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Coverage of Known Spaces: The Boustrophedon Cellular Decomposition

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Abstract. Coverage path planning is the determination of a path that a robot must take in order to pass over each point in an environment. Applications include de-mining, floor scrubbing, and inspection. We developed the boustrophedon cellular decomposition, which is an exact cellular decomposition approach, for the purposes of coverage. Essentially, the boustrophedon decomposition is a generalization of the trapezoidal decomposition that could allow for non-polygonal obstacles, but also has the side effect of having more "efficient" coverage paths than the trapezoidal decomposition. Each cell in the boustrophedon decomposition is covered with simple back and forth motions. Once each cell is covered, then the entire environment is covered. Therefore, coverage is reduced to finding an exhaustive path through a graph which represents the adjacency relationships of the cells in the boustrophedon decomposition. This approach is provably complete and experiments on a mobile robot validate this approach.

Keywords: vacuum cleaning, coverage, floor coverage, boustrophedon, cellular decomposition, motion planning, complete, algorithm, mobile robot

1. Introduction

Coverage path planning determines a path that guarantees that an agent will pass over every point in a given environment. This procedure allows for a variety of applications. Naval applications include mine-countermeasure missions and continental shelf oceanographic mapping. Commercial applications include contamination cleanup, floor scrubbing, crop plowing, and bridge inspection. Without a coverage algorithm, these applications cannot be handled. Most current coverage path planners are rudimentary at best because they are based on heuristics. Using such approaches for mine sweeping is akin to performing this operation with a faulty mine detector. Therefore, the coverage path planning alogirithm described in this paper is *complete*; that is, in finite time, it will find a coverage path in a connected component of the robot's free space.

Our approach exploits a geometric structure termed an *exact cellular decomposition*, which is the union of non-intersecting regions composing the target environment. Each region is termed a *cell*, and the union of the cells fills the environment. In each cell, a coverage path can be readily determined, such as simple back-and-forth motions; thus coverage path planning reduces to planning motions from one cell to another. This work will develop a new cellular decomposition, termed the *boustrophedon decomposition* and apply it to coverage path planning. We use the term boustrophedon, which was first used in the English language in 1699, because it literally means, "the way of the ox." Typically, when an ox drags a plow in a field, it crosses the full length of the field in a straight line, turns around, and then traces a new straight line path adjacent to the previous one. By repeating this procedure, the ox is guaranteed to cover (and thereby to plow) the entire field. See Fig. 1.

The boustrophedon cellular decomposition is a new type of decomposition where the robot's free space is broken down into cells such that the robot can cover each cell with back-and-forth boustrophedon motions. Once a robot covers each cell, it has covered the entire free region of an environment. This approach has been validated with simulations and a Nomadic 200 mobile robot base.

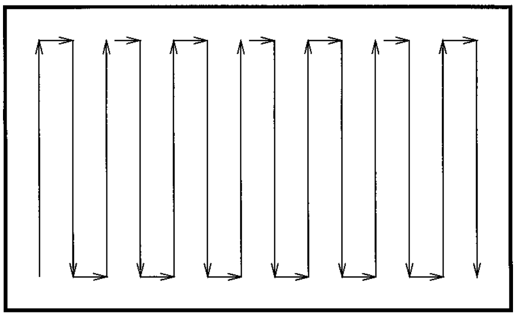

Figure 1. Boustrophedon path.

2. Background Work

2.1. *Prior Work in Coverage*

Previous work in coverage include applications such as floor maintenance (Colegrave and Branch, 1994). In these approaches, the path must be explicitly programmed into the robot; i.e., they do not use an algorithm to generate the coverage path, but instead prescribe one "by hand." Furthermore, these algorithms rely on landmarks deployed in the environment. Some modern agricultural operations represent a significant opportunity for coverage applications whose coverage path is easy to automatically generate. The Demeter project (Ollis and Stentz, 1996) is used to harvest large fields; in this approach the robot simply uses vision to guide its path alongside the previous cut crop line and can only cover rectangular fields.

A floor coverage approach that considers non-holonomic constraints is described in Hofner and Schmidt (1995). In this work, a set of templates is used to cover only a bounded region that is free of obstacles. These templates are used to accommodate the non-holonomic constraints of the robot, and thus may be useful for planning back and forth motions within each cell of the proposed approach. However, its limitation is that it cannot plan paths when obstacles are present.

