

**VIETNAM NATIONAL UNIVERSITY, HO CHI MINH CITY**

**HO CHI MINH CITY UNIVERSITY OF TECHNOLOGY**

**OFFICE FOR INTERNATIONAL STUDY PROGRAMS**



**FACULTY OF TRANSPORTATION**

**Bachelor Project**

**Modelling and simulation using Matlab/Simulink and its  
applications in Electric Power Steering system.**

**Instructor: Ngô Đức Việt**

**Class: TR4091 – CC01**

**Name: Trịnh Tiến Long**

**Student ID: 1852047**

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## **I/ Introduction:**

### **1) Objective:**

During the past ten years, EPS has been introduced in gradually increasing numbers. Although electric power steering system offer significant advantages over their hydraulic counterparts, electric motor technology and controls had not reached the point where they could be used in this application until just recently. Electrically assisted power steering is replacing the traditional hydraulic system where the pressure is provided via a pump driven by the vehicles engine. The hydraulic system is constantly running and by using the EPS the fuel consumption can be reduced. In electric and hybrid vehicles, the engine does not run continuously so electric power steering is the only possible solution.

Simulink is a simulation and model-based design environment for dynamic and embedded systems, which are integrated with MATLAB. Simulink was developed by a computer software company MathWorks. Furthermore, it allows to incorporate MATLAB algorithms into models as well as export the simulation results into MATLAB for further analysis.

In Simulink, it is very straightforward to represent and then simulate a mathematical model representing a physical system. Models are represented graphically in Simulink as block diagrams. A wide array of blocks are available to the user in provided libraries for representing various phenomena and models in a range of formats. One of the primary advantages of employing Simulink (and simulation in general) for the analysis of dynamic systems is that it allows us to quickly analyze the response of complicated systems that may be prohibitively difficult to analyze analytically. Simulink is able to numerically approximate the solutions to mathematical models that we are unable to, or don't wish to, solve "by hand."

In general, the mathematical equations representing a given system that serve as the basis for a Simulink model can be derived from physical laws. The focus of this project is that we can get used to MATLAB Simulink with some examples and then apply that knowledge to simulate a simple Electric Power Steering system that usually been used in modern vehicles.

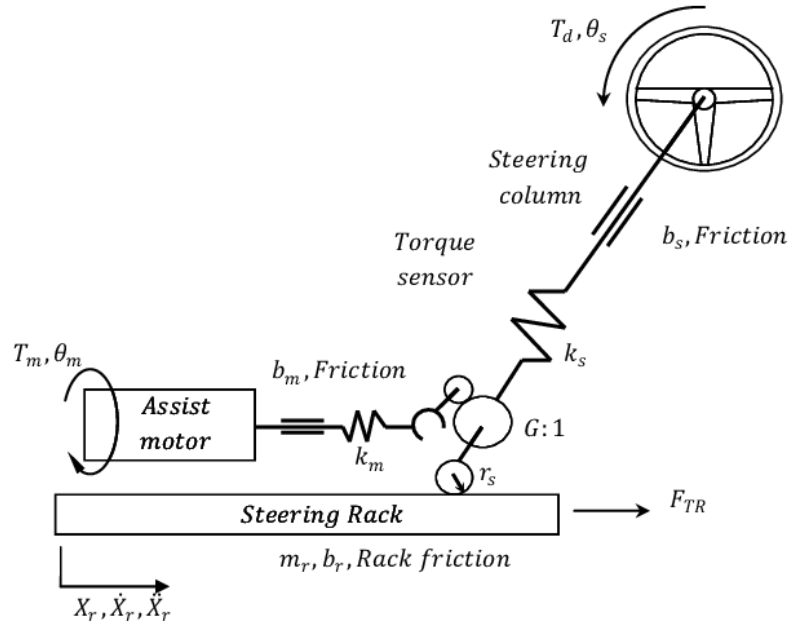


Figure 1: Electric Power Steering system structure

2) Scope of implementation:

First part of this project focus on learning how to use MATLAB Simulink with some physical model: The Mass-Spring-Damper model

Last part of this project focus on Electric Power Steering model which developed to present vehicle behaviour when driving in normal condition of roads and cars, so it may not be reliable in non-linear conditions (When the vehicle is driven up to its limits). The model developed in this project does not represent the steering condition in parking situations . The model is developed by assuming that the wheels are in contact with the road surface. So, the wheel lift phenomenon is assumed negligible in this model.

