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ABSTRACT

An Electric Power Steering (EPS) System will be considered in this report. The modeling of this dynamic system will be achieved with both simplicity and usability taken into account. As such, a Reduced Order Model that reveals the important dynamic distinctions of the system will be developed from a more complex one. This model will be used to analyze various closed loop effects such as torque performance, disturbance rejection, noise rejection, road feel, and stability. These fundamental effects (compromises) are used toward the design of a desired control system. This modeling philosophy together with a comprehensive understanding of system compromises is essential to an optimized EPS system.

INTRODUCTION

Electric Power Steering is without a doubt, the most exciting improvement to steering systems since the introduction of hydraulic power steering, some 50 years ago. It achieves the almost impossible: simplifies the system - provides additional benefits - introduces virtually no side effects or costs. The details of all these benefits as provided by Delphi's EPS system, is outlined in [1]. Here, it suffices to merely point them out in the following categories:

- Engine Independence / Fuel Economy
- Tunability of Steering Feel
- Modularity / Quick Assembly
- Compact Size
- Environmental Friendliness

In this paper, we shall take a technical dive into the second bullet. Our focus will be to look at some of the modeling techniques and other technical tools to achieve a desired steering feel. Therefore, following a brief description of the E•Steer™ system, Reduced Order Modeling will be introduced. With this model validated, the closed loop characteristics of the system can be analyzed. Finally, with a thorough understanding of these closed loop behaviors, control system Algorithms are defined. With proper tuning a particular steering feel is achieved and demonstrated toward the end of this paper. Conclusion will look back at what has been accomplished. Some thoughts for continuous improvements are also given.

SYSTEM ARCHITECTURE

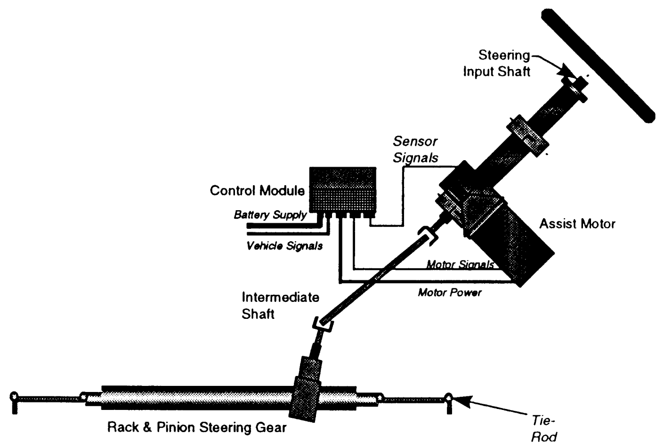

Figure 1. E•Steer™ incorporated in the steering system

An E•Steer™ system in its column assist configuration is shown in Figure 1. The system is made up of: a Steering Column, a Gear Assist mechanism attached to this column, a Brushless Motor, a Controller and a Sensor within the assist housing. The rest of the steering system: Steering Wheel (or hand wheel, HW), Intermediate Shaft (I-Shaft), Rack & Pinion, and the Tie Rods are also shown.

The main purpose of any power steering system is, of course, to provide assist to the driver. This is achieved by the torque sensor, which measures the driver's torque and sends a signal to the controller proportional to this torque. The controller also receives steering position information from the position sensor that is collocated with the torque sensor and together they make up the Sensor. The torque and position information is processed in the controller and an assist command is generated. This assist command is further modulated by the vehicle speed signal, which is also received by the controller. This command is given to the motor, which provides the torque to the assist mechanism. The gear mechanism amplifies this torque, and ultimately the loop is closed by applying the assist torque to the steering column.

The power source from the battery and motor position signals (i.e. Hall-effect signals coming from the motor assembly which are used to commutate the brushless motor) are also shown in Figure 1.

Other EPS architectures such as Rack Assist, Pinion Assist, etc. have also been proposed by Delphi and others. These different configurations, while important with respect to packaging, environmental effects, etc. are of little consequence to the discussion of this paper. Therefore, the Column Assist architecture shown in Figure 1 is the primary candidate considered for the upcoming analysis.

REDUCED ORDER MODELING

From a mechanical point of view, the steering system is made of (or maybe modeled with) many masses or inertias lumped together with springs and dampers (or friction elements). Figure 2 sketches a complete (Full Order) model of an EPS system.

