UM-SJTU JOINT INSTITUTE
PHYSICS LABORATORY
(VP 241)

LABORATORY REPORT

EXERCISE 2

THE HALL PROBE: CHARACTERISTICS AND APPLICATIONS

Name: Weikai Zhou	**ID: 518021911039**	**Group: 8**
Name: Yingtao Zou	ID: 518370910069	Group: 8

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1. Introduction [1]

1.1. Objectives

- Try to study the principle of the Hall effect and its applications by using a Hall probe.
- Try to verify that the Hall voltage is proportional to the magnetic field
- Try to study the sensitivity of an integrated Hall probe by calculating the magnetic field at the center of a solenoid.
- Try to measure the magnetic field distribution along the axis of the solenoid and compare it with the corresponding theoretical curve.

1.2. Theoretical background

The phenomenon that when an electric current passes through a sample placed in a magnetic field, electric potential difference proportional to the current and to the magnetic field appears across the material in the direction perpendicular to both the current and the magnetic field is observed by E.H. Hall in 1879. This effect is known as the Hall effect, and since its discovery it has led to many practical applications. The principle of the Hall effect is used in devices for magnetic field measurements as well as in position and motion detectors.

1.2.1. Hall effect

If we consider a conducting sheet, which is made of a metal or a semiconductor, placed in a magnetic field so that the plane of the sheet is perpendicular to the direction of the magnetic field **B** (Figure 1). An electric potential difference between the sides *a* and *b* of the sheet will be generated, if the electric current *I* passes through the sheet in the direction as shown in Figure 1. We call this phenomenon as the Hall effect, and the electric potential difference is called the Hall voltage U_H.

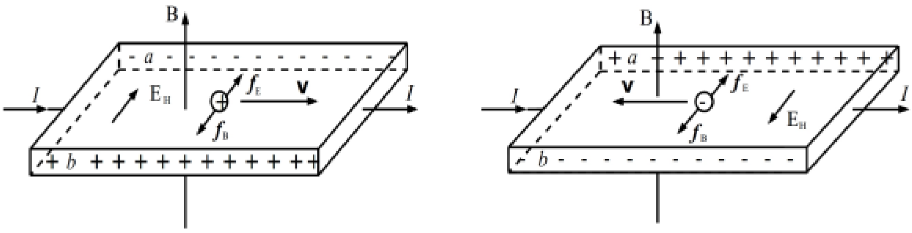

Figure 1. The principle of the Hall effect

From a microscopical perspective, the Hall effect is caused by the Lorentz force, which is a force acting on charges moving in a magnetic field. The Lorentz force F_B leads to the deflection of the moving charges. These charges accumulate on one side of the sheet and increase the magnitude of the transverse electric field E_H (the Hall field). There will be an electric force F_E acting upon the charges due to this field. A balance will be eventually reached and U_H will stabilize since F_B and F_E act in opposite direction. The sign of the charge carriers (positive or negative) will determine the sign of U_H. Therefore, we can analyze the sign of U_H to determine the type of charge carriers in semiconductors.

The Hall voltage is proportional to both the current and the magnitude of the magnetic field, and inversely proportional to the thickness of the sheet d if the external magnetic field is not too strong

$$U_H = R_H \frac{IB}{d} = KIB \tag{1}$$

where R_H is the so-called Hall coefficient and $K = R_H/d = K_H/I$, where K_H is the so-called sensitivity of the Hall element.

1.2.2. Integrated hall probe

If we measure Hall voltage with a Hall probe when the sensitivity K_H and the current I are fixed, we can easily find the magnitude of the magnetic field. We should amplify the Hall voltage before the measurement since it is usually very small.

Silicon can be used to design both the Hall probe and the integrated circuits, so it is convenient to arrange the Hall probe and the electric circuits into a single device. And we usually call such a device an integrated Hall probe.

The integrated Hall probe SS495A consists of a Hall sensor, an amplifier, and a voltage compensator (Figure 2). We can read the output voltage U ignoring the residual voltage. The working voltage $U_S = 5\,V$, and the output voltage U_0 is approximately $2.5\,V$ when the magnetic field is zero. The relation between the output voltage U and the magnetic field is

$$B = \frac{U - U_0}{K_H} \tag{2}$$

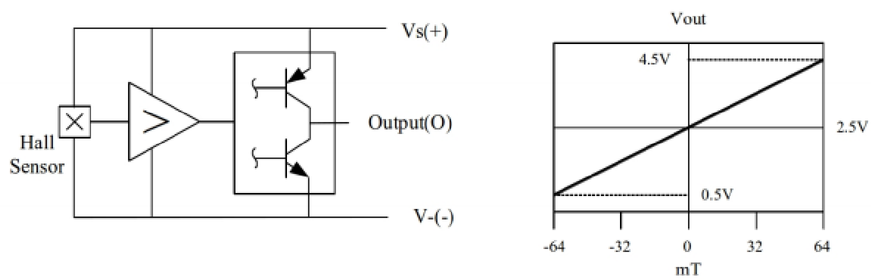

Figure 2. The integrated Hall probe SS495A (left). The relation between the output voltage U and the magnitude of the magnetic field B (right).

1.2.3. Magnetic field distribution inside a solenoid

We use the following formula to calculate the magnetic field distribution on the axis of a single layer solenoid

$$B(x) = \mu_0 \frac{N}{L} I_M \left\{ \frac{L + 2x}{2[D^2 + (L + 2x)^2]^{\frac{1}{2}}} + \frac{L - 2x}{2[D^2 + (L - 2x)^2]^{\frac{1}{2}}} \right\} = C(x) I_M \tag{3}$$

where N is the number of turns of the solenoid, L is its length, I_M is the current through the solenoid wire, and D is the solenoid's diameter. The magnetic permeability of vacuum is $\mu_0 = 4\pi \times 10^{-7}\ H/m$.

We use the solenoid with ten layers for this experiment. The magnetic field $B(x)$ for each layer can be calculated using Eq. (3). We can add contributions of all layers to find the net magnetic on the axis of the solenoid. The theoretical value of the magnetic field inside the solenoid with $I_M = 0.1\ A$ is given in Table 1.

x [cm]	B [mT]	x [cm]	B [mT]
± 0.0	1.4366	± 8.0	1.4057
± 1.0	1.4363	± 9.0	1.3856
± 2.0	1.4356	± 10.0	1.3478
± 3.0	1.4343	± 11.0	1.2685
± 4.0	1.4323	± 11.5	1.1963
± 5.0	1.4292	± 12.0	1.0863
± 6.0	1.4245	± 12.5	0.9261
± 7.0	1.4173	± 13.0	0.7233

Table 1. Theoretical value of the magnetic field inside the solenoid

1.2.4. Study of the geomagnetic field with a Hall probe (optional)

The geomagnetic field is the magnetic associated with the Earth. The geomagnetic field lines are shown schematically in Figure 3. The Earth's magnetic field is similar to that of a bar magnet tilted about 11.5 degrees from the spin axis of the Earth.

