The optimization of electric power assisted steering to improve vehicle performance

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Abstract: To investigate the effects of electric power assisted steering (EPAS) on a vehicle's steering performance, this paper explicitly proposes several indices, such as steering sensitivity and road feel, for evaluation of steering performance. Then a single-track car model with lateral and yaw movement is considered and an EPAS model is built. The mathematical expressions for the steering performance indices are deduced and a stability criterion is established by integrated analysis of the vehicle's dynamics and the EPAS dynamics. By means of frequency domain analysis, the effect of each parameter of EPAS on steering performance indices is analysed. Finally, an optimizing model used to improve these steering indices is put forward and the result is given.

Keywords: electric power assisted steering, dynamics, steering sensitivity, road feel, optimization

NOTATION

a, b	distances from respectively the front and rear tyre's centre to vehicle's centre of gravity
A_y	vehicle's lateral acceleration
B_h, B_m, B_w	coefficients of viscous damping at handwheel, motor and front wheel
C_f, C_r	front and rear tyre cornering coefficients
E	road feel
G_1, G_2	ratios of gear assist mechanism and steering gears
I_h, I_m, I_w	moment of inertia of handwheel, motor and front wheel
I_z	yaw moment of inertia about z axis
k_e, k_t	back electromagnetic force constant and torque constant of motor
k_s	torque sensor stiffness
l	$=a+b$
m	total mass of vehicle
r	yaw rate of vehicle
s	Laplace operator
T_h, T_r', T_m, T_a	torque at handwheel, equivalent reaction torque at column, electromagnetic torque of motor and assisting torque
u, i_a	voltage and current of motor
V	vehicle speed at centre of gravity
α	slip angle of tyre
β	slip angle at centre of gravity of vehicle
δ	steering angle of front wheel
θ_h, θ_m, θ_w	handwheel angle, motor angle and equivalent angle of front wheel
λ	assist gain of controller

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1 INTRODUCTION

In recent years vehicle technology has been advancing on improving handling performance, decreasing fuel consumption and reducing the pollution of the environment. Electric power assisted steering (EPAS) is just one of the technologies that meet these requirements. It can lower the driver's steering effort without raising the steering gear ratio. Besides, it consumes less fuel than hydraulic power steering (HPS) because it uses power only when steering operation is needed. What is more, it is easy for EPAS to vary the feel only by adjusting the torque boost curve according to the vehicle speed. In addition to these benefits, it has a compact package, high reliability with a self-diagnostic system and causes little to no environmental pollution. Just because it possesses all these advantages over HPS, EPAS will surely become an important item of equipment on most kinds of vehicles, especially on electric vehicles.

With the development of EPAS, studies on its design and control have become more and more practically important. However, research on the effects of EPAS on the steering performance and on how to optimize the parameters of EPAS to obtain good steering performance has not yet drawn enough attention. Therefore, this paper will focus on these problems. It is organized as follows. Section 2 gives a review of papers on EPAS. In Section 3 the indices for evaluation of steering performance including steering sensitivity, road feel and stability are presented. In Section 4 the model of the vehicle is given. In Section 5 a model of the EPAS is built, which comprises models of motor, steering system and controller. In Section 6 the mathematical expressions of steering sensitivity, road feel are deduced and the stability criterion is established, and each parameter of the EPAS that influences these steering performances is analysed by means of frequency domain analysis. On the basis of the analysis in Section 6, a method to optimize the parameters of EPAS is proposed and the result is calculated in Section 7. Section 8 comes to the conclusion.

2 CURRENT LITERATURE REVIEW

Most papers on EPAS have focused on its controlling problem. Two compensators were presented in reference [**1**]: one was adequate in the case of a high bandwidth actuator, and the other was used to stabilize a low bandwidth actuator with better robustness properties. Optimum design of an EPAS controller was discussed in reference [**2**], including optimization of DC gain, phase lag, zero and pole locations to obtain high level steering system performance. In reference [**3**], the stability of a vehicle at high speed in wet road conditions and the influence of the EPAS's motor parameters variation on the vehicle's stability were investigated. On the basis of the yaw and lateral acceleration state feedback, the vehicle stability was achieved. Chen [**4**] put forward a method to characterize the dynamic characteristic of EPAS by the transmissibility from rack load to steering wheel torque and made sure that proportional and differential (PD) control was adequate for EPAS to generate the desired dynamic performance. Kurishige *et al.* [**5**] developed a new control strategy based on damping for a specified frequency using two kinds of motor angular velocity estimator, to eliminate the undesirable steering vibration resulting from high gains of the controller. To obtain the optimum steering feel, using a systematic control design with a 'generalized plant' and the H-infinity method, Sugitani *et al.* [**6**] proposed a method of supplying better road information to the driver. In reference [**7**], based on the H-infinity method and a torque estimator, multiple objectives including fast response to the driver torque command, good driver feel and robustness could be ensured.

