

A quick note on implied volatility

Applied Stochastic Processes (FIN 514)

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Overview

- The Black-Scholes-Merton (BSM), Bachelier (Normal) models are seldom used to *predict* the option price. The option prices are determined from the supply and demand of market.
- However, the pricing models are still important because they provide a consistent measure to intuitively understand the prices of the options with different strikes, time to maturity, etc.
- **Implied volatility (IV)** is the value of the volatility in pricing model which returns (or solve) the price of an option given (i.e., from market):

$$C(K, S_0, \sigma, T_e) = \text{Price}$$

- How can you compare the two prices?

$$C(K = 105, S_0 = 100, T_e = 1) = 5.9 \quad P(K = 98, S_0 = 100, T_e = 1) = 8.7$$

The implied volatility is same as 20%.

General rule

- Option prices (both call and put) increase as the volatility increases: $C(K, S_0, \sigma, T_e)$ is monotonically increasing function.
- Option value = Time value + Intrinsic Value
 - Intrinsic value: the value you get by exercising option now. (> 0)
 - Time value: the extra value from the change of the underlying price until the expiry. (> 0)
- The intrinsic value can be understood as the option value with $\sigma = 0$, $C(K, S_0, \sigma = 0, T_e)$, hence the minimum value.
- The call option value as $\sigma \rightarrow \infty$:
 - BSM model ($S_T \geq 0$): S_0 . The underlying stock is always worth more than any call option with $K > 0$.
 - Bachelier (Normal) model (S_T can be negative): ∞ . Call option can protect the (infinite) loss.

IV computation

- The computation of IV depends on numerical root-solving method. [Demo]
 - Newton method: using vega,

$$\sigma^{(k+1)} = \sigma^{(k)} - \frac{C(\sigma^{(k)}, \dots) - \text{Price}}{V(\sigma^{(k)})}, \quad V(\sigma) = \frac{\partial C(\sigma, \dots)}{\partial \sigma}$$

- Brent's method: [Demo]
- BSM model:
 - Newton's method with good initial guess (PyFeng package)
 - Let's Be Rational ([Jackel, 2015](#)): Machine epsilon error within two step iterations.
- Normal model:
 - [Choi et al. \(2007\)](#). Numerical Approximation of the Implied Volatility Under Arithmetic Brownian Motion:
Polynomial approximation with error ($< 10^{-9}$)

IV under the Bachelier Model (Choi et al., 2007)

$$\sigma_N = \sqrt{\frac{\pi}{2T}} (2C - \theta(F_0 - K)) h(\eta),$$

where C is the undiscounted price of either call ($\theta = 1$) or for put ($\theta = -1$) option. The function $h(\eta)$ and the argument η are defined in a sequence,

$$h(\eta) = \sqrt{\eta} \frac{\sum_{k=0}^7 a_k \eta^k}{1 + \sum_{k=1}^9 b_k \eta^k}, \quad \eta = \frac{v}{\tanh^{-1}(v)}, \quad \text{and} \quad v = \frac{|F_0 - K|}{2C - \theta(F_0 - K)},$$

with the coefficients,

a_0	=	3.99496	16873	45134	e-1	b_1	=	4.99053	41535	89422	e+1
a_1	=	2.10096	07950	68497	e+1	b_2	=	3.09357	39367	43112	e+1
a_2	=	4.98034	02178	55084	e+1	b_3	=	1.49510	50083	10999	e+3
a_3	=	5.98876	11026	90991	e+2	b_4	=	1.32361	45378	99738	e+3
a_4	=	1.84848	96954	37094	e+3	b_5	=	1.59891	96976	79745	e+4
a_5	=	6.10632	24078	67059	e+3	b_6	=	2.39200	88917	20782	e+4
a_6	=	2.49341	52853	49361	e+4	b_7	=	3.60881	71083	75034	e+3
a_7	=	1.26645	80513	48246	e+4	b_8	=	-2.06771	94864	00926	e+2
						b_9	=	1.17424	05993	06013	e+1.