

UM-SJTU Joint Institute
Physics laboratory
Vp241

Lab Report 4

Polarization of Light

Name: Zheng Hao Cheng

ID: 518370910181

Date: 22/11/2019

I. Objective

Understand some properties of light, in particular to study the polarization phenomenon and verify Malus' law, as well as to understand the way half- and quarter-wave plates work in optical systems. Generation and detection of elliptically and circularly polarized light will also be investigated.

II. Theoretical Background

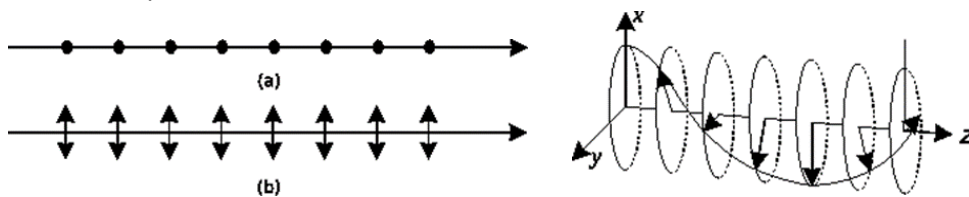
Light can be described in terms of electromagnetic waves, with the plane of oscillations of the electric field vector (as well as the magnetic field vector) perpendicular to the direction of light propagation. Therefore, light is an example of a transverse wave. For light sources producing the so-called natural light, the emitted light is a random mixture of waves with the electric field vector oscillating in all possible transverse directions. This is due to the randomness of the radiation mechanism. Such natural light is also called unpolarized light. For unpolarized light the distribution of the directions of the electric field vector, in the plane perpendicular to the direction of propagation, is uniform. If the distribution is not uniform, the light is said to be polarized. Studies of the polarization of light played an important role in the development of wave optics. They have resulted in a wide range of applications in numerous areas, such as optical measurement techniques, crystal structure research, and experimental stress analysis.

1. Polarization of Light

The electric field vector E , which in the context of electromagnetic waves corresponding to the visible part of the spectrum is sometimes referred to as the light vector describes a time-dependent, propagating electric field. In the plane perpendicular to the propagation direction of a light wave, the light vector may have different directions along which its magnitude oscillates. The light, for which the light vector maintains a certain oscillation direction, is called linearly polarized and the axis defining the direction is called the polarization axis.

The light with the light vector direction rotating about the propagation direction, so that its endpoint traces a circle, is called circularly polarized light. If the vector traces an ellipse, the light is said to be elliptically polarized.

Light emitted from ordinary light sources (natural light) is unpolarized. However, it can be regarded as a statistical equal-weight mixture of linearly polarized waves with equal amplitudes. There the light may be also partially polarized, which means it can be regarded as a combination of a polarized and the natural (unpolarized) light. The direction corresponding to the maximum amplitude of the light vector of such partially polarized light is the oscillation direction of the polarized component.



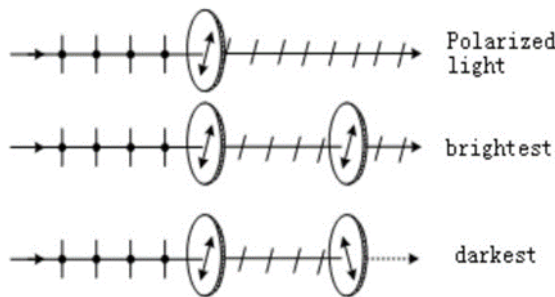
2. Polarizer

A device commonly used to produce polarized light is a polaroid (also called a polarizer). It polarizes the light using the principle of dichroism: a selective absorption mechanism tends to allow the light polarized in a certain direction (direction of the crystal alignment) to pass through the material, while the light polarized in all other directions is absorbed. This turns the incident natural light into linearly polarized.

A polarization device can not only change incident natural light to polarized light (it then acts as a polarizer), but may also be used to detect and analyze linearly polarized, natural, and partially polarized light (it is then called an analyzer).

3. Malus' law

A visible effect in the light coming out of a polarization device is a change of the light brightness.



Suppose that we have two polarizers arranged so that their planes are parallel — the left one plays the role of a polarizer, the other one is an analyzer (see Figure 3). Let the angle between their transmission directions (polarization axes) be θ . The light is incident normally on the polarizer and then continues to the analyzer. The intensity of the linearly polarized light leaving the analyzer is: $I_1 = I_0 \cos^2 \theta$, where $I_{\text{light},0}$ is the intensity of the linearly polarized light incident on the analyzer. Obviously, for a single polarizer, if polarized light is incident on it, then the transmitted light intensity will change periodically when rotating the polarizer. If the incident light is partially or elliptically polarized, the minimum intensity will not be zero as there will be always some component of the light polarized in the transmission direction. The incident light must be natural or circularly polarized if the intensity does not change at all. Hence, by using a polarizer, one can distinguish linearly polarized light from the natural and circularly polarized light.

