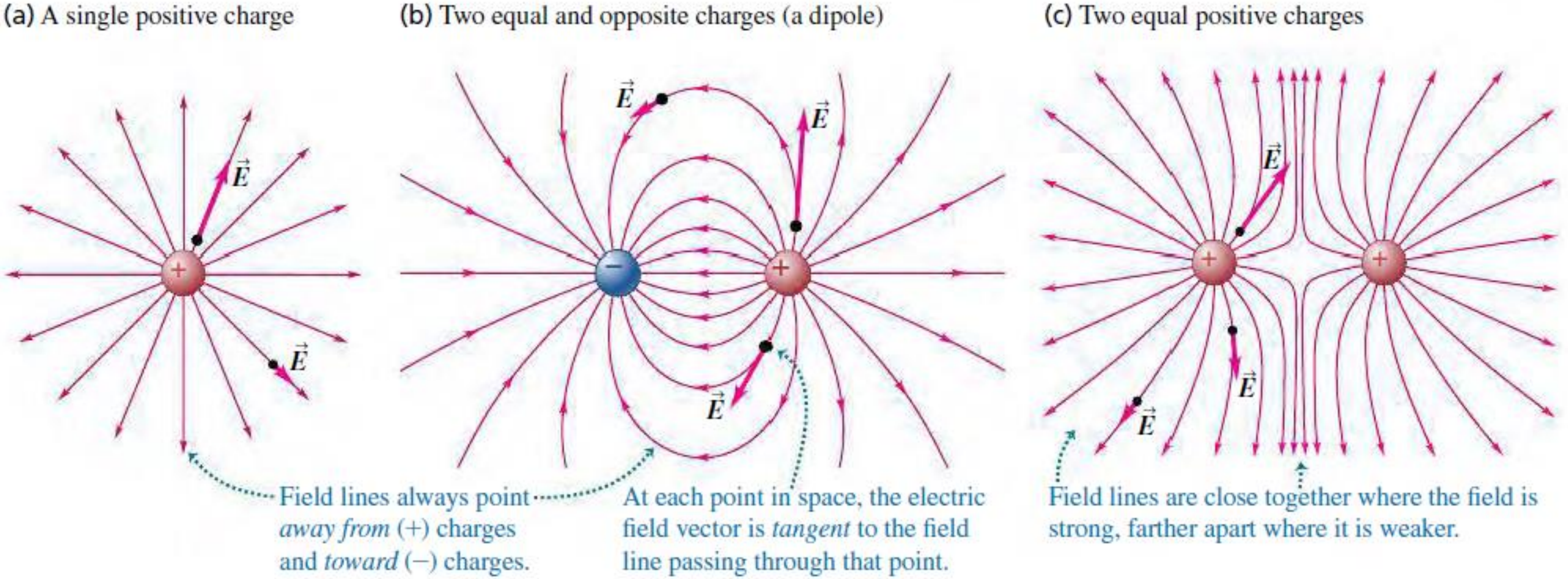
Superposition (not uniform)



$$\lambda(y) = Ay^2$$

Electric Field Lines

21.28 Electric field lines for three different charge distributions. In general, the magnitude of $\vec{E}$ is different at different points along a given field line.



Divergence

Definition

Suppose the following vector field is in Cartesian coordinates and is differentiable

$$\mathbf{F}(x, y, z) = P(x, y, z)\mathbf{e}_x + Q(x, y, z)\mathbf{e}_y + R(x, y, z)\mathbf{e}_z$$

that is, the component functions are all differentiable, then

$$\operatorname{div} \mathbf{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

is called the divergence of $\mathbf{F}$ or the divergence of the vector field defined by $\mathbf{F}$.

$$\operatorname{div} \mathbf{F} = \nabla \cdot \mathbf{F}$$

From Prof. Liu Jing's slides (vv255 2018 summer)

Curl

Definition

Suppose the following vector field in Cartesian coordinates is differentiable,

$$\mathbf{F}(x, y, z) = P(x, y, z)\mathbf{e}_x + Q(x, y, z)\mathbf{e}_y + R(x, y, z)\mathbf{e}_z$$

that is, the component functions are all differentiable, then

$$\operatorname{curl}\mathbf{F} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\mathbf{e}_x + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\mathbf{e}_y + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\mathbf{e}_z$$

is called the curl of $\mathbf{F}$ or the curl of the vector field defined by $\mathbf{F}$.