3) Working condition:

Continuously change to adapt with various conditions.

4) Technical requirement:

Working normally in above condition.

5) Limitation

This project skips on the affect of motor that will be finish in next stage of project.

## II/ Theoretical basics:

### 1. Mass damper theory:

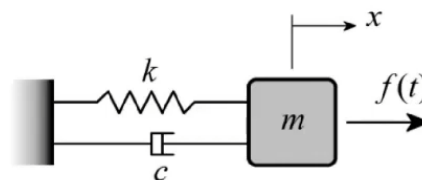


Figure 2: Mass - damper model

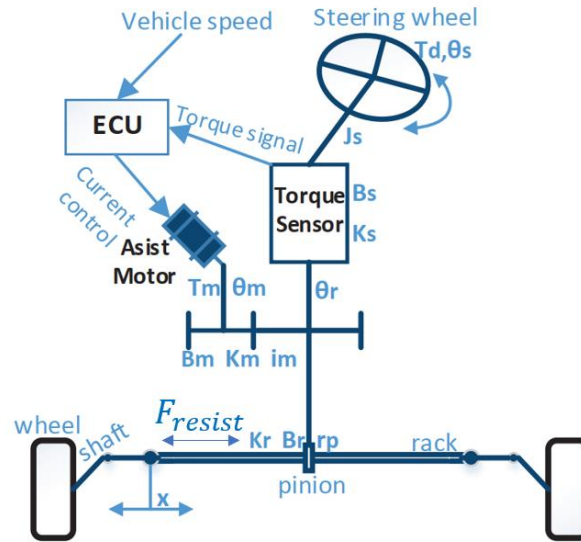
The Mass-Spring-Damper model is one of the most common models used by engineers to model kinematic systems. From human tissue to bridges, this straightforward model features three mechanisms and can be summarized as the following second-order differential equation: Here  $x$  represents the displacement of the object with mass  $m$  away from its resting position. The

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

$$\Rightarrow \ddot{x} = \frac{1}{m}(f(t) - c\dot{x} - kx)$$

mass is subject to some spring force characterized by spring constant  $k$  and a damping force that resists change in motion with damping coefficient  $c$ . The function  $g$  here can be thought of as some input to the system that could depend on position, velocity, or time. For this example, we can vary  $f(t)$  as an input.

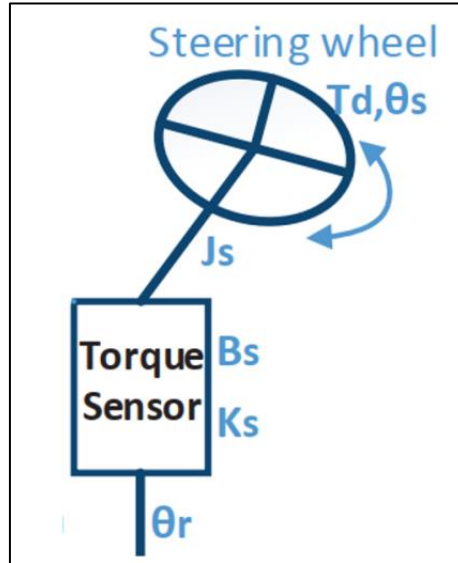
## 2. Electric Power Steering Dynamic equations:



**Figure 3:** The overall Electric Power Steering dynamic factors

**Figure 3** illustrates the physical structure of a steering system. The structure components are a column type steering system which include the steering wheel, steering column, the rack and the pinion mechanism. The assistance motor is a permanent magnet synchronous motor, connected to the steering shaft through gears and provides the assisting torque needed by the driver to steer the vehicle. The input torque from the steering wheel is measured by a torque sensor mounted on the steering column and connected to the electronic control unit. The assistance torque produced by the motor act on the wheel via rack and pinion system. Different amount of assistance torque is applied depends on the driving conditions, which is realized with a specific control logic implemented in the ECU.

