Macroprudential Policy with Long-Term Debts*

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Abstract

Fluctuations in long-term credit have important macroeconomic consequences over boom-bust cycles. This paper studies the crisis management when corporate entities can borrow long-term debts. With long-term debts, there exists a dynamic interaction between *ex post* interventions and *ex ante* macroprudential policies. Using the Bianchi (2016) framework, we characterize the optimal time-consistent macroprudential policies and show that the desirability of macroprudential policy could be weakened or strengthened due to a distinct incentive effect of long-term debts. We numerically show that there is a welfare loss of 0.6% if this incentive effect is not accounted for.

JEL classification: E44, G18, H23

Keywords: macroprudential regulation, debt maturity

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1 Introduction

Why is macroprudential policy necessary and how should it be designed? Most of the literature answers this question by focusing on the pecuniary externality when firms borrow short-term debts (e.g., Bianchi (2011); Bianchi and Mendoza (2018)). The discussion of debt maturity in the optimal design of macroprudential policy is largely missing.

However, in US manufacturing firm data, long-term debts account for the majority (over 60%) of firm's total debts. In addition, empirical evidence shows that firms with a larger fraction of long-term debts prespecified to mature right after the onset of the financial crisis cut spending more than otherwise similar firms (Almeida et al., 2011). The above facts naturally raise the question regarding whether long-term debts increase the desirability of macroprudential policy. What is the interaction between *ex ante* intervention and the *ex post* intervention in the presence of long-term debts? How should optimal macroprudential policies be designed?

In this paper, we present stylized facts on the prevalence and the distinct features of the long-term borrowing. We find that, for manufacturing firms, long-term debts account for more than 65% of the total debts and 30% of the total liabilities, and that long-term debt ratio declines in economic downturns. More importantly, we find that unlike short-term debts, U.S. firms' long-term debt holding decreases in anticipation to expected future interest rate decline. In addition, we find that such decline is more pronounced for firms that are more financially constrained. The fact that firms seem to defer long-term borrowing in response to future decrease in interest rate is interesting.

To explain these facts, we extend the small-open-economy (SOE) framework of the Bianchi (2016) to allow for long-term borrowing. In the model, productive firms finance labor hiring and capital investment by using an individually optimal combination of equity and long-term debts. In doing so, they are fully aware that their investment increases long-term borrowing and restricts the future short-term debt capacity. Nevertheless, they fail to recognize that their investment will collectively raise the general equilibrium factor price (wage externality and capital price externality). Increased factor prices prompt all firms to borrow more long-term debts, which in turn reduces the future debt capacity and eventually leads to economy-wide distress for the corporate sector. The complementarity of capital and labor in firms' production function implies that the two externalities reinforce one another, forming a joint force that results in inefficient credit cycles that demand policy interventions. In other words, it is the pre-committed repayment that squeezes firms' future debt capacity by a micro-founded collateral constraint. The reductions in the debt capacity depress real activities when the economy is hit by an adverse shock.

We show how debt maturity plays a crucial role in the moral hazard debate of crisis management. The conventional wisdom that *ex post* interventions always induce *ex ante* risk taking applies only when debt is short-term. If debt maturity is longer than one period, there exists an distinct incentive effect: future liquidity provision in the form of monetary stimulus disciplines *ex ante* borrowing incentives of private agents, and future macroprudential policies encourage *ex ante* borrowing.

We characterize the corrective policies based on the dynamic model, and show they admit a state-dependent, sufficient-statistic-like formula. We analytically decompose the prudential intervention into three components. First, there is a static term represents the contemporaneous effect of price on the current financing constraint; second, a dynamic component captures how the current investment affects the future capital price and future financial constraint through capital accumulation; and third, a wedge captures the interaction of debt maturity and private agents' action toward *ex post* interventions.

We numerically illustrate the mechanism of the dynamic model in a parsimonious calibration by solving the model using the global method and conduct the following exercises. First, we compare the policy rules for the labor and investment choice between the competitive equilibrium and the constrained efficiency. We find that, in normal times, the social planner would choose a lower level of investment. Then, we show that our model is able to capture the "displinary" effect in both credit tightening and credit loosening periods. Further, we simulate our model and find that the optimal macroprudential policy can suppress the *ex ante* borrowing and investment incentives. In addition, a crisis experiment shows that the model's internal propagation mechanism is able to generate typical crisis dynamics. Under these prudential interventions, investment drops by 20% less. The optimal policy eventually induces a $0.6\% \sim 0.7\%$ welfare gain. If the incentive effect of long-term debt were not considered, the welfare gain would be less than 0.1%.

Literature. This paper connects to two strands of the literature. First and foremost, it relates to the literature on pecuniary externalities with financial frictions. Since the work of pioneering work of Mendoza (2010) and Bianchi (2011), the literature has mainly focused on the debt deflation mechanism and the pecuniary externality arising from the fact that the borrowing capacity is tied to the value of domestic output. While most of the literature along this line has focused on the efficiency properties of a competitive equilibrium when firms borrow short-term debts, the investigation of long-term debts is largely missing. As most of the debts in the manufacturing sectors are long-term, long-term debts are shown to be important to the understanding of the real impact of financal crisis (Miao and Wang, 2010). Some papers explicitly analyze long-term debts. Korinek (2018) analyze a general liability structure with state-contingent payoffs and establish a mapping between competitive equilibrium under arbitrary maturity structure and that under short-term debts. Under longer maturities, the externality kernel converges toward an ergodic steady state that describes the long-run externalities in the economy. In another paper, Jeanne and Korinek (2018) shows, under long-term debts with collateral constraint, the usual debt deflation dynamics in the pecuniary models are mitigated. Our contribution is the characterization of the distinct incentive effect when debt maturity is longer than one period, and how this effect plays a role in the optimal design of prudential intervention.

Our paper adds to the recent theoretical literature that analyzes the interactions between *ex post* interventions and *ex ante* risk-taking incentives. The key to this interaction is the *ex ante* incentive effects of the *ex post* liquidity provision. Jeanne and Korinek (2020) systematically study the optimal mix of both *ex ante* regulations and *ex post* interventions based on a tractable model with fire-sale externality. They show the effects of more generous liquidity provision on the optimal macroprudential tax are actually ambiguous. On one hand, the liquidity provision increases *ex ante* risk-taking; on the other hand, it reduces private agents' vulnerability to crises and therefore makes it efficient for private agents to take on greater leverage. There are also papers focusing on specific aspects of the interaction between *ex ante* and *ex post* interventions. Liu, Wang and Yang (2024) studies a clean continuous-time model with strategic asset sales. They show that financial crisis is delayed due to the negative externality when individual banks wait to liquidate assets, and the dynamic interactions of the externality imply inefficiency could have different signs. While they emphasize the strategic interactions of agents, we focus on the maturity dimension of the debt and its dynamic policy implications. In addition, our paper is also related to Dong and Xu (2022) who study the trade-offs between the *ex ante* and *ex post* intervention when assset bubbles endogenously arise in an infinitely horizon production environment. Compared to their paper, we highlight the distinct incentive role of collateralized debts of longer maturity.

Benigno et al. (2016) provide an integrated analysis of a wide spectrum of policy tools, including fiscal, monetary, and macroprudential policies, based on the framework of Mendoza (2010). They show that when the exchange rate policy is costless, the desirability of capital controls is reduced. Bornstein and Lorenzoni (2017) challenge the moral hazard view with a clean three-period model with aggregate demand externalities. They identify the conditions under which aggregate demand externalities are completely corrected by using a countercyclical interest rate policy. The policy implication obtained in these papers is close to ours, in that optimally designed crisis management provides a stabilizing effect without distorting *ex ante* incentives. Similar to the above work, the pecuniary externality in our model is also induced by a price-based collateral constraint. The important difference is that we focus on the maturity aspect of the interaction between crisis management and prudential regulations.

Our paper also relates to the strand of literature studying the macroeconomic consequences of long-term debts. Miao and Wang (2010) study a real business cycle model where corporates borrows long-term debts. They show how credit risk amplifies aggregate technology shocks. Our modeling of long-term debts is similar to theirs. The main difference is the analysis of efficiency properties and the characterization of optimal time-consistent policies.

The remainder of this paper is structured as follows. Section 2 presents data and stylized facts that motivate our theoretical discussion. Section 3 presents the dynamic model where we characterize the equilibrium and discuss policy implications. Section 4 shows a quantitative and welfare analysis. Section 5 concludes the paper.

2 Stylized Facts

In this section, we present stylized facts that motivate our later theoretical analysis. Specifically, we show that for US manufacturing firms, long-term debt holding accounts for a major fraction, and their incentive of borrowing long-term debts is different from that of short-term debts in response to future interest rate expectations. We first briefly describe the data and then show the results in later subsections.

2.1 Data Descriptions

Our main dataset is the Compustat Quarterly North America database, which provides rich accounting information on firms' debt holdings for different maturities. In the baseline sample, we restrict our attention to US manufacturing firms from the first quarter of 1986 to the fourth quarter of 2019.[1] The sample selection follows Poeschl (2023), and the units of our analysis are firm-quarter pairs.

As our empirical exercise involves the investigation of how debt responds to expected future cost of issuing debts, we merge the Compustat database with the average price of federal funds futures from the Datastream Future Contract database.[2] Specifically, we define the quarterly interest rate expectation as 100 minus the average price of all federal funds future contracts that are traded in the current quarter and will be settled in the next quarter, and

[1] We choose the sample period from 1986 ot 2019 for the reason that observations before 1986 are limited and the data quality is not satisfactory.

[2] According to Gürkaynak, Sack and Swanson (2007), the federal funds futures outperform the other securities in measuring monetary policy expectations.

take the first difference of this expectation series for stationarity. Detailed data description is relegated to Appendix A.1 and A.2.

2.2 Empirical Results

As a starting point, we first show the time series of average long-term debt share of US manufacturing firms, together with the total nonfinancial corporate debt growth rate and the nominal GDP growth rate in Figure 1.[3] We define a firm's long-term debt share as the ratio of its long-term debt holdings to its total debt holdings. Figure 1 suggests two messages. First, long-term borrowing is important in terms of magnitude for manufacturing firms since, on average, long-term debts account for a substantial fraction (about 65%) of total debts for the past three decades, and the ratio continues to grow recently.[4]

Second, the share of long-term debts over total debts drops in recession episodes. For example, during the 2008 financial crisis, the growth rate of total corporate debts of the manufacturing sector decreased 3% $\sim$ 4%, with long-term debts dropping further (i.e., the average long-term debt ratio over total debts also fell about 3%). Fact 1 concludes our findings here.

Fact 1. *On average, long-term debts of manufacturing firms account for more than* 65% *of the total debts (long-term plus short-term) and have larger declines during economic recessions than short-term debts.*

In what follows, we investigate the incentive effects separately for both long-term and short-term debts. In particular, we are interested in how long-term debt growth responds to expected future interest rates which measure the cost of future borrowing and how this response is different from that of short-term debt growth.

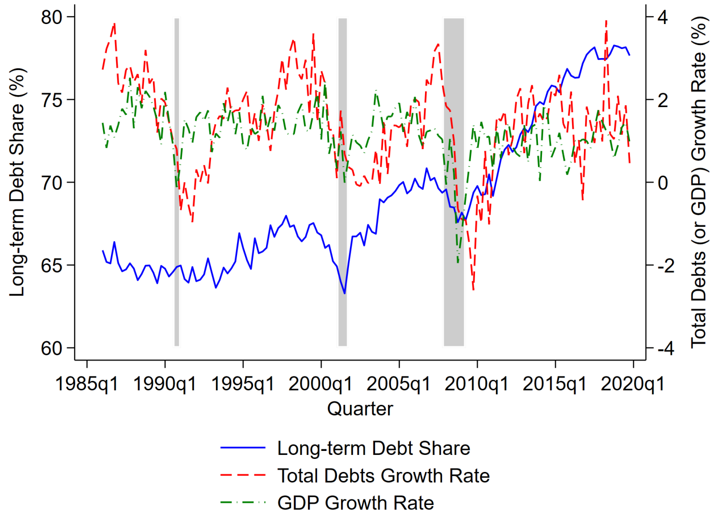
To do this, we use the binscatter method to plot the relationship between the change in interest rate expectation and the long-term (and short-term) debt growth rate.[5] In doing so, we control for fixed effects of firms to address the potential concern of omitted firm-specific time-invariant characteristics.

[3] Instead of using Compustat, we use the term *Nonfinancial Corporate Business: Debt Securities and Loans* from Federal Reserve Economic Data (FRED) to calculate total debt growth rate. The nominal GDP data is also from FRED. The formula for computing growth rate is $h_t \equiv (x_t - x_{t-1})/x_{t-1}$ where x_t denotes the total debts or nominal GDP at time t.

[4] For robustness, we use the ratio of long-term debts to total liabilities to proxy long-term debt share and the result can be found in Figure A.2.

[5] Using the quarterly data, we define firms' long-term (short-term) debt growth rate by $g_{i,t} \equiv 2(d_{i,t} - d_{i,t-1})/(d_{i,t} + d_{i,t-1})$ where $d_{i,t}$ denotes total long-term debts (*dlttq*) or current debts (*dlcq*) of firm i in quarter t. This measure of growth rate has some attractive advantages and is widely used in the literature (Davis, Haltiwanger and Schuh, 1998; Cravino and Levchenko, 2017).

Figure 1: U.S. Manufacturing Corporate Debts



Note: This figure shows the time series of average long-term debt share of US manufacturing firms, US nonfinancial corporate debts growth rate, and US nominal GDP growth rate from 1986q1 to 2019q4. Shaded areas denote recessions adopted from NBER recession series.

Figure 2 reports the results. Panel A exhibits a positive relationship between long-term debt growth and the change in expected future interest rates. The result shows that a 1% decline in expected interest rate change is associated with a 1.24% decline in long-term debt growth. The positive correlation seems to be at odds with the conventional moral hazard narrative, where firms should borrow more, instead of less, in response to a favorable future interest rate. The pattern of the short-term debts seems to be weakly consistent with this standard intuition (i.e., the conventional moral hazard narrative), as we observe a slightly negative relationship in panel B.

Furthermore, we separately look at two different groups of firms, one financially constrained and the other not. For our baseline analysis, we follow Kaplan and Zingales (1997) and Lamont, Polk and Saaá-Requejo (2001) to compute each firm's annual Kaplan-Zingales (KZ) index. We sort this index from high to low by year and define a firm that is in the top 20% in the previous year as a financially constrained firm.[6] As shown in the red line in panel C, this correlation is more profound for financially constrained firms. Quantitatively, a 1% decline in interest rate expectation change will lead to a 2.68% decrease in the long-term debt growth rate. However, the pattern is less clear for short-term debts. In panel D, the

[6]Lamont, Polk and Saaá-Requejo (2001) use tertiles as the cutoff points and assign three values to this indicator, i.e., 1 =financially constrained, −1 =not financially constrained, 0 =otherwise. We do not exactly follow them since a two-value dummy makes our results more interpretable.

correlation between these two variables are mildly positive and not significant.[7]

Next, we conduct a regression analysis using the standard panel regression model with a set of firm-level controls. The purpose of this exercise is to further address the concern of omitted variables that may bring endogeneity and contaminate our previous estimates. The specific model is as follows

$$g_{i,t} = \beta_0 + \beta_1 \Delta e_t + \beta_2 f_{i,t} + \beta_3 \Delta e_t \times f_{i,t} + \boldsymbol{\gamma}' \boldsymbol{x}_{i,t} + \eta_i + \epsilon_{i,t}, \tag{1}$$

where $g_{i,t}$ denotes the growth rate of long-term (or short-term) debts; Δe_t denotes the change in future interest rate expectation; $f_{i,t}$ denotes a dummy indicating whether a firm is financially constrained; $\Delta e_t \times f_{i,t}$ denotes the interaction term that captures the heterogeneous effects; $\boldsymbol{x}_{i,t}$ denotes a vector of potential control variables, including time-variant firm characteristics, the lagged growth rate $g_{i,t-1}$, and the contemporaneous monetary policy shock; η_i denotes the firm i's fixed effect, and $\epsilon_{i,t}$ denotes the error term.[8] In the above regression, the standard errors are clustered at the firm level.