Very simply put, Reduced Order Modeling, means consider the Full Order Model of Figure 2 and ask the following question: Can the number of masses (or inertias) be reduced by combining two or more of these masses into one? The answer to this question lies in the existence of overly stiff elements that connect these masses. If there are elements that are much stiffer than others, the answer is perhaps yes. This reduction in modeling order stems from the fact that higher stiffnesses contribute to higher frequency modes that usually are inconsequential to the fundamental behavior of the system (dominated by lower frequency modes).

The mechanics of model reduction can be explained by considering the rack and tie rod connection of Figure 2. The rack mass (M_R) is linked to each one of the tie rods masses (M_{TR}) through the stiff inner ball joint stiffness K_{IBJ} to be eliminated (see Figure 3). Note that in the Full Order Model there are 3 degrees of freedom. In the Reduced Order Model, the tie rod masses are eliminated, and thus, there is one degree of freedom (X_R) left. The equations of motion for the Full Order Model are:

$$F_P - 2K_{IBJ}(X_{RTR} - X_R) = M_R \ddot{X}_R$$
$$F_{ROBJ} + K_{IBJ}(X_R - X_{RTR}) = M_{TR} \ddot{X}_{RTR}$$
$$F_{LOBJ} + K_{IBJ}(X_R - X_{LTR}) = M_{TR} \ddot{X}_{LTR}$$

where:

F_P	=	Force from pinion
F_{ROBJ}	=	Force from right outer ball joint
F_{LOBJ}	=	Force from left outer ball joint
X_R	=	Rack displacement
X_{RTR}	=	Right tie rod displacement
X_{LTR}	=	Left tie tod displacement