All the papers described above studied only the control question of EPAS and did not discuss its dynamic modelling and analysis, which references [**8**] and [**9**] attempted. However, reference [**8**] only gave a simplified dynamics model and used this model to analyse various closed-loop effects such as disturbance rejection, road feel and stability; a mathematical model was not given. In reference [**9**], only the effects of the motor inertia, vehicle speed and closed-loop gain on the open-loop system response were simulated, without the torque sensor's stiffness or the ratio of gear assist mechanism being considered.

The following findings arise from the above review of the literature. Although the steering performance such as road feel was considered [**2**, **4**, **6**, **7**], explicit mathematical expressions for these performances were not presented or deduced. Furthermore, how the variation of the EPAS parameters affects these steering performances was also not dealt with. Although it is important to study control problems, to some degree the component design and parameter optimization determine the characteristic of EPAS and the steering performance. So, if the EPAS parameters can be adjusted before the controller design, an enhanced dynamic performance of the EPAS and vehicle steering performance will be achieved.

3 THE INDICES FOR EVALUATION OF STEERING PERFORMANCE

To investigate the effects of EPAS on steering performance, the definition of steering performance must be decided first. It is well known that a good steering system must meet the following goals: good steering sensitivity, suitable road feel and sound stability. Therefore, the indices for evaluation of these steering performances must be defined first.

3.1 Steering sensitivity

This index, which represents the vehicle's response to a steering command, is very important in evaluating the steering performance of a vehicle. It is necessary to investigate the effects of EPAS on steering performance. There were many definitions for steering sensitivity, such as steering torque gradient $\mathrm{d}T/\mathrm{d}A_y$ and steering angle gradient $\mathrm{d}\theta/\mathrm{d}A_y$ [**10**]. Others [**11**] defined steering sensitivity in terms of vehicle yaw rate (r) for a given steering angular input (θ). The last of these definitions is found to be more practical and was adopted in this paper.

3.2 Road feel

Road feel, regarded as the road information transferred to the driver, is of great importance because it reflects the condition of the road and the state of the vehicle. There are also many definitions for road feel, such as the frequency band of tyre/steering wheel transmission characteristics [**6**], the transmissibility from rack load to steering wheel torque [**4**] and the steering torque for a given rack load [**10**]. This paper adopts the final definition because it connects more directly to the evaluation of variance of road feel.

3.3 Stability

For a normal vehicle, the handling stability depends on the vehicle's speed and the parameter itself. The case is not the same for a vehicle equipped with EPAS, in which the motor-driven EPAS is actively involved in the stabilization of the vehicle dynamics [**3**]. However, reference [**3**] only analysed the effect of the motor's parameter on the stability of the vehicle, the stability criterion was not given. Therefore, it is of great importance to study the stability, which is a combination of vehicle system and EPAS system, and to establish the stability criterion for the variation of the EPAS parameters. This paper discusses this kind of stability and analyses the effect of each parameter of EPAS on such stability in Section 6.3. Then the stability criterion is proposed.

4 MODEL OF THE VEHICLE LATERAL DYNAMICS

To simplify the problem, only a single-track car model with lateral and yaw movement is considered. Also, the relationship between the lateral forces generated by the front or rear tyre and slip angles is assumed to be linear. The car is running at speed V at the centre of gravity (CG). From reference [**12**] the following equations can be obtained:

$$mV\frac{\mathrm{d}\beta}{\mathrm{d}t} + (C_\mathrm{f} + C_\mathrm{r})\beta + \left[mV + \frac{1}{V}(ac_\mathrm{f} - bC_\mathrm{r})\right]r = C_\mathrm{f}\delta \tag{1}$$

$$(aC_\mathrm{f} - bC_\mathrm{r})\beta + I_z\frac{\mathrm{d}r}{\mathrm{d}t} + \frac{(a^2C_\mathrm{f} + b^2C_\mathrm{r})r}{V} = aC_\mathrm{f}\delta \tag{2}$$

where C_f, C_r are the front and rear tyre cornering coefficients and m, I_z, a, b are the inherent parameters of the vehicle. With Laplace transformation and zero initial condition, the following equations can be derived from equations (1) and (2):

$$\beta(s) = \frac{X(s)}{Z(s)}\delta(s), \qquad r(s) = \frac{Y(s)}{Z(s)}\delta(s) \tag{3}$$

where

$$X(s) = C_\mathrm{f}\left(I_z s + \frac{bl}{V}C_\mathrm{r} - amV\right)$$

$$Y(s) = C_\mathrm{f}(amVs + lC_\mathrm{r})$$

$$Z(s) = mI_zVs^2 + [m(a^2C_\mathrm{f} + b^2C_\mathrm{r}) + I_z(C_\mathrm{f} + C_\mathrm{r})]s + \frac{C_\mathrm{f}C_\mathrm{r}l^2}{V} - mV(aC_\mathrm{f} - bC_\mathrm{r})$$

