JOINT INSTITUTE
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Fall 2019 VV256_Assignment 4

Series solution. Fourier series. Sturm-Liouville BVPs. **(130 points)**

Deadline: 2019-11-23

Problem 1 **(30 points)**

a. Determine the general solution of the equation

$$y'' - 2x^2 y = 0$$

in terms of power series about $x = 0$.

b. Determine two linearly independent solutions of the equation

$$x^2 y'' + (x - 2x^2)y' - xy = 0$$

using the series solution approach.

Problem 2 **(45 points)**

a. Sketch the graph of the given function for three periods.

b. Find the Fourier series for the given function.

c. Plot the partial sums $S_N(x)$ for $N = 1, 5, 10$.

$$\mathbf{1.}\ f(x) = \begin{cases} x & -\pi \le x \le 0 \\ 2x & 0 < x \le \pi \end{cases} \quad f(x + 2\pi) = f(x)$$

$$\mathbf{2.}\ f(x) = \begin{cases} x & 0 \le x \le 1 \\ 1 & 1 < x < 2 \end{cases}\ \textit{cosine series}\ \ T = 4; \quad \mathbf{3.}\ f(x) = -x, -\pi < x < 0 \quad \textit{sine series}, T = 2\pi$$

Problem 3 **(20 points)**

a. Show that for all values of α there is an infinite sequence of positive eigenvalues of the problem

$$y''(x) + \lambda y(x) = 0$$
$$\alpha\, y(0) + y'(0) = 0, \qquad y(1) = 0 \quad (\alpha = const)$$

b. Find eigenvalues of the problem if $\alpha = 1$.

Problem 4 **(35 points)**

a. Find normalized eigenfunctions of the problem

$$y''(x) + \lambda y(x) = 0, \quad y'(0) = 0, \qquad y(1) + y'(1) = 0$$

b. Find the coefficients b_n in the eigenfunction expansion

$$\sum_{n=1}^{\infty} b_n\, \varphi_n(x)$$

of the function $y = x \quad 0 \le x \le 1$, using the normalized eigenfunctions from part a.

c. Solve the problem

$$y''(x) + 2y(x) = -x, \qquad y'(0) = 0, \qquad y(1) + y'(1) = 0$$

by means of an eigenfunction expansion.

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