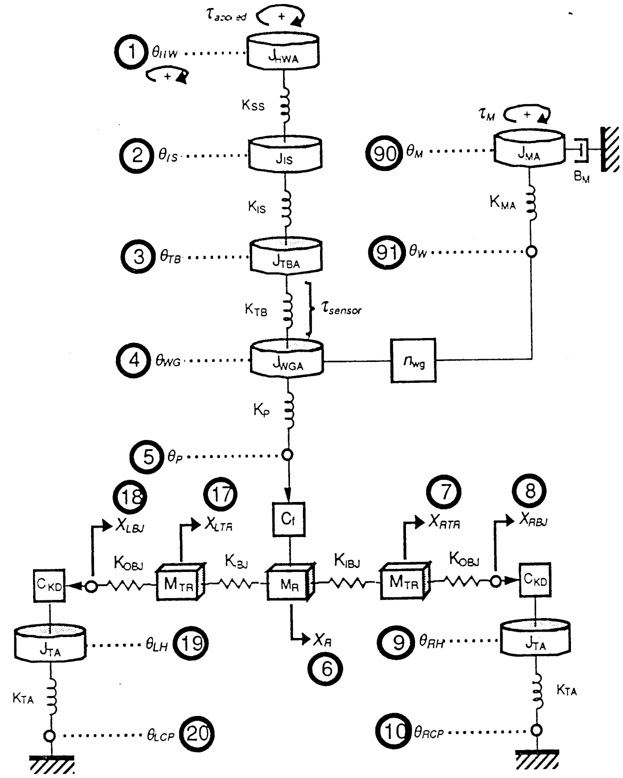

Figure 2. Full order model of an EPS system

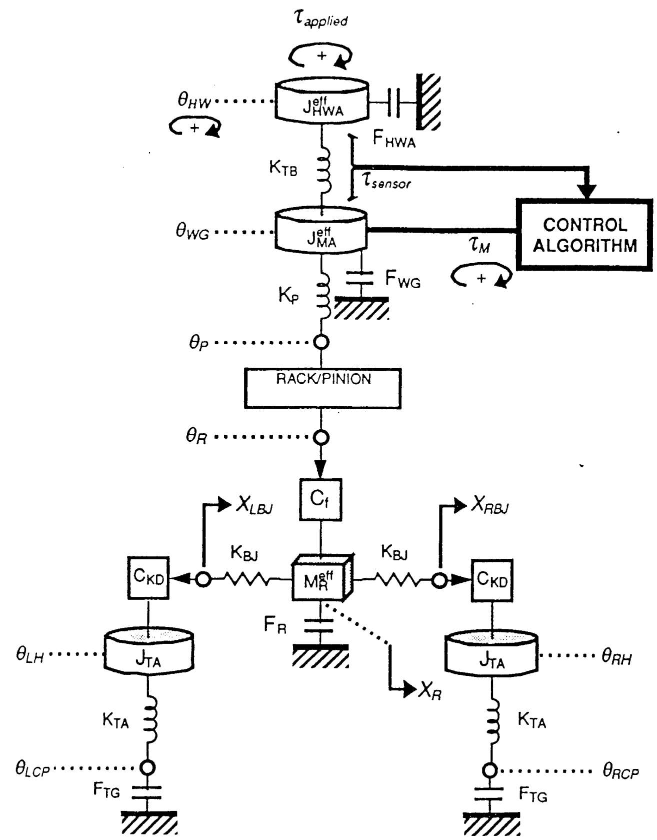

Figure 3. Reduced Order Model of an EPS system

Now, to reduce this, assume $X_R = X_{RTR} = X_{LTR}$ (i.e. K_{IBJ} is very stiff.) The equation of motion for the Reduced Order Model becomes:

$$F_p + F_{ROBJ} + F_{LOBJ} = (M_R + 2M_{TR})\ddot{X}_R = M_{R_{eff}}\ddot{X}_R$$

The algebra gets a bit more complicated if power-transforming elements such as gears are involved in the reduction process. The method is the same, however, and the result is simplification.

But why do we want to reduce the model anyway? What's wrong with having a complicated (and arguably more accurate) model? The primary answer to this question is that unnecessary complexity (i.e. high frequency dynamics) brings with it undue confusion. Attention may be given to unimportant dynamics as opposed to important ones. In other words, “the forest may be missed for the branches.” Furthermore, with a complex model simulations take more time to run and even the resulting compensation design could become more complex than it needs to be. The latter could pose all kinds of implementation issues, also.

Of course, too much reduction is dangerous too. At the end, it becomes an engineering call as to how much model reduction is just right. Perhaps, the single most important reason for model reduction is that through the search for finding the “Optimal Reduced Order Model” an invaluable understanding of the dynamics of the system is acquired.

So far, we have been focusing on model reduction with respect to mechanical elements. The same philosophy may be applied to electrical effects. Should we include motor induction effects, or not? Can we neglect the Pulse Width Modulation (PWM) effects, or not?

Figure 4 shows the step response comparisons between the Full and Reduced Order models. As it can be seen, the two simulations give identical results (solid and dashed lines are indistinguishable). Yet, the Reduced Order Model simulation took roughly one tenth of the time of the Full Order Model to finish.

CLOSED LOOP CHARACTERISTICS

Along with model reduction, the other necessary step before useful results can be deduced from the model, is indeed, model validation. Sometimes, before decisions about model reduction can be made, preliminary model validation must be ascertained. Therefore, model reduction and validation could very well be an iterative process.

There are many ways of validating a model. Without getting into a discussion on various means of achieving this goal, we have found the frequency response test of a vehicle equipped with the E•Steer™ to be most useful and convenient. Figure 5 helps to visualize this test. The idea is to validate the open loop plant or the *Steering System Model* block. This is achieved by nullifying the *Control* algorithms block to a known constant gain, disconnecting (or zeroing) other inputs such as vehicle speed and position, opening the torque loop between the *Sensor* and the *Control* algorithms block. Using a signal analyzer, a sinusoidal signal is injected to the controller at the opened node of the loop and the response of the system is measured from the sensor at the same node. The input frequency is varied (i.e. swept) through a defined range and the plots of output/input magnitudes and phase vs. the frequency is generated (see Figure 6).