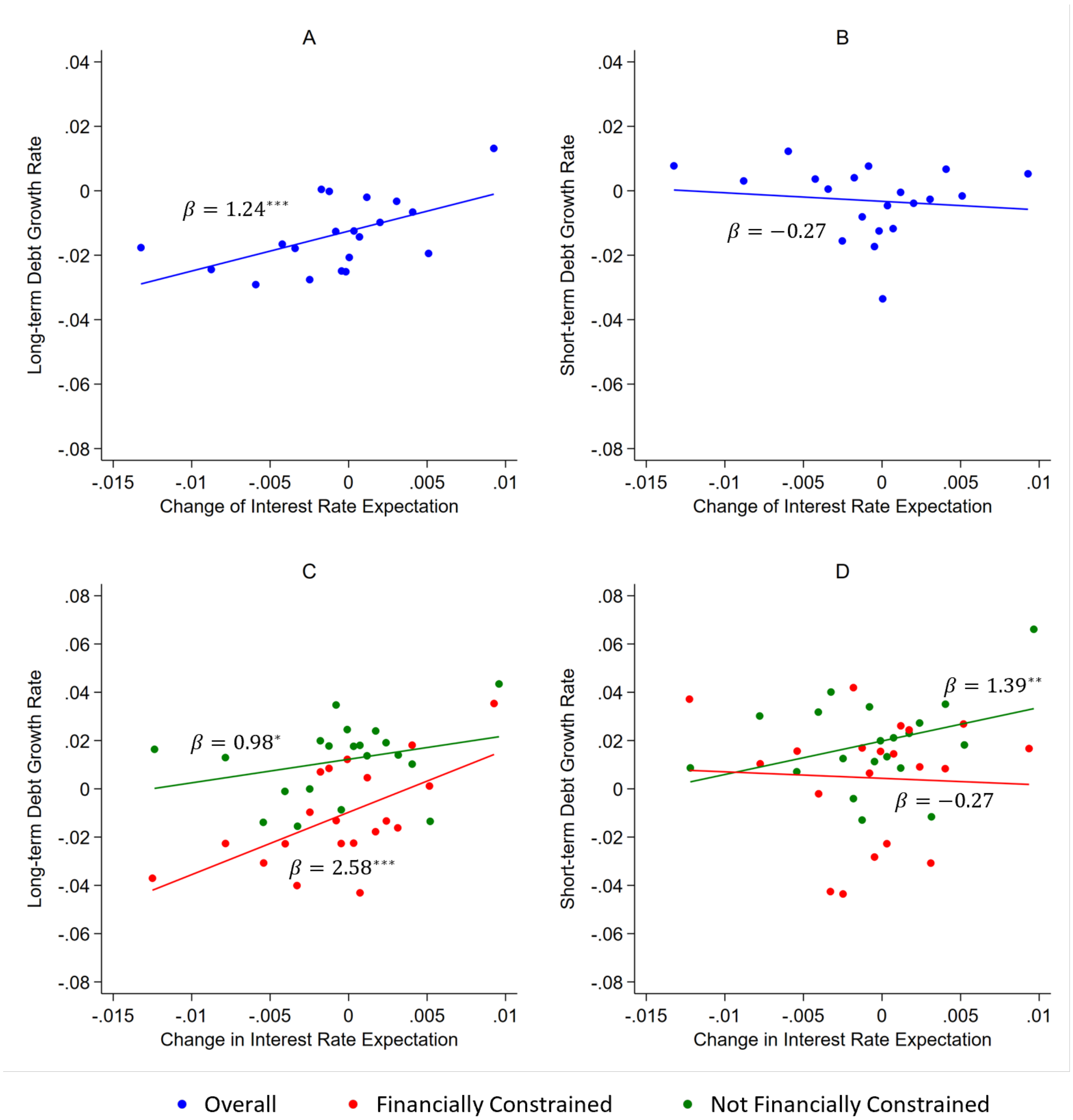
Table 1 shows the results. With different control variables, column (1)$\sim$(3) of panel A present the estimates of the overall effect of changes in interest rate expectation on the long-term debt growth rate. The coefficients are all significantly positive and the magnitude (1.43% $\sim$ 2.17%) is consistent with that in panel A of Figure 2. Column (4)$\sim$(6) of panel A incorporate the financial constraint dummy and the interaction term. The overall effect is slightly weakened but the heterogeneous effect for financially constrained firms is significantly positive and its magnitude is even larger than that in panel C of Figure 2, which quantitatively implies that the long-term debt growth rate of these firms declines more than 3% when their changes in expectations fell 1%. Panel B presents the estimates of the effects on short-term debt growth rate to echo with panel B and D in Figure 2. The overall effect of expectation change is still ambiguous but the heterogeneous effects for financially constrained firms are negative, although they are not significant. Fact summarizes our findings mentioned above.

Fact 2. *Expected future interest rate decline is associated with the decrease in long-term debt growth, and this fact is more pronounced if firms are more financially constrained.*

[7]In Figure A.3, we also provide raw scatter plots where variables are taken average on quarterly level and firms' fixed effects are not considered. The results are consistent.

[8]Specifically, the control variables include firm size, i.e., log total assets, leverage ratio, cash ratio, and sales ratio. Appendix A.3 provides the distribution of $g_{i,t}$ and the summary statistics for these control variables. We control the lagged growth rate since the growth rate may have persistency. We also consider monetary shocks which are directly borrowed from Bu, Rogers and Wu (2021) for controlling contemporaneous interest rates that may be correlated with the error term.

Figure 2: Changes in Interest Rate Expectation and Debt Growth Rate (Binscatters)



Note: This figure consists of four binscatter plots and displays the correlation between the change in interest rate expectation and the long-term (short-term) debt growth rate of US manufacturing firms. Panel A and B show the results of the overall sample. Panel C and D show the results of financially constrained firms and financially non-constrained firms. Firms' fixed effects are controlled. βs are the coefficients of corresponding panel regressions with firm fixed effects. ***, **, and * denote 1%, 5%, and 10% statistical significance, respectively.

Table 1: Changes in Interest Rate Expectation and Debt Growth Rate (Regressions)

	A. Long-term Debt Growth Rate, $g_{i,t}$					
	(1)	(2)	(3)	(4)	(5)	(6)
Δe_t	1.484***	1.433***	2.170***	1.051**	1.016**	1.605**
	(0.264)	(0.243)	(0.317)	(0.513)	(0.465)	(0.625)
$f_{i,t}$				-0.104***	-0.096***	-0.097***
				(0.010)	(0.010)	(0.012)
$\Delta e_t \times f_{i,t}$				2.387**	2.291**	2.102
				(1.076)	(1.002)	(1.285)
Constant	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes
Firm Characteristics	Yes	Yes	Yes	Yes	Yes	Yes
Lagged Growth Rate	No	Yes	Yes	No	Yes	Yes
MP Shocks	No	No	Yes	No	No	Yes
Observations	181,165	175,441	135,608	49,599	48,769	38,267
Clusters	5,553	5,447	4,754	2,907	2,878	2,512
R^2	0.039	0.072	0.078	0.089	0.129	0.137
	B. Short-term Debt Growth Rate, $g_{i,t}$					
	(1)	(2)	(3)	(4)	(5)	(6)
Δe_t	0.217	-0.136	-0.439	1.610**	1.274*	0.852
	(0.327)	(0.322)	(0.412)	(0.703)	(0.682)	(0.874)
$f_{i,t}$				-0.110***	-0.092***	-0.087***
				(0.012)	(0.012)	(0.014)
$\Delta e_t \times f_{i,t}$				-1.019	-0.413	-0.985
				(1.511)	(1.506)	(1.855)
Constant	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes
Firm Characteristics	Yes	Yes	Yes	Yes	Yes	Yes
Lagged Growth Rate	No	Yes	Yes	No	Yes	Yes
MP Shocks	No	No	Yes	No	No	Yes
Observations	175,878	157,158	120,559	48,726	44,990	35,189
Clusters	5,589	5,242	4,565	2,955	2,789	2,430
R^2	0.028	0.033	0.037	0.061	0.062	0.066

Note: This table shows the estimation results of the specification (1) by using OLS. The dependent variable of panel A is long-term debt growth rate. The dependent variable of panel B is short-term debt growth rate. The standard deviations are clustered on firm level. ***, **, and * denote 1%, 5%, and 10% statistical significance, respectively.

2.3 Robustness

In this subsection, we check the robustness to address concerns regarding the above baseline analysis. In what follows, we provide discussions on each of the robustness exercises and relegate results in the Appendix A.4.

Alternative expectation measure. The Michigan's Survey of Consumers (MSC) provides another commonly used measurement on interest rate expectations. We check whether our results are robust to this alternative measures of interest rate expectation. Table A.2 shows the regression results. In panel A, the pure effect of interest rate expectation on long-term debt growth rate is still significantly positive, and it seems to be driven by only financially constrained firms. In panel B, although short-term debt growth rate is weakly positively correlated with interest rate expectation, the heterogeneous effect for financially constrained firms is not significant. The economic magnitude in this case is not comparable to that in the benchmark due to the metric difference, but the direction and the statistical significance of the coefficients are basically consistent with the benchmark results.

Alternative measure of financial constraints. In the literature, a concern of KZ index is that it may not reflect firms' true financial status (Hadlock and Pierce, 2010; Farre-Mensa and Ljungqvist, 2016). Instead, Hadlock and Pierce (2010) find the simpler firm size measure is highly correlated with financial conditions, with larger firms less likely to be constrained. The regression results are in Table A.3 where we use firm size to proxy financial constraints. The pure effects of expected interest rate change are stable. For long-term debts, the coefficient in front of the interaction term is significantly negative, while for short-term debts, the heterogeneous effect is not strongly significant.

Sorting KZ index by industry and year. Whether firms may face financial constraints varies from sector to sector (Rajan and Zingales, 1998). We reconstruct the financial constraint dummy variable by sorting KZ index by both industry and year. Consistent regression results are listed in Table A.4 and our findings are still robust.

All non-financial firms in Compustat. To alleviate the concerns of the sample selection, we now focus on a larger firm sample where only financial firms, regulated firms and non-profit firms are excluded. Similar regressions are conducted and we find that the results shown in Table A.5 are robust as well.

3 Dynamic Model

This section introduces long-term debt to the Bianchi (2016) framework and studies how the existence of long-term debts affects the optimal design of macroprudential policy.

3.1 Model Setting

The model is in a small-open-economy (SOE) setting, where the price of long-term debts exogenously fluctuates. The model consists of production firms, households, and capital producers. Production firms purchase both capital and labor to produce final consumption goods. Capital is supplied in a competitive market by capital producers, and labor is supplied by households. In the model, only capital producers have access to the technology for converting consumption to investment goods.

Households are the ultimate owners of capital producers, and they also own production firms through equity contracts.[9] In what follows, we describe the problems of households, capital producers, and production firms in turn.

Households. The representative household chooses consumption c_t, and purchases equity s_{t+1}, to maximize their lifetime utility from Greenwood, Hercowitz and Huffman (1988). Specifically, the maximization problem is given by

$$\max_{\{c_t, n_t, s_{t+1}\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u \left(c_t - \chi \frac{n_t^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \right) \right\}, \tag{2}$$

where $u(\cdot)$ is the flow utility, n_t is the labor supply, and γ measures the Frisch elasticity. The maximization problem is subject to the following flow-of-funds constraint

$$c_t + p_t s_{t+1} = W_t n_t + (d_t + p_t) s_t + \pi_t, \tag{3}$$

where s_t is the share of equity holding in production firms, d_t is the dividend payment per share, p_t is the equity price, and π_t is the profit from capital producers which we describe momentarily. Following Bianchi (2016), we assume that households do not have access to

[9] This setting is particularly convenient. As the households sector is the only owner of the economy, there is no issue of Pareto weighting across agents in the social planner's program.

the bond market. The optimization condition on c_t, n_t, and s_{t+1} yields

$$
\begin{aligned}
c_t: &\quad u'_{c_t} = \lambda_t, && (4)\\
n_t: &\quad W_t = \chi n_t^{\frac{1}{\gamma}}, && (5)\\
s_{t+1}: &\quad p_t = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(d_{t+1} + p_{t+1}\right) \right\}. && (6)
\end{aligned}
$$

The first equation equates marginal utility with the shadow value of households' wealth; the second equation describes the labor supply; and the third equation represents the equity holding decision.

Capital producers. Capital producers supply capital period-by-period. Their problem is static,

$$\pi_t \equiv \max_{i_t} \left\{ P_t i_t - G\left(i_t\right) \right\}, \tag{7}$$

where i_t denotes quantities for investment good producers, and $G\left(i_t\right)$ measures the cost of capital production that satisfies $G(0) = 0$, $G'(i_t) > 0$, $G'(0) = 0$, and $G''(i_t) > 0$. The first-order condition on i_t yields

$$P_t = G'\left(i_t\right). \tag{8}$$

Production firms. Production firms are representative and of unit measure. They operate a constant return to scale technology that combines capital and labor to produce consumption goods,

$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha}, \tag{9}$$

where capital k_t is pre-determined, and z_t represents the time-varying aggregate productivity.

The timeline is as follows. A firm starts in period t with k_t units of pre-determined capital stock and b_t units of long-term debt (in quantities). It then hires labor to produce consumption goods. After production, the firm pays dividends d_t, purchases investment i_t from capital producers at the competitive market price P_t, and issues non-state-contingent long-term debt l_t.

The repayment schedule of the long-term debt follows Gomes, Jermann and Schmid (2016); Bianchi, Hatchondo and Martinez (2018) and is given as follows. The term b_t represents the amount of outstanding debt at the beginning of period t. A fraction, λ, of the principal is repaid in every period, while the remainder, $(1 - \lambda)$, is left outstanding, implying that this debt has an average maturity of $1/\lambda$.[10] In addition to principal amortization, the firm is also required to pay a periodic coupon payment of ξ per unit of outstanding debt.

[10]In order to save notations, we abuse λ to denote the debt maturity.

The debt accumulation equation is given by

$$b_{t+1} = (1 - \lambda)\, b_t + l_t, \tag{10}$$

where l_t denotes new issuance. The firm's capital accumulation equation is given by

$$k_{t+1} = (1 - \delta)\, k_t + i_t, \tag{11}$$

where δ is the depreciation rate. Overall, a firm's flow-of-funds constraint is given by

$$d_t + P_t i_t = z_t k_t^{\alpha} n_t^{1-\alpha} - W_t n_t - (\xi + \lambda)\, b_t + Q_t l_t, \tag{12}$$

where d_t is the dividend repayment, $P_t i_t$ represents capital expenditure, P_t is the capital price, W_t is the wage rate, $(\xi + \lambda)$ is the coupon and principal repayment, and Q_t is a time-varying bond price.[11]

Firm financing decisions are subject to occasionally binding constraints. First, bond issuance is subject to a collateral constraint,

$$Q_t b_{t+1} \leq \theta_t k_{t+1}, \tag{13}$$

which states that the total debt issuance cannot exceed the fraction, θ_t, of the capital holding. Here, fluctuations in θ_t measure the credit conditions (Jermann and Quadrini, 2012). By equation (10), this constraint can be equivalently written as

$$Q_t l_t \leq \theta_t k_{t+1} - Q_t\, (1 - \lambda)\, b_t, \tag{14}$$

so that a higher long-term debt b_t, ceteris paribus, constrains a firm's ability to issue new debt. Note that in this inequality, new issuance guarantees that the constraint is satisfied every period so that firms find no incentive to default.

In addition to the above constraint, we follow Bianchi (2016) to assume equity issuance is also subject to the following constraint

$$d_t \geq \bar{d}, \tag{15}$$

which states that firms' dividend payments need to satisfy a lower bound. A special case is $\bar{d} = 0$, where it becomes the usual limited liability constraint.[12]

[11] We assume $\beta[(\xi + \lambda) + (1 - \lambda)Q_t] < Q_t$ to generate the persistent motive for borrowing.

