PROBLEM SET 6

Due: 28 October 2019, 12.30 p.m.

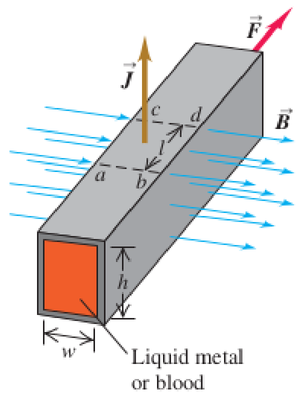
Problem 1. A particle with mass m and positive charge q moves in antiparallel electric and magnetic fields $\mathbf{E} = (-E_0, 0, 0)$ and $\mathbf{B} = (B_0, 0, 0)$, where E_0 and B_0 are positive constants. Assuming the initial conditions: $\mathbf{v}(0) = (v_{0x}, v_{0y}, 0)$ and $\mathbf{r}(0) = (0, 0, 0)$, find the velocity $\mathbf{v}(t)$ and position $\mathbf{r}(t)$ for $t > 0$.

(*6 points*)

Problem 2. Magnetic forces acting on conducting fluids provide a convenient means of pumping these fluids. For example, this method can be used to pump blood without the damage to the cells that can be caused by a mechanical pump. A horizontal tube with rectangular cross section (height h, width w) is placed at right angles to a uniform magnetic field with magnitude B so that a length l is in the field (see the figure). The tube is filled with a conducting liquid, and an electric current of density J is maintained in the third mutually perpendicular direction.

(a) Show that the difference of pressure between a point in the liquid on a vertical plane through ab and a point in the liquid on another vertical plane through cd, under conditions in which the liquid is prevented from flowing, is $\Delta p = JlB$.

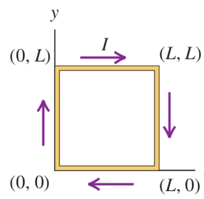
(b) What current density is needed to provide a pressure difference of 1 atm between these two points if $B = 2.2$ T and $l = 35$ mm?



(*2 + 1 points*)

Problem 3. As we discussed in class, the net force on a current loop in a uniform magnetic field is zero. But what if B is not uniform?

The figure shows a square loop of wire that lies in the xy-plane. The loop has corners at $(0,0), (0,L), (L,0)$, and $(L,0)$. It carries a constant current I in the clockwise direction. The magnetic field has no x-component but has both y- and z-components: $\mathbf{B}(\mathbf{r}) = \frac{B_0 z}{L}\,\hat{n}_y + \frac{B_0 y}{L}\hat{n}_z$, where B_0 is a positive constant.



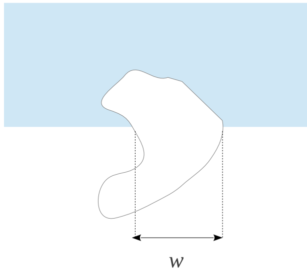
(a) use a computer (e.g. *Wolfram Mathematica* or *Matlab*) to sketch the magnetic field lines in the yz-plane.

(b) Find the magnitude and direction of the magnetic force exerted on each of the sides of the loop.

(c) Find the magnitude and direction of the net magnetic force on the loop.

(*1 + 2 + 1 points*)

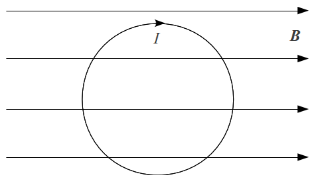
Problem 4. A plane wire loop of irregular shape is situated so that part of it is in a uniform magnetic field $\mathbf{B}$ (in the figure below the field occupies the shaded region and points perpendicular to the plane of the loop). The loop carries the current I. Show that the magnitude of the net magnetic force on the loop is $F = IBw$, where w is the chord subtended.



What is the direction of the force?

(*3 points*)

Problem 5. A circular loop of radius R carries a clockwise electric current I. The loop is placed in a uniform magnetic field $\mathbf{B}$ (see the figure).



(a) What is the net force on the current loop?

(b) Find the torque on the current loop with respect to the axis of symmetry of the loop perpendicular to the vector $\mathbf{B}$.

(*1 + 3 points*)

Problem 6. In the Bohr model of the hydrogen atom, in the lowest energy state the electron orbits the proton at a speed of 2.2×10^6 m/s in a circular orbit of radius 5.3×10^{-11} m.

(a) What is the orbital period of the electron?

(b) If the orbiting electron is considered to be a current loop, what is the current I?

(c) What is the magnetic moment of the atom due to the orbital motion of the electron?

(*1 + 1/2 + 1/2 points*)