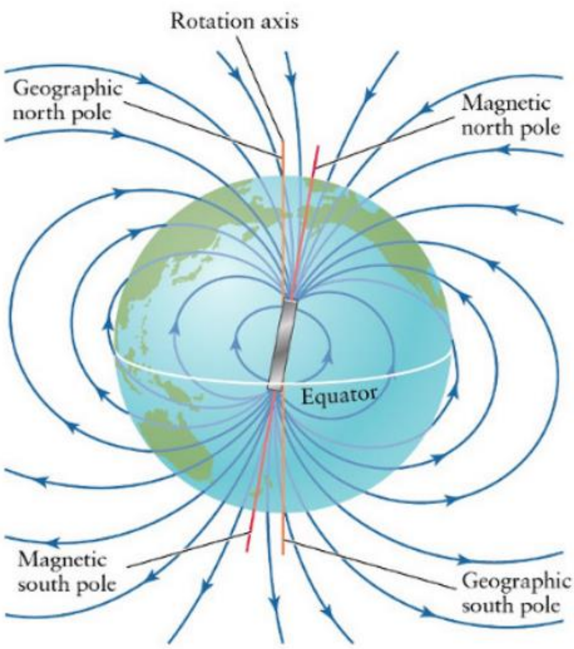

Figure 3. Magnetic field of the Earth

The figure below (Figure 4) shows the geomagnetic field distribution of China in 1970. We can see that the magnetic inclination is about 44.5°and the magnitude of the magnetic field in Shanghai is about 48000 nT.

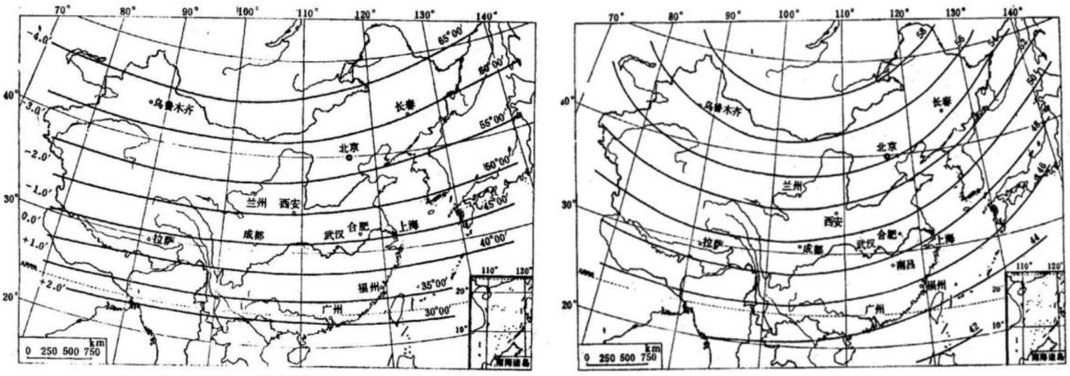

Figure 4. Geomagnetic inclination in China, 1970 (left). The magnitude of the geomagnetic field in China, 1970 (right).

2. Apparatus [1]

2.1. Experimental setup

The experimental setup shown in Figure 5 consists of an integrated Hall probe SS495A (Figure 6) with $K_H = 31.25 \pm 1.25\, V/T$ (at the working voltage 5 V) or $K_H = 3.125 \pm 0.125\, mV/G$, a solenoid, a power supply, a voltmeter, a DC voltage divider, and a set of connecting wires.

Figure 5. Measurement setup

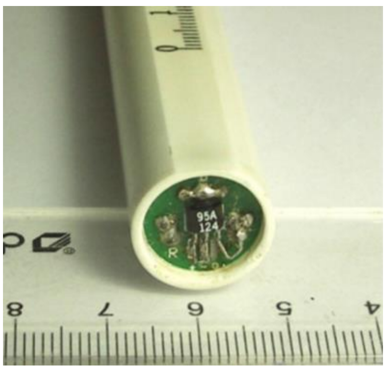

Figure 6. Integrated Hall probe SS495A

2.2. Precision or uncertainty

U_S $(voltage\ source)$	0.5% [V]
U_0 & U (voltimeter)	$0.05\% + 6 \times 10^{-3}\ or\ 6 \times 10^{-4}\ [V]$
I_M $(current\ source)$	$2\%\ [mA]$
x $(distance)$	$0.05\ [cm]$

Table 2. precision or uncertainty

3. Measurement procedure [1]

3.1. Relation between sensitivity K_H and working voltage U_S

3.1.1. We place the integrated Hall probe at the center of the solenoid. Then, we set the working voltage at 5 V and measure the output voltage U_0 (I_M = 0) and U (I_M = 250 mA). We take the theoretical value of B (x = 0) from Table 1 and calculate the sensitivity of the probe K_H by using Eq. (2).

3.1.2. Then, we measure K_H for different values of U_S (form 2.8 V to 10 V). We should calculate K_H/U_S and plot the curve K_H/U_S vs. U_S.

3.2. Relation between output voltage U and magnetic field B

3.2.1. We should connect the 2.4~2.6 V output terminal of the DC voltage divider and the negative port to the voltmeter with $B = 0$, $U_S = 5\ V$. then, we should adjust the voltage until $U_0 = 0$.

3.2.2. We should place the integrated Hall probe at the center of the solenoid and measure the output voltage U for different values of I_M ranging from 0 to 500 mA, with intervals of 50 mA.

3.2.3. We should explain the relation between B (x = 0) and the Hall voltage U_H. We should pay attention to the fact that the output voltage U is the amplified signal from U_H. The theoretical value of B (x = 0) can be found from Table 1.

3.2.4. We should plot the curve U vs. B and find the sensitivity K_H by a linear fit (use a computer). We should compare the value we obtained with the theoretical value in given in the Apparatus section and the value we have found in the first part.

3.3. Magnetic field distribution inside the solenoid

3.3.1. We should measure the magnetic field distribution along the axis of the solenoid for I_M = 250 mA, record the output voltage U and the corresponding position x. Then we should find $B = B(x)$. (We can use the value of K_H found in the previous part of the experiment).

3.3.2. We should use a computer to plot the theoretical and the experimental curve showing the magnetic field distribution inside the solenoid. We should use dots for the data measured and a solid line for the theoretical curve. The origin of the plot should be at the center of the solenoid.

3.4. Measurement of the geomagnetic field (optional)

3.4.1. We should use the integrated Hall probe to measure the magnitude and the direction of the geomagnetic field.

3.5. Cautions

3.5.1. We should make sure that the V + port and the V − port of the integrated Hall probe are connected correctly, otherwise the probe will get damaged.

3.5.2. After turning the power supply on, we should wait for 5 minutes before starting measurements, in order to let the power supply temperature reaches a steady state.

3.5.3. Working voltage U_S must be lower than 10 V for measurements described in part 3.1..

3.5.4. We should set the output voltage and the current to zero before turning off the power supply.

3.5.5. We should turn off the power supply before disassembling the equipment.

4. Results

4.1. Relation between sensitivity K_H and working voltage U_S

The data we get are listed below (Table 3&4).