Using Newton's law and neglecting no necessary factors the equations of EPS can be derived:



**Figure 4:** Overall structure of the steering input

- The dynamic equations from the steering wheel to steering column:

$$J_s \times \frac{d^2 \theta_s}{dt} = T_d - K_s(\theta_s - \theta_r) - B_s \times \frac{d\theta_s}{dt}$$

While:

$J_s$ : Inertia of steering wheel and steering column ( $\text{kg.m}^2$ )

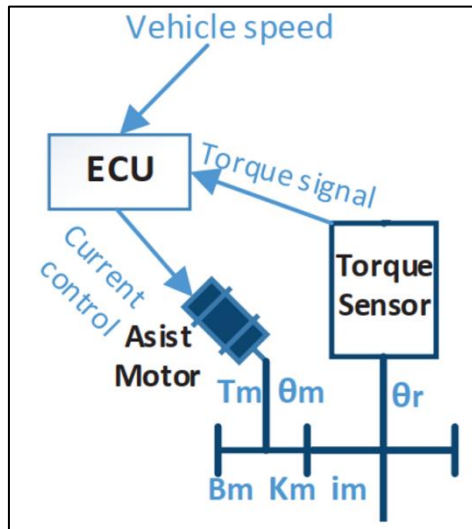
$B_s$ : Viscous damping coefficient of steering column (deboost of steering column) ( $\text{Nm.rad}^{-1}$ )

$K_s$ : Rigidity of torsional bar ( $\text{Nm.rad}^{-1}$ )

$\theta_s$ : Turn angle of steering wheel (rad)

$\theta_r$ : Turn angle of output steering axle (rad)

$T_d$ : Input torque of steering wheel (N.m)



**Figure 5:** Overall input, output signal and structure of assist motor section

- The dynamics of assistance section, which is showed in Figure 5, is described by following equation:

$$J_m \times \frac{d^2 \theta_m}{dt^2} = T_m - K_m(\theta_m - i_m \theta_r) - B_m \times \frac{d\theta_m}{dt}$$

While:

$J_m$ : the moment of inertia of the motor and clutch section ( $\text{kg.m}^2$ )

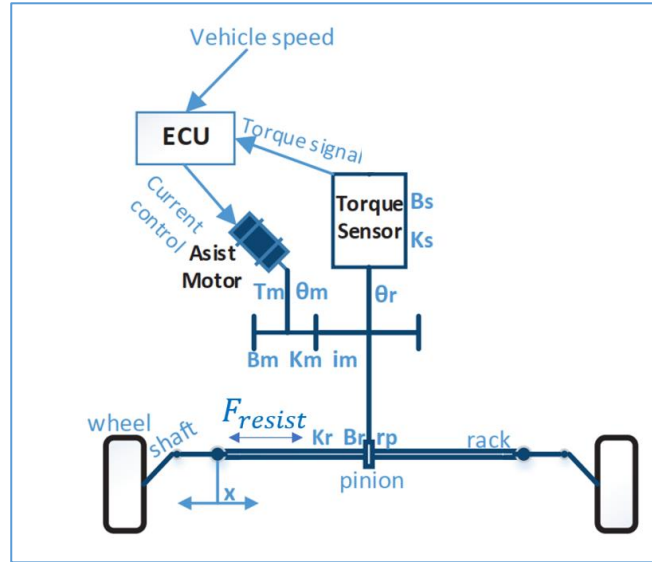
$B_m$ : Viscous damping coefficient of the motor (deboost of the motor) ( $\text{Nm.rad}^{-1}$ )

$K_m$ : Rigidity of the motor and reducer ( $\text{Nm.rad}^{-1}$ )

$\theta_m$ : Turn angle of motor (rad)

$\theta_r$ : Turn angle of output steering axle (rad)

$i_m$ : Reduction ratio of reducer



**Figure 6: Overall Rack and Pinion Dynamics**

- Finally, rack and pinion section is illustrated in Figure 6 and governed by the equation:

$$m \times \frac{d^2 x}{dt^2} = \frac{1}{r_p} [K_m(\theta_m - i_m \theta_r) i_m + K_s(\theta_s - \theta_r)] - B_r \times \frac{dx}{dt} - K_r \times x - F_{resist}$$

While:

$m$ : mass of the pinion and rack (kg)

$r_p$ : pinion radius ( $m$ )

$B_r$ : Viscous damping coefficient of rack and pinion (deboost of the rack and pinion) ( $\text{Nm.rad}^{-1}$ )

$K_m$ : Rigidity of the motor and reducer ( $\text{Nm.rad}^{-1}$ )

$\theta_m$ : Turn angle of motor (rad)

$\theta_r$ : Turn angle of output steering axle (rad)

$i_m$ : Reduction ratio of reducer

$\theta_s$ : Turn angle of steering wheel (rad)

