JOINT INSTITUTE
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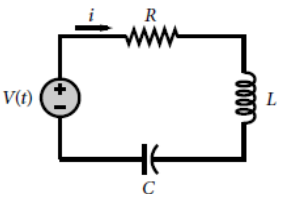
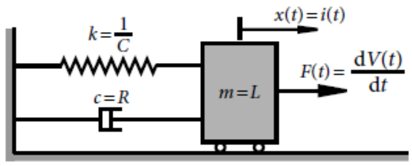
Fall 2019 VV256_Assignment 3

Higher-order ODEs with variable and constant coefficients **(130 points)**

Deadline: 2019-11-11

Introduction to Problem 1





A circuit consisting of a resistor R, an inductor L, a capacitor C, and a voltage source $V(t)$ connected in series is called the series RLC circuit. The series RLC circuit is equivalent to a mass-damper-spring system as shown. Applying Kirchhoff's Voltage Law,

$$-V(t)+Ri+L\frac{di}{dt}+\frac{1}{C}\int_{-\infty}^{t} i\,dt=0.$$

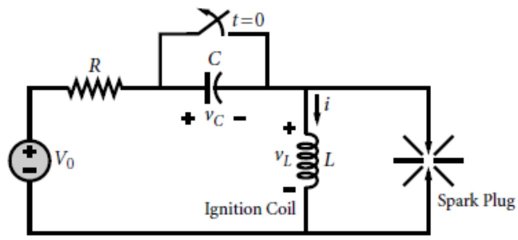
Differentiating with respect to t,

$$L\frac{d^2i}{dt^2}+R\frac{di}{dt}+\frac{1}{C}i=\frac{dV(t)}{dt}$$

$$\frac{d^2i}{dt^2}+2\xi\omega_0\frac{di}{dt}+\omega_0^2 i=\frac{1}{L}\frac{dV(t)}{dt},\qquad \omega_0^2=\frac{1}{LC},\qquad \xi\omega_0=\frac{R}{2L}.$$

Problem 1 (10 points)

An automobile ignition system is modeled by the circuit shown in the figure.



The voltage source V_0 represents the battery and alternator. The resistor R models the resistance of thewiring, and the ignition coil is modeled by the inductor L.
The capacitor C, known as the condenser, is in parallel with the switch, which is known as the electronic ignition. The switch has been closed for a long time prior to $t<0^-$.

a. Determine the voltage v_L across the inductor for $t>0$.

Remarks:

1. For $t<0$, the switch is closed, the capacitor behaves as an open circuit and the inductor behaves as a short circuit $\Rightarrow i(0^-)=\frac{V_0}{R}, v_C(0^-)=0$.
2. At $t=0$, the switch is open. The current in an inductor and the voltage across a capacitor cannot change abruptly $\Rightarrow i(0^+)=i(0^-)=\frac{V_0}{R},\ v_C(0^+)=\ v_C(0^-)=0$

$$-V_0+Ri(0^+)+v_C(0^+)+v_L(0^+)=0\ \Rightarrow v_L(0^+)=V_0-Ri(0^+)=0, v_L(0^+)=L\frac{di(0^+)}{dt}\Rightarrow i'(0^+)=0$$

b. For $V_0=12\,V, R=4\Omega\ C=1\mu F, L=8mH$, determine the maximal inductor voltage and the time when it is reached (Answer: $-259\,V, t=1.405\times10^{-4}sec$)

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Problem 2 ($5 \times 4 \times 6 = 120$ points)

Find the general solution of the following ODEs using the indicated method. A particular solution is given in some questions.

The Wronskian	Order reduction (see lecture notes)
a. $ty'' - (2t+1)y' + (t+1)y = 0$ b. $t^2(t+1)y'' - 2y = 0,\ y_1 = 1 + \frac{1}{t}$ c. $ty'' + 2y' - ty = 0,\ y_1 = \frac{\exp(t)}{t}$ d. $t(t-1)y'' - ty' + y = 0.$	a. $y'' + y'^2 = 2e^{-y}$ b. $yy'' + 1 = y'^2$ c. $ty'' = y' + t\sin(y'/t)$. d. $t^2y'' = y'^2$
Homogeneity (see lecture notes)	**Undetermined coefficients**
a. $t^2yy'' = (y - ty')^2$ b. $y(ty'' + y') = ty'^2(1-t)$ c. $yy'' = y'^2 + 15y^2\sqrt{t}$ d. $t^2yy'' + y'^2 = 0$	a. $y'' + 2y' + 2y = 5\cos t + 10\sin 2t$ b. $y'' - 4y' + 4y = (1+t)e^t + 2e^{2t} + 3e^{3t}$ c. $y'' - 3y' + 4y = 12e^{2t} + 4e^{3t}$ d. $y''' - 2y' - 4y = 50(\sin t + e^{2t})$ $y(0) = 1, \quad y'(0) = -1, \quad y''(0) = 0$
Euler's equation	**Variation of constants**
a. $\rho^2 \frac{d^2u}{d\rho^2} + \rho\frac{du}{d\rho} - u = 0$ b. $x^2y'' - xy' + y = 3x^2 \ \ (y' = \frac{dy}{dx})$ c. $(2t+1)^2y'' - 2(2t+1)y' - 12y = 0$ $(y' = \frac{dy}{dt})$ d. $x^3y''' - 6x^2y'' + 18xy' - 24y = 0$ $(y' = \frac{dy}{dx})$	a. $y'' - y = t^{-1} - 2t^{-3}$ b. $y'' + 3y' + 2y = \sin e^t$ c. $y'' + 4y = 2\tan t$ d. $y''\cos\frac{t}{2} + \frac{1}{4}y\cos\frac{t}{2} = 1$