[12] The microfoundation for the case in which $d > 0$ follows the classical corporate finance literature, which documents the agency friction nonpecuniary benefit associated with paying dividends to shareholders due to

Let $V(k_t, b_t, \boldsymbol{x}_t)$ denote the cum-dividend market firm value for a firm with three groups of states $\{k_t, b_t, \boldsymbol{x}_t\}$, where $\boldsymbol{x}_t = \{z_t, \theta_t, Q_t\}$ denotes the exogenous shock states. The optimization program can be written as

$$V(k_t, b_t, \boldsymbol{x}_t) = \max_{\{c_t, i_t, n_t, l_t, k_{t+1}, b_{t+1}\}} \left\{ d_t + \beta E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} V(k_{t+1}, b_{t+1}, \boldsymbol{x}_{t+1}) \right] \right\}, \tag{16}$$

where $\Lambda_t = \lambda_t$ is the aggregate household's marginal utility. The associated constraints are

$$(\varsigma_t): \quad d_t - Q_t l_t + P_t i_t = z_t k_t^{\alpha} n_t^{1-\alpha} - W_t n_t - (\xi + \lambda) b_t, \tag{17}$$

$$(\varrho_t): \quad k_{t+1} = (1 - \delta) k_t + i_t, \tag{18}$$

$$(\vartheta_t): \quad b_{t+1} = (1 - \lambda) b_t + l_t, \tag{19}$$

$$(\mu_t): \quad Q_t l_t \leq \theta_t k_{t+1} - Q_t (1 - \lambda) b_t, \tag{20}$$

$$(\psi_t): \quad d_t \geq \bar{d}, \tag{21}$$

where equation (17) is the budget constraint, equation (18) is the capital accumulation, equation (19) is the debt accumulation, equation (20) is the collateral constraint, and equation (21) is the equity constraint. Let ς_t, ϱ_t, ϑ_t, μ_t, and ψ_t denote the multipliers associated with constraints (17) to (21). Taking the first-order conditions on $\{c_t, i_t, n_t, l_t, k_{t+1}, b_{t+1}\}$ yields

$$d_t: \quad 1 + \psi_t = \varsigma_t, \tag{22}$$

$$n_t: \quad (1 - \alpha) z_t k_t^{\alpha} n_t^{-\alpha} = W_t, \tag{23}$$

$$i_t: \quad -\varsigma_t P_t + \varrho_t = 0, \tag{24}$$

$$l_t: \quad \varsigma_t Q_t = \vartheta_t, \tag{25}$$

$$k_{t+1}: \quad \varrho_t = \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \left[\varsigma_{t+1} \alpha \frac{y_{t+1}}{k_{t+1}} + (1 - \delta) (\varsigma_{t+1} P_{t+1}) \right] \right\} + \theta_t \mu_t, \tag{26}$$

$$b_{t+1}: \quad \vartheta_t = \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \varsigma_{t+1} \left[(\xi + \lambda) + (1 - \lambda) Q_{t+1} \right] \right\} + Q_t \mu_t, \tag{27}$$

where equation (22) states that, because of the presence of the equity constraint, the value of increasing a firm's beginning-of-period wealth by \$1 is (weakly) larger than one; equation (23) represents labor demand; equations (24) and (25) equalize the benefit of investment and debt issuance to their costs; and, finally, equations (26) and (27) characterize the optimal

information friction (Lintner, 1956; Kalay, 1980; Brickley, 1983; Ambarish, John and Williams, 1987; Brav et al., 2005). Our formulation of an *ad hoc* equity constraint follows that of Bianchi (2016). Although our formulation comes at the cost of abstracting away from richer settings, where shocks or policies may affect the nature of equity constraints, it nevertheless delivers analytical tractability, which will be valuable for our later analysis.

demand for the next period's capital and debt, conditional on their prices. In addition, two complementary slackness conditions are associated with constraints (20) and (21),

$$\mu_t \left(\theta_t k_{t+1} - Q_t b_{t+1}\right) = 0, \tag{28}$$

$$\psi_t \left(d_t - \bar{d}\right) = 0. \tag{29}$$

3.2 Competitive Equilibrium and Constrained Efficiency

Definition of competitive equilibrium. In this economy, the endogenous state variables are firms' bonds and capital holdings. Given their initial values, $\{K_{-1}, B_{-1}\}$, the competitive equilibrium is defined as the sequence of allocations $\{C_t, Y_t, I_t, L_t, K_{t+1}, B_{t+1}, \Pi_t, N_t\}_{t=0}^{\infty}$ and prices $\{P_t, W_t, p_t\}_{t=0}^{\infty}$ that satisfy (i) the household's optimization; (ii) the capital producer's optimization; (iii) firms' optimization; and (iv) the market-clearing conditions for capital, consumption, labor, and equity shares:

$$
\begin{aligned}
P_t &= G'\left(I_t\right), && (30)\\
C_t &= Y_t - G\left(I_t\right) + Q_t B_{t+1} - \left[\left(\xi + \lambda\right) + \left(1 - \lambda\right) Q_t\right] B_t, && (31)\\
\chi N_t^{\frac{1}{\gamma}} &= \left(1 - \alpha\right) \frac{Y_t}{N_t}, && (32)\\
S_{t+1} &= 1. && (33)
\end{aligned}
$$

The following equation characterizes the equilibrium labor,

$$\left(1 - \alpha\right) z_t K_t^{\alpha} N_t^{-\alpha} = \chi N_t^{\frac{1}{\gamma}}, \tag{34}$$

where the labor choice only depends on pre-determined capital K_t and exogenous z_t. The equilibrium investment is determined by

$$G'\left(I_t\right) = \frac{\theta_t \mu_t}{1 + \psi_t} + \beta E_t \left\{ \frac{\Lambda'_{t+1}}{\Lambda'_t} \frac{1 + \psi_{t+1}}{1 + \psi_t} \left[\alpha \frac{Y_{t+1}}{K_{t+1}} + \left(1 - \delta\right) G'\left(I_{t+1}\right) \right] \right\}. \tag{35}$$

Equation (35) is a dynamic capital pricing equation. In this equation, the benefit of investing, which is the marginal value of increasing the household's future wealth plus the shadow value of relaxing the collateral constraint, equals the cost of investing, which is the capital price.

Definition of constrained efficiency. A social planner chooses allocations to maximize social welfare on behalf of firms and households. In doing so, the planner respects equity and collateral constraints, as well as market-clearing conditions. The planner does not have access to the lump-sum redistribution between firms and households.[13] In particular, the planner's problem is given by (to facilitate a comparison with the competitive equilibrium, we place a superscript sp on the planner's allocation)

$$\max_{\left\{C_t^{sp}, N_t^{sp}, K_{t+1}^{sp}, B_{t+1}^{sp}\right\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u \left[C_t^{sp} - \chi \frac{\left(N_t^{sp}\right)^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \right] \right\}, \tag{36}$$

subject to the following constraints:

$$\begin{aligned} C_t^{sp} &= Y_t^{sp} - G\left(I_t^{sp}\right) + Q_t B_{t+1}^{sp} - \left[(\xi+\lambda) + (1-\lambda) Q_t\right] B_t^{sp}, && (37)\\ \bar{d} &\leq Y_t^{sp} - \chi \left(N_t^{sp}\right)^{1+\frac{1}{\gamma}} + Q_t B_{t+1}^{sp} - \left[(\xi+\lambda) + (1-\lambda) Q_t\right] B_t^{sp} - G'\left(I_t^{sp}\right) I_t^{sp}, && (38)\\ Q_t B_{t+1}^{sp} &\leq \theta_t K_{t+1}^{sp}, && (39) \end{aligned}$$

where the first equation is the resource constraint, which combines the flow-of-funds constraints for households and firms. The second constraint is the equity constraint for firms, where we plug in the labor market equilibrium condition $W_t^{sp} = \chi \left(N_t^{sp}\right)^{\frac{1}{\gamma}}$ and the capital market equilibrium condition $P_t^{sp} = G'\left(I_t^{sp}\right)$. The third constraint is the borrowing constraint. Let λ_t^{sp}, ψ_t^{sp}, and μ_t^{sp} denote the Lagrange multiplier associated with constraints (37) to (39). The planner's allocation is then given by

$$\begin{aligned} C_t^{sp}: \quad & \lambda_t^{sp} = \left(u_t'\right)^{sp}, && (40)\\ N_t^{sp}: \quad & (1-\alpha) \frac{Y_t^{sp}}{N_t^{sp}} = \left(1 + \frac{1}{\gamma} \frac{\psi_t^{sp}}{1+\psi_t^{sp}}\right) \chi \left(N_t^{sp}\right)^{\frac{1}{\gamma}}, && (41)\\ K_{t+1}^{sp}: \quad & \left[1 + \frac{\psi_t^{sp}}{1+\psi_t^{sp}} \frac{G''\left(I_t^{sp}\right) I_t^{sp}}{G'\left(I_t^{sp}\right)}\right] G'\left(I_t^{sp}\right) && (42)\\ & = \frac{\theta_t \mu_t^{sp}}{1+\psi_t^{sp}} + \beta E_t \left\{ \frac{\lambda_{t+1}^{sp}}{\lambda_t^{sp}} \frac{1+\psi_{t+1}^{sp}}{1+\psi_t^{sp}} \left[\alpha \frac{Y_{t+1}^{sp}}{K_{t+1}^{sp}} + (1-\delta) G'\left(I_{t+1}^{sp}\right)\right] \right\},\\ B_{t+1}^{sp}: \quad & \left(1 - \frac{\mu_t^{sp}}{1+\psi_t^{sp}}\right) Q_t = \beta E_t \left\{ \frac{\lambda_{t+1}^{sp}}{\lambda_t^{sp}} \frac{1+\psi_{t+1}^{sp}}{1+\psi_t^{sp}} \left[(\xi+\lambda) + (1-\lambda) Q_{t+1}\right] \right\}. && (43) \end{aligned}$$

Comparing equations (34) and (35) with (41) and (42), one clearly observes two wedges in the planner's allocation that do not appear in the competitive equilibrium. In the competitive equilibrium, firms fail to internalize the collective effect of their individually optimal

[13] In deriving the planner's problem, we follow the primal approach by substituting prices and policies. This planner's full problem involves choosing various allocations, prices, and policies subject to the resource, collateral, equity, and implementability constraints. Here, we only present the reduced problem of a planner who only chooses real allocations. We relegate the discussion of policies implementing the constrained efficient allocation to the next subsection. The detailed proof of the equivalence between the full program and the relaxed program appears in Appendix B.1.

borrowing decisions during credit booms on raising wages and capital prices. When the equity constraints bind, the benefit of reducing wages involves the relaxation of their equity constraint. The competitive equilibrium is constrained inefficient, as agents are price takers.

Interaction of capital and labor market wedges. The modeling of capital and labor induces two wedges, which represent simultaneous inefficiencies in the labor and capital markets. This section shows that these inefficiencies also reinforce each other in a dynamic setting. On the one hand, note that by plugging (41) into equation (42), we get

$$\left[1+\frac{\psi_t^{sp}}{1+\psi_t^{sp}}\frac{G''(I_t^{sp})I_t^{sp}}{G'(I_t^{sp})}\right]G'(I_t^{sp})=\frac{\theta_t\mu_t^{sp}}{1+\psi_t^{sp}}+\beta E_t\left\{\frac{\lambda_{t+1}^{sp}}{\lambda_t^{sp}}\frac{1+\psi_{t+1}^{sp}}{1+\psi_t^{sp}}[\mathrm{mpk}^{sp}+(1-\delta)G'(I_{t+1}^{sp})]\right\}, \tag{44}$$

where mpk represents the marginal return to capital, which is given by

$$\mathrm{mpk}^{sp}=\alpha z_{t+1}(K_{t+1}^{sp})^{\alpha-1}\left[\frac{(1-\alpha)z_{t+1}(K_{t+1}^{sp})^{\alpha}}{\chi\left(1+\frac{1}{\gamma}\frac{\psi_{t+1}^{sp}}{1+\psi_{t+1}^{sp}}\right)}\right]^{\frac{\gamma-\alpha\gamma}{1+\alpha\gamma}}. \tag{45}$$

Conditional on capital holding, non-internalized labor market externalities increase tomorrow's marginal return on capital, which feeds into today's capital price and results in an even larger investment. On the other hand, heightened investment today, due to pecuniary externalities, will in turn increase tomorrow's marginal return to labor. This feedback between the wedges in the capital and labor markets forms a joint force that exacerbates the distortion between hiring and investment. To see this, suppose that the (deterministic) steady state features a binding equity constraint and a non-binding collateral constraint. Then, the steady-state capital holding is given by

$$K^{sp}=\left\{\frac{\beta\alpha\left(1-\alpha\right)^{\frac{\gamma(1-\alpha)}{1+\alpha\gamma}}z^{\frac{1+\gamma}{1+\alpha\gamma}}\left[\chi\left(1+\frac{1}{\gamma}\frac{\psi^{sp}}{1+\psi^{sp}}\right)\right]^{\frac{\gamma(\alpha-1)}{1+\alpha\gamma}}}{\left[1-\beta\left(1-\delta\right)+\frac{\psi}{1+\psi}\frac{G''(I^{sp})}{G'(I^{sp})}I^{sp}\right]G'\left(I^{sp}\right)}\right\}^{\frac{1+\alpha\gamma}{1-\alpha}}, \tag{46}$$

where $\frac{\psi^{sp}}{1+\psi^{sp}}\frac{G''(I^{sp})I^{sp}}{G'(I^{sp})}$ represents the capital price externality and $\frac{1}{\gamma}\frac{\psi^{sp}}{1+\psi^{sp}}$ represents the wage externality. In equation (46), two externalities interact multiplicatively, forming a joint force that leads the planner to invest less.

3.3 Policy Implications

The presence of pecuniary externalities in the labor and capital markets calls for a policy intervention. We assume that the planner has access to a macroprudential tax on firms' payroll and a tax on borrowing on a period-by-period basis. In particular, the planner uses taxes on firms' payroll to restore the efficiency of labor demand and uses taxes on debt issuance to correct investment demand. Let τ_t^n denote the payroll tax, and τ_t^l denote the tax on *new borrowing*. Taking these policies as given, the flow-of-funds constraint for individual firms becomes

$$d_t - \left(1 - \tau_t^l\right) Q_t l_t + P_t i_t = z_t k_t^\alpha n_t^{1-\alpha} - \left(1 + \tau_t^n\right) W_t n_t - \left(\xi + \lambda\right) b_t + T_t, \tag{47}$$

where positive τ_t^n and τ_t^l represent taxes, otherwise they mean a subsidies. These policies have been adopted in practice in real-world crisis management (Farhi and Tirole, 2012; Dávila and Korinek, 2018). Note that τ_t^n and τ_t^l only affect firms' budget constraints. They do not affect the equity or collateral constraints. In other words, policy makers respect these two constraints as the market does. T_t is the lump-sum rebate that equals (net) tax revenue.

Optimal macroprudential policies. The time-varying Pigouvian taxes τ_t^l and τ_t^n are the policies required for the planner to restore efficiency. We follow the primal approach to first analyze the optimal taxes. The following results summarize.

Proposition 1. *The macroprudential taxes that implements the constrained efficient allocation are given by (all variables are evaluated at planner's allocations)*

$$\begin{aligned}
\tau_t^n &= \frac{1}{\gamma}\frac{\psi_t}{1+\psi_t}, && (48)\\
\tau_t^l &= \frac{\zeta_t}{1+\zeta_t} - \beta E_t\left\{\frac{\Lambda_{t+1}}{\Lambda_t}\frac{1+\psi_{t+1}}{1+\psi_t}\left[\frac{\zeta_{t+1}}{1+\zeta_t}\frac{\left(\xi+\lambda\right)+\left(1-\lambda\right)Q_{t+1}}{Q_t}\right]\right\} && (49)\\
&\quad +\beta E_t\left\{\frac{\Lambda_{t+1}}{\Lambda_t}\frac{1+\psi_{t+1}}{1+\psi_t}\frac{1+\zeta_{t+1}}{1+\zeta_t}\frac{\left(1-\lambda\right)\tau_{t+1}^l Q_{t+1}}{Q_t}\right\},
\end{aligned}$$

where τ_t^n is the tax on payroll; τ_t^l is the tax on new debt issuance; Λ_t and ψ_t are the shadow values of the planner's budget and equity constraints; and ζ_t captures the size of the capital price externality that satisfies

$$\zeta_t = \frac{\psi_t}{1+\psi_t}\frac{G''\left(I_t\right)}{G'\left(I_t\right)}I_t + \beta E_t\left\{\frac{\Lambda_{t+1}}{\Lambda_t}\frac{1+\psi_{t+1}}{1+\psi_t}\zeta_{t+1}\frac{\alpha Y_{t+1}/K_{t+1}+\left(1-\delta\right)G'\left(I_{t+1}\right)}{G'\left(I_t\right)}\right\}. \tag{50}$$

Interpreting the prudential policy. The macroprudential tax on hiring is a combination of two groups of sufficient statistics. One is the inverse of the labor supply elasticity $1/\gamma$, and the other is the time-varying shadow value of the planner's equity constraint. Together, they reflect the labor market wedge between the competitive and planner's outcomes.

The macroprudential tax on long-term credit issuance involves a static component, a dynamic component, and a third component that captures private agents' *ex ante* incentive toward *ex post* interventions. The term $\zeta_t/\left(1+\zeta_t\right)$ captures the static aspect. When current investment increases, there is a larger capital price externality (by equation (50)). However, increased current investment also reduces future capital demand, which leads to a smaller future capital price externality. Therefore, the second term is negative. Finally, the third term captures the *ex ante* incentive effects (i.e., the $(1-\lambda)\tau_{t+1}^l Q_{t+1}$ term), which is tied to the fact that debts are long-term (i.e., $\lambda \neq 1$).

To explain the intuition of this term, consider a snap-shot of the dynamic model in three periods. In the first period, firms borrow long-term to invest. In doing so, firms correctly perceive her long-term borrowing decision today will squeeze future borrowing (through collateral constraint). Therefore, a high future borrowing cost (i.e., $\tau_{t+1}^l > 0$) will effectively reduce the cost of borrowing today, and leads firms to borrow even more and increases the necessity of using macroprudential policy; Conversely, a lower future borrowing cost (i.e., $\tau_{t+1}^l < 0$) increases the cost of borrowing today, which disaplines agents' incentive to invest and reduces the necessity of macroprudential policy. In Appendix B.4, we formalize such a three-period model to clearly illustrate the above intuition, and the following corollary concludes.