5 EPAS DESCRIPTION AND MODELLING

At present, most of the EPAS systems consist of a speed sensor, a torque sensor, an electronic control unit (ECU), an assisting motor, a clutch and a gear assist mechanism as shown in Fig. 1. An EPAS works as follows. The ECU processes the signals from speed sensor and torque sensor and generates a command to drive the motor when needed. Then the gear assist mechanism amplifies the output torque from the motor to provide an assisting torque for steering. In an EPAS system, the worm gears are usually used to transfer the assisting torque to the manual steering system at the position of the column, as is also the assisting mode regarded in this paper. Also, the normal assisting strategy adopted is proportional control, i.e. the control voltage of the motor is proportional to the torque measured by the torque sensor.

On the basis of the above assumptions, a simplified model of EPAS is put forward as in Fig. 1. The front tyre and steering mechanism are equivalently transformed to the steering column. Representing the rotating angles of the handwheel, motor and front wheel by $\theta_\mathrm{h}, \theta_\mathrm{m}, \theta_\mathrm{w}$, coefficients of viscous damping at handwheel, front wheel and motor are B_h, B_w, B_m, moments of inertia at handwheel, front wheel and motor are $I_\mathrm{h}, I_\mathrm{w}, I_\mathrm{m}$, and the ratios of the gear assist mechanism and the steering gears are G_1, G_2, the following result

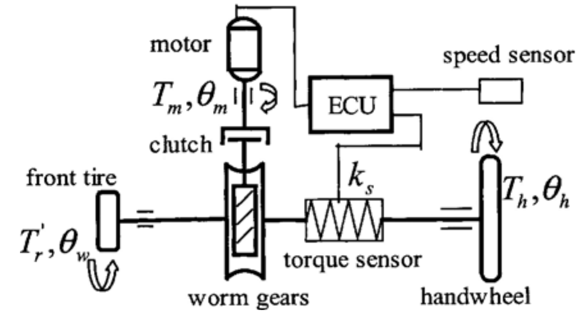

Fig. 1 The dynamic model of the EPAS system

can be inferred on the basis of the equivalent transformation of the front tyre:

$$\theta_{\mathrm{w}} = G_2 \delta \tag{4}$$

5.1 The motor's electrical model

Assuming that a DC motor with armature resistance R and inductance L is used in the system, it can be described by [**13**]

$$u = Ri_{\mathrm{a}} + L\frac{\mathrm{d}i_{\mathrm{a}}}{\mathrm{d}t} + k_{\mathrm{e}}\frac{\mathrm{d}\theta_{\mathrm{m}}}{\mathrm{d}t} \tag{5}$$

where k_{e} is the motor back electromagnetic force constant and u, i_{a} are the voltage and current of the armature.

5.2 The motor's mechanical model

Considering the dynamics of a motor with a load of T_{a}/G_1, it can be found from the Newton law that

$$T_{\mathrm{m}} - \frac{T_{\mathrm{a}}}{G_1} = I_{\mathrm{m}}\frac{\mathrm{d}^2\theta_{\mathrm{m}}}{\mathrm{d}t^2} + B_{\mathrm{m}}\frac{\mathrm{d}\theta_{\mathrm{m}}}{\mathrm{d}t} \tag{6}$$

where $T_{\mathrm{m}} = k_{\mathrm{t}} i_{\mathrm{a}}$ and k_{t} is the torque constant.

5.3 Models of the handwheel and the front tyre

From Fig. 1, the following equations can be derived with the Newton law:

$$T_{\mathrm{h}} - k_{\mathrm{s}}(\theta_{\mathrm{h}} - \theta_{\mathrm{w}}) = I_{\mathrm{h}}\frac{\mathrm{d}^2\theta_{\mathrm{h}}}{\mathrm{d}t^2} + B_{\mathrm{h}}\frac{\mathrm{d}\theta_{\mathrm{h}}}{\mathrm{d}t} \tag{7}$$

$$T_{\mathrm{a}} + k_{\mathrm{s}}(\theta_{\mathrm{h}} - \theta_{\mathrm{w}}) = T_{\mathrm{r}}' + I_{\mathrm{w}}\frac{\mathrm{d}^2\theta_{\mathrm{w}}}{\mathrm{d}t^2} + B_{\mathrm{w}}\frac{\mathrm{d}\theta_{\mathrm{w}}}{\mathrm{d}t} \tag{8}$$

