# Modeling and Simulation of Automotive Electric Power Steering System

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#### Abstract

The working principle and constitution of Electric Power Steering System (EPS) was introduced. The mathematic model was established and the state equation was listed. The influence factors and the corresponding solutions were analyzed. The close-loop control strategies of PID and PWM were adopted to control the target current of the motor for EPS. The simulations were made based on MATLAB/Simulink tools. The results indicated that the steering wheel with EPS system was easy to control and to return promptly with perfect steering feel. The model for EPS was precise, simple and utility and could afford further study on optimum design and control, and system debugging. It was also helpful to research the robust control research for EPS.

## 1. Introduction

Steering system is one of the essential parts in a vehicle. There are two types of steering systems. One is Hydraulic Power Steering (HPS), and the other is Electric Power Steering System (EPS). Compared with HPS, EPS has some advantages: lower energy, lower pollution and fewer mechanical components. And EPS can be adjusted and assembled easily. So as a new type of steering system, EPS is now gradually taking the place of HPS. In this paper, the architecture and principle of EPS is introduced in detail. Then based on the architecture and principle the state equation of EPS is built. And last conclusions are made depending on simulation figures <sup>[1, 2]</sup>.

## 2. Architecture and principle of EPS

When a vehicle with EPS turns, torque sensor detects two signals: torque and steering direction from steering wheel. Then these two signals are sent to Electric Control Unit (ECU). The ECU sends control instructions to motor controller depending on the parameters of steering torque, steering direction, vehicle speed and other dates, so the

motor controller gives corresponding steering torque to create assistant torque. When needn't to turn, the ECU doesn't send any signal to motor controller. So the motor doesn't need work.

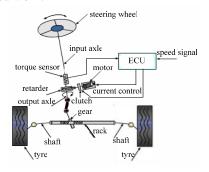


Figure 1 Constitution of EPS system

An EPS consists of speed sensor, torque sensor, steering speed sensor, ECU, power drive module, clutch and motor (See Figure 1). The ECU decides steering direction and the most appropriative assistant torque and sends control signals to motor and clutch. These signals make the motor running by power drive module and protection module. A retarder reduces motor speed to make the torque amplified. This torque can drive the gear to generate the corresponding assistant torque. The EPS can adjust this torque arbitrarily by precise algorithm and make the gear obtain assistant torque what the driver wants <sup>[3]</sup>.

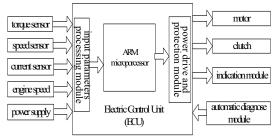


Figure 2 Framework of EPS



The framework of EPS is made up of various input parameters and their processing modules, ECU, power drive and protection module, motor, clutch, indication module and automatic diagnose module (See Figure 2). The power drive module consists of full-bridge circuit, gate drive circuit, relay drive circuit and clutch drive circuit.

## 3. Mathematic model of EPS system

Creating EPS mathematic model is a absolutely necessary step for study of EPS dynamic performance and drive performance. The main mechanical components of EPS have five parts: steering wheel, input axle (also called steering axle), motor, retarder and gear. Its dynamic model is shown as Figure 3.

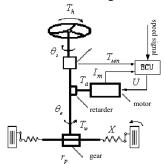


Figure 3 EPS dynamic model

Using Newton's law and neglecting no-necessary factors the equations of EPS can be derived<sup>[4]</sup>.

Considering the moment of inertia of steering wheel and the viscous damping of input axle, there is

$$J_{\rm s} \frac{\mathrm{d}^2 \theta_{\rm s}}{\mathrm{d}t^2} + B_{\rm s} \frac{\mathrm{d}\theta_{\rm s}}{\mathrm{d}t} = T_{\rm h} - T_{\rm sen} \tag{1}$$

Where  $J_{\rm s}$  is the moment of inertia of input wheel,  $B_{\rm s}$  is the viscous damping coefficient of input axle,  $\theta_{\rm s}$  is the angle of input axle,  $T_{\rm h}$  is the steering torque working on the input wheel,  $T_{\rm sen}$  is the anti-torque working on the torsion bar.

Because torque sensor is forced by the torsion bar, torque working on the torsion bar is directly proportional to the angle of the difference between input axle and output axle, hence there is

$$T_{\rm sen} = K_{\rm s}(\theta_{\rm s} - \theta_{\rm e}) \tag{2}$$

Where  $K_{\rm s}$  is rigidity coefficient of torsion bar,  $\theta_{\rm e}$  is the angle of output axle.

