



JOINT INSTITUTE  
交大密西根学院

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PHYSICS LABORATORY  
VP241

EXERCISE 4

POLARIZATION OF LIGHT

# 1 Pre-lab Reading

Chapter 33, in particular section 33.5 (Young and Freedman)

## 2 Objectives

The objective of this exercise is to understand some properties of light, in particular to study the polarization phenomenon and verify Malus' law, as well as to understand the way half- and quarter-wave plates work in optical systems. Generation and detection of elliptically and circularly polarized light will also be investigated.

## 3 Theoretical Background

Light can be described in terms of electromagnetic waves, with the plane of oscillations of the electric field vector (as well as the magnetic field vector) perpendicular to the direction of light propagation. Therefore, light is an example of a transverse wave. For light sources producing the so-called *natural light*, the emitted light is a random mixture of waves with the electric field vector oscillating in all possible transverse directions. This is due to the randomness of the radiation mechanism. Such *natural light* is also called *unpolarized light*. For unpolarized light the distribution of the directions of the electric field vector, in the plane perpendicular to the direction of propagation, is uniform. If the distribution is not uniform, the light is said to be *polarized*. Studies of the polarization of light played an important role in the development of wave optics. They have resulted in a wide range of applications in numerous areas, such as optical measurement techniques, crystal structure research, and experimental stress analysis.

### 3.1 Polarization of Light

The electric field vector  $\mathbf{E}$ , which in the context of electromagnetic waves corresponding to the visible part of the spectrum is sometimes referred to as the light vector describes a time-dependent, propagating electric field. In the plane perpendicular to the propagation direction of a light wave, the light vector may have different directions along which its magnitude oscillates. The light, for which the light vector maintains a certain oscillation direction, is called *linearly polarized* and the axis defining the direction is called the polarization axis (see Figure 5).

The light with the light vector direction rotating about the propagation direction, so that its endpoint traces a circle, is called *circularly polarized* light. If the vector traces an ellipse, the light is said to be *elliptically polarized* (see Figure 2).

Light emitted from ordinary light sources (natural light) is unpolarized. However it can be regarded as a statistical equal-weight mixture of linearly polarized waves with equal amplitudes. There the light may be also partially polarized, which means it can be regarded as a combination of a polarized and the natural (unpolarized) light. The

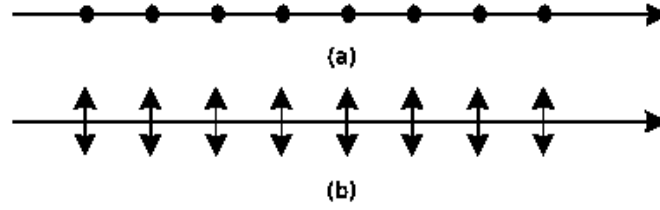


Figure 1. (a) Linearly polarized light with the polarization axis perpendicular to the page plane. (b) Linearly polarized light with the polarization axis parallel to the page plane.

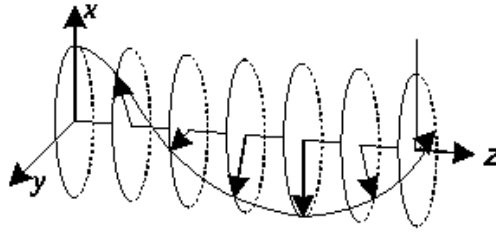


Figure 2. Elliptically polarized light propagating in the  $z$  direction. The light is polarized in the  $xy$  plane.

direction corresponding to the maximum amplitude of the light vector of such partially polarized light is the oscillation direction of the polarized component.

### 3.2 Polarizer

A device commonly used to produce polarized light is a polaroid (also called a polarizer). It polarizes the light using the principle of dichroism: a selective absorption mechanism tends to allow the light polarized in a certain direction (direction of the crystal alignment) to pass through the material, while the light polarized in all other directions is absorbed. This turns the incident natural light into linearly polarized.

A polarization device can not only change incident natural light to polarized light (it then acts as a polarizer), but may also be used to detect and analyze linearly polarized, natural, and partially polarized light (it is then called an analyzer).

### 3.3 Malus' law

A visible effect in the light coming out of a polarization device is a change of the light brightness.

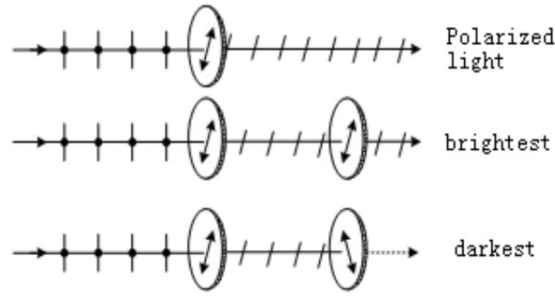


Figure 3. Change in the brightness of the light depends on the mutual orientation of the polarizer and the analyzer.

