JOINT INSTITUTE
交大密西根学院

Recitation Class 8
(Examples)

Teaching Assistant
Xie Jinglei

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Mutual Inductance

Example 7.10

A short solenoid (length l and radius a, with n_1 turns per unit length) lies on the axis of a very long solenoid (radius b, n_2 turns per unit length) as shown in Fig. 7.31. Current I flows in the short solenoid. What is the flux through the long solenoid?

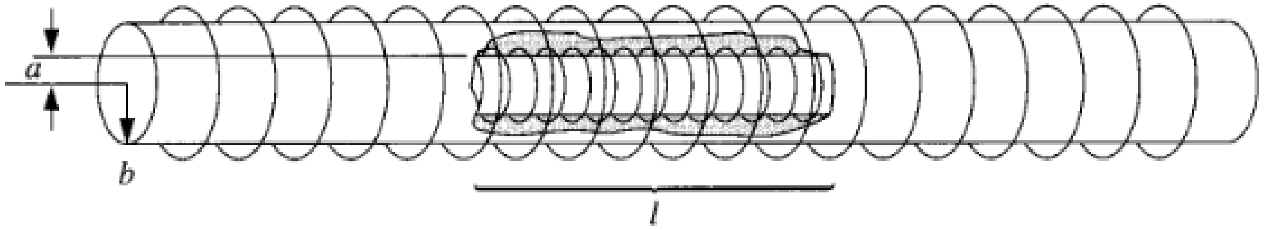

Figure 7.31

Solution: Since the inner solenoid is short, it has a very complicated field; moreover, it puts a different amount of flux through each turn of the outer solenoid. It would be a *miserable* task to compute the total flux this way. However, if we exploit the equality of the mutual inductances, the problem becomes very easy. Just look at the reverse situation: run the current I through the *outer* solenoid, and calculate the flux through the *inner* one. The field inside the long solenoid is constant:

$$B = \mu_0 n_2 I$$

(Eq. 5.57), so the flux through a single loop of the short solenoid is

$$B\pi a^2 = \mu_0 n_2 I \pi a^2.$$

There are $n_1 l$ turns in all, so the total flux through the inner solenoid is

$$\Phi = \mu_0 \pi a^2 n_1 n_2 l I.$$

This is also the flux a current I in the *short* solenoid would put through the *long* one, which is what we set out to find. Incidentally, the mutual inductance, in this case, is

$$M = \mu_0 \pi a^2 n_1 n_2 l.$$

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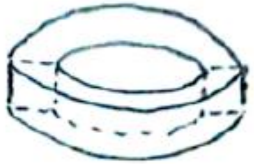
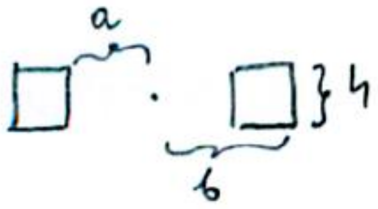
Self Inductance

Toroidal solenoid

Example (inductance of a toroidal coil with rectangular cross-section)



cross-section



Magnetic field inside $\boxed{B = \frac{\mu_0 I N}{2\pi r}}$ (see rec. class)

