VP 240 Homework 4 [illegible] 518021911039

Problem 1

(a).
$$\begin{cases} \frac{4}{3}\pi r^3 \rho g = qE \\ V_{AB} = Ed \end{cases} \Rightarrow \frac{4}{3}\pi r^3 \rho g = q\frac{V_{AB}}{d} \Rightarrow q = \frac{4\pi}{3}\cdot\frac{\rho r^3 g d}{V_{AB}}$$

(b).
$$6\pi\eta r v_\infty = mg = \frac{4}{3}\pi r^3 \rho g \Rightarrow r = \sqrt{\frac{9}{2}\cdot\frac{\eta v_\infty}{\rho g}} = \frac{3\sqrt{2}}{2}\sqrt{\frac{\eta v_\infty}{\rho g}}$$

$$q = \frac{\cancel{4\pi}}{\cancel{3}}\cdot\frac{\cancel{\rho g} d}{V_{AB}}\cdot\frac{9\eta v_\infty}{\cancel{2\rho g}}\cdot\frac{3\sqrt{2}}{\cancel{2}}\sqrt{\frac{\eta v_\infty}{\rho g}}$$

$$= \frac{9\sqrt{2}\,\pi d\eta v_\infty}{V_{AB}}\sqrt{\frac{\eta v_\infty}{\rho g}} = 18\pi\frac{d}{V_{AB}}\sqrt{\frac{\eta^3 v_\infty^3}{2\rho g}}$$

(c).
$$v_\infty = \frac{10^{-3}}{39.3} = 2.545\times10^{-5}\ m/s$$

$$q = 18\pi\frac{10^{-3}}{9.16}\sqrt{\frac{\left(\frac{10^{-3}}{39.3}\right)^3(1.81\times10^{-5})^3}{2\times824\times9.8}} = 4.80\times10^{-19}\ C$$

Therefore, there are three excess electrons.

From (b), we can get that
$$r = \frac{3\sqrt{2}}{2}\sqrt{\frac{1.81\times10^{-5}\times\frac{10^{-3}}{39.3}}{824\times9.8}} = 5.07\times10^{-7}\ m$$

Problem 2

(a).
$$C = \frac{Q}{V_{ab}} = \varepsilon_0\frac{A}{x}$$

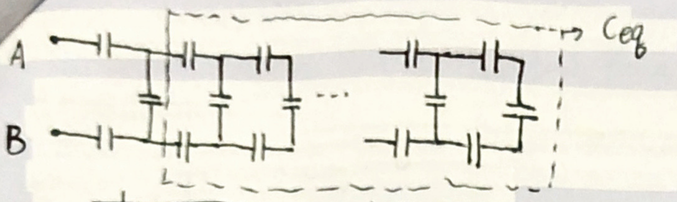
$$U = \frac{Q^2}{2C} = \frac{Q^2 x}{2\varepsilon_0 A}$$

(b).
$$\Delta U = \frac{Q^2(x+dx)}{2\varepsilon_0 A} - \frac{Q^2 x}{2\varepsilon_0 A} = \frac{Q^2 dx}{2\varepsilon_0 A}$$

(c).
$$F\,dx = \frac{Q^2 dx}{2\varepsilon_0 A} \Rightarrow F = \frac{Q^2}{2\varepsilon_0 A}$$
(The direction of F is vertical to the plate and pointing outwards)

(d). E depends on both of the plates, and it is only true between the two plates, so the force exerted by the electric field on one plate does not equal to EQ. F depends on one plate and it equals to the force to the other plate in magnitude.

3.



$$\cancel{\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C_0} + \frac{1}{C}}$$

$$\frac{1}{C_{eq}+C} + \frac{1}{C} + \frac{1}{C} = \frac{1}{C_{eq}}$$

$$C C_{eq} + 2C_{eq}^2 + 2C C_{eq} = C C_{eq} + C^2$$

$$2C_{eq}^2 + 2C C_{eq} - C^2 = 0$$

$$C_{eq} = \frac{\sqrt{3}-1}{2} C$$

Problem 4.

(a). $$\frac{\varepsilon_1 \frac{L^2}{D}}{\varepsilon_0 \frac{L^2}{D}} = \varepsilon_r \Rightarrow \varepsilon_1 = \varepsilon_r \varepsilon_0$$

$$C = \varepsilon_r \varepsilon_0 \frac{Lx}{D} + \varepsilon_0 \frac{L(L-x)}{D} = \frac{\varepsilon_0 L}{D}(\varepsilon_r x + L - x)$$

(b). $$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2$$

$$dU = \frac{1}{2}\left(\varepsilon_r\varepsilon_0 \frac{L(x+dx)}{D} + \varepsilon_0 \frac{L(L-x-dx)}{D}\right)V^2 - \frac{1}{2}\left(\varepsilon_r\varepsilon_0 \frac{Lx}{D} + \varepsilon_0 \frac{L(L-x)}{D}\right)V^2$$

$$= \frac{1}{2}\left(\varepsilon_r\varepsilon_0 \frac{L\,dx}{D} - \varepsilon_0 \frac{L\,dx}{D}\right)V^2 = \frac{(\varepsilon_r - 1)\varepsilon_0 V^2 L}{2D}dx$$

(C). $$Q = CV = \frac{\varepsilon_0 L V}{D}(\varepsilon_r x + L - x)$$

$$\cancel{dU = \frac{Q^2}{2\frac{\varepsilon_0 L}{D}}\left(\frac{1}{\varepsilon_r(x+dx)+L-x-dx} - \frac{1}{\varepsilon_r x + L - x}\right) = \frac{\varepsilon_0 L V^2(\varepsilon_r x + L - x)}{2D} \cdot \frac{(1-\varepsilon_r)dx}{\varepsilon_r x + L - x + \varepsilon_r dx - dx}}$$

$$dU = \frac{Q^2}{2C_1} - \frac{Q^2}{2C} = \frac{Q^2}{2}\left(\frac{1}{C_1} - \frac{1}{C}\right) = \frac{Q^2}{2}\left(-\frac{1}{C^2}\right)dC = \frac{Q^2}{2}\left(-\frac{1}{C^2}\right)\frac{dC}{dx}dx\,I(b)$$

$$= -\frac{Q^2}{2C^2}\frac{\varepsilon_0 L}{D}(\varepsilon_r - 1)dx = -\frac{(\varepsilon_r - 1)\varepsilon_0 V^2 L}{2D}dx = -dU(b)$$

r (b). $\frac{(\varepsilon_r-1)\varepsilon_0 V^2 L}{2D} dx = -\vec{F} dx \Rightarrow \vec{F} = -\frac{(\varepsilon_r-1)\varepsilon_0 V^2 L}{2D}$, Minus means opposite with $\vec{x}$

which suggests that the electric force on the slab pushes it out of the capacitor

For (c). $-\frac{(\varepsilon_r-1)\varepsilon_0 V^2 L}{2D} dx = -\vec{F} dx \Rightarrow \vec{F} = \frac{(\varepsilon_r-1)\varepsilon_0 V^2 L}{2D}$, Positive means the same with $\vec{x}$,

which suggests that the electric force on the slab pulls it into the capacitor.

(e). The connected battery does work when the slab is inserted an additional distance.

$$|\vec{F}| = \frac{\varepsilon_0 L V^2 (\varepsilon_r - 1)}{2D}$$