ECO5002 Introduction to Economics

Lecture 4: The Theory of Consumer Choice

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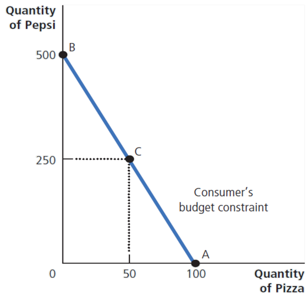
Before Start

- You should have already got a big picture about Economics: it's all about demand and supply.
- How are demand and supply formed? Ad hoc?
- The following two lectures will focus on consumers and firms and show you how to derive demand curve and supply curve.

I. Budget Constraint

- **Budget Constraint** is the the limit on the consumption bundles that a consumer can afford. $|\text{Slope}| = \text{Relative Price}$.

Number of Pizzas	Liters of Pepsi	Spending on Pizza	Spending on Pepsi	Total Spending
100	0	$1,000	$ 0	$1,000
90	50	900	100	1,000
80	100	800	200	1,000
70	150	700	300	1,000
60	200	600	400	1,000
50	250	500	500	1,000
40	300	400	600	1,000
30	350	300	700	1,000
20	400	200	800	1,000
10	450	100	900	1,000
0	500	0	1,000	1,000



- Formally, we indicate the consumption bundle by $(x1, x2)$. So the budget constraint can be written as: $p_1x_1 + p_2x_2 \leq m$ where p denotes price and m denotes income.

I. Budget Constraint

- How the budget line changes?
  - changes in income m: parallel shift outward/inward.
  - changes in relative price p_1/p_2: changes in slope (rotate).
- Consider taxes or subsidies.
  - quantity tax/subsidy: a certain amount for each unit of the good.

$$(p_1 + t)x_1 + p_2 x_2 \leq m$$

  - value tax/subsidy: a percentage term on price.

$$(1 + \tau)p_1 x_1 + p_2 x_2 \leq m$$

  - lump-sum tax/subsidy: fixed amount of money.

$$p_1 x_1 + p_2 x_2 \leq m - T$$

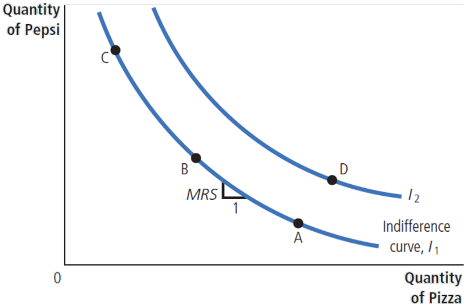
- Consider rationing.
  - If good 1 is rationed, the section of the budget set beyond the rationed quantity will be lopped off.

II. Preference and Utility

- **Preference** is a technical term usually used in relation to choosing between alternatives. For example, someone prefers A over B if they would rather choose A than B.
- Notations for preference:
  - strictly prefer: $(x_1, x_2) \succ (y_1, y_2)$.
  - indifferent: $(x_1, x_2) \sim (y_1, y_2)$.
  - weakly prefer: $(x_1, x_2) \succeq (y_1, y_2)$.
- Assumptions about preference:
  - complete: any two bundles can be compared.
  - reflexive: at least as good as itself.
  - transitive: if $(x_1, x_2) \succeq (y_1, y_2)$ and $(y_1, y_2) \succeq (z_1, z_2)$, then $(x_1, x_2) \succeq (z_1, z_2)$.