$K_s$ : Rigidity of torsional bar ( $\text{Nm.rad}^{-1}$ )

We can see in above equation we have the resist force apply on the rack. This force that resists the motion of the rack when the driver is steering

### III. MATLAB/SIMULINK SIMULATION

#### 1. Mass – damper system

Parameter	Value
Mass of solid (m)	1.0 kg
Spring constant (k)	100 N/m
Damping coeff. (C)	0.15 N/(m/s)
Force applied (F)	100 N

Figure 7 Parameter of mass-damper system

##### 1.1) Simulink block diagram

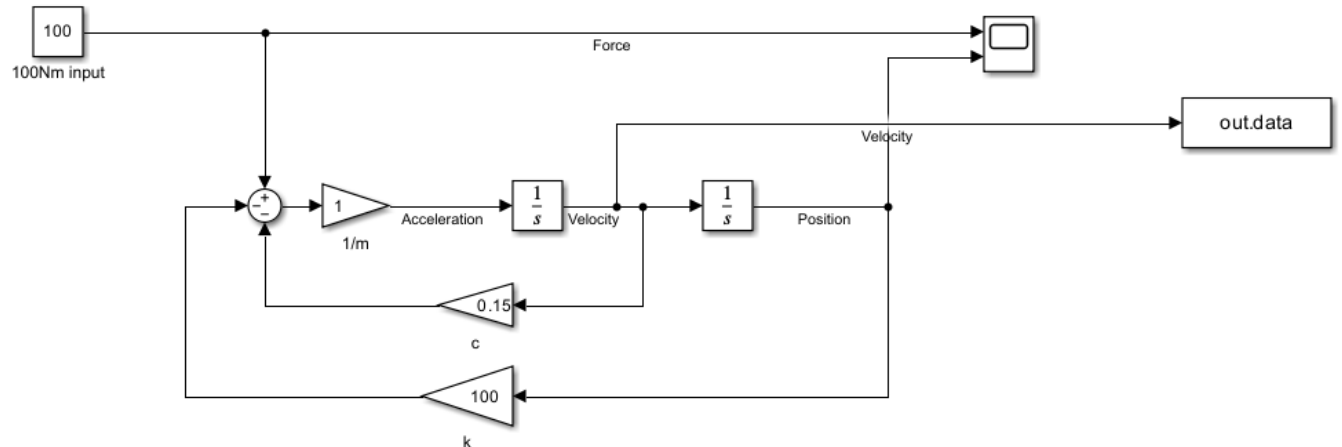


Figure 8: Mass-damper block diagram

##### 1.2) Simulation result and discussion

- The position and velocity of the spring-damper system are generated by solving the developed transfer function using MATLAB. The displacement graph is shown in Fig xxx while the velocity graph is shown in Fig xxx.
- For the given parameters of the system, the position vs. time response in MATLAB gives the maximum value equal to 1.961 m. Similarly, the maximum velocity is found to be 9.761 m/s.
- Simulink model uses the solver ode45 for solving the differential equation for the spring-mass-damper system.



