

Stochastic Finance (FIN 519)

Homework Solutions

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RN and **RV** stand for random number and random variable respectively.

1. **HW 1-1** An RN X follows the gamma distribution with parameters (a, b) . The PDF is given by

$$f(x) = \frac{b^a}{\Gamma(a)} x^{a-1} e^{-bx},$$

where the gamma function $\Gamma(a)$ is defined by

$$\Gamma(a) = \int_0^\infty x^{a-1} e^{-x} dx \quad (a > 0).$$

- (a) If a is a positive integer, show that $\Gamma(a) = (a-1)! = (a-1)(a-2) \cdots 1$.
- (b) Find the MGF of X , that is, $E(e^{tX})$.
- (c) From (b), show that

$$E(X) = \frac{a}{b} \quad \text{and} \quad \text{Var}(X) = \frac{a}{b^2}.$$

- (d) Find the skewness and ex-kurtosis of X .