VP240-1 Recitation class
Week #10

Bicheng Gan
ganbicheng@sjtu.edu.cn

SJTU - UMJI — November 25, 2019

Brief introduction

- I will skip the RLC circuit part, but it will be covered in the final exam. It is also discussed in the VE215 course.

1 Short Review

1. Mutual Inductance
   - If the current in coil 1 is changing, the changing flux through coil 2 induces an emf in coil 2.
   - $$\mathcal{E}_2 = -N_2 \frac{d\Phi_{B2}}{dt}$$
     We introduce a proportionality constant M_{21}, called the mutual inductance of the two coils, we write:
     $$N_2 \Phi_{B2} = M_{21} i_1$$
     where Φ_{B2} is the flux through a single turn of coil 2. From this,
     $$N_2 \frac{d\Phi_{B2}}{dt} = M_{21} \frac{di_1}{dt}$$
     and we can rewrite as: $\mathcal{E}_2 = -M_{21} \frac{di_1}{dt}$.
   - If the coils are in vacuum, the flux Φ_{B2} through each turn of coil 2 is directly proportional to the current i1 . Then the mutual inductance M_{21} is a constant that depends only on the geometry of the two coils (the size, shape, number of turns, and orientation of each coil and the separation between the coils).
   - We can repeat our discussion for the opposite case in which a changing cur- rent i_2 in coil 2 causes a changing flux Φ_{B1} and an emf $\mathcal{E}_1$ in coil 1. It turns out that the corresponding constant M_{12} is always equal to M_{21}, even though in general the two coils are not identical and the flux through them is not the same.
   - **Only a time-varying current in a coil** can induce an emf and hence a current in a second coil. The above equation shows that the induced emf in each coil is directly proportional to the rate of change of the current in the other coil, not to the value of the current. A steady current in one coil, no matter how strong, cannot induce a current in a neighboring coil.
   - The SI unit of mutual inductance is called the henry (1 H).
2. Self-inductance

   An important related effect occurs in a single isolated circuit. A current in a circuit sets up a magnetic field that causes a magnetic flux through the same circuit; this flux changes when the current changes. Thus any circuit that carries a varying current has an emf induced in it by the variation in its own magnetic field. Such an emf is called a self-induced emf.

Self-induced emfs can occur in any circuit, since there is always some magnetic flux through the closed loop of a current-carrying circuit. But the effect is greatly enhanced if the circuit includes a coil with N turns of wire

$$L = \frac{N\Phi_B}{i}$$

The self=induced emf: $\mathcal{E} = -L\frac{di}{dt}$

Self-induced emf opposes changes in current Note that the self-induced emf does not oppose the current i itself; rather, it opposes any change (di/dt) in the current. Thus the circuit behavior of an inductor is quite different from that of a resistor.

3. magnetic field energy

   The total energy U supplied while the current increases from zero to a final value I is:

$$U = \dot{L}\int_0^I idi = \frac{1}{2}LI^2$$

   Don't confuse the behavior of resistors and inductors where energy is concerned. Energy flows into a resistor whenever a current passes through it, whether the current is steady or varying; this energy is dissipated in the form of heat. By contrast, energy flows into an ideal, zero-resistance inductor only when the current in the inductor increases. This energy is not dissipated; it is stored in the inductor and released when the current decreases. When a steady current flows through an inductor, there is no energy flow in or out.

4. Transformer

   A transformer, used in alternating-current circuits to raise or lower voltages, is fundamentally no different from the two coils.Time-varying alternating current in one coil of the transformer produces an alternating emf in the other coil; the value of M, which depends on the geometry of the coils, determines the amplitude of the induced emf in the second coil and hence the amplitude of the output voltage.

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

5. More should be discussed in the later recitation class

2 Discussion

- Q31.1 Household electric power in most of western Europe is supplied at 240 V, rather than the 120 V that is standard in the United States and Canada. What are the advantages and disadvantages of each system?

- Q30.5 Two identical, closely wound, circular coils, each having self-inductance L, are placed next to each other, so that they are coaxial and almost touching. If they are connected in series, what is the self-inductance of the combination? What if they are connected in parallel? Can they be connected so that the total inductance is zero? Explain.

- Q30.7 You are to make a resistor by winding a wire around a cylindrical form. To make the inductance as small as possible, it is proposed that you wind half the wire in one direction and the other half in the opposite direction. Would this achieve the desired result? Why or why not?

- Q30.9 In an R-C circuit, a resistor, an uncharged capacitor, a dc battery, and an open switch are in series. In an R-L circuit, a resistor, an inductor, a dc battery, and an open switch are in series. Compare the behavior of the current in these circuits (a) just after the switch is closed and (b) long after the switch has been closed. In other words, compare the way in which a capacitor and an inductor affect a circuit.

- Q31.1 Household electric power in most of western Europe is supplied at 240 V, rather than the 120 V that is standard in the United States and Canada. What are the advantages and disadvantages of each system?
- Q31.4 Equation (31.14) was derived by using the relationship i = dq/dt between the current and the charge on the capacitor. In Fig. 31.9a the positive counterclockwise current increases the charge on the capacitor. When the charge on the left plate is positive but decreasing in time, is i = dq/dt still correct or should it be i = -dq/dt? Is i = dq/dt still correct when the right-hand plate has positive charge that is increasing or decreasing in magnitude? Explain.
- Q31.8 In an L-R-C series circuit, can the instantaneous voltage across the capacitor exceed the source voltage at that same instant? Can this be true for the instantaneous voltage across the inductor? Across the resistor? Explain.
- E30.2 In Example 30.1, suppose the current i2 in the outer coil is given by $i2 = (2.0 * 10^6 A/s)t$. (Currents in wires can indeed increase this rapidly for brief periods.) (a) At t = 3.0 μ s, what is the average magnetic flux through each turn of the solenoid (coil 1) due to the current in the outer coil? (b) What is the induced emf in the solenoid?
- E30.4 If the current in the toroidal solenoid in Example 30.3 increases uniformly from 0 to 6.0 A in 3.0μs, find the magnitude and direction of the self-induced emf.
- Application Page1052 Figure 31.12 shows an application of the above discussion to a loudspeaker system. Low-frequency sounds are produced by the woofer, which is a speaker with large diameter; the tweeter, a speaker with smaller diameter, produces highfrequency sounds. In order to route signals of different frequency to the appropriate speaker, the woofer and tweeter are connected in parallel across the amplifier output. The capacitor in the tweeter branch blocks the low-frequency components of sound but passes the higher frequencies; the inductor in the woofer branch does the opposite.

3 Problems and exercises

Question 1

Exercises: 30.9, 30.11, 30.21, 30.32, 30.36, 30.40

Question 2

Problems: 30.45 (!), 30.47, 30.62, 30.67, 30.77 (time-permitting)

Question 3

Exercises: 31.2, 31.13, 31.21, 31.34

Question 4

Problems: 31.49, 31.61