4. Generation of Elliptically and Circularly Polarized Light. Half-wave and Quarter-wave Plates

Suppose that linearly polarized light is incident normally on a crystal plate whose

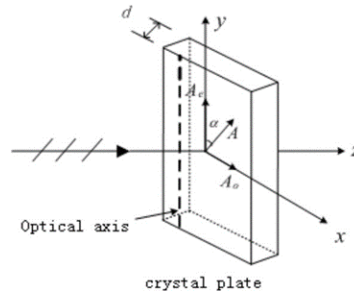
surface is parallel to its optical axis, and the angle between the polarizing axis and the optical axis of the plate is α . Then the linearly polarized light is resolved into two waves: an e-wave with the oscillation direction parallel to the optical axis of the plate (extraordinary axis) and an o-wave whose oscillation direction is perpendicular to the optical axis (ordinary axis). They propagate in the same direction, but with different speeds. The resulting optical path difference over the thickness d of the plate is:

$$\Delta = (n_e - n_o)d$$

and, consequently, the phase difference

$$\delta = \frac{2\pi}{\lambda}$$

where λ is the wavelength, n_e is the refractive index for the extraordinary axis, and n_o is the refractive index for the ordinary axis. In a so-called positive crystal $\delta > 0$, whereas in a negative one $\delta < 0$.



when the light propagates through the crystal plate, the two components of the light vector are

$$E_x = A_o \cos \omega t$$

$$E_y = A_e \cos(\omega t + \delta),$$

where $A_e = A \cos \alpha$, $A_o = A \sin \alpha$. Eliminating time from the above equations one obtains

$$\frac{E_x^2}{A_o^2} + \frac{E_y^2}{A_e^2} - 2 \frac{E_x E_y}{A_o A_e} \cos \delta = \sin^2 \delta.$$

Note that is the equation of an ellipse for $\delta = \pi/2$.

When the thickness of the plate changes, the optical path difference changes as well.

Some cases of particular interest, are discussed below:

If $\Delta = k\lambda$, where $k = 0, 1, 2, \dots$, the phase difference $\delta = 0$, and Eq. (2) reduces to

$$E_y = \frac{A_e}{A_o} E_x$$

which is a linear equation. Hence the transmitted light is linearly polarized with the oscillation direction remaining unchanged. A waveplate that satisfies this condition is called a full-wave plate. The light goes through a full-wave plate without changing its polarization state.

If $\Delta = (2k + 1)\lambda/2$, where $k = 0, 1, 2, \dots$, the phase difference $\delta = \pi$, then:

$$E_y = -\frac{A_e}{A_o} E_x$$

The transmitted light is also linearly polarized with the polarization axis rotated by the angle of 2α . A waveplate that satisfies the condition is called 1/2-wave plate or half-wave plate. When a polarized light passes through a half-wave plate, its polarization axis gets rotated by an angle 2α . If $\alpha = \pi/4$, then the polarization axis of the transmitted light is perpendicular to that of the incident light.

Finally, if $\Delta = (2k + 1)\lambda/4$, where $k = 0, 1, 2, \dots$, the phase difference $\delta = \pi/2$, then:

$$\frac{E_x^2}{A_o^2} + \frac{E_y^2}{A_e^2} = 1.$$

The transmitted light is elliptically polarized. A waveplate that satisfies the above condition is called a 1/4-wave plate or a quarter-waveplate and is an important optical element in many polarization experiments.

If $A_e = A_o = A$, then $E_x^2 + E_y^2 = A^2$, and the transmitted light is circularly polarized. Since the amplitudes of the *o*-wave and the *e*-wave are both functions of α , the polarization state after passing through a 1/4-wave plate will vary, depending on the angle:

- if $\alpha = 0$, the transmitted light is linearly polarized with the polarization axis parallel to the optical axis of the 1/4-wave plate;
- if $\alpha = \pi/2$, the transmitted light is linearly polarized with the polarization axis perpendicular to the optical axis of the 1/4-wave plate;
- if $\alpha = \pi/4$, the transmitted light is circularly polarized;
- otherwise, the transmitted light is elliptically polarized.

III. Apparatus

The measurement setup consists of: a semiconductor laser, a tungsten iodine lamp, a silicon photo-cell, a UT51 digital universal meter, as well as two polarizers, 1/2-wave and 1/4-wave plates (the uncertainty of the the angle is 2σ) and a lens with a glass sheet. The elements are placed on an optical bench.

IV. Procedure

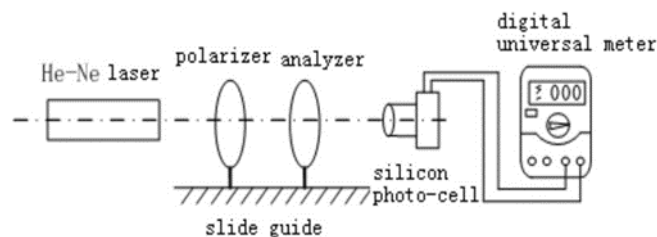
1. Adjustment

- (a) Adjust the photo-cell by choosing the appropriate aperture. There are different apertures on the photo-cell (see the figure below) used in different experiments. In this experiment, only the $\varnothing 6.0$ aperture, which preserves the incident light intensity, is needed. If other aperture is chosen, the intensity of light may get reduced, resulting in a zero reading on the universal meter. Therefore, before proceeding to the next steps, adjust the laser and the photo-cell so the light can pass through the $\varnothing 6.0$ aperture.

- (b) With the laser fixed at one of the ends of the bench, place the lens and the glass sheet in front of it. Make sure that the light passes through the center of the lens
- (c) Rotate the polarizer for at least whole circle so that the current in the digital meter can reach 0 and the maximum current should be more than $0.800\ \mu\text{A}$, which ensure that the change of current could be obvious in the following experiments.

2. Demonstration of Malus' Law

- (a) Assemble the measurement setup as shown in Figure below. Make sure that the laser ray passes through the polarizer to generate linearly polarized light before continuing to the analyzer and the silicon photo-cell.
- (b) Rotate the analyzer for 360° and observe a change in the light intensity to find the maximum electric current I_0 . The distance between the light source and the photovoltaic device is 120 cm; Measure the solar power by the provided solar power meter.
- (c) Set the angle of analyzer to 90° and adjust the angle of the polarizer until the electric current measured by the multimeter reaches its minimum. At this point, the polarizing axes of the polarizer and the analyzer are perpendicular to each other. The distance between the light source and the photovoltaic device is 120 cm, with two devices in series.
- (d) Rotate the analyzer from 90° to 0° and record the magnitude of the current I every 5° . Record the values in a table and plot the graph I/I_0 vs. $\cos^2 \theta$. Perform linear fitting and compare the data with the theoretical result.



