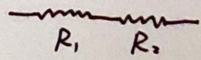
VP 240 Homework 5 周维凯 518021911039

Problem 1.

Suppose the length and cross sectional area are l and A.

$\rho_1 = \rho_{01}[1+\alpha_1(T-0)] = \alpha_1\rho_{01}T + \rho_{01}$, $\rho_2 = \rho_{02}[1+\alpha_2(T-0)] = \alpha_2\rho_{02}T+\rho_{02}$

(a). $R_{eq} = R_1 + R_2$

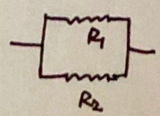


$\rho_{eq}\frac{2l}{A} = \rho_1\frac{l}{A} + \rho_2\frac{l}{A}$

$\rho_{eq} = \frac{1}{2}(\rho_1+\rho_2) = \frac{\alpha_1\rho_{01}+\alpha_2\rho_{02}}{2}T + \frac{\rho_{01}+\rho_{02}}{2}$

Therefore $\boxed{\alpha_{eq} = \frac{\alpha_1\rho_{01}+\alpha_2\rho_{02}}{\rho_{01}+\rho_{02}}}$

(b). $R_{eq} = \frac{R_1R_2}{R_1+R_2}$

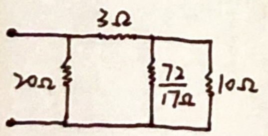


$\rho_{eq}\frac{l}{2A} = \frac{\rho_1\rho_2\frac{l^2}{A^2}}{(\rho_1+\rho_2)\frac{l}{A}} = \frac{\rho_1\rho_2}{\rho_1+\rho_2}\cdot\frac{l}{A}$

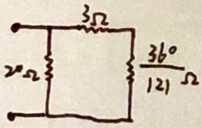
$\rho_{eq} = \frac{2(\alpha_1\rho_{01}T+\rho_{01})(\alpha_2\rho_{02}T+\rho_{02})}{(\alpha_1\rho_{01}+\alpha_2\rho_{02})T+\rho_{01}+\rho_{02}}$

Therefore $\boxed{\alpha_{eq} = \frac{(\rho_{01}+\rho_{02})\alpha_1\alpha_2T + \rho_{02}\alpha_1+\rho_{01}\alpha_2}{\rho_{01}+\rho_{02}+(\rho_{02}\alpha_2+\rho_{01}\alpha_1)T}}$

Problem 2.



$\Rightarrow$



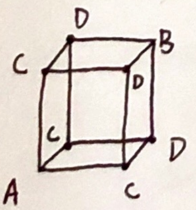
$4+6=10\,\Omega$

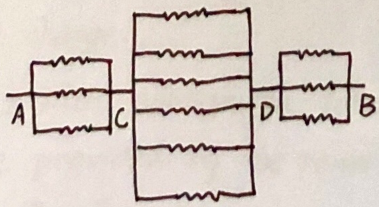
$9\|18 = \frac{72}{17}\,\Omega$

$\frac{72}{17}\|10 = \frac{360}{121}\,\Omega$

$\boxed{R_{eq} = 20\|(3+\frac{360}{121}) = 4.6\,\Omega}$

Problem 3.





$\boxed{R_{eq} = \frac{R}{3}+\frac{R}{6}+\frac{R}{3} = \frac{5}{6} \approx 0.83\,\Omega}$

4.

(a). $\boxed{I = \frac{\varepsilon_1 - \varepsilon_2}{2r+R} = 0.4A}$

(b). $P_R = I^2 \cdot R = \boxed{1.28W}$

$P_r = 2 \cdot I^2 \cdot r = \boxed{0.32W}$

(c). The ε_1 one

$P_1 = \varepsilon_1 I = 4.8W$

(d). The ε_2 one

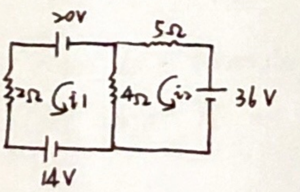
$P_2 = \varepsilon_2 I = 3.2W$

(e). Production $P = P_1 = 4.8\,W$

Consumption $P = P_R + P_r + P_2 = 4.8W$

Therefore $P_{production} = P_{consumption}$

Problem 5.



$$\begin{cases} -20 + 2i_1 + 14 + 4(i_1 - i_2) = 0 \\ 5i_2 + 4(i_2 - i_1) - 36 = 0 \end{cases} \Rightarrow \begin{cases} i_1 = \frac{99}{19}\,A \\ i_2 = \frac{120}{19}\,A \end{cases}$$

$$\boxed{\begin{aligned} I_1 &= i_1 = 5.21\,A \\ I_2 &= i_2 - i_1 = 1.11\,A \\ I_3 &= i_2 = 6.32\,A \end{aligned}}$$

Problem 6.

Assume the resistance of each bulb is R and it doesn't change.

(a). The brightness will not change.

After we close the switch, although the total current increases due to the new branch, the voltage provided by the power source keeps the same since it has no internal resistance. Therefore, the voltage applied on the two branches of bulbs is the same. So is the current. Hence, the brightness will not change.

se the internal resistance of the battery is r and the value of the resistance onnected with S is R_0.

$$R_{total\_original} = \frac{1}{\frac{1}{2R}+\frac{1}{R}} + r > R_{total-present} = \frac{1}{\frac{1}{2R}+\frac{1}{R}+\frac{1}{R_0}} + r$$

According to $I = \frac{\varepsilon}{R_{total}}$, we can get $I_{original} < I_{present}$

Then according to $V = \varepsilon - Ir$, we can get $V_{original} > V_{present}$.

Therefore, after we close the switch, the voltage applied on the two branches of bulbs decreases. So does the current. Hence, the brightness of the bulbs decreases.

Problem 7.

(a). $q(t) = 7\times10^{-6} \times e^{-\frac{t}{470\times10^{3}\times0.92\times10^{-6}}} \leq 1.6\times10^{-19}$

$t \geq 19.36S$

Therefore, $\boxed{t_d = 19.36S}$

(b). $q(t) = Q_{max}\, e^{-\frac{t}{RC}} \leq 1.6\times10^{-19}$

$t \geq RC\left(\ln 1.6\times10^{-19} - \ln Q_{max}\right)$

From this formula, we can find that t_d is actually related to R and C. The larger R or C is, the larger t_d is.

Problem 8.

$$\begin{cases} V_B = V_C \\ V_B = \varepsilon - N\frac{\varepsilon}{N+M} \\ V_C = \varepsilon - P\frac{\varepsilon}{P+X} \end{cases} \Rightarrow \varepsilon - N\frac{\varepsilon}{N+M} = \varepsilon - P\frac{\varepsilon}{P+X}$$

$$N(P+X) = P(N+M)$$

$$\boxed{X = \frac{MP}{N}}$$