Active Steering for Vehicle System Using Sliding Mode Control

Conference Paper · July 2006

DOI: 10.1109/SCORED.2006.4339349 · Source: IEEE Xplore

3 authors:

Nor Maniha Ghani
Universiti Malaysia Pahang

Yahaya Md Sam
Universiti Teknologi Malaysia

Anuar Ahmad
Universiti Teknologi Malaysia

ACTIVE STEERING FOR VEHICLE SYSTEM USING SLIDING MODE CONTROL

NOR MANIHA BTE ABD GHANI

A project report submitted in partial fulfilment of the
requirements for the award of the degree of
Master of Engineering (Electrical-Mechatronics and Automatic Control)

Faculty of Electrical Engineering
Universiti Teknologi Malaysia

MAY 2006

PSZ 19:16 (Pind. 1/97)

UNIVERSITI TEKNOLOGI MALAYSIA

BORANG PENGESAHAN STATUS TESIS$^{\upsilon}$

JUDUL: **ACTIVE STEERING FOR VEHICLE SYSTEM USING SLIDING MODE CONTROL**

SESI PENGAJIAN: 2005/2006

SAYA **NOR MANIHA BTE ABD GHANI**
(HURUF BESAR)

mengaku membenarkan tesis (~~PSM~~/Sarjana/~~Doktor Falsafah~~)* ini disimpan di Perpustakaan Universiti Teknologi Malaysia dengan syarat-syarat kegunaan seperti berikut:

1. Tesis adalah hakmilik Universiti Teknologi Malaysia.
2. Perpustakaan Universiti Teknologi Malaysia dibenarkan membuat salinan untuk tujuan pengajian sahaja.
3. Perpustakaan dibenarkan membuat salinan tesis ini sebagai bahan pertukaran antara institusi pengajian tinggi.
4. ** Sila tandakan (✓)

☐	SULIT	(Mengandungi maklumat yang berdarjah keselamatan atau kepentingan Malaysia seperti yang termaktub di dalam AKTA RAHSIA RASMI 1972)
☐	TERHAD	(Mengandungi maklumat TERHAD yang telah ditentukan oleh organisasi/badan di mana penyelidikan dijalankan)
✓	TIDAK TERHAD	

Disahkan oleh

(TANDATANGAN PENULIS) (TANDATANGAN PENYELIA)

ALAMAT TETAP:
NO 53 JALAN 32
TAMAN TAS KUANTAN
PAHANG

NAMA PENYELIA:
P. M. DR. YAHAYA MD SAM

TARIKH: **4 MEI 2006** TARIKH: **4 MEI 2006**

CATATAN: * Potong yang tidak berkenaan.
** Jika tesis ini SULIT atau TERHAD, sila lampirkan surat daripada pihak berkuasa/organisasi berkenaan dengan menyatakan sekali sebab dan tempoh tesis ini perlu dikelaskan sebagai SULIT atau TERHAD.
υ Tesis dimaksudkan sebagai tesis bagi Ijazah Doktor Falsafah dan Sarjana secara penyelidikan, atau disertasi bagi pengajian secara kerja kursus dan penyelidikan, atau Laporan Projek Sarjana Muda (PSM).

"I hereby, declare that I have read this thesis and in my
opinion this thesis is sufficient in terms of scope
and quality for the award of degree of
Master of Engineering (Electrical-Mechatronics and Automatic Control)

Signature : ______________________

Name of Supervisor : ASSOC. PROF DR. YAHAYA BIN MD. SAM

Date : 4 MAY 2006

I declare that this thesis *"Active Steering for Vehicle System Using Sliding Mode Control"* is the result of my own research except for works that have been cited in the reference. The thesis has not been accepted any degree and not concurrently submitted in candidature of any other degree.

Signature : ________________________

Name of Author : NOR MANIHA BTE ABD GHANI

Date : 4 MAY 2006

Dedicated to my mother, father, sister and brother

ACKNOWLEDGEMENT

Alhamdullillah, I am greatly indebted to ALLAH SWT on His blessing upon completing this project successfully.

I would like to express my gratitude to honorable Associate Professor Dr. Yahaya Md Sam for his guidance, support and encouragement throughout this study. Many thanks are due to him for the helpful suggestions and valuable discussion.

Lastly, I would like to dedicate my gratitude to my parents, my family and friends especially my coursemates who helped me directly or indirectly in the completion of this project. The encouragement and unlimited support they bestowed mean a lot to me.

PUBLICATIONS

The following papers have been submitted to Conference:

1. Nor Maniha Abd Ghani , Yahaya Md Sam and Adizul Ahmad. “ACTIVE STEERING FOR VEHICLE SYSTEM USING ROBUST CONTROL STRATEGY”, 4th Student Conference on Research Development SCOReD 2006. (Accepted for Oral presentation)

2. Nor Maniha Abd Ghani and Yahaya Md Sam and Adizul Ahmad. “SLIDING MODE CONTROL OF AN ACTIVE STEERING FOR VEHICLE STABILITY”, First International Conference on Man-Machine Systems ICoMMS 2006. (Under review)

ABSTRACT

The objectives of this thesis are to present a modeling and control of a single-track car model for active steering vehicle system. The sliding mode control strategy will be utilized to overcome various coefficients of road frictions and external disturbances on the system. In order to compensate the disturbances, side slip angles and yaw rate of the vehicle will be observed. The model presented take into account different friction of road coefficients of the system. From the mathematical derivation it is found that the system has fulfilled a matching condition. Extensive computer simulations are performed for various types of disturbances such as crosswind and braking torque. From the simulation results the effect of disturbance attenuation will be observed. The performance of the proposed controller will be compared to the linear quadratic regulator and pole placement techniques. The results showed that the sliding mode control scheme is effectively in attenuating various disturbances for different road coefficients as compared to the LQR and pole placement control schemes. Furthermore, the simulation results also showed that the system is insensitive to the external disturbances and capable to overcome 'late action' by the driver due to sudden disturbance on any road conditions.

ABSTRAK

Tesis ini bertujuan untuk memperkenalkan model matematik dan teknik kawalan sistem kemudi aktif untuk model kereta separuh. Kawalan ragam gelincir digunakan untuk mengatasi masalah pelbagai nilai pekali geseran jalan dan gangguan luar yang bertindak ke atas sistem. Oleh itu, untuk mengatasi masalah gangguan ini, gelinciran sisi dan kadar golengan sistem ini akan dikenalpasti. Sistem kawalan ini dipilih sebagai strategi kawalan untuk memperbaiki pelbagai pekali geseran jalan dan gangguan luar yang bertindak ke atas sistem. Daripada model matematik, sistem ini didapati memenuhi keadaan terpadan. Penyelakuan computer telah dijalankan dan didapati masalah gangguan luar yang berbeza dapat diatasi seperti angin lintang dan daya brek. Prestasi pengawal yang diperkenalkan dianalisis dengan pengawal jenis pengatur kuadratik linear dan penempatan kutub. Keputusan yang diperolehi menunjukkan pengawal yang dicadangkan berupaya memperbaiki gangguan luar pada pekali jalan yang berbeza jika dibandingkan dengan pengawal lain. Keputusan juga menunjukkan bahawa sistem ini tidak lagi sensitif kepada gangguan luaran dan berupaya mengatasi kesan lengah daripada pemandu hasil daripada gangguan mengejut pada semua permukaan jalan.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

α_F	-	Sideslip angles at front tire
α_R	-	Sideslip angles at rear tire
β_F	-	Front chassis sideslip angle
β_R	-	Rear chassis sideslip angle
c_F	-	Front cornering stiffness
c_R	-	Rear cornering stiffness
δ_F	-	Front steering angle
δ_R	-	Rear steering angle
F_{ytF}	-	Lateral force at front tire
F_{ytR}	-	Lateral force at rear tire
ℓ_F	-	Distance between the center of gravity (CG) and the front axle
F_{yF}	-	Dominant component in chassis coordinates for front tire
F_{yR}	-	Dominant component in chassis coordinates for rear tire
ℓ_R	-	Distance between the center of gravity (CG) and the rear axle
F_X	-	longitudinal force component
J	-	Moment of inertia for the car body
m	-	Mass of the car body
v	-	Velocity of the car
M_{zD}	-	Disturbances
μ	-	Coefficient of road friction

$\sigma(t)$	-	Sliding surface for a single track model
ρ	-	Sliding gain for a single track model
δ	-	Boundary layer thickness for a single track model
β	-	Side slip angle
r	-	Yaw rate
S	-	Generalized sliding surface

LIST OF ABBREVIATIONS

LSF	Linear State Feedback
DOF	Degree of Freedom
LQR	Linear Quadratic Regulator
VSC	Variable Structure Control
SMC	Sliding Mode Control
MRAC	Model Reference Adaptive Control

CHAPTER 1

INTRODUCTION

1.1 Vehicle stability

Vehicle handling is essentially the response provided by the vehicle due to forces that act on it. For example, driver steering, wind, degree of horizontal curvature and vehicle suspension affect the handling characteristic and destabilized a vehicle. The performance of the vehicle stability system has been greatly increased due to increasing vehicle capabilities. This include active suspension system and active steering system in order to stabilize a vehicle.

A new generation of stability control systems help the driver and passengers to maintain ride comfort and safety. A stability control system goes one step further by actually detecting when a driver has lost some degree of control. It then automatically stabilize the vehicle to help the driver regain control back.

The investigation of vehicle suspension system are much interested in recent years and it promised some comfort issue. Steering control for vehicle motion is crucial for vehicle safety. The goal of this research is concentrated on active steering for vehicle system. The basic problems associated with vehicle handling are subject to the vehicle desired path and vehicle stabilization on this path.

Vehicle handling and ride characteristics combined with the mechanics of road-tire interaction greatly influence the vehicle stability. Finally, to ensure the vehicle stability, all four tires must remain contact with the road surface. The associated friction between the mediums is also plays a role in the vehicle handling and ultimately ride experience.

Even on a ‘good’ road, sudden movement of the steering could make a car skid will caused a critical driving situation. For example, a child crossing the road unexpectedly will force the driver to an evasive action. The inexperienced driver who caught in this kind of situation is easily overreact and destabilizes the car.

A skid normally occurs when the speed of the car is too fast for the normal road conditions. A skid hardly ever occurs at a slow speed. Severe braking can also cause a skid. Many dangerous situations occurred on the roads because a car driver does not react fast enough at the beginning of skidding or rollover. Automatic feedback systems can assist the driver to overcome such dangerous situations. A further step is automatic driving by following a lane reference. In both cases, robustness of the control system with respect to the uncertain road-tire contact is very important. Further uncertain parameters may be due to the vehicle mass, velocity and acceleration, slip-angle and yaw angle.

1.2 Project Overview

The existing driver assistant systems use a braking system that been applied on the individual wheels [1]. These systems are cheap, because the hardware consists of the existing ABS braking system with an additional yaw rate sensor and do not require a new actuator.

There are few reasons why an actuator is needed in the new design of the driver assistant system. The first reason is to generate torque that be able to compensate yaw disturbance torques. Torques is tire force times lever arm. Then, the second reason is that different friction coefficient μ on the left and right sides (μ-split braking) may be the cause of the disturbance torque. In contrast, a steering torque can compensate the braking torque and achieve a straight short braking path. A third reason for driver assistance by steering (rather than by braking) is energy conservation, reduced wear of tires and brakes and smooth operation around zero correction.

Practically, braking systems cannot immediately compensate small errors, but it have to intervene relatively late in detected emergency situations when the car is close to skidding and the driver is unpleasantly surprised by the intervention. Only the steering systems are feasible for continuous operation. Also for better comfort under continuous disturbances.

It is well known that vehicle dynamics are subjected to various uncertainties due to modelling inaccuracies [3]. Thus, conventional linear control approaches is not capable to handle this situation. Therefore, the robust performance capabilities against uncertainties may be overcome by applying a robust controller to the vehicle system design.

The aim of this study is to address the automatic steering control of passenger cars for general lane-following manoeuvres. A lateral vehicle control system using a 2-DOF controller based on H_∞ loop-shaping methodology is successfully designed [3]. This robust controller provides good lane-keeping and lane-change abilities on both curved and straight road segments. Furthermore, it offers a computationally efficient algorithm and does not require explicit knowledge of the vehicle uncertainty. However, the test results demonstrate that higher vehicle speed has a destabilizing effect on the vehicle system.

1.3 Objective of study

The objectives of this research are as follows:

I. To establish a single-track car model.

II. To develop a controller that base on the robust control strategy that will overcome uncertainties and disturbances of a road handling that will prevent the car skidding problem.

III. To evaluate and analyzed the performance of the system with a proposed controller.

To achieve these objectives various parameters such as tire slip angle and yaw rate will be observed by using the computer simulations. Performance of the proposed controller will be compared to LQR and Sliding Mode Control techniques.

Theoretical verification of the proposed controller on its stability and rechability will be accomplished by using a Lyapunov's second method theory. The performance of the active steering system will be observed by using extensive computer simulation that will be performed using MATLAB software and SIMULINK Toolbox subjected to various types of parameter.

1.4 Scope of Project

The work undertaken in this project are limited to the following aspects:

I. Mathematical establishment of a single track car model as described by Ackermann J. et all.

II. An active car steering system is evaluated on straight road due to various disturbance profiles and coefficient of road friction.

III. Design a controller for a single track car model using sliding mode control technique to compensate disturbances.

IV. Perform a simulation works by using a MATLAB/Simulink to observe effectiveness and robustness of the controller.

V. Compare the performance of the proposed SMC with pole placement and Linear Quadratic Regulator (LQR) techniques.

1.5 Research Methodology

The methodology of this research is shown in the flow chart in Figure 1.1 below:-

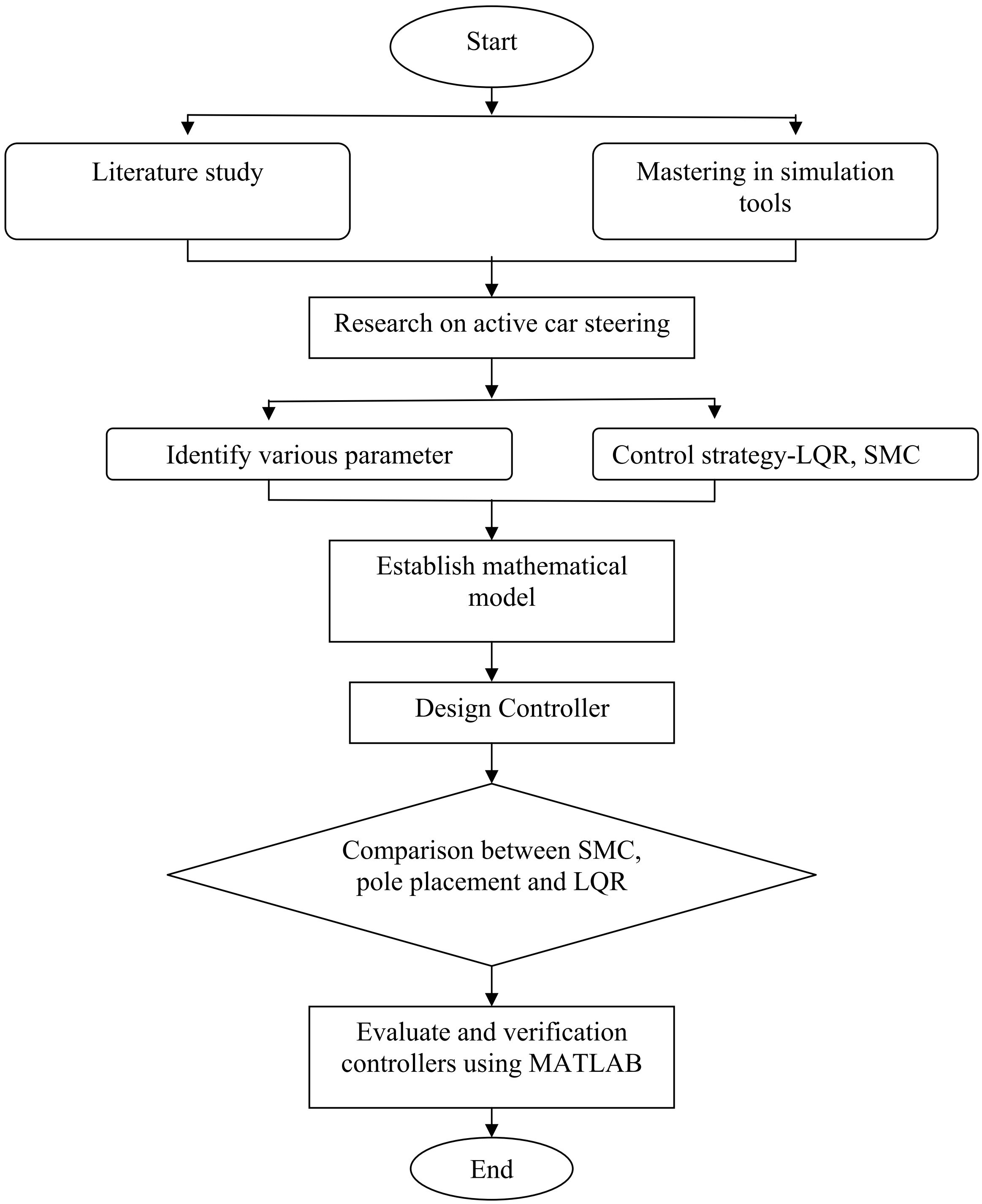

Figure 1.1 Research methodology flow chart

In the beginning, there are two tasks that need to be done simultaneously. Besides of doing literature review, exercises and tutorials need to be done for mastering the simulations tools and some preliminary simulations need to be carried out as well.

Then, the problem of this research will be stressed on a car manoeuvres on straight road due to various disturbances and coefficient of road friction to ensure stability and prevent skidding. The task is to remain the car at the centerline planned lane as accurately as possible with the changes of disturbances. This will be considered of the dynamics relationship of a car and take into account the yaw movement and lateral motion that make the car unstable.

Thus, the steering control strategy is required to minimize the lateral displacement, lateral acceleration and yaw rate error due to step changes in disturbances. Various parameters will be observed such as relationship between slip angles and lateral forces on tire and uncertainties on the friction of the road surfaces.

After the preliminary research, the mathematical model will be established for lane-keeping manoeuvres (straight path). First, the state space representation of the dynamic model of the car with forces input will be outlined. Based on the dynamic models of the actuators, the state space representation of the active steering car will be derived. Finally the effect of the disturbance torque as an undesired yaw rate that should be compensated by an automatic control system will be presented.

Then, new control strategy for active steering car system will be proposed based on the sliding mode control approach. The design of the sliding mode control which consists of selecting the sliding surfaces and the controller will be discussed. Various types of the sliding surfaces are outlined. Several types of controllers that drive the state trajectory onto the sliding surface will be presented. The applications of sliding mode control in control engineering world will be clarified to show the importance of the sliding mode control strategy and applicability.

The sliding mode controller is proposed to improve the performance of the ride comfort and road handling characteristic of the car steering system. To assure the robustness of the proposed controller, various disturbances and coefficient of road friction will be applied to the system. The chattering and boundary layer effect on the controller will also be outlined by varying the parameters in the continuous switching gain.

Finally, the proposed controller will be evaluated with extensive simulation work to determine the performance. Then, the performance of the system using SMC will be compared with pole placement and Linear Quadratic Regulator (LQR) techniques. The results will be verified and analyzed in time domain related to disturbance attenuation such as reduce in magnitude, overshoot, rise time and settling time.

1.6 Literature Review

In the past years, various control strategies have been proposed by researchers to improve the vehicle stability in the presence of parameter variations of the vehicles. These control strategies may be grouped into techniques and approaches how it will be used. In the following, some of these control approaches that have been reported in the literature will be briefly presented.

Most of the approaches have assumed a linearized model in the design and do not consider nonlinearity of the tire characteristics. These approaches may yields good results as long as the vehicle remains within the linear region of the tire characteristics. However the controller may even worsen the driving situation drastically compared to conventional vehicle, as soon as the nonlinear region of the tire characteristics is entered [4]. Therefore, vehicle stability system has to be designed and taking into account the uncertain and nonlinear tire characteristics which are determined by the road-tire-contact.

A robust decoupling of car steering dynamics with arbitrary mass distribution is presented [2]. The restrictive mass distribution assumption is abandoned and a generalized decoupling control law for arbitrary mass distribution has been derived. The result of this paper provides an interface between the modelling of the steering dynamics of a single car by two masses and the higher level control problems of automatic steering and distance keeping of single mass models in a platoon of cars. However, there are some restrictive assumption in this paper which is the constant velocity, small sideslip and steering angles.

H_∞ control approach is proposed to overcome robust stabilization and uncertain plants [6]. A linear matrix inequalities based on H_∞ methodology has been designed [7]. Then $H\infty$ loop-shaping design procedure was proposed [8]. The results showed that this method provides a computationally efficient algorithm and does not require explicit knowledge of the uncertainty.

The combination of H_∞ loop shaping and 2-DOF has been reported in [3] [7] to achieve high performance control system for vehicle handling. It has been shown that this algorithm allowed separate processing of the robust stabilization problem and reference signals. The test results the robust control scheme offers a computationally efficient method and does not require explicit knowledge of the vehicle uncertainty. The presented system exhibits the required performances and robustness properties under parameter variations while maintaining passenger comfort. However, the test results demonstrate that higher vehicle speed has a destabilizing effect on the vehicle system.

A model reference adaptive control (MRAC) technique of 2WS cars which is realized by steer-by-wire technology has been reported [9]. The aim of MRAC is to make the output of varies parameter asymptotically approach the output of a user defined reference model that represents a desired characteristics. The study introduce first-order system whose output is D*, defined as the combination of yaw rate and lateral acceleration. This method can treat the nonlinear relationships between the slip angles and the lateral forces on tires, and the uncertainties on the friction of the road surface.

