GAMES101-现代计算机图形学入门

1. Transformation

Assume v is a 3 dimension vector, k is a unit vector of rotation axis.

Then, v revolves around k with θ° $$v_{rot} = cos\theta v + (1-cos\theta) (v\cdot k)k + sin\theta v \times k$$

2. View and Camera Transformation

2.1 How to take a photo ?

💡 **Model - View - Projection**

2.2 How to perform view transformation ?

1. Define the camera, $\vec{e}$ is position, $\vec{g}$ is gaz direction, $\vec{t}$ is up direction.
2. locate the coordinate of objects under $\vec{e},\vec{g},\vec{t}$
3. transfer $\vec{e},\vec{g},\vec{t}$ to $\vec{x},\vec{y},\vec{z}$ with $\vec{g} = \vec{-z}$, $\vec{t} = \vec{y}$ and $\vec{x} = \vec{g}\times \vec{t}$

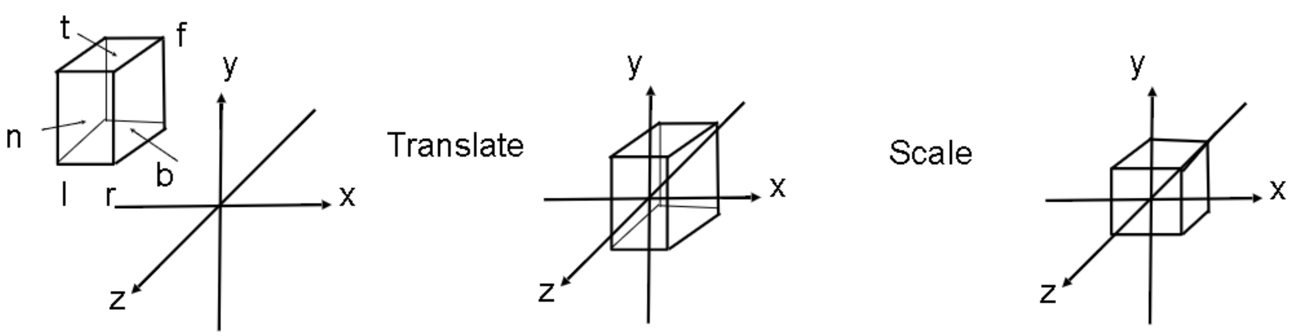
2.3 Rotation Matrix

For every affine transformation: $$\vec{y} = A\vec{x}+\vec{b}$$ which is equal to: $$\begin{pmatrix} \vec{y}\ 1 \end{pmatrix} = \begin{pmatrix} A & \vec{b}\ \vec{0} & 1 \end{pmatrix}\begin{pmatrix} \vec{x}\ 1 \end{pmatrix} $$ We know that transfor $\vec{x}\vec{y}\vec{z}$ to $\vec{e}\vec{g}\vec{t}$ with $$\begin{pmatrix} R & \vec{b}\ \vec{0} & 1 \end{pmatrix} $$ and $$R^{-1} = \begin{bmatrix} x_{\hat{g} \times \hat{t}} & x_{\hat{t}} & x_{-\hat{g}}\ y_{\hat{g} \times \hat{t}} & y_{\hat{t}} & y_{-\hat{g}}\ z_{\hat{g} \times \hat{t}} & z_{\hat{t}} & z_{-\hat{g}}\
\end{bmatrix} $$ and we know that $R = (R^{-1})^{T}$