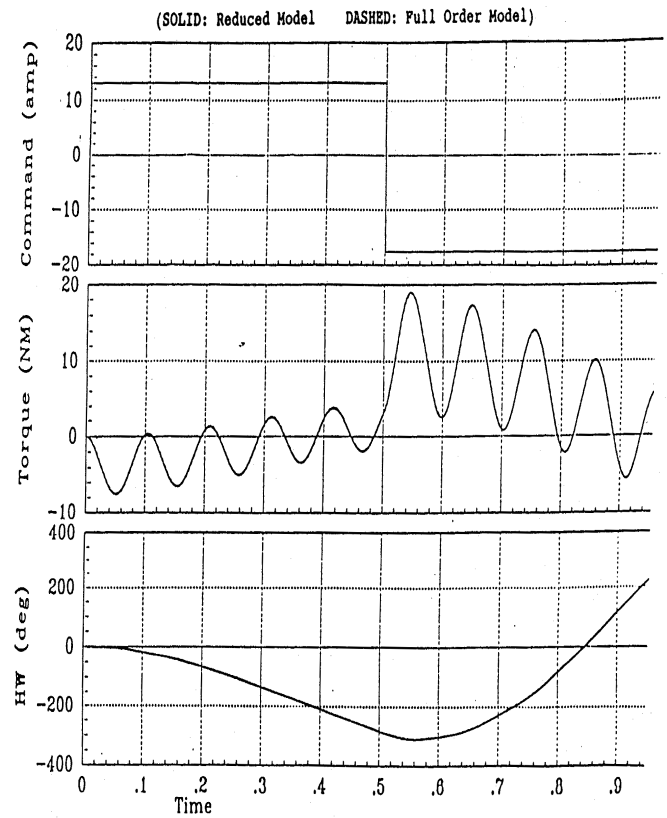

Figure 4. Comparison of full and reduced order models

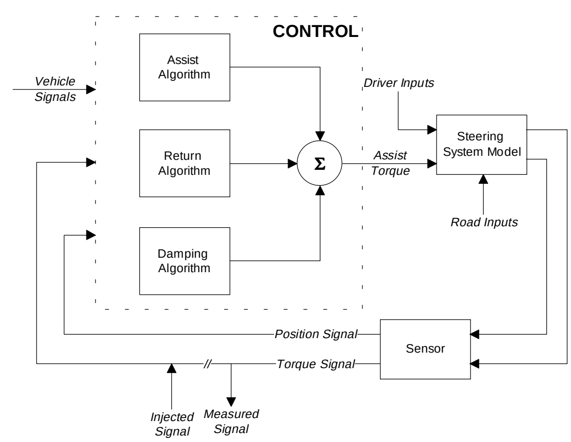

Figure 5. Closed Loop Block Diagram and Open Loop validation test setup

The frequency response of the actual steering system is then compared with that of our mathematical model. If discrepancies exist, parameters (that are unknown or hard to measure) are tuned to resolve the discrepancies. Sometimes unmodeled dynamics may need to be added to achieve a better match. Figure 6 is the resulting comparison between our models and the actual system. This is an excellent match and indeed verifies that our model is validated.

Figure 6 shows that the fundamental frequencies of the system are below 30 Hz. Thus, neglecting the higher frequencies (which are in the hundreds or thousands of Hz) through model reduction, is more than justified. It should be noted that how the steering wheel is held during the frequency response test affects the very low frequency region of the response. Above 10 Hz and especially at the cross over frequency (~25 Hz) the firmness by which the wheel is held is of no consequence to the response. Therefore, it is best to do this test with hands off the wheel.

With the open loop plant (the *Steering System Model*) validated, it is now time to close the loop and analyze the closed loop compromises, and then navigate toward an acceptable design.

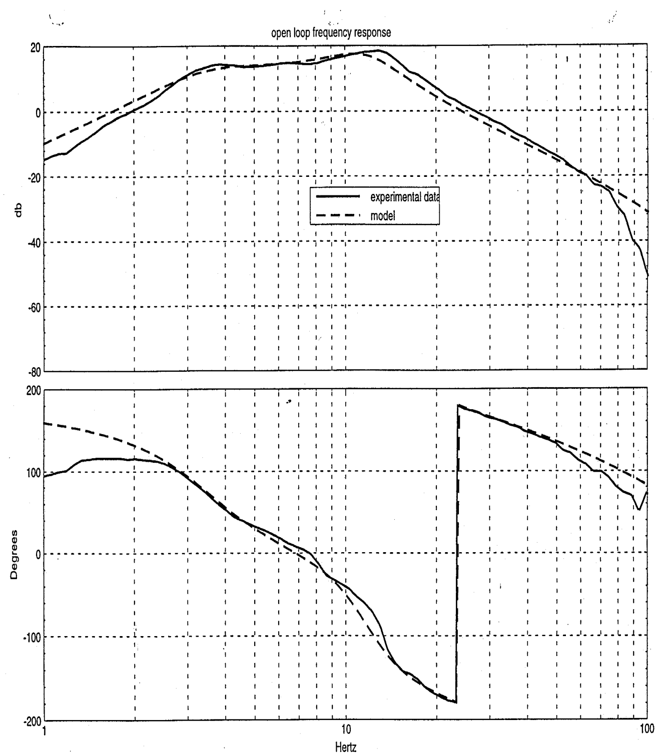

Figure 6. Model validation of the open loop system

The first objective of any power assist system is to provide high levels of assist. In this case, the assist comes from having high gains in the torque loop. In fact, we must achieve gains of 10 times that of shown in Figure 6. With the 0 dB line crossing the magnitude plot at around the cross over frequency (25 Hz) already, a compensation scheme is required to give us a factor of 10 or 20 dB of gain margin. Besides stability at high gains there are other closed loop behaviors that must be watched for.