The coverage algorithm described in Zelinsky et al. (1993) is well suited to unstructured environments. Although it is complete, it achieves floor coverage in a discretized environment (i.e., it is resolution complete). A similar approach, without proof, with cooperating robots was mentioned in Kurabayashi et al. (1996). Finally, Hert et al. (1996) produced an algorithm that is similar to the proposed approach in the planar case. Although the proposed algorithm produces nearly the same path as Lumelsky' group's in the planar case, the approach described in this paper is easier to implement because it has two cases whereas their approach contains a series of special cases. Finally, the approach in Hert et al. (1996) is not complete. The primary contribution of the algorithm supplied by Hert et al. (1996) is that it is incremental, and thus may lead to a sensor based implementation on a mobile robot.

2.2. *Exact Cellular Decomposition*

The approach to coverage used in this paper is an adaptation of an existing complete motion planning scheme, termed an *exact cellular decomposition*. Cellular decomposition is a motion planning technique in which the free configuration space (set of all robot configurations where the robot does not overlap an obstacle) is decomposed into cells such that the union of the cells is the original free space. Each cell can be represented as a node in a graph, where adjacent cells have an edge connecting their corresponding nodes. This graph is called an adjacency graph. If each cell can be covered by the robot, then the floor coverage problem reduces to determining a walk through the adjacency graph that visits each node at least once, i.e., the traveling salesman problem, for which a solution (possibly sub-optimal) always exists.

One popular cellur decomposition technique, which can yield a complete coverage path solution, is the trapezoidal decomposition (Latombe, 1991) (also known as the slab method (Preparata and Shamos, 1985)) in which the robot's free space is decomposed into trapezoidal cells. Since each cell is a trapezoid, coverage in each cell can easily be achieved with simple back and forth motions (See Fig. 1). Coverage of the environment is achieved by visiting each cell in the adjacency graph.

The trapezoidal decomposition approach assumes that a vertical line, termed a *slice*, sweeps left to right through a bounded environment which is populated with *polygonal* obstacles. Cells are formed via a sequence of *open* and *close* operations which occur when the slice encounters an *event*, an instance in which a slice intersects a vertex of a polygon. There are three types of events: IN, OUT, and MIDDLE. Loosely speaking, at an IN event the current cell is closed (thereby completing its construction) and two new cells are opened (thereby initiating their construction) See Fig. 2. An OUT event is the reverse: two cells are closed, and a new one is opened. See Fig. 3. The

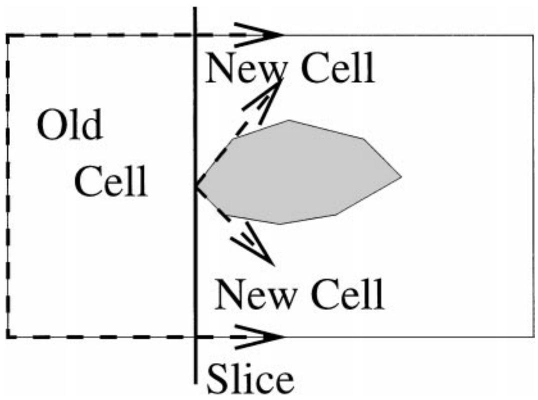

Figure 2. In event.

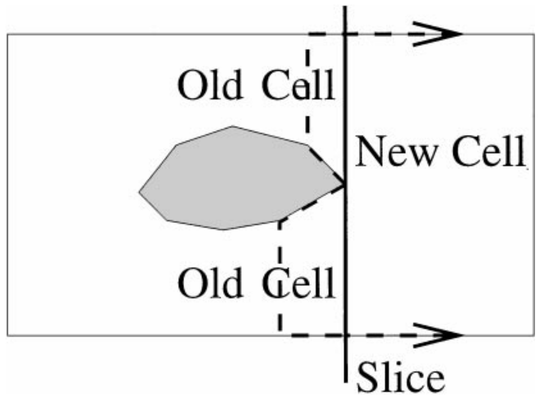

Figure 3. Out event.

IN event can be viewed as one cell breaking up into two cells, whereas the OUT event is when two cells merge into one. At a MIDDLE event, the current cell is closed, and a new one is formed. The result of these operations is a freespace that is broken down into trapezoidal cells.

The terrain coverage system of VanderHeide and Rao (1995) is based on a trapezoidal decomposition of a planar environment populated with one or two well-separated obstacles. The advantage of this system is that it is sensor based because it uses line of sight information.

Unfortunately, the trapezoidal approach requires too many redundant back and forth motions to guarantee completeness. In the left hand side of Fig. 4 the robot needs to make on additional lengthwise motion to cover the remaining portion of the trapezoidal cell. This can be viewed as part of the cost in guaranteeing the robot exhaustively covers the entire environment. Another drawback of the trapezoidal approach is that it requires the environment to be polygonal.