3. Linearly Polarized Light and the Half-wave Plate

- (a) Set up the equipment on the optical bench as shown in below Figure. A is the analyzer and P is the polarizer. Set the polarizing axes of A and P perpendicular to each other before placing the $1/2$ -wave plate in the apparatus; extinction of the light can be observed on screen.
- (b) After inserting the $1/2$ -wave plate, rotate it to make the light extinction appear again and set this position as the initial position.
- (c) Rotate the $1/2$ -wave plate for $\alpha = 10^\circ$ from the initial position and the light extinction will be broken. Then rotate A to make the light extinction appear again, record the angle of rotation $\Delta\theta$ in a table.
- (d) Rotate the $1/2$ -wave plate for 10° from the previous position (now $\alpha = 20^\circ$) and repeat Step (c) Repeat this step (increase α) for 8 times. Plot the graph $\Delta\theta$ vs. θ .

(e) Analyze the data

4. Circularly and Elliptically Polarized Light and the 1/4-wave Plate

(a) Keep the position of other plates remaining. Replace the 1/2-wave plate by the 1/4-wave plate, rotate it to make the current in the electrical meter to be 0. Set the this position to be $\theta = 90^\circ$, rotate the analyzer for 360° and record the light intensity for every 10°

(b) Rotate the 1/4-wave plate for 20° , repeat the step 1.

(c) Rotate the 1/4-wave plate for 45° , repeat the step 1.

(d) Rotate the Rotate the 1/4-wave plate for 70° , rotate the analyzer and record its position and the magnitude of the current when the light intensity reaches maximum.

V. Results & Analysis

1.Measurement Uncertainty for equipment:

| QUANTITY | PRECISION |
|----------------------|--------------------------|
| The electric Current | $\pm 0.001[\mu\text{A}]$ |
| angle | $2 [^\circ]$ |

2. Demonstration of Malus Law

Below shows the origin data:

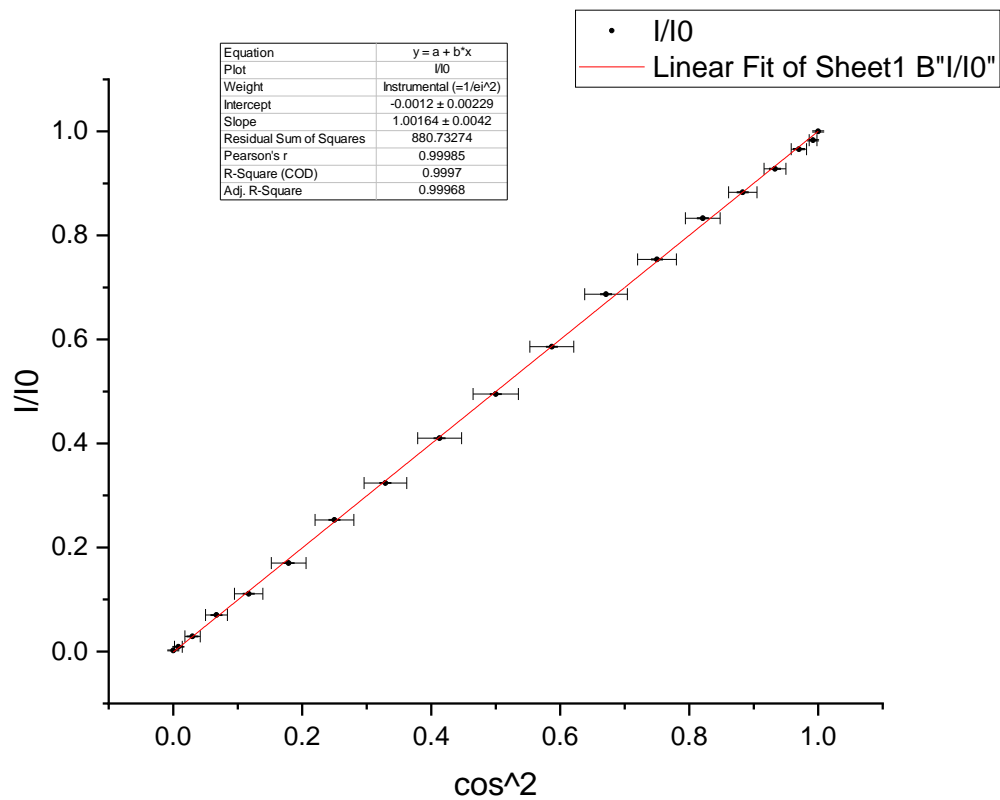
| Maximum Electric Current $I_0 = 1.315 \pm 0.001[\mu\text{A}]$ | | | |
|---|--|------------|--|
| θ | $I [\mu\text{A}] \pm 0.001[\mu\text{A}]$ | θ | $I [\mu\text{A}] \pm 0.001[\mu\text{A}]$ |
| 0° | 1.315 | 50° | 0.539 |
| 5° | 1.293 | 55° | 0.426 |
| 10° | 1.270 | 60° | 0.333 |
| 15° | 1.220 | 65° | 0.223 |
| 20° | 1.161 | 70° | 0.146 |
| 25° | 1.096 | 75° | 0.093 |
| 30° | 0.991 | 80° | 0.038 |
| 35° | 0.903 | 85° | 0.012 |
| 40° | 0.771 | 90° | 0.002 |
| 45° | 0.651 | | |