Flux through a single turn

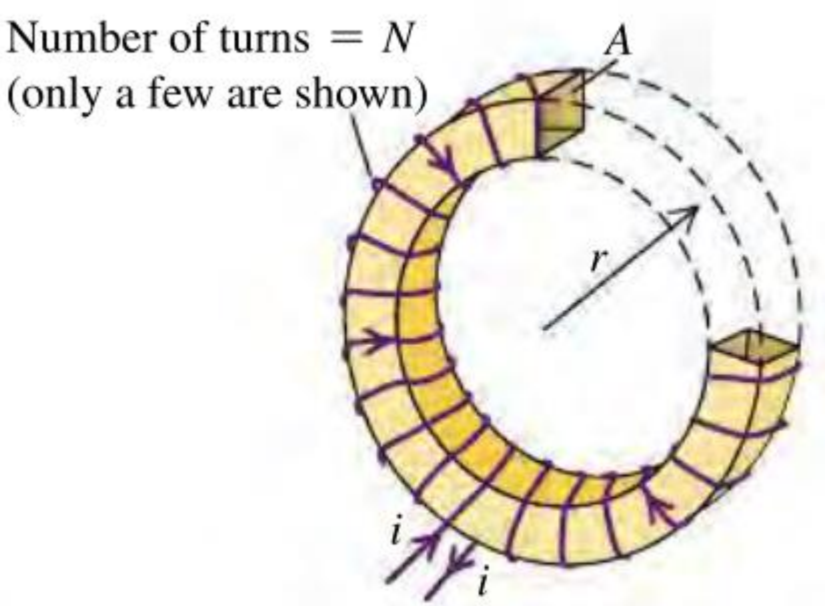
$dS = h\,dr$

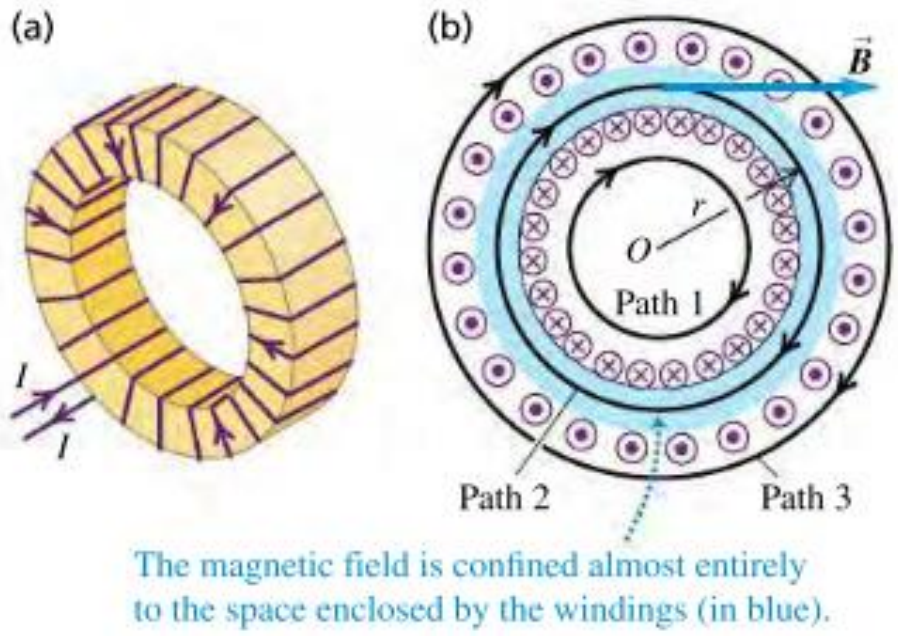
$$\Phi_B^{(1)} = \int \bar{B}\cdot d\bar{S} = \frac{\mu_0 I N}{2\pi} h \int_a^b \frac{dr}{r} = \frac{\mu_0 I N h}{2\pi} \ln\left(\frac{b}{a}\right)$$

Total flux

$$\Phi_B = \underbrace{N}_{\text{\# of turns}} \cdot \Phi_B^{(1)} = \underbrace{\frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{b}{a}\right)}_{L} I \; ; \qquad \boxed{L = \frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{b}{a}\right)}$$

Toroidal solenoid





Obtain emf using L

If the current in the toroidal solenoid in Example 30.3 increases uniformly from 0 to 6.0 A in 3.0 μs, find the magnitude and direction of the self-induced emf.

$$|\mathcal{E}| = L\left|\frac{di}{dt}\right| = (40 \times 10^{-6}\ \mathrm{H})(2.0 \times 10^{6}\ \mathrm{A/s}) = 80\ \mathrm{V}$$

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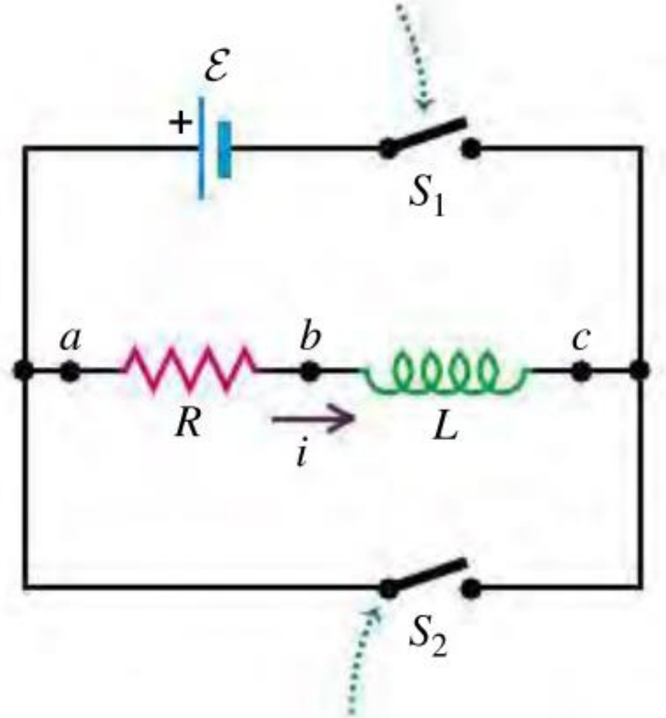
RL circuits

RL circuits

Closing switch S_1 connects the R-L combination in series with a source of emf $\mathcal{E}$.



Closing switch S_2 while opening switch S_1 disconnects the combination from the source.