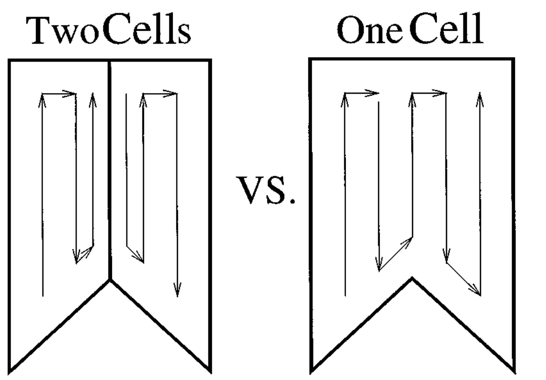

Figure 4. Fewer cells is better.

3. Contributions

The boustrophedon cellular decomposition, introduced in this paper, is an enhancement of the trapezoidal decomposition and is designed to minimize the number of excess lengthwise motions, as described in the previous paragraph. In essence, all cells between IN and OUT events are merged into one cell. Compare the trapezoidal decomposition in Fig. 5 with the boustrophedon decomposition in Fig. 6. Note that the boustrophedon decomposition has a fewer number of cells.

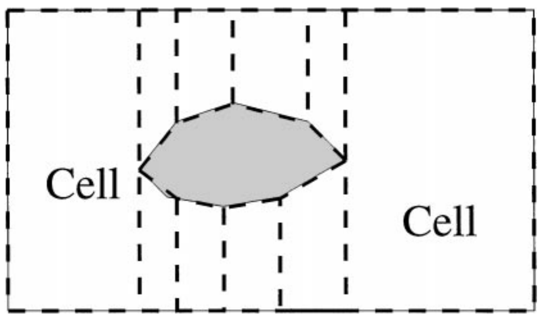

Figure 5. Trapezoidal decomposition.

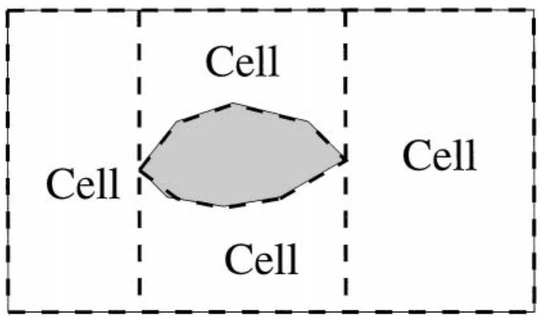

Figure 6. Boustrophedon decomposition.

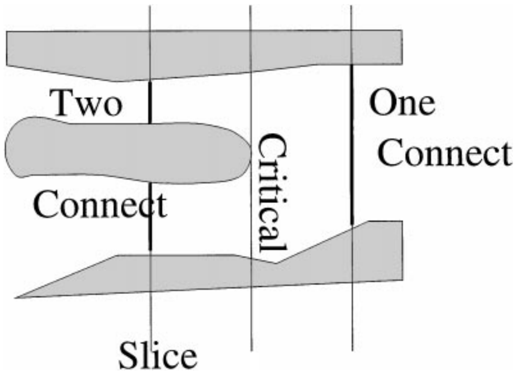

Figure 7. Critical points are points where the connectivity of a slice changes, i.e., they are events.

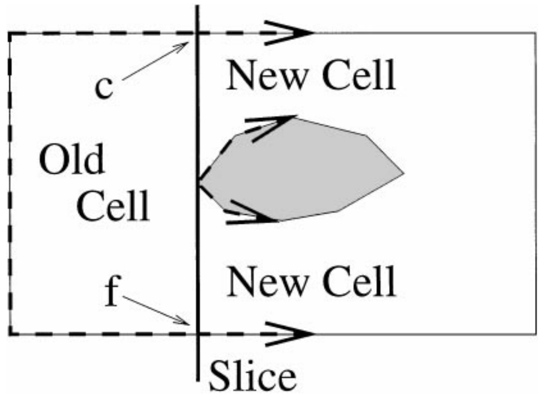

Figure 8. In event.

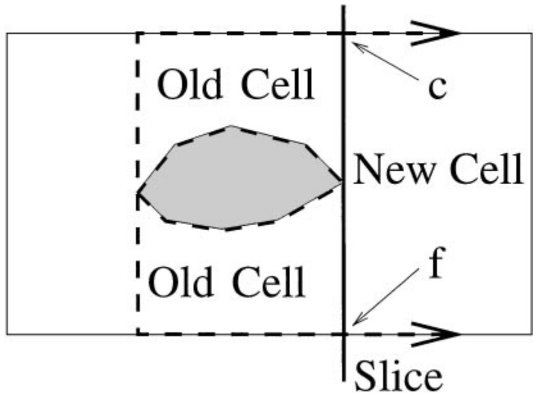

Figure 9. Out event.