$U_S[V] \pm 0.5\%[V]$	U_0 (I_M = 0) [V] $\pm 0.05\% + 6 \times 10^{-3}/10^{-4}$ [V]	U (I_M = 250 mA) [V] $\pm 0.05\% + 6 \times 10^{-3}/10^{-4}$ [V]
5.00	2.513	2.634

Table 3. Data for U_0 and U with $U_S = 5\ V$

	$U_S[V] \pm 0.5\%[V]$	U_0 (I_M = 0) [V] $\pm 0.05\% + 6 \times 10^{-3}/10^{-4}$ [V]	U (I_M = 250 mA) [V] $\pm 0.05\% + 6 \times 10^{-3}/10^{-4}$ [V]
1	2.80	1.3970	1.4677
2	3.20	1.5983	1.6796
3	3.60	1.8050	1.8964
4	4.00	2.0053	2.1061
5	4.40	2.2103	2.317
6	4.80	2.410	2.525
7	5.20	2.614	2.738
8	5.60	2.815	2.942
9	6.00	3.013	3.148
10	6.40	3.213	3.350
11	6.80	3.406	3.546
12	7.20	3.601	3.745
13	7.60	3.791	3.936
14	8.00	3.980	4.127
15	8.40	4.167	4.312
16	8.80	4.355	4.501
17	9.40	4.632	4.784
18	10.00	4.915	5.066

Table 4. Data for U_0 and U with different U_S

From Table 1, we can get that the theoretical value of $B(x = 0) = 1.4366\ mT = 1.4366 \times 10^{-3}\ T$ with $I_{M\_t} = 0.1\ A$. While in our experiment, $I_{M\_e} = 250\ mA = 0.25\ A$, we can calculate B in this part as follows

$$B = B(x = 0) \times \frac{I_{M\_e}}{I_{M\_t}} = 0.00359 \pm 0.00007\ \mathrm{T}$$

4.1.1. Data for $\boldsymbol{U_0}$ and $\boldsymbol{U}$ with $\boldsymbol{U_S = 5\ V}$

For the data recorded in Table 3, we can use Eq. (2) to get that

$$K_H = \frac{U - U_0}{B} = \frac{2.634 - 2.513}{0.00359} = 34 \pm 3\ \mathrm{V/T}$$

with relative uncertainty as $\frac{3}{34} \times 100\% = 8.8\%$. Although it is larger than 5%, we may still think this part is relatively successful because it is still within 10%. The reasons may be the precision of the device. For example, the unit of I_M is [A] when we adjust it on the device, while the unity required is [mA], which will increase error. If we can use better device, the result will be better. Besides, we only measure the data for once. If we measure more times, it will be better.

4.1.2. Data for $\boldsymbol{U_0}$ and $\boldsymbol{U}$ with different $\boldsymbol{U_S}$

Using the first measurement as an example, we can calculate as follows

$$K_H = \frac{U - U_0}{B} = \frac{1.4677 - 1.3970}{0.00359} = 19.7 \pm 0.7\ \mathrm{V/T}$$

$$\frac{K_H}{U_S} = \frac{\frac{U - U_0}{B}}{U_S} = \frac{U - U_0}{BU_S} = \frac{1.4677 - 1.3970}{0.00359 \times 2.80} = 7.0 \pm 0.2\ \mathrm{T}^{-1}$$

The whole table is listed below for the answer of the rest calculation (Table 5).

	U_S [V]	u_{U_S} [V]	K_H [V/T]	u_{K_H} [V/T]	$\frac{K_H}{U_S}$ [T^{-1}]	$u_{\frac{K_H}{U_S}}$ [T^{-1}]
1	2.80	0.014	19.7	0.7	7.0	0.2
2	3.20	0.016	22.6	0.7	7.1	0.2
3	3.60	0.018	25.4	0.8	7.1	0.2
4	4.00	0.02	28.1	0.9	7.0	0.2
5	4.40	0.02	30	2	6.8	0.5
6	4.80	0.02	32	3	6.7	0.6
7	5.20	0.03	35	3	6.6	0.6
8	5.60	0.03	35	3	6.3	0.5
9	6.00	0.03	38	3	6.3	0.5
10	6.40	0.03	38	3	6.0	0.5
11	6.80	0.03	39	3	5.7	0.5
12	7.20	0.04	40	3	5.6	0.4
13	7.60	0.04	40	3	5.3	0.4
14	8.00	0.04	41	3	5.1	0.4
15	8.40	0.04	40	3	4.8	0.4
16	8.80	0.04	41	3	4.6	0.4
17	9.40	0.05	42	3	4.5	0.4
18	10.00	0.05	42	3	4.2	0.3

Table 5. Uncertainty for U_S and $\frac{K_H}{U_S}$

Then, based on this table, we can plot the curve for $\frac{K_H}{U_S}$ vs. U_S with origin (Figure 7).

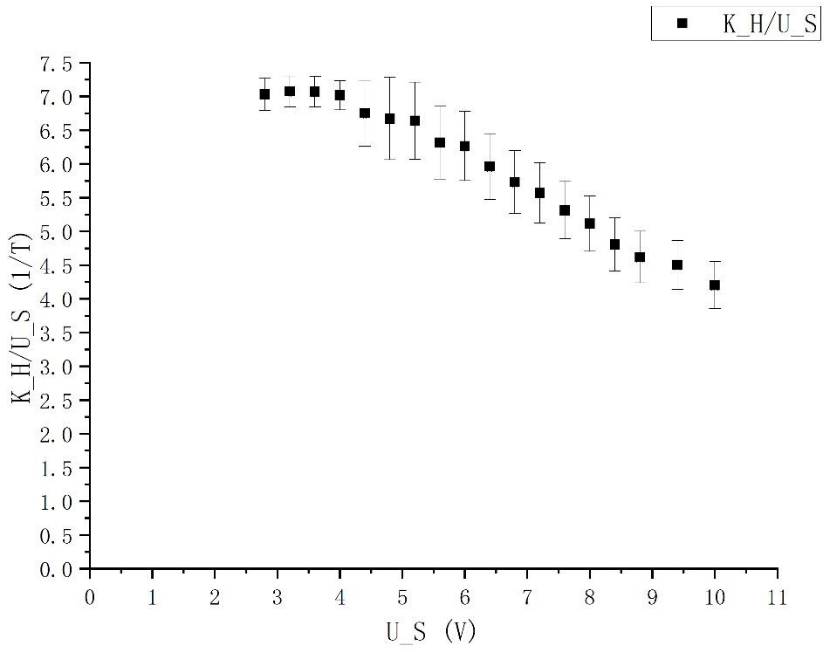

Figure 7. $\frac{K_H}{U_S}$ vs. U_S

Theoretically, K_H should be proportional to U_S, which means the graph we get should be like a horizontal line. In this figure, we can find that it tends to decline. The reasons for this may be due to the internal resistance of the wire or the precision of the device. But generally speaking, it still maintains in some range, which can prove that K_H should be proportional to U_S.

4.2. Relation between output voltage U and magnetic field B

The data we get are listed below (Table 6).

	I_M [mA] $\pm 2\%$ [mA]	U [mV] $\pm 0.05\% + 6 \times 10^{-3}/10^{-4}$ [V]
1	0	0.00
2	50	27.35
3	100	49.86
4	150	73.21
5	200	98.44
6	250	119.66
7	300	143.82
8	350	165.28
9	400	186.52
10	450	209.82
11	500	233.4

Table 6. Measurement data for the I_M vs. U relation

Using the second measurement as an example, we can calculate the magnetic field as follows

$$B_2 = \frac{B(x=0)}{I_{M\_t}} \times I_{M\_2} = \frac{1.4366 \times 10^{-3}}{0.1} \times 50 \times 10^{-3} = 0.000718 \pm 0.000014\, T$$

The whole table is listed below for the answer of the rest calculation (Table 7).