Recently, intelligent based techniques such as fuzzy logic, neural network and genetic algorithm have been applied to the active steering system [10]. The papers presented a fuzzy-rule-based cornering force estimator to avoid using an uncertain highly nonlinear expression, and neural network compensator is additionally utilized for the estimator to correctly find cornering force. The result indicated that the proposed control system is robust against the uncertainty in vehicle dynamic model disturbances such as a side wind gust and road conditions.

A fuzzy logic controller with Hardware-In-the-Loop Simulation (HILS) simulator has been proposed by [11] to evaluate the performance of the system on a slippery road. HILS simulator is composed of hardware (steering wheel) and software (vehicle simulation tool and steering control system). This method used fuzzication, fuzzy inference and defuzzication technique. It can be observed that this controller able to maintain the steering maneuverability on slippery road and very useful to correct the vehicle’s route when the vehicle’s direction is biased due to side wind or obstacles.

However, the proposed steering control system is similar as the ABS system braking does.

1.7 Layout of Thesis

This section outlines the structure of the thesis. Chapter 2 deals with the mathematical modelling of a single track car model. In this chapter, detailed derivations on the modelling of the active car steering systems with linearization model plant are presented. A mathematical derivation is well established followed by active car steering in state space form. In addition, the assumptions and limitations that been added to the model will be described.

Chapter 3 discusses control algorithm design for controlling active car steering system. Analysis regarding on performance of designed controller will be conducted.

Chapter 4 discusses the simulation results due to various road coefficients and disturbance profiles. The performance of the sliding mode control is compared with pole placement and LQR techniques. The chattering and boundary layer effect of the controller will be presented by varying the parameter in the continuous switching gain. The simulation work is performed and evaluated by using Matlab/Simulink Toolbox.

The summary of the result and recommendation for future works will be presented in Chapter 5.

CHAPTER 2

SYSTEM MODEL

2.1 Introduction

In this chapter, detailed derivations on the modeling of the active car steering systems with linearization model plant are presented. A mathematical derivation is well established followed by active car steering in state space form. Moreover, the assumptions and limitations that have been added to the single track car model will be described. Finally, disturbance profiles which are considered as the disturbance input torque are also presented in this chapter.

2.2 Mathematical Modeling For A Single Track Model

Basically, there are three motion or rotation components of a car which are x, y and z rotations. Figure 2.1 shows a typical vehicle axis system. Motion along the y-axis is referred to as sideslip, yaw is rotation about the z-axis while roll is rotation about x-axis, which intuitively is also known as a rotation axis [11].

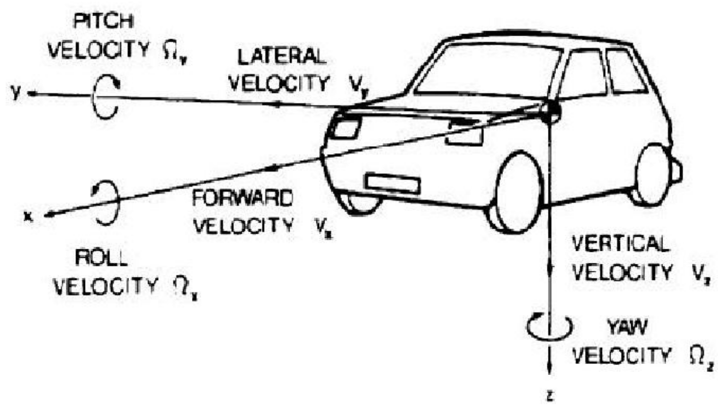

Figure 2.1 Vehicle axis system

The implementation of a steering control system schematic is shown in Figure 2.2. The main steering angle δ_S is commanded by the driver from the steering wheel. A small corrective steering angle is set by an actuator with input from a feedback controller. The main feedback signal is the yaw rate and the superposition $\delta_F = \delta_S + \delta_C$ may be done mechanically.

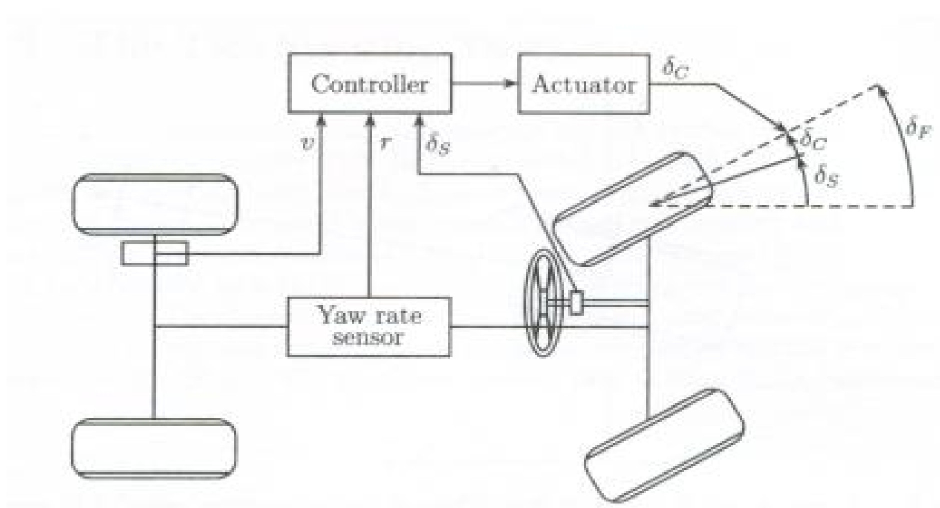

Figure 2.2 Feedback controlled additive steering angle δ_C

The dynamics of vehicle steering may be described by the single track model. The single track model is obtained by lumping the two front wheels into one wheel in the center line of the car, it is similar for the two rear wheels. Then, the car model is reduced as in Figure 2.3.

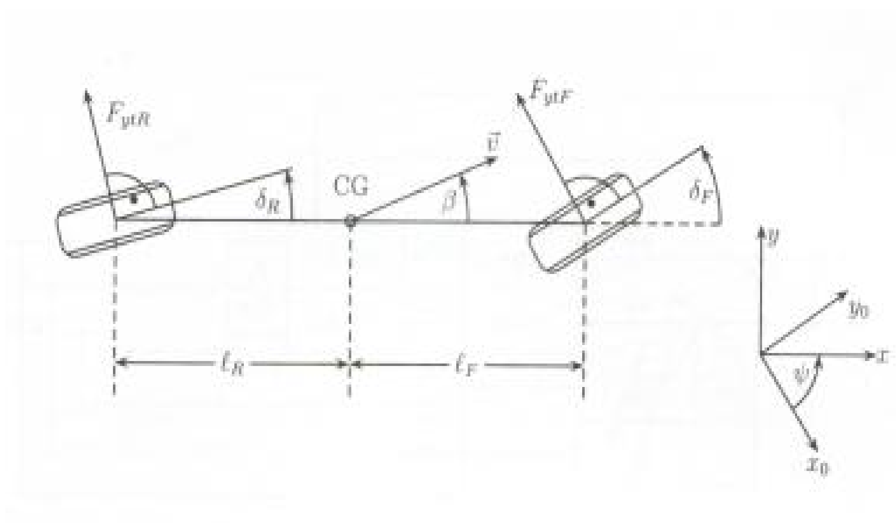

Figure 2.3 Single-track model for car steering

The angles δ_F and δ_R are the front and rear steering angles. The distance between the center of gravity (CG) and the front axle is ℓ_F and rear axle is ℓ_R and the sum $\ell=\ell_R + \ell_F$ is the wheelbase. The angle β between the vehicle center line and the velocity vector v at the CG is called the vehicle sideslip angle. In the horizontal plane of Figure 2.3, an initially fixed coordinates system (x_o,y_o) is shown together with a vehicle fixed coordinates system (x,y) that is rotated by a yaw angle ψ. In the dynamic equations, the yaw rate $\psi := r$ is taken as one of the system state variables.

The yaw angle r will be included in the model is of automatic car steering, where the position of the vehicle relative to the lane is considered. The forces transmitted between the road surface and the car chassis via the wheels are represented in Figure 2.3

by the side forces F_{ytF} and F_{ytR} respectively. The forces in the longitudinal direction of the tires are assumed zero, *i.e* the wheels are freely spinning.

Figure 2.4 shows the velocity vector v_F under a sideslip angle β_F at the front axle with respect to the longitudinal axis (x-axis) of the chassis. The lateral force F_{ytF} in tire coordinates is a function of the tire slip angle $\alpha_F = \delta_F - \beta_F$. Its dominant component in chassis coordinates is

$$F_{yF} = F_{ytF} \cos \delta_{F.} \tag{2.1}$$

The small retarding component $F_{xF} = -F_{ytF} \sin \delta_F$ does not generate a yaw torque, if it occurs symmetrically at the left and right wheel. Its longitudinal effect is compensated by speed control (automatic or by the driver). In a static tire description, the tire side force F_{ytF} is a function of the tire slip angle α_F as follows,

$$F_{ytF} = f(\alpha_F) = f(\delta_F - \beta_F), \tag{2.2}$$

$$F_{yF} = f(\delta_F - \beta_F) \cos \delta_{F.} \tag{2.3}$$

The index F indicates the front wheels; it is replaced by R for the rear wheels. If the velocity vector v_F is aligned with the tire, then the lateral force is zero, $f(0) = 0$. For $\alpha_A > 10^{\mathrm{o}}$, the lateral force is close to saturation. Ideally, a control system cannot overcome such physical limits. Therefore, it is very important to design steering controllers such that only small tire sideslip angles occur.

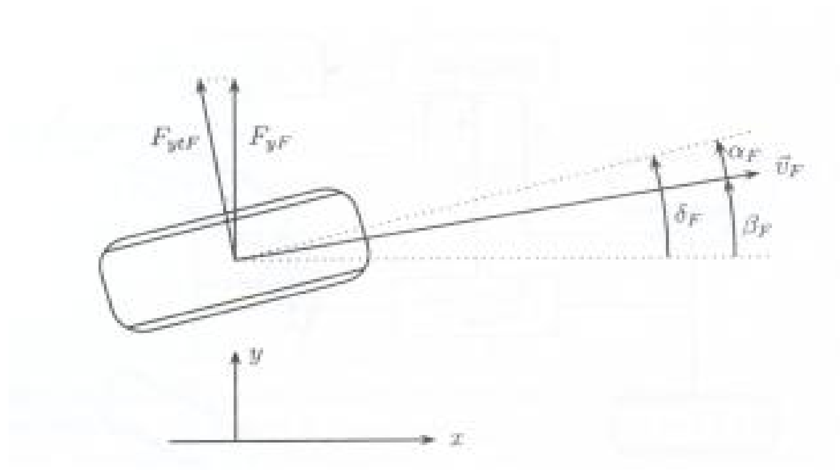

Figure 2.4 Lateral forces F_{ytF} at the front wheel in tire coordinates and F_{yF} in chassis coordinates.

Input to the vehicle dynamics are the lateral forces at the front and rear axles:

$$
\begin{aligned}
F_{yF} &= F_{ytF} \cos \delta_F \\
F_{yR} &= F_{ytR} \cos \delta_R
\end{aligned}
\tag{2.4}
$$

And a longitudinal force component:

$$
F_x = -F_{ytF} \sin \delta_F - F_{ytR} \sin \delta_R \tag{2.5}
$$

Note that these forces represent the sum of the forces at the left and right tire. From the dynamics model, the forces control state variables are *β, v, r* respectively. The equations of motions for 3DOF in the horizontal plane are:

a) Lateral motion

$$
mv(\dot{\beta}+\dot{\psi})\cos\beta + m\dot{v}\sin\beta = F_{yF} + F_{yR} \tag{2.6}
$$

b) Longitudinal motion

$$-mv(\dot{\beta}+\dot{\psi})\sin\beta + m\dot{v}\cos\beta = F_x \tag{2.7}$$

c) Yaw motion

$$J\ddot{\psi} = F_{yF}\ell_F - F_{yR}\ell_R + M_{zD} \tag{2.8}$$

With $r := \dot{\psi}$, and from equations (2.6) to (2.8), the following motion equations can be obtained

$$\begin{bmatrix} mv(\dot{\beta}+r) \\ m\dot{v} \\ J\dot{r} \end{bmatrix} = \begin{bmatrix} -\sin\beta & \cos\beta & 0 \\ \cos\beta & \sin\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} F_x \\ F_{yF} + F_{yR} \\ F_{yF}\ell_F - F_{yR}\ell_R + M_{zD} \end{bmatrix} \tag{2.9}$$

Then, the sideslip angles α_F and α_R at the front and rear tires are obtained by a kinematic model from the steering angles δ_F, δ_R and from the state variables β, r and v. The front and rear chassis sideslip angles are β_F and β_R. The velocity components in the direction of the longitudinal center line of the vehicle must be equal:

$$v_R \cos\beta_R = v_F \cos\beta_F = v \cos\beta \tag{2.10}$$

The velocity components perpendicular to the center line depend on the yaw rate r, as

$$\begin{aligned} v_F \sin\beta_F &= v \sin\beta + \ell_F r, \\ v_R \sin\beta_R &= v \sin\beta - \ell_R r. \end{aligned} \tag{2.11}$$

The velocity terms v_F and v_R are eliminated by division by the corresponding terms from equation (2.10). Thus, the kinematic models are

$$\tan\beta_F = \frac{v\sin\beta + \ell_F r}{v\cos\beta} = \tan\beta + \frac{\ell_F r}{v\cos\beta}$$

$$\tan\beta_R = \frac{v\sin\beta - \ell_R r}{v\cos\beta} = \tan\beta - \frac{\ell_F r}{v\cos\beta} \tag{2.12}$$

and the tire sideslip angles as shown in Figure 2.4 are

$$\alpha_F = \delta_F - \beta_F$$
$$\alpha_R = \delta_R - \beta_R \tag{2.13}$$

The feedback-structured model is now completed by the non-linear tire model:

$$F_{ytF} = f_F\,(\alpha_F)$$
$$F_{ytR} = f_R\,(\alpha_R) \tag{2.14}$$

2.3 Linearization for Constant Velocity and Small Angles

The vehicle dynamics represented in equation (2.9) are non-linear. These equations may be linearized by the assumption $\dot{v} = 0$. This is justified, because the velocity v is changing more slower than the state variables r and β. The velocity v is now treated as an uncertain constant parameter. Also, the force component component $F_x \sin\beta$ is neglected. Then, the linearized version of equation (2.9) is:-

$$\begin{bmatrix} mv(\dot{\beta}+r) \\ J\dot{r} \end{bmatrix} = \begin{bmatrix} (F_{yF}+F_{yR})\cos\beta \\ F_{yF}\ell_F - F_{yR}\ell_R + M_{zD} \end{bmatrix} \tag{2.15}$$

The chassis sideslip angles β, β_F, β_R are small and $\cos\beta = 1$ hence:-

$$\begin{aligned} \beta_F &= \beta + \ell_F r / v \\ \beta_R &= \beta - \ell_R r / v \end{aligned} \tag{2.16}$$

The steering angles δ_F, δ_R are small, cos $\delta_F = 1$ and cos $\delta_R = 1$ for the rear wheels. Hence,

$$\begin{aligned} F_{yF} &= F_{yF}(\alpha_F), \\ F_{yR} &= F_{yR}(\alpha_R), \end{aligned} \tag{2.17}$$

are the unknown characteristics. Equation (2.15) yields

$$\begin{bmatrix} mv(\dot{\beta}+r) \\ J\dot{r} \end{bmatrix} = \begin{bmatrix} F_{yF}(\alpha_F)+F_{yR}(\alpha_R) \\ F_{yF}(\alpha_F)\ell_F - F_{yR}(\alpha_R)\ell_R + M_{zD} \end{bmatrix} \tag{2.18}$$

where $\alpha_F = \delta_F - \beta_F$, $\alpha_R = \delta_R - \beta_R$.

Under the feedback control strategy according to Figure 2.2, the steering angle δ_F is the sum of the driver command δ_s and a corrective angle δ_C, are generated by the feedback system. The relationship between δ_s, δ_c *and* α_F, β_F is illustrated by Figure 2.4.

In order to allow a linear analysis approach is applied to the car, the lateral tire forces are now linearized about a zero tire sideslip angle as follows:

$$F_{yF}(\alpha_F) = \mu c_F(\alpha_F), \qquad (2.19)$$
$$F_{yR}(\alpha_R) = \mu c_R(\alpha_R),$$

The slope c_F and c_R of the tire characteristic are called cornering stiffness. The friction coefficient $\mu \leq 1$ is assumed to be similar for the front and rear wheels. Typical values of the friction coefficient μ are [1]:

$\mu = 1$ (dry road)
$\mu = 0.5$ (wet road)
$\mu = 0.15$ (ice)

By solving the equation (2.18) for $\dot{\beta}$ and $\dot{r}$ and substitute the linearized tire characteristics F_{yF} and F_{yR} yields,

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{\mu}{mv}(c_F\alpha_F + c_R\alpha_R) - r \\ \frac{\mu}{J}(c_F\ell_F\alpha_F + c_R\ell_R\alpha_R) + \frac{1}{J}M_{zD} \end{bmatrix} \qquad (2.20)$$

then, substitute equations (2.13) and (2.16) gives:

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{\mu}{mv}[c_F(\delta_F - \beta - \ell_F r/v) + c_R(\delta_R - \beta + \ell_R r/v)] - r \\ \frac{\mu}{J}[c_F\ell_F(\delta_F - \beta - \ell_F r/v) - c_R\ell_R(\delta_R - \beta + \ell_R r/v)] + M_{zD}/J \end{bmatrix} \qquad (2.21)$$

The linear state space model is written as

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \delta_F \\ \delta_R \end{bmatrix} + \begin{bmatrix} 0 \\ b_z \end{bmatrix} M_{zD} \tag{2.22}$$

where the non-zero elements of A, B and D matrices are:

$a_{11} = -\mu(c_R + c_F)/mv$, $a_{12} = -1 + \mu(c_R \ell_R - c_F \ell_F) mv^2$,

$a_{21} = \mu(c_R \ell_R - c_F \ell_F)/J$, $a_{22} = -\mu(c_R \ell_R^{\,2} + c_F \ell_F^{\,2}) Jv$,

$b_{11} = \mu c_F / mv$, $b_{12} = \mu c_R / mv$,

$b_{21} = \mu c_F \ell_F / J$, $b_{22} = \mu c_R \ell_R / J$,

$b_z = 1/J$

The parameters of the active car steering system is shown in Table 2.1 [1]

Table 2.1: Parameter value for the active steering car system (BMW 735i)

Mass of the car body, m	: 1864 kg
Moment of inertia for the car body, J	: 3654 kgm^2
Velocity of the car, v	: 70 m/s
Cornering stiffness of the rear axle, c_R	: 213800 N/rad
Cornering stiffness of the front axle, c_F	: 101600 N/rad
Wheelbase of the rear axle, ℓ_R	: 1.32 m
Wheelbase of the front axle, ℓ_F	: 1.51 m

2.4 Disturbance Profiles

To study the robustness of the active car steering that subject to various disturbance types irregularities, various set of disturbance profiles will be used in this study. The disturbance profiles *f(t)* can be classified into two types which are crosswind and μ-split braking torque. Figures 2.5 and 2.6 show a braking torque and crosswind profiles that represents the external disturbance that generated to the active steering car system [14]. These disturbance profiles are as defined in the following subsections.

2.4.1 Disturbance Profile 1

Disturbance profile 1 representing braking torque as shown in Figure 2.5.

2.4.2 Disturbance Profile 2

Disturbance profile 2 representing crosswind as shown in Figure 2.6.

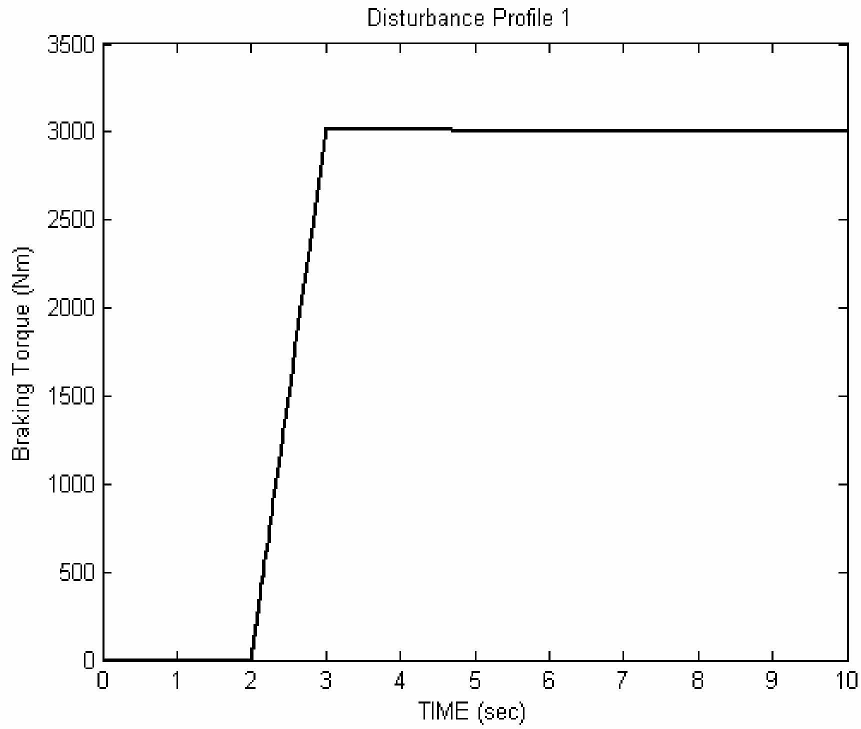

Figure 2.5 Disturbance profile 1 represented a braking torque

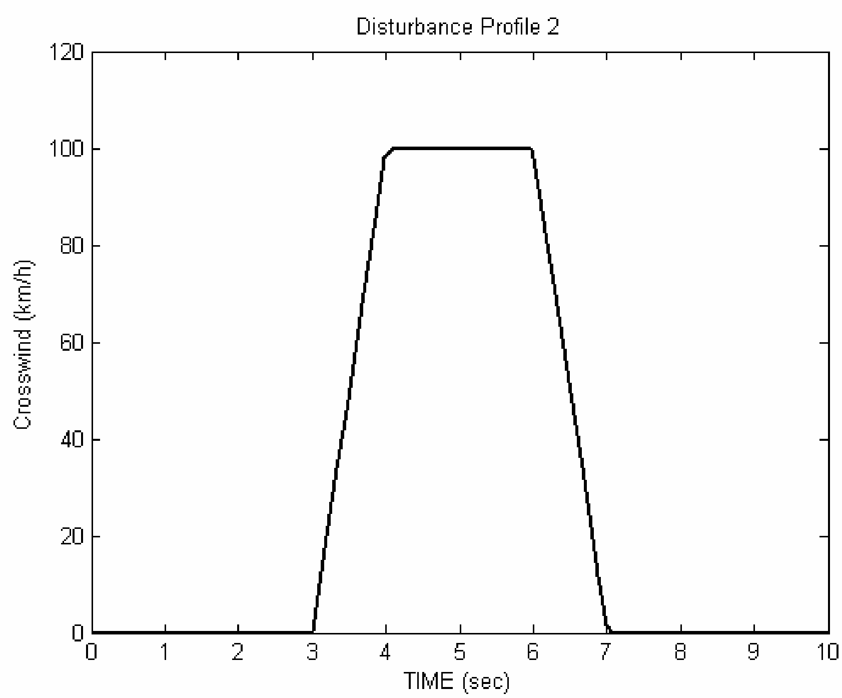

Figure 2.6 Disturbance profile 2 represented a crosswind

These two types of disturbance profiles will be used for both road condition, μ=1 (dry road) and μ=0.5 (wet road) [1].

