Fall_2019 **VV256_Assignment 5: Laplace transforms. Systems of ODEs.**

Deadline: 2019-12-06

Problem 1 **[20 points]**

Find the Laplace transform for the following functions

$$\boldsymbol{a})\quad f(t)=\begin{cases}0, & t<0\\ 2+t-t^2+t^3+e^t\sin t, & t>0\end{cases}\qquad \boldsymbol{b})\quad f(t)=\begin{cases}2t, & 0\le t<\frac{1}{2}\\ -2\pi, & \frac{1}{2}\le t<1\\ 0, & t<0 \ \ or \ \ t\ge 1\end{cases}$$

Problem 2 **[10 points]**

Find the inverse Laplace transform for the image

$$\bar{f}(s)=\frac{3s+13}{(s-1)(s^2+2s+5)}$$

Problem 3 **[10+15× 4 =70 points]**

Use the Laplace transform to solve the initial value problems

$\boldsymbol{a})$ $\ddot{y}(t)-5\dot{y}(t)+6y(t)=0, y(0)=2, \dot{y}(0)=1,$ $\boldsymbol{b})$ $\ddot{y}(t)-y(t)=te^{2t}, y(0)=0, \dot{y}(0)=1.5$

$\boldsymbol{c})\begin{cases}\dot{x}+y=0\\ \dot{y}+x=0\end{cases}, x(0)=1, y(0)=-1,$ $\boldsymbol{d})$ $4\ddot{y}(t)+4\dot{y}(t)+5y(t)=\begin{cases}4 & 0\le t<\pi\\ 0 & t\ge\pi\end{cases},\quad y(0)=\dot{y}(0)=0,$

$\boldsymbol{e})$ $\ddot{y}(t)+2\dot{y}(t)+2y(t)=\delta(t-\pi), y(0)=\dot{y}(0)=0$

Problem 4 **[50 points]**

Solve the following linear systems of ODEs with constant coefficients

$$\boldsymbol{a})\quad\begin{cases}x'=2y-3x\\ y'=\ \ y-2x\end{cases}\qquad \boldsymbol{b})\quad\begin{cases}x'=3x-2y-\ \ z,\\ y'=3x-4y-3z\\ \ \ z'=2x-4y.\end{cases}\qquad \boldsymbol{c})\quad\begin{cases}x'=5x-3y+2e^{3t},\\ y'=x+y+5e^{-t}.\end{cases}$$

$\boldsymbol{d})\quad\begin{cases}x''=3x+4y,\\ y''=-x-y.\end{cases}$ (This system is not in the normal form) $\boldsymbol{e})\quad\begin{cases}x'=2y-x+1,\\ y'=3y-2x.\end{cases}$

For the systems a) and e) sketch the phase portrait. Show the direction of motion along phase trajectories.

Problem 5 **[50 points]**

Sketch phase trajectories for the following equations. What happens with the solution as $t\to+\infty$?

$\boldsymbol{a})$ $\ddot{x}-x+x^2=0$ $\boldsymbol{b})$ $\ddot{x}+2x^3=0$ $\boldsymbol{c})$ $\ddot{x}+2x^3-2x=0$ $\boldsymbol{d})$ $\ddot{x}+2^x+x+1=0$

$\boldsymbol{e})$ $\ddot{x}+2\dot{x}+5=0$ $\boldsymbol{f})$ $\ddot{x}+\dot{x}+2x-x^2=0$ $\boldsymbol{g})$ $\ddot{x}+\sqrt{x^2+\dot{x}^2}-1=0$