Corollary 1. *When $\lambda \neq 1$, an ex post subsidy (i.e., $\tau_{t+1}^l < 0$), reduce the ex ante desirability of prudential taxes, and an ex post macroprudential tax (i.e., $\tau_{t+1}^l > 0$) increases the ex ante desirability of macroprudential taxes.*

Note that in Proposition 1, any policy intervention τ_t^l satisfying equation (49) achieves constrained efficiency. In addition, we show the time-consistency property of the macroprudential policy. Specifically, when prudential intervention is unrestricted, it can be set to eliminate the moral hazard induced by any *ex post* interventions, and time-consistency is achieved because of this. We thus have the following corollary.

Corollary 2. *τ_t^n and τ_t^l are time consistent when they are unrestricted.*

4 Quantitative Analysis

In this section, we perform quantitative analysis of the dynamic model. We first parameterize the model and briefly describe the solution method. Then, we discuss and compare the policy rules for the competitive equilibrium and the constrained efficiency. Next, we simulate the model and show how the model fits the stylized facts reported in the empirical section. Finally, we study the real effects and welfare implications of the macroprudential policy.

4.1 Model Quantification

Functional forms and parameter values. In the following analyses, we assume the capital producer's cost takes the form below,

$$G(i_t) = \eta \frac{i_t^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}, \tag{51}$$

where η is the cost of producing investment, and ν represents the capital supply elasticity. In addition, we assume households have log utility, i.e., $u(\cdot) = \log(\cdot)$.

We set the debt maturity to three years (i.e., $\lambda = 0.33$).[14] We set the discount factor as $\beta = 0.96$, the depreciation rate as $\delta = 0.10$, and the share of capital as $\alpha = 0.33$. We normalize labor disutility, χ, to 0.50 and set the Frisch elasticity parameter as $\gamma = 1.50$. We set the dividend payout limit, $\bar{d}$, to be 0.33 to match the average dividend-to-revenue ratio of 7.5% (Factset S&P 500 (2000–2015)), and we set the investment cost parameter, $\eta = 0.97$, and capital supply elasticity parameter, $\nu = 2.00$. For simplicity, we set the coupon rate as $\xi = 0.00$. Table 2 lists the parameter assignments. We assume that the support for each shock processes (i.e., bond price Q_t, productivity z_t and credit condition θ_t) takes two states: high (H) and low (L). We adopt the support and the joint transition matrix (a 8×8 square matrix) from Bianchi (2016, p. 3657).

Table 2: Parameters Table

Parameters	Symbols	Values
Discount Rate	β	0.96
Depreciation Rate	δ	0.10
Equity Constraint	$\bar{d}$	0.33
Investment Cost	η	0.97
Labor Aversion	χ	0.50
Capital Elasticity	ν	2.00
Capital Share	α	0.33
Frisch Elasticity	γ	1.50
Coupon Rate	ξ	0.00
Debt Maturity	$1/\lambda$	3.00

Note: This table shows the model parameter choices in the quatitative analyses.

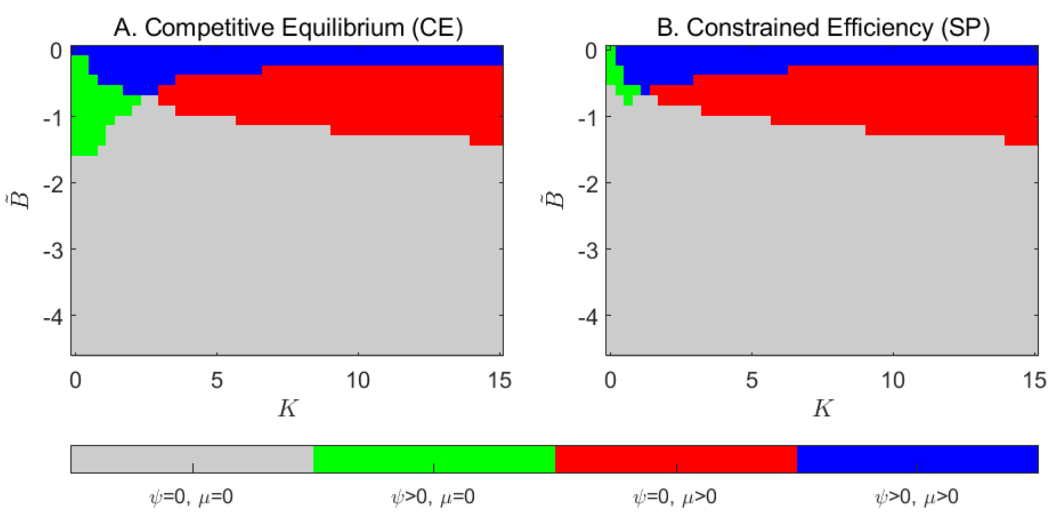
Solution method. The model has three groups of state variables (capital, bonds, and shocks) and two occasionally binding constraints. To globally solve the model, we use the

[14]The median U.S. corporate debt maturity is around three years (Barclay and Smith, 1995).

Euler equation iteration method to iterate the model's optimality condition on a $100 \times 50 \times 8$ meshed grid (bonds $\times$ capital $\times$ shocks). To rectangularize the state space, we normalize bond grid by subtracting the natural limit of debts. The competitive equilibrium allocation and the constrained efficiency allocation are solved separately. We then use the same method to calculate optimal macroprudential policies based on the constrained efficiency allocation. The details of the numerical solution procedure are relegated to Appendix C.

Constraint regions and policy rules. We exhibit the solution to our model in this part. Following Bianchi (2016), we first plot the occasionally binding/nonbinding constraint regions for both the competitive equilibrium and the social planner, and then we plot firm's individual investments and labor policy rules in these different regions. In Figure 3, the blue areas in both panel A and panel B are the cases when equity and collateral constraints bind simultaneously (i.e., $\psi > 0, \mu > 0$). The red regions are the cases when only the collateral constraint binds (i.e., $\psi = 0, \mu > 0$). The equity constraint is more likely to bind when firms' capital stock is low (i.e., green regions where $\psi > 0, \mu = 0$). Firms are unconstrained if they have enough cash flow and do not have a debt burden (i.e., grey regions where $\psi = 0, \mu = 0$).

Figure 3: Constraint Regions



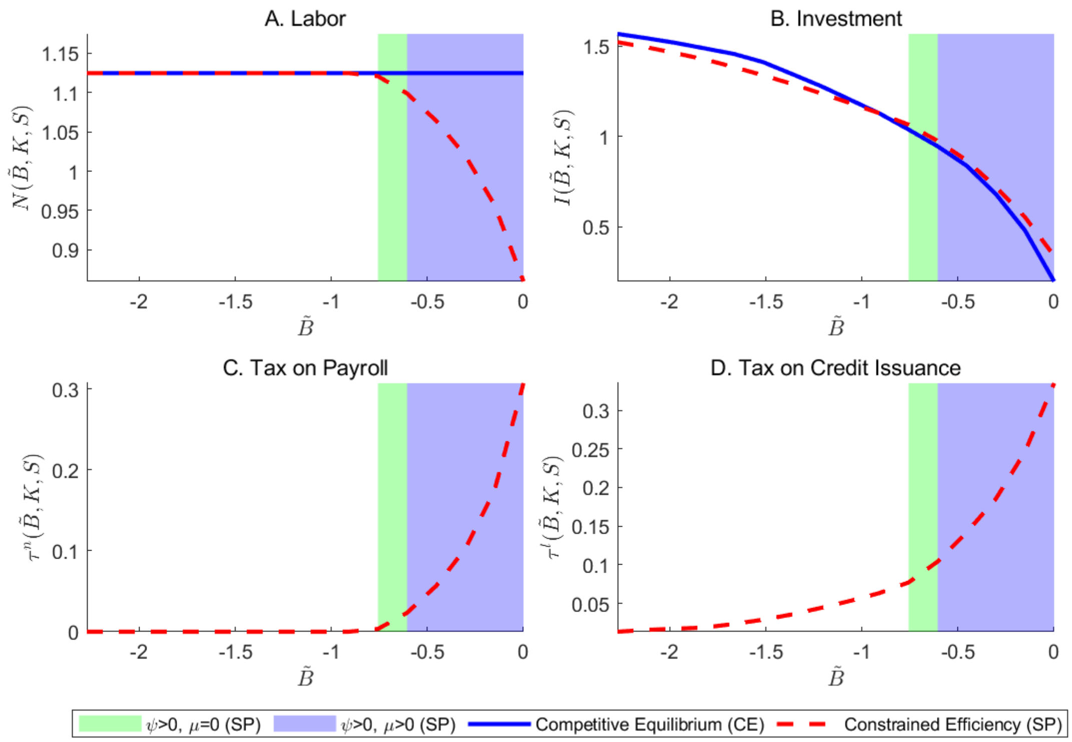
Note: This figure plots the regions of different constraints in competitive equilibrium and in constrained efficiency. ψ and μ are multipliers corresponding to the equity constraint and the collateral constraint, respectively.

In Figure 4, we compare policies rules between the competitive equilibrium and the social planner, and we shade the planner's constraint regions (only equity binding in light green and both binding in light blue). In panel A, the labor decision keeps constant in the competitive equilibrium, but the social planner will choose a lower level of employment when the equity constraint begins to bind. This is consistent with the implication of the social planner's labor choice (i.e., equation (41) in which a wedge exists between the labor demand and labor supply). In panel B, the comparason of the investment decisions are relatively less straightforward. When neither of the constraints is binding, there is overinvestment in the competitive equilibrium, but when the equity constraint starts to bind, the situation is the opposite (i.e., firms underinvest in the competitive equilibrium). That is, the social planner would choose a higher level of investment. Intuitively, as labor is reduced by a large amount, investment expenditures increase by the budget constraint. In Appendix B.3, we provide a detailed analysis on this point. Panels C and D show the optimal Pigovian tax that corrects the inefficiency. In panel C, the optimal labor tax is zero in non-binding regions but will be positive if the equity constraint starts to bind. In panel D, the optimal tax on debt issuance is positive even if both constraints are not binding today. The reason is that, by equation (49), the optimal tax on debt issuance is not only a function of today's state variables, but also a function of future taxes, i.e., τ_{t+1}^{l}.

To further investigate the forces behind the τ_t^l term, we follow the discussions in Proposition 1 to decompose this macroprudential tax into (i) a static term, (ii) a dynamic term, and (iii) a term that corrects agents' expectation of future policies. The blue dash line in Figure 5 shows the static term. Because firms' individual investment decision fails to internalize the capital price effect, this term is always positive. The size of this term, however, depend on whether the firm is constrained. When the firm is far-away from the equity/collateral constraint, the tax rate is low (around 4%). This number increases to 20% $\sim$ 40% when the economy is in the constraint region. The first positive term is offsetted by the second negative term (shown in the red dot line), which captures the effect that more current investment may increase the future cash flow and relax the future equity constraint. The green solid line captures the third term which highlights the necessity of a correction for agent's response towards future interventions.

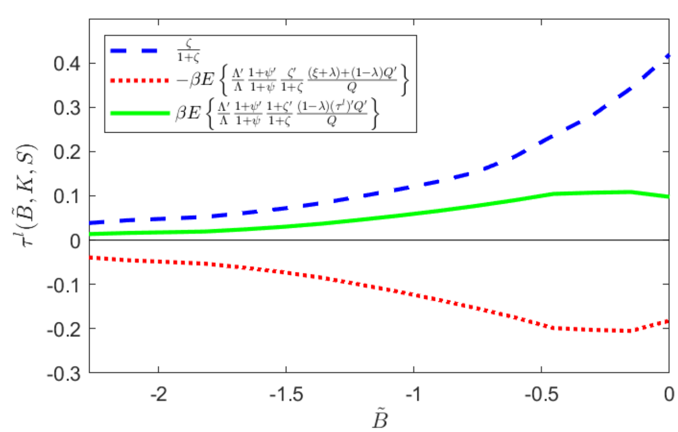
Fitting stylized facts. We then show that our model is able to replicate the positive relationship between the interest rate expectation and the long-term debt growth rate. First, we use the numerically solved policy rules (of the competitive equilibirum) and the transition matrix to simulate the model. In particular, we simulate a panel of 500 observations, each lasting for 10,000 periods. For each of the observations, we keep the last 2,000 periods and drop the initial 8,000 periods as burn-in. Table 3 exhibits the model's prediction and compares it with the results from the data. For the overall sample, the correlation between the bond growth rate and the interest rate expectation is 2.34 in the model (compared with 1.24 in the data). This positive relationship is anticipated, as cheaper borrowing induces

Figure 4: Policy Rules



Note: This figure plots a slice of policy rules for investment, labor, tax on payroll, and tax on credit issuance ($k = 0.61$, $z = z^L$, $\theta = \theta^L$, $Q = Q^H$). The blue solid lines represent the results for the competitive equilibrium, while the red dash lines represent the results for the constrained efficiency. The green and blue shades denote, repectively, regions where only the equity constraint binds, and regions where both constraints bind (in the constrained efficiency).

Figure 5: Decomposition of Tax on Credit Issuance



Note: This figure plots the decomposition of tax on credit issuance (panel D of Figure 4) according to equation (49). The blue, red and green curves are respectively correponding to the static, the dynamic and the third term that corrects agents' expectation of future policies.

firms to wait. For observations that are financially constrained, this relationship becomes 3.76 (compared with 2.58 in the data). These results suggest that our model can capture the salient facts discussed in Section 2.

Table 3: Correlation between Bond Growth Rate and Expected Interest Rate

	Data	Model
Overall Sample	1.24	2.34
Financially Constrained Observations	2.58	3.76

Notes: This table shows the correlation between the bond growth rate and the expected interest rate in the model and compares it with that in the empirical results in Figure 2. Specifically, we define the interest rate expectation in the model as $\hat{e}_t = [(\xi + \lambda) + (1 - \lambda)\mathbb{E}_t Q_{t+1}]/Q_t$ and define the bond growth rate same as before. To keep consistency with empirical analyses, we identify a financially constrained observation when its bond holding in $t-1$ period ranks in the top 20%.

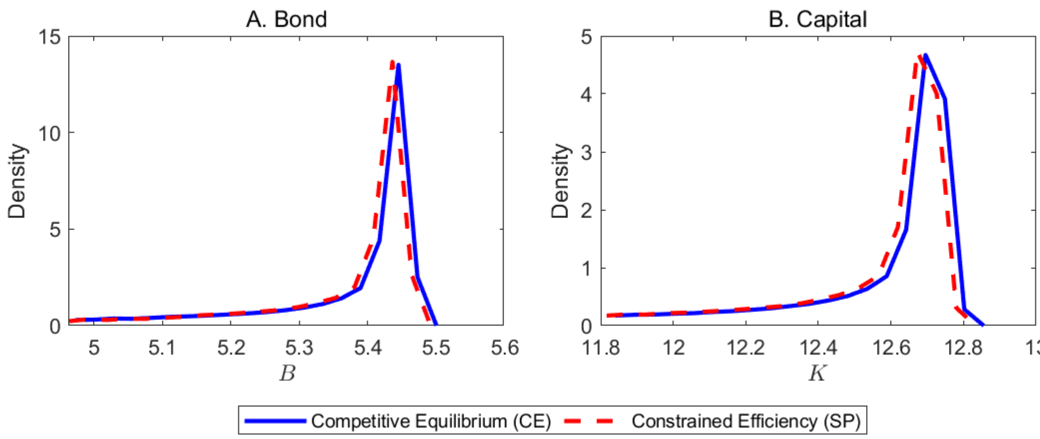
4.2 Effects of Optimal Policies and Welfare Implications

Based on our previous quantitative analysis, we first show how the optimally designed macroprudential policy helps to prevent firms from overborrowing. Then, we present the effect of macroprudential policy in dampening the crisis. Finally, we show the welfare implication of the macroprudential policy by contrasting it with a misspecified policy that misses the distinct incentive effects of long-term debts.

Ergodic distributions. To begin with, we show the ergodic distribution of bond and capital holdings with and without the optimal macroprudential policies. Again, we simulate a panel of 500 observations with 10,000 periods and keep the last 2,000 periods. Figure 6 reports the ergodic distributions of firms' decisions in booms (when $\theta = \theta^H$), with panel A showing the results for bond and panel B showing the results for capital. One can find that the social planner would like to choose a lower level (quantitatively $0.005 \sim 0.01$ less) of bonds and capital, especially on the right tails. The result indicates that the optimal macroprudential policy can suppress the *ex ante* borrowing and investment incentives.

Impulse response functions. Next, we show the effect of optimal macroprudential policies in dampening the crisis. Specifically, we follow similar steps to simulate 10,000 periods and set $\theta = \theta^L$ (which is a negative credit shock) at the $8,000^{\text{th}}$ period for all the observations. The financial shock then naturally evolves following the transition matrix. Figure 7 shows the response of macro variables in the shock window for both the planner's solution and the competitive equilibrium. Particularly, when the optimal macroprudential policy

Figure 6: Ergodic Distributions



Note: This figure shows the ergodic distribution of bonds and capital assets when $\theta = \theta^H$. The blue solid lines represent the results for competitive equilibrium. The red dash lines represent the results for constrained efficiency.

is implemented, output drops by around 1% less (panel A). This dampening effect mainly comes from a 20% less drop in investment (panel B). In addition, the model also implies (slightly) less volatile debt, as firms' bond holding falls by nearly 21% in the competitive equilibrium but 19% in the planner's case (panel C).

