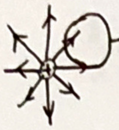
VP 240 Homework 2 周绵凯 518021911039

Problem 1.

(a). Since $\vec{E}$ is uniform, then for any Gauss's surface $\phi = 0$.

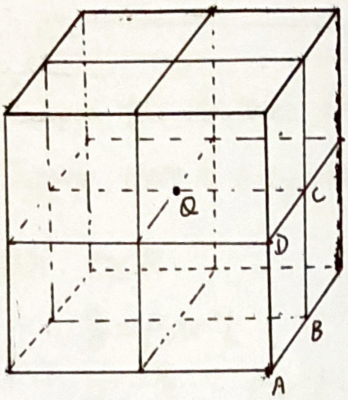
However, if there is any excess charge, $\phi = \frac{q}{\varepsilon_0} \neq 0$, which is a contradiction.

Therefore it is electrically neutral.

(b).  In this region, there is no charge but $\vec{E}$ is not uniform.

Therefore it is false.

Problem 2.



If we add seven more cubes like on the left.

Then $\phi_{total} = \frac{Q}{\varepsilon_0}$

Therefore $\phi_{ABCD} = \frac{1}{24} \phi_{total} = \frac{Q}{24\varepsilon_0}$

Problem 3.

(a). $Q = \int_a^b \frac{k}{r} 4\pi r^2 dr = 2\pi k (b^2 - a^2)$

(b). (i). $r < a$

According to Gauss's law, $\phi = \frac{q}{\varepsilon_0} = 0$ and $\vec{E} = \vec{0}$

(ii) $a < r < b$

$Q_1 = \int_a^r \frac{k}{r} 4\pi r^2 dr = 2\pi k (r^2 - a^2)$

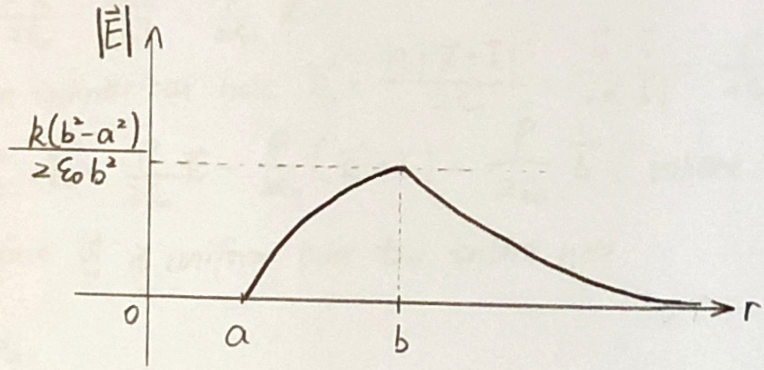
$\phi_1 = \frac{Q_1}{\varepsilon_0} = \frac{2\pi k (r^2 - a^2)}{\varepsilon_0} = 4\pi r^2 E \Rightarrow E = \frac{k(r^2 - a^2)}{2\varepsilon_0 r^2}$

$\vec{E} = \frac{k(r^2 - a^2)}{2\varepsilon_0 r^2} \cdot \frac{\vec{r}}{|\vec{r}|}$ where $\vec{r}$ is the radius vector pointing outward ii). $r > b$

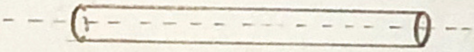
$$\phi = \frac{Q}{\varepsilon_0} = \frac{2\pi k(b^2-a^2)}{\varepsilon_0} = 4\pi r^2 E \Rightarrow E = \frac{k(b^2-a^2)}{2\varepsilon_0 r^2}$$

$$\vec{E} = \frac{k(b^2-a^2)}{2\varepsilon_0 r^2} \cdot \frac{\vec{r}}{|\vec{r}|}$$, where $\vec{r}$ is the radius vector pointing outward

(C).



Problem 4.



Suppose the distance from the axis of the cylinder is r.

Suppose there is a cylinder with radius r and length L.

(a). $r \leq R$

$$q = \rho \pi r^2 L$$

$$\phi = \frac{q}{\varepsilon_0} = \frac{\rho \pi r^2 L}{\varepsilon_0} = 2\pi r L E \Rightarrow E = \frac{\rho r}{2\varepsilon_0}$$

$$\vec{E} = \frac{\rho r}{2\varepsilon_0} \cdot \frac{\vec{r}}{|\vec{r}|}$$, where $\vec{r}$ is the radius vector perpendicular to the axis and pointing outward

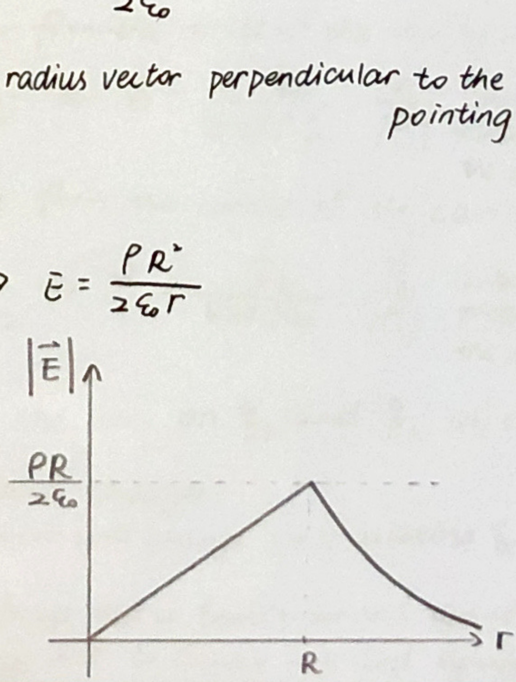
(b). $r > R$

$$q = \rho \pi R^2 L$$

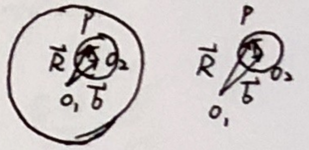
$$\phi = \frac{q}{\varepsilon_0} = \frac{\rho \pi R^2 L}{\varepsilon_0} = 2\pi r L E \Rightarrow E = \frac{\rho R^2}{2\varepsilon_0 r}$$

$$\vec{E} = \frac{\rho R^2}{2\varepsilon_0 r} \cdot \frac{\vec{r}}{|\vec{r}|}$$

, where $\vec{r}$ is the radius vector perpendicular to the axis and pointing outward



Problem 5.