2.5 Conclusion

In this chapter, the mathematical model of the active car steering has been established. The complete model of the system will be used in the design of a new controller and simulation to study the effectiveness of the proposed controller. Various road coefficient parameters and disturbance profiles have also been discussed at the end of this chapter. These parameters will be used in the simulation to study the robustness of the proposed controller which will be presented in the following chapters.

CHAPTER 3

SLIDING MODE CONTROL DESIGN FOR ACTIVE STEERING VEHICLE SYSTEM

3.1 Introduction

The problem of controlling uncertain dynamical systems that are subjected to external disturbances is a topic which is of considerable interest to control researchers. One approach to solve this problem is by means of variable structure control (VSC). The VSC is utilized a high-speed switching control law to drive the nonlinear plant's state trajectory onto a specified and user chosen surface in the state space, and to maintain the plant's state trajectory on this surface for all subsequent time. This surface is called the switching surface because if the state trajectory of the plane is above the surface, control path has one gain, and has a different gain if the trajectory drops below the surface [16]. Then, the control action is called sliding mode control (SMC). The plant dynamics restricted to this surface represent the control system's behavior. By a proper design of the sliding surface, SMC attains the conventional goals of control such as stabilization, tracking, regulation, etc.

In this study, Sliding Mode Control (SMC) has been proposed to control a vehicle with active steering system. In this chapter, the formulation of sliding mode control is presented to control the vehicle with active steering system. In the mathematical development of the proposed controller, the stability, reachability and the sensitivity factors of the proposed controller will also be considered and presented. It will be shown through computer simulation that the proposed controller is robust to overcome the active steering problem.

3.2 Overview on Sliding Mode Control

Variable Structure Control System (VSCS) has been developed from the pioneering work in Russian in the early 1960s. A good tutorial introduction to VSC gave an insightful view on the rapid development of this control strategy [16]. Various researchers have utilized the VSC schemes to control a number of nonlinear and uncertain systems due to its inherent insensitivity property to parameter variations and external disturbances.

The SMC strategies are to force the system states to reach, and subsequently remain on a predefined surface within the state space [15]. In order to achieve these strategies, the design of the SMC scheme is divided into two components:

i. The design of sliding surface in the state space so that the reduced order sliding motion satisfies the specifications imposed by the designer.

ii. The synthesis of the control law such that the trajectories of the closed loop motion are directed towards the sliding surface and remains thereafter.

3. 2. 1 Sliding Mode Control Design

In this study, the sliding mode control design is presented to control the active steering vehicle system. The design is started with the selection of the sliding surface that can ensure the stability of the system. Then, the controller based on the SMC technique is designed to drive the state trajectory onto the sliding surface. It will be proven mathematically that the reachability condition of the proposed controller is achieved if the proposed theorem is satisfied to ensure that the state trajectory slides onto the sliding surface and remains thereafter.

The typical structure of a sliding-mode controller is composed of a nominal part and additional terms to deal with model uncertainty and disturbances. The way SMC deals with uncertainty is to drive the plant's state trajectory onto a sliding surface and maintain the error trajectory on this surface for all subsequent times. The significant advantage of SMC is that the controlled system becomes insensitive to disturbances such as sinusoidal signals and random noises, and variations of state variables while on the sliding surface.

Throughout the study, $\|Z\|$ denotes the Euclidean norm when Z is a vector:

$$\|Z\| == \sqrt{Z^T Z} \tag{3.1}$$

and an induced norm when Z is a matrix:

$$\|Z\| == \sqrt{\lambda_{\max} Z^T Z} \tag{3.2}$$

Where $\lambda_{max}(Z)$ is the largest eigenvalue of the matrix Z.
Consider the uncertain linear time invariant system given by

$$\dot{x}(t) = Ax(t) + Bu(t) + Df(t) \tag{3.3}$$

Where $x(t) = \Re^n$ is the state vector, $u(t) = \Re^m$ is the control input, the uncertain function or disturbance vector $f(t) \in \Re^l$, n, m and l are the number of states, inputs and disturbances, respectively. $A \in \Re^{nxn}$ is the system matrix, $B \in \Re^{nxm}$ is the input matrix and $D \in \Re^{nxl}$ is the disturbance matrix. Without loss of generality, it can be assumed that the input disturbance matrices B and D are of full rank. The following assumptions are taken as standard throughout this study:

i) The pair (A, B) is controllable

ii) The input distribution matrix B has full rank

iii) System with uncertainties satisfy the matching condition, i. e. rank [B|$f(t)$] = rank [B]

3. 2. 2 Switching Surface Design

The conventional sliding surface $\sigma(t)$ is defined as follows,

$$\sigma(t) = Cx(t) \tag{3.4}$$

where $C \in \Re^{m\times n}$ is a full rank constant matrix. The matrix C is chosen such that $CB \in \Re^{m\times n}$ is nonsingular. The matrix C has the following structure,

$$C=[c_{11}\ c_{12}\ ;\ c_{21}\ c_{22}] \tag{3.5}$$

Then, the differential of equation (3.4) gives,

$$\dot{\sigma}(t) = C\dot{x}(t) \tag{3.6}$$

In describing the method of equivalent control it will initially be assumed that the uncertain function in equation (3.3) is identically zero [14]:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{3.7}$$

Substituting equation (3.7) into equation (3.6) gives,

$$\dot{\sigma}(t) = C[Ax(t) + Bu(t)] \tag{3.8}$$

Equating equation (3.8) to zero gives the equivalent control, $u_{eq}(t)$ as follows,

$$u_{eq}(t) = -[CB]^{-1}CAx(t) \tag{3.9}$$

Substituting equation (3.9) into equation (3.7) gives the equivalent dynamics equation of the system in sliding mode as follows,

$$\dot{x}(t) = Ax(t) + B[-[CB]^{-1} CAx(t)]$$

$$\dot{x}(t) = [I_n - B[CB]^{-1} C]Ax(t) \qquad \text{for all } t \geq t_s \text{ and } Sx(t_s)=0 \qquad (3.10)$$

Where I_n is an *nxn* identity matrix.

It can be observed in equation (3.10) that the dynamics of the system during sliding mode is affected by matrix *C*.

The design of the controller scheme that drives the trajectories of the system in equation (3.10) onto the sliding surface $\sigma(t) = 0$ and the system remains in it thereafter will be presented in the next section.

3. 2. 3 Stability during Sliding Mode

During the sliding mode, the uncertain system with matching condition, System with uncertainties satisfy the matching condition, i. e. rank [B|$f(t)$] = rank [B] is asymptotically stable provided the following theorem is satisfied.

Define

$$P_S = (I_n - B(CB)^{-1} C) \qquad (3.11)$$

then, P_s is a projection operator and satisfies two simple yet very important (from the viewpoint of sliding mode control) equations:

$$SP_s = 0 \qquad \text{and} \qquad P_sB = 0 \tag{3.12}$$

From equation (3.9) the equivalent control can be considered to be the linear state feedback component. It would be tempting to think of using the signal

$$u(t)=Kx(t) \tag{3.13}$$

where $K=-(CB)^{-1}CA$ as a state feedback control law. Then, specifically consider the uncertain function of the system given in (3.3), the new equivalent dynamic equation become:

$$u_{eq}(t) = -(CB)^{-1}[CAx(t) + CDf(t)] \qquad for\ t \geq t_s \tag{3.14}$$

<u>Theorem 1</u>

The ideal sliding motion is totally insensitive to the uncertain function *f(t)* in equation 3.3 if $R(D) \subset R(B)$

Proof:

Substituting the equivalent control law equation (3.14) into the uncertain system representing equation (3.3), it follows that the sliding motion satisfies

$$\dot{x}(t) = P_s Ax(t) + P_s Df(t) \qquad for\ t \geq t_s \text{ and } Sx(t_s) = 0 \tag{3.15}$$

where P_s is the projection operator defined in equation (3.11). Now $R(D) \subset R(B)$,then there exists a matrix of elementary column operations $R \in \Re^{mxl}$ such that $D=BR$. As a result it follows that $P_sD = P_s(BR) = (P_sB)R = 0$ by the projection property described in equation (3.12). The reduced order motion as given in equation (3.15) reduces to

$$\dot{x}(t) = P_s A x(t) \qquad \text{for } t \geq t_s \text{ and } Sx(t_s) = 0 \qquad (3.16)$$

which does not depend on the exogenous signal or uncertain function. Any uncertainty which can be expressed as in equation (3.3) where $R(D) \subset R(B)$ is described as matched uncertainty. Any uncertainty which does not lie within the range space of the input distribution matrix is described as unmatched uncertainty.

Remark. For the system with uncertainties satisfy the matching condition, i. e. rank $[B|f(t)]$ = rank [B], then equation (3.10) can be reduced to $\dot{x}(t) = P_s A x(t)$. Thus asymptotic stability of the system during sliding mode is assured [15].

Now the design of the control scheme that drives the state trajectories of the system in equation (3.10) onto the sliding surface $\sigma(t) = 0$ and the system remains in it there after will be presented.

3. 2. 4 Controller design

Controller design is the second phase of the SMC design procedure. The objective of the controller is to determine switched feedbacks gains that will drive the plant state trajectory to the switching surface and maintain the sliding mode condition [16] reported that the controller can be determined by using one of the following methods:

i. Diagonalization method
ii. Hierarchy Control method
iii. Equivalent Control method

There are two different approaches to design a controller under the diagonalization method. The essential future of these methods is the conversion of the multi-input design problem into m single-input design problem. The first approach requires a transformation of the input signal *u(t)* into a new input signal *u*(t)* via a nonsingular transformation as follows

$$u^*(t) = Q^{-1}(t,x)\left[\frac{\partial\sigma(x)}{\partial x}\right]B(t,x)u(t) \tag{3.17}$$

where *Q(t,x*) is an arbitrary *mxm* diagonal matrix with elements $q_i(t,x), i=1,\ldots\ldots,m,$ such that $|q_i(t,x)>0|$ for all *x*. Frequently, *Q(t,x)* is chosen as an identity matrix. In the second approach, a nonsingular transformation of the sliding surface *σ(t,x)* into a new sliding surface *σ*(t,x)* is needed to yield

$$\sigma^*(t,x) = \Omega(t,x)\sigma(t) = 0 \tag{3.18}$$

For an appropriate transformation matrix *Ω(t,x)*. *u*(t)* is then obtained as in equation (3.17).

In the hierarchy control approach, the controller is divided into several control actions. For example, for an *m*-inputs system, the first control u_1 drives the system from an initial condition onto the surface σ_1=0. The second control u_2 then drives the system onto the intersection of σ_1=0 and σ_2=0, while u_1 maintains a sliding mode on $\sigma_1=0$. The third control u_3 drives the system along the intersection of the surfaces $\sigma_1=0$ and $\sigma_2=0$ to the intersection of the first three switching surfaces. This hierarchy of controls is continued until the last control u_m drives the system to a sliding mode on the intersection of all the m switching surfaces. Further discussions on the diagonalization and control hierarchy methods are available in [16].

The objective in the designing the control scheme is to drives the state trajectories of the system in Equation 3. 15 onto the sliding surface $\sigma(t)$ and the system remains in it thereafter. In order to fulfill the reaching condition, the control input $u(t)$ is divided into two part as described in equation 3. 19. A linear component which equal to the equivalent control input, $u_{eq}(t)$, and a nonlinear component which is equal to the switching control input, $u_s(t)$

$$u(t) = u_{eq}(t) + u_s(t) \tag{3.19}$$

The equivalent control $u_{eq}(t)$ is as given in equation (3.9). The following switching control $u_s(t)$ is used in this study:

$$u_s(t) = -(CB)^{-1} \rho \frac{\sigma(t)}{|\sigma(t)| + \delta} \tag{3.20}$$

where δ is the boundary layer thickness that to be selected to reduce the chattering effect and ρ is a design parameter which will be specified by designer.

A continuous function is proposed for the nonlinear switching control component instead of discontinuous function. Practically, such a discontinuous control is undesirable because the sgn($\sigma(t)$) function may cause chattering problem [17]. Ideally, a control strategy is required to ensure that the system dynamic is both close to $\sigma(t)$ and as close as possible to the ideal sliding mode dynamics. Thus, a control effort is required to ensure $\sigma(t)$ or a neighborhood of $\sigma(t)$ is reached and maintained without the chattering problem. Thus, a continuous, usually nonlinear, control input is employed to ensure that the system dynamics is within a region containing *σ(t)*.

In this study, the controller that will be determined use combination both Hierarchy Control method and Equivalence Control method. Since we have 2-inputs system, σ_1=0 for u_1 and σ_2=0 for u_2 need to be derived. Therefore, we can justify the function of equation (3.6) by choosing a control inputs as:

$$u_1(t) = -(CB)^{-1}CAx(t) - (CB)^{-1}\rho\frac{\sigma_1(t)}{\left|\sigma_1(t)\right| + \delta} \tag{3.21}$$

and

$$u_2(t) = -(CB)^{-1}CAx(t) - (CB)^{-1}\rho\frac{\sigma_2(t)}{\left|\sigma_2(t)\right| + \delta} \tag{3.22}$$

The proposed control input should be designed such that the reaching condition $\dot{V} = \sigma\dot{\sigma} \le 0$ is satisfied. It is well known that if the system is fulfilled the sliding mode condition, hence *σ(t) =0*.

By defining a Lyapunov function:

$$V(\sigma) = 1/2[\sigma]^2 \tag{3.23}$$

It can be guaranteed that the sliding surface $\sigma(t) = 0$ is reached in finite time by choosing equation 3.6 to ensure that $\dot{V} = \sigma\dot{\sigma} \le 0$

$$\sigma\dot{\sigma} = -\rho\,|\,\sigma\,| \qquad \text{or} \qquad \dot{\sigma} = -\rho\,\mathrm{sgn}(\sigma) \tag{3.24}$$

where ρ is a tunable parameter.

<u>Theorem 2</u>

The hitting condition of the sliding surface equation (3.4) is satisfied if $\rho > 0$.

Proof.

The reaching condition is evaluated as follows,

$$\sigma(t)\dot{\sigma}(t) = \sigma(t)[C\dot{x}(t)]$$

$$= \sigma(t)[C\{Ax(t) + Bu(t)\}]$$

$$= \sigma(t)[CAx(t) + CBu(t)] \tag{3.25}$$

Substituting equation 3.21 for $u_1(t)$ and equation (3.22) for $u_2(t)$ into equation 3.25, gives

$$\sigma(t)\dot{\sigma}(t) = \sigma(t)[CAx(t) + (CB)\{-(CB)^{-1}CAx(t) - (CB)^{-1}\rho\frac{\sigma(t)}{|\sigma(t)|\delta}\}]$$

$$= \sigma_1(t)\left[-\rho\frac{\sigma_1(t)}{|\sigma_1(t)| + \delta}\right] \qquad \text{for } \sigma_1(t) \tag{3.26}$$

$$= \sigma_2(t)\left[-\rho\frac{\sigma_2(t)}{|\sigma_2(t)| + \delta}\right] \qquad \text{for } \sigma_2(t) \tag{3.27}$$

The sliding condition is established if $\rho > 0$. The theorem 2 ensures that the proposed control law drives the system state trajectory onto the sliding surface and remains on it thereafter. It follows that $\dot{V}(t) \leq 0$ if theorem 2 is satisfied. Thus the reaching condition is satisfied.

3. 3 Conclusion

In this chapter, the mathematical and stability theorem of the proposed controller have been established and outlined. This active steering system is classified as matched condition. The proposed Sliding Mode Control will be presented in the following chapter and the performances of the system will be observed and compared to LQR as well as pole placement technique.

CHAPTER 4

SIMULATIONS

4.1 Simulations

The single track model for car steering has been obtained in Chapter 2. The mathematical model in state space form is given by equation (2.22). The parameters for the systems are considered to be linear. In the modeling, it is assumed that the road disturbance is an input to the system.

The performance of this system will be investigated base on the effect of the active steering system on various disturbance profiles and varying coefficient of road friction. The system can be separated into two road condition, which are μ=0.5 for wet road condition and μ=1 for dry road condition respectively.

By substituting the parameter values of the car steering system as tabulated in Table 2.1 into equation 2.22 for μ=0.5 and velocity is 70m/s, the state space model can be transformed into:

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -1.2086 & -0.9929 \\ 17.6245 & 1.18105 \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} 0.38935 & 0.8193 \\ 20.9929 & -38.6174 \end{bmatrix} \begin{bmatrix} \delta_F \\ \delta_R \end{bmatrix} + \begin{bmatrix} 0 \\ 0.0002737 \end{bmatrix} f(t) \tag{4.1}$$

while for μ=1, the state space model can be transformed into:

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -2.4172 & -0.9859 \\ 35.2490 & 2.3621 \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} 0.7787 & 1.6386 \\ 41.9858 & -77.2348 \end{bmatrix} \begin{bmatrix} \delta_F \\ \delta_R \end{bmatrix} + \begin{bmatrix} 0 \\ 0.0002737 \end{bmatrix} f(t) \tag{4.2}$$

Based on equations (3.4) and (3.22) or (3.23) , the calculation for the sliding surface and controller parameters are performed according to the following steps:

Step 1: Choose the proper value for matrix C in equations (3.4), (3.21) and (3.22) using trial and error method.

Step 2: Choose the proper values for ρ and δ in equations (3.21) and (3.22) using trial and error method.

Step 3: Perform the simulation and observe the performance of the system. Repeat Step 1 and 2 until the performance is satisfactory.

4.2 Results and Discussion

To prove the robustness of the proposed controller in improving the road handling and the ride comfort of the active car steering system, various disturbance profiles will be utilized. The performance of the systems using SMC technique will be compared to the pole placement and LQR techniques. The friction coefficient and different vehicle velocity will be observed as the varying parameters. Performance of each case, in terms of its ability to attenuate disturbances in the yaw rate as well as side slip angle will be simulated. The performance of an active car steering with the proposed controller will be investigated and compared through computer simulations.

4.2.1 Performance of SMC on Various Disturbance Profiles

The road handling performance of active car steering may be observed through yaw rate and side slip angle characteristics. External disturbance to the system is generated by using side wind and braking torque profiles.

The simulation is perform following the guidelines as specified in section 4.1:

Step 1: The following matrix C is:
C=[0.0005 0; 0.0005 0]

Step 2: The following values for ρ and δ have been used:
ρ=0.09 and δ=0.005.

For comparison purposes, the performance of the SMC is compared to the linear quadratic regulator (LQR) and pole placement method. Assumed a quadratic performance index in the form of

$$J = \frac{1}{2}\int_0^t (x^T Qx + u^T Ru)dt \tag{4.3}$$

In the design of the LQR controller, weighting matrices Q and R are selected as Q = diag($q1,q2,q3,q4$) where $q1=q3=1$, $q2=q4=0$ and R = [100 0; 0 100]. The above values have been chosen to ensure that the performance of the road handling is improved in term of disturbance attenuation. The performance of the SMC is also compared to a linear state feedback control approach, where the following linear state feedback law is used:

$$u_{LSF}(t)=-Kx(t) \tag{4.4}$$

4.2.2 An active steering system on wet road.

In order to observe the robustness of the system, various disturbance profiles are generated to active car steering system. Figures 4.1 and 4.2 show the sliding surface and control input for the system on wet road, μ=0.5 for disturbance profile 1 whereas Figures 4.3 and 4.4 show sideslip angle and yaw rate performances respectively. The results show that the disturbance has been significantly attenuated by using the SMC controller.

Figures 4.5 - 4.8 illustrate control input, phase portrait, sideslip angle and yaw rate respectively under disturbance profile 2. The results show that the performance of the proposed controller is the best to compare with another two controllers.