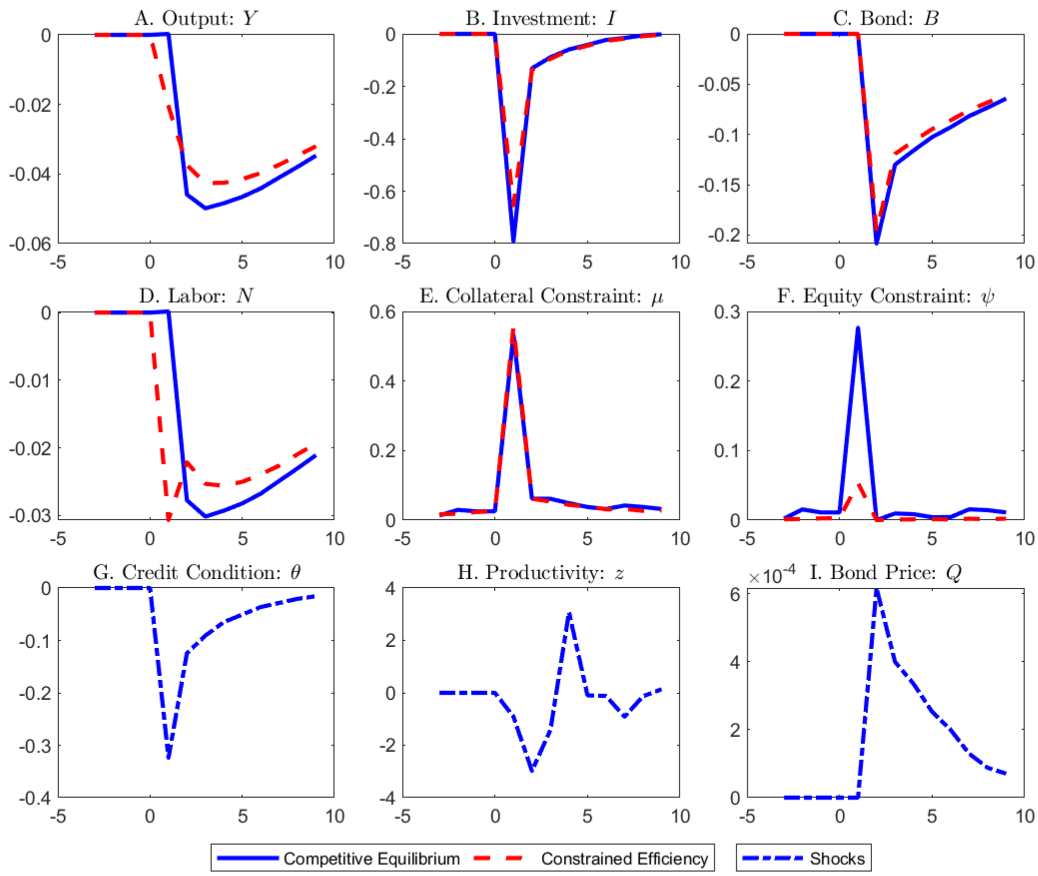
It is worth mentioning that the output reacts immediately in the social planner's case but reacts one period later in the competitive equilibrium. This is because the labor choice in the constrained efficiency drops promptly due to a positive multiplier of equity constraint, but that in the competitive equilibrium shortly keeps unchanged since it only relies on the predetermined capital (panel D). Although the planner chooses a even lower level of employment on impact, the labor hours recover more quickly in the following periods.

Welfare computation. Finally, we show the welfare implications of the optimally designed macroprudential policy. To do so, we first define welfare gains as consumption increases that render representative households indifferent between living in economies under the competitive equilibrium and the social planner's allocations. Specifically, we compute the consumption equivalence ω such that

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \log \left(c_t^{sp} - \chi \frac{(n_t^{sp})^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \right) \right\} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \log \left(c_t \left(1+\omega\right) - \chi \frac{n_t^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \right) \right\}, \tag{52}$$

where c_t^{sp} and n_t^{sp} represent the social planner's allocations, and c_t and n_t represent the corresponding allocations under the competitive equilibrium. Following Bianchi (2016), we

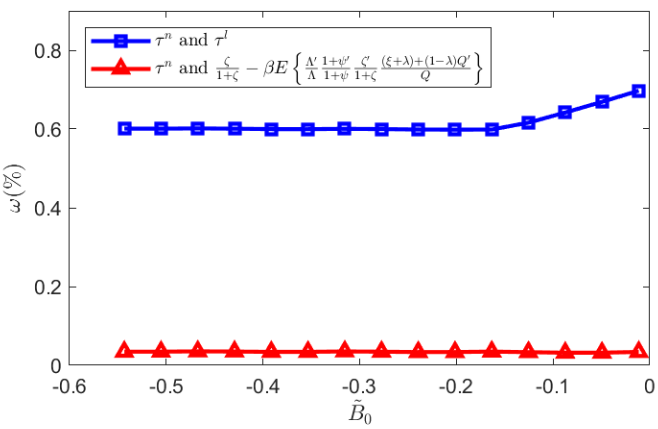
Figure 7: Impulse Response Functions



Notes: This figure plots the dynamics for (1) real allocations including output, investment, bond and employment, (2) multipliers for two constraints, and (3) three exogenous shocks. At time $t = 0$, there is a negative credit shock. The vertical axis is in percentage. The results for the competitive equilibrium are shown in blue solid lines, while the results for the constrained efficiency are shown in red dash lines.

fix the initial capital stock to be 10% below its long-run mean, and experiment different initial (normalized) bond holdings $\tilde{B}_0$ around its long-run mean. Figure 8 reports the results. The blue line shows the optimally designed macroprudential policy following equation (49). Quantitatively, we show that this policy induces a 0.6% $\sim$ 0.7% welfare gain in the consumption equivalent measure, and this number is increasing in $\tilde{B}_0$ when the overborrowing distortion becomes more severe. When the planner fails to take the incentive effect (i.e., the third term of equation (49)) into account, the red line shows that the welfare improvement reduces to less than 0.1%. This suggests the importance of incorporating the incentive effects of long-term debts into the optimal designing of macroprudential policy.

Figure 8: Welfare Gains



Notes: This figure plots the welfare gains in two cases. In the first case (shown in blue), the optimal policies are used. In the second case (shown in red), the incentive effect of long-term debts (i.e., the third term of equation (49)) is ignored.

5 Conclusion

Fluctuations in long-term credit have important macroeconomic consequences over boom-bust cycles. This paper studies the efficiency properties of a model in which firms can issue long-term debts. The model highlights the role of firms' collective investment in raising asset prices during booms, which in turn squeezes the debt capacity that restricts real activities during crises. It offers a novel mechanism by which the incentive effects of liquidity provisions crucially depend on the maturity structure of credits.

The analysis presented in this paper can be extended in the following dimensions. First, it would be interesting to consider the case where firms' collateral constraint is tied to the price of capital. Doing so will provide another amplification mechanism for asset fire sales. This would be an interesting extension since the deleveraging of long-term debts and fire sales may interact to form a joint force during recessions. The current model focuses on flexible prices. Another direction of enquiry would be to extend the model to allow for nominal rigidities so that output can be demand driven. Gomes, Jermann and Schmid (2016) believe that the long-term debt is essential for macroeconomic models to have slow recovery. In a sticky price setting, one can investigate how debt maturity interacts with aggregate demand channels and provide a more comprehensive account of macroprudential policy and crisis management design.

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Appendices

A Data and Empirics Appendix

In this section, we first show detailed data descriptions for firms and interest rate expectations. We then present the summary statistics of variables used in our empirical analyses. Finally, we provide additional empirical results.

A.1 Firm Data

We clean the quarterly data by the following steps:

1. Only US manufacturing firms (with SIC code 2000∼3999) from the first quarter of 1986 to the last quarter of 2019 are selected.

2. Observations with missing or negative total assets (atq), negative common stock ($cstkq$), negative total stockholders' equity ($teqq$), negative common share issued ($cshiq$), and negative long-term debt ($dlttq$) or current debt ($dlcq$) are deleted. Observations whose cash and short-term investments ($cheq$) are greater than total assets are also dropped.

3. Firm characteristics used in the regressions include firm size (natural log of total assets, $\ln(atq)$), leverage (total liabilities over total assets, ltq/atq), cash ratio (cash and short-term investments over total assets, $cheq/atq$), and sales ratio (sales over total assets, $saleq/atq$). To avoid the influence of unreasonable outliers, we winsorize these four variables together with long-term debt and current debt at the 1% and 99% levels.

4. Firms should have at least five consecutive observations in the sample period.

We clean the annual data by the following steps:

1. Only US manufacturing firms from 1985 to 2019 are selected.

2. Observations with missing or negative total assets (at) are deleted.

3. To keep consistency with Lamont, Polk and Saaá-Requejo (2001), observations with missing or negative real sales growth rate are deleted when we generate the KZ index.

A.2 Interest Rate Expectations

Market-based Measure. For benchmark, we use the price of federal funds futures from Datastream Future Contract to construct interest rate expectations. We select the sample from 1985 and only focus on the future contracts whose first three digits of *DSMnem* (an ID for a future contract) are "CFF" which uniquely identifies the 30-day federal funds futures. For each contract, the daily price is defined as the average of the highest price and the lowest price. Then the daily series are transformed into quartly series. For example, the price in quarter 1 is the average price of all contracts that are traded in quarter 1 and will be settled in quarter 2, the price in quarter 2 is the average price of all contracts that are traded in quarter 2 and will be settled in quarter 3, so on and so forth. Next, we subtract prices from 100 and normalize the results by 100 to transform prices into interest rates. Finally, we take the first difference of this quarterly interest rate series for stationarity.

Survey-based Measure. For robustness, we also construct the interest rate expectation by using the Michigan's Survey of Consumers (MSC). The corresponding question in the questionnaire is:

No one can say for sure, but what do you think will happen to interest rates for borrowing money during the next 12 months – will they go up, stay the same, or go down?

For a specific quarter, there are 100 interviewees. This database provides an indicator, *Relative*, which is the number of interviewees who chose "go down" minus the number of interviewees who chose "go up" plus 100. A higher *Relative* means more consumers thought that interest rate would go down in the next 12 months. Otherwise, it means more consumers believed that interest rate would go up. To keep consistency, we put a minus sign in front of this measure and make it normalized by 100 as well. This survey-based variable is highly correlated (coef.=0.8397) with the first difference in the market-based expectation series in that it already captures the average expectation of interest rate change.

A.3 Descriptive Statistics

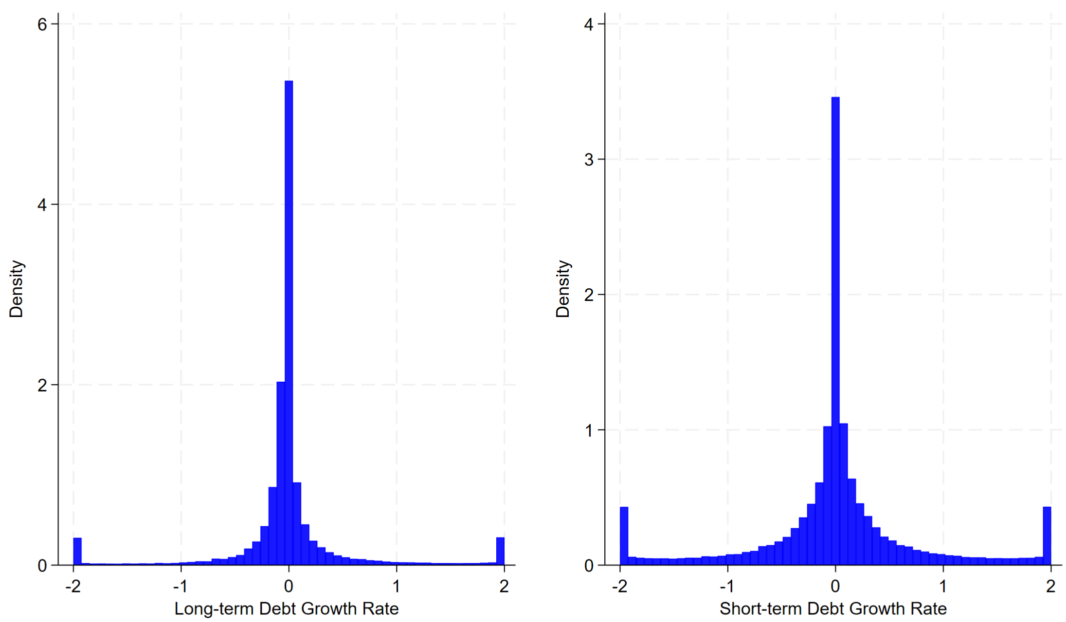
The summary statistics for firms' characteristics are shown in the following table and figures.

Table A.1: Summary Statistics of US Manufacturing Firm Sample

	Observations	Mean	SD	Min	Max
Long-term Debt Growth Rate	221,946	-0.01	0.56	-2.00	2.00
Short-term Debt Growth Rate	214,774	0.00	0.76	-2.00	2.00
ln(atq)	286,292	4.71	2.15	-1.61	10.30
ltq/atq	286,218	0.50	0.35	0.03	5.25
$cheq/atq$	285,958	0.18	0.22	0.00	0.97
$saleq/atq$	285,484	0.26	0.17	0.00	0.97

Note: This table shows the descriptive statistics for dependent variables and control variables on firm-quarter level. The number of observations, mean, standard deviation, minimum and maximum value for each variable are reported.

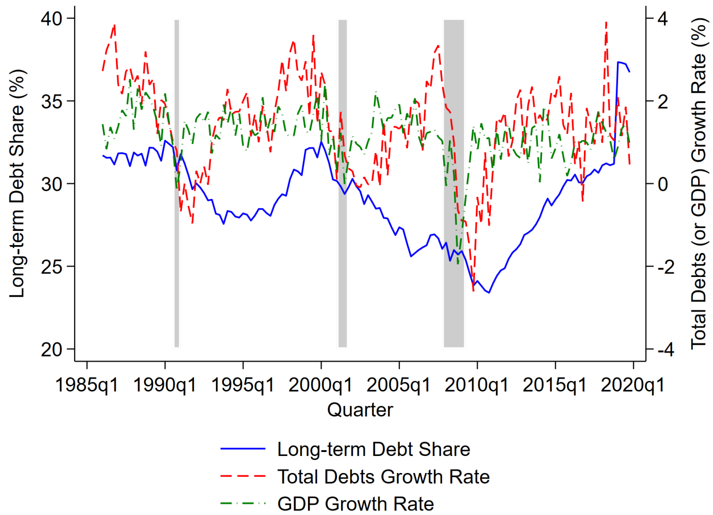
Figure A.1: Distribution of Long-term (Short-term) Debt Growth Rate



Note: The figure shows the distribution of long-term (short-term) debt growth rate.

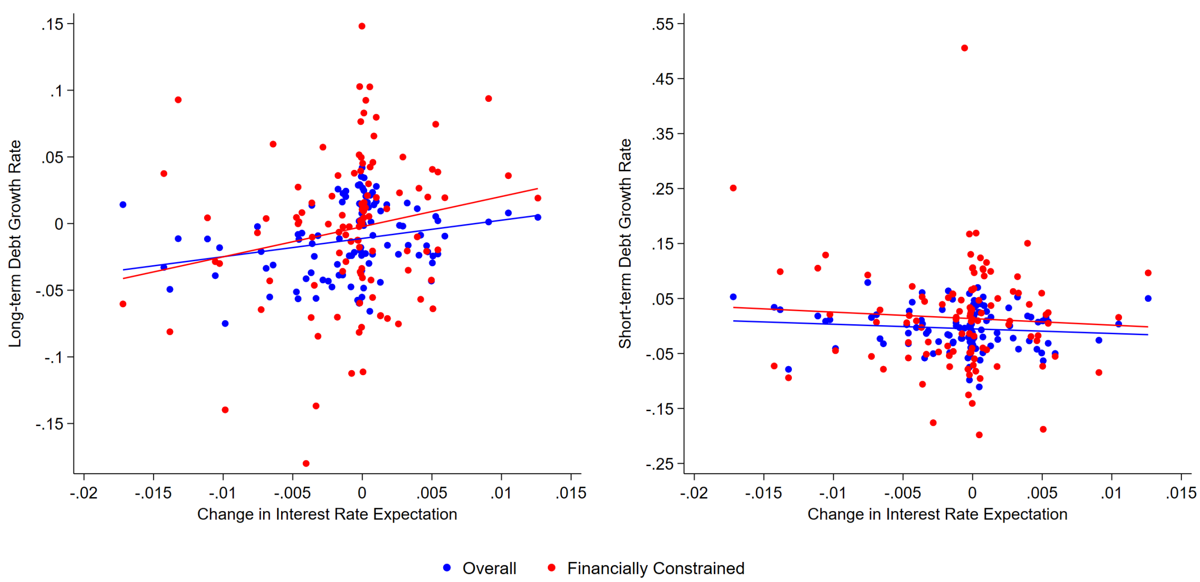
A.4 Supplementary Results

Figure A.2: Alternative Long-term Debt Share of US Manufacturing Firms



Note: This figure shows the time series of average long-term debt share of US manufacturing firms from 1986q1 to 2019q4. The long-term debt share is measured by the ratio of long-term debts to total liabilities. Shaded areas denote recessions adopted from NBER recession series.