Suppose that we have two polarizers arranged so that their planes are parallel — the left one plays the role of a polarizer, the other one is an analyzer (see Figure 3). Let the angle between their transmission directions (polarization axes) be  $\theta$ . The light is incident normally on the polarizer and then continues to the analyzer. The intensity of the linearly polarized light leaving the analyzer is

$$I_{\text{light}} = I_{\text{light},0} \cos^2 \theta, \quad (1)$$

where  $I_{\text{light},0}$  is the intensity of the linearly polarized light incident on the analyzer. Equation (1), named after Étienne-Louis Malus as the Malus' law, was derived in 1809.

Obviously, for a single polarizer, if polarized light is incident on it, then the transmitted light intensity will change periodically when rotating the polarizer. If the incident light is partially or elliptically polarized, the minimum intensity will not be zero as there will be always some component of the light polarized in the transmission direction. The incident light must be natural or circularly polarized if the intensity does not change at all. Hence, by using a polarizer, one can distinguish linearly polarized light from the natural and circularly polarized light.

### 3.4 Generation of Elliptically and Circularly Polarized Light. Half-wave and Quarter-wave Plates

Suppose that linearly polarized light is incident normally on a crystal plate whose surface is parallel to its optical axis, and the angle between the polarizing axis and the optical axis of the plate is  $\alpha$ . Then the linearly polarized light is resolved into two waves: an *e*-wave with the oscillation direction parallel to the optical axis of the plate (*extraordinary axis*) and an *o*-wave whose oscillation direction is perpendicular to the optical axis (*ordinary axis*). They propagate in the same direction, but with different speeds. The resulting optical path difference over the thickness  $d$  of the plate is

$$\Delta = (n_e - n_o)d,$$

and, consequently, the phase difference

$$\delta = \frac{2\pi}{\lambda} (n_e - n_o)d,$$

where  $\lambda$  is the wavelength,  $n_e$  is the refractive index for the extraordinary axis, and  $n_o$  is the refractive index for the ordinary axis. In a so-called positive crystal  $\delta > 0$ , whereas in a negative one  $\delta < 0$ .

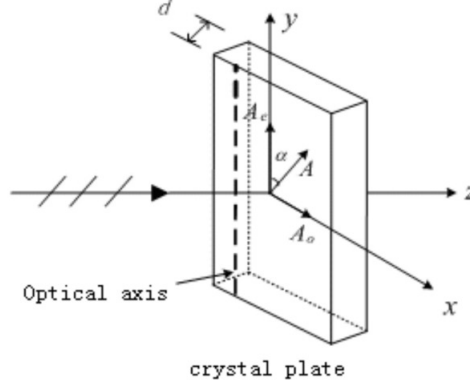


Figure 4. Linearly polarized light passing through a waveplate.

As shown in Figure 4, when the light propagates through the crystal plate, the two components of the light vector are

$$\begin{aligned} E_x &= A_o \cos \omega t \\ E_y &= A_e \cos(\omega t + \delta), \end{aligned}$$

where  $A_e = A \cos \alpha$ ,  $A_o = A \sin \alpha$ . Eliminating time from the above equations one obtains

$$\frac{E_x^2}{A_o^2} + \frac{E_y^2}{A_e^2} - 2 \frac{E_x E_y}{A_o A_e} \cos \delta = \sin^2 \delta. \quad (2)$$

Note that is the equation of an ellipse for  $\delta = \pm\pi/2$ .

When the thickness of the plate changes, the optical path difference changes as well. Some cases of particular interest, are discussed below:

- If  $\Delta = k\lambda$ , where  $k = 0, 1, 2, \dots$ , the phase difference  $\delta = 0$ , and Eq. (2) reduces to

$$E_y = \frac{A_e}{A_o} E_x,$$

which is a linear equation. Hence the transmitted light is linearly polarized with the oscillation direction remaining unchanged. A waveplate that satisfies this condition is called a *full-wave plate*. The light goes through a full-wave plate without changing its polarization state.

- If  $\Delta = (2k + 1)\lambda/2$ , where  $k = 0, 1, 2, \dots$ , the phase difference  $\delta = \pi$ , and Eq. (2) simplifies to

$$E_y = -\frac{A_e}{A_o} E_x.$$

The transmitted light is also linearly polarized with the polarization axis rotated by the angle of  $2\alpha$ . A waveplate that satisfies the condition is called *1/2-wave plate* or *half-wave plate*. When a polarized light passes through a half-wave plate, its polarization axis gets rotated by an angle  $2\alpha$ . If  $\alpha = \pi/4$ , then the polarization axis of the transmitted light is perpendicular to that of the incident light.