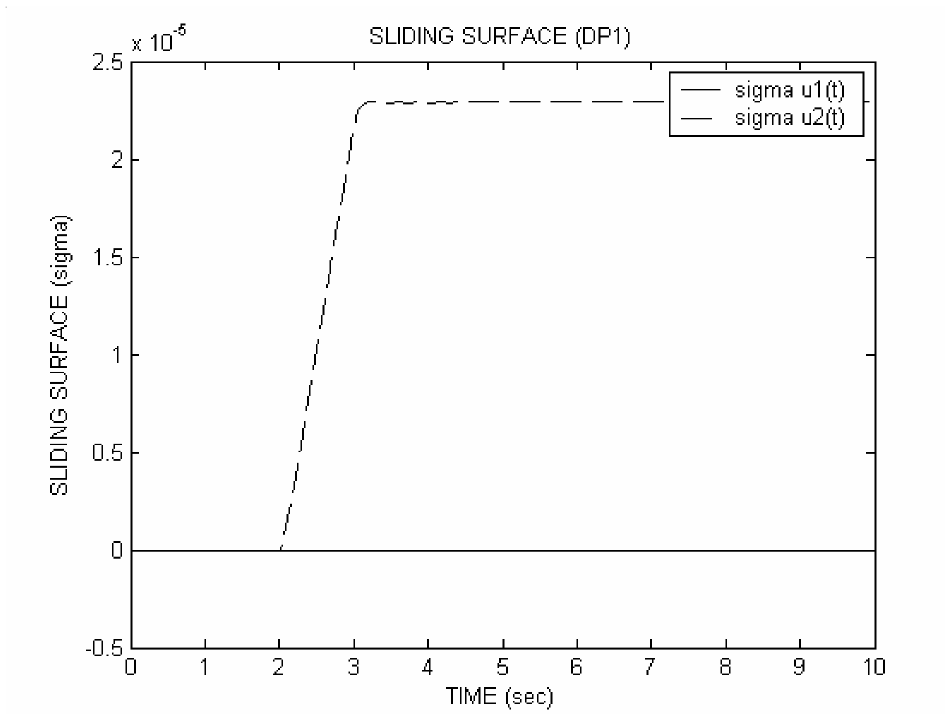

Figure 4.1 Sliding surface for DP1

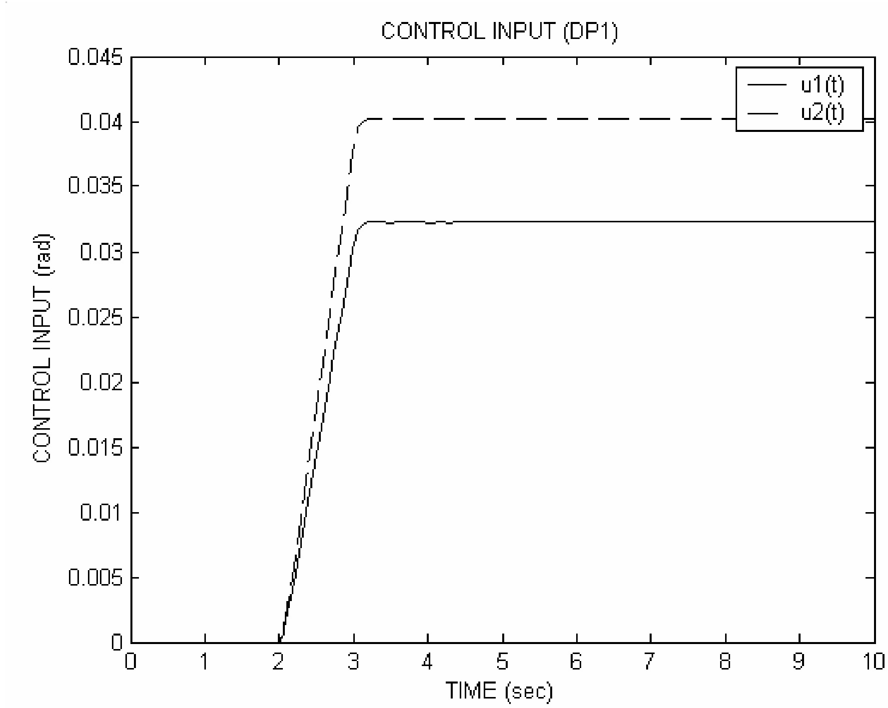

Figure 4.2 Control Input for DP1

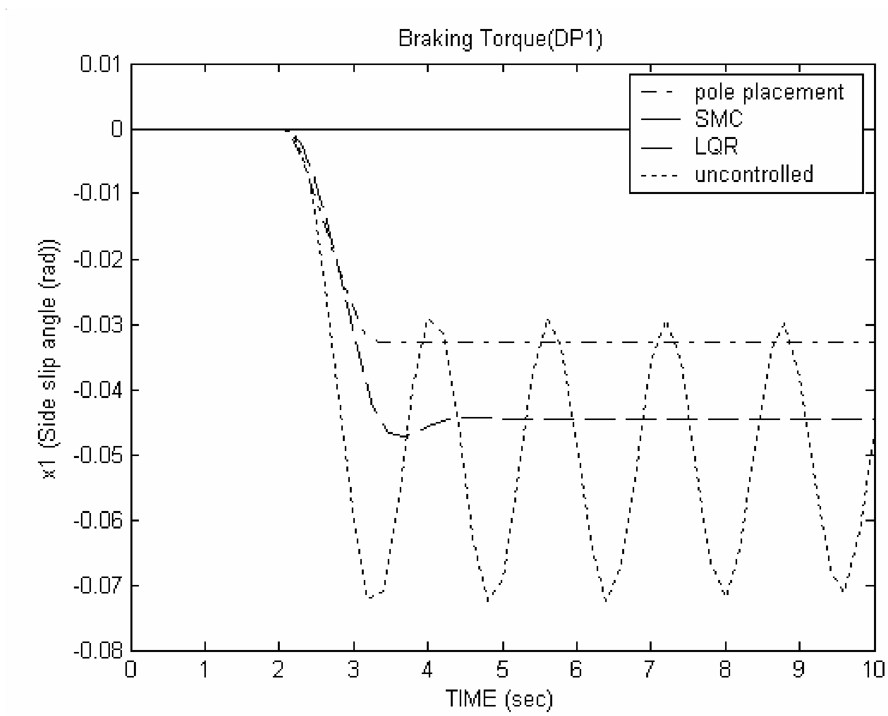

Figure 4.3 Side Slip angle for DP1

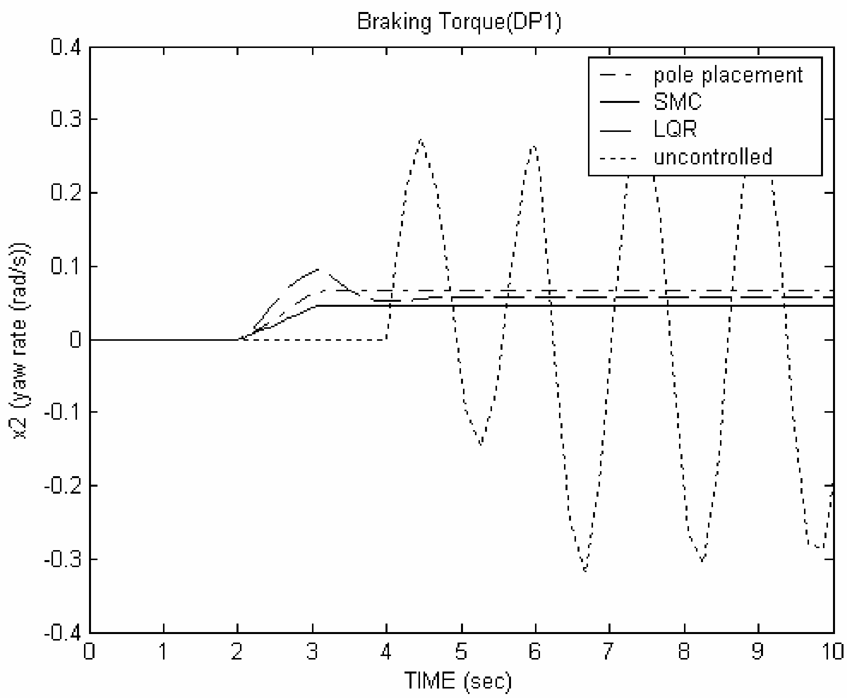

Figure 4.4 Yaw rate for DP1

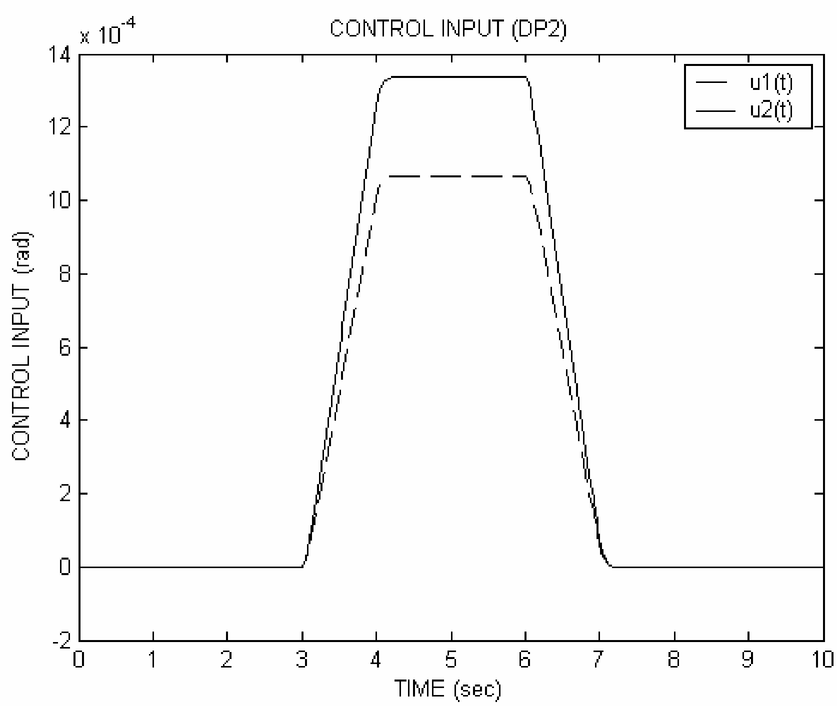

Figure 4.5 Control Input for DP2

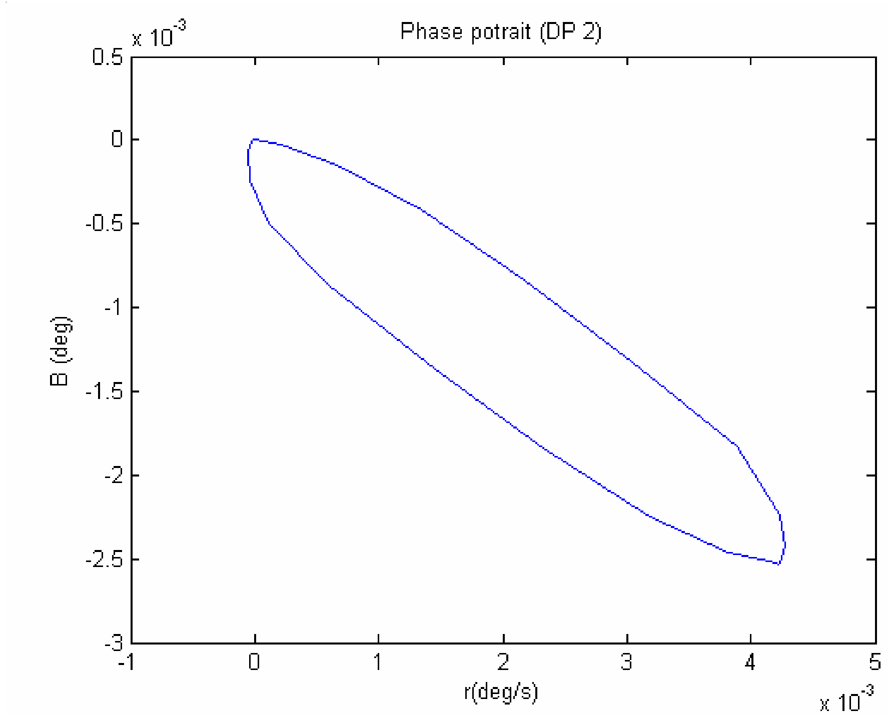

Figure 4.6 Phase portrait for DP2

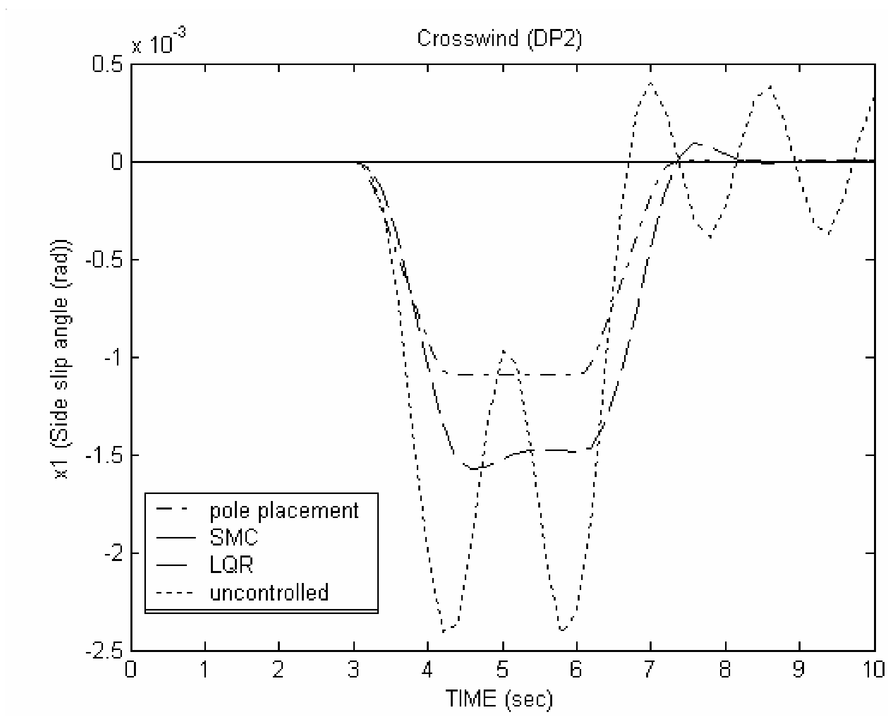

Figure 4.7 Side Slip angle for DP2

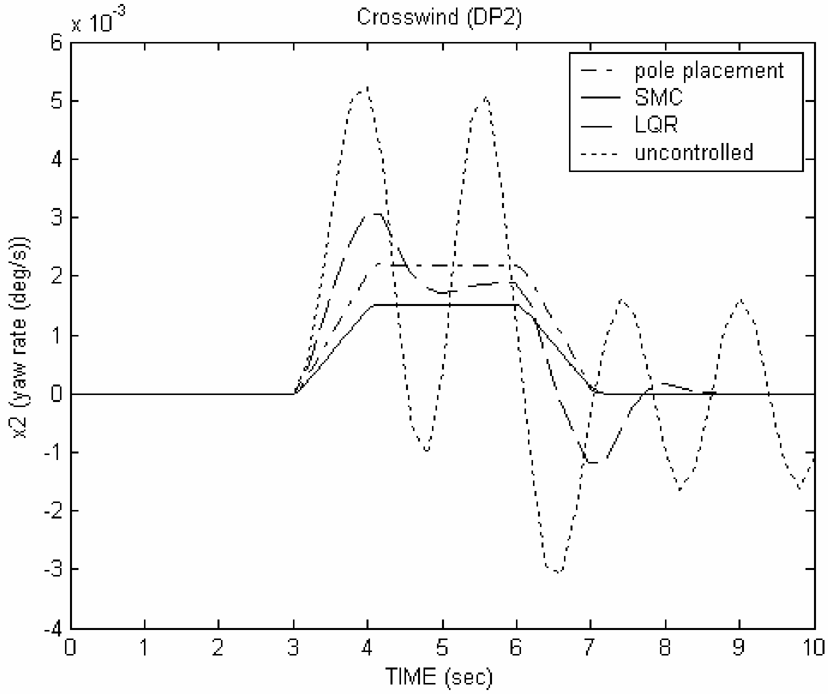

Figure 4.8 Yaw rate for DP2

4.2.3 An active steering system on dry road.

Then, the system are tested on dry road, μ=1 and maintain vehicle velocity at 70m/s. Figures 4.9 and 4.10 show the sliding surface and control input to the system on dry road for disturbance profile 1 respectively whereas Figures 4.11 and 4.12 illustrate the output which are side slip angle and yaw rate for this road condition. Against, it is observed that the SMC technique is effectively suppressed the disturbance to compare with the LOR and Pole-placement techniques.

Figures 4.13 - 4.16 show sliding surface, control input, sideslip angle and yaw rate under disturbance profile 2, respectively. The results show that performance of the proposed controller is the best to compare with another two controllers for the dry road condition.