5.4 Models of torque sensors and controller

To simplify the problem, the torque sensor is assumed to be ideal without phase lag and distortion, and the torque measured by the torque sensor is $T_{\mathrm{s}} = k_{\mathrm{s}}(\theta_{\mathrm{h}} - \theta_{\mathrm{w}})$. The pure proportional control strategy is adopted here, so the control voltage can be obtained as follows:

$$u = \lambda k_{\mathrm{s}}(\theta_{\mathrm{h}} - \theta_{\mathrm{w}}) \tag{9}$$

Here λ is the assist gain of the controller. In addition, the motor's angle must be compatible with the steering column's angle. Therefore they must satisfy

$$\theta_{\mathrm{m}} = G_1 \theta_{\mathrm{w}} \tag{10}$$

6 MATHEMATICAL EXPRESSIONS OF THE STEERING PERFORMANCE'S INDICES

6.1 Steering sensitivity

Before the steering sensitivity deduction, the expression for the front tyre's reaction torque exerted by the road will be decided first. Under a small slip angle of the tyre, the reaction torque is $T_{\mathrm{r}} = dC_{\mathrm{f}}\alpha$ [**12**], where d is the caster angle offset distance at the front tyre. On the basis of an analysis of the geometry of the front wheel [**12**], the slip angle is $\alpha = \delta - (a/V)r - \beta$. Then the equivalent reaction torque at steering column is

$$T_{\mathrm{r}}' = \frac{dC_{\mathrm{f}}}{G_2}\left(\delta - \frac{a}{V}r - \beta\right) \tag{11}$$

From equations (4) and (11) and Laplace transformation with zero initial condition, the following result can be obtained:

$$T_{\mathrm{r}}'(s) = \frac{dC_{\mathrm{f}}}{G_2^2}\frac{VZ(s) - aY(s) - VX(s)}{VZ(s)}\theta_{\mathrm{w}}(s) \tag{12}$$

Let $T_{\mathrm{r}}'(s) = [P(s)/Q(s)]\theta_{\mathrm{w}}(s)$; from equations (3) to (10) and (12), it can be deduced that

$$\frac{\theta_{\mathrm{w}}(s)}{\theta_{\mathrm{k}}(s)} = \frac{M(s)}{N(s)} \tag{13}$$

where

$$M(s) = k_{\mathrm{s}}(Ls + R + \lambda G_1 k_{\mathrm{t}})$$

$$N(s) = \left[(G_1^2 I_{\mathrm{m}} + I_{\mathrm{w}})s^2 + \left(G_1^2 B_{\mathrm{m}} + B_{\mathrm{w}} + \frac{G_1^2 k_{\mathrm{e}} k_{\mathrm{t}}}{Ls + R}\right)s + k_{\mathrm{s}} + \frac{\lambda G_1 k_{\mathrm{t}} k_{\mathrm{s}}}{Ls + R}\right](Ls + R) + \frac{P(s)}{Q(s)}(Ls + R)$$

According to the definition of steering sensitivity in Section 3.1 and equations (3), (10) and (13), the steering sensitivity can be expressed as

$$\frac{r(s)}{\theta_{\mathrm{h}}(s)} = \frac{r(s)}{\delta(s)}\frac{\delta(s)}{\theta_{\mathrm{w}}(s)}\frac{\theta_{\mathrm{w}}(s)}{\theta_{\mathrm{h}}(s)} = \frac{Y(s)M(s)}{G_2 Z(s)N(s)} \tag{14}$$

Equation (14) shows that the parameters affecting steering sensitivity are torque sensor stiffness, electric characteristics of the motor, moment of inertia of the motor, ratio of the gear assist mechanism, assist gain of the controller and the inherent parameters of vehicle. The objective of this paper is studying the effects of the EPAS parameters rather than studying the effects of the vehicle's inherent parameters on vehicle steering performance, so the parameters related to the vehicle are not under discussion.

At a constant velocity (30 m/s), the effects of each parameter of EPAS on the steering sensitivity are given

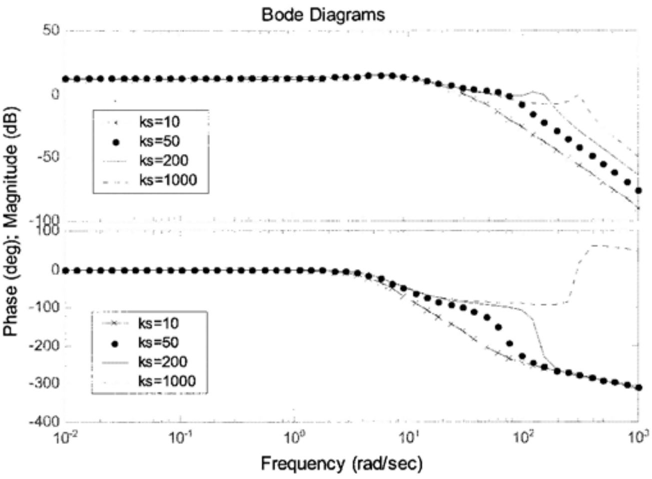

Fig. 2 The effects of the torque sensor's stiffness on steering sensitivity in a Bode plot (the assist gain $\lambda = 10$)