$$\operatorname{curl}\mathbf{F} = \nabla \times \mathbf{F}$$

From Prof. Liu Jing's slides (vv255 2018 summer)

Stokes' Theorem

Stokes' theorem

Let $\mathcal{S}$ be an oriented piecewise smooth surface that is bounded by a positively oriented, piecewise smooth, simple, closed boundary curve $\mathcal{C}$. Let $\mathbf{F}$ be a vector field whose components have continuous partial derivatives in an open region in $\mathbb{R}^3$ that contains $\mathcal{S}$. Then

$$\oint_{\mathcal{C}} \mathbf{F} \cdot \, d\mathbf{r} = \oint_{\mathcal{C}} \mathbf{F} \cdot \mathbf{T} \, ds = \iint_{\mathcal{S}} (\operatorname{curl} \mathbf{F}) \cdot \mathbf{n} \, dS = \iint_{\mathcal{S}} \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$$

where $\mathbf{n}$ is the unit normal vector of $\mathcal{S}$.

From Prof. Liu Jing's slides (vv255 2018 summer)

The Divergence Theorem

The Divergence Theorem

Let $\mathcal{S}$ be a piecewise smooth closed surface with outward orientation and $\mathcal{E}$ be a region in $\mathbb{R}^3$ that is enclosed by $\mathcal{S}$. Suppose $\mathbf{F}$ is a vector field with a continuous partial derivatives in an open region containing $\mathcal{E}$, then

$$\iint_{\mathcal{S}} \mathbf{F} \cdot \mathbf{n}\, dS = \iiint_{\mathcal{E}} \operatorname{div} \mathbf{F}\, dV$$

where $\mathbf{n}$ is the outward unit normal vector of $\mathcal{S}$.

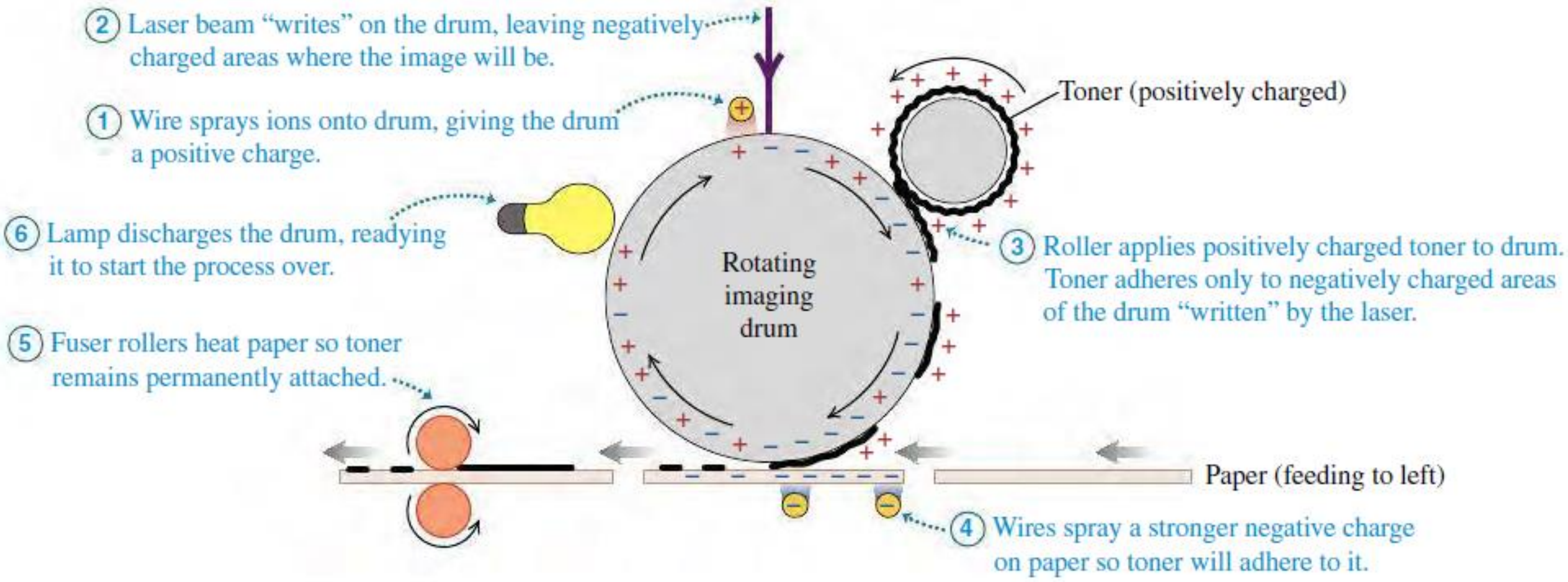
From Prof. Liu Jing's slides (vv255 2018 summer)