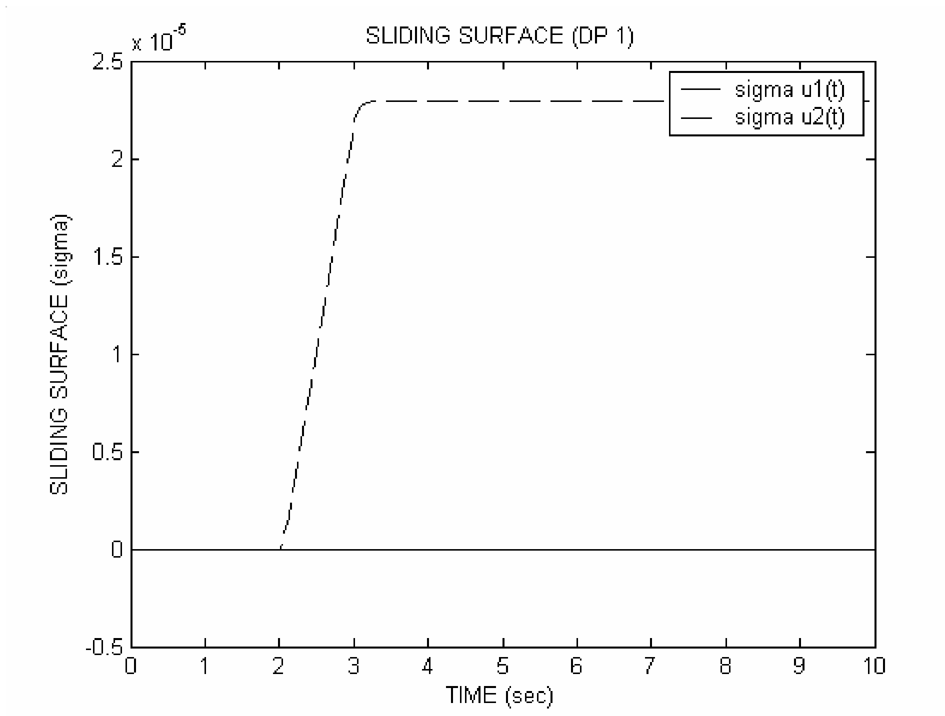

Figure 4.9 Sliding surface for DP1

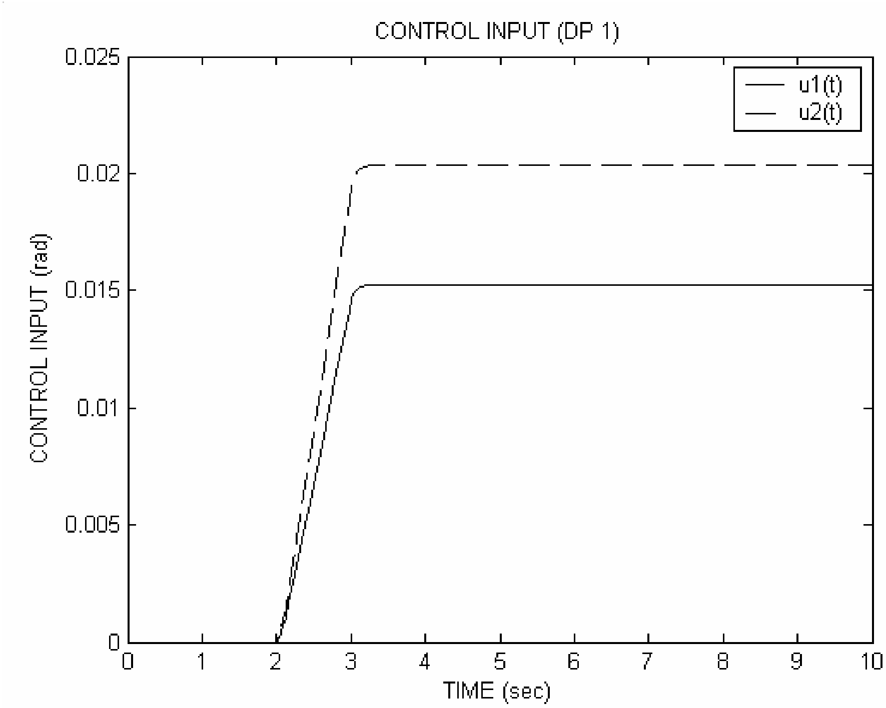

Figure 4.10 Control Input for DP1

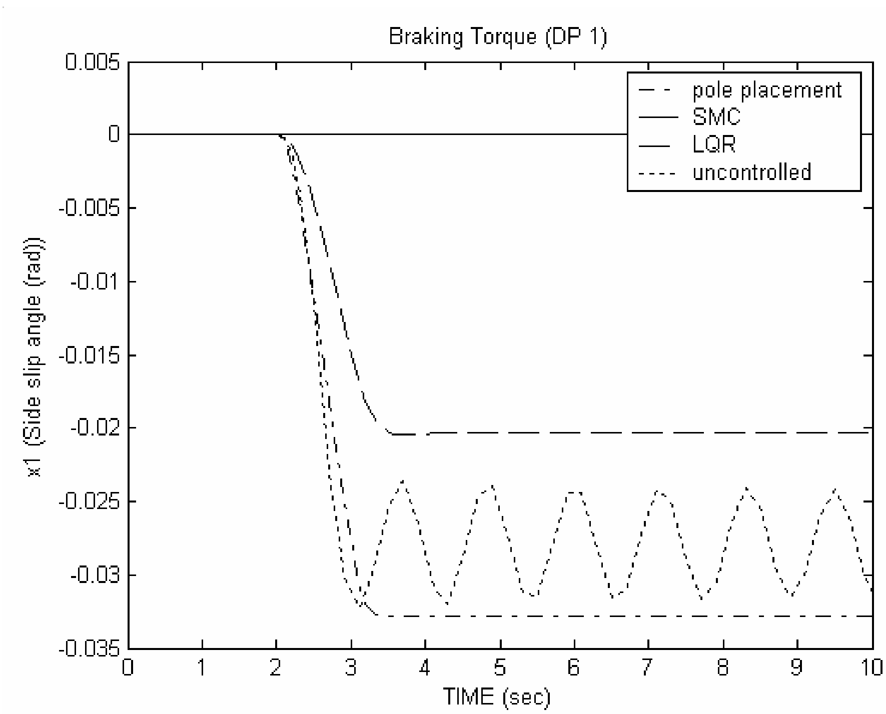

Figure 4.11 Side slip angle for DP1

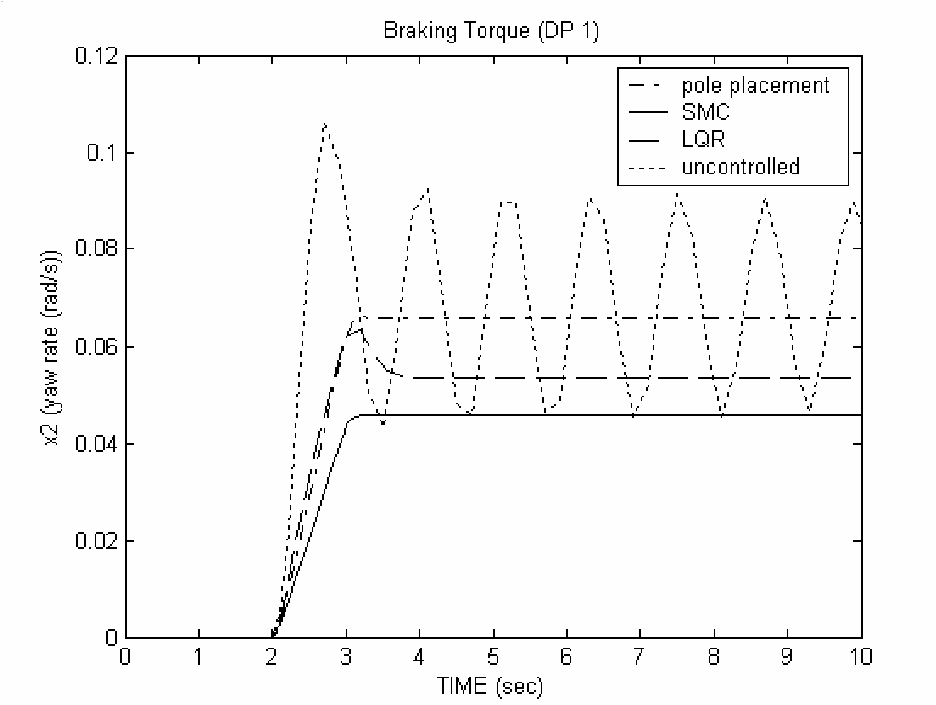

Figure 4.12 Yaw rate for DP1

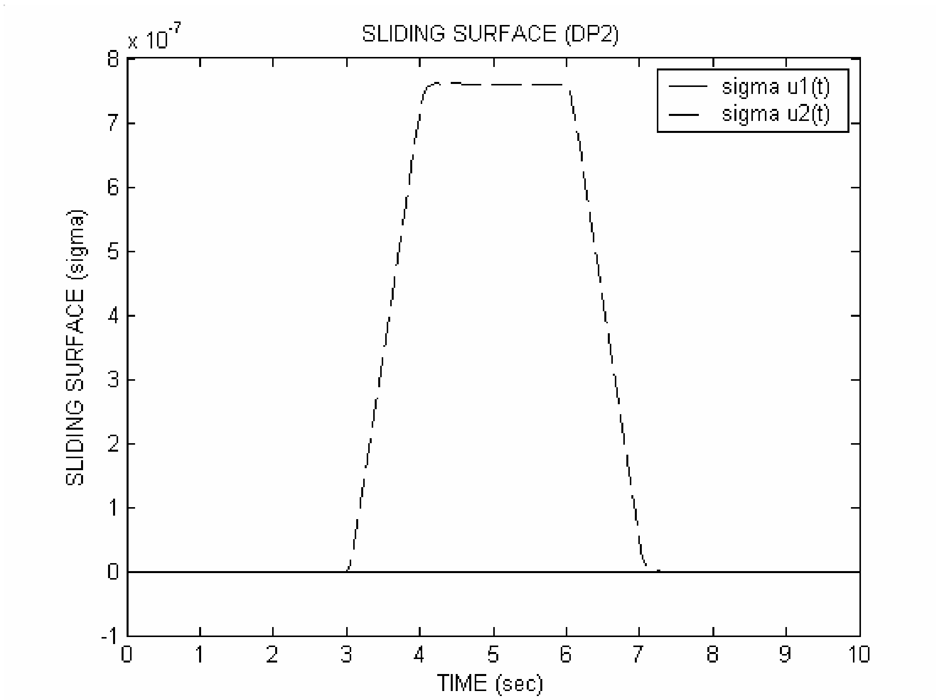

Figure 4.13 Sliding surface for DP2

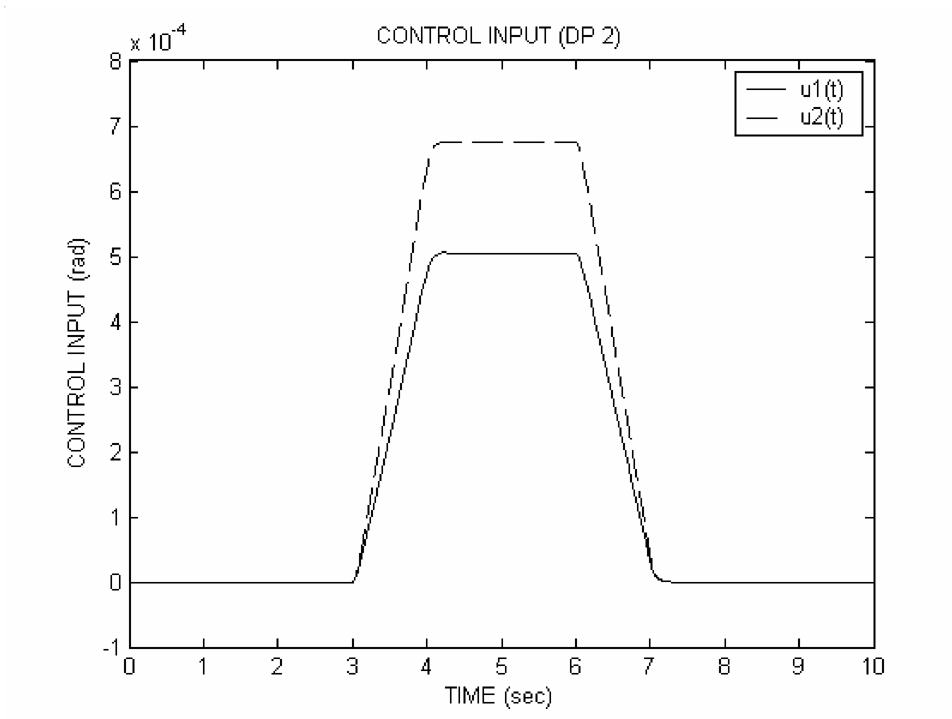

Figure 4.14 Control Input for DP2

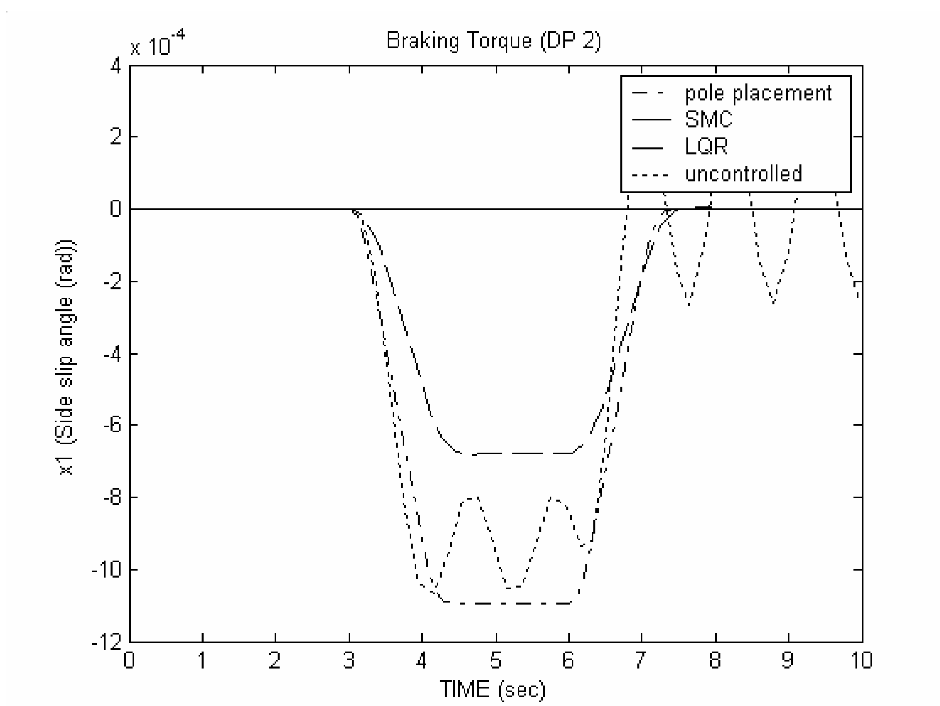

Figure 4.15 Side slip angle for DP2

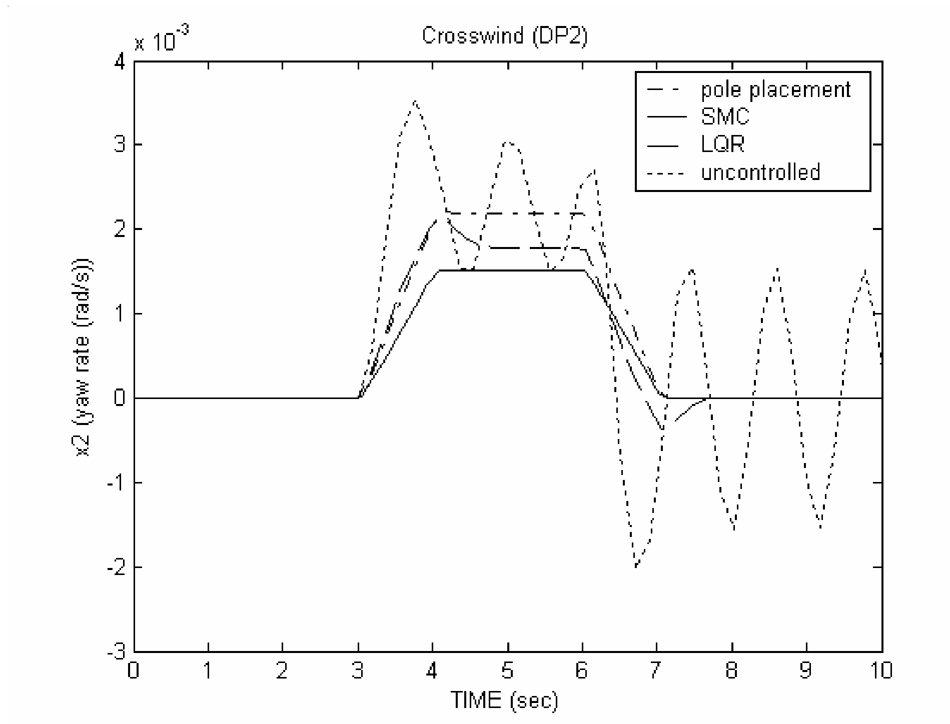

Figure 4.16 Yaw rate for DP2

The sliding mode control strategy design has been successfully implemented to the active steering system. The results clearly shown that the sliding mode control scheme is robust in compensating various disturbances and road coefficient in the system. From the simulation results, there is a significant in vehicle performance between dry road and wet road. Also the simulation results indicated that, the performance for the side slip angle and yaw rate on wet road is worse than the performance on dry road which exhibits high in overshoot and magnitude. Thus, the vehicle tends to skid on the wet road than the dry road.

In the simulation study, it is demonstrated that the SMC strategy gives best performance compared to LQR and pole placement controller. Again the magnitude is drastically reduced and the overshoot is avoided. Its effect is that the driver has to care much less about disturbance attenuation. The important quick reaction to disturbances is done by the automatic feedback system. Using Lyapunov stability theory, it is shown that the estimation assured that the matched uncertainty system is ultimately asymptotically stable. However, the active steering system and the proposed controller is capable to overcome ‘late action’ by the driver due to sudden disturbance on any road conditions.

4.2.4 Effect of the Reaching Mode Condition on Varying Sliding Gain, ρ.

In the following simulations, the effect of varying the constant ρ for sliding mode controller is investigated. The constant ρ is called a sliding gain and the reaching mode condition of sliding mode controller is affected by varying the sliding gain, ρ as in equation (3.21) and equation (3.22). The reaching mode condition is satisfied if $\rho>0$.

The following values of sliding gain ρ have been considered in the simulation:

Positive sliding gain : $\rho=0.09$ (satisfying the reaching mode condition)

Negative sliding gain : $\rho= -0.09$ (not satisfying the reaching mode condition)

This sliding gain, ρ is tested for active steering system on dry road which is for road coefficient,$\mu=1$. Other constant parameters in the sliding surface equation and SMC controller are similar as in Table 2.1.

Figures 4.17 - 4.20 illustrate the sliding surface for both positive and negative sliding gain for disturbance profile 1 and disturbance profile 2 respectively. It can be seen from the figure that for the positive sliding gain (satisfying the reaching mode condition), the state trajectory slide on the sliding surface. In contrary, for the negative sliding gain (not satisfying the reaching mode condition), the state trajectory does not slide on the sliding surface. Thus, the reaching mode condition is satisfied for the positive sliding gain. Figures 4.21 - 4.32 illustrate the effects of varying the sliding gain on the side slip angle, yaw rate and control input for both disturbance profile 1 and disturbance profile 2.

The simulation results demonstrate that the reaching mode condition of the sliding mode controller is satisfied if $\rho>0$. The results show that if the sliding mode condition is not satisfied, the controller will not give a satisfactory performance of the active steering system as desired.

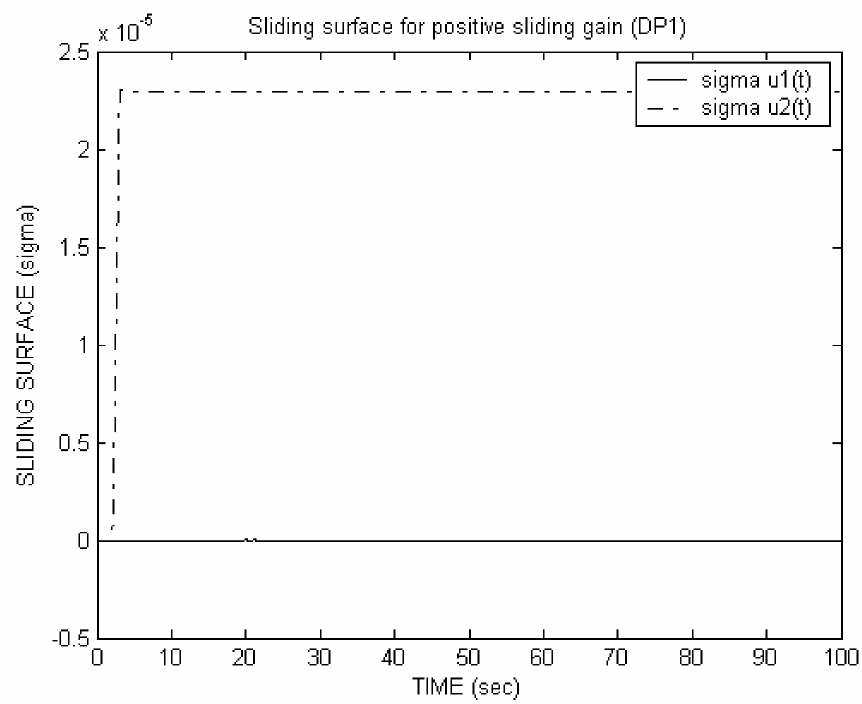

Figure 4.17 Sliding surface for positive sliding gain (DP1)

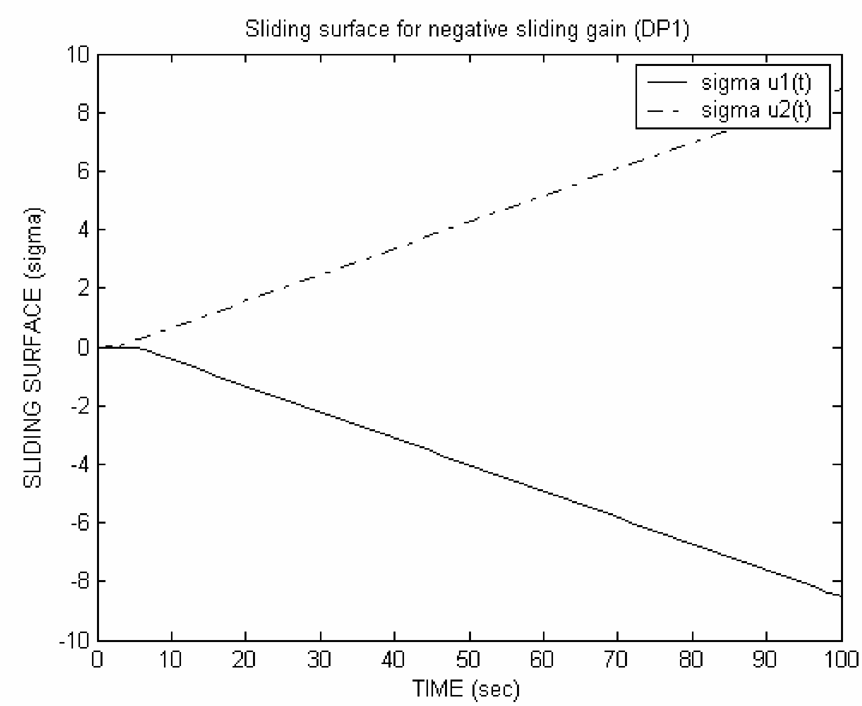

Figure 4.18 Sliding surface for negative sliding gain (DP1)

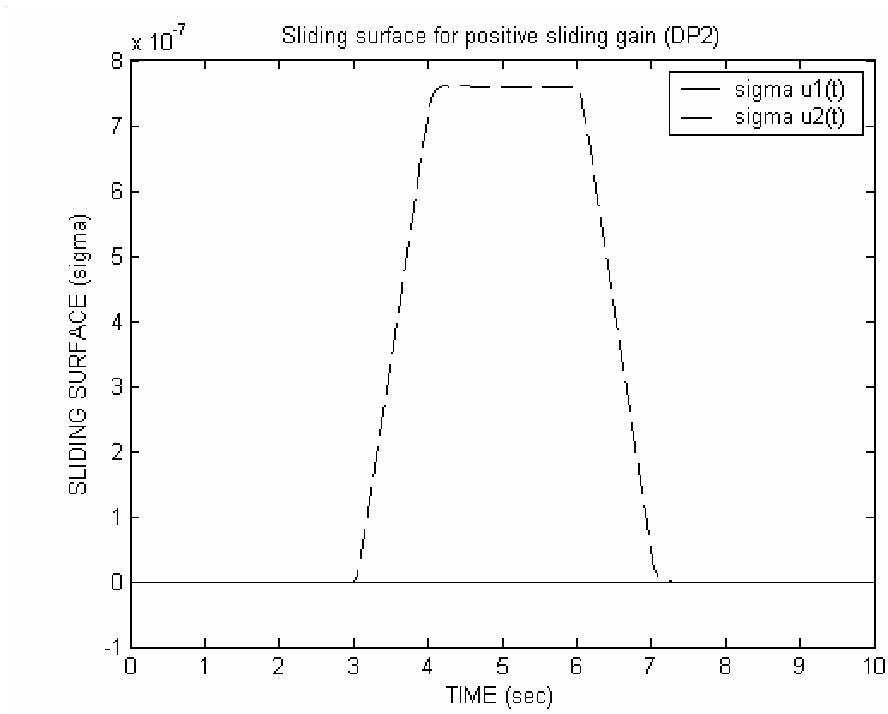

Figure 4.19 Sliding surface for positive sliding gain (DP2)

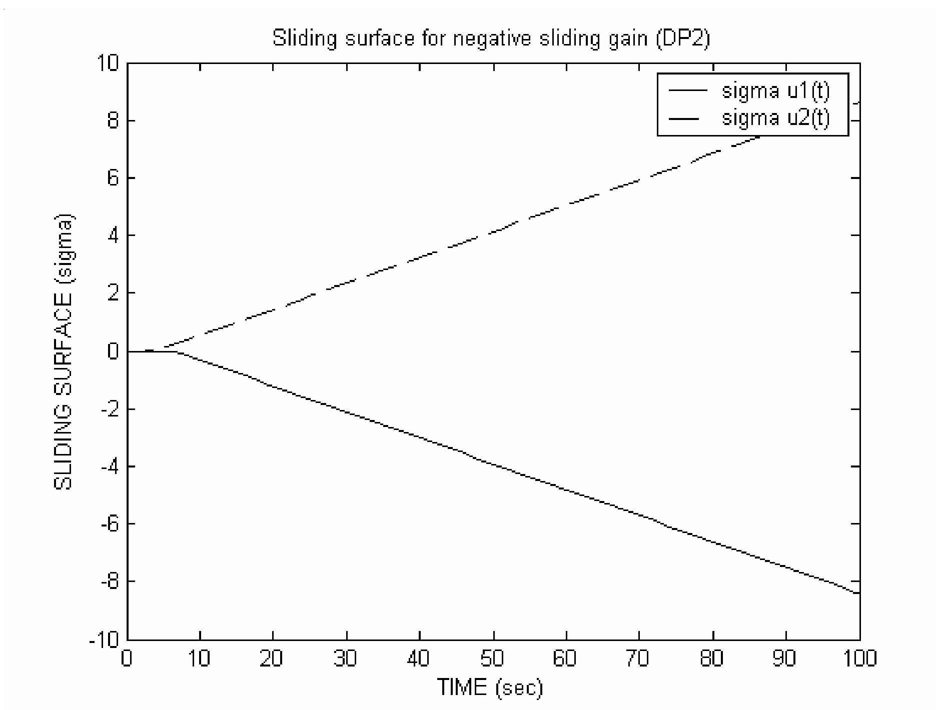

Figure 4.20 Sliding surface for negative sliding gain (DP2)