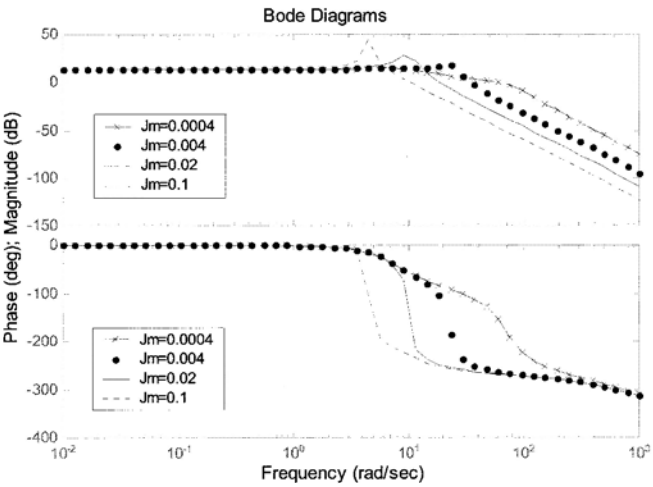

Fig. 3 The effects of the motor's inertia on steering sensitivity in a Bode plot (the assist gain $\lambda = 10$)

in Figs 2 to 5 in Bode plots, in which only the parameter investigated is allowed to vary. The parameters used in the analysis are listed in Table 1. Because the motor cannot be optionally selected and the coefficients of viscous damping also cannot be optionally adjusted, in Figs 2 to 5 only the effects of the following parameters on the steering sensitivity are investigated.

1. *Torque sensor stiffness*. At a constant assist gain of the controller (gain of 10), an increase in the torque sensor stiffness results in a decrease in the phase lag and an increase in the bandwidth as shown in Fig. 2. However, when the stiffness exceeds a certain value, it leads to a resonance in the magnitude–frequency curve and even loss of stability.
2. *Inertia of motor*. At a constant assist gain of the controller (gain of 10), an increase in the inertia of the motor results in an increase in the phase lag and a decrease in the bandwidth as shown in Fig. 3. As for the case of torque sensor stiffness, there is a resonance in the magnitude–frequency curve and stability is lost when the inertia of motor is high enough.

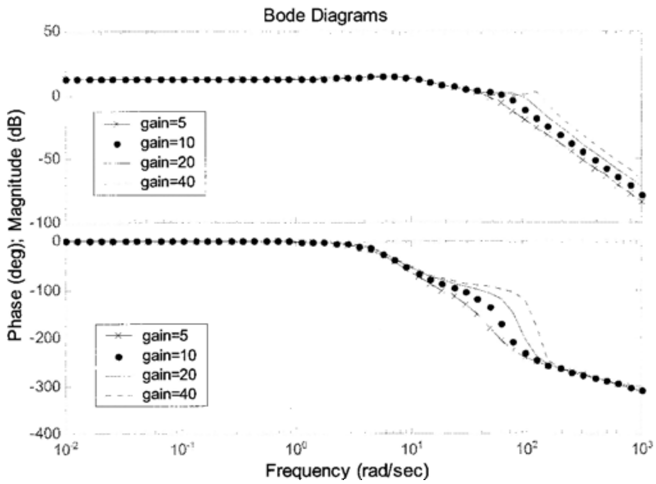

Fig. 4 The effect of the assist gain of the controller on steering sensitivity in a Bode plot

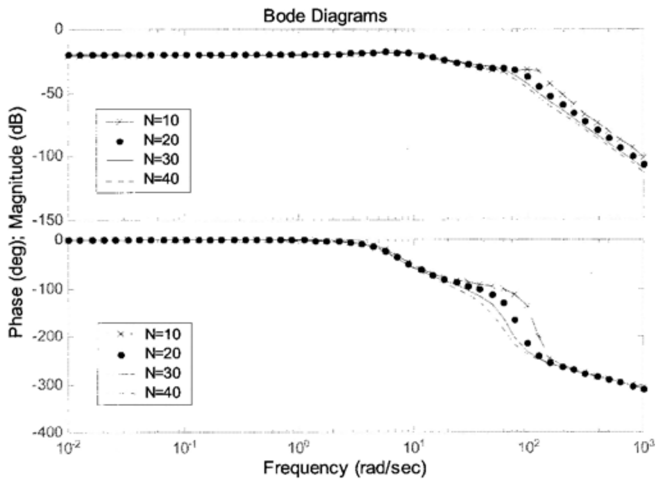

Fig. 5 The effect of the ratio of the gear assist mechanism on steering sensitivity in a Bode plot

3. *Assist gain of controller*. An increase in the assist gain of the controller results in a decrease in the phase lag as shown in Fig. 4. However, a resonance appears and even stability is lost when the assist gain is high enough.
4. *Ratio of gear assist mechanism*. At a constant assist gain of controller (gain of 10), an increase of the ratio results in an increase in the phase lag as shown in Fig. 5; however, the resonance of the magnitude–frequency curve decreases with the increase of this ratio.