The motor for EPS is a permanent magnetic field DC motor. The relationship among voltage U across it,

inductance L , armature resistance R , back electromotive force  $K_{\rm b}$  ,motor speed  $\frac{{
m d} heta_{
m m}}{{
m d} t}$  ,current I and time t is

$$U = L \frac{\mathrm{d}I}{\mathrm{d}t} + RI + K_{\mathrm{b}} \frac{\mathrm{d}\theta_{\mathrm{m}}}{\mathrm{d}t} \tag{3}$$

Electromagnetic torque  $T_{\rm m}$  which is generated by motor can be expressed as a formula:  $T_{\rm m}=K_{\rm a}I$ , where  $K_{\rm a}$  is torque coefficient of the motor.

To analyze the force working on the mechanism for the motor, an equation can be obtained as follows:

$$J_{\rm m} \frac{\mathrm{d}^2 \theta_{\rm m}}{\mathrm{d}t^2} + B_{\rm m} \frac{\mathrm{d}\theta_{\rm m}}{\mathrm{d}t} = T_{\rm m} - T_{\rm a} \tag{4}$$

Where  $J_{\rm m}$  is the moment of inertia of the motor and clutch,  $B_{\rm m}$  is the viscous damping coefficient of the motor,  $\theta_{\rm m}$  is the angle for the motor,  $T_{\rm m}$  is steering torque working on the input wheel,  $T_{\rm a}$  is the torque for the motor.

To simplify the motor model, assist torque for the motor can be expressed as:  $T_{\rm a} = K_{\rm m} (\theta_{\rm m} - G \theta_{\rm e})$ , where  $K_{\rm m}$  is rigidity coefficient of the retarder for the motor. Furthermore, to analyze the force working on the output axle for the motor, an equation can be obtained as follows:

$$J_{\rm e} \frac{\mathrm{d}^2 \theta_e}{\mathrm{d}t^2} + B_{\rm e} \frac{\mathrm{d}\theta_{\rm e}}{\mathrm{d}t} = T_{\rm sen} + GT_{\rm a} - T_{\rm w} \tag{5}$$

Where  $J_{\rm e}$  is the moment of inertia of output wheel,  $B_{\rm e}$  is the viscous damping coefficient of output axle, G is the reduction ratio of the retarder,  $T_{\rm w}$  is the anti-torque working on the output axle.

To analyze the gear, such equation as follows can be obtained:

$$m_{\rm r} \frac{{\rm d}^2 x_{\rm r}}{{\rm d}t^2} + b_{\rm r} \frac{{\rm d}x_{\rm r}}{{\rm d}t} = \frac{T_{\rm w}}{r_{\rm p}} - F_{\rm TR}$$
 (6)

where  $m_{\rm r}$  is equivalent mass of the gear and rack,  $b_{\rm r}$  is damping coefficient of the rack,  $x_{\rm r}$  is displacement of the rack,  $r_{\rm p}$  is radius of the gear and  $F_{\rm TR}$  is the force working on the rack by steering resistance and aligning torque.

When a vehicle steers, steering resistance is primarily influenced by the friction created by the tyre and the surface of ground and various other frictions. And it is also influenced by vehicle speed, road condition, turning radius, air resistances, and the speed of steering wheel. Hence steering resistance can be simplified as:

$$F_{\rm TR} = k_{\rm r} x_{\rm r} + F_{\delta} \tag{7}$$

Where  $k_{\rm r}$  is the elastic coefficient for the equivalent spring,  $F_{\delta}$  is random signal from the road and  $x_{\rm r}$  can be expressed as:

$$x_{\rm r} = \theta_{\rm e} x_{\rm p} \tag{8}$$

Combine the equations (1)-(8), equation (9)-(11) can be obtained as follows.

$$J_{s} \frac{\mathrm{d}^{2} \theta_{s}}{\mathrm{d}t^{2}} + B_{s} \frac{\mathrm{d} \theta_{s}}{\mathrm{d}t} + K_{s} \theta_{s} = T_{h} + K_{s} \frac{x_{r}}{r_{p}}$$

$$\tag{9}$$

$$J_{\rm m} \frac{\mathrm{d}^2 \theta_{\rm m}}{\mathrm{d}t^2} + B_{\rm m} \frac{\mathrm{d}\theta_{\rm m}}{\mathrm{d}t} + K_{\rm m} \theta_{\rm m} = T_{\rm m} + G K_{\rm m} \frac{x_{\rm r}}{r_{\rm s}}$$
(10)

$$M_{\rm r} \frac{{\rm d}^2 x_{\rm r}}{{\rm d}t^2} + B_{\rm r} \frac{{\rm d}x_{\rm r}}{{\rm d}t} + K_{\rm r} x_{\rm r} = \frac{GK_{\rm m}}{r_{\rm p}} \theta_{\rm m} + \frac{K_{\rm s}}{r_{\rm p}} \theta_{\rm s} - F_{\rm d}$$
 (11)

Where  $M_{\rm r}$  (expressed as:  $M_{\rm r} = m_{\rm r} + \frac{J_{\rm e}}{r_{\rm p}^2}$ ),  $B_{\rm r}$  (expressed

as: 
$$B_r = b_r + \frac{B_e}{r_p^2}$$
) and  $K_r$  (expressed as:  $K_r = k_r + \frac{K_s + G^2 K_m}{r_p^2}$ )

represent respectively the equivalent mass, the equivalent damping coefficient and the equivalent elastic coefficient of the retarder, the gear and the rack.

