Fall_2019 VV256_Assignment 2: Nonlinear first-order ODEs and elements of linear algebra.

Deadline: 2019-10-15

Problem 1 [20 points]

State Bihari's lemma and find the interval of existence of the solution to the initial-value problem

$$y' = 2y^2 - x, \qquad y(1) = 1.$$

Problem 2 [20 points]

Find all solutions of the following equations including singular solutions if they exist. Illustrate your answer with graphs of solutions.

a) $(y')^2 - y^2 = 0,$ b) $8(y')^3 = 27y,$ c) $y + xy' = 4\sqrt{y'},$ d) $y' = e^{\frac{ty'}{y}}$

Problem 3 [20 points]

a) Prove that the system of elements of a linear space is linearly dependent if and only if one of those elements can be expressed as a linear combination of others.

b) Determine if the following elements are linearly dependent or independent:

1. $(1,2,-2),\ (3,1,0),\ (2,-1,1),\ (4,3,-2)$ in $\mathbb{R}^3$.
2. $e^{\alpha_1 t}, e^{\alpha_2 t}, \dots, e^{\alpha_n t}$ (all α_i are distinct) in $C[a,b]$

Problem 4 [15 points]

Solve the following systems of linear equations:

$$a)\ \begin{cases} x - z = 0 \\ 3x + y + z = 0 \\ -x + y + 2z = 0 \end{cases} \qquad b)\ \begin{cases} x + 2 - z = 1 \\ 2x + y + z = 1 \\ x - y + 2z = 1 \end{cases} \qquad c)\ \begin{cases} x - z = 0 \\ 3x + y + z = 1 \\ -x + y + 2z = 2 \end{cases}$$

Problem 5 [15 points]

For the matrix $A = \begin{pmatrix} 0 & 1 & -2 \\ -1 & 0 & 2 \\ 2 & -2 & 0 \end{pmatrix}$, find

a) Its inverse A^{-1} b)eigenvalues and corresponding eigenvectors

c) $||A||_1, ||A||_2, ||A||_\infty$.

Problem 6 [10 points]

State and prove Young's, Holder's and Minkowski's inequalities. (Use recommended reference books.)

Dr. Olga Danilkina 中国 上海闵行区东川路 800 号 邮编 200240

Tel: +86-21-34206765 Ext. 4372 http://umji.sjtu.edu.cn