After calculating, we can generate the following table:

| θ | $\cos^2 \theta$ | $\frac{I}{I_0}$ |
|------------|-----------------|-----------------|
| 0° | 1.000 | 1 |
| 5° | 0.992 | 0.983 |
| 10° | 0.970 | 0.966 |
| 15° | 0.933 | 0.928 |

| | | |
|-----|-------|-------|
| 20° | 0.883 | 0.883 |
| 25° | 0.821 | 0.833 |
| 30° | 0.750 | 0.754 |
| 35° | 0.671 | 0.687 |
| 40° | 0.587 | 0.586 |
| 45° | 0.500 | 0.495 |
| 50° | 0.413 | 0.410 |
| 55° | 0.329 | 0.324 |
| 60° | 0.250 | 0.253 |
| 65° | 0.179 | 0.170 |
| 70° | 0.117 | 0.111 |
| 75° | 0.067 | 0.070 |
| 80° | 0.030 | 0.029 |
| 85° | 0.008 | 0.009 |
| 90° | 0.000 | 0.002 |

Then using origin to make the linear fit plot:



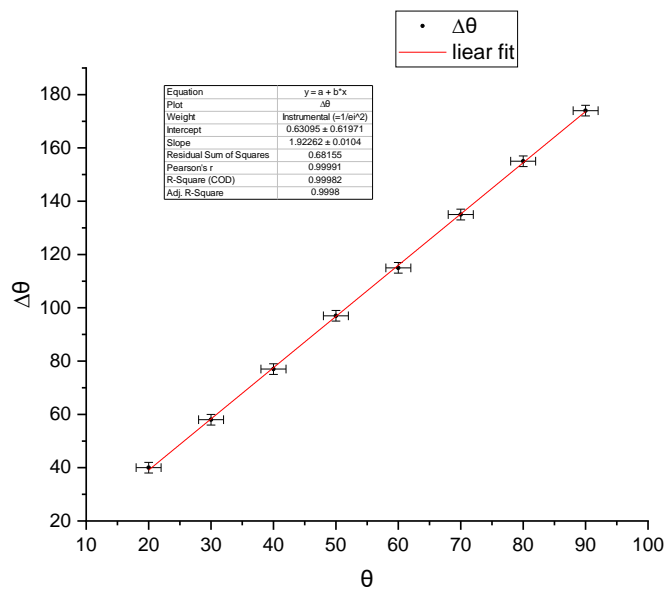
Where slope = 1.00164

3. Linearly polarized light with Half-wave Plate input solar power

Below shows the origin data:

| Rotation angle of the $\frac{1}{2}$ -wave plate | Rotation angle of the analyzer |
|---|--------------------------------|
| initial | 0 |
| 10° | 20° |
| 20° | 40° |
| 30° | 58° |
| 40° | 77° |
| 50° | 97° |
| 60° | 115° |
| 70° | 135° |
| 80° | 155° |
| 90° | 174° |

Then using origin to make the linear fit plot:



Where slope = 1.92262

4. Circularly & Elliptically Polarized Light with $\frac{1}{4}$ -wave plate: 0°

Below shows the origin data:

| Rotation angle of $\frac{1}{4}$ -wave plate: 0° | | | |
|--|-------------------------------|----------|-------------------------------|
| Maximum Electric Current $I_0 = 1.005 \pm 0.001 [\mu A]$ | | | |
| θ | $I [\mu A] \pm 0.001 [\mu A]$ | θ | $I [\mu A] \pm 0.001 [\mu A]$ |
| 0° | 0.004 | 180° | 0.007 |
| 10° | 0.035 | 190° | 0.056 |

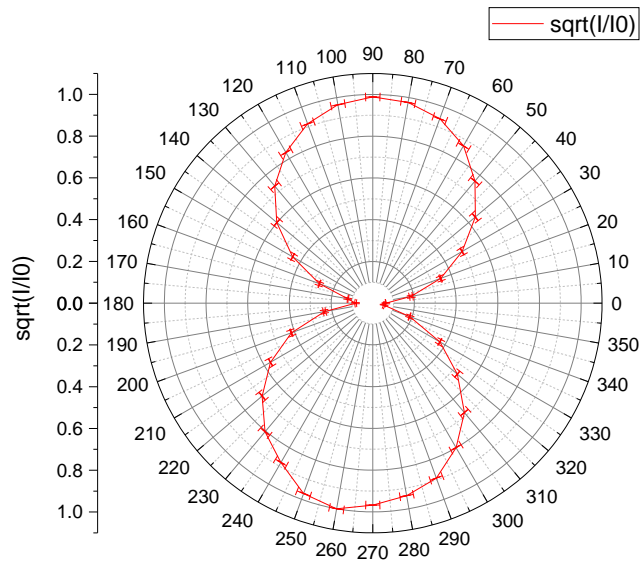
| | | | |
|------|-------|------|-------|
| 20° | 0.121 | 200° | 0.174 |
| 30° | 0.246 | 210° | 0.323 |
| 40° | 0.414 | 220° | 0.483 |
| 50° | 0.585 | 230° | 0.650 |
| 60° | 0.760 | 240° | 0.780 |
| 70° | 0.888 | 250° | 0.945 |
| 80° | 0.958 | 260° | 1.005 |
| 90° | 0.981 | 270° | 0.940 |
| 100° | 0.935 | 280° | 0.880 |
| 110° | 0.840 | 290° | 0.800 |
| 120° | 0.699 | 300° | 0.641 |
| 130° | 0.538 | 310° | 0.470 |
| 140° | 0.363 | 320° | 0.282 |
| 150° | 0.198 | 330° | 0.139 |
| 160° | 0.075 | 340° | 0.036 |
| 170° | 0.015 | 350° | 0.003 |