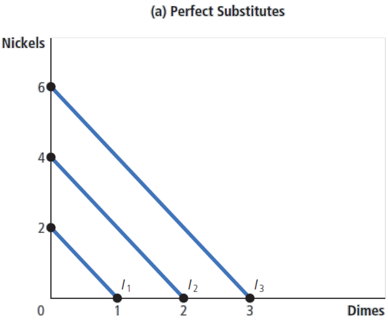
II. Preference and Utility

- We describe preferences graphically by **indifference curves**.
  - bundles that give the consumer the same level of satisfaction.
  - (1) higher indifference curves are preferred to lower ones.
  - (2) indifference curves are downward-sloping.
  - (3) indifference curves cannot cross. (prove by contradiction)
  - (4) indifference curves are bowed inward.
- $|\text{Slope of tangent line}| = \text{Marginal rate of substitution (MRS)}$



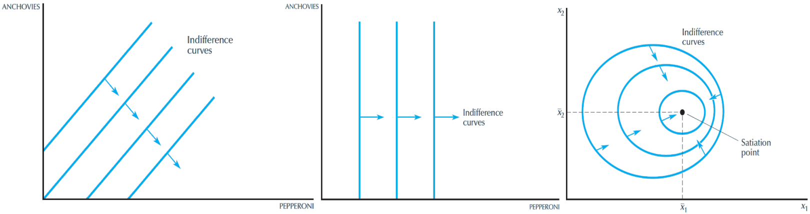
II. Preference and Utility

- Perfect substitutes v.s. Perfect complements.



II. Preference and Utility

- Bads, Neutral Goods, and Satiation.
  - bads: a commodity that the consumer doesn't like.
  - neutral Goods: the consumer doesn't care about it one way or the other.
  - satiation: there is some overall <u>best</u> bundle for the consumer.



II. Preference and Utility

- **Utility** is a way to describe preferences. We can assign a number to every possible consumption bundle such that more-preferred bundles get assigned larger numbers than less-preferred bundles.
- Ordinal v.s. Cardinal: only order matters.
- Some examples:
  - Cobb-Douglas: $u(x_1, x_2) = x_1^a x_2^b$.
  - perfect substitutes: $u(x_1, x_2) = ax_1 + bx_2$.
  - perfect complements: $u(x_1, x_2) = \min\{ax_1, bx_2\}$.
  - quasi-linear: $u(x_1, x_2) = v(x_1) + x_2$.
- **Marginal Utility** is defined as:

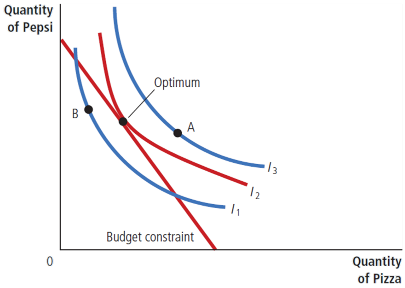
$$MU_1 = \frac{\partial u(x_1, x_2)}{\partial x_1}, \quad MU_2 = \frac{\partial u(x_1, x_2)}{\partial x_2}.$$

- Relation between MU and MRS: $\mathrm{d}x_2/\mathrm{d}x_1 = -MU_1/MU_2$.

III. Optimal Choice

- In general, the consumer chooses consumption of the two goods so that the marginal rate of substitution (MRS) equals the relative price. (Tangent point of BC and IC).

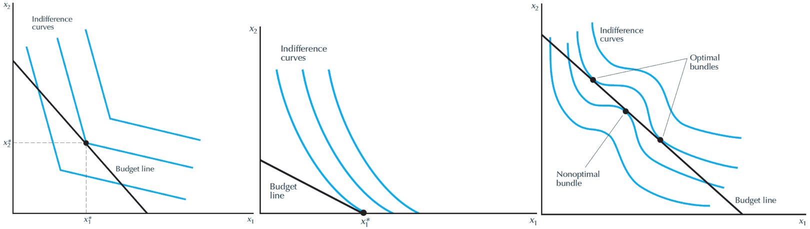
$$|MRS| = \frac{MU_1}{MU_2} = \frac{p_1}{p_2}$$



- What about point A and B?