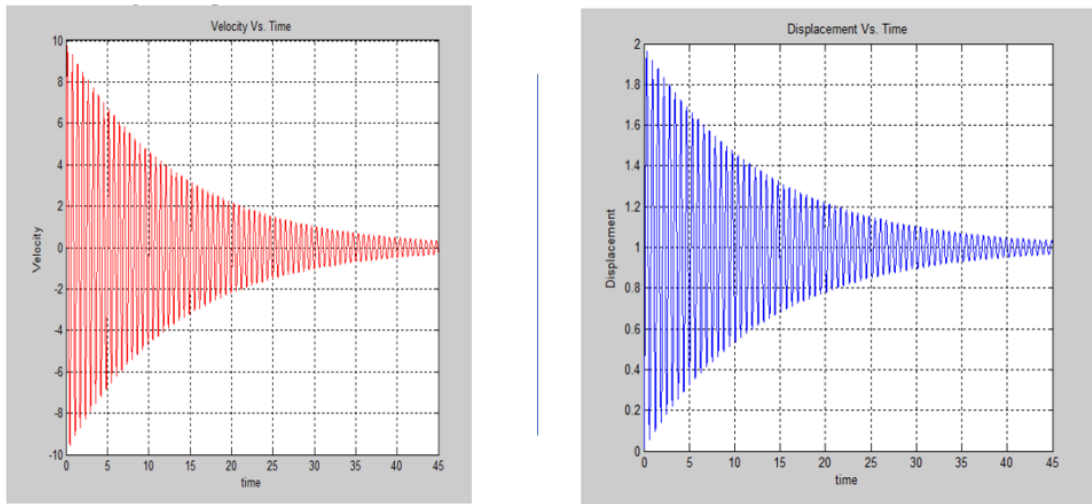


Figure 9: Simulation result

## 2. EPS system

### 2.1) Simulink block diagram

Symbols	Value	Name
$J_s$	0.0012 [ $\text{kgm}^2$ ]	Inertia of steering wheel and steering column
$B_s$	0.26 [ $\text{Nmrad}^{-1}$ ]	Viscous damping coefficient of steering column
$K_s$	115 [ $\text{Nmrad}^{-1}$ ]	Rigidity of torsional bar
$\theta_s$	[Rad]	Turn angle of steering wheel
$\theta_r$	[Rad]	Turn angle of output steering axle
$T_d$	[Nm]	Input torque of steering wheel
$T_m$	[Nm]	Output torque of the motor
$K_m$	125 [ $\text{Nmrad}^{-1}$ ]	Rigidity of the motor and reducer
$B_m$	[ $\text{Nmrad}^{-1}$ ]	Viscous damping coefficient of the motor
$i_m$	7.225	Reduction ratio of reducer
$\theta_m$	[Rad]	Turn angle of motor
$K_r$	91064 [ $\text{Nm}^{-1}$ ]	Linear rigidity
$B_r$	653.203 [ $\text{Nmrad}^{-1}$ ]	Viscous damping coefficient of rack and pinion
$r_p$	0.007783 [m]	Pinion radius
x	m	Rack displacement
$m_r$	32 [kG]	Mass of the rack and pinion system



## 2.2) Simulation result and discussion

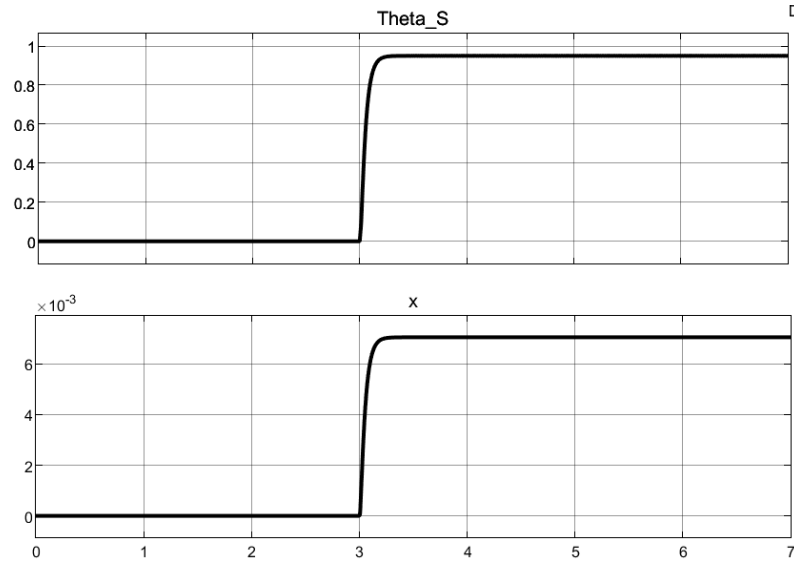


Figure 11: Result of simulation

The inputs to this system are the torque generated by the driver. Since we present a model of the power steering system, details of the tire dynamics are neglected. The outputs of the mechanical subsystem are the displacement of the rack,  $X$ , and the rotational displacement of the steering column  $\theta_s$ .

As we can see from Fig 11, with  $\theta_s$  changed from 0 to nearly 1 rad,  $x$  jumped to approximately  $7 \times 10^{-3}$  m. Input torque at 3s is 5Nm.

## V. CONCLUSION

A dynamic model of a power steering system is developed. The model can be used for performance evaluation and can be easily adapted to fit in a larger vehicle handling model. It can also be used for the design of other power steering systems, where it allows the designer to test changes in dynamic conditions. Other variations, such as pinion radius, stiffness of the torsion bar or piston area can be made as well, and the results tested in the same manner. The simulation results agree to a great extent with the real test results.

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