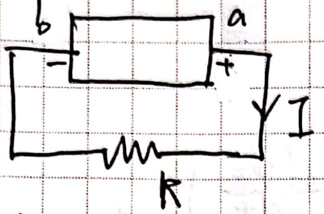
Mid 2 Review

I. DC circuits

Slides

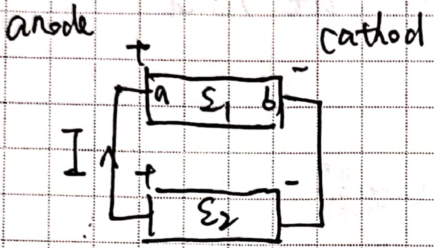
EMF:



$V_{ab} = \mathcal{E} - Ir$

$P = V_{ab} I = (\mathcal{E} - Ir) I = \mathcal{E} I - I^2 r$

P → provided by source



$\mathcal{E}_2 > \mathcal{E}_1$

$V_{ab} = \mathcal{E}_1 + Ir$

$P = V_{ab} I = \mathcal{E}_1 I + I^2 r$

P → power input to source with $\mathcal{E}_1$

electrical energy → non-electrical energy at a rate $\mathcal{E}_1 I$

II. Magnetic field

slides

1. Magnetic force on a moving charge

$\vec{F} = q\vec{v} \times \vec{B}$

$= qvB\sin\phi = qv_{\perp}B$

$v_{\perp}$ → perpendicular component

E · B both present

$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

2. Circular motion with constant speed

$F = |q|vB = m\frac{v^2}{R}$ (centripital force)

$R = \frac{mv}{|q|B}$

3. Hall effect

refer to lecture notes

4. Magnetic force on straight wire with I

$$\vec{F} = I\,\underbrace{\vec{l}}_{\text{current direction}} \times \vec{B}$$

not straight wire

$$F = \int I\,d\vec{l} \times \vec{B}$$

5. Torque

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = 2 \cdot \frac{b}{2} F \sin\varphi = Fb \sin\varphi = \frac{IaB}{F}\, b \sin\varphi$$

Define $\mu = IA$ $(A = ab)$ * solenoid : nIA

μ: magnetic dipole moment

$$\tau = \mu B \sin\varphi$$

$$\Rightarrow \quad \vec{\tau} = \vec{\mu} \times \vec{B}$$

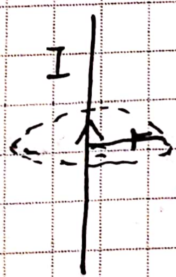
potential energy $\quad U = -\vec{\mu} \cdot \vec{B}$

III. Sources of magnetic field

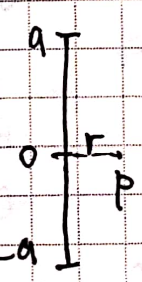
1. Slide $\quad c^2 = \dfrac{1}{\varepsilon_0 \mu_0}$

2. Important formulas

① I (infinite)



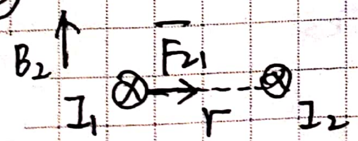
$$B = \frac{\mu_0 I}{2\pi r}$$



$$B = \frac{\mu_0 I}{4\pi} \frac{2a}{r\sqrt{r^2 + a^2}}$$

$a \to \infty$

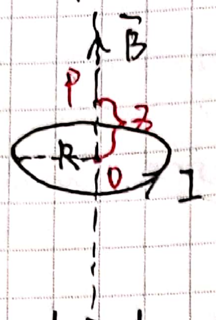
②



$$|\vec{F}_{21}| = I_1 l \frac{\mu_0 I_2}{2\pi r}$$

$$= I_2 l \frac{\mu_0 I_1}{2\pi r} = |\vec{F}_{12}|$$

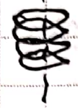
③ circular current loop



$$B_p = \frac{\mu_0 I}{2} \frac{R^2}{(z^2+R^2)^{\frac{3}{2}}}$$

center: $\frac{\mu_0 I}{2R}$

multi-loops



N loops

$$B = \frac{\mu_0 I}{2} \frac{R^2}{(z^2+R^2)^{\frac{3}{2}}} \cdot N$$

$$B_{max} = \frac{N\mu_0 I}{2R} \quad (\text{center})$$

3. slide

4. Ampere's law

$$\oint_{\Gamma} \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

I_{net} $\odot$ $\vec{B}$ $\otimes$ $\vec{B}$

* Construct whatever loop

$$\oint \vec{E} \cdot d\vec{l} = 0 \quad (\oint \vec{F} \cdot d\vec{l} = 0, \text{ conservative})$$

but $\oint \vec{B} \cdot d\vec{l} \neq 0$

($\vec{F} = q\vec{v} \times \vec{B}$ not conservative field)

① example

$$\oint \vec{B} \cdot d\vec{l} = 2\pi r B = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

make sure all B around the current is equal.

② cylindrical conducter

refer to lecture notes

③ solenoid (slide)

take bc. ad to be very long

$\Rightarrow$ let $B_{cd} = 0$

B_{bc} & $B_{ad} = 0$ (Assume $\vec{B}$ is perpendicular to bc & ad)

$\Rightarrow \oint \vec{B} \cdot d\vec{l} = B_{ab} \cdot L = \mu_0 n I$

IV. Electromagnetic Induction

1. Motional EMF

$\mathcal{E} = -\frac{d\phi_B}{dt}$ → changing rate of magnetic flux

$\mathcal{E} = -N \frac{d\phi_B}{dt}$ for N loops tightly bined together

direction : slide

2. Slide-wire Generator

$F_B = BIL = B \frac{\mathcal{E}}{R} L = \frac{B^2 L^2 v}{R}$

Power
- field → wire,
  $P_{ext} = F_B v = \frac{B^2 L^2 v^2}{R}$
- current :
  $P_{diss} = I^2 R = \left(\frac{\mathcal{E}}{R}\right)^2 R = \frac{B^2 L^2 v}{R}$

Mechenical energy ⇔ electrical energy

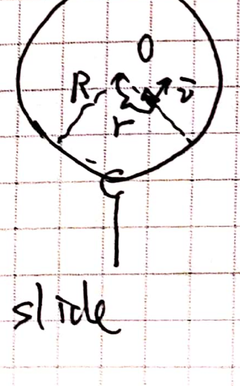
3. Faraday's disk

linear velocity

$v = \omega r$

$\vec{F_B} = q\vec{v} \times \vec{B}$ $\quad \frac{\vec{F_B}}{q} = \vec{v} \times \vec{B} = \boxed{\omega r} B \hat{r}$ → radial direction

$\vec{B} \odot$



$d\mathcal{E} = \omega r B dr$ EMF induced across the segment

$\mathcal{E} = \int_0^R \omega r B dr = \frac{1}{2} \omega B R^2$

4. displacement current

$$i_D = \varepsilon \frac{d\boxed{\Phi_E}}{dt}$$

$\uparrow$

permittivity of material in capacitor ($\varepsilon = \varepsilon_r \varepsilon_0$)

Generalized Ampere's Law

$$\oint_{\Gamma} \boxed{\vec{B}} \cdot d\vec{l} = \mu_0 (i_C + i_D)_{end}$$

(either i_C or i_D is 0)

☆ changing electric field

$\longrightarrow$ magnetic field.