Simplex Solver

November 11, 2022

Problem

Given the following linear system and objective function, find the optimal solution.

$$\max 32x_1 + 30x_2 + 35x_3 + 30x_4 + 20x_5$$

$$\begin{cases} 2x_1 + x_2 + 5x_3 + 3x_5 \leq 100 \\ 2x_2 \leq 80 \\ 2x_3 + x_4 \leq 85 \\ 7x_1 + 2x_4 + x_5 \leq 200 \\ 2x_2 + 10x_3 + x_5 \leq 100 \end{cases}$$

Solution

Add slack variables to turn all inequalities to equalities.

$$\begin{cases} 2x_1 + x_2 + 5x_3 + 3x_5 + s_1 = 100 \\ 2x_2 + s_2 = 80 \\ 2x_3 + x_4 + s_3 = 85 \\ 7x_1 + 2x_4 + x_5 + s_4 = 200 \\ 2x_2 + 10x_3 + x_5 + s_5 = 100 \end{cases}$$

Create the initial tableau of the new linear system.

$$\left[\begin{array}{ccccccccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & s_1 & s_2 & s_3 & s_4 & s_5 & b & \\ \hline 2 & 1 & 5 & 0 & 3 & 1 & 0 & 0 & 0 & 0 & 100 & s_1 \\ 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 80 & s_2 \\ 0 & 0 & 2 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 85 & s_3 \\ 7 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 1 & 0 & 200 & s_4 \\ 0 & 2 & 10 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 100 & s_5 \\ \hline -32 & -30 & -35 & -30 & -20 & 0 & 0 & 0 & 0 & 0 & 0 & \end{array}\right]$$

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is x_3 and the departing variable is s_5.

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

x_1	x_2	x_3	x_4	x_5	s_1	s_2	s_3	s_4	s_5	b	
2	0	0	0	5/2	1	0	0	0	−1/2	50	s_1
0	2	0	0	0	0	1	0	0	0	80	s_2
0	−2/5	0	1	−1/5	0	0	1	0	−1/5	65	s_3
7	0	0	2	1	0	0	0	1	0	200	s_4
0	1/5	1	0	1/10	0	0	0	0	1/10	10	x_3
−32	−23	0	−30	−33/2	0	0	0	0	7/2	350	

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is x_1 and the departing variable is s_1.

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

x_1	x_2	x_3	x_4	x_5	s_1	s_2	s_3	s_4	s_5	b	
1	0	0	0	5/4	1/2	0	0	0	−1/4	25	x_1
0	2	0	0	0	0	1	0	0	0	80	s_2
0	−2/5	0	1	−1/5	0	0	1	0	−1/5	65	s_3
0	0	0	2	−31/4	−7/2	0	0	1	7/4	25	s_4
0	1/5	1	0	1/10	0	0	0	0	1/10	10	x_3
0	−23	0	−30	47/2	16	0	0	0	−9/2	1150	

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is x_4 and the departing variable is s_4.

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

x_1	x_2	x_3	x_4	x_5	s_1	s_2	s_3	s_4	s_5	b	
1	0	0	0	5/4	1/2	0	0	0	−1/4	25	x_1
0	2	0	0	0	0	1	0	0	0	80	s_2
0	−2/5	0	0	147/40	7/4	0	1	−1/2	−43/40	105/2	s_3
0	0	0	1	−31/8	−7/4	0	0	1/2	7/8	25/2	x_4
0	1/5	1	0	1/10	0	0	0	0	1/10	10	x_3
0	−23	0	0	−371/4	−73/2	0	0	15	87/4	1525	

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is x_5 and the departing variable is s_3.

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

x_1	x_2	x_3	x_4	x_5	s_1	s_2	s_3	s_4	s_5	b	
1	20/147	0	0	0	−2/21	0	−50/147	25/147	17/147	50/7	x_1
0	2	0	0	0	0	1	0	0	0	80	s_2
0	−16/147	0	0	1	10/21	0	40/147	−20/147	−43/147	100/7	x_5
0	−62/147	0	1	0	2/21	0	155/147	−4/147	−38/147	475/7	x_4
0	31/147	1	0	0	−1/21	0	−4/147	2/147	19/147	60/7	x_3
0	−695/21	0	0	0	23/3	0	530/21	50/21	−113/21	2850	

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is x_2 and the departing variable is s_2.

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

x_1	x_2	x_3	x_4	x_5	s_1	s_2	s_3	s_4	s_5	b	
1	0	0	0	0	−2/21	−10/147	−50/147	25/147	17/147	250/147	x_1
0	1	0	0	0	0	1/2	0	0	0	40	x_2
0	0	0	0	1	10/21	8/147	40/147	−20/147	−43/147	2740/147	x_5
0	0	0	1	0	2/21	31/147	155/147	−4/147	−38/147	12455/147	x_4
0	0	1	0	0	−1/21	−31/294	−4/147	2/147	19/147	20/147	x_3
0	0	0	0	0	23/3	695/42	530/21	50/21	−113/21	87650/21	

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is s_5 and the departing variable is x_3.

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

x_1	x_2	x_3	x_4	x_5	s_1	s_2	s_3	s_4	s_5	b	
1	0	−17/19	0	0	−1/19	1/38	−6/19	3/19	0	30/19	x_1
0	1	0	0	0	0	1/2	0	0	0	40	x_2
0	0	43/19	0	1	7/19	−7/38	4/19	−2/19	0	360/19	x_5
0	0	2	1	0	0	0	1	0	0	85	x_4
0	0	147/19	0	0	−7/19	−31/38	−4/19	2/19	1	20/19	s_5
0	0	791/19	0	0	108/19	231/19	458/19	56/19	0	79410/19	

There are no negative elements in the bottom row, so we know the solution is optimal. Thus, the solution is:

$$s_1 = 0, s_2 = 0, s_3 = 0, s_4 = 0, s_5 = \frac{20}{19}, x_1 = \frac{30}{19}, x_2 = 40, x_3 = 0, x_4 = 85, x_5 = \frac{360}{19}, z = \frac{79410}{19}$$