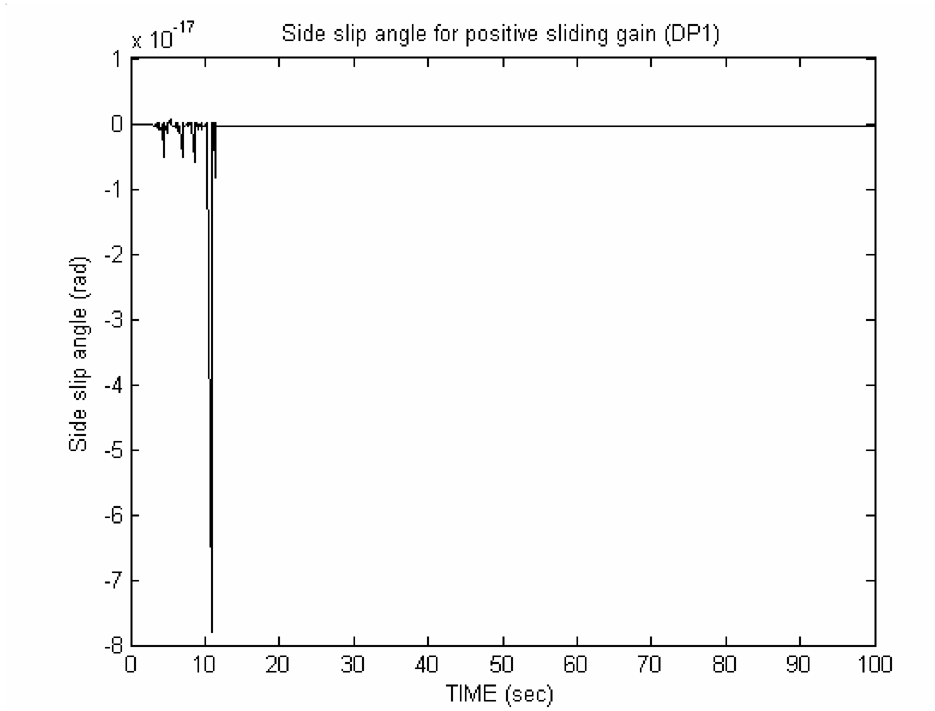

Figure 4.21 Side slip angle for positive sliding gain (DP1)

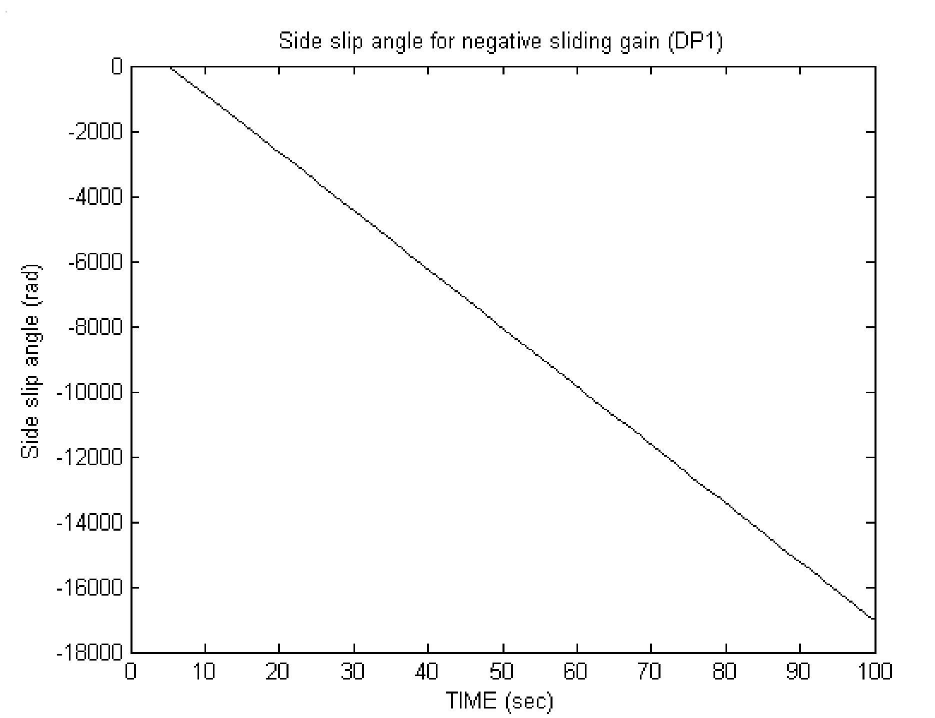

Figure 4.22 Side slip angle for negative sliding gain (DP1)

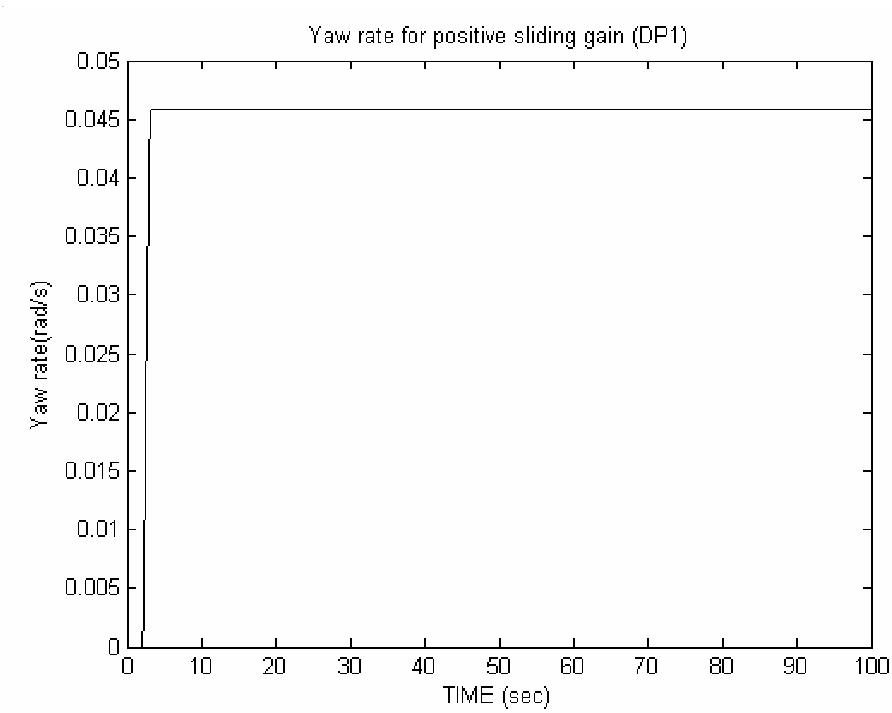

Figure 4.23 Yaw rate for positive sliding gain (DP1)

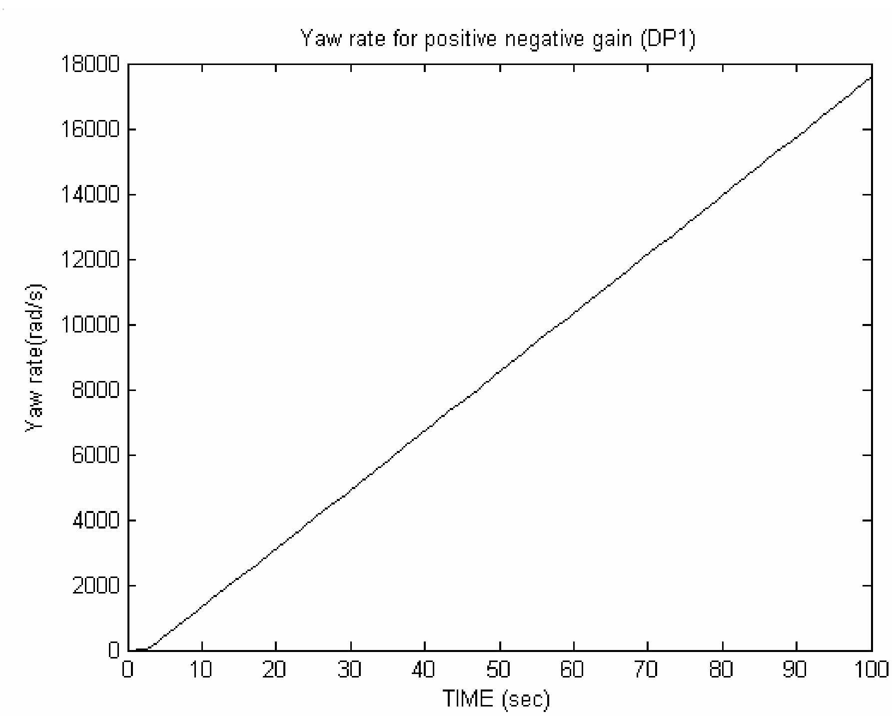

Figure 4.24 Yaw rate for negative sliding gain (DP1)

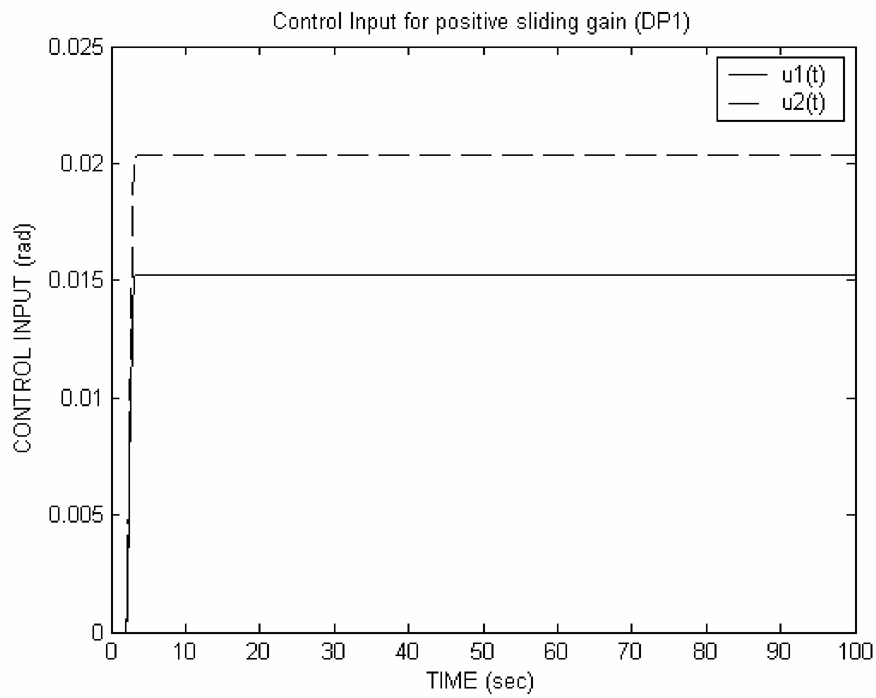

Figure 4.25 Control Input for positive sliding gain (DP1)

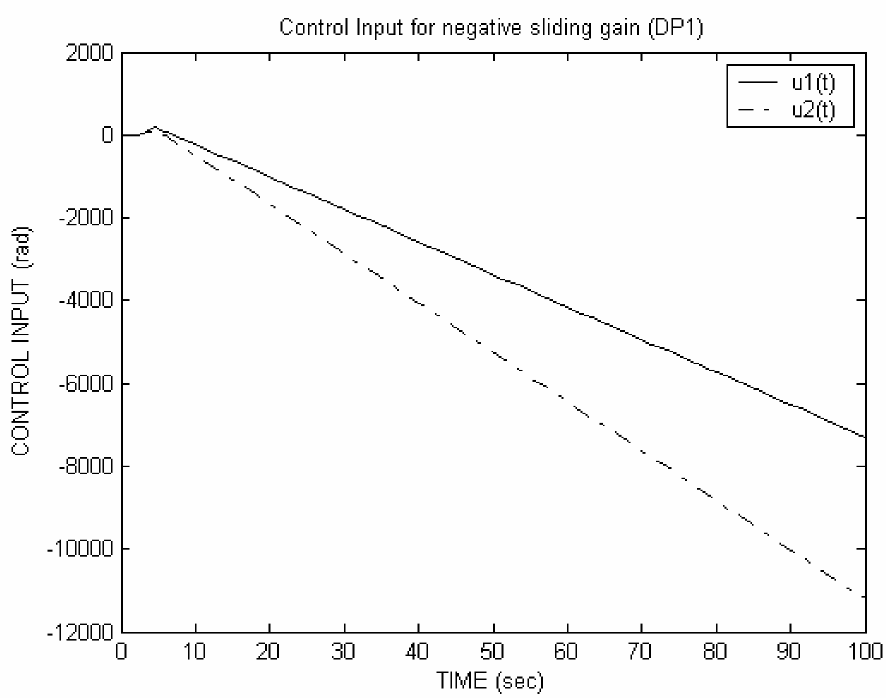

Figure 4.26 Control Input for negative sliding gain (DP1)

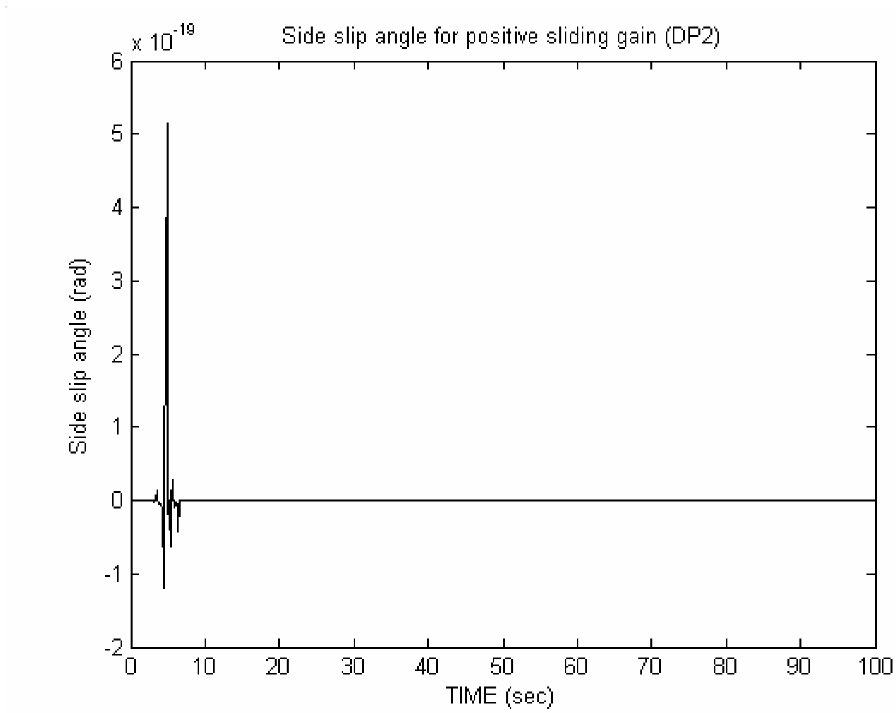

Figure 4.27 Side slip angle for positive sliding gain (DP2)

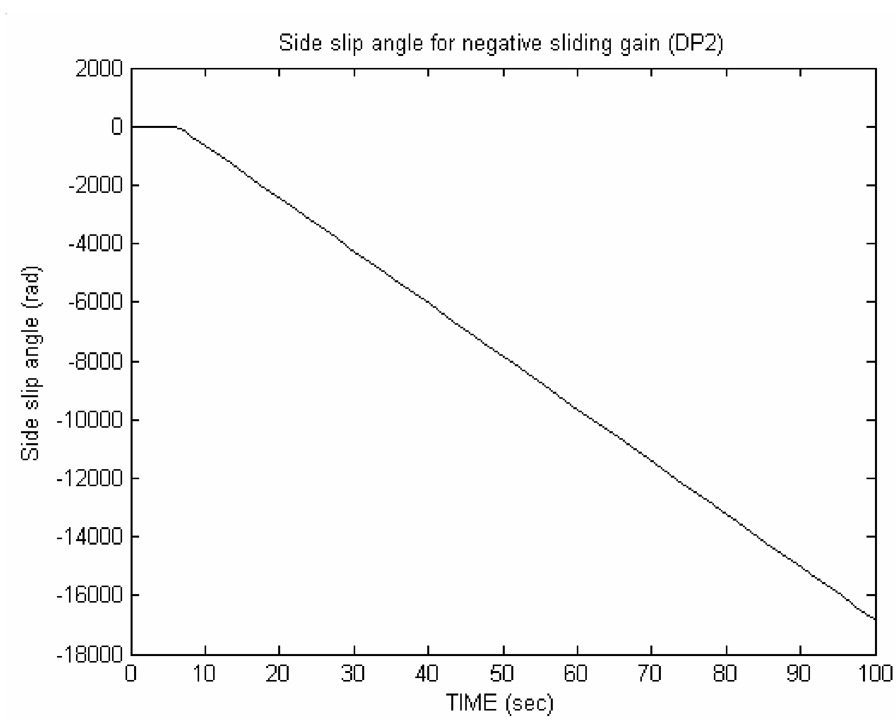

Figure 4.28 Side slip angle for negative sliding gain (DP2)

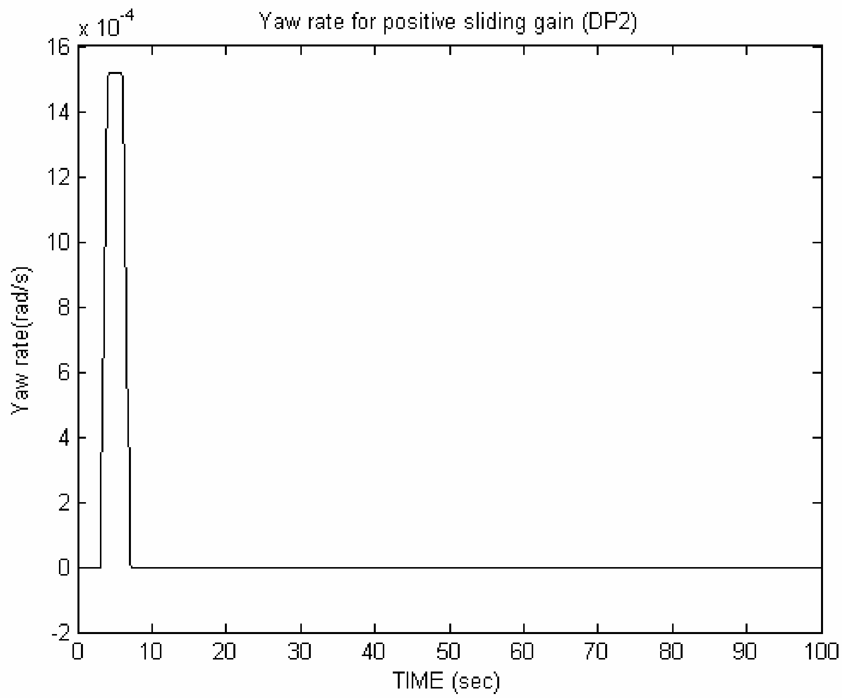

Figure 4.29 Yaw rate for positive sliding gain (DP2)

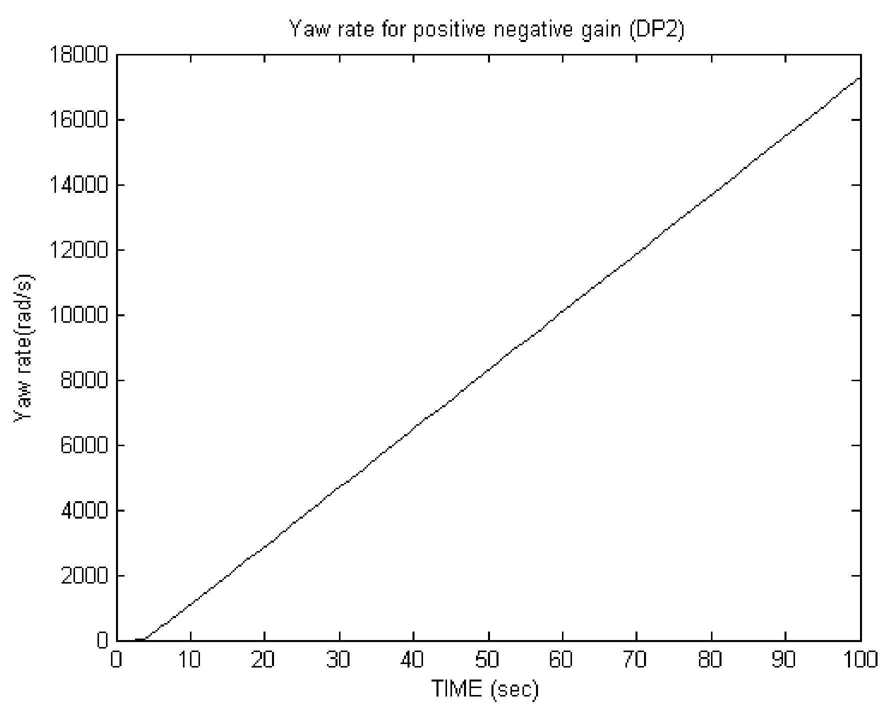

Figure 4.30 Yaw rate for negative sliding gain (DP2)

