Fall_2019 **vv256_Assignment 1:** Linear and nonlinear first-order ODEs.

Deadline: 2019-09-27

Problem 1 [20 points]

a) Give a brief description of the math model of Newton's law of cooling with the constant temperature of the environment, and apply it to the following problem

b) A body at temperature $40°C$ is placed in a room at temperature $20°C$. It cools to $30°C$ in 10 minutes. What is the temperature of the body after 20 minutes?

c) Explain what happens with the solution if the environment temperature is a function of time.

Problem 2 [20 points]

a) Derive the mathematical model of radioactive elements decay and solve the arising differential equation. Eliminate the constant of proportionality in favor of the half-life t^*.

b) All organisms contain two isotopes of Carbon: the stable ^{12}C and the radioactive ^{14}C (half-life of approximately 5,580 years). While alive, the amount of ^{14}C in an organism is constantly replenished through inhaling, and so the ratio of the two Carbon isotopes remains constant. After death this ratio changes because ^{14}C decays and is not replenished. A fossil is unearthed and it is found that the amount of ^{14}C is 30% of what it would be for a living organism of similar size. How old is the fossil?

Problem 3 [30 points]

Consider a reservoir of constant volume $V(m^3)$ with an inlet admitting a constant flow I (m^3/s) and an outlet through which the flow is the same I. At time t the reservoir contains pollutant $P(t)$ (m^3).

a) Derive a model to describe the rate of change of pollutant in the reservoir if the pollutant concentration in the inlet is a function of time (let the concentration of pollutant in the outflow be the same as in the reservoir). *Hint: the rate of change of pollutant equals inflow of pollutant minus outflow.*

b) Find the general solution of the problem. Explain what happens if there are no pollutant flows into the reservoir at all and if the pollutant concentration in the inflow is a constant.

c) Take $V = 2 \times 10^6\ m^3$, $I = 2\ m^3/s$ and assume that the reservoir is originally free of pollution. Between $t_0 = 0\ s$ and $t_1 = 10^6\ s$ a constant pollutant concentration of 10^{-4} is present in the inflow. What is the pollutant concentration in the reservoir for all times? Plot the result using Matlab.

Problem 4 [30 points]

a) Solve the following ODE [5 points each problem]

$$1)\ x^2 - y^2 + 2xyy' = 0, \qquad 2)\ ln(y^2+1) + \frac{2y(x-1)}{y^2+1}y' = 0, \qquad 3)\ y' - \frac{y}{2x} = 10x^2y^5,$$

$$4)\ 2x^2y' = (x-1)(y^2-x^2) + 2xy$$

b) Describe the method of integrating factors applied to non-exact ODEs (use the recommended textbooks for a reference).

c) Solve the DE $y - xy' = 0$ using the method of integrating factors.

Dr. Olga Danilkina 中国 上海闵行区东川路 800 号 邮编 200240

Tel: +86-21-34206765 Ext. 4372 http://umji.sjtu.edu.cn