After calculating, we can generate the following table:

| θ | $\sqrt{\frac{I}{I_0}}$ | θ | $\sqrt{\frac{I}{I_0}}$ |
|----------|------------------------|----------|------------------------|
| 0° | 0.063 | 180° | 0.083 |
| 10° | 0.187 | 190° | 0.236 |
| 20° | 0.347 | 200° | 0.416 |
| 30° | 0.495 | 210° | 0.567 |
| 40° | 0.642 | 220° | 0.693 |
| 50° | 0.763 | 230° | 0.804 |
| 60° | 0.870 | 240° | 0.881 |
| 70° | 0.940 | 250° | 0.970 |
| 80° | 0.976 | 260° | 1.000 |
| 90° | 0.988 | 270° | 0.967 |
| 100° | 0.965 | 280° | 0.936 |
| 110° | 0.914 | 290° | 0.892 |
| 120° | 0.834 | 300° | 0.799 |
| 130° | 0.732 | 310° | 0.684 |
| 140° | 0.601 | 320° | 0.530 |
| 150° | 0.444 | 330° | 0.372 |
| 160° | 0.273 | 340° | 0.189 |

| | | | |
|------|-------|------|-------|
| 170° | 0.122 | 350° | 0.055 |
|------|-------|------|-------|

Then using origin to make the polar coordinate plot:



5. Circularly & Elliptically Polarized Light with $\frac{1}{4}$ -wave plate: 20°

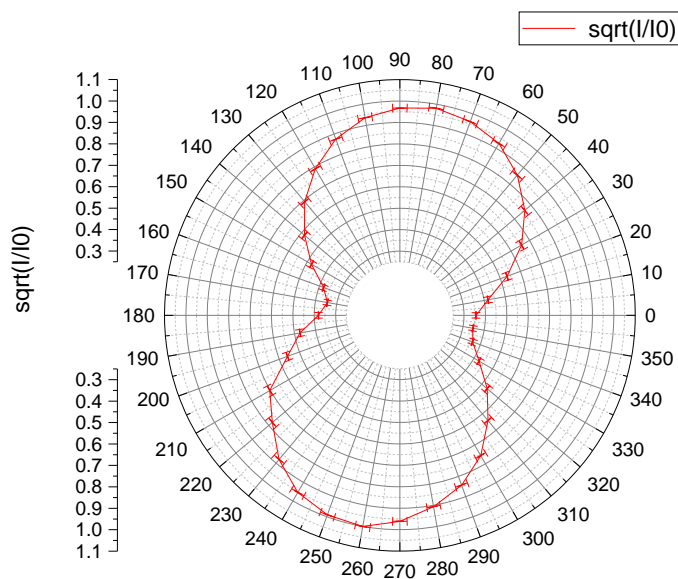
Below shows the origin data:

| Rotation angle of $\frac{1}{4}$ -wave plate: 20° | | | |
|---|------------------------------|----------|------------------------------|
| Maximum Electric Current $I_0 = 0.914 \pm 0.001[\mu A]$ | | | |
| θ | $I [\mu A] \pm 0.001[\mu A]$ | θ | $I [\mu A] \pm 0.001[\mu A]$ |
| 0° | 0.116 | 180° | 0.133 |
| 10° | 0.160 | 190° | 0.205 |
| 20° | 0.260 | 200° | 0.286 |
| 30° | 0.395 | 210° | 0.450 |
| 40° | 0.531 | 220° | 0.553 |
| 50° | 0.666 | 230° | 0.705 |
| 60° | 0.784 | 240° | 0.832 |
| 70° | 0.845 | 250° | 0.896 |
| 80° | 0.880 | 260° | 0.914 |
| 90° | 0.855 | 270° | 0.844 |
| 100° | 0.801 | 280° | 0.750 |
| 110° | 0.700 | 290° | 0.649 |
| 120° | 0.570 | 300° | 0.523 |
| 130° | 0.438 | 310° | 0.376 |
| 140° | 0.310 | 320° | 0.260 |
| 150° | 0.208 | 330° | 0.167 |
| 160° | 0.132 | 340° | 0.120 |
| 170° | 0.108 | 350° | 0.110 |

After calculating, we can generate the following table:

| θ | $\sqrt{\frac{I}{I_0}}$ | θ | $\sqrt{\frac{I}{I_0}}$ |
|----------|------------------------|----------|------------------------|
| 0° | 0.356 | 180° | 0.381 |
| 10° | 0.418 | 190° | 0.474 |
| 20° | 0.533 | 200° | 0.559 |
| 30° | 0.657 | 210° | 0.702 |
| 40° | 0.762 | 220° | 0.778 |
| 50° | 0.854 | 230° | 0.878 |
| 60° | 0.926 | 240° | 0.954 |
| 70° | 0.962 | 250° | 0.990 |
| 80° | 0.981 | 260° | 1.000 |
| 90° | 0.967 | 270° | 0.961 |
| 100° | 0.936 | 280° | 0.906 |
| 110° | 0.875 | 290° | 0.843 |
| 120° | 0.790 | 300° | 0.756 |
| 130° | 0.692 | 310° | 0.641 |
| 140° | 0.582 | 320° | 0.533 |
| 150° | 0.477 | 330° | 0.427 |
| 160° | 0.380 | 340° | 0.362 |
| 170° | 0.344 | 350° | 0.347 |

Then using origin to make the polar coordinate plot:



6. Circularly & Elliptically Polarized Light with $\frac{1}{4}$ -wave plate: 45°

Below shows the origin data:

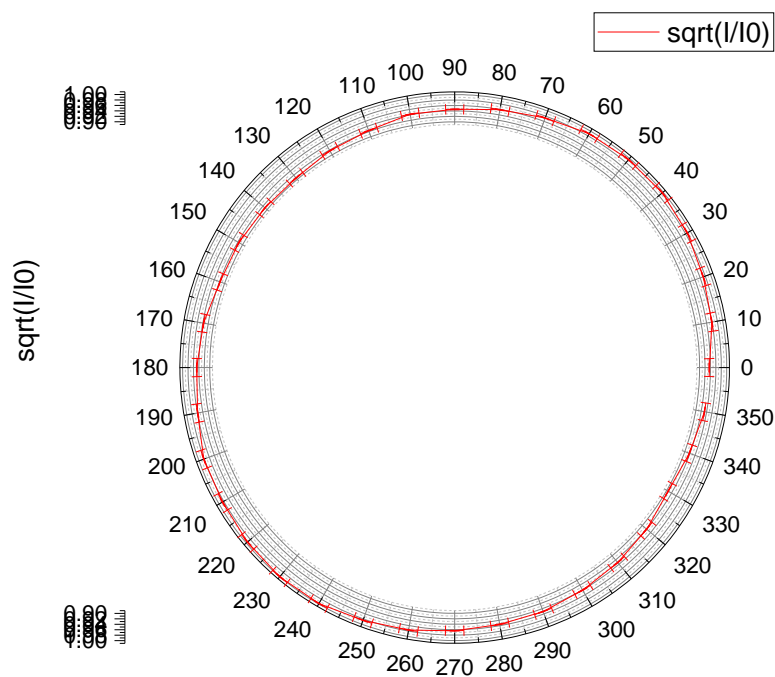
| Rotation angle of $\frac{1}{4}$ -wave plate: 45° | | | |
|---|------------------------------|-------------|------------------------------|
| Maximum Electric Current $I_0 = 0.570 \pm 0.001[\mu A]$ | | | |
| θ | $I [\mu A] \pm 0.001[\mu A]$ | θ | $I [\mu A] \pm 0.001[\mu A]$ |
| 0° | 0.499 | 180° | 0.512 |
| 10° | 0.525 | 190° | 0.524 |
| 20° | 0.538 | 200° | 0.548 |
| 30° | 0.555 | 210° | 0.553 |
| 40° | 0.566 | 220° | 0.563 |
| 50° | 0.563 | 230° | 0.570 |
| 60° | 0.557 | 240° | 0.568 |
| 70° | 0.540 | 250° | 0.553 |
| 80° | 0.524 | 260° | 0.543 |
| 90° | 0.510 | 270° | 0.530 |
| 100° | 0.506 | 280° | 0.520 |
| 110° | 0.487 | 290° | 0.515 |
| 120° | 0.479 | 300° | 0.505 |
| 130° | 0.469 | 310° | 0.495 |
| 140° | 0.472 | 320° | 0.484 |
| 150° | 0.478 | 330° | 0.468 |
| 160° | 0.480 | 340° | 0.480 |
| 170° | 0.500 | 350° | 0.495 |

After calculating, we can generate the following table:

| θ | $\sqrt{\frac{I}{I_0}}$ | θ | $\sqrt{\frac{I}{I_0}}$ |
|-------------|------------------------|-------------|------------------------|
| 0° | 0.936 | 180° | 0.948 |
| 10° | 0.960 | 190° | 0.959 |
| 20° | 0.972 | 200° | 0.981 |
| 30° | 0.987 | 210° | 0.985 |
| 40° | 0.996 | 220° | 0.994 |
| 50° | 0.994 | 230° | 1 |
| 60° | 0.989 | 240° | 0.998 |
| 70° | 0.973 | 250° | 0.985 |
| 80° | 0.959 | 260° | 0.976 |
| 90° | 0.946 | 270° | 0.964 |
| 100° | 0.942 | 280° | 0.955 |

| | | | |
|------|-------|------|-------|
| 110° | 0.924 | 290° | 0.951 |
| 120° | 0.917 | 300° | 0.941 |
| 130° | 0.907 | 310° | 0.932 |
| 140° | 0.910 | 320° | 0.921 |
| 150° | 0.916 | 330° | 0.906 |
| 160° | 0.918 | 340° | 0.918 |
| 170° | 0.937 | 350° | 0.932 |

Then using origin to make the polar coordinate plot:



7. Circularly & Elliptically Polarized Light with $\frac{1}{4}$ -wave plate: 70°

The data of the rotation angle and the current is listed below:

| | |
|--|-------|
| Rotation angle of $\frac{1}{4}$ -wave plate: 70° | |
| $\theta [^\circ] \pm [2^\circ]$ | 100° |
| $I [\mu A] + 0.001 [\mu A]$ | 0.805 |

VI. Uncertainty Analysis

1. Demonstration of Malus Law

| QUANTITY | PRECISION |
|----------------------|--------------------------|
| The electric Current | $\pm 0.001[\mu\text{A}]$ |
| angle | 2 [°] |

For $u_{\frac{I}{I_0}}$, we have, $u_{\frac{I}{I_0}} = \frac{I}{I_0} \sqrt{\left(\frac{1}{I}\right)^2 u_I^2 + \left(\frac{1}{I_0^2}\right)^2 u_{I_0}^2} = 0.001 \sqrt{\left(\frac{1}{I_0}\right)^2 + \left(\frac{I}{I_0^2}\right)^2}$

Therefore, we can have the following table,

| θ | $u_{\frac{I}{I_0}}$ |
|----------|---------------------|
| 0° | 0.0011 |
| 5° | 0.0011 |
| 10° | 0.0011 |
| 15° | 0.0010 |
| 20° | 0.0010 |
| 25° | 0.0010 |
| 30° | 0.0010 |
| 35° | 0.0009 |
| 40° | 0.0009 |
| 45° | 0.0008 |
| 50° | 0.0008 |
| 55° | 0.0008 |
| 60° | 0.0008 |
| 65° | 0.0008 |
| 70° | 0.0008 |
| 75° | 0.0008 |
| 80° | 0.0008 |
| 85° | 0.0008 |
| 90° | 0.0008 |