Based on the above equations the state equation is built as:

$$\frac{dx}{dt} = Ax + Bu \qquad , \qquad \text{where}$$

$$v = Cx + Du$$

$$x = [\theta_{s} \quad \frac{\mathrm{d}\theta_{s}}{\mathrm{d}t} \quad x_{r} \quad \frac{\mathrm{d}x_{r}}{\mathrm{d}t} \quad \theta_{m} \quad \frac{\mathrm{d}\theta_{m}}{\mathrm{d}t}]^{T}, u = [T_{h} \quad T_{m} \quad F_{d}]^{T} \text{ an}$$

$$d \quad y = [T_{a} \quad T_{sen} \quad \theta_{m} \quad \frac{\mathrm{d}\theta_{m}}{\mathrm{d}t} \quad x_{r}]^{T} \text{ are the system state,}$$

control input and output, respectively, A, B, C and D are constant matrices. A, B, C and D are represented as follows:

## 4. Control strategy and simulation

Generally, there are three types of control strategy of motor torque for EPS. They are current control strategy, voltage control strategy and torque control strategy. Current control strategy is adopted in this system. This control strategy only needs wheel torque signal and vehicle speed signal. Depending on these two signals and pre-established assistant torque curves, vehicle Electric control unit decides target current of motor (expressed as  $I_{\rm T}$ ) and sends motor's rotor current measured by current sensor back to ECU. The difference between target current (expressed as  $I_{\rm T}$ ) and feedback current (expressed as  $I_{\rm F}$  ) equals zero by using PID control algorithm. Furthermore, the friction and the moment of inertia of motor cause road feeling to disappear, return ability delay, steeling wheel vibration and low sensitivity such problems. So it is necessary to compensate assistant torque<sup>[5,6]</sup>.

Current control strategy is used to control motor torque and simulation frame for EPS is built based on MATLAB depending on the mathematic model of EPS at last <sup>[4]</sup>.

When vehicle speed is 0km/h and 10km/h respectively, and motor speed is 1rad/s, their handiness simulation figures are shown as figure 4 and figure 5. Figure 4 indicates that when vehicle speed is 0km/h the mechanical maximum steering torque is 21.6N.m while the steering torque is 7.4N.m with assistant torque. The electrical assistant torque is 34% of mechanical torque. Figure 5 indicates that when vehicle speed is 10km/h the mechanical maximum steering torque is 11.3N.m while the steering torque is 5.6N.m with assistant torque. The electrical assistant torque is 48% of mechanical torque. So we can conclude that the vehicle's handiness with EPS is improved and its assistant gain is reduced with the vehicle speed acceleration

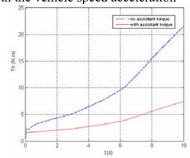


Figure 4 Torque comparison of 0km/h

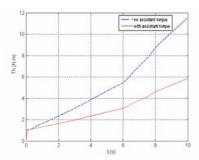


Figure 5 Torque comparison of 10km/h

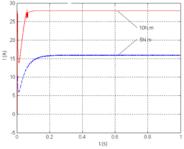


Figure 6 Step response of motor current

When vehicle speed is 0km/h and the force step input on the steering wheel is 10N.m and 5N.m respectively, the corresponding motor current figure is shown as figure 6. The figure indicates that the motor responses promptly and has high stability. Figure 7 is a pre-defined steering torque. Figure 8 is the corresponding current figure. From these two figures it shows the current control strategy has better tracking performance.

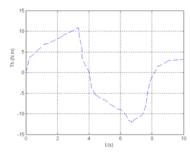


Figure 7 Pre-defined steering torque

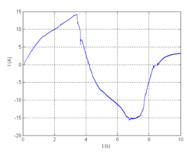


Figure 8 Corresponding current of steering torque

### 5. Conclusions and discussions

- (1) The model for EPS in this paper is precise, simple and utility. The model can be used to analyze and simulate the parameters in this system.
- (2) The simulation results indicate that the model can afford further study on optimum design and control, and system debugging. It is also helpful to research the robust control research for EPS.
- (3) Assistant characteristic in EPS is difficult to confirm because of vehicle speed, vehicle model, road condition and driver condition such factors. In this paper, a simple linear assistant characteristic is used. A more precise assistant characteristic should be further researched.

### 6. References

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