The advantage of having a fewer number of cells is that the number of back-and-forth boustrophedon motions can be minimized. For example, consider two adjacent trapezoidal cells whose widths are each two and a half times the width of the robot. In order to cover each trapezoid, the robot must make three passes, for a total of six lengthwise motions. With the boustrophedon decomposition approach, the two cells are merged into one cell which is a monotone polygon and requires five passes to cover. See Fig. 4.

Instead of exploiting the structure of polygons to determine IN and OUT events, this approach will rely on changes in connectivity of a slice to determine the existence of an event. Typically, this is called a critical point, which is used in roadmap motion planning techniques such as Canny and Lin's "Opportunistic Path Planner" (OPP) (Canny and Lin, 1990, 1993), which is itself based on Canny's Roadmap Algorithm (Canny, 1988). Now, the robot can perform coverage in curved, or even sampled, environments. See Fig. 7.

4. Algorithm Overview

The goal here is to determine a path that covers a bounded and connected component of the robot's free space. Without loss of generality, assume there is only one bounded and connected component of free space for the sake of explanation. Just like the trapezoidal method, a slice is swept through the bounded free space. Starting from "negative infinity," the first event occurs when the slice initially encounters the free space. This instantiates the formation of the first cell of the decomposition. The slice then continues to sweep throught the free space. Just like the trapezoidal decomposition, at an IN event, where the connectivity of the lice increases, the current cell is closed and two new cells are opened (Fig. 8). Conversely, at an OUT event, where the connectivity of the slice decreases, the two current cells are closed and one new cell is opened (Fig. 9). Finally, this procedure terminates when the slice leaves the bounded free space. The difference between the trapezoidal decomposition and boustrophedon decomposition approach is with the middle events: at the MIDDLE events, do not open nor close a cell, but rather simply update the current cell. Essentially, cells are opened and closed when there is a change in connectivity of slice. See Fig. 6.

As the decomposition is computed, the adjacency graph is also determined. Again, each cell is a node in the graph and an edge connects the nodes of adjacent cells. A depth-first-like graph search algorithm outputs a path list that represents an exhaustive walk through the adjacency graph. A walk through the path list constitutes an exhaustive walk through the adjacency graph.

Finally, the actual path for the robot to take is computed using the above described path list. When the robot enters an "uncleaned" cell, the boustrophedon motion is planned, and then a path to the next cell in the path list in planned. When the robot enters a "cleaned" cell, it simply plans a path through that cell to the next cell in the path list. These two actions are repeated until the end of the path list is reached, i.e., until each cell has been cleaned.

5. Experiments

We performed experiments on a Nomad base in a 300 square foot environment and then in a 800 square foot environment. Figure 10 contains a floor plan of the 300 square foot environment that has two obstacles. The obstacles are made out of cardboard but are drawn as black polygons in the Nomad display software. Figures 11 and 12 contains intermediate results of the floor coverage algorithm. In Fig. 11, two cells are already covered and in Fig. 12, all but two cells are covered. The final coverage result can be seen in Fig. 13.

It can be seen in Fig. 13 that this approach has some problems near obstacle edges that form acute angles with the boustrophedon back and forth paths. To partially alleviate this problem, when the robot travels through a cleaned cell, it travels near the boundary of an obstacle. Future implementation will include one additional pass where each obstacle is specially circumnavigated. For applications such as floor cleaning, this additional approach is reasonable because most floor cleaning applications require a different mechanism near the boundary of the environment.

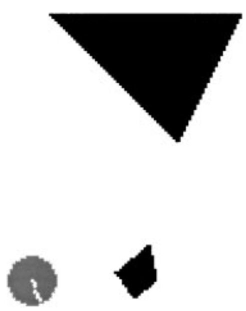

Figure 10. Floor plan bounded above and below by line segments. Black polygons are obstacles.

Figure 11. Two cells have been covered.

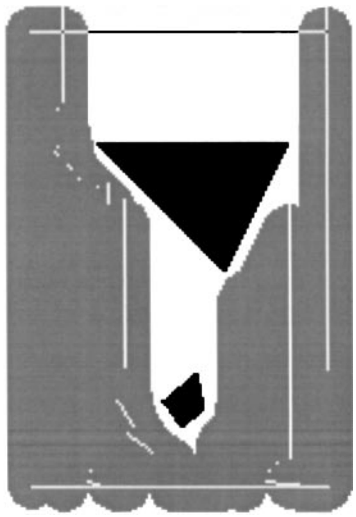

Figure 12. Coverage almost complete.