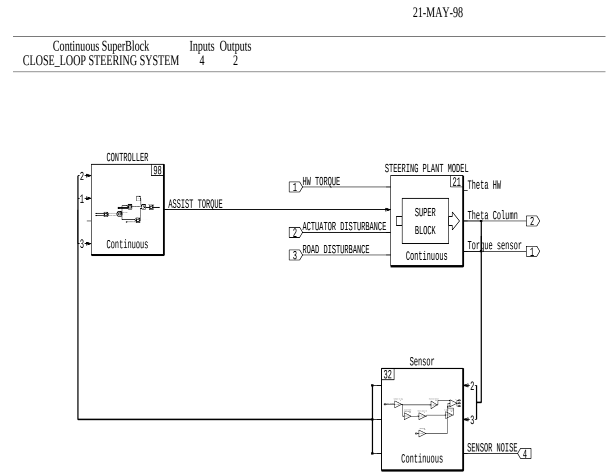

Figure 7. Closed loop block diagram of the steering system

Figure 7 shows the top-level block diagram of the E•Steer™ system with all its inputs that must either be accepted or rejected by the closed loop system. Figure 8 through Figure 11 show these closed loop frequency responses. For all of these, the torque sensor torque is the output as it is the best available indicator of the Steering Feel. In addition, all of these plots are uncompensated for us to begin to better understand the natural tendencies of the closed loop system.

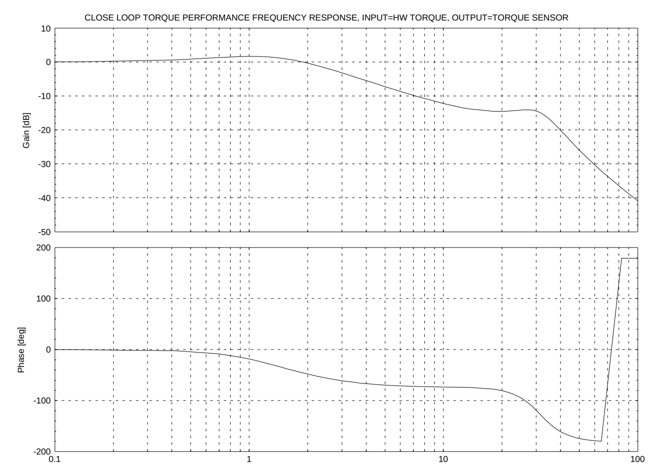

Figure 8. Closed loop torque frequency response

Figure 8 illustrates the torque performance. The plot shows that the response to the driver's torque (HW torque) is flat in both phase and gain plots to 3 Hz and thereafter the system acts like a filter and rolls it off. Since a good driver is capable of maintaining inputs of 3 to 5 Hz, it is important that the initial flatness be maintained in the compensated system, as well. This means that compensation schemes that achieve stability (at high gains) by sacrificing phase at lower frequencies are already doomed.