3. Projection Transformation

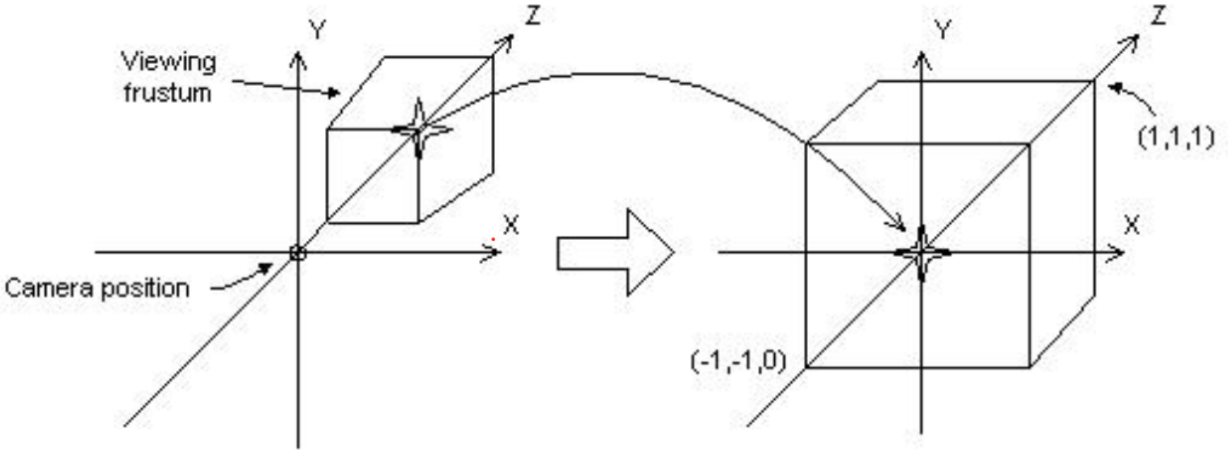
3.1 Orthographic Projection

1. move the centre of cube to the original coordinate
2. scale the cube to $2\times 2\times 2$



3.2 Perspective Projection

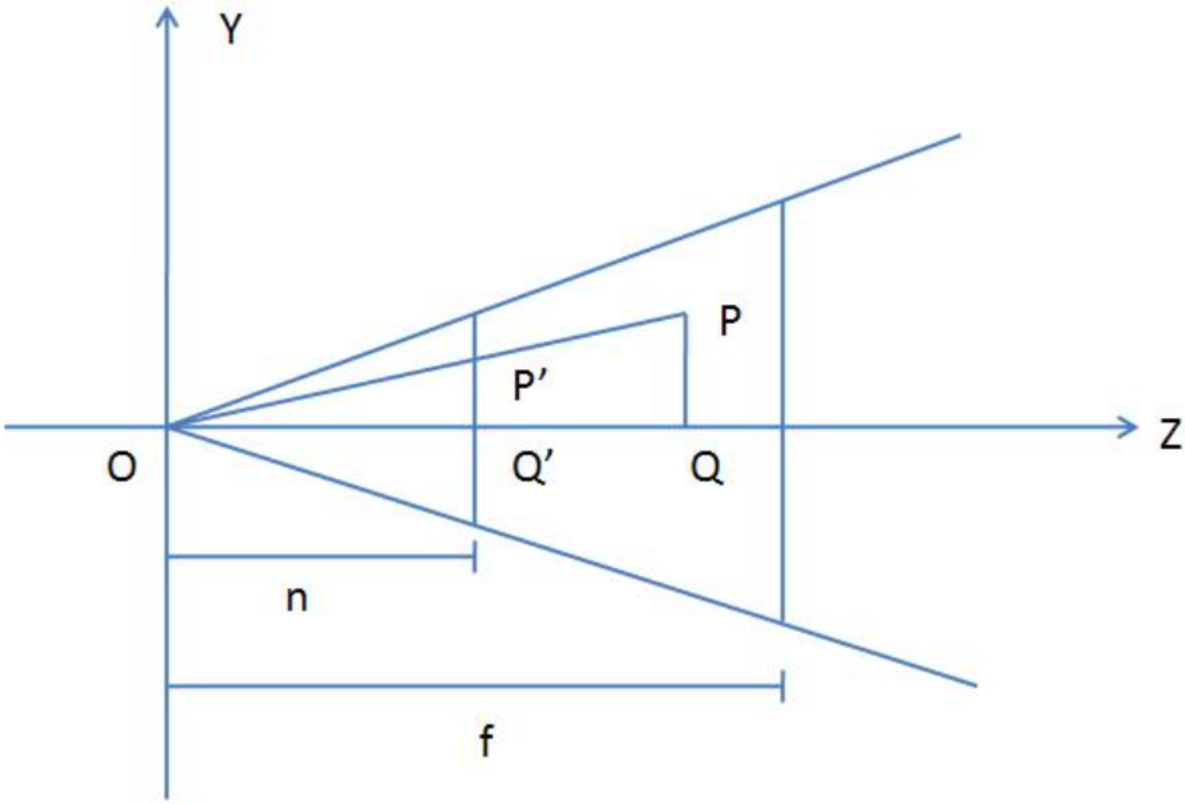
All we should do is:



We can divide this process into two parts:

1. Orthographic Projection $p\rightarrow p'$
2. scaling $p'\rightarrow p''$

For step1:



we know that $y' = \frac{ny}{z}$ and $x' = \frac{nx}{z}$

For step2:
Assume the hight of projection is H, and the hight of cube in the end is 2. Hence, $$\frac{y'}{y''} = \frac{H}{2}$$

and the transfer matrix is $$\begin{bmatrix} \frac{cot\theta}{Aspect} & 0 & 0 & 0\ 0 & cot\theta & 0 & 0\ 0 & 0 & \frac{f}{f-n} & 1\ 0 & 0 & \frac{fn}{n-f} & 0
\end{bmatrix} $$ with $Aspect = \frac{x}{y}$, $cot\theta = \frac{n}{0.5H}$

