VP240-1 Recitation class
Week #2

Bicheng Gan
ganbicheng@sjtu.edu.cn

SJTU - UMJI — September 23, 2019

Brief introduction

- About homework one: Briefly discuss consequences of this fact (qualitative discussion is sufficient). You should relate it with the simple harmonic oscillation, instead of discuss the phenomenon.
- About homework one: Approximation. Do it at the final step, not do it within your progress.
- Keep in mind of the application of Gauss's law, the difficulties will begin right now.
- Don't be hesitate. If you have any problem, please ask your friends as well as our TAs or instructors.
- I'll leave the electric potential on the next RC to maintain the completeness.
- I assume you read the notes at least once and listen to the lectures.

1 Short Review

1. Electric flux and enclosed charge

   There is an alternative relationship between charge distributions and electric fields. (not just vector sum). If the electric-field pattern is known in a given region, what can we determine about the charge distribution in that region?

   > **Info:**
   >
   > - **electric flux**.
   > - Positive charge inside the box goes with an outward electric flux through the box's surface, and negative charge inside goes with an inward electric flux.
   > - The net electric flux through the surface of the box **is directly proportional to** the magnitude of the net charge enclosed by the box. (How to explain it?)
   > - the net electric flux due to a single point charge inside the box **is independent of** the size of the box and depends only on the net charge inside the box.

2. Qualitative statement of Gauss's Law

   To summarize, for the special cases of a closed surface in the shape of a rectangular box and charge distributions made up of point charges or infinite charged sheets, we have found:

Info:

- Whether there is a net outward or inward electric flux through a closed surface depends on the sign of the enclosed charge.
- Charges outside the surface do not give a net electric flux through the surface.
- The net electric flux is directly proportional to the net amount of charge enclosed within the surface but is otherwise independent of the size of the closed surface.

3. Calculating electric flux

   Consider first a flat area A perpendicular to a uniform electric field $\overrightarrow{\boldsymbol{E}}$. We define the electric flux through this area to be the product of the field magnitude E and the area A :

$$\Phi_E = EA \tag{1}$$

Info:

- We use the fluid-flow analogy: water fluid.
- $\Phi_E = \overrightarrow{\boldsymbol{E}} \cdot \overrightarrow{\boldsymbol{A}}$ (electric flux for uniform $\overrightarrow{\boldsymbol{E}}$, flat surface)
- A surface has two sides, so there are two possible directions for $\hat{n}$ and $\vec{A}$. We must always specify which direction we choose. We once related the charge inside a closed surface to the electric flux through the surface. With a closed surface we will always choose the direction of $\hat{\boldsymbol{n}}$ to be outward, and we will speak of the flux out of a closed surface. Thus what we called "outward electric flux" corresponds to a positive value of Φ_E, and what we called "inward electric flux" corresponds to a negative value of Φ_E .

4. Flux of a nonuniform electric field

   We calculate the electric flux through each element and integrate the results to obtain the total flux:

$$\Phi_E = \int E \cos\phi dA = \int E_\perp dA = \int \overrightarrow{\boldsymbol{E}} \cdot d\overrightarrow{\boldsymbol{A}} \tag{2}$$

Info:

- We call this integral the surface integral of the component $E_\perp$ over the area, or the surface integral of $\overrightarrow{\boldsymbol{E}} \cdot d\overrightarrow{\boldsymbol{A}}$.
- Choose the form that suits the situation. Not stick to one formula.

5. General Form of a Gauss's Law

   Now comes the final step in obtaining the general form of Gauss's law. Suppose the surface encloses not just one point charge q but several charges $q_1, q_2,\ q_3, \ldots$ The total (resultant) electric field $\vec{E}$ at any point is the vector sum of the E fields of the individual charges. Let Q_{encl} be the total charge enclosed by the surface: $Q_{\text{encl}} = q_1 + q_2 + q_3 + \cdots$ Also let $\vec{E}$ be the total field at the position of the surface area element $d\vec{A}$, and let E_1 be its component perpendicular to the plane of that element (that is, parallel to dA). Then we can write an equation for each charge and its corresponding field and and the results. When we do, we obtain the general statement of Gauss's law:

$$\Phi_E = \oint \overrightarrow{\boldsymbol{E}} \cdot d\overrightarrow{\boldsymbol{A}} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

Info:

- **The total electric flux through a closed surface is equal to the total (net) electric charge inside the surface, divided by ε_0.**
- It provides a relationship between the electric field on a closed surface and the charge distribution within that surface.

Notice: Gaussian surfaces are imaginary. Remember that the closed surface in Gauss's law is imaginary; there need not be any material object at the position of the surface. We often refer to a closed surface used in Gauss's law as a Gaussian surface.

6. Applications of Gauss's Law

   Gauss's law is valid for any distribution of charges and for any closed surface.

Info:

- When excess charge is placed on a solid conductor and is at rest, it resides entirely on the surface, not in the interior of the material.
- Conductor with the cavity.

2 Discussion

- A hemispherical open surface of radius R is placed in a uniform field of magnitude E. What would be the flux through the entire closed surface? What is the flux through the flat end?
- If the electric field of a point charge were proportional to $1/r^3$ instead of $1/r^2$, would Gauss's law still be valid? Explain your reasoning. (Hint: Consider a spherical Gaussian surface centered on a single point charge.)
- Explain this statement: "In a static situation, the electric field at the surface of a conductor can have no component parallel to the surface because this would violate the condition that the charges on the surface are at rest." Would this statement be valid for the electric field at the surface of an insulator? Explain your answer and the reason for any differences between the cases of a conductor and an insulator.
- How to argue that central forces $F(r) = f(r)r$ are conservative?

3 Problems and exercises

Question 1

Consider an insulating ball with constant density of charge $\rho < 0$. Imagine that a part of the ball has been removed, leaving an empty spherical bubble inside the ball. The distance between the center of the ball and the center of the bubble is r_0 .
Use the superposition principle to find the electric field (both magnitude and direction) inside the bubble.

Question 2

Electric charge is distributed uniformly along an infinitely long, thin wire. The charge per unit length is λ (assumed positive). Find the electric field by using Gauss's law.

Question 3

An infinite cylinder of radius R is charged non-uniformly with the density of charge $\rho = Ar$, where A is a positive constant. Find the electric field at any point of space (consider both: $r < R$ and $r > R$). (Pay attention to the discussion of mutual orientation of vectors E and dA on the Gaussian surface, and their contribution to the electric flux.)

Question 4

Negative charge -Q is distributed uniformly over the surface of a thin spherical insulating shell with radius R. Calculate the force (magnitude and direction) that the shell exerts on a positive point charge q located a distance (a) $r < R$ from the center of the shell (outside the shell); (b) $r < R$ from the center of the shell (inside the shell).

Question 5

Which statement is true about $\overrightarrow{E}$ inside a negatively charged sphere as described here? (a) It points from the center of the sphere to the surface and is largest at the center. (b) It points from the surface to the center of the sphere and is largest at the surface. (c) It is zero. (d) It is constant but not zero.