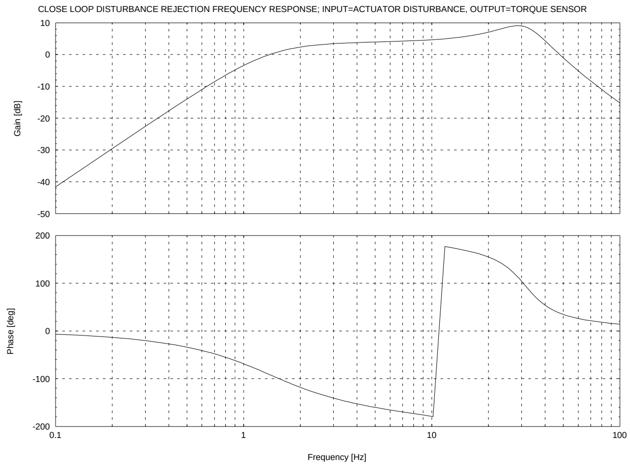

Figure 9. Closed loop actuator disturbance rejection

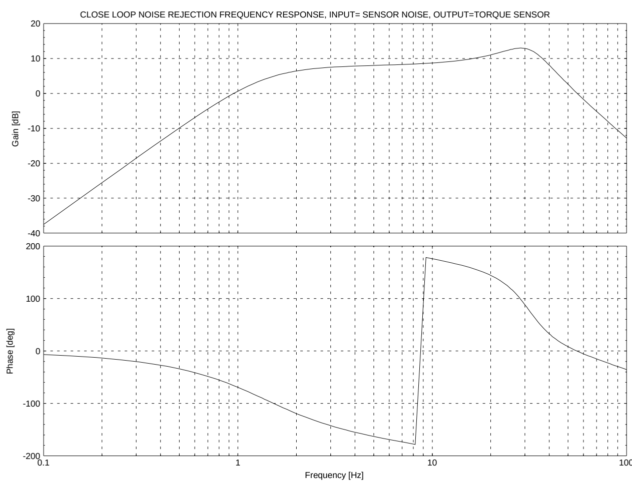

Figure 10. Closed loop noise disturbance rejection

Figure 9 and Figure 10 show the two main disturbances that should be rejected (preferably at all frequencies.) These are actuator disturbances (torque ripple, cogging, etc.) and noise signals (sensor, controller, etc.). Mathematically the transfer function for the actuator disturbance rejection is:

$$\frac{T_s}{T_d} = \frac{g}{1 + g\, g_c h}$$

where:

- T_s = Torque sensor torque
- T_d = Actuator disturbance torque
- g = Open loop transfer function from assist torque to torque sensor torque
- g_c = Controller transfer function (torque loop, only)
- h = Sensor transfer function (torque loop, only)

At the same time, the noise rejection transfer function is:

$$\frac{T_s}{N} = \frac{g_c g}{1 + g\, g_c h}$$

where:

N = Noise signal

Indeed, the classical duel between actuator disturbance rejection and noise rejection is obvious from the above two equations. For actuator disturbance rejection we need high gains (high g_c), while for noise rejection we need low gains (low g_c.) Luckily, there exist a nice separation between noise and actuator inputs in terms of their excitation frequencies. Noise usually has high frequency content, while actuator disturbances are lower in frequency.

So far we have seen a need for high gains at lower frequencies for providing assist and rejecting actuator disturbances and low gains for stability (at mid frequencies; cross over range) and noise rejection (at higher frequencies).

The road feel transfer function (Figure 11) is similar in shape to the other two rejection transfer functions (with the exception of some scaling factors.) Yet, it is different from the others in that at low (and perhaps mid) frequency ranges it needs to be accepted and at higher frequencies, it needs to be rejected. This is because drivers like to “feel the road” at the lower frequencies. This is in contrast to the assist and actuator disturbance requirements. Luckily, here too a nice separation exists. This time, vehicle speed comes to the rescue. At low speeds and parking situations), high gains can be used to provide good assist and reject disturbances at a cost to a dull road feel. At higher speeds (highway situations) gains are lowered to give the driver a nice road feel at a cost to a heavier driving condition.

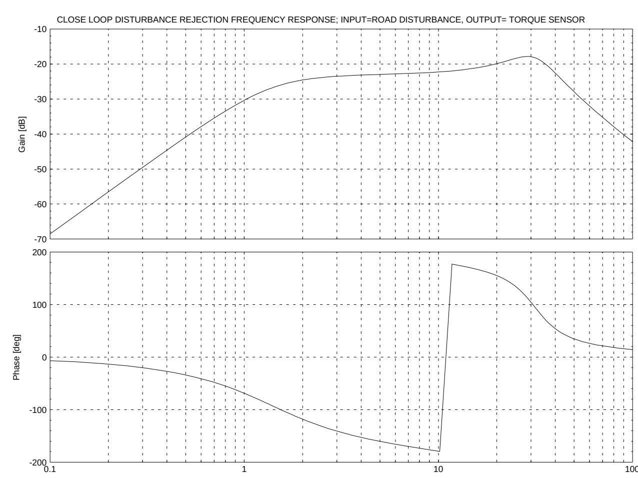

Figure 11. Closed loop road disturbance rejection

ALGORITHM DEFINITION AND RESULTS

All of the closed loop issues in the previous section are addressed in the *Assist Algorithm* block (see Figure 5) and are implemented in the E•Steer™ Controller (see Figure 1). Two main other algorithms are also implemented. The first is the *Return Algorithm* (see Figure 5), and the second is the *Damping Algorithm*. These two algorithms primarily utilize position information and provide additional steering functionality.