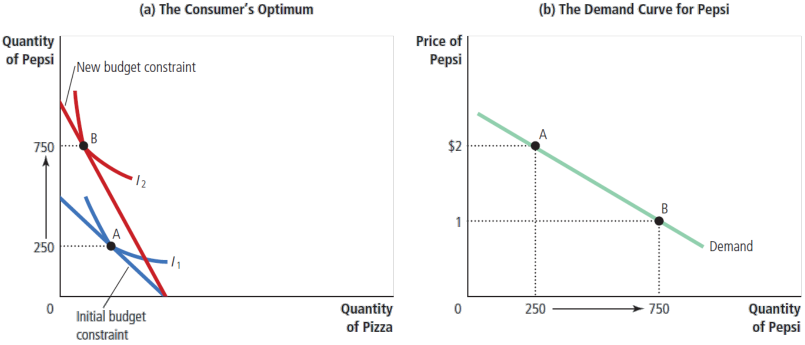
III. Optimal Choice

- Some special cases:
  - kink solution.
  - corner solution.
  - multiple solutions.



III. Optimal Choice

- Deriving the demand curve:
  - suppose that the price of Pepsi falls from \$2 to \$1.
  - the consumer's optimum moves from point A to point B.
  - the quantity of Pepsi consumed rises from 250 to 750 liters.
- Consumers can sometimes violate the law of demand and buy more of a good when the price rises, e.g., Giffen goods.



III. Optimal Choice

- **Example 1**

$$\max_{\{x_1, x_2\}} u(x_1, x_2) = x_1^c x_2^d$$

$$\text{s.t.} \quad p_1 x_1 + p_2 x_2 = m$$

- Step 1: Write down the Lagrangian

$$\mathcal{L} = x_1^c x_2^d - \lambda(p_1 x_1 + p_2 x_2 - m)$$

- Step 2: First-order conditions

$$\frac{\partial \mathcal{L}}{\partial x_1} = c x_1^{c-1} x_2^d - \lambda p_1 = 0$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = d x_1^c x_2^{d-1} - \lambda p_2 = 0$$

III. Optimal Choice

- Step 3: Combine two FOCs

$$\frac{p_1}{p_2} = \frac{cx_2}{dx_1} \quad \Rightarrow \quad x_2 = \frac{p_1}{p_2} \cdot \frac{d}{c} \cdot x_1$$

- Step 4: Plug into budget constraint and get demand functions

$$x_1 = \frac{c}{c+d} \cdot \frac{m}{p_1}$$

$$x_2 = \frac{d}{c+d} \cdot \frac{m}{p_2}$$

  - which reflect the relation between x and p and have downward slopes.

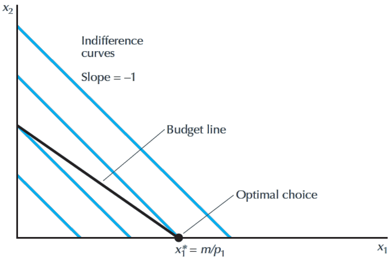
III. Optimal Choice

- **Example 2**

$$\max_{\{x_1, x_2\}} u(x_1, x_2) = x_1 + x_2$$

$$\text{s.t.} \quad p_1 x_1 + p_2 x_2 = m$$

- the slope of IC = -1
- if $p_1 > p_2$, only purchase good 2 (m/p_2).
- if $p_1 < p_2$, only purchase good 1 (m/p_1).
- if $p_1 = p_2$, indifferent.



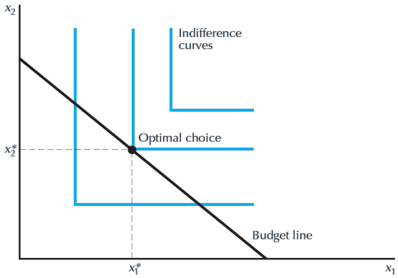
III. Optimal Choice

- **Example 3**

$$\max_{\{x_1,x_2\}} u(x_1,x_2) = \min\{x_1,x_2\}$$

$$\text{s.t.} \quad p_1x_1 + p_2x_2 = m$$

- optimality occurs when $x_1 = x_2$.
- demand functions: $x_1 = x_2 = \frac{m}{p_1+p_2}$.