6.2 Road feel

Provided that the handwheel is fixed as in the case of constant angle steering, the torque exerted by the driver

Table 1 Parameters of vehicle and EPAS system

Vehicle		
m	Mass	2000 kg
I_z	Inertia	3000 kg m^2
a	Front tyre distance from CG	1.0 m
b	Rear tyre distance from CG	1.8 m
V	Speed	30 m/s
$C_f = C_r$	Cornering stiffness of tyre	1.4×10^5 N/rad
Torque sensor		
k_s	Torsional stiffness	40 N m/rad
Motor		
R	Resistance of armature	0.15 Ω
L	Inductance	1.5×10^{-5} H
k_t	Torque constant	0.02 N m/A
k_e	Back electromagnetic force constant	0.02 V s
I_m	Inertia	4.7×10^{-4} kg m^2
B_m	Viscous damping coefficient	0.02 N s/rad
Front wheel		
I_w	Inertia	4.4×10^{-3} kg m^2
B_w	Viscous damping coefficient	0.03 N m s/rad
Ratio of steering gear and the gear assist mechanism		
G_1	Ratio of gear assist mechanism	30
G_2	Ratio of steering gear	15

can be simplified from equation (7):

$$T_h = k_s(\theta_h - \theta_w) = -k_s\theta_w \tag{15}$$

With equations (5), (6), (9) and (15) put into equation (8) and Laplace transformation with zero initial condition, the reaction torque can be expressed as follows:

$$T'_r = -\left[(G_1^2 I_m + I_w)s^2 + \left(G_1^2 B_m + B_w + \frac{G_1^2 k_e k_t}{Ls + R}\right)s + k_s\left(1 + \frac{\lambda G_1 k_t}{Ls + R}\right)\right]\theta_w \tag{16}$$

According to the definition in Section 3.2, road feel can be expressed as

$$E = \frac{T_h(s)}{T'_r(s)} = \frac{k_s}{M'(s)} \tag{17}$$

Here

$$M'(s) = (G_1^2 I_m + I_w)s^2 + \left(G_1^2 B_m + B_w + \frac{G_1^2 k_e k_t}{Ls + R}\right)s + k_s\left(1 + \frac{\lambda G_1 k_t}{Ls + R}\right)$$

From the expression for the road feel in equations (17), it can be inferred that the parameters of EPAS affecting steering sensitivity are torque sensor stiffness, electric characteristics of the motor, moment of inertia of the motor, ratio of the gear assist mechanism and assist gain of the controller. For the same reason as in Section 6.1, only the following parameters influencing road feel are investigated in Figs 6 to 9, from which the following conclusions can be drawn.