Figure A.3: Debt Growth Rate v.s. Changes in Interest Rate Expectation



Note: This figure consists of two raw scatter plots and displays the correlation between the change in interest rate expectation and the long-term (short-term) debt growth rate of US manufacturing firms. Variables are averaged at quarterly level.

Table A.2: Regression Results with a Survey-based Expectation

	A. Long-term Debt Growth Rate, $g_{i,t}$					
	(1)	(2)	(3)	(4)	(5)	(6)
Δe_t	0.046***	0.028***	0.038***	0.019	0.005	0.001
	(0.006)	(0.005)	(0.007)	(0.012)	(0.011)	(0.014)
$f_{i,t}$				-0.068***	-0.064***	-0.061***
				(0.017)	(0.016)	(0.018)
$\Delta e_t \times f_{i,t}$				0.056**	0.050**	0.056*
				(0.026)	(0.024)	(0.029)
Constant	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes
Firm Characteristics	Yes	Yes	Yes	Yes	Yes	Yes
Lagged Growth Rate	No	Yes	Yes	No	Yes	Yes
MP Shocks	No	No	Yes	No	No	Yes
Observations	221,133	211,109	151,076	58,623	57,434	42,076
Clusters	6,277	6,206	5,092	3,236	3,208	2,653
R^2	0.036	0.068	0.075	0.083	0.124	0.134
	B. Short-term Debt Growth Rate, $g_{i,t}$					
	(1)	(2)	(3)	(4)	(5)	(6)
Δe_t	0.020***	0.012*	0.018**	0.024	0.022	0.025
	(0.007)	(0.007)	(0.008)	(0.016)	(0.015)	(0.019)
$f_{i,t}$				-0.095***	-0.069***	-0.063**
				(0.022)	(0.021)	(0.024)
$\Delta e_t \times f_{i,t}$				0.023	0.037	0.042
				(0.033)	(0.032)	(0.039)
Constant	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes
Firm Characteristics	Yes	Yes	Yes	Yes	Yes	Yes
Lagged Growth Rate	No	Yes	Yes	No	Yes	Yes
MP Shocks	No	No	Yes	No	No	Yes
Observations	213,999	188,928	133,806	57,243	52,770	38,517
Clusters	6,316	6,016	4,921	3,283	3,110	2,571
R^2	0.026	0.032	0.036	0.056	0.057	0.062

Note: This table shows the estimation results of the specification (1) by using OLS. The dependent variable is the long-term (short-term) debt growth rate. A survey-based measure of interest rate expecation is used. The standard deviations are clustered on firm level. ***, **, and * denote 1%, 5%, and 10% statistical significance, respectively.

Table A.3: Regression Results with Firms Size as Financial Constraint Indicator

	A. Long-term Debt Growth Rate, $g_{i,t}$					
	(1)	(2)	(3)	(4)	(5)	(6)
Δe_t	1.484***	1.433***	2.170***	3.704***	3.671***	5.837***
	(0.264)	(0.243)	(0.317)	(0.782)	(0.713)	(0.996)
$\Delta e_t \times \ln(atq)_{i,t}$				-0.455***	-0.456***	-0.707***
				(0.129)	(0.118)	(0.158)
Constant	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes
Firm Characteristics	Yes	Yes	Yes	Yes	Yes	Yes
Lagged Growth Rate	No	Yes	Yes	No	Yes	Yes
MP Shocks	No	No	Yes	No	No	Yes
Observations	181,165	175,441	135,608	181,165	175,441	135,608
Clusters	5,553	5,447	4,754	5,553	5,447	4,754
R^2	0.039	0.072	0.078	0.039	0.072	0.078
	B. Short-term Debt Growth Rate, $g_{i,t}$					
	(1)	(2)	(3)	(4)	(5)	(6)
Δe_t	0.217	-0.136	-0.439	1.567*	0.499	-0.311
	(0.327)	(0.322)	(0.412)	(0.810)	(0.810)	(1.074)
$\Delta e_t \times \ln(atq)_{i,t}$				-0.288*	-0.131	-0.025
				(0.155)	(0.156)	(0.197)
Constant	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes
Firm Characteristics	Yes	Yes	Yes	Yes	Yes	Yes
Lagged Growth Rate	No	Yes	Yes	No	Yes	Yes
MP Shocks	No	No	Yes	No	No	Yes
Observations	175,878	157,158	120,559	175,878	157,158	120,559
Clusters	5,589	5,242	4,565	5,589	5,242	4,565
R^2	0.028	0.033	0.037	0.028	0.033	0.037

Note: This table shows the estimation results of the specification (1) by using OLS. The dependent variable is the long-term (short-term) debt growth rate. We directly use firm size to proxy the financial constraints. The standard deviations are clustered on firm level. ***, **, and * denote 1%, 5%, and 10% statistical significance, respectively.

Table A.4: Regression Results with KZ Index Sorted by Industry and Year

	A. Long-term Debt Growth Rate, $g_{i,t}$					
	(1)	(2)	(3)	(4)	(5)	(6)
Δe_t	1.484***	1.433***	2.170***	1.287**	1.296***	2.049***
	(0.264)	(0.243)	(0.317)	(0.546)	(0.496)	(0.662)
$f_{i,t}$				-0.058***	-0.049***	-0.050***
				(0.007)	(0.007)	(0.008)
$\Delta e_t \times f_{i,t}$				0.933	0.699	0.095
				(0.968)	(0.897)	(1.176)
Constant	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes
Firm Characteristics	Yes	Yes	Yes	Yes	Yes	Yes
Lagged Growth Rate	No	Yes	Yes	No	Yes	Yes
MP Shocks	No	No	Yes	No	No	Yes
Observations	181,165	175,441	135,608	49,599	48,769	38,267
Clusters	5,553	5,447	4,754	2,907	2,878	2,512
R^2	0.039	0.072	0.078	0.087	0.128	0.135
	B. Short-term Debt Growth Rate, $g_{i,t}$					
	(1)	(2)	(3)	(4)	(5)	(6)
Δe_t	0.217	-0.136	-0.439	1.669**	1.390*	1.158
	(0.327)	(0.322)	(0.412)	(0.751)	(0.730)	(0.926)
$f_{i,t}$				-0.064***	-0.049***	-0.046***
				(0.009)	(0.009)	(0.010)
$\Delta e_t \times f_{i,t}$				-0.494	-0.392	-1.399
				(1.371)	(1.344)	(1.657)
Constant	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes
Firm Characteristics	Yes	Yes	Yes	Yes	Yes	Yes
Lagged Growth Rate	No	Yes	Yes	No	Yes	Yes
MP Shocks	No	No	Yes	No	No	Yes
Observations	175,878	157,158	120,559	48,726	44,990	35,189
Clusters	5,589	5,242	4,565	2,955	2,789	2,430
R^2	0.028	0.033	0.037	0.060	0.062	0.065

Note: This table shows the estimation results of the specification (1) by using OLS. The dependent variable is the long-term (short-term) debt growth rate. To generate financial constraint dummy, we sort KZ index by both industry and year. The standard deviations are clustered on firm level. ***, **, and * denote 1%, 5%, and 10% statistical significance, respectively.

Table A.5: Regression Results with All Non-financial Firms

	A. Long-term Debt Growth Rate, $g_{i,t}$					
	(1)	(2)	(3)	(4)	(5)	(6)
Δe_t	1.610***	1.586***	2.344***	1.553***	1.368***	2.036***
	(0.203)	(0.188)	(0.242)	(0.246)	(0.226)	(0.288)
$f_{i,t}$				-0.077***	-0.072***	-0.069***
				(0.004)	(0.004)	(0.005)
$\Delta e_t \times f_{i,t}$				0.392	1.297**	1.157
				(0.645)	(0.596)	(0.760)
Constant	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes
Firm Characteristics	Yes	Yes	Yes	Yes	Yes	Yes
Lagged Growth Rate	No	Yes	Yes	No	Yes	Yes
MP Shocks	No	No	Yes	No	No	Yes
Observations	313,092	302,246	235,909	263,718	257,461	203,662
Clusters	10,251	10,021	8,742	9,099	8,959	7,867
R^2	0.041	0.076	0.082	0.044	0.079	0.085

	B. Short-term Debt Growth Rate, $g_{i,t}$					
	(1)	(2)	(3)	(4)	(5)	(6)
Δe_t	0.437*	0.095	-0.207	1.119***	0.621**	0.310
	(0.256)	(0.251)	(0.317)	(0.309)	(0.298)	(0.371)
$f_{i,t}$				-0.095***	-0.078***	-0.074***
				(0.005)	(0.005)	(0.005)
$\Delta e_t \times f_{i,t}$				-2.058***	-1.864**	-2.371***
				(0.743)	(0.737)	(0.919)
Constant	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes
Firm Characteristics	Yes	Yes	Yes	Yes	Yes	Yes
Lagged Growth Rate	No	Yes	Yes	No	Yes	Yes
MP Shocks	No	No	Yes	No	No	Yes
Observations	302,640	269,861	209,355	255,529	230,554	181,378
Clusters	10,250	9,616	8,374	9,111	8,580	7,517
R^2	0.031	0.036	0.039	0.033	0.038	0.041

Note: This table shows the estimation results of the specification (1) by using OLS with all non-financial firms. The dependent variable is the long-term (short-term) debt growth rate. The standard deviations are clustered on firm level. ***, **, and * denote 1%, 5%, and 10% statistical significance, respectively.

B Model Appendix

B.1 Proof of Proposition (1)

We prove this proposition as follows. We first define a program where a Ramsey planner who chooses allocations, prices, and policies, subject to resource, collateral, equity, and implementability constraints. Second, we show a relaxed program in which the planner only chooses allocations. Third, we establish equivalence between the full program and the relaxed program by constructing a competitive equilibrium based on the planner's allocation. In particular, we show that all implementability constraints do not bind in the full problem.

Constrained planner's full program. Consider a Ramsey planner's problem in which she does not have access to lump-sum redistributional policies across households and firms, but has access to macroprudential policies. In particular, this planner chooses a sequence of allocation and policies to maximize households' (the ultimate owners') utility, and the problem can be written as follows (for easy illustration, we drop the *sp* superscript)

$$\max_{\left\{C_t,N_t,K_{t+1},B_{t+1},W_t,P_t,\tau_t^n,\tau_t^l,T_t,\chi_t,\psi_t,\mu_t,p_t\right\}_{t=0}^{\infty}} E_0\left\{\sum_{t=0}^{\infty}\beta^t u\left(C_t-\chi\frac{N_t^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}\right)\right\}, \tag{B.1}$$

subject to the resource constraint

$$C_t-\left(1-\tau_t^l\right)Q_tL_t=Y_t-\left(1+\tau_t^n\right)W_tN_t+W_tN_t-G\left(I_t\right)-\left(\xi+\lambda\right)B_t+T_t, \tag{B.2}$$

where $I_t=K_{t+1}-(1-\delta)K_t$, and $L_t=B_{t+1}-(1-\lambda)B_t$, the equity constraint

$$Y_t-\left(1+\tau_t^n\right)W_tN_t-\left(\xi+\lambda\right)B_t\geq P_tI_t-\left(1-\tau_t^l\right)Q_tL_t+\bar{d}, \tag{B.3}$$

the collateral constraint

$$Q_tB_{t+1}\leq\theta_tK_{t+1}, \tag{B.4}$$

the budget constraint

$$T_t=\tau_t^nW_tN_t+\tau_t^lQ_tL_t, \tag{B.5}$$

the implementability constraints, including (i) households' maximization conditions w.r.t. labor, investment, and equity holding, i.e.,

$$W_t=\chi N_t^{\frac{1}{\gamma}}, \tag{B.6}$$

$$P_t = G'(I_t), \tag{B.7}$$

$$p_t = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} (d_{t+1} + p_{t+1}) \right], \tag{B.8}$$

and (ii) firms' maximization conditions w.r.t. labor, capital and bond holding, i.e.,

$$(1-\alpha) z_t K_t^{\alpha} N_t^{-\alpha} = (1 + \tau_t^n) W_t, \tag{B.9}$$

$$P_t = \frac{\theta_t \mu_t}{1+\psi_t} + \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \frac{1+\psi_{t+1}}{1+\psi_t} \left[\alpha \frac{Y_{t+1}}{K_{t+1}} + (1-\delta) P_{t+1} \right] \right\}, \tag{B.10}$$

$$\left(1 - \tau_t^l - \frac{\mu_t}{1+\psi_t}\right) = E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \frac{1+\psi_{t+1}}{1+\psi_t} \left[\frac{(\xi+\lambda) + (1-\lambda)\left(1-\tau_{t+1}^l\right) Q_{t+1}}{Q_t} \right] \right\}, \tag{B.11}$$

and complementary slackness conditions associated with equity and collateral constraints

$$0 = \psi_t \left[Y_t - (1+\tau_t^n) W_t N_t - (\xi+\lambda) B_t - P_t I_t + \left(1-\tau_t^l\right) Q_t L_t - \bar{d} \right], \tag{B.12}$$

$$0 = \mu_t (\theta_t K_{t+1} - Q_t B_{t+1}), \tag{B.13}$$

$$\psi_t \geq 0, \tag{B.14}$$

$$\mu_t \geq 0. \tag{B.15}$$

The relaxed program. In what follows, we first drop the implementability constraint and define a relaxed maximization program where the Ramsey planner chooses allocations $\{C_t, N_t, K_{t+1}, B_{t+1}\}_{t=0}^{\infty}$ only, i.e.,

$$\max_{\{C_t, N_t, K_{t+1}, B_{t+1}\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u \left(C_t - \chi \frac{N_t^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \right) \right\}, \tag{B.16}$$

and the planner needs to respect resource constraint and two occasionally binding constraints,

$$C_t - Q_t L_t = Y_t - G(I_t) - (\xi+\lambda) B_t, \tag{B.17}$$

$$Y_t - \bar{d} - \chi N_t^{1+\frac{1}{\gamma}} - (\xi+\lambda) B_t \geq G'(I_t) I_t - Q_t L_t, \tag{B.18}$$

$$Q_t B_{t+1} \leq \theta_t K_{t+1}, \tag{B.19}$$

where $I_t = K_{t+1} - (1-\delta) K_t$, and $L_t = B_{t+1} - (1-\lambda) B_t$. Let the Lagrangian equation be

$$\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} u\left(C_t - \chi \frac{N_t^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}\right) \\ +\Lambda_t \left[Y_t - G(I_t) - (\xi+\lambda) B_t - C_t + Q_t L_t\right] \\ +\Lambda_t \psi_t \left[G'(I_t) I_t - Q_t L_t + Y_t - \bar{d} - \chi N_t^{1+\frac{1}{\gamma}} + (\xi+\lambda) B_t\right] \\ +\Lambda_t \mu_t \left(\theta_t K_{t+1} - Q_t B_{t+1}\right) \end{array} \right\} \right\}. \tag{B.20}$$