IV. The Effect of Price Change

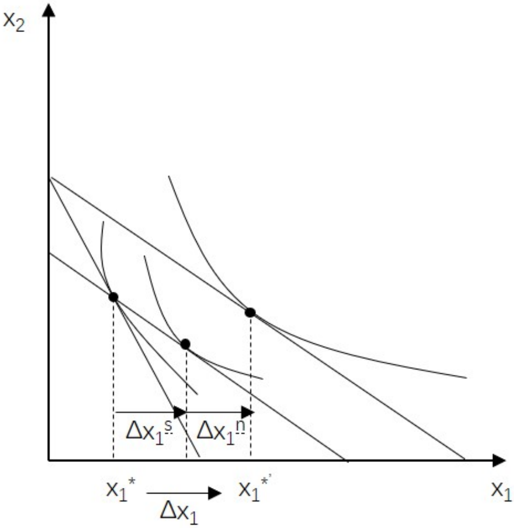
- We can use the trick of price change to draw the Marshallian demand curve: normally, price $\downarrow \Rightarrow$ quantity demanded $\uparrow$.
- Then, let us decompose this price effect into two parts.
- **Substitution effect**
  - if the price of good 1 decreases, $p_1 \downarrow$, but p_2 doesn't change.
  - good 1 becomes relatively cheaper than good 2.
  - a consumer would like to buy more good 1.
- **Income effect**
  - if the price of good 1 decreases, $p_1 \downarrow$, but p_2 doesn't change.
  - it seems that the consumer's income increases
  - a consumer may like to buy more good 1 and good 2.

IV. The Effect of Price Change

Slutsky Decomposition:

- **Total effect**: if the price of good 1 decreases, the quantity demanded for good 1 moves (increases) from x_1^* to $x_1^{*\prime}$.
- Step 1: rotate the original budget constraint parallel to the new one and let it still cross the old consumption bundle. The old bundle is still feasible. This intermediate budget line will be tangent to another indifference curve and the quantity increment is called **substitution effect**, i.e., Δx_1^s.
- Step 2: shift the intermediate budget constraint towards the new position, and the quantity increment is called **income effect**, i.e., Δx_1^n.

IV. The Effect of Price Change

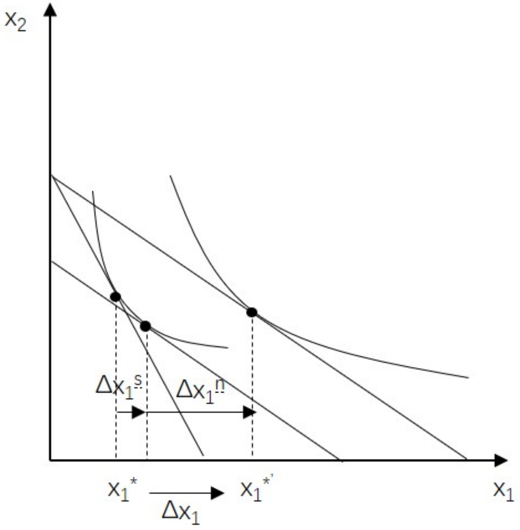


IV. The Effect of Price Change

Hicksian Decomposition:

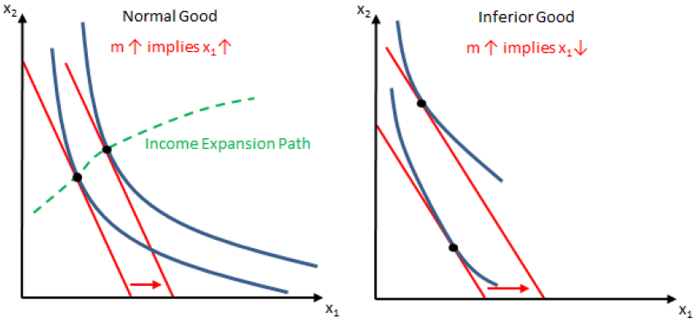
- **Total effect**: if the price of good 1 decreases, the quantity demanded for good 1 moves (increases) from x_1^* to $x_1^{*\prime}$.
- Step 1: rotate the original budget constraint parallel to the new one and let it still be tangent to the old curve. The old bundle is infeasible. The quantity increment is called **substitution effect**, i.e., Δx_1^s.
- Step 2: shift the intermediate budget constraint towards the new position, and the quantity increment is called **income effect**, i.e., Δx_1^n.

IV. The Effect of Price Change



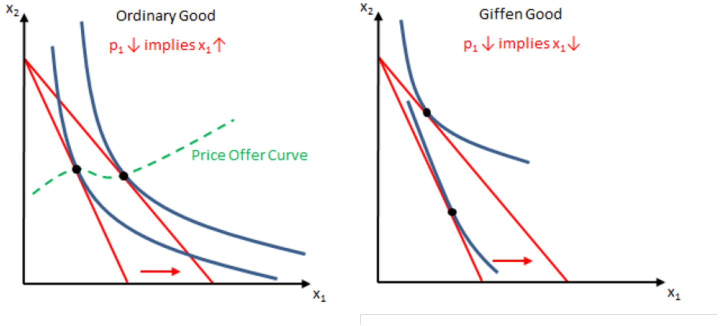
IV. The Effect of Price Change

Normal goods v.s. Inferior goods



IV. The Effect of Price Change

Ordinary goods v.s. Giffen goods



- $\Delta x_1 = \Delta x_1^s + \Delta x_1^n$.
- The substitution effect is always negative: $p_1 \downarrow$, $\Delta x_1^s > 0$.
- The income effect could be positive or negative.
- A Giffen good must be an inferior good. But an inferior good may not be a Giffen good.

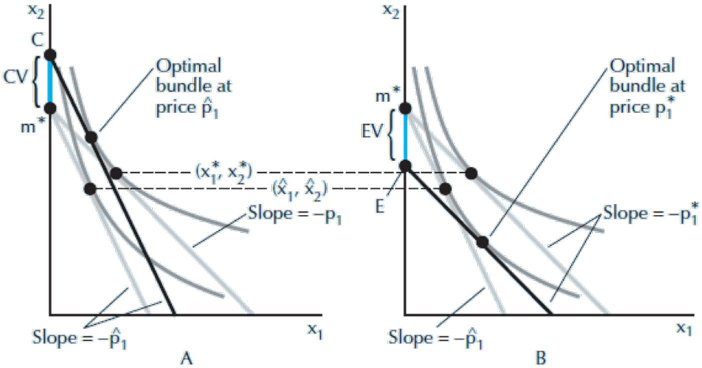
V. Compensating and Equivalent Variation

- Consumer surplus is a way to measure the welfare change when the price changes.

$$CS = \int_{p_1^1}^{p_1^2} x_1(p_1, p_2, m)\mathrm{d}p_1$$

- There are two alternative criteria:
  - **Equivalent Variations**: it is the change in income that the consumer needs at the old prices p_1^1 to be as well off as at the new price p_1^2.
  - **Compensating Variations**: it is the change in income that the consumer needs at the new prices p_1^2 to be as well off as at the old prices p_1^1.

V. Compensating and Equivalent Variation



- Old price: p_1, new price: $\hat{p}_1 > p_1$. Normalize $p_2 = 1$.
- Old optimal bundle: (x_1^*, x_2^*), new optimal bundle: $(\hat{x}_1, \hat{x}_2)$.

Reading

- Chapter 21, *Principles of Economics* by Mankiw.
- Chapter 2 $\sim$ 6, *Intermediate Microeconomics* by Varian.
- Chapter 8 and 14, *Intermediate Microeconomics* by Varian.