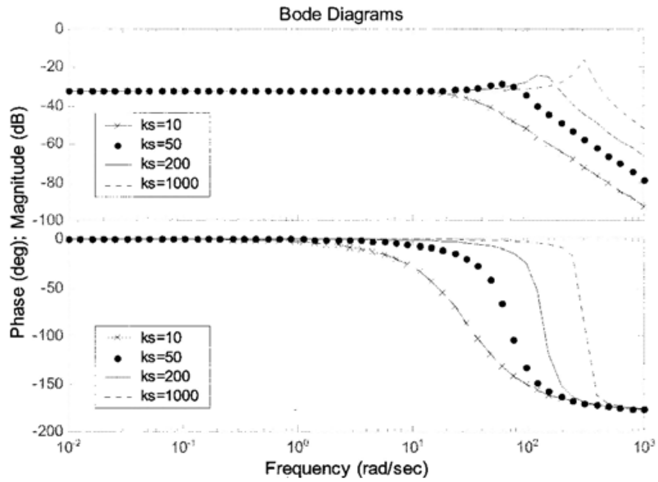

Fig. 6 The effect of the torque sensor's stiffness on road feel in a Bode plot

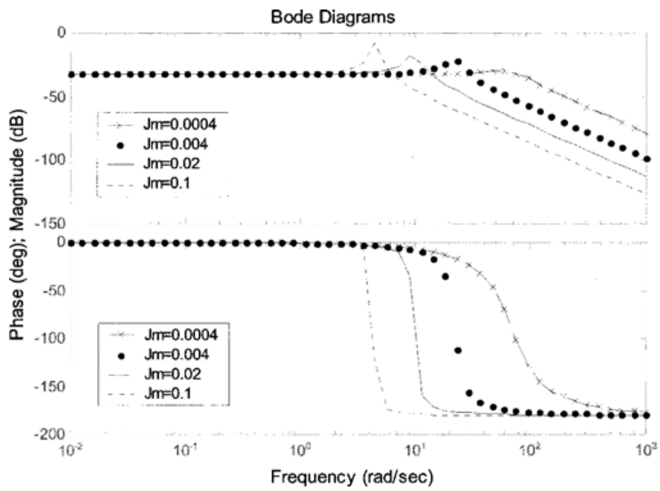

Fig. 7 The effect of the motor's inertia on road feel in a Bode plot

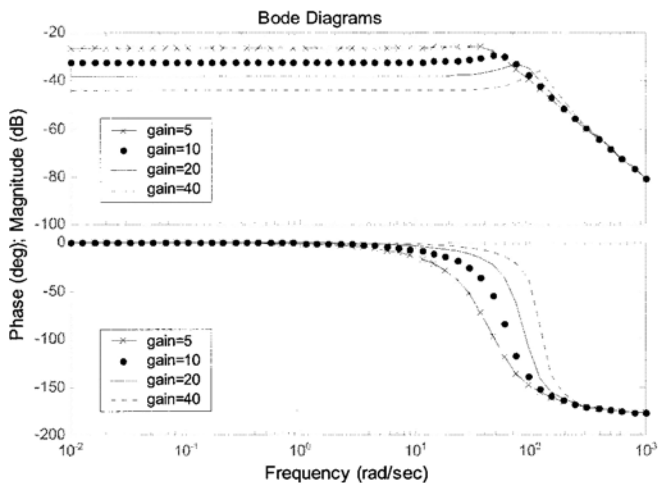

Fig. 8 The effect of the assist gain of the controller on road feel in a Bode plot

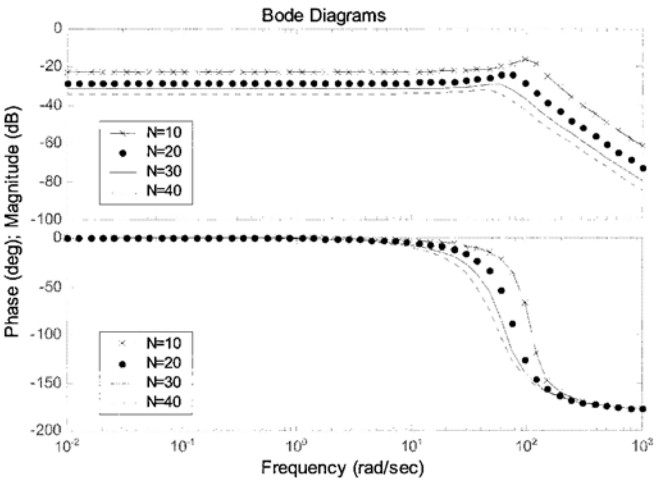

Fig. 9 The effect of the ratio of the gear assist mechanism on road feel in a Bode plot

1. *Torque sensor stiffness*. An increase in the torque sensor stiffness results in a decrease in the phase lag and does not lead to a notable variance in the magnitude as shown in Fig. 6. Also, a resonance in the magnitude–frequency curve appears when the stiffness exceeds a certain value.
2. *Inertia of motor*. An increase in the inertia of the motor results in an increase in the phase lag and does not lead to a notable variety in the magnitude as shown in Fig. 7. As in the case of stiffness, a resonance appears when the inertia exceeds a certain value.
3. *Assist gain of controller*. An increase in the assist gain of controller results in a decrease in the phase lag and magnitude as shown in Fig. 8, but a resonance appears when the assist gain is high enough.
4. *Ratio of gear assist mechanism*. An increase in the ratio results in an increase in the phase lag and a decrease in the magnitude as shown in Fig. 9; however, the variance is not so notable. Also, a resonance appears when the ratio is too small.

6.3 Stability

From the analysis of Sections 6.1 and 6.2, it can be inferred that the torque sensor stiffness, inertia of motor and assist gain of controller have substantial effects on steering sensitivity and road feel; the variation of these parameters can even lead to a loss of stability in steering sensitivity and resonance in road feel that makes it poor for the driver. Therefore, it is of importance to know under what conditions these parameters result in a loss of stability as defined in Section 3.3. This kind of stability is exhibited very strongly in the steering sensitivity, which is a combination of EPAS dynamics and vehicle dynamics. So the stability of the steering sensitivity will be used to analyse the stability of the whole system.