	I_M $[A]$	u_{I_M} $[A]$	U $[V]$	u_U $[V]$	B $[T]$	u_B $[T]$
1	0	0	0.00000	0.0006	0	0
2	0.050	0.0010	0.02735	0.0006	0.000718	0.000014
3	0.100	0.002	0.04986	0.0006	0.00144	0.00003
4	0.150	0.003	0.07321	0.0006	0.00215	0.00004
5	0.200	0.004	0.09844	0.0006	0.00287	0.00006
6	0.250	0.005	0.11966	0.0007	0.00359	0.00007
7	0.300	0.006	0.14382	0.0007	0.00431	0.00009
8	0.350	0.007	0.16528	0.0007	0.00503	0.00010
9	0.400	0.008	0.18652	0.0007	0.00575	0.00011
10	0.450	0.009	0.20982	0.0007	0.00646	0.00013
11	0.500	0.010	0.2334	0.0007	0.00718	0.00014

Table 7. Calculated uncertainty for I_M, U and B

Based on this table, we can plot the relation of U vs. B with Origin (Figure 8).

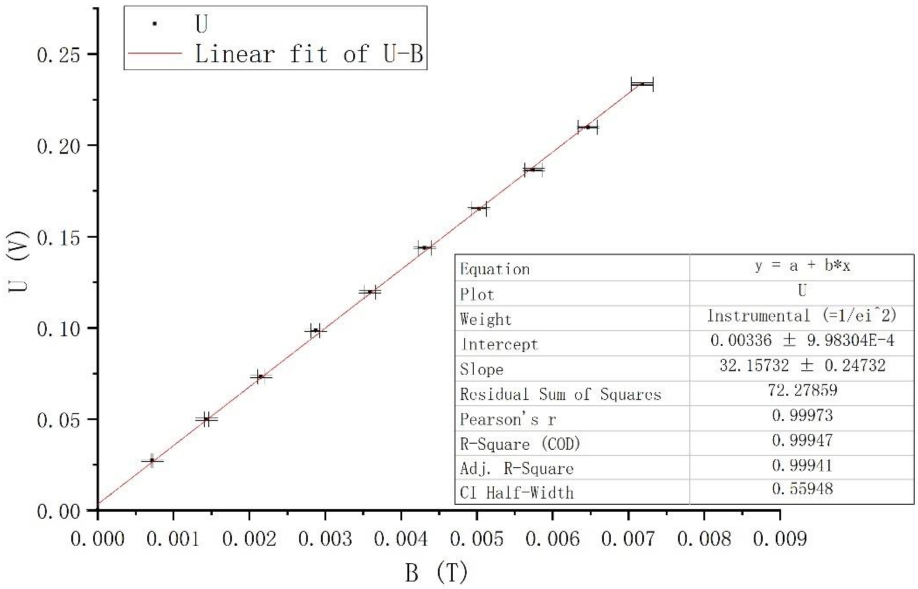

Figure 8. Linear fit of U vs. B

Form this figure, we know that U and B are linearly dependent since $r^2 = 0.99947$ and Pearson's r = 0.99973, which are close to 1. And $U = a \times B + b$, where $a = 32.15732$ and $b = 0.003366$.

Therefore, we can get that

$$K_H = 32.2 \pm 0.6\ \mathrm{V/T}$$

Theoretically, $K_H = 31.25 \pm 1.25\ V/T$, and the value we get in the previous part is $K_H = 34 \pm 3\ \mathrm{V/T}$. So, we can calculate the relative error respectively as

$$\frac{32.2 - 31.25}{32.2} \times 100\% = 2.95\%$$

$$\frac{34 - 32.2}{32.2} \times 100\% = 5.59\%$$

We can see that the relative error is small, so we may assume that this part is quite successful.

4.3. Magnetic field distribution inside the solenoid

The data we get are listed below (Table 8).

	x [cm] $\pm 0.05\ [cm]$	U [mV] $\pm 0.05\%$ $+6 \times 10^{-3}/10^{-4}$ [V]		x [cm] $\pm 0.05\ [cm]$	U [mV] $\pm 0.05\%$ $+6 \times 10^{-3}/10^{-4}$ [V]
1	0.00	10.91	27	17.00	119.62
2	0.50	14.04	28	18.00	119.59
3	1.00	19.27	29	19.00	119.31
4	1.50	26.75	30	20.00	119.29
5	2.00	38.60	31	21.00	119.05
6	2.50	53.68	32	22.00	118.36
7	3.00	70.97	33	23.00	117.63
8	3.50	85.49	34	24.00	116.15
9	4.00	96.32	35	25.00	114.10
10	4.50	103.43	36	25.50	112.51
11	5.00	107.81	37	26.00	110.44
12	0.25	12.33	38	26.50	106.80
13	0.75	16.49	39	27.00	101.31
14	1.25	22.74	40	27.50	93.87
15	1.75	30.81	41	27.75	88.43
16	6.00	113.23	42	28.00	81.80
17	7.00	115.77	43	28.25	72.80
18	8.00	117.29	44	28.50	66.13
19	9.00	118.12	45	28.75	56.73
20	10.00	118.80	46	29.00	47.58
21	11.00	119.18	47	29.25	40.91
22	12.00	119.31	48	29.50	34.29
23	13.00	119.43	49	29.75	29.03
24	14.00	119.30	50	30.00	24.11
25	15.00	119.50	51	2.25	46.49
26	16.00	119.71	52	2.75	62.37

Table 8. Data for the U vs. x relation

Using the first measurement as an example, we can calculate that

$$B = \frac{U}{K_H} = \frac{10.91 \times 10^{-3}}{32.2} = 0.00034 \pm 0.00002\ \mathrm{T}$$

The whole table is listed below for the answer of the rest calculation (Table 9).

