Simplex Solver

November 11, 2022

Problem

Given the following linear system and objective function, find the optimal solution.

$$\max 150x_1 + 104x_2 + 69x_3 + 167x_4 + x_5$$

$$\begin{cases} 18y_1 + 23y_3 + 167/2y_4 + y_5 \geq 500 \\ 37y_1 + 52y_2 \geq 360 \\ 21y_1 + 52y_2 + 23y_3 + 167/4y_4 + y_5 \geq 400 \\ 75y_1 + y_5 \geq 300 \\ 23y_3 + 167/4y_4 \geq 300 \end{cases}$$

Solution

Add slack variables to turn all inequalities to equalities.

$$\begin{cases} 18y_1 + 37y_2 + 21y_3 + 75y_4 + s_1 = 150 \\ 52y_2 + 52y_3 + s_2 = 104 \\ 23y_1 + 23y_3 + 23y_5 + s_3 = 69 \\ 167/2y_1 + 167/4y_3 + 167/4y_5 + s_4 = 167 \\ y_1 + y_3 + y_4 + s_5 = 1 \end{cases}$$

Create the initial tableau of the new linear system.

y_1	y_2	y_3	y_4	y_5	s_1	s_2	s_3	s_4	s_5	b	
18	37	21	75	0	1	0	0	0	0	150	s_1
0	52	52	0	0	0	1	0	0	0	104	s_2
23	0	23	0	23	0	0	1	0	0	69	s_3
167/2	0	167/4	0	167/4	0	0	0	1	0	167	s_4
1	0	1	1	0	0	0	0	0	1	1	s_5
−500	−360	−400	−300	−300	0	0	0	0	0	0	

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is y_1 and the departing variable is s_5.

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

y_1	y_2	y_3	y_4	y_5	s_1	s_2	s_3	s_4	s_5	b	
0	37	3	57	0	1	0	0	0	-18	132	s_1
0	52	52	0	0	0	1	0	0	0	104	s_2
0	0	0	-23	23	0	0	1	0	-23	46	s_3
0	0	$-167/4$	$-167/2$	$167/4$	0	0	0	1	$-167/2$	$167/2$	s_4
1	0	1	1	0	0	0	0	0	1	1	y_1
0	-360	100	200	-300	0	0	0	0	500	500	

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is y_2 and the departing variable is s_2.

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

y_1	y_2	y_3	y_4	y_5	s_1	s_2	s_3	s_4	s_5	b	
0	0	-34	57	0	1	$-37/52$	0	0	-18	58	s_1
0	1	1	0	0	0	$1/52$	0	0	0	2	y_2
0	0	0	-23	23	0	0	1	0	-23	46	s_3
0	0	$-167/4$	$-167/2$	$167/4$	0	0	0	1	$-167/2$	$167/2$	s_4
1	0	1	1	0	0	0	0	0	1	1	y_1
0	0	460	200	-300	0	$90/13$	0	0	500	1220	

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is y_5 and the departing variable is s_3.

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

y_1	y_2	y_3	y_4	y_5	s_1	s_2	s_3	s_4	s_5	b	
0	0	-34	57	0	1	$-37/52$	0	0	-18	58	s_1
0	1	1	0	0	0	$1/52$	0	0	0	2	y_2
0	0	0	-1	1	0	0	$1/23$	0	-1	2	y_5
0	0	$-167/4$	$-167/4$	0	0	0	$-167/92$	1	$-167/4$	0	s_4
1	0	1	1	0	0	0	0	0	1	1	y_1
0	0	460	-100	0	0	$90/13$	$300/23$	0	200	1820	

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is y_4 and the departing variable is y_1.

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

y_1	y_2	y_3	y_4	y_5	s_1	s_2	s_3	s_4	s_5	b	
-57	0	-91	0	0	1	$-37/52$	0	0	-75	1	s_1
0	1	1	0	0	0	$1/52$	0	0	0	2	y_2
1	0	1	0	1	0	0	$1/23$	0	0	3	y_5
$167/4$	0	0	0	0	0	0	$-167/92$	1	0	$167/4$	s_4
1	0	1	1	0	0	0	0	0	1	1	y_4
100	0	560	0	0	0	$90/13$	$300/23$	0	300	1920	

There are no negative elements in the bottom row, so we know the solution is optimal. Thus, the solution is:

$$s_1 = 1, s_2 = 0, s_3 = 0, s_4 = \frac{167}{4}, s_5 = 0, x_1 = 0, x_2 = \frac{90}{13}, x_3 = \frac{300}{23}, x_4 = 0, x_5 = 300, y_1 = 0, y_2 =$$