Choose a point P in the cylindrical hole.

If the large cylinder doesn't have the cylindrical hole:

$$\vec{E}_1 = \frac{\rho R}{2\varepsilon_0} \cdot \frac{\vec{R}}{|\vec{R}|} = \frac{\rho}{2\varepsilon_0}\vec{R}$$

For the cylindrical hole $\vec{E}_2 = \frac{\rho|\vec{R}-\vec{b}|}{2\varepsilon_0} \cdot \frac{\vec{R}-\vec{b}}{|\vec{R}-\vec{b}|} = \frac{\rho}{2\varepsilon_0}(\vec{R}-\vec{b})$

$\vec{E} = \vec{E}_1 - \vec{E}_2 = \frac{\rho}{2\varepsilon_0}\vec{R} - \frac{\rho}{2\varepsilon_0}(\vec{R}-\vec{b}) = \frac{\rho}{2\varepsilon_0}\vec{b}$. where $\vec{b}$ is a constant vector

Therefore $\vec{E}$ is uniform over the entire hole.

Problem 6.

(a). $4\pi r_a^2 \cdot \sigma_a = -q_a \Rightarrow \sigma_a = \frac{-q_a}{4\pi r_a^2}$

$4\pi r_b^2 \cdot \sigma_b = -q_b \Rightarrow \sigma_b = \frac{-q_b}{4\pi r_b^2}$

$4\pi R^2 \cdot \sigma_R = q_a + q_b \Rightarrow \sigma_R = \frac{q_a + q_b}{4\pi R^2}$

(b). Suppose distance from the center of the conductor is $r > R$.

$\phi = \frac{q_a + q_b}{\varepsilon_0} = 4\pi r^2 E \Rightarrow E = \frac{q_a + q_b}{4\pi r^2 \varepsilon_0}$

Therefore, $\vec{E} = \frac{q_a + q_b}{4\pi r^2 \varepsilon_0} \cdot \frac{\vec{r}}{|\vec{r}|}$, where $\vec{r}$ is vector pointing outward from the center to the position.

(c). For cavity a, suppose the distance from the center of the cavity is $r_1 < r_a$.

$\phi = \frac{q_a}{\varepsilon_0} = 4\pi r_1^2 E \Rightarrow E = \frac{q_a}{4\pi r_1^2 \varepsilon_0} \Rightarrow \vec{E} = \frac{q_a}{4\pi r_1^2 \varepsilon_0} \cdot \frac{\vec{r_1}}{|\vec{r_1}|}$, where $\vec{r_1}$ is the vector pointing outward from the center

For cavity b, suppose the distance from the center of the cavity is $r_2 < r_b$.

$\phi = \frac{q_b}{\varepsilon_0} = 4\pi r_2^2 E \Rightarrow E = \frac{q_b}{4\pi r_2^2 \varepsilon_0} \Rightarrow \vec{E} = \frac{q_b}{4\pi r_2^2 \varepsilon_0} \cdot \frac{\vec{r_2}}{|\vec{r_2}|}$, where $\vec{r_2}$ is the vector pointing outward from the center

(d). Due to Gauss's law and symmetry, the force on q_a and q_b all equal zero.

(e). ① σ_R will change due to the induced charges.

② The electric field outside the conductor will change since adding q_c will change electric field lines.

③ The electric field within cavities will not change due to Gauss's law and symmetry

④ The force on q_a and q_b will not change due to Gauss's law and symmetry roblem 7.

(a). Suppose the Gaussian surface is the surface of the ball with radius r. And M is at the center of the ball.

$$\oint_{\Sigma} E_G \hat{n} d_A = E\, 4\pi r^2 = -4\pi GM \Rightarrow E = -\frac{GM}{r^2} \Rightarrow \vec{E} = -\frac{GM}{r^2} \cdot \frac{\vec{r}}{|\vec{r}|},$$

where $\vec{r}$ is the radius vector pointing outward.

(b). ① $r > R$

$$E\, 4\pi r^2 = -4\pi GM \Rightarrow E = -\frac{GM}{r^2} \Rightarrow \vec{E} = -\frac{GM}{r^2} \cdot \frac{\vec{r}}{|\vec{r}|},$$ where $\vec{r}$ is the radius vector pointing outward.

② $r < R$

$$E\, 4\pi r^2 = -4\pi G\left(\frac{r}{R}\right)^3 M \Rightarrow E = -\frac{GMr}{R^3} \Rightarrow \vec{E} = -\frac{GMr}{R^3} \cdot \frac{\vec{r}}{|\vec{r}|},$$ where $\vec{r}$ is the radius vector pointing outward.

(c). ① $r > R$

$$E\, 4\pi r^2 = -4\pi GM \Rightarrow E = -\frac{GM}{r^2} \Rightarrow \vec{E} = -\frac{GM}{r^2} \cdot \frac{\vec{r}}{|\vec{r}|},$$ where $\vec{r}$ is the radius vector pointing outward.

② $r < R$

$$\vec{E} = \vec{0}$$