	x [cm]	u_x [cm]	U [V]	u_U [V]	B [T]	u_B [T]
1	0.00	0.05	0.01091	0.0006	0.00034	0.00002
2	0.50	0.05	0.01404	0.0006	0.00044	0.00002
3	1.00	0.05	0.01927	0.0006	0.00060	0.00002
4	1.50	0.05	0.02675	0.0006	0.00083	0.00002
5	2.00	0.05	0.03860	0.0006	0.00120	0.00003
6	2.50	0.05	0.05368	0.0006	0.00167	0.00004
7	3.00	0.05	0.07097	0.0006	0.00220	0.00005
8	3.50	0.05	0.08549	0.0006	0.00265	0.00005
9	4.00	0.05	0.09632	0.0006	0.00299	0.00006
10	4.50	0.05	0.10343	0.0007	0.00321	0.00006
11	5.00	0.05	0.10781	0.0007	0.00335	0.00007
12	0.25	0.05	0.01233	0.0006	0.00038	0.00002
13	0.75	0.05	0.01649	0.0006	0.00051	0.00002
14	1.25	0.05	0.02274	0.0006	0.00071	0.00002
15	1.75	0.05	0.03081	0.0006	0.00096	0.00003
16	6.00	0.05	0.11323	0.0007	0.00352	0.00007
17	7.00	0.05	0.11577	0.0007	0.00360	0.00007
18	8.00	0.05	0.11729	0.0007	0.00364	0.00007
19	9.00	0.05	0.11812	0.0007	0.00367	0.00007
20	10.00	0.05	0.11880	0.0007	0.00369	0.00007
21	11.00	0.05	0.11918	0.0007	0.00370	0.00007
22	12.00	0.05	0.11931	0.0007	0.00371	0.00007
23	13.00	0.05	0.11943	0.0007	0.00371	0.00007
24	14.00	0.05	0.11930	0.0007	0.00370	0.00007
25	15.00	0.05	0.11950	0.0007	0.00371	0.00007
26	16.00	0.05	0.11971	0.0007	0.00372	0.00007
27	17.00	0.05	0.11962	0.0007	0.00371	0.00007
28	18.00	0.05	0.11959	0.0007	0.00371	0.00007
29	19.00	0.05	0.11931	0.0007	0.00371	0.00007
30	20.00	0.05	0.11929	0.0007	0.00370	0.00007
31	21.00	0.05	0.11905	0.0007	0.00370	0.00007
32	22.00	0.05	0.11836	0.0007	0.00368	0.00007
33	23.00	0.05	0.11763	0.0007	0.00365	0.00007
34	24.00	0.05	0.11615	0.0007	0.00361	0.00007
35	25.00	0.05	0.11410	0.0007	0.00354	0.00007
36	25.50	0.05	0.11251	0.0007	0.00349	0.00007
37	26.00	0.05	0.11044	0.0007	0.00343	0.00007
38	26.50	0.05	0.10680	0.0007	0.00332	0.00007
39	27.00	0.05	0.10131	0.0007	0.00315	0.00006
40	27.50	0.05	0.09387	0.0006	0.00292	0.00006

41	27.75	0.05	0.08843	0.0006	0.00275	0.00005
42	28.00	0.05	0.08180	0.0006	0.00254	0.00005
43	28.25	0.05	0.07280	0.0006	0.00226	0.00005
44	28.50	0.05	0.06613	0.0006	0.00205	0.00004
45	28.75	0.05	0.05673	0.0006	0.00176	0.00004
46	29.00	0.05	0.04758	0.0006	0.00148	0.00003
47	29.25	0.05	0.04091	0.0006	0.00127	0.00003
48	29.50	0.05	0.03429	0.0006	0.00106	0.00003
49	29.75	0.05	0.02903	0.0006	0.00090	0.00003
50	30.00	0.05	0.02411	0.0006	0.00075	0.00002
51	2.25	0.05	0.04649	0.0006	0.00144	0.00003
52	2.75	0.05	0.06237	0.0006	0.00194	0.00004

Table 9. Calculated uncertainty for *x*, *U*, *B*

For the theoretical value, we can calculate them as

$$B = \frac{B(x)}{I_{M\_t}} \times I_M = \frac{1.4366 \times 10^{-3}}{0.1} \times 0.25 = 0.00359 \pm 0.00007\ \mathrm{T}$$

The whole table is listed below for the answer of the rest calculation (Table 10).

x [cm]	$B(x)$ [mT]	B [T]	u_B [T]
0.0	0.7233	0.00181	0.00004
0.5	0.9261	0.00232	0.00005
1.0	1.0863	0.00272	0.00005
1.5	1.1963	0.00299	0.00006
2.0	1.2685	0.00317	0.00006
3.0	1.3478	0.00337	0.00007
4.0	1.3856	0.00346	0.00007
5.0	1.4057	0.00351	0.00007
6.0	1.4173	0.00354	0.00007
7.0	1.4245	0.00356	0.00007
8.0	1.4292	0.00357	0.00007
9.0	1.4323	0.00358	0.00007
10.0	1.4343	0.00359	0.00007
11.0	1.4356	0.00359	0.00007
12.0	1.4363	0.00359	0.00007
13.0	1.4366	0.00359	0.00007
14.0	1.4363	0.00359	0.00007
15.0	1.4356	0.00359	0.00007
16.0	1.4343	0.00359	0.00007
17.0	1.4323	0.00358	0.00007
18.0	1.4292	0.00357	0.00007
19.0	1.4245	0.00356	0.00007
20.0	1.4173	0.00354	0.00007
21.0	1.4057	0.00351	0.00007
22.0	1.3856	0.00346	0.00007
23.0	1.3478	0.00337	0.00007
24.0	1.2685	0.00317	0.00006
24.5	1.1963	0.00299	0.00006
25.0	1.0863	0.00272	0.00005
25.5	0.9261	0.00232	0.00005
26.0	0.7233	0.00181	0.00004

Table 10. calculated uncertainty for B

Based on Table 9 and 10, we can get the figure below (Figure 9)

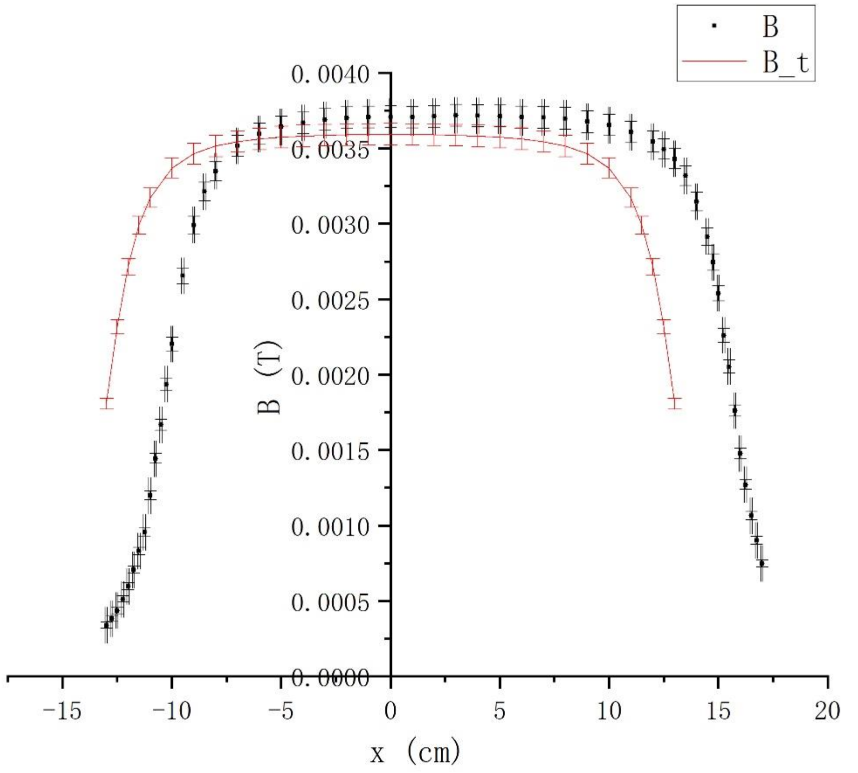

Figure 9. The theoretical and experimental B vs. x

From this figure, we know that the magnetic field usually is its largest at the center of the solenoid while becomes smaller at the end of the solenoid. Generally, the trend of the experimental one and theoretical one are similar. But there are still some differences. For example, the experimental one shifted to the right compared with the theoretical one. We may think that the actual center of the solenoid is not at 13.0 cm but around 16.0 cm according to the data we have, and we can repeat the experiment for many times to get the position. Besides, during the experiment, we find that the readings on the meter are not stable, this may be because of the precision of the device.

