PROBLEM SET 7

Due: 12 November 2019, 12.30 p.m.

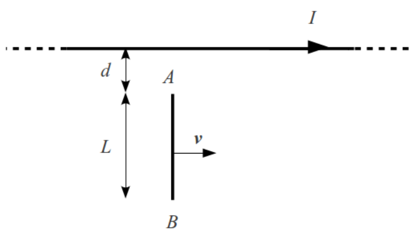
Problem 1. Consider a solenoid of a *finite* length l and radius R, with the current I running through it. The number of turns per unit length of the solenoid is n. Find the magnitude B of the magnetic field inside the solenoid, on its axis of symmetry as a function of the position x with respect to the center ($x = 0$). Plot the graph of $B(x)/B(0)$ vs. x/R for $l = 10R$ and $l = 20R$, attach the plots to your homework.

Hint. Approximate the solenoid as consisting of a large number of circular loops stacking together and use the result for a single loop that we obtained in class when discussing the law of Biot and Savart.

Comment. You test this formula in one of the labs.

(*4 points*)

Problem 2. A long straight wire carries constant current I. A metal bar of length L is moving at constant velocity $\mathbf{v}$, as shown in the figure. Point A is a distance d from the wire.



What is the electric potential difference between points A and B? Which point is at higher potential?

(*3 points*)

Problem 3. A metal bar of length L, mass m and resistance R is placed on long frictionless metal rails that are inclined at an angle φ above the horizontal. The rails have negligible resistance. A uniform magnetic field of magnitude B is directed downward as in the figure below. The bar is released from rest and slides down the rails.

(a) What is the terminal speed of the bar?

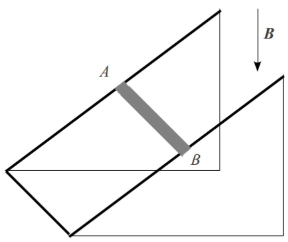
(b) Is the direction of the current induced in the bar from A to B or from B to A?

(c) What is the induced current in the bar when the terminal speed has been reached?

(d) After the terminal speed has been reached, at what rate is electrical energy being converted to thermal energy in the resistance of the bar?

(e) After the terminal speed has been reached, at what rate is work being done on the bar by gravity? Compare your answer to that in part (d).

(*2 + 1/2 + 1/2 + 1 + 1 points*)



Problem 4. A square shaped conducting loop lies in the xy–plane. The coordinates of its vertices are: $(0,0,0)$, $(0,a,0)$, $(a,a,0)$, and $(a,0,0)$. A magnetic field $\mathbf{B}(\mathbf{r},t) = (0,0,4t^2y)$ is applied. Find the emf and the direction of the resulting current at any instant $t > 0$.

(*4 points*)

Problem 5. A rectangular loop of wire of length a, width b, and resistance R is initially ($t = 0$) placed next to an infinitely long wire carrying current I, so that the side with length a is a distance d from the wire. The loop moves away from the long wire with velocity $\mathbf{v}$ pointing in the direction lying in the plane of the loop and perpendicular to the wire. Find (a) the magnitude of the magnetic flux through the loop, (b) the current I_{loop} induced in the loop at any instant of time $t > 0$.

(*4 + 1 points*)

Problem 6. A capacitor has two parallel plates with area A separated by a distance d. The space between plates is filled with a material having relative dielectric permittivity ε_{r}. The material is not a perfect insulator, but has resistivity ρ. The capacitor is initially charged with charge of magnitude Q_0 on each plate, which gradually discharges by conduction through the dielectric.

(a) Calculate the conduction current density $J_{\text{c}}(t)$ in the dielectric.

(b) Show that at any instant the displacement current density in the dielectric is equal in magnitude to the conduction current density but opposite in direction, so the *total* current density is zero at every instant.

(*2 + 3 points*)

Problem 7. It is impossible to have a uniform electric field that abruptly drops to zero in a region of space in which the magnetic field is constant. Prove this statement.

You may construct an indirect proof: assume that it is possible, and then show that your assumption contradicts a law of nature.

(a) In the bottom half of a piece of paper, draw evenly spaced horizontal lines representing a uniform electric field to your right. Use dashed lines to draw a rectangle $ABCDA$ with horizontal side AB in the electric field region and horizontal side CD in the top half of your paper where $\mathbf{E} = 0$.

(b) Show that integration along the loop formed by your rectangle contradicts Faraday's law.

(*2 points*)