The *Return Algorithm* brings the steering wheel to the center. This is especially nice when inherent friction (or build tolerances) prevent the wheel to come back to the exact center position. The *Damping Algorithm* allows for the wheel to come back to center in a nice damped way and avoid the “free control” oscillations [2] that are usually present at high vehicle speeds.

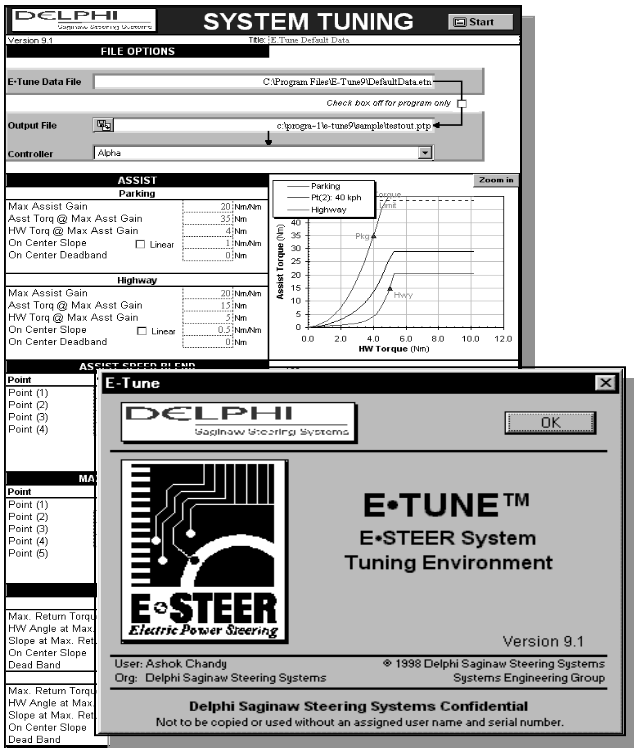

Figure 12. E•Tune™ software

The tuning of the steering feel as a function of sensor signals and vehicle speed is achieved through the use of a proprietary software called E•Tune™ (see Figure 12). This visual software allows for the gains within the assist, return, and damping algorithms to be set and thus achieve a desired steering system feel. E•Tune™ configurations can be downloaded from a laptop computer into the controller while driving the vehicle. This versatile tool allows the driver to gain immediate feedback of the changes he/she has made. Hence, a quick tuning (within minutes) of the steering feel is possible without ever touching the hardware.

Of course, the compromises mentioned in the previous section are still in place and serve as good guidelines for tuning. Within these bounds, however, a great deal of room is left for properly adjusting the steering feel per customer’s requirements. Notice (Figure 12) that the typical bathtub-shaped assist function of the hydraulic systems are preserved here. Additionally, improved road feel is achieved by lowering the assist curves for highway situations (i.e. Variable Effort Steering, VES, comes with E•Steer™ at no cost).

Figure 13 through Figure 15 show some of the results of a successful control algorithm design and tuning. First, Figure 13 shows that the compensated system (dashed line) is stable. The uncompensated system (solid line) is quite oscillatory at this high gain situation. Note that when the torque sensor (representing the driver’s effort) picks up 5 Nm (dashed line), the assist torque might be as high as 50 Nm. Figure 13, therefore, shows that the closed loop system has achieved stability at high gain conditions.