5. Conclusion [1]

5.1. Relation between sensitivity K_H and working voltage U_S

In this part, we first calculate the magnetic field for $I_M = 250$ mA

$$B = 0.00359 \pm 0.00007 \text{ T}$$

Then, for $U_S = 5$ V, we calculate

$$K_H = \frac{U - U_0}{B} = \frac{2.634 - 2.513}{0.00359} = 34 \pm 3 \text{ V/T}$$

with relative uncertainty as $\frac{3}{34} \times 100\% = 8.8\%$. Although it is larger than 5%, we may still think this part is relatively successful because it is still within 10%. The reasons may be the precision of the device. For example, the unit of I_M is [A] when we adjust it on the device, while the unity required is [mA], which will increase error. If we can use better device, the result will be better. Besides, we only measure the data for once. If we measure more times, it will be better.

For varying U_S, we first calculate $\frac{K_H}{U_S}$, and then plot the curve for $\frac{K_H}{U_S}$ vs. U_S. In this figure, we can find that it tends to decline. Theoretically, K_H should be proportional to U_S, which means the graph we get should be like a horizontal line. The reasons for this may be due to the internal resistance of the wire or the precision of the device. But generally speaking, it still maintains in some range, which can prove that K_H should be proportional to U_S.

5.2. Relation between output voltage U and magnetic field B

In this part, we first calculate magnetic field, the plot the relation of *U* vs. *B* with Origin.

Form the figure, we know that *U* and *B* are linearly dependent since $r^2 = 0.99947$ and Pearson's r = 0.99973, which are close to 1. And $U = a \times B + b$, where $a = 32.15732$ and $b = 0.003366$.

Therefore, we can get that

$$K_H = 32.2 \pm 0.6\ \mathrm{V/T}$$

Theoretically, $K_H = 31.25 \pm 1.25\ V/T$, and the value we get in the previous part is $K_H = 34 \pm 3\ \mathrm{V/T}$. So, we can calculate the relative error respectively as

$$\frac{32.2 - 31.25}{32.2} \times 100\% = 2.95\%$$

$$\frac{34 - 32.2}{32.2} \times 100\% = 5.59\%$$

We can see that the relative error is small, so we may assume that this part is quite successful.

5.3. Magnetic field distribution inside the solenoid

In this part, we first calculate magnetic field of experimental and theoretical one. Then we plot *B* vs. *x*. From the figure, we know that the magnetic field usually is its largest at the center of the solenoid while becomes smaller at the end of the solenoid. Generally, the trend of the experimental one and theoretical one are similar. But there are still some differences. For example, the experimental one shifted to the right compared with the theoretical one. We may think that the actual center of the solenoid is not at 13.0 cm but around 16.0 cm according to the data we have, and we can repeat the experiment for many times to get the position. Besides, during the experiment, we find that the readings on the meter are not stable, this may be because of the precision of the device.

6. References

[1] Exercise 2 - lab manual [rev 3.9], UM-JI SJTU. Edited by Qin Tian, Wang Zhiyu, Lin Yiqiao, Bao Yufan, Mateusz Krzyzosiak.

[2] Uncertainty analysis handbook, UM-JI SJTU.

A. Uncertainty analysis [2]

A.1. Uncertainty for relation between sensitivity K_H and working voltage U_S

Since $B(x=0) = 1.4366 \times 10^{-3}\ T$ and $I_{M\_t} = 0.1\ A$ are theoretical value, we can assume that $u_{B(x=0)} = 0$ and $u_{I_{M\_t}} = 0$. For $I_{M\_e} = 250$ mA, according to the precision or uncertainty of the device, we can calculate its uncertainty as follows

$$u_{I_{M\_e}} = 250 \times 2\% = 5 \text{ mA} = 0.005 \text{ A}$$

Then, according to the $B = B(x=0) \times \frac{I_{M\_e}}{I_{M\_t}}$, we can calculate the uncertainty for B as

$$u_B = \sqrt{\left(\frac{\partial B}{\partial B(x=0)} \cdot u_{B(x=0)}\right)^2 + \left(\frac{\partial B}{\partial I_{M\_t}} \cdot u_{I_{M\_t}}\right)^2 + \left(\frac{\partial B}{\partial I_{M\_e}} \cdot u_{I_{M\_e}}\right)^2}$$

$$= \sqrt{\left(\frac{B(x=0)}{I_{M\_t}} \cdot u_{I_{M\_e}}\right)^2} = \sqrt{\left(\frac{1.4366 \times 10^{-3}}{0.1} \cdot 0.005\right)^2} = 0.00007 \text{ T}$$

A.1.1. Data for U_0 and U with $U_S = 5$ V

Then, according to the $K_H = \frac{U-U_0}{B}$, we can calculate the uncertainty for K_H as

$$u_{K_H} = \sqrt{\left(\frac{\partial K_H}{\partial U} \cdot u_U\right)^2 + \left(\frac{\partial K_H}{\partial U_0} \cdot u_{U_0}\right)^2 + \left(\frac{\partial K_H}{\partial B} \cdot u_B\right)^2}$$

$$= \sqrt{\left(\frac{u_U}{B}\right)^2 + \left(\frac{u_{U_0}}{B}\right)^2 + \left[\frac{(U-U_0) \cdot u_B}{B^2}\right]^2}$$

For u_U and u_{U_0}, according to the precision or uncertainty of the device, we can calculate as follows

$$u_U = 2.634 \times 0.05\% + 6 \times 10^{-3} = 0.007 \text{ V}$$
$$u_{U_0} = 2.513 \times 0.05\% + 6 \times 10^{-3} = 0.007 \text{ V}$$

Then, we can get u_{K_H} as

$$u_{K_H} = \sqrt{\left(\frac{0.007}{0.00359}\right)^2 + \left(\frac{0.007}{0.00359}\right)^2 + \left[\frac{(2.634 - 2.513) \cdot 0.00007}{0.00359^2}\right]^2} = 3 \text{ V/T}$$

A.1.1. Data for U_0 and U with different U_S

Using the first measurement as an example. For u_{U_S}, u_U and u_{U_0}, according to the precision or uncertainty of the device, we can calculate as follows

$$u_{U_S} = 2.80 \times 0.5\% = 0.014\ \mathrm{V}$$
$$u_{U_0} = 1.3970 \times 0.05\% + 6 \times 10^{-4} = 0.0013\ \mathrm{V}$$
$$u_U = 1.4677 \times 0.05\% + 6 \times 10^{-4} = 0.0013\ \mathrm{V}$$

According to the $K_H = \frac{U-U_0}{B}$, we can calculate the uncertainty for K_H as

$$u_{K_H} = \sqrt{\left(\frac{\partial K_H}{\partial U} \cdot u_U\right)^2 + \left(\frac{\partial K_H}{\partial U_0} \cdot u_{U_0}\right)^2 + \left(\frac{\partial K_H}{\partial B} \cdot u_B\right)^2}$$
$$= \sqrt{\left(\frac{u_U}{B}\right)^2 + \left(\frac{u_{U_0}}{B}\right)^2 + \left[\frac{(U-U_0) \cdot u_B}{B^2}\right]^2}$$
$$= \sqrt{\left(\frac{0.0013}{0.00359}\right)^2 + \left(\frac{0.0013}{0.00359}\right)^2 + \left[\frac{(1.4677 - 1.3970) \times 0.00007}{0.00359^2}\right]^2}$$
$$= 0.7\ \mathrm{V/T}$$