For $u_{\cos^2 \theta}$, we have,

$$u_{\cos^2 \theta} = 2 \cos \theta \sin \theta u_{\theta},$$

Hence, we can have the following table.

| θ | $u_{\cos^2 \theta}$ |
|----------|---------------------|
| 0° | 0.000 |
| 5° | 0.006 |
| 10° | 0.012 |

| | |
|-----|-------|
| 15° | 0.017 |
| 20° | 0.022 |
| 25° | 0.027 |
| 30° | 0.030 |
| 35° | 0.033 |
| 40° | 0.034 |
| 45° | 0.035 |
| 50° | 0.034 |
| 55° | 0.033 |
| 60° | 0.030 |
| 65° | 0.027 |
| 70° | 0.022 |
| 75° | 0.017 |
| 80° | 0.012 |
| 85° | 0.006 |
| 90° | 0.000 |

2. Linearly polarized light with Half-wave Plate

The uncertainty of the rotation angle is $u_{\theta} = 2^{\circ}$

3. Circularly and elliptically polarized light with $\frac{1}{4}$ -wave plate (0°)

For $u_{\sqrt{\frac{I}{I_0}}}$, we have,

$$u_{\sqrt{\frac{I}{I_0}}} = \sqrt{\frac{I}{I_0} * \frac{u_{\frac{I}{I_0}}}{2 \frac{I}{I_0}}} = \frac{0.001}{2} \sqrt{\frac{1}{I * I_0} + \frac{I}{I_0^3}}$$

Hence, we can have the following table.

| θ | $u_{\sqrt{\frac{I}{I_0}}}$ | θ | $u_{\sqrt{\frac{I}{I_0}}}$ |
|----------|----------------------------|----------|----------------------------|
| 0° | 0.0079 | 180° | 0.0060 |
| 10° | 0.0027 | 190° | 0.0021 |
| 20° | 0.0014 | 200° | 0.0012 |
| 30° | 0.0010 | 210° | 0.0009 |
| 40° | 0.0008 | 220° | 0.0008 |
| 50° | 0.0008 | 230° | 0.0007 |
| 60° | 0.0007 | 240° | 0.0007 |
| 70° | 0.0007 | 250° | 0.0007 |

| | | | |
|------|--------|------|--------|
| 80° | 0.0007 | 260° | 0.0007 |
| 90° | 0.0007 | 270° | 0.0007 |
| 100° | 0.0007 | 280° | 0.0007 |
| 110° | 0.0007 | 290° | 0.0007 |
| 120° | 0.0007 | 300° | 0.0007 |
| 130° | 0.0008 | 310° | 0.0008 |
| 140° | 0.0009 | 320° | 0.0010 |
| 150° | 0.0011 | 330° | 0.0014 |
| 160° | 0.0018 | 340° | 0.0026 |
| 170° | 0.0041 | 350° | 0.0091 |

4. Circularly and elliptically polarized light with ¼-wave plate (20°)

For $u \sqrt{\frac{I}{I_0}}$, we have,

$$u \sqrt{\frac{I}{I_0}} = \sqrt{\frac{I}{I_0} * \frac{u_I}{2 \frac{I}{I_0}}} = \frac{0.001}{2} \sqrt{\frac{1}{I * I_0} + \frac{I}{I_0^3}}$$

Hence, we can have the following table.

| θ | $u \sqrt{\frac{I}{I_0}}$ | θ | $u \sqrt{\frac{I}{I_0}}$ |
|----------|--------------------------|----------|--------------------------|
| 0° | 0.0015 | 180° | 0.0014 |
| 10° | 0.0013 | 190° | 0.0012 |
| 20° | 0.0010 | 200° | 0.0010 |
| 30° | 0.0009 | 210° | 0.0009 |
| 40° | 0.0008 | 220° | 0.0008 |
| 50° | 0.0008 | 230° | 0.0008 |
| 60° | 0.0008 | 240° | 0.0008 |
| 70° | 0.0008 | 250° | 0.0008 |
| 80° | 0.0008 | 260° | 0.0008 |
| 90° | 0.0008 | 270° | 0.0008 |
| 100° | 0.0008 | 280° | 0.0008 |
| 110° | 0.0008 | 290° | 0.0008 |
| 120° | 0.0008 | 300° | 0.0008 |
| 130° | 0.0009 | 310° | 0.0009 |
| 140° | 0.0010 | 320° | 0.0011 |
| 150° | 0.0012 | 330° | 0.0013 |
| 160° | 0.0015 | 340° | 0.0015 |

| | | | |
|------|--------|------|--------|
| 170° | 0.0016 | 350° | 0.0016 |
|------|--------|------|--------|

5. Circularly and elliptically polarized light with ¼-wave plate (45°)

For $u \sqrt{\frac{I}{I_0}}$, we have,

$$u \sqrt{\frac{I}{I_0}} = \sqrt{\frac{I}{I_0} * \frac{u_I}{2 \frac{I}{I_0}}} = \frac{0.001}{2} \sqrt{\frac{1}{I * I_0} + \frac{I}{I_0^3}}$$

Hence, we can have the following table.

