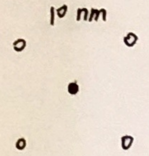
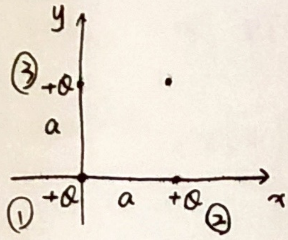
VP240 Homework 3 周旅凯 518021911039

Problem 2.



$$W = u_1 - u_2$$
$$= \frac{4}{4\pi\varepsilon_0}\cdot\frac{qq_0}{\frac{\sqrt{2}}{2}a} - \frac{2}{4\pi\varepsilon_0}\cdot\frac{qq_0}{\frac{1}{2}a} - \frac{2}{4\pi\varepsilon_0}\cdot\frac{qq_0}{\frac{\sqrt{5}}{2}a}$$
$$= \frac{qq_0}{4\pi\varepsilon_0 a}\left(\frac{8}{\sqrt{2}} - 4 - \frac{4}{\sqrt{5}}\right)$$
$$= 0.038\ \text{eV}$$

Problem 3.



(a). Let's set a coordinate like shown on the left

$$\vec{E} = \left[\frac{1}{4\pi\varepsilon_0}\frac{Q}{a^2} + \frac{\sqrt{2}}{2}\frac{1}{4\pi\varepsilon_0}\frac{Q}{(\sqrt{2}a)^2}\right]\hat{n_x} + \left[\frac{1}{4\pi\varepsilon_0}\frac{Q}{a^2} + \frac{\sqrt{2}}{2}\frac{1}{4\pi\varepsilon_0}\frac{Q}{(\sqrt{2}a)^2}\right]\hat{n_y}$$
$$= \frac{(4+\sqrt{2})Q}{16\pi\varepsilon_0 a^2}\hat{n_x} + \frac{(4+\sqrt{2})Q}{16\pi\varepsilon_0 a^2}\hat{n_y}$$

(b). $V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{a}\times 2 + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{\sqrt{2}a} = \frac{(4+\sqrt{2})Q}{8\pi\varepsilon_0 a}$ (We set infinity as zero).

(c). $W = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{a}\times 2 + \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{\sqrt{2}a} - 0 = \frac{(4+\sqrt{2})Q^2}{8\pi\varepsilon_0 a}$

(d). When assembling the first one, $W_1 = 0$.

When assembling the second one, $W_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{a}$

When assembling the third one, $W_3 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{a} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{\sqrt{2}a}$

$$W = W_1 + W_2 + W_3 = 0 + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{a} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{a} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{\sqrt{2}a} = \frac{(4+\sqrt{2})Q^2}{8\pi\varepsilon_0 a}$$

Problem 4.

From the previous homework, we know that

$$\vec{E} = \begin{cases} \vec{0} & , r<a \\ \frac{k}{2\varepsilon_0 r^2}(r^2-a^2)\cdot\frac{\vec{r}}{|\vec{r}|} & , a<r<b \\ \frac{k}{2\varepsilon_0 r^2}(b^2-a^2)\cdot\frac{\vec{r}}{|\vec{r}|} & , r>b \end{cases}$$

We set the infinity as zero.

When $r<a$, $V(r) - V(\infty) = \int_r^\infty \vec{E} d\vec{r} = \int_r^a \vec{E} d\vec{r} + \int_a^b \vec{E} d\vec{r} + \int_b^\infty \vec{E} d\vec{r}$

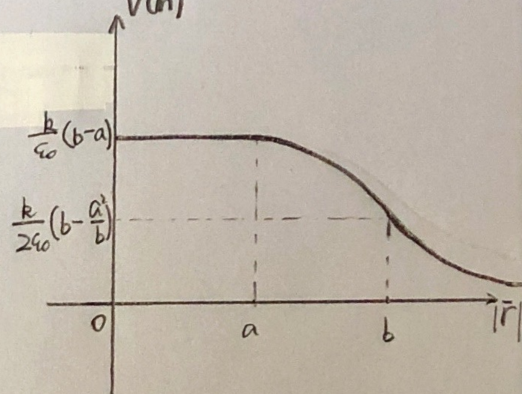
$$= \frac{k}{2\varepsilon_0}\left(b+\frac{a^2}{b}-2a\right) + \frac{k}{2\varepsilon_0}\left(b-\frac{a^2}{b}\right) = \frac{k}{\varepsilon_0}(b-a)$$

When $a<r<b$, $V(r)-V(\infty) = \int_r^b \vec{E} d\vec{r} + \int_b^\infty \vec{E} d\vec{r} = \frac{k}{2\varepsilon_0}\left(b+\frac{a^2}{b}-r-\frac{a^2}{r}\right) + \frac{k}{2\varepsilon_0}\left(b-\frac{a^2}{b}\right)$

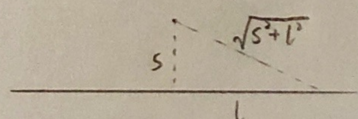
$$= \frac{k}{2\varepsilon_0}\left(2b-r-\frac{a^2}{r}\right)$$

When $r>b$, $V(r)-V(\infty) = \int_r^\infty \vec{E} d\vec{r} = \frac{k(b^2-a^2)}{2\varepsilon_0 r}$

Therefore $V(|r|) = \begin{cases} \frac{k}{\varepsilon_0}(b-a) & r<a \\ \frac{k}{2\varepsilon_0}\left(2b-r-\frac{a^2}{r}\right) & a<r<b \\ \frac{k}{2\varepsilon_0 r}(b^2-a^2) & r>b \end{cases}$



Problem 5.



Let's choose a point where is s_0 from the wire as zero.

$$V(s) - V(s_0) = \int_s^{s_0} \vec{E} d\vec{s}$$

$$E \cdot 2\pi s l = \frac{q}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = \frac{\lambda}{2\pi s \varepsilon_0}$$

$$V(s) = \int_s^{s_0} \frac{\lambda}{2\pi\varepsilon_0 s} ds = \frac{\lambda}{2\pi\varepsilon_0}(\ln s_0 - \ln s) = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{s_0}{s}$$

$$|\nabla V| = \frac{\lambda}{2\pi\varepsilon_0 s}$$

According to Gauss's law, $\vec{E} = \frac{\lambda}{2\pi s\varepsilon_0}\frac{\vec{s}}{|\vec{s}|}$

Therefore the gardient of the poten tial yields the correct field.

oblem 6

Assume the density of charge is ρ

$$\rho \cdot \frac{4}{3}\pi R^3 = q \Rightarrow \rho = \frac{3q}{4\pi R^3}$$

$$dq = \rho 4\pi r^2 dr = \frac{3qr^2}{R^3} dr, \quad q_0 = \frac{4}{3}\pi r^3 \cdot \rho = \frac{r^3}{R^3} q$$

$$dW = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq \cdot q_0}{r} = \frac{3q^2 r^4}{4\pi\varepsilon_0 R^6} dr$$

$$W = \int_0^R dW = \int_0^R \frac{3q^2 r^4}{4\pi\varepsilon_0 R^6} dr = \frac{3q^2}{20\pi\varepsilon_0 R}$$