Let

$$\frac{P(s)}{Q(s)} = K\frac{s^2+ms+n}{s^2+gs+h}$$

and combine this with equation (14). Then an equation expressing stability can be obtained from the denominator of equation (14):

$$a_1s^5+a_2s^4+a_3s^3+a_4s^2+a_5s+a_6=0 \tag{18}$$

where

$$a_1 = L(G_1^2I_m + I_w)$$

$$a_2 = (gL+R)(G_1^2I_m+I_w)+L(G_1^2B_m+B_w)$$

$$a_3 = (hL+gR)(G_1^2I_m+I_w) + (gL+R)(G_1^2B_m+B_w)+k_sL+G_1^2k_ek_t$$

$$a_4 = hR(G_1^2I_m+I_w)+(hL+gR)(G_1^2B_m+B_w) + (gL+R)k_s+G_1k_t(\lambda k_s+gG_1k_e)+K$$

$$a_5 = hR(G_1^2B_m+B_w)+k_s(hL+gR) + hG_1^2k_ek_t+\lambda gG_1k_tk_s+Km$$

$$a_6 = h(k_sR+\lambda G_1k_tk_s)+Kn$$

According to the Routh stability criterion [**14**], the following inequalities can be obtained:

$$a_2a_3-a_1a_4>0 \tag{19}$$

$$a_2a_5-a_1a_6>0 \tag{20}$$

On the basis of relations (19) and (20), two inequalities about the stiffness of the torque sensor, inertia of the motor, assist gain of the controller, electric constants of the motor and ratio of gear assist mechanism are obtained:

$$f_1(\mathbf{X}_1)>0 \tag{21}$$

$$f_2(\mathbf{X}_1)>0 \tag{22}$$

where

$$\mathbf{X}_1 = [k_s\ I_m\ \lambda\ L\ R\ G_1]^{\mathrm{T}}$$

This is the stability criterion for the EPAS system. In the case of practical design, the assist gain of the controller cannot be optionally chosen, so the design must be made under the least disadvantageous conditions, i.e. maximum assist gain. In this condition, only if the vector $\mathbf{X}_1$ satisfies relations (21) and (22), can the stability be guaranteed.

7 DISCUSSION

In Section 6 the effects of the parameters of EPAS on steering sensitivity and road feel are presented, but it is still important to study how to optimize these parameters to improve the steering performance. The essence of the problem is to find the optimum value for the vector $\mathbf{X}_1$ satisfying inequalities (21) and (22). However, it must be made clear that the electric constants such as L and R are determined by the motor selected and cannot be easily changed, so they are not taken into account. Similarly, the assist gain of the controller is always varying according to the states of EPAS and it cannot be chosen in advance. Therefore, optimizing the parameters of EPAS is equivalent to optimizing the torque sensor stiffness, the inertia of the motor and the ratio of the gear assist mechanism. If the integration of the effects of these EPAS parameters on steering sensitivity analysed in Section 6.1 and on road feel analysed in Section 6.2 is considered, the following inferences can be drawn.

1. *Stiffness of torque sensor*. An increase in the stiffness results in a decrease in the phase lag and an increase in the bandwidth of the steering sensitivity and of the road feel. When the stiffness exceeds a certain value, the steering sensitivity will become unstable and the road feel will become poor. Therefore, only if the system is stabilized, can the stiffness be as high as possible.
2. *Inertia of motor*. An increase in the inertia of the motor leads to an increase in the phase lag and a decrease in the bandwidth of the steering sensitivity and of the road feel. When the inertia surpasses a certain value, the steering sensitivity will lose stability and the road feel will become poor. Therefore, the inertia can be as small as possible.
3. *Ratio of gear assist mechanism.* An increase in the ratio results in an increase in the phase lag and a decrease in the bandwidth of the steering sensitivity and of the road feel. The magnitude of road feel also decreases as the ratio increases. However, the resonance in the magnitude of road feel becomes smaller. Therefore, the ratio should be moderate.

On the basis of the inferences above and the stability criterion in Section 6.3, the optimization problem of these parameters of EPAS can be expressed as the following optimizing model, considering that the inertia of the motor cannot be optionally adjusted in practice:

$$\text{maximize } k_s \tag{23}$$

$$\text{such that} \quad f_1(\mathbf{X}_1) > 0$$
$$f_2(\mathbf{X}_1) > 0$$

Because the space for installation is small, in this design the ratio of the gear assist mechanism is 30. Also, the motor selected determines the electrical constants of the motor, which are listed in Table 1. On the basis of the optimizing model (23), the optimized result for the stiffness of the torque sensor is 165.44 N m/rad.

8 CONCLUSION

In summary, three explicit indices, steering sensitivity, road feel and stability, for the evaluation of steering performance are put forward. The expressions for steering sensitivity and road feel are derived and the stability criterion is established after the analysis of EPAS dynamics and of vehicle dynamics. Then, by means of frequency domain analysis, the effect of each parameter on these indices is given. This shows that the torque sensor stiffness, the inertia of the motor and assist gain of the controller have a significant effect on steering sensitivity and road feel. To obtain the best steering performance, an optimizing model is used to find the optimum value for parameters of the EPAS system and the result is given.

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