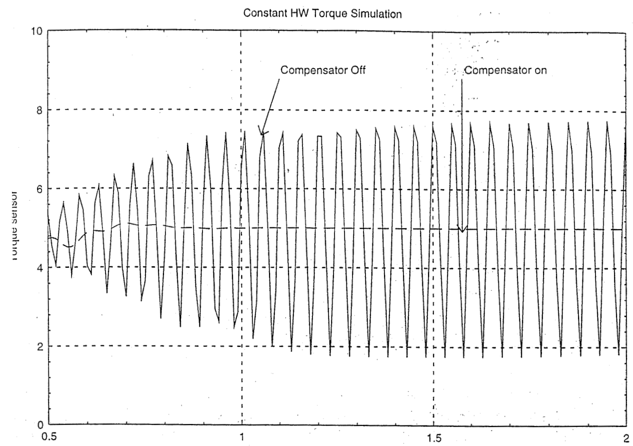

Figure 13. Effect of compensation on stability at high gains

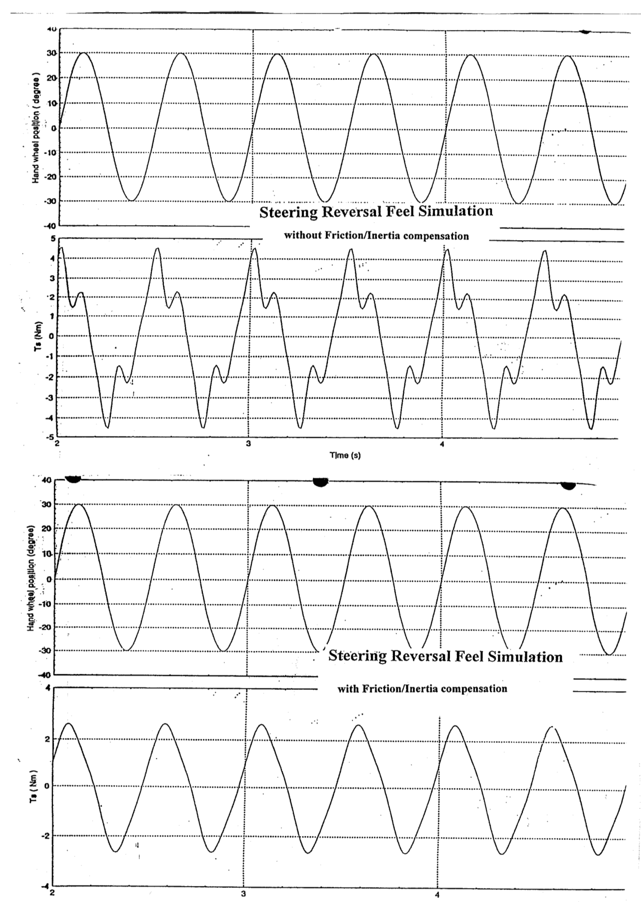

Figure 14. Effect of the friction/inertia compensation on reversals

Another closed loop requirement was the flatness or bandwidth of the torque transfer function. However, due to the lowering of gains at reversals, the closed loop system loses bandwidth and an “inertia” or a “hump” feel is possible during a reversal maneuver (see Figure 14). The friction/inertia compensation of the *Assist Algorithm* (see Figure 5) is designed to address this situation. Notice the difference in Figure 14 between the smooth results of the compensated case and the lumpiness of the uncompensated case.

Finally, return to center must be achieved without excessive overshoots at higher vehicle speeds. Figure 15 shows that, indeed, the *Damping Algorithm* achieves this important function of the steering system. The need for this algorithm is greater at higher vehicle speeds. At 80 kph, this algorithm is necessary, whereas the 40 kph case is perhaps tolerable without it.

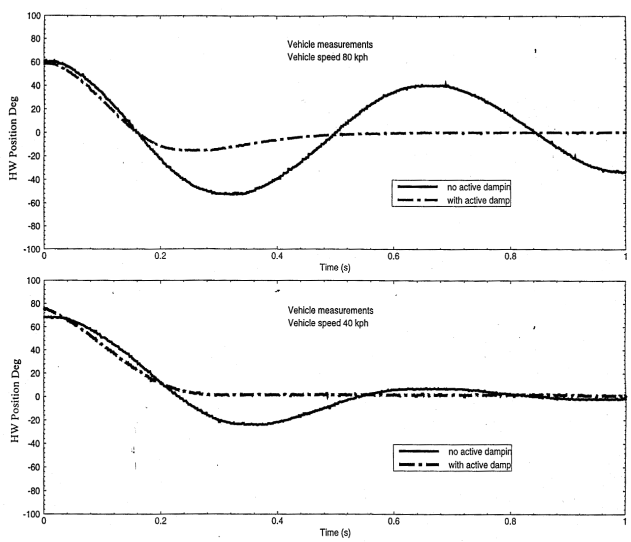

Figure 15. Effect of the damping algorithm on free control oscillations

CONCLUSION

In this paper the need for using a Reduced Order Model as related to an Electric Power Steering system, E•Steer™, was explained. The model was then validated through a simple frequency response test.

Closed loop transfer functions where used to understand the basic compromises of this system and to set requirements for the compensation design.

Control algorithms and a versatile tuning capability, E•Tune™, was shown to be highly effective in arriving at an optimal Steering Feel.

Concurrent and or future work in this arena include improved motor technologies, advanced tools such as Rapid Prototyping Controllers or Hardware in the Loop Diagnostics setups, and the use of this technology in Automated Highways and Steer-by-Wire projects.

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