The associated optimality conditions on $\{C_t, N_t, K_{t+1}, B_{t+1}\}$ are given by,

$$(1-\alpha) z_t K_t^{\alpha} N_t^{-\alpha} = \left(1 + \frac{1}{\gamma} \frac{\psi_t}{1+\psi_t}\right) \chi N_t^{\frac{1}{\gamma}}, \tag{B.21}$$

$$\left[1 + \frac{\psi_t}{1+\psi_t} \frac{G''(I_t)}{G'(I_t)} I_t\right] G'(I_t) = \frac{\theta_t \mu_t}{1+\psi_t} + \beta E_t \left\{\frac{\Lambda_{t+1}}{\Lambda_t} \frac{1+\psi_{t+1}}{1+\psi_t} \left[\alpha \frac{Y_{t+1}}{K_{t+1}} + (1-\delta) P_{t+1}\right]\right\}, \tag{B.22}$$

$$\left(1 - \frac{\mu_t}{1+\psi_t}\right) = \beta E_t \left\{\frac{\Lambda_{t+1}}{\Lambda_t} \frac{1+\psi_{t+1}}{1+\psi_t} \frac{(\xi+\lambda) + (1-\lambda) Q_{t+1}}{Q_t}\right\}, \tag{B.23}$$

$$\psi_t \left(Y_t - \chi N_t^{1+\frac{1}{\gamma}} - (\xi+\lambda) B_t - G'(I_t) I_t + Q_t L_t - \bar{d}\right) = 0, \tag{B.24}$$

$$\mu_t \left(\theta_t K_{t+1} - Q_t B_{t+1}\right) = 0. \tag{B.25}$$

Construct the decentralized allocations. We construct the decentralized allocations by setting multipliers using policies, and meanwhile keep all real allocations identical across planner's and decentralized equilibrium. Specifically, we set and later verify

$$\Lambda_t^{ce} = \Lambda_t, \tag{B.26}$$
$$I_t^{ce} = I_t, \tag{B.27}$$
$$N_t^{ce} = N_t, \tag{B.28}$$
$$K_{t+1}^{ce} = K_{t+1}, \tag{B.29}$$
$$B_{t+1}^{ce} = B_{t+1}, \tag{B.30}$$
$$C_t^{ce} = C_t, \tag{B.31}$$
$$\psi_t^{ce} = (1+\zeta_t) \psi_t + \zeta_t, \tag{B.32}$$
$$\mu_t^{ce} = \mu_t, \tag{B.33}$$

where ζ_t satisfies

$$\zeta_t = \frac{\psi_t}{1+\psi_t}\frac{G''(I_t)}{G'(I_t)}I_t + \beta E_t\left\{\frac{\Lambda_{t+1}}{\Lambda_t}\frac{1+\psi_{t+1}}{1+\psi_t}\zeta_{t+1}\frac{\alpha\frac{Y_{t+1}}{K_{t+1}} + (1-\delta)G'(I_{t+1})}{G'(I_t)}\right\}, \tag{B.34}$$

and the policies to implement these allocations are

$$\tau_t^n = \frac{1}{\gamma}\frac{\psi_t}{1+\psi_t}, \tag{B.35}$$

$$\begin{aligned}\tau_t^l = \frac{\zeta_t}{1+\zeta_t} &- \beta E_t\left\{\frac{\Lambda_{t+1}}{\Lambda_t}\frac{1+\psi_{t+1}}{1+\psi_t}\left[\frac{\zeta_{t+1}}{1+\zeta_t}\frac{(\xi+\lambda)+(1-\lambda)Q_{t+1}}{Q_t}\right]\right\} \\ &+ \beta E_t\left\{\frac{\Lambda_{t+1}}{\Lambda_t}\frac{1+\psi_{t+1}}{1+\psi_t}\frac{1+\zeta_{t+1}}{1+\zeta_t}\frac{(1-\lambda)\tau_{t+1}^l Q_{t+1}}{Q_t}\right\}.\end{aligned} \tag{B.36}$$

Verification. We verify that the constructed decentralized allocations satisfy agents' optimality conditions (i.e., it is indeed an competitive equilibrium). First, households' and capital producers' optimality conditions (i.e., constraints (B.6), (B.7), and (B.8)) can be shown to be slack as agents set

$$W_t^{ce} = \chi N_t^{\frac{1}{\gamma}}, \tag{B.37}$$

$$P_t^{ce} = G'(I_t), \tag{B.38}$$

$$p_t^{ce} = \beta E_t\left[\frac{\lambda_{t+1}^{ce}}{\lambda_t^{ce}}\left(d_{t+1}^{ce} + p_{t+1}^{ce}\right)\right]. \tag{B.39}$$

Entrepreneurs' optimality conditions are also slack. Note that their labor demand is satisfied from equation (B.35). For capital demand, note that the competitive equilibrium is

$$(1+\psi_t^{ce})P_t = \theta_t\mu_t^{ce} + \beta E_t\left\{\frac{\Lambda_{t+1}}{\Lambda_t}\left(1+\psi_{t+1}^{ce}\right)\left[\alpha\frac{Y_{t+1}}{K_{t+1}} + (1-\delta)P_{t+1}\right]\right\} \tag{B.40}$$

$$\Updownarrow$$

$$(1+(1+\zeta_t)\psi_t+\zeta_t)P_t = \theta_t\mu_t + \beta E_t\left\{\frac{\Lambda_{t+1}}{\Lambda_t}\left(1+(1+\zeta_{t+1})\psi_{t+1}+\zeta_{t+1}\right)\left[\alpha\frac{Y_{t+1}}{K_{t+1}} + (1-\delta)P_{t+1}\right]\right\} \tag{B.41}$$

$$\Updownarrow$$

$$(1+\zeta_t)P_t = \frac{\theta_t\mu_t}{1+\psi_t} + \beta E_t\left\{\frac{\Lambda_{t+1}}{\Lambda_t}\frac{1+\psi_{t+1}}{1+\psi_t}(1+\zeta_{t+1})\left[\alpha\frac{Y_{t+1}}{K_{t+1}} + (1-\delta)P_{t+1}\right]\right\} \tag{B.42}$$

$$\Updownarrow$$

$$\left[1+\frac{\psi_t}{1+\psi_t}\frac{G''(I_t)}{G'(I_t)}I_t\right]G'(I_t) = \frac{\theta_t\mu_t}{1+\psi_t} + \beta E_t\left\{\frac{\Lambda_{t+1}}{\Lambda_t}\frac{1+\psi_{t+1}}{1+\psi_t}\left[\alpha\frac{Y_{t+1}}{K_{t+1}} + (1-\delta)P_{t+1}\right]\right\}, \tag{B.43}$$

which is the planner's capital Euler equation. In the first step, we plug in equation (B.32). In the second step, we divide both sides by $(1+\psi_t)$. In the last step, we use equation (B.34).

Then, the planner's bond Euler equation construction can be shown to be consistent with that of the competitive equilibrium due to the construction of τ_t^l in equation (B.36). To see this, note that for competitive equilibrium

$$\left(1-\tau_t^l\right)\left(1+\psi_t^{ce}\right)-\mu_t^{ce}=\beta E_t\left\{\frac{\Lambda_{t+1}}{\Lambda_t}\left(1+\psi_{t+1}^{ce}\right)\frac{(\xi+\lambda)+(1-\lambda)\left(1-\tau_{t+1}^l\right)Q_{t+1}}{Q_t}\right\} \tag{B.44}$$

$$\Updownarrow$$

$$\left(1-\tau_t^l\right)\left(1+\zeta_t\right)-\frac{\mu_t}{1+\psi_t}=\beta E_t\left\{\frac{\Lambda_{t+1}}{\Lambda_t}\frac{1+\psi_{t+1}}{1+\psi_t}\left(1+\zeta_{t+1}\right)\frac{(\xi+\lambda)+(1-\lambda)\left(1-\tau_{t+1}^l\right)Q_{t+1}}{Q_t}\right\} \tag{B.45}$$

$$\Updownarrow$$

$$1-\frac{\mu_t}{1+\psi_t}=\beta E_t\left\{\frac{\Lambda_{t+1}}{\Lambda_t}\frac{1+\psi_{t+1}}{1+\psi_t}\frac{(\xi+\lambda)+(1-\lambda)Q_{t+1}}{Q_t}\right\}, \tag{B.46}$$

where the first step plugs in equation (B.32), and the last step uses equation (B.36).

Finally, the equity and collateral constraints are also satisfied as the planner respects them. By construction in equation (B.32) and (B.33), the multipliers of the equity and collateral constraints are both positive, so that equations (B.12)∼(B.15) are satisfied.

B.2 Proof of Corollary (2)

Note that from the above proof, the relaxed program is equivalent to the full program. Since the relaxed program is time consistent, the full program is time consistent. Therefore, the optimal macroprudential policies are time consistent as well.

B.3 Intuition of Two-Sided Result

Panel B of Figure 4 shows when both the equity and collateral constraints bind, firms underinvest. To explain this result, we restrict our attention to a special case in which capital fully depreciates (i.e., $\delta=1$) and firms cannot borrow. Identifying the region in which the competitive equilibrium overinvests is relatively straightforward: comparing equations (35) and (42), one can see that the non-internalized capital price effect implies that investment is higher in competitive equilibrium. When both constraints bind, the competitive equilibrium solution of investment K_{t+1} is determined by

$$\eta K_{t+1}^{1+\frac{1}{\nu}}-\theta_t K_{t+1}=z_t K_t^\alpha N_t^{1-\alpha}-W_t N_t-\bar{d}, \tag{B.47}$$

where the optimal labor demand is $N_t=\left[\left((1-\alpha)/\chi\right)z_t K_t^\alpha\right]^{\frac{\gamma}{1+\alpha\gamma}}$. The planner's solution for K_{t+1}^{sp} is given by the same conditions,

$$\eta\left(K_{t+1}^{sp}\right)^{1+\frac{1}{\nu}}-\theta_t K_{t+1}^{sp}=z_t\left(K_t^{sp}\right)^\alpha\left(N_t^{sp}\right)^{1-\alpha}-\chi\left(N_t^{sp}\right)^{1+\frac{1}{\gamma}}-\bar{d}, \tag{B.48}$$

except that her labor is determined by $N_t^{sp} = \left[(1-\alpha)z_t\left(K_t^{sp}\right)^{\alpha} / \left(\chi\left(1+\frac{1}{\gamma}\frac{\psi_t^{sp}}{1+\psi_t^{sp}}\right)\right)\right]^{\frac{\gamma}{1+\alpha\gamma}}$. Note that the solution of N_t and N_t^{sp} implies that the production revenue (i.e., net payroll) is larger in the planner's case than in the competitive equilibrium (note that revenue reaches its maximum at $N_t^* = [((1-\alpha)/(\chi(1+1/\gamma)))z_t(K_t^{sp})^{\alpha}]^{\frac{\gamma}{1+\alpha\gamma}}$),

$$z_t\left(K_t^{sp}\right)^{\alpha}\left(N_t^{sp}\right)^{1-\alpha} - \chi\left(N_t^{sp}\right)^{1+\frac{1}{\gamma}} > z_t K_t^{\alpha} N_t^{1-\alpha} - \chi N_t^{1+\frac{1}{\gamma}}, \tag{B.49}$$

so that when K is large enough, the function $\eta K^{1+\frac{1}{\nu}} - \theta K$ is increasing, and, according to equations (B.47) and (B.48),

$$K_{t+1} < K_{t+1}^{sp}, \tag{B.50}$$

and given the binding borrowing constraints,

$$B_{t+1} < B_{t+1}^{sp}. \tag{B.51}$$

In other words, firms' individually optimal hiring decisions drive up equilibrium wages, reduce wealth, and, consequently, lead to lower investment when the collateral constraint binds.

B.4 Three-Period Model

In this subsection, we consider a three-period model $(t, t+1, t+2)$ that inherits the dynamic version to illustrate how taxes on new credit issuance affect real terms. In the following discussion, we restrict our attention to only taxes on credit issuance (i.e., τ^l), and hence set taxes on payroll to zero (i.e., $\tau^n = 0$). To get a closed-form solution, we assume that entrepreneurs have a linear utility function and maximize their life-time utility

$$d_t + \beta d_{t+1} + \beta^2 d_{t+2}, \tag{B.52}$$

subject to budget constraints,

$$d_t + P_t i_t = y_t - W_t n_t - (\xi+\lambda) b_t + (1-\tau_t^l) Q_t l_t, \tag{B.53}$$

$$d_{t+1} + P_{t+1} i_{t+1} = y_{t+1} - W_{t+1} n_{t+1} - (\xi+\lambda) b_{t+1} + (1-\tau_{t+1}^l) Q_{t+1} l_{t+1}, \tag{B.54}$$

$$d_{t+2} = y_{t+2} - W_{t+2} n_{t+2} - (\xi+\lambda) b_{t+2}, \tag{B.55}$$

$$b_{t+1} = (1-\lambda) b_t + l_t, \tag{B.56}$$

$$b_{t+2} = (1-\lambda) b_{t+1} + l_{t+1}, \tag{B.57}$$

$$k_{t+1} = (1-\delta) k_t + i_t, \tag{B.58}$$

$$k_{t+2} = (1-\delta) k_{t+1} + i_{t+1}, \tag{B.59}$$

where $y_{t+j} = z_{t+j} k_{t+j}^{\alpha} n_{t+j}^{1-\alpha}, \forall j \in \{0,1,2\}$ and occasionally binding constraints,

$$Q_t b_{t+1} \leq \theta_t k_{t+1}, \tag{B.60}$$

$$0 = d_t, \tag{B.61}$$

$$Q_{t+1} b_{t+2} = \theta_{t+1} k_{t+2}, \tag{B.62}$$

$$0 = d_{t+1}. \tag{B.63}$$

To highlight the intuition, we further assume that only the equity constraint binds at time t and both constraints bind at time $t+1$.[15] At time $t+2$, neither of the constraints binds so that firms pay dividend only in this last period. In what follows, we derive firms' optimal investment decision at time t (i.e., i_t or equivalently k_{t+1}) and show how the capital price and the investment are determined in equilibrium.

Due to the Cobb-Douglas technology, for any given wage rate W_{t+j}, firms' labor demand is a linear function of the capital holding, i.e.,

$$n_{t+j} = \left[\frac{W_{t+j}}{(1-\alpha) z_{t+j}} \right]^{-\frac{1}{\alpha}} k_{t+j}, \quad \forall j \in \{0,1,2\}, \tag{B.64}$$

which implies that the budget constraints (B.53), (B.54) and (B.55) can be simplified into

$$d_t + P_t i_t = R_t k_t - (\xi + \lambda) b_t + (1 - \tau_t^l) Q_t l_t, \tag{B.65}$$

$$d_{t+1} + P_{t+1} i_{t+1} = R_{t+1} k_{t+1} - (\xi + \lambda) b_{t+1} + (1 - \tau_{t+1}^l) Q_{t+1} l_{t+1}, \tag{B.66}$$

$$d_{t+2} = R_{t+2} k_{t+2} - (\xi + \lambda) b_{t+2}, \tag{B.67}$$

where $R_{t+j} \equiv \frac{\alpha}{1-\alpha} (1-\alpha)^{\frac{1}{\alpha}} z_{t+j}^{\frac{1}{\alpha}} W_{t+j}^{\frac{\alpha-1}{\alpha}}, \forall j \in \{0,1,2\}$. Plugging equations (B.56)∼(B.59) into (B.65)∼(B.67) and combining the results, we have

$$\begin{aligned} d_{t+2} = & \left[R_{t+2} - \frac{(\xi+\lambda)\theta_{t+1}}{Q_{t+1}} \right] \left[\frac{R_{t+1} + (1-\delta) P_{t+1}}{P_{t+1} - \theta_{t+1}} - \frac{P_t}{Q_t} \cdot \frac{(\xi+\lambda) + (1-\tau_{t+1}^l)(1-\lambda) Q_{t+1}}{P_{t+1} - \theta_{t+1}} \right] \cdot k_{t+1} \\ & + \left[R_{t+2} - \frac{(\xi+\lambda)\theta_{t+1}}{Q_{t+1}} \right] \frac{(\xi+\lambda) + (1-\tau_{t+1}^l)(1-\lambda) Q_{t+1}}{P_{t+1} - \theta_{t+1}} \cdot \frac{R_t + (1-\delta) P_t}{Q_t} \cdot k_t \\ & - \left[R_{t+2} - \frac{(\xi+\lambda)\theta_{t+1}}{Q_{t+1}} \right] \frac{(\xi+\lambda) + (1-\tau_{t+1}^l)(1-\lambda) Q_{t+1}}{P_{t+1} - \theta_{t+1}} \cdot \frac{(\xi+\lambda) + (1-\tau_t^l)(1-\lambda) Q_t}{Q_t} \cdot b_t. \end{aligned} \tag{B.68}$$

[15] This is the case when θ_t is relatively high but θ_{t+1} is low. Correspondingly, due to the correlation between Q_t and θ_t, Q_t is relatively low, but Q_{t+1} is high.

where d_{t+2} is linear in k_{t+1}. The optimal choice of k_{t+1} is

$$k_{t+1} = \begin{cases} \infty & \text{if} \quad \hat{P}_t < x^*_{t+1} \\ \forall a \in \mathbb{R} & \text{if} \quad \hat{P}_t = x^*_{t+1} \\ 0 & \text{if} \quad \hat{P}_t > x^*_{t+1} \end{cases} . \tag{B.69}$$

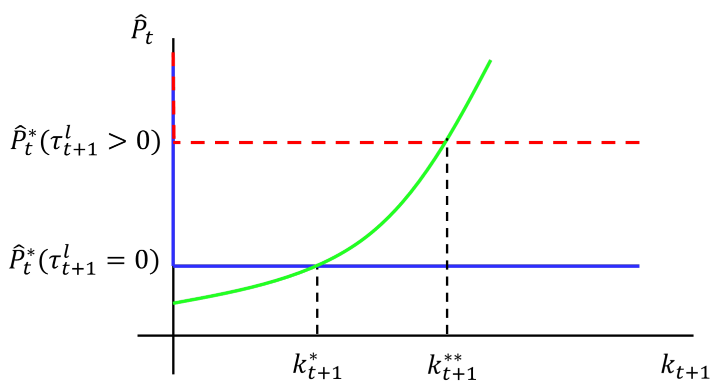
where $\hat{P}_t \equiv P_t/Q_t$ is the price of capital relative to bond, and

$$x^*_{t+1} \equiv \frac{R_{t+1} + (1-\delta)P_{t+1}}{(\xi+\lambda) + (1-\tau^l_{t+1})(1-\lambda)Q_{t+1}} \tag{B.70}$$

is the cut-off point which is increasing in the credit issuance tax at time $t+1$, i.e., τ^l_{t+1}.[16]

We then plot the relationship between k_{t+1} and $\hat{P}_t$ with and without policy interventions at time $t+1$ in Figure B.1 below. We show that a tax on credit issuance at time $t+1$ (i.e., a positive τ^l_{t+1}) will result in an increase of the cut-off point, and hence the capital demand curve will move upward (from blue solid line to red dash line). Given an upward sloping capital supply curve (derived from the capital producer), the equilibrium level of capital will grow from k^*_{t+1} to k^{**}_{t+1}. This implies that the *ex ante* investment incentive is positively correlated with the *ex post* intervention on interest rate. Notice that the effect of τ^l_{t+1} depends on the fact that the firm can borrow long-term debts (i.e., $0 < \lambda < 1$). Once only short-term debts are available (i.e., $\lambda = 1$), this effect will disappear. The above results echo the implications of Equation (49).