According to the equation that $\frac{K_H}{U_S} = \frac{U-U_0}{BU_S}$, we can calculate the uncertainty as

$$u_{\frac{K_H}{U_S}} = \sqrt{\left(\frac{\partial \frac{K_H}{U_S}}{\partial U} \cdot u_U\right)^2 + \left(\frac{\partial \frac{K_H}{U_S}}{\partial U_0} \cdot u_{U_0}\right)^2 + \left(\frac{\partial \frac{K_H}{U_S}}{\partial B} \cdot u_B\right)^2 + \left(\frac{\partial \frac{K_H}{U_S}}{\partial U_S} \cdot u_{U_S}\right)^2}$$
$$= \sqrt{\left(\frac{u_U}{BU_S}\right)^2 + \left(\frac{u_{U_0}}{BU_S}\right)^2 + \left(\frac{U-U_0}{B^2U_S} \cdot u_B\right)^2 + \left(\frac{U-U_0}{BU_S^2} \cdot u_{U_S}\right)^2}$$
$$= \sqrt{\begin{array}{c}\left(\frac{0.0013}{0.00359 \times 2.80}\right)^2 + \left(\frac{0.0013}{0.00359 \times 2.80}\right)^2 + \\ \left(\frac{1.4677 - 1.3970}{0.00359^2 \times 2.80} \cdot 0.00007\right)^2 + \left(\frac{1.4677 - 1.3970}{0.00359 \times 2.80^2} \cdot 0.014\right)^2\end{array}}$$
$$= 0.2\ \mathrm{T}^{-1}$$

The table below (Table 11) lists the uncertainty calculated for rest data.

	U_S [V]	u_{U_S} [V]	U_0 [V]	u_{U_0} [V]	U [V]	u_U [V]	K_H [V/T]	u_{K_H} [V/T]	$\frac{K_H}{U_S}$ $[\mathrm{T}^{-1}]$	$u_{\frac{K_H}{U_S}}$ $[\mathrm{T}^{-1}]$
1	2.80	0.014	1.3970	0.0013	1.4677	0.0013	19.7	0.7	7.0	0.2
2	3.20	0.016	1.5983	0.0014	1.6796	0.0014	22.6	0.7	7.1	0.2
3	3.60	0.018	1.8050	0.0015	1.8964	0.0015	25.4	0.8	7.1	0.2
4	4.00	0.02	2.0053	0.0016	2.1061	0.0017	28.1	0.9	7.0	0.2
5	4.40	0.02	2.2103	0.0017	2.317	0.007	30	2	6.8	0.5
6	4.80	0.02	2.410	0.007	2.525	0.007	32	3	6.7	0.6
7	5.20	0.03	2.614	0.007	2.738	0.007	35	3	6.6	0.6
8	5.60	0.03	2.815	0.007	2.942	0.007	35	3	6.3	0.5
9	6.00	0.03	3.013	0.008	3.148	0.008	38	3	6.3	0.5
10	6.40	0.03	3.213	0.008	3.350	0.008	38	3	6.0	0.5
11	6.80	0.03	3.406	0.008	3.546	0.008	39	3	5.7	0.5
12	7.20	0.04	3.601	0.008	3.745	0.008	40	3	5.6	0.4
13	7.60	0.04	3.791	0.008	3.936	0.008	40	3	5.3	0.4
14	8.00	0.04	3.980	0.008	4.127	0.008	41	3	5.1	0.4
15	8.40	0.04	4.167	0.008	4.312	0.008	40	3	4.8	0.4
16	8.80	0.04	4.355	0.008	4.501	0.008	41	3	4.6	0.4
17	9.40	0.05	4.632	0.008	4.784	0.008	42	3	4.5	0.4
18	10.00	0.05	4.915	0.008	5.066	0.009	42	3	4.2	0.3

Table 11. Calculated uncertainty for U_S, U_0, U, K_H and $\frac{K_H}{U_S}$

A.2. Uncertainty for relation between output voltage U and magnetic field B

Using the second measurement as an example, we can calculate the uncertainty for I_M and U as follows

$$u_{I_M} = 50 \times 2\% = 1.0\ \mathrm{mA} = 0.0010\ \mathrm{A}$$

$$u_U = 27.35 \times 10^{-3} \times 0.05\% + 6 \times 10^{-4} = 0.0006\ \mathrm{V}$$

Since $B(x=0) = 1.4366 \times 10^{-3}\ T$ and $I_{M\_t} = 0.1\ A$ are theoretical value, we can assume that $u_{B(x=0)} = 0$ and $u_{I_{M\_t}} = 0$. According to the equation that $B_2 = \frac{B(x=0)}{I_{M\_t}} \times I_{M\_2}$, we can calculate the uncertainty for B_2 as follows

$$u_{B_2} = \sqrt{\left(\frac{\partial B_2}{\partial B(x=0)} \cdot u_{B(x=0)}\right)^2 + \left(\frac{\partial B_2}{\partial I_{M\_t}} \cdot u_{I_{M\_t}}\right)^2 + \left(\frac{\partial B_2}{\partial I_{M\_2}} \cdot u_{I_{M\_2}}\right)^2}$$

$$= \sqrt{\left(\frac{B(x=0)}{I_{M\_t}} \times u_{I_{M\_2}}\right)^2} = \frac{1.4366 \times 10^{-3}}{0.1} \times 0.001 = 0.000014 \text{ T}$$

The table below (Table 12) lists the uncertainty calculated for rest data.

	I_M $[A]$	u_{I_M} $[A]$	U $[V]$	u_U $[V]$	B $[T]$	u_B $[T]$
1	0	0	0.00000	0.0006	0	0
2	0.050	0.0010	0.02735	0.0006	0.000718	0.000014
3	0.100	0.002	0.04986	0.0006	0.00144	0.00003
4	0.150	0.003	0.07321	0.0006	0.00215	0.00004
5	0.200	0.004	0.09844	0.0006	0.00287	0.00006
6	0.250	0.005	0.11966	0.0007	0.00359	0.00007
7	0.300	0.006	0.14382	0.0007	0.00431	0.00009
8	0.350	0.007	0.16528	0.0007	0.00503	0.00010
9	0.400	0.008	0.18652	0.0007	0.00575	0.00011
10	0.450	0.009	0.20982	0.0007	0.00646	0.00013
11	0.500	0.010	0.2334	0.0007	0.00718	0.00014

Table 12. Calculated uncertainty for I_M, U and B

A.3. Uncertainty for magnetic field distribution inside the solenoid

From the previous part, we know that $K_H = 32.2 \pm 0.6$ V/T. Using the first measurement as an example, we can calculate that

$$u_U = 10.91 \times 10^{-3} \times 0.05\% + 6 \times 10^{-4} = 0.0006 \text{ V}$$

Since $B = \frac{U}{K_H}$, we can calculate that

$$u_B = \sqrt{\left(\frac{\partial B}{\partial U} \cdot u_U\right)^2 + \left(\frac{\partial B}{\partial K_H} \cdot u_{K_H}\right)^2} = \sqrt{\left(\frac{u_U}{K_H}\right)^2 + \left(\frac{U}{K_H^2} \cdot u_{K_H}\right)^2}$$

$$= \sqrt{\left(\frac{0.0006}{32.2}\right)^2 + \left(\frac{10.91 \times 10^{-3}}{32.2^2} \cdot 0.6\right)^2} = 0.00002 \text{ T}$$