| θ | $u \sqrt{\frac{I}{I_0}}$ | θ | $u \sqrt{\frac{I}{I_0}}$ |
|----------|--------------------------|----------|--------------------------|
| 0° | 0.0012 | 180° | 0.0012 |
| 10° | 0.0012 | 190° | 0.0012 |
| 20° | 0.0012 | 200° | 0.0012 |
| 30° | 0.0012 | 210° | 0.0012 |
| 40° | 0.0012 | 220° | 0.0012 |
| 50° | 0.0012 | 230° | 0.0012 |
| 60° | 0.0012 | 240° | 0.0012 |
| 70° | 0.0012 | 250° | 0.0012 |
| 80° | 0.0012 | 260° | 0.0012 |
| 90° | 0.0012 | 270° | 0.0012 |
| 100° | 0.0012 | 280° | 0.0012 |
| 110° | 0.0012 | 290° | 0.0012 |
| 120° | 0.0012 | 300° | 0.0012 |
| 130° | 0.0012 | 310° | 0.0012 |
| 140° | 0.0012 | 320° | 0.0012 |
| 150° | 0.0012 | 330° | 0.0012 |
| 160° | 0.0012 | 340° | 0.0012 |
| 170° | 0.0012 | 350° | 0.0012 |

VII. Conclusion

1. Answer to the question in the procedure,

- (a) 4 times
- (b) 2 times
- (c) The transmitted light will be linearly polarized with the polarized axis rotated by the angle of 2 alpha.

2. Demonstration of Malus' Law

From the linear fit, it can be figure out that there is a linear relationship between I/I_0 and $\cos^2 \theta$ which is correspond to the Malus's law. With slope equals to 1.00164 in our result,

we can calculate the relative error:

$$E = \frac{|\eta_{exp} - \eta_{theory}|}{\eta_{theory}} \times 100\% = E = \frac{|1 - 1.00164|}{1} \times 100\% = 0.164\%,$$

, which is consider quite low.

3.Linearly Polarized Light and the Half-wave Plate

From the graph, we have slope: 1.92262 ± 0.029 , with the theorem $\Delta\theta = 2\theta$, which indicate that slope between $\Delta\theta$ and 2θ should be 2. Then, we have $E = \frac{|2 - 1.92262|}{2} \times 100\% = 3.869\%$, which is small that still can help us verified the relationship successful.

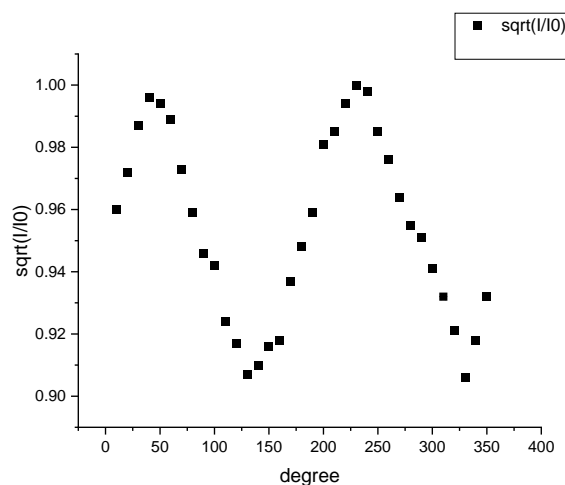
4.Circularly and elliptically polarized light with $\frac{1}{4}$ -wave plate (0, 20, 45, 70 degree)

For rotation angle of 0 degree and 20 degrees, the maximum current should occur twice with theta 180 degree between them. In 0 degree, the maximums happen in 90 and 260 degrees, which has little shift. In 20 degrees, the maximums happen in 80 and 260 with perfect 180 degrees different.

However, the maximum current of 0 degree occur when $\theta = 0$ or 180 degree while the 20 degree maximum current occur when $\theta = 20$ or 200 degree. The reason that our data can't meet the theory is due to the additional step during experiments. Our group adjust the current = 0 as the initial condition which is unnecessary. So the maximum current will shift in our data.

Theoretically, when alpha is 0 degree, the transmitted light will be linearly polarized. As the graph in the result part, it is corresponded to the theorem. For rotation angle of 20 degree, the transmitted light will be elliptically polarized. As in the result part, it is corresponded to the theorem.

For rotation angle of 45 degrees, the result change within a quite small range(0.9~1.0), which means the light intensity remain almost the same. It is in an acceptable range of relative error. Hence, we have verified the relationship successful.



VIII. Discussion

There are two main reasons that might affect the error: the light from the cellphone, the unstable current source and human adjustment error.

During the experiment, there might be a chance for students to shake the light that influences the detector. It is quite hard to avoid while some students want to observe the current value on the detector, while others want to light their own machine to adjust it. Therefore, it causes unavoidable error.

During the lab, our group found that our current source generates a current that gradually decreases. In a $\frac{1}{4}$ wave-plate, the maximum current at 70 degrees is 0.850, which is further smaller than the 20 degrees of a $\frac{1}{4}$ wave-plate. I have asked one of my classmates and found the same problem. Some current sources do generate unstable current.

Due to the fact that the lab was operating in the dark with light from a cellphone, there might be a great chance for people to adjust the angle with error. With an uncertainty of angle of 2 degrees, which is already considered not small, the human error might cause much more. Maybe there can be some electrical device to help students adjust the angle in the dark with much more precision in the future.

IX. Reference

1. Exercise 4 –Lab Manual.pdf
2. [https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=2&cad=rja&uact=8&ved=2ahUKEwi0ivGzwYzmAhVCIqYKHUK5AoAQFjABegQIDxAE&url=https%3A%2F%2Fen.wikipedia.org%2Fwiki%2FPolarization_\(waves\)&usg=AOvVaw3yPUPmRQH6yrnlfavTk6Jd](https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=2&cad=rja&uact=8&ved=2ahUKEwi0ivGzwYzmAhVCIqYKHUK5AoAQFjABegQIDxAE&url=https%3A%2F%2Fen.wikipedia.org%2Fwiki%2FPolarization_(waves)&usg=AOvVaw3yPUPmRQH6yrnlfavTk6Jd)

X. Data sheet

Attach at end