Figure B.1: Illustration of the Three-Period Model



Note: This figure shows how equilibrium of capital is determined. The green solid curve represents the capital supply. The blue solid curve represents the capital demand without policy interventions. The red dash curve represents the capital demand with a tax on credit issuance.

[16] Acccording to the assumptions mentioned previously, bond price Q_{t+1} is high enough and credit condition θ_{t+1} is low enough. Thus, $P_{t+1} > \theta_{t+1}$ and $Q_{t+1}R_{t+2} > (\xi+\lambda)\theta_{t+1}$ can always hold. In addition, to ensure that d_{t+2} will not be negative when $k_{t+1} = 0$, we can further require that $\left[R_t + (1-\delta)P_t\right] k_t \geq \left[(\xi+\lambda) + (1-\tau^l_t)(1-\lambda)Q_t\right] b_t$.

C Quantitative Appendix

In this section, we first summarize conditions for the competitive equilibrium and then describe detailed algorithms. Other algorithms for solving the constrained efficiency and optimal policies are omitted here for neatness, and one can refer to the attached code.

C.1 Summary Conditions

We use $\mathcal{S} = \{K, B, z, \theta, Q\}$ to denote a set of state variables in our model. The competitive equilibrium consists of seven allocation functions $Y(\mathcal{S}), C(\mathcal{S}), N(\mathcal{S}), K^{'}(\mathcal{S}), B^{'}(\mathcal{S}), I(\mathcal{S}), D(\mathcal{S})$ that are, respectively, output, consumption, hours, capital, bond holding, investment, and dividend; two price functions $W(\mathcal{S}), P(\mathcal{S})$ that are wage rate and capital price; two functions for multipliers $\{\psi(\mathcal{S}), \mu(\mathcal{S})\}$ of the equity constraint and the collateral constraint; and one auxiliary function $\{\Lambda(\mathcal{S})\}$ that is marginal utility of households. These twelve variables satisfy the following twelve conditions, i.e.,

$$N = \left[\frac{(1-\alpha)zK^{\alpha}}{\chi}\right]^{\frac{\gamma}{1+\alpha\gamma}}, \tag{C.1}$$

$$W = \chi N^{\frac{1}{\gamma}}, \tag{C.2}$$

$$Y = zK^{\alpha}N^{1-\alpha}, \tag{C.3}$$

$$I = \left(\frac{P}{\eta}\right)^{v}, \tag{C.4}$$

$$K' = I + (1-\delta)K, \tag{C.5}$$

$$C = Y - \eta\frac{I^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} + QB^{'} - \left[(\xi+\lambda) + (1-\lambda)Q\right]B, \tag{C.6}$$

$$D = C + \eta\frac{I^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} - \chi N^{1+\frac{1}{\gamma}} - PI, \tag{C.7}$$

$$\Lambda = \left(C - \chi\frac{N^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}\right)^{-1}, \tag{C.8}$$

$$P - \frac{\theta\mu}{1+\psi} = \beta\mathbb{E}\left\{\frac{\Lambda^{'}}{\Lambda}\frac{1+\psi'}{1+\psi}\left[\alpha\frac{Y'}{K'} + (1-\delta)P'\right]\right\}, \tag{C.9}$$

$$1 - \frac{\mu}{1+\psi} = \frac{\beta}{Q}\mathbb{E}\left\{\frac{\Lambda^{'}}{\Lambda}\frac{1+\psi'}{1+\psi}\left[(\xi+\lambda) + (1-\lambda)Q'\right]\right\}, \tag{C.10}$$

$$0 = \mu\left(\theta K' - QB'\right), \tag{C.11}$$

$$0 = \psi\left(D - \bar{d}\right). \tag{C.12}$$

C.2 Solution Algorithms

C.2.1 Feasible State Space

Step 1. Define a grid for shock triples, $X = \left\{\varepsilon^z, \varepsilon^\theta, \varepsilon^Q\right\} \in \mathbf{X}$ where $\mathbf{X}$ has 8 possible values since each shock has binary realizations. Define grids with 100 nodes for bonds and with 50 nodes for capital, i.e., $\mathbf{B} = \{B^1 < B^2 < \ldots < B^{100}\}$, $\mathbf{K} = \{K^1 < K^2 < \ldots < K^{50}\}$. The state space contains $100 \times 50 \times 8$ elements and is defined by points $(B, K, X) \in \mathbf{B} \otimes \mathbf{K} \otimes \mathbf{X}$.

Step 2. For iteration j, guess a feasibility boundary $\hat{g}_j(K)$, which reflects the relationship between bond holdings B and capital K and does not rely on shock triples X. An initial guess could be an arbitrary constant, e.g., $\hat{g}_0(K) = 100$.

Step 3. Given each state $(B, K, X) \in \mathbf{B} \otimes \mathbf{K} \otimes \mathbf{X}$, use equations (C.4), (C.5), (C.6), and (C.7) (replacing D by the minimum equity payment $\bar{d}$) and the borrowing constraint to determine the maximum level of debt that firms can issue. Notice that $Y(B, K, X)$, $N(B, K, X)$, Q and θ are determined as long as the state (B, K, X) is specified, but $B^{'}$ and $K^{'}$ should be determined by

$$B^{\mathrm{m}}_{j+1}(\cdot) = \max_{\left\{B^{'}, K^{'}\right\}} \left\{ \frac{Y(\cdot) - \eta\left[K^{'} - (1-\delta)K\right]^{\frac{1}{v}+1} - \chi N(\cdot)^{1+\frac{1}{\gamma}} + QB^{'} - \bar{d}}{(\xi + \lambda) + (1-\lambda)Q} \right\} \tag{C.13}$$

$$\text{s.t.} \quad QB^{'} \leq \theta K^{'} \tag{C.14}$$

$$B^{'} \leq \hat{g}_j\left(K^{'}\right) \tag{C.15}$$

The notation $(\cdot)$ is used to simplify (B, K, X). Use B^1 and $(1-\delta)K$ as the initial guess for $B^{'}$ and $K^{'}$. Linear interpolation should be used since $K^{'}$ is not necessary in $\mathbf{K}$.

Step 4. The new boundary function is then defined as

$$g_{j+1}(K) = \max\left\{B \in \mathbf{B} : B \leq \min_s \left[B^{\mathrm{m}}_{j+1}(\cdot)\right]\right\}. \tag{C.16}$$

Step 5. Update the guess for the boundary function by a convex combination,

$$\hat{g}_{j+1}(K) = (1-\rho_g) \cdot \hat{g}_j(K) + \rho_g \cdot g_{j+1}(K), \tag{C.17}$$

where ρ_g controls the learning step. Then, move to **Step 2** and let $\hat{g}_j(K) = \hat{g}_{j+1}(K)$ until the difference between $\hat{g}_j(K)$ and $g_{j+1}(K)$ is sufficiently small, i.e.,

$$\|\hat{g}_j(K) - g_{j+1}(K)\| < \varsigma_g = 10^{-4}. \tag{C.18}$$

Step 6. Let $g(K)$ denote the converged feasibility boundary function, and let $\overline{K}^{'}(K)$ denote the corresponding solution of next period's capital. Keep the structure of the capital grid but redefine the bond grid by subtracting $g(K)$, i.e., $\tilde{B} = B - g(K)$. Therefore, for each state K, the upper bound for normalized bond is equal to 0, making the feasible state space an exact rectangular and easy to be interpolated. The normalized bond grid is defined as $\tilde{\mathbf{B}} = \left\{\tilde{B}^1 < \tilde{B}^2 < \ldots < \tilde{B}^{100} = 0\right\}$. All elements $(\tilde{B}, K, X)$ in this new state space $\tilde{\mathbf{B}} \otimes \mathbf{K} \otimes \mathbf{X}$ are feasible. Now, use $(\cdot)$ as an abbreviation of $(\tilde{B}, K, X)$.

C.2.2 Auxiliaries and Initial Guesses

Step 1. Define the following two auxiliary variables, $\mathcal{K}$ and $\mathcal{B}$, that will be used as main ingredients in the main loop of the following algorithm, i.e.,

$$\mathcal{K} \equiv \beta \mathbb{E}\left\{\Lambda^{'}(1+\psi')\left[\alpha \frac{Y'}{K'} + (1-\delta)P'\right]\right\}, \tag{C.19}$$

$$\mathcal{B} \equiv \frac{\beta}{Q}\mathbb{E}\left\{\Lambda^{'}(1+\psi')\left[(\xi+\lambda) + (1-\lambda)Q'\right]\right\}. \tag{C.20}$$

Equations (C.9) and (C.10) are transformed into

$$\left[(1+\psi)P - \theta\mu\right]\Lambda = \mathcal{K}, \tag{C.21}$$

$$(1+\psi-\mu)\Lambda = \mathcal{B}. \tag{C.22}$$

Step 2. Consider an extreme case where, for each $(\tilde{B}, K, X)$, the conjectured policy functions for capital and for bond holding are

$$K_0^{'}(\cdot) = \overline{K}^{'}(K), \tag{C.23}$$

$$\tilde{B}_0^{'}(\cdot) = \min\left\{\frac{\theta}{Q}K_0^{'}(\cdot), g\left(K_0^{'}(\cdot)\right)\right\} - g\left(K_0^{'}(\cdot)\right), \tag{C.24}$$

which means that the capital of the next period always takes the value when the bond hits the feasibility boundary, and the bond holding of the next period is restricted either by the feasibility boundary or by the borrowing constraint.

Step 3. Given $K_0^{'}(\cdot)$ and $\tilde{B}_0^{'}(\cdot)$, $I_0(\cdot)$, $P_0(\cdot)$, $C_0(\cdot)$ and $\Lambda_0(\cdot)$ are sequentially determined by equations (C.5), (C.4), (C.6), and (C.8).

Step 4. According to equations (C.19) and (C.20), use standard bi-linear interpolation method to obtain the initial guesses for two auxiliary variables, i.e., $\hat{\mathcal{K}}_0$ and $\hat{\mathcal{B}}_0$.

C.2.3 Solve for Competitive Equilibrium

Step 1. Since $N(\cdot)$, $W(\cdot)$, and $Y(\cdot)$ only depend on the state $(\tilde{B}, K, X)$, we can calculate them before the main loop. **(Main loop)** Start iteration j with the guesses $\hat{\mathcal{K}}_j$ and $\hat{\mathcal{B}}_j$. The initial guesses are $\hat{\mathcal{K}}_0$ and $\hat{\mathcal{B}}_0$ obtained from previous sector. For each $(\tilde{B}, K, X)$ in the state space, consider **case 1**: assume $\mu_{j+1}(\cdot) = 0$ and $\psi_{j+1}(\cdot) = 0$; $P_{j+1}(\cdot)$ is given by $P_{j+1}(\cdot) = \hat{\mathcal{K}}_j/\hat{\mathcal{B}}_j$; $I_{j+1}(\cdot)$ and $K_{j+1}^{'}(\cdot)$ are given by equations (C.4) and (C.5); $\Lambda_{j+1}(\cdot)$ is given by $\Lambda_{j+1}(\cdot) = \hat{\mathcal{B}}_j$; $C_{j+1}(\cdot)$ is given by equation (C.8); $\tilde{B}_{j+1}^{'}(\cdot)$ and $D_{j+1}(\cdot)$ are given by equations (C.6) and (C.7).

Step 2. Check two constraints. If (i) neither of the constraint binds and (ii) $\tilde{B}_{j+1}^{'}(\cdot) \leq 0$, then set flag $= 0$ and move to step 6. Otherwise, go to case 2: assume $\mu_{j+1}(\cdot) = 0$ and $\psi_{j+1}(\cdot) > 0$; $P_{j+1}(\cdot)$ is still given by $P_{j+1}(\cdot) = \hat{\mathcal{K}}_j/\hat{\mathcal{B}}_j$; $I_{j+1}(\cdot)$ and $K_{j+1}^{'}(\cdot)$ are given by equations (C.4) and (C.5); $C_{j+1}(\cdot)$ and $\Lambda_{j+1}(\cdot)$ are given by equations (C.7) and (C.8); $\tilde{B}_{j+1}^{'}(\cdot)$ is given by equation (C.6); $\psi_{j+1}(\cdot)$ is given by $\psi_{j+1}(\cdot) = \hat{\mathcal{B}}_j/\Lambda_{j+1}(\cdot) - 1$.

Step 3. Check the borrowing constraint. If (i) it does not bind, (ii) $\tilde{B}_{j+1}^{'}(\cdot) \leq 0$, and (3) the equity constraint binds in **case 1**, then set flag $= 1$ and move to step 6. Otherwise, go to case 3: assume $\mu_{j+1}(\cdot) > 0$ and $\psi_{j+1}(\cdot) = 0$; there are seven unknowns $\mu_{j+1}(\cdot)$, $P_{j+1}(\cdot)$, $I_{j+1}(\cdot)$, $K_{j+1}^{'}(\cdot)$, $\tilde{B}_{j+1}^{'}(\cdot)$, $C_{j+1}(\cdot)$, and $\Lambda_{j+1}(\cdot)$ which can be solved from a system of seven nonlinear equations including (C.4), (C.5), (C.6), (C.8), (C.21), (C.22) and the borrowing constraint; $D_{j+1}(\cdot)$ is given by equation (C.7).

Step 4. Check the equity constraint. If (1) it does not bind, (2) $\tilde{B}_{j+1}^{'}(\cdot) \leq 0$, (3) the borrowing constraint binds in **case 1**, and (4) flag $\neq 1$, then set flag $= 2$ and move to step 6. Otherwise, go to case 4: assume $\mu_{j+1} > 0$ and $\psi_{j+1} > 0$; $D_{j+1}(\cdot) = \bar{d}$ and $Q\left(\tilde{B}^{'}(\cdot) + g(K^{'}(\cdot))\right) = \theta K^{'}(\cdot)$ must hold; there are four unknowns $K_{j+1}^{'}(\cdot)$, $I_{j+1}(\cdot)$, $P_{j+1}(\cdot)$, and $C_{j+1}(\cdot)$ which can be solved from four nonlinear equations (C.4), (C.5), (C.6), and (C.7); $\tilde{B}^{'}(\cdot)$ is given by the binding borrowing constraint; $\Lambda_{j+1}(\cdot)$ is given by equation (C.8); $\mu_{j+1}(\cdot)$ and $\psi_{j+1}(\cdot)$ are solved from equations (C.21) and (C.22).

Step 5. Set flag = 3 and move to to step 6.

Step 6. Solve for new auxiliaries $\mathcal{K}_{j+1}$ and $\mathcal{B}_{j+1}$ using standard bi-linear interpolation.

Step 7. Update guesses for the auxiliary variables by a convex combination,

$$\hat{\mathcal{K}}_{j+1} = (1 - \rho_{\mathcal{K}}) \cdot \hat{\mathcal{K}}_j + \rho_{\mathcal{K}} \cdot \mathcal{K}_{j+1}, \tag{C.25}$$

$$\hat{\mathcal{B}}_{j+1} = (1 - \rho_{\mathcal{B}}) \cdot \hat{\mathcal{B}}_j + \rho_{\mathcal{B}} \cdot \mathcal{B}_{j+1}. \tag{C.26}$$

Then set $\hat{\mathcal{K}}_j = \hat{\mathcal{K}}_{j+1}$ and $\hat{\mathcal{B}}_j = \hat{\mathcal{B}}_{j+1}$, and move back to step 1 until

$$\left\| \hat{\mathcal{K}}_j - \mathcal{K}_{j+1} \right\| < \varsigma_{\mathcal{K}} = 10^{-4}, \tag{C.27}$$

$$\left\| \hat{\mathcal{B}}_j - \mathcal{B}_{j+1} \right\| < \varsigma_{\mathcal{B}} = 10^{-4}. \tag{C.28}$$