2

Discussions

Working Principle of a Laser Printer

21.2 Schematic diagram of the operation of a laser printer.



Question 21.10

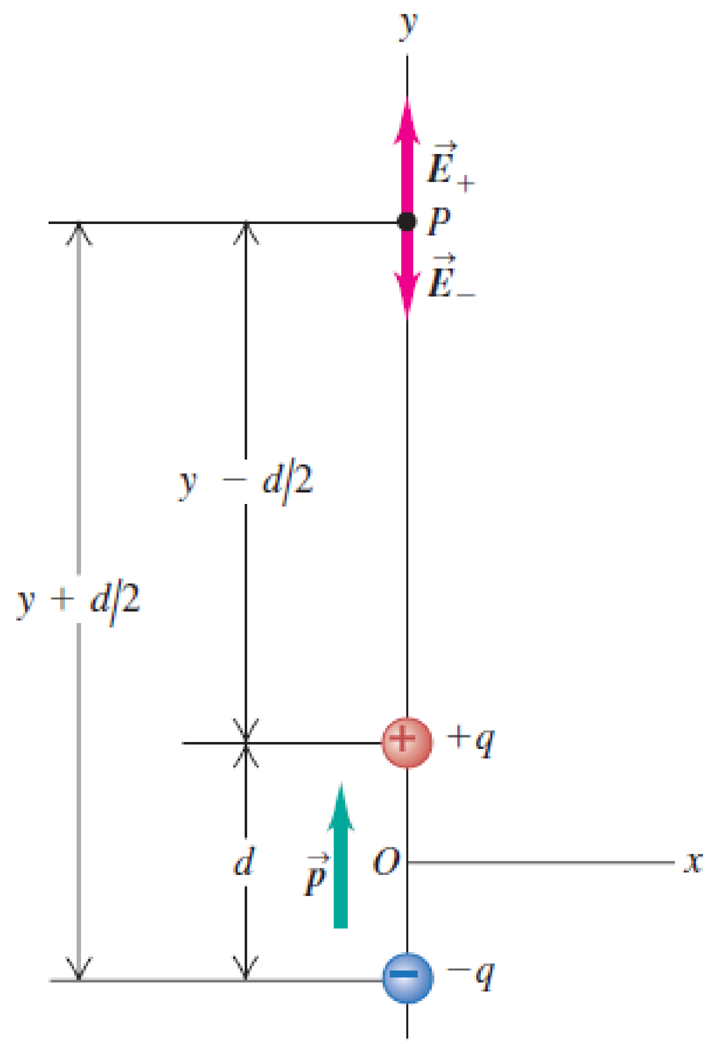
Two identical metal objects are mounted on insulating stands. Describe how you could place charges of **opposite sign** but **exactly equal magnitude** on the two objects.

3

Exercises

Electric Dipole

An electric dipole is centered at the origin, with $\vec{p}$ in the direction of the $+y$-axis (**Fig. 21.33**). Derive an approximate expression for the electric field at a point P on the y-axis for which y is much larger than d. To do this, use the binomial expansion $(1 + x)^n \cong 1 + nx + n(n - 1)x^2/2 + \cdots$ (valid for the case $|x| < 1$).



Electric Field

21.97 •• CALC Negative charge $-Q$ is distributed uniformly around a quarter-circle of radius a that lies in the first quadrant, with the center of curvature at the origin. Find the x- and y-components of the net electric field at the origin.

Coulomb Force

21.68 •• CP Two identical spheres with mass m are hung from silk threads of length L, as shown in Fig. P21.68. Each sphere has the same charge, so $q_1 = q_2 = q$. The radius of each sphere is very small compared to the distance between the spheres, so they may be treated as point charges. Show that if the angle θ is small, the equilibrium separation d between the spheres is $d = (q^2L/2\pi\epsilon_0 mg)^{1/3}$. (*Hint:* If θ is small, then $\tan\theta \cong \sin\theta$.)

Figure **P21.68**