We then performed an experiment in a larger environment, 800 square feet, as depicted in Fig. 14. The light grey trace represents the path that the robot was commanded to follow, whereas the dark trace represents the actual path that the robot followed; the grey trace (command path) is drawn on top of the actual path. As can be seen in Fig. 14, the first cell (left-most cell) was covered fairly accurately, but by the time the robot starts to cover the third cell, it has accrued substantial dead-reckoning. In the fourth cell, the robot crashes into an obstacle because of dead-reckoning error.

This experiment indicates that even if full knowledge of the environment were available, the robot still must

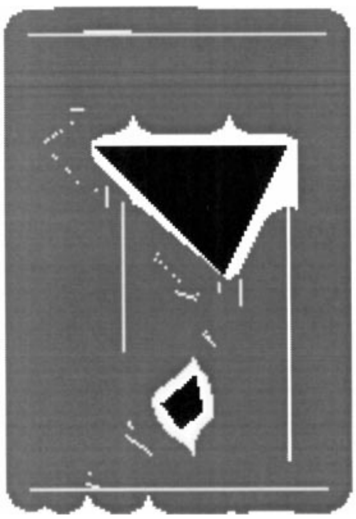

Figure 13. Complete coverage.

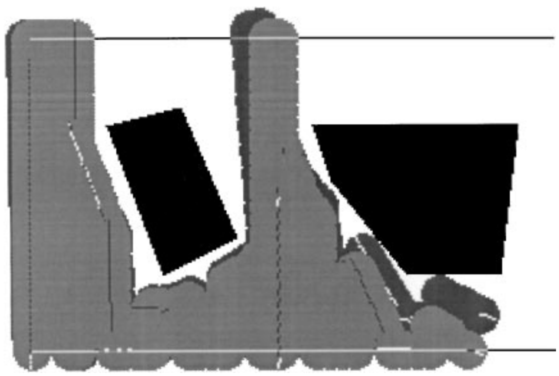

Figure 14. Localization error while covering.

use sensor information to guide the coverage operation. For example, this approach can benefit from the vision based techniques used in the Demeter project (Ollis and Stentz, 1996). Demeter can use the crop-line from the previous pass to guide is coverage path.

6. Conclusion and Future Work

This paper describes a new type of coverage algorithm that is complete. The term complete is used in the motion planning sense (Latombe, 1991), not in the operating research field sense. In other words, a robot, in theory, is guaranteed to follow a path such that every point in the environment is passed over by the robot. The algorithm is based on a new type of exact cellular decomposition approach termed the boustrophedon cellular decomposition. Boustrophedon means the way of the ox; boustrophedon motion is back and forth ox-like motions. A robot can easily plan a boustrophedon motion in each cell of the boustrophedon decomposition. Once each cell is covered, the entire environment is covered.

Experimental results on a mobile robot has validated this approach and pointed out some avenues of future work. This approach sometimes skips portions of the environment near the boundaries of obstacles because of the discretization of the side step. If the robot has a shorter side step, these uncovered areas are reduced. There is a tradeoff time/coverage here. Nevertheless, a simple obstacle following algorithm, after the main coverage is completed, will alleviate this problem. Such a solution is consistent with normal floor cleaning where the portion of the floor near the walls requires an additional pass.

An open issue in coverage path planning in general is determining an "optimal" path. At the very least, determining an optimal path is an NP-complete problem and may very well prove to be intractable even when restrictive assumptions are put in place. This is why we were relegated to a depth-first search of the adjacency graph. However, the path planning algorithm described in this paper can be employed to determine a "user-guided near-optimal path." We can vary parameters, such a sweep direction, and then apply a metric, such as path length, to each path produced by the boustrophedon decomposition. Using the algorithm described in this paper, the user can "try" different paths with little time and effort. By tuning parameters, the user can converge on a near-optimal path.

Near-term research also includes improving the boustrophedon cellular decomposition by allowing for the definition of bigger (and thus fewer) cells. For example, in Fig. 6 the bottom three cells can be merged into one cell in which boustrophedon motion can be planned. Furthermore, since a cell is open or closed when there is a change in the connectivity of the sweep line, this algorithm can be easily modified to environments with curved obstacles.

Also, there are issues in optimization that need to be considered. The first issue deals with the development of metrics which gauge heuristic graph searches of the adjacency graph. Such metrics include: path length, area of re-covered floor space, time, etc. Another optimization issue deals with determining the angle for the sweep-line; some environments may be better suited to a horizontal sweep line.

Another issue deals with material removal. In the case of the