The table below (Table 13) lists the uncertainty calculated for rest data.

	x [cm]	u_x [cm]	U [V]	u_U [V]	B [T]	u_B [T]
1	0.00	0.05	0.01091	0.0006	0.00034	0.00002
2	0.50	0.05	0.01404	0.0006	0.00044	0.00002
3	1.00	0.05	0.01927	0.0006	0.00060	0.00002
4	1.50	0.05	0.02675	0.0006	0.00083	0.00002
5	2.00	0.05	0.03860	0.0006	0.00120	0.00003
6	2.50	0.05	0.05368	0.0006	0.00167	0.00004
7	3.00	0.05	0.07097	0.0006	0.00220	0.00005
8	3.50	0.05	0.08549	0.0006	0.00265	0.00005
9	4.00	0.05	0.09632	0.0006	0.00299	0.00006
10	4.50	0.05	0.10343	0.0007	0.00321	0.00006
11	5.00	0.05	0.10781	0.0007	0.00335	0.00007
12	0.25	0.05	0.01233	0.0006	0.00038	0.00002
13	0.75	0.05	0.01649	0.0006	0.00051	0.00002
14	1.25	0.05	0.02274	0.0006	0.00071	0.00002
15	1.75	0.05	0.03081	0.0006	0.00096	0.00003
16	6.00	0.05	0.11323	0.0007	0.00352	0.00007
17	7.00	0.05	0.11577	0.0007	0.00360	0.00007
18	8.00	0.05	0.11729	0.0007	0.00364	0.00007
19	9.00	0.05	0.11812	0.0007	0.00367	0.00007
20	10.00	0.05	0.11880	0.0007	0.00369	0.00007
21	11.00	0.05	0.11918	0.0007	0.00370	0.00007
22	12.00	0.05	0.11931	0.0007	0.00371	0.00007
23	13.00	0.05	0.11943	0.0007	0.00371	0.00007
24	14.00	0.05	0.11930	0.0007	0.00370	0.00007
25	15.00	0.05	0.11950	0.0007	0.00371	0.00007
26	16.00	0.05	0.11971	0.0007	0.00372	0.00007
27	17.00	0.05	0.11962	0.0007	0.00371	0.00007
28	18.00	0.05	0.11959	0.0007	0.00371	0.00007
29	19.00	0.05	0.11931	0.0007	0.00371	0.00007
30	20.00	0.05	0.11929	0.0007	0.00370	0.00007
31	21.00	0.05	0.11905	0.0007	0.00370	0.00007
32	22.00	0.05	0.11836	0.0007	0.00368	0.00007
33	23.00	0.05	0.11763	0.0007	0.00365	0.00007
34	24.00	0.05	0.11615	0.0007	0.00361	0.00007
35	25.00	0.05	0.11410	0.0007	0.00354	0.00007
36	25.50	0.05	0.11251	0.0007	0.00349	0.00007
37	26.00	0.05	0.11044	0.0007	0.00343	0.00007
38	26.50	0.05	0.10680	0.0007	0.00332	0.00007
39	27.00	0.05	0.10131	0.0007	0.00315	0.00006
40	27.50	0.05	0.09387	0.0006	0.00292	0.00006

41	27.75	0.05	0.08843	0.0006	0.00275	0.00005
42	28.00	0.05	0.08180	0.0006	0.00254	0.00005
43	28.25	0.05	0.07280	0.0006	0.00226	0.00005
44	28.50	0.05	0.06613	0.0006	0.00205	0.00004
45	28.75	0.05	0.05673	0.0006	0.00176	0.00004
46	29.00	0.05	0.04758	0.0006	0.00148	0.00003
47	29.25	0.05	0.04091	0.0006	0.00127	0.00003
48	29.50	0.05	0.03429	0.0006	0.00106	0.00003
49	29.75	0.05	0.02903	0.0006	0.00090	0.00003
50	30.00	0.05	0.02411	0.0006	0.00075	0.00002
51	2.25	0.05	0.04649	0.0006	0.00144	0.00003
52	2.75	0.05	0.06237	0.0006	0.00194	0.00004

Table 13. Calculated uncertainty for x, U and B

Since $B(x)$ and I_{M_t} are theoretical value, we can assume that $u_{B(x)} = 0$ and $u_{I_{M\_t}} = 0$. For $I_{M\_e} = 250\ \mathrm{m}A$, according to the precision or uncertainty of the device, we can calculate its uncertainty as follows

$$u_{I_{M\_e}} = 250 \times 2\% = 5\ \mathrm{mA} = 0.005\ \mathrm{A}$$

Then, according to the $B = B(x) \times \frac{I_{M\_e}}{I_{M\_t}}$, we can calculate the uncertainty for B as

$$u_B = \sqrt{\left(\frac{\partial B}{\partial B(x=0)} \cdot u_{B(x=0)}\right)^2 + \left(\frac{\partial B}{\partial I_{M\_t}} \cdot u_{I_{M\_t}}\right)^2 + \left(\frac{\partial B}{\partial I_{M\_e}} \cdot u_{I_{M\_e}}\right)^2}$$

$$= \sqrt{\left(\frac{B(x)}{I_{M\_t}} \cdot u_{I_{M\_e}}\right)^2} = \sqrt{\left(\frac{B(x)}{0.1} \cdot 0.005\right)^2} = 0.05B(x)$$

$$= 0.05 \times 1.4366 \times 10^{-3} = 0.00007\ \mathrm{T}$$

The whole table is listed below (Table 14)

x [cm]	$B(x)$ [mT]	B [T]	u_B [T]
0.0	0.7233	0.00181	0.00004
0.5	0.9261	0.00232	0.00005
1.0	1.0863	0.00272	0.00005
1.5	1.1963	0.00299	0.00006
2.0	1.2685	0.00317	0.00006
3.0	1.3478	0.00337	0.00007
4.0	1.3856	0.00346	0.00007
5.0	1.4057	0.00351	0.00007
6.0	1.4173	0.00354	0.00007
7.0	1.4245	0.00356	0.00007
8.0	1.4292	0.00357	0.00007
9.0	1.4323	0.00358	0.00007
10.0	1.4343	0.00359	0.00007
11.0	1.4356	0.00359	0.00007
12.0	1.4363	0.00359	0.00007
13.0	1.4366	0.00359	0.00007
14.0	1.4363	0.00359	0.00007
15.0	1.4356	0.00359	0.00007
16.0	1.4343	0.00359	0.00007
17.0	1.4323	0.00358	0.00007
18.0	1.4292	0.00357	0.00007
19.0	1.4245	0.00356	0.00007
20.0	1.4173	0.00354	0.00007
21.0	1.4057	0.00351	0.00007
22.0	1.3856	0.00346	0.00007
23.0	1.3478	0.00337	0.00007
24.0	1.2685	0.00317	0.00006
24.5	1.1963	0.00299	0.00006
25.0	1.0863	0.00272	0.00005
25.5	0.9261	0.00232	0.00005
26.0	0.7233	0.00181	0.00004

Table 14. Calculated uncertainty for B