4. Drawing to Raster Displays

4.1 Pixel Values Approximate A Triangle By Sampling

1. Algorithm for discretizing a function by sampling a function:

```
for (i in domian(x); i++)
     output[x] = f(x)
```

2. Algorithm of a pixel is in triangle:

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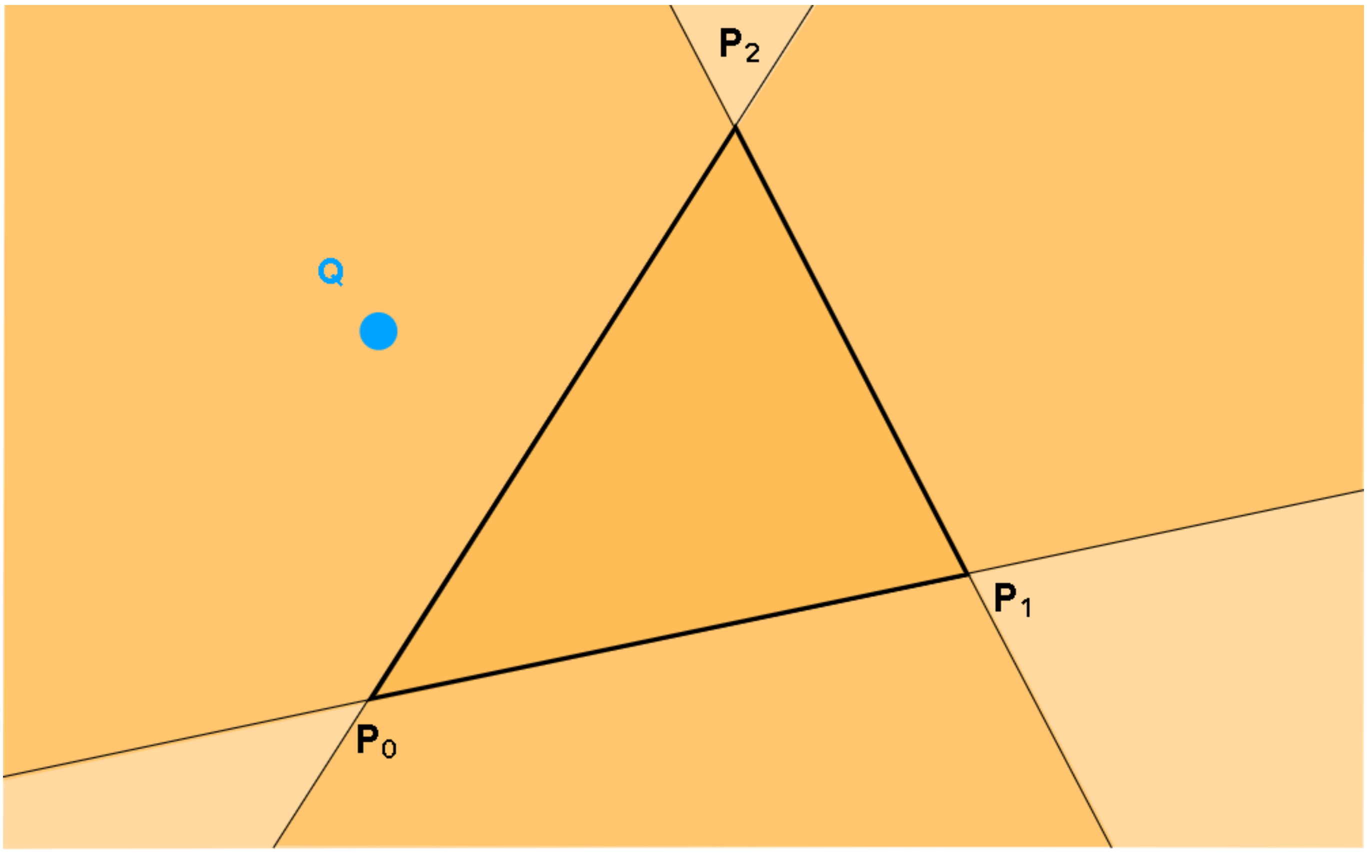
for (i in domain(x); i++)
    for(y in domain(y); y++)
        output[x][y] = isinside(tri, x+0.5, y+0.5)
```

3. How to evaluate if a pixel is in triangel:



If $\vec{QP_{1}}\times \vec{QP_{2}}$ has the same direction with $\vec{P_0P_{1}}\times \vec{P_0P_{2}}$, then that shows they are in the same side. Similarly, if a point stand in the same side with P_0,P_1,P_2, which means it is in the triangle.