- Finally, if  $\Delta = (2k + 1)\lambda/4$ , where  $k = 0, 1, 2, \dots$ , the phase difference  $\delta = \pm\pi/2$ , and Eq. (2) transforms into

$$\frac{E_x^2}{A_o^2} + \frac{E_y^2}{A_e^2} = 1.$$

The transmitted light is elliptically polarized. A waveplate that satisfies the above condition is called a *1/4-wave plate* or a *quarter-waveplate* and is an important optical element in many polarization experiments.

If  $A_e = A_o = A$ , then  $E_x^2 + E_y^2 = A^2$ , and the transmitted light is circularly polarized. Since the amplitudes of the *o*-wave and the *e*-wave are both functions of  $\alpha$ , the polarization state after passing through a 1/4-wave plate will vary, depending on the angle:

- if  $\alpha = 0$ , the transmitted light is linearly polarized with the polarization axis parallel to the optical axis of the 1/4-wave plate;
- if  $\alpha = \pi/2$ , the transmitted light is linearly polarized with the polarization axis perpendicular to the optical axis of the 1/4-wave plate;
- if  $\alpha = \pi/4$ , the transmitted light is circularly polarized;
- otherwise, the transmitted light is elliptically polarized.

## 4 Measurement Setup and Procedure

### 4.1 Apparatus

The measurement setup consists of: a semiconductor laser, a tungsten iodine lamp, a silicon photo-cell, a UT51 digital universal meter, as well as two polarizers, 1/2-wave and 1/4-wave plates (the uncertainty of the the angle is  $2^\circ$ ) and a lens with a glass sheet. The elements are placed on an optical bench.

### 4.2 Measurement Procedure and Data Analysis

#### 4.2.1 Apparatus Adjustment

1. Adjust the photo-cell by choosing the appropriate aperture. There are different apertures on the photo-cell (see the figure below) used in different experiments. In this experiment, only the  $\varnothing 6.0$  aperture, which preserves the incident light intensity, is needed. If other aperture is chosen, the intensity of light may get reduced, resulting in a zero reading on the universal meter.

Therefore, before proceeding to the next steps, adjust the laser and the photo-cell so that the light can pass through the  $\varnothing 6.0$  aperture.

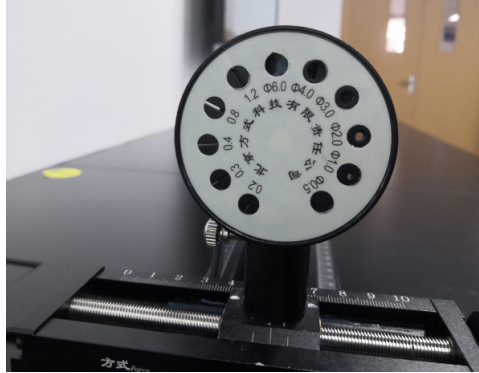


Figure 5. Photo-cell.

2. With the laser fixed at one of the ends of the bench, place the lens and the glass sheet in front of it. Make sure that the light passes through the center of the lens.
3. Adjust the distance between the lens and the laser to the focal length of the lens.
4. Move the glass sheet along the bench. If the size of the light spot on the glass varies significantly, repeat Step 2.
5. Remove the glass sheet. Set the digital universal meter in the appropriate mode and range.

#### 4.2.2 Demonstration of Malus' Law

1. Assemble the measurement setup as shown in Figure 6. Make sure that the laser ray passes through the polarizer to generate linearly polarized light before continuing to the analyzer and the silicon photo-cell.
2. Rotate the analyzer for  $360^\circ$  and observe a change in the light intensity to find the maximum electric current  $I_0$ .
3. Set the angle of analyzer to  $90^\circ$  and adjust the angle of the polarizer until the electric current measured by the multimeter reaches its minimum. At this point, the polarizing axes of the polarizer and the analyzer are perpendicular to each other.
4. Rotate the analyzer from  $90^\circ$  to  $0^\circ$  and record the magnitude of the current  $I$  every  $5^\circ$ . Record the values in a table and plot the graph  $I/I_0$  vs.  $\cos^2 \theta$ . Perform linear fitting and compare the data with the theoretical result.

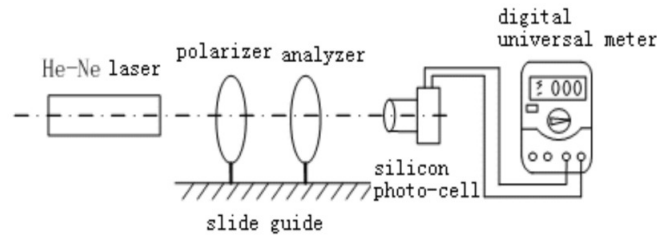


Figure 6. Experimental setup for a demonstration of Malus' law.