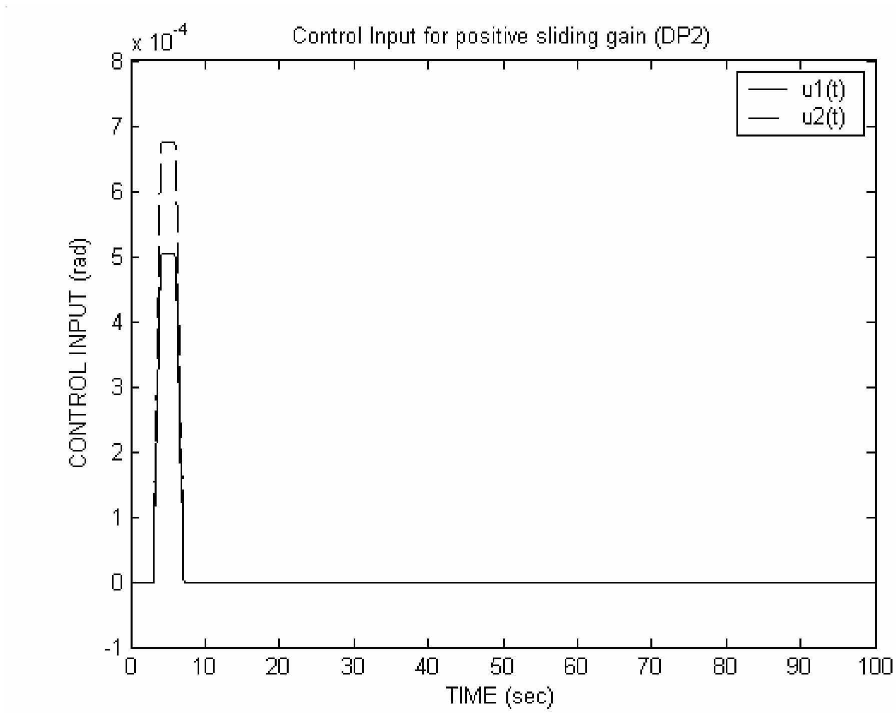

Figure 4.31 Control Input for positive sliding gain (DP2)

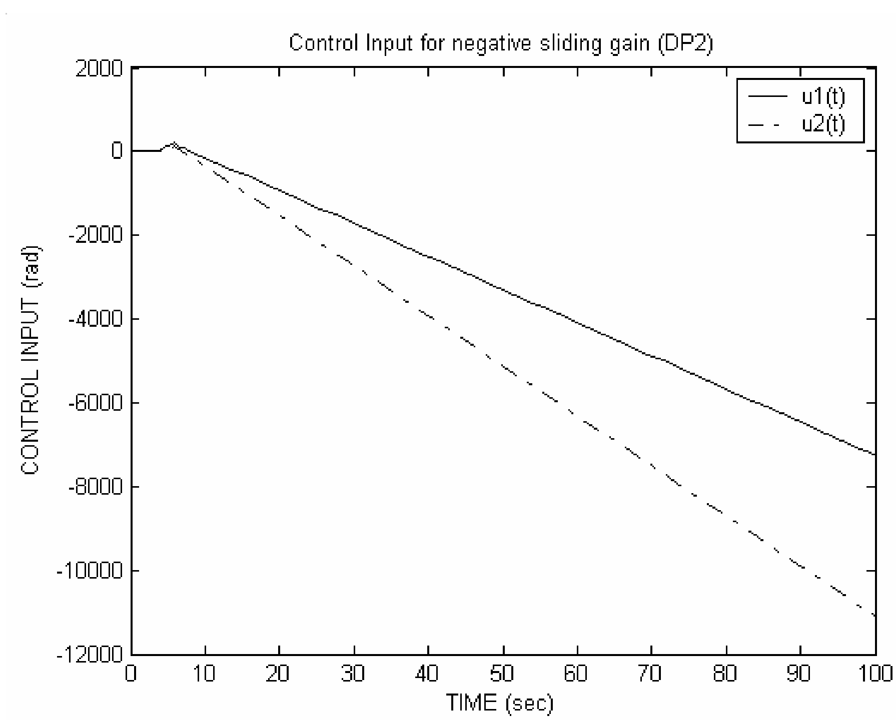

Figure 4.32 Control Input for negative sliding gain (DP2)

4.2.5 Effect on varying the value of boundary layer thickness, δ

In the following simulations, the effect of varying the boundary layer thickness, δ is studied and viewed to observe the responses. This section discuss the effect of the small and large values of δ on side slip angle, yaw rate, sliding surface and the control input.

The following values of δ have been considered in the simulations:

Case 1: δ=0.005

Case 2: δ=0.5

Figures 4.33 - 4.48 show the simulation results for both cases and disturbances. For case 1 and disturbance profile 1, the simulation results on the side slip angle, yaw rate, control input and sliding surface as shown in Figures 4.33, 4.35, 4.37 and 4.39 respectively which give fast response occur after the disturbance was applied to the system. In contrary, it exhibits slow response for case 2 as illustrated in Figures 4.34, 4.36, 4.38 and 4.40. Thus, a small value of δ created faster response as compared to a large value of δ. The slow response occurred because the small value of δ produce a very large control input as shown in Figure 4.38 and figure 4.48 for both disturbance 1 and disturbance 2 respectively. Thus, in applying the proposed SMC technique to the active steering system, a proper selection of the δ values must be taken such that best performance is achieved.

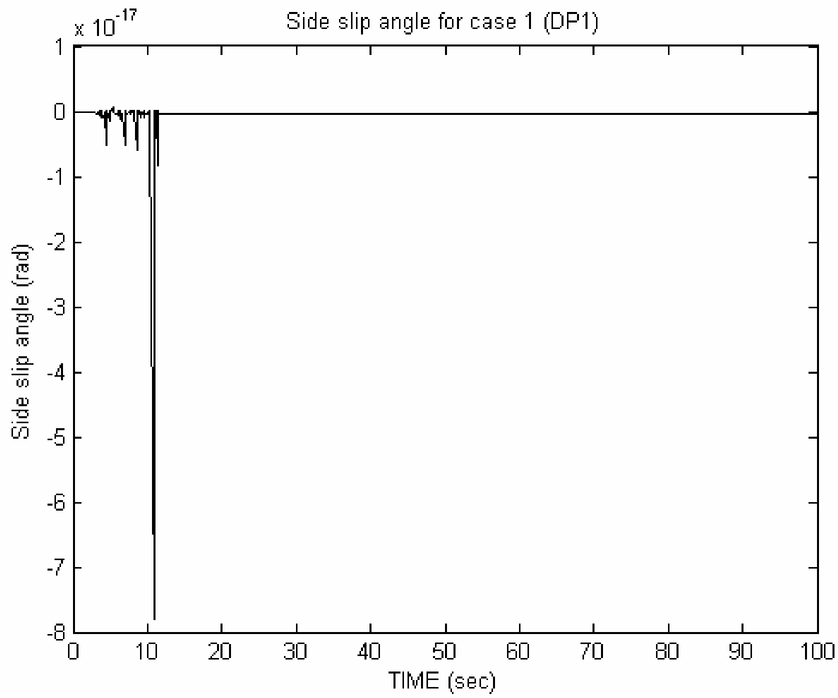

Figure 4.33 Side slip angle for case 1 (DP1)

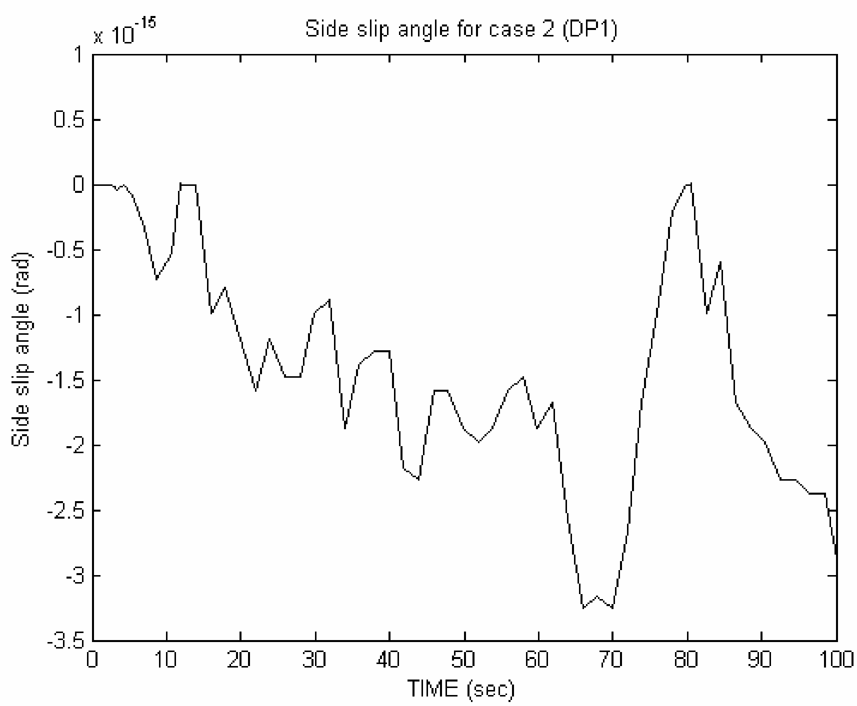

Figure 4.34 Side slip angle for case 2 (DP1)

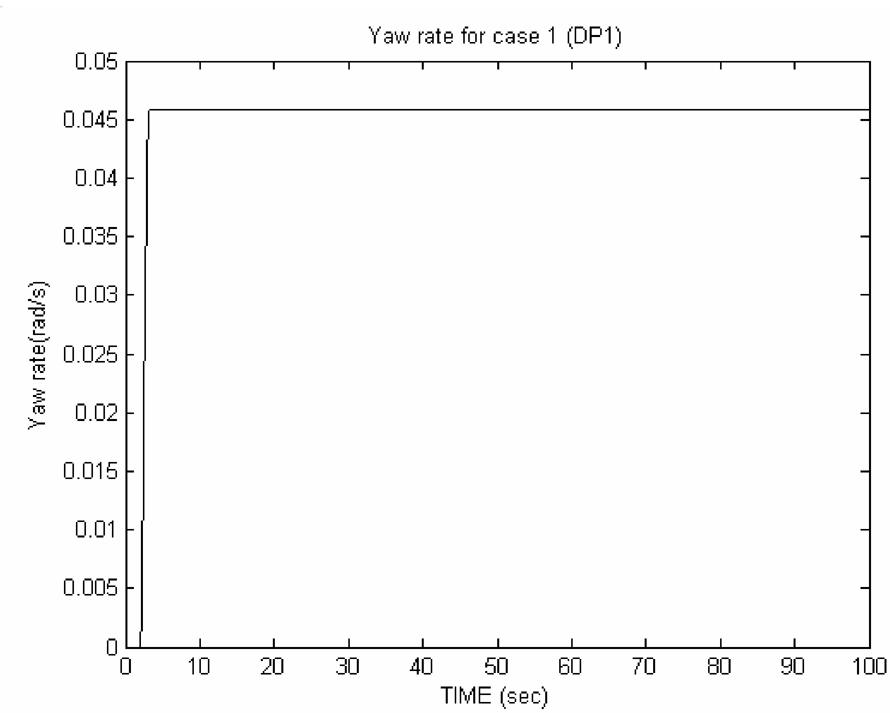

Figure 4.35 Yaw rate for case 1 (DP1)

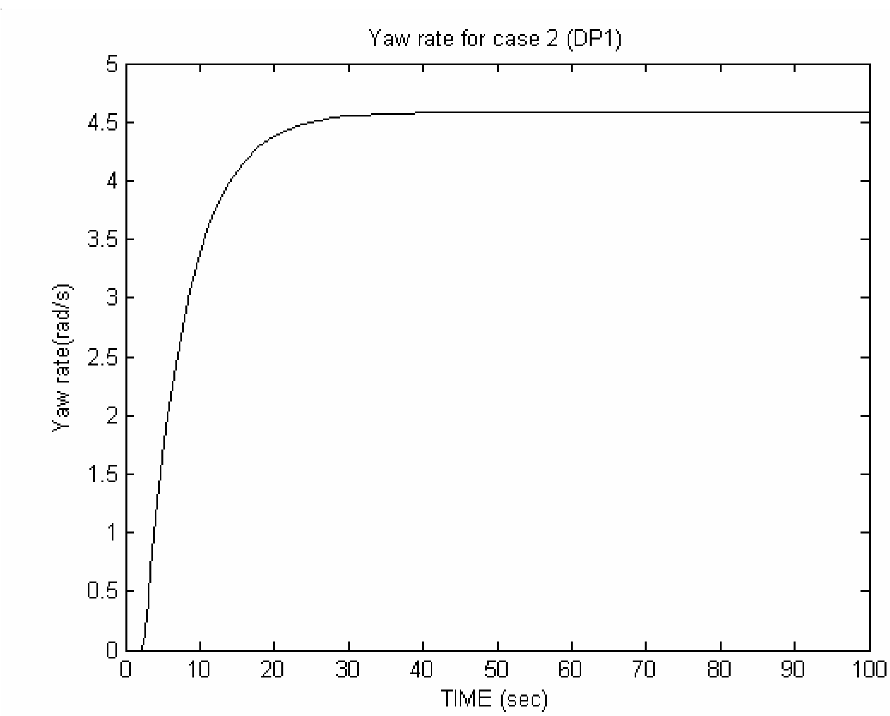

Figure 4.36 Yaw rate for case 2 (DP1)

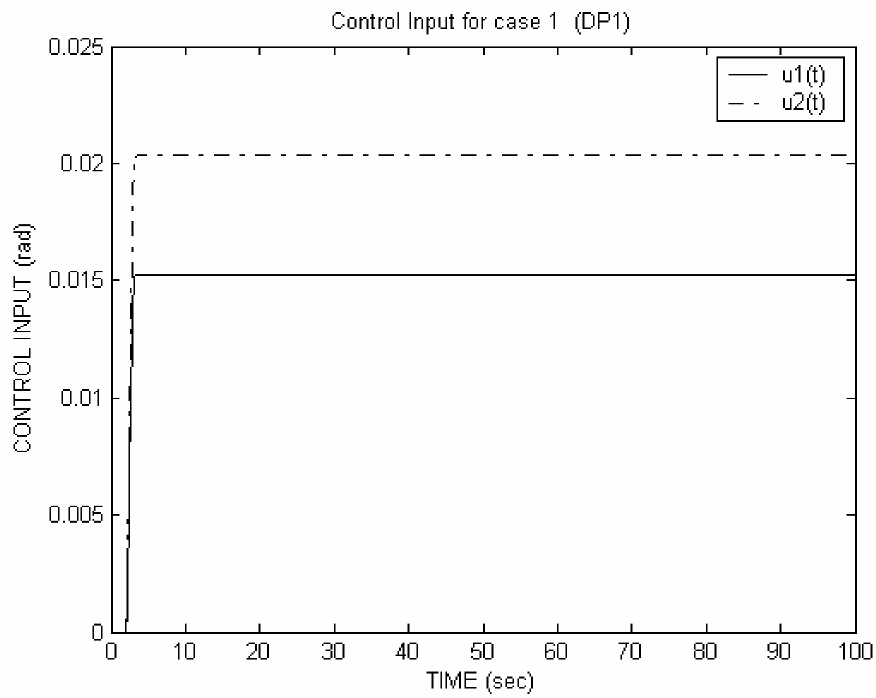

Figure 4.37 Control input for case 1 (DP1)

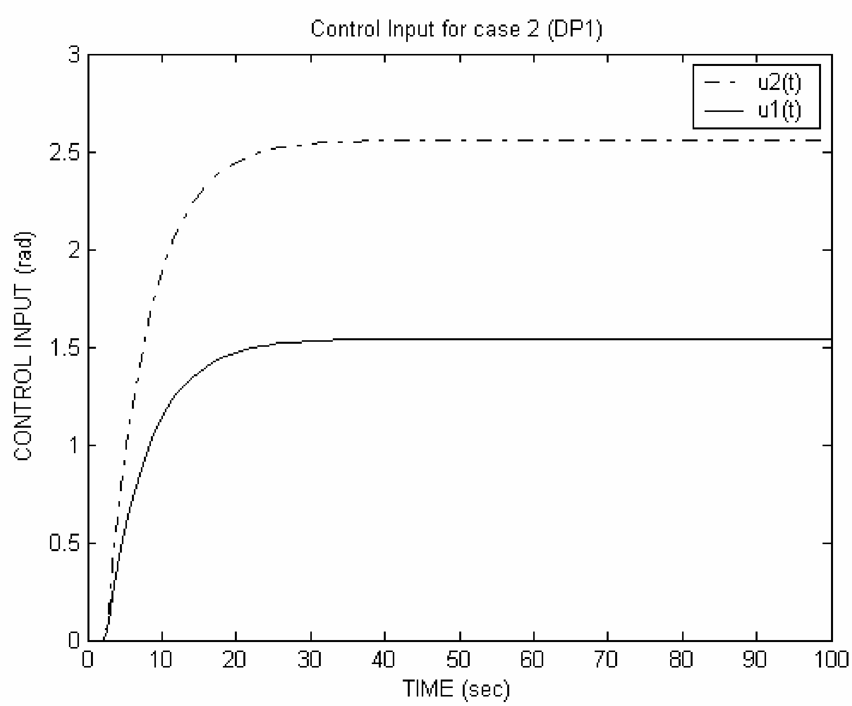

Figure 4.38 Control input for case 2 (DP1)

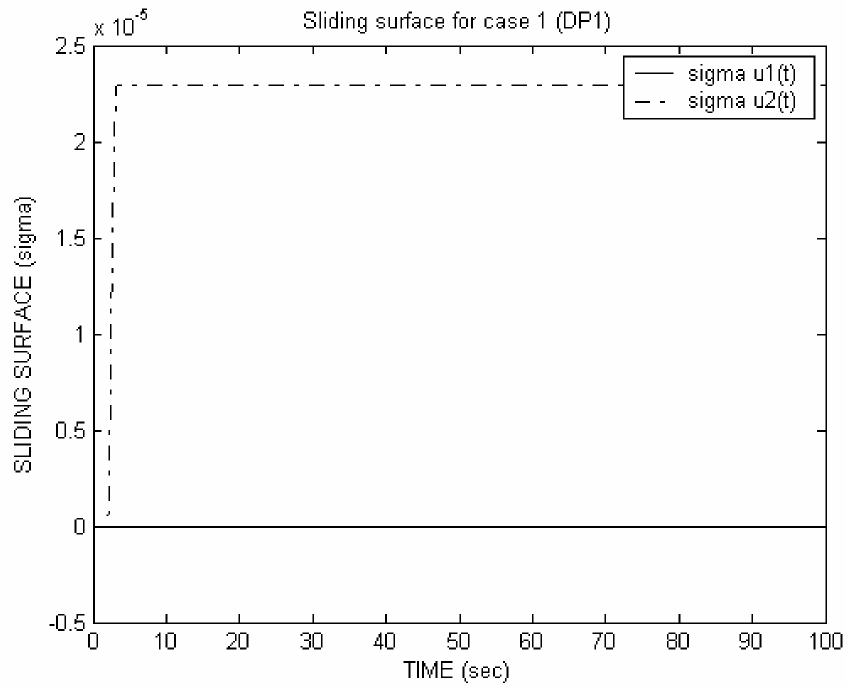

Figure 4.39 Sliding surface for case 1 (DP1)

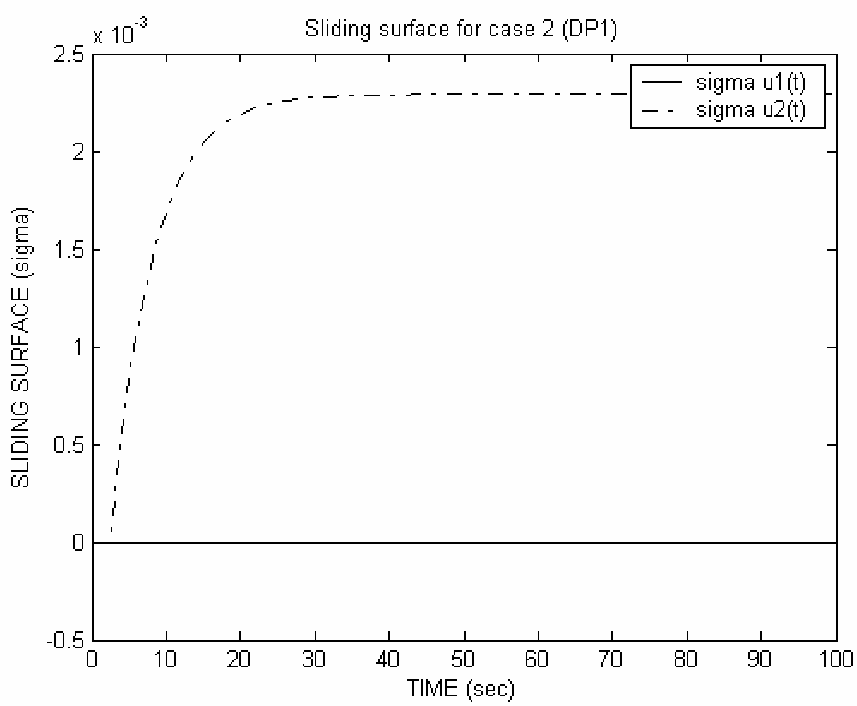

Figure 4.40 Sliding surface for case 2 (DP1)

