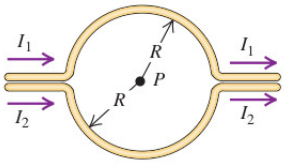
PROBLEM SET 7

Due: 4 November 2019, 12.30 p.m.

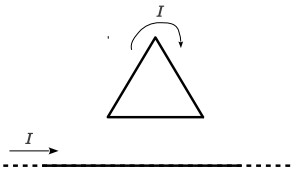
Problem 1. Calculate the magnitude of the magnetic field at point P in terms of R and the currents I_1 and I_2 flowing through the long wires with semicircular sections shown in the figure below. What does your expression give if $I_1 = I_2$?

(3 points)



Problem 2. What is the force on the equilateral triangle loop with side a placed a distance s from a long, straight–line wire? The electric current in both is I.

(3 points)



Problem 3. A thin disk made of dielectric material with radius a has total charge $Q > 0$ distributed uniformly over its surface. It rotates n times per second about the axis perpendicular to the surface of the disk and passing through its center. Find the magnetic field at the center of the disk.

(3 points)

Problem 4. An infinitely long conducting tape of width L and negligible thickness lies in a horizontal plane and carries a uniform current I (in the direction of the long dimension).

(a) Show that at on the axis of symmetry of the tape, at a distance y from its surface, the magnitude of the magnetic field is equal to $B(y) = (\mu_0 I/\pi L) \arctan L/2y$.

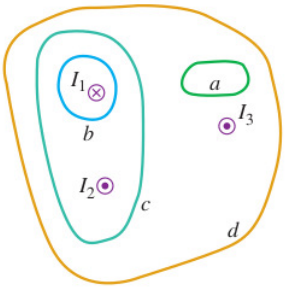
(b) Discuss the result in the limit $y \gg L$.

(3 + 1/2 points)

Problem 5. The figure below shows, in cross section, several conductors carrying currents with magnitudes I_1, I_2, and I_3 through the plane of the figure in the directions shown. Four loops labeled a through d are shown there, too.

What is the circulation of the magnetic field $\oint \mathbf{B} \circ d\mathbf{l}$ for each loop traversed in the counterclockwise direction?

(4 × 1/2 point)



Problem 6. A long, straight, solid cylinder of radius a, oriented with its axis in the z–direction, carries an electric current of density

$$\mathbf{J}(\mathbf{r}) = \begin{cases} \frac{b}{r} \exp\left(\frac{r-a}{\delta}\right) \hat{\mathbf{k}} & \text{for } r \leq a \\ 0 & \text{otherwise,} \end{cases}$$

where r is the radial distance from the axis of the cylinder and $a, b, \delta > 0$ are constants (what are their units?).

(a) Let I_0 be the total current passing through the entire cross section of the wire. Obtain an expression for I_0 in terms of a, b, δ.

(b) Use Ampère's law to find the magnetic field $\mathbf{B}$ in the region $r > a$. Express your answer in terms of I_0 rather than b.

(c) Obtain an expression for the current I through a circular cross section of radius $r \leq a$ and centered at the cylinder axis. Express your answer in terms of I_0.

(d) Use Ampère's law to find the magnetic field $\mathbf{B}$ in the region $r \leq a$.

(3/2 + 2 + 1 + 2 points)