#### 4.2.3 Linearly Polarized Light and the Half-wave Plate

1. Set up the equipment on the optical bench as shown in Figure 7.  $A$  is the analyzer and  $P$  is the polarizer. Set the polarizing axes of  $A$  and  $P$  perpendicular to each other before placing the  $1/2$ -wave plate in the apparatus; extinction of the light can be observed on screen.
2. After inserting the  $1/2$ -wave plate, rotate it to make the light extinction appear again and set this position as the initial position.
3. Rotate the  $1/2$ -wave plate for  $\alpha = 10^\circ$  from the initial position and the light extinction will be broken. Then rotate  $A$  to make the light extinction appear again, record the angle of rotation  $\Delta\theta$  in a table.
4. Rotate the  $1/2$ -wave plate for  $10^\circ$  from the previous position (now  $\alpha = 20^\circ$ ) and repeat Step 3. Repeat this step (increase  $\alpha$ ) for 8 times. Plot the graph  $\Delta\theta$  vs.  $\theta$ .
5. Analyze the data and answer the following questions in your lab report:
  - (a) How many times can the light extinction be observed when the  $1/2$ -wave plate rotates for  $360^\circ$ ?
  - (b) How many times can the light extinction be observed when the analyzer rotates for  $360^\circ$ ?
  - (c) Explain the polarization state of linearly polarized light after passing through the  $1/2$ -wave plate.

#### 4.2.4 Circularly and Elliptically Polarized Light and the $1/4$ -wave Plate

1. Set up the equipment on the optical bench as shown in Figure 7.  $A$  is the analyzer and  $P$  is the polarizer. Set the polarizing axes of  $A$  and  $P$  perpendicular to each other before placing the  $1/4$ -wave plate in the apparatus; extinction of the light can be observed on screen. At this point the angle  $\theta = 90^\circ$ .
2. After inserting the  $1/4$ -wave plate, rotate it to make the light extinction appear again and set this position as the initial position. At this point  $\alpha = 0^\circ$ . Rotate the  $1/4$ -wave plate and observe the change in the light intensity.



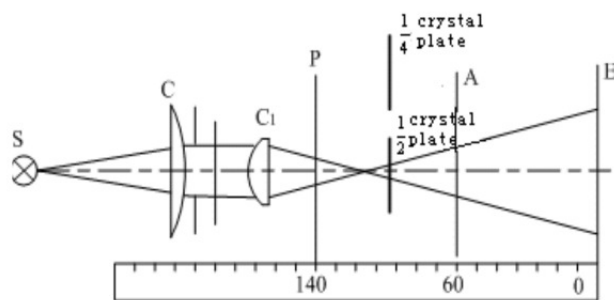


Figure 7. Experimental setup for the 1/2-wave plate.

3. Rotate the analyzer for  $360^\circ$  and record the light intensity (which is indicated by the current  $I$ ) for every  $10^\circ$ . Record the data in a table.
4. Rotate the 1/4-wave plate for  $20^\circ$ , repeat Step 3.
5. Rotate the 1/4-wave plate for  $45^\circ$ , repeat Step 3.
6. Rotate the 1/4-wave plate for  $70^\circ$ . Then rotate the analyzer and record its position and the magnitude of the current when the light intensity reaches a maximum.
7. Use a computer to plot the relation between the rotation angle of the analyzer and the light amplitude in polar coordinates. Normalize the amplitude by its maximum value. Mark the position recorded in Step 6 and compare it with the data recorded in Step 4.

Pay attention to the fact that the light intensity is found indirectly by measuring the electric current, and the intensity is proportional to the amplitude squared. The current indicates the intensity, not the amplitude.

8. Compare the result of Step 5 with that for the circular polarization. Plot a linear fit to the data when the angle is  $45^\circ$ .

## 5 Cautions

- Do not direct the laser beam into the eye.
- Do not touch the surface of the polarizers or the wave plates.
- Please leave the equipment in order before leaving.

## 6 Preview Questions

- ▶ What is natural light?
- ▶ Two polarizers are arranged so that their planes are parallel and their polarizing axes are perpendicular to each other. Natural light is incident normally on the first polarizer and the intensity of light leaving it is  $I_{\text{light},0}$ . What is the intensity of the light leaving the second polarizer? How does the result change if we place a third polarizer between the two, so that its polarizing axis forms the angle  $\pi/4$  with the polarizing axis of the first polarizer? What is the law you used to answer these questions.
- ▶ What is linearly polarized light? How to generate linearly polarized light?
- ▶ What is a  $1/2$ -wave plate? Briefly explain its function.
- ▶ What is a  $1/4$ -wave plate? How to use it to generate and analyze circularly polarized light and elliptically polarized light?
- ▶ How to distinguish whether light is natural or circularly polarized?
- ▶ Suppose that linearly polarized light is incident normally on a crystal plate with thickness  $d$  whose surface is parallel to its optical axis. What is the phase difference between the ordinary and the extraordinary waves leaving the crystal plate? Identify all symbols.