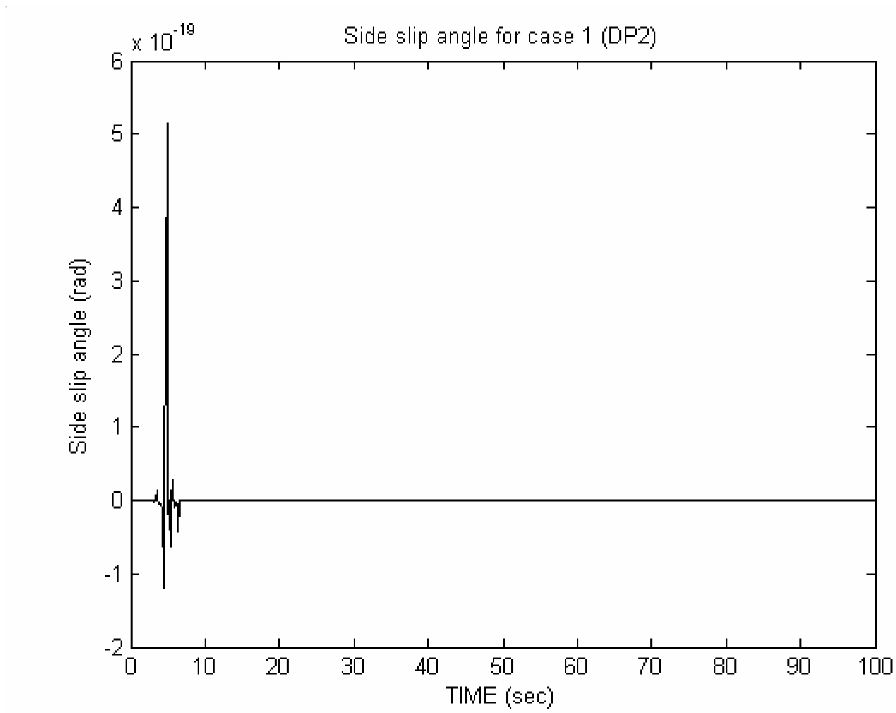

Figure 4.41 Side slip angle for case 1 (DP2)

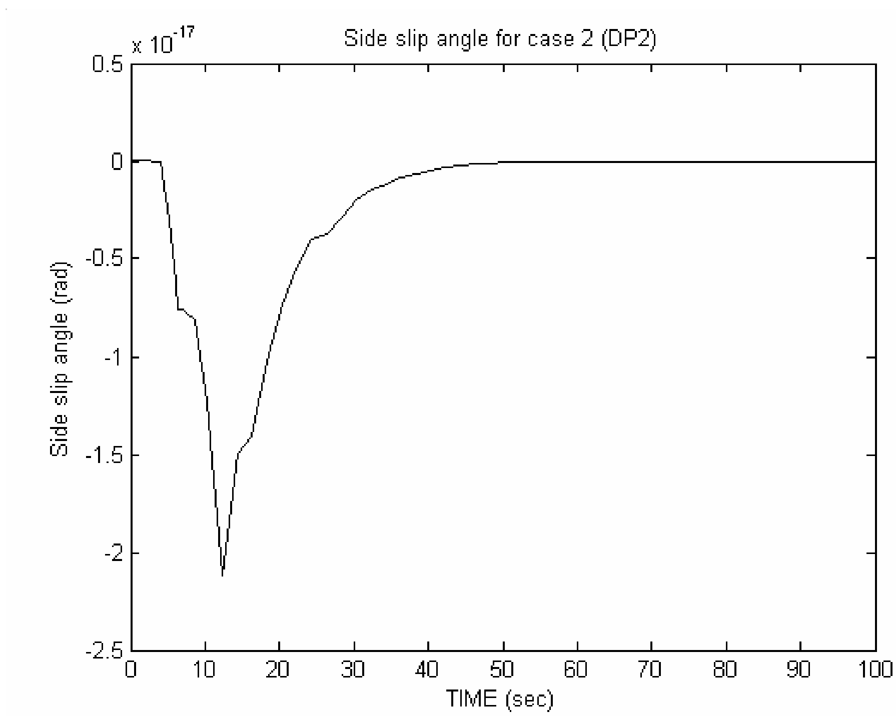

Figure 4.42 Side slip angle for case 2 (DP2)

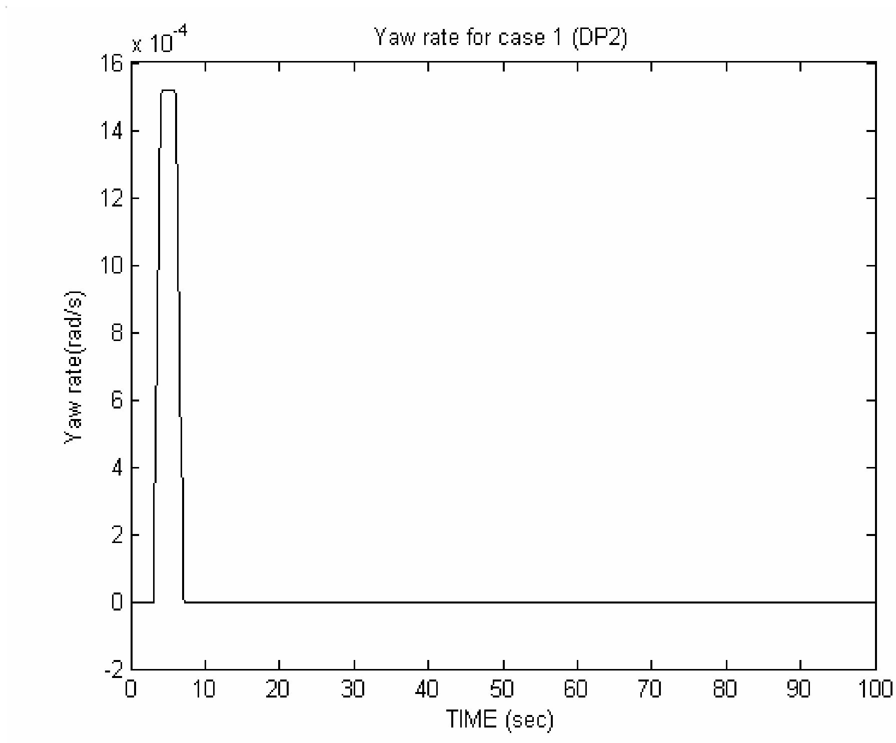

Figure 4.43 Yaw rate for case 1 (DP2)

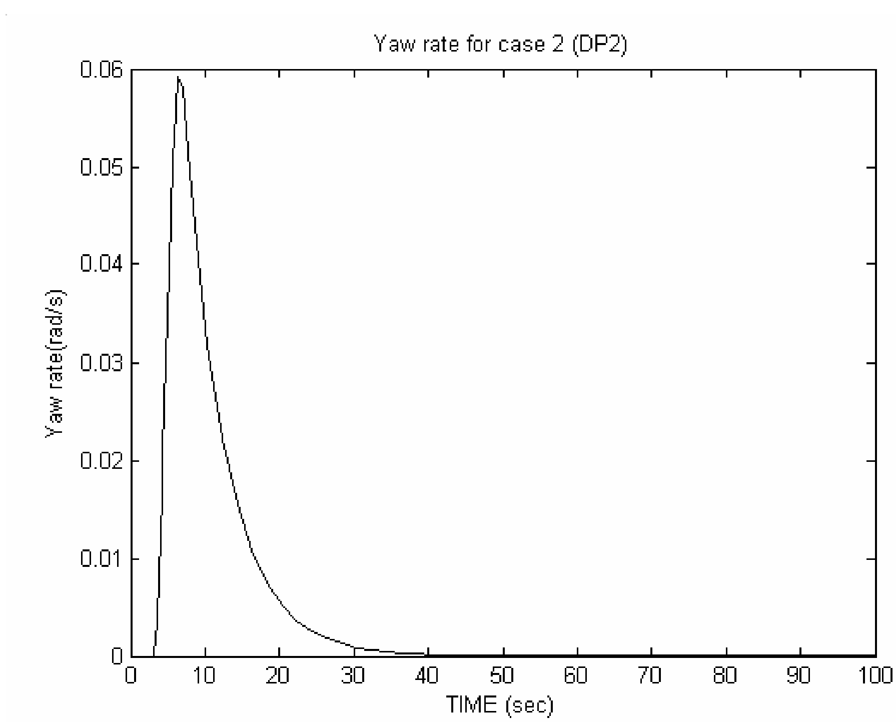

Figure 4.44 Yaw rate for case 2 (DP2)

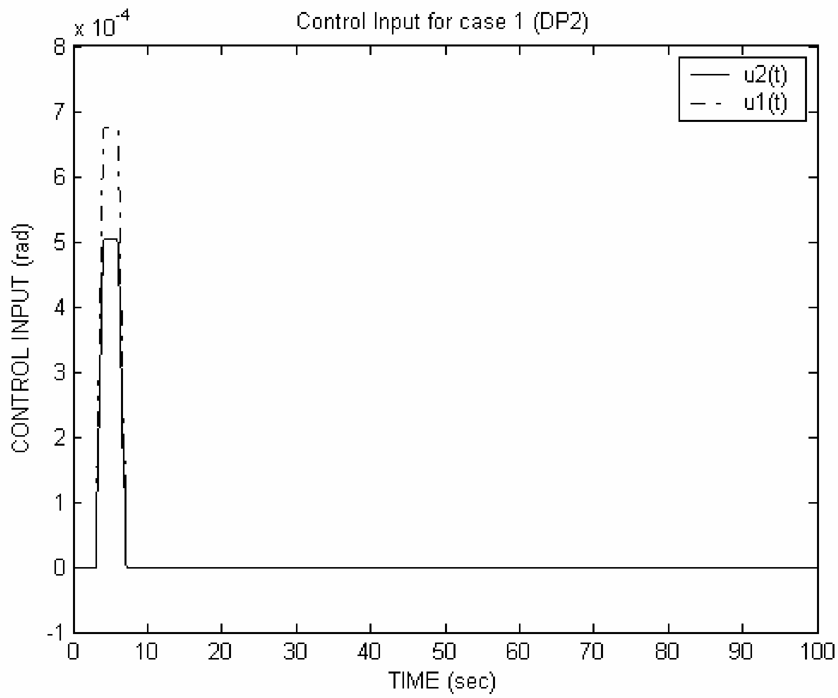

Figure 4.45 Control Input for case 1 (DP2)

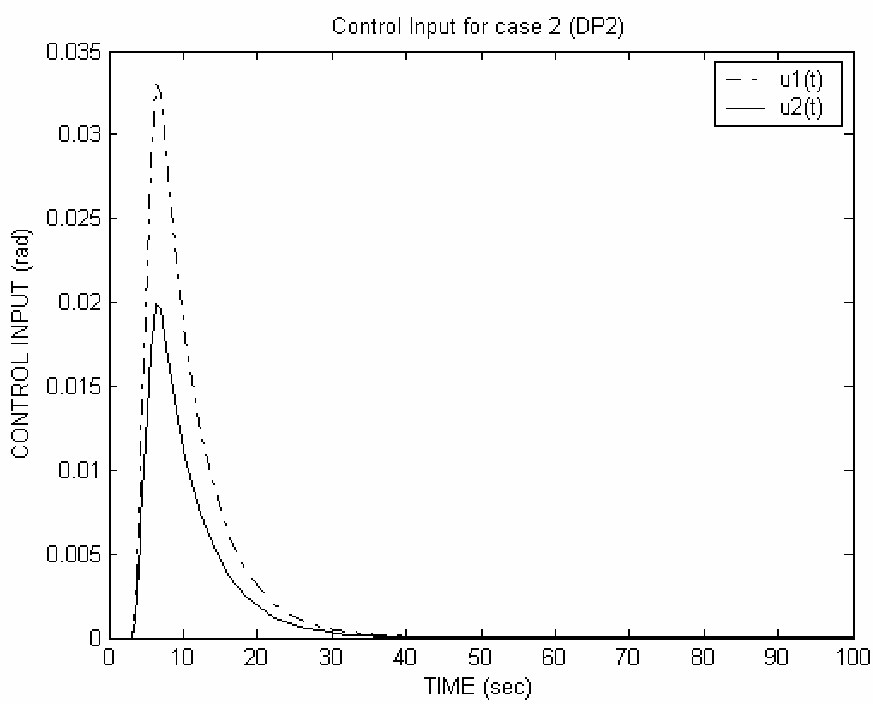

Figure 4.46 Control Input for case 2 (DP2)

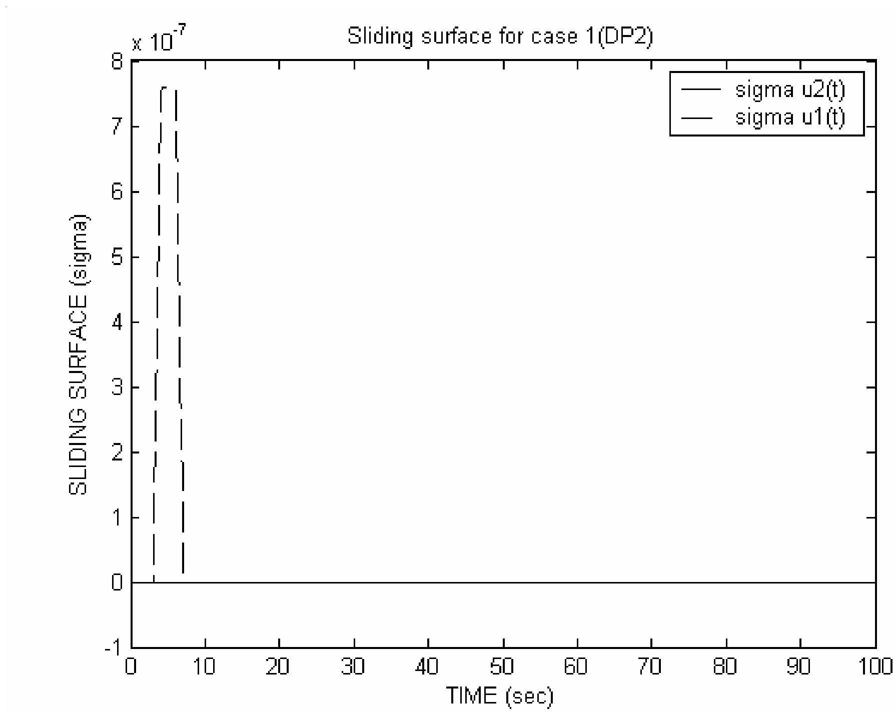

Figure 4.47 Sliding surface for case 1 (DP2)

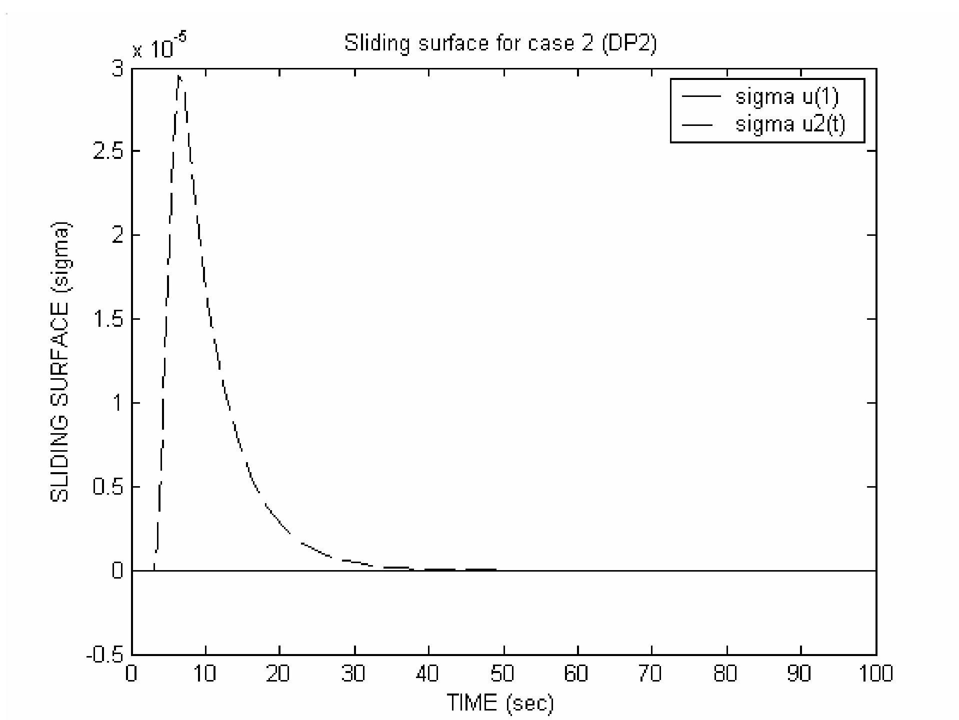

Figure 4.48 Sliding surface for case 2 (DP2)

4.3 Conclusion

Based on the sliding mode control approach, a framework for designing a controller for active car steering of a single track car model has been proposed. The active steering system is classified as matching condition. It has been shown theoretically and computer simulations that the proposed controller improved road handling performances using the active steering system compared to the uncontrolled car. Fast response has been overcome by using the continuous switching function and the boundary layer that can be adjusted by varying the sliding gain in the controller.

Furthermore, the sliding mode controller is robust to various types of disturbances and road coefficient. Various parameters such as friction of road coefficient and various disturbances have been varied to observe the effectiveness and robustness of the proposed sliding mode controller in the simulation. From the simulation results, there is a significant in vehicle performance between dry road and wet road. The performance for the side slip angle and yaw rate on wet road is worse than the performance on dry road which exhibits high in overshoot and magnitude. The vehicle tends to skid on wet road than on dry road.

In the simulation study, it is demonstrated that the SMC strategy gives best performance compared to LQR and pole placement controller. Again the magnitude is drastically reduced and the overshoot is avoided. Its effect is that the driver has to care much less about disturbance attenuation. The control action is finished faster than uncontrolled vehicle before the driver is even capable of starting his countersteering due to disturbances, thus can avoid car skidding.

CHAPTER 5

CONCLUSION AND SUGGESTIONS

5.1 Conclusion

In this thesis, the sliding mode control strategy has been successfully implemented to the active steering system. A single track car model is used to investigate the active steering performance under the proposed controller to avoid car skidding and improve performance of the system. A mathematical modeling of a single track car model has been established in this study. It has been shown that this system with uncertainties satisfy the matching condition.

Therefore, the sliding mode control has been proposed in solving the matched uncertainties system. By using the Lyapunov stability theory, it is shown that the sliding surface assured that the matched uncertainty system is asymptotically stable during the sliding mode. Furthermore, the study showed that the proposed sliding mode controller which is based on the equivalent method, assured that the reachability condition of the states trajectories are satisfied.

The results clearly shown that the sliding mode control scheme is robust and effective in compensating various disturbances and road coefficient in the system. In the simulation study, it was demonstrated that the SMC strategy gives best performance compared to LQR and pole placement techniques.

5.2 Suggestion For Future Work

Modeling of the active steering car system considered as linear and use full track car model. With this consideration, the analysis will be more realistic because the dynamics is close to their real system.

In this study, the constant matrices C in sliding mode switching surface is determined using trial and error approach. Proper selection of these matrices is very important because it will eliminate the uncertainties in this system. The tuning process is time consuming although this method delivered satisfactory results. Therefore, it is suggested that, these matrices be determined by using a specific and more reliable method.

REFERENCES

[1] Ackermann, J., et all, 2002: Robust Control: The Parameter Space Approach, Springer, pp.171-285.

[2] Ackermann, J., 1994: Robust Decoupling Of Car Steering Dynamics With Arbitrary Mass Distribution. *Proc. American Control Conference. (Maryland).*

[3] You, S.S and Jeong, S.K., 2004: Automatic Steering Controllers For General Lane-Following Manoeuvres Of Passenger Cars Using 2- DOF Robust Control Synthesis. *Transaction Of The Institute of Measurement And Control 26*,4 pp.273-292.

[4] Sienel,W. 1995: Robust Decoupling For Active Car Steering Holds For Arbitrary Dynamic Tire Characteristics. *Proc. Third European Control Conference. (Rom)*, pp.744-748.

[5] Ackermann, J., 1997: Robust Control Prevents Car Skidding. *IEEE Control Systems Technology* 3, pp.23-31.

[6] Doyle, J., Glover, K., Khargonekar, P. and Francis, B. 1989: State Space Solutions To Standard H2 and H∞ control problems. *IEEE Transactions Automatic Control 34,* pp.831-847.

[7] You, S.-S. and Jeong, S.-K. 2002: Controller Design And Analysis For Automatic Steering Of Passenger Cars. *Mechatronics* 12, pp.427-46.

[8] McFarlene, D.C. and Glover, K. 1990: *Robust controller design using normalized coprime factor plant descriptions.* Berlin: Springer-Verlag.

[9] Fukao, T., Miyasaka, S., Mori, K., Adachi, N. and Osuka, K. 2001: Active steering systems based on model reference adaptive nonlinear control. *IEEE Intelligent Transportation System Conference Proc.(USA),* pp.502-506.

[10] Park, M.K., Suh, I.H., Byoun, S.J., and Oh, S.R. 1996: An intelligent coordinated control system for steering and traction of electric vehicles. *IEEE Control System Technology.* pp.1972-1976.

[11] Huh, K., Seo, C., Kim, J. and Hong, D. 1999: Active steering control based on the estimated tire forces. *Proc. American Control Conference*. pp.729-733.

[12] Wong.J.Y., 1978, *Theory of Ground Veh*icles, Wiley and Sons Inc., New.

[13] T.P. Newcomb and R.T.Spurr, 1979: *Commercial Vehicle Braking*, Butterworth & Co (Publishers) Ltd.

[14] Oliver Hanke, Torsten Bertram and Manfred Hiller, 2001: Analysis and Control of Vehicle Dynamics Under Crosswind Conditions. *IEEE/ASME International Conference on Advance Intelligent Mechatronics Proceedings, Corno Italy*,pp.331-336.

[15] Christopher Edwards and Sarah K. Spurgeon, *Sliding Mode Control:Theory and Applications*, London: Taylor & Francis Group Ltd, 1998.

[16] DeCarlo, R.A., Zak, S.H.. and Matthews, G.P. Variable Structure Control of Nonlinear Multivariable System: A Tutorial. Proceedings of the IEEE. 1988. 76(3); pp.212-232.

[17] Lee, J.H., Allire, P.U, Tao, G., Decker, J.A. and Zhang, X. Experiment Study of Sliding Mode Control for a Benchmark Magnetic Bearing System and Artificial Heart Pump Suspension. IEEE Trans. On Control System Technology. 2003. 11; pp.128-138.

[18] Yahaya Md Sam. (2004). “Modeling And Control of Active Suspension System Using PI Sliding Mode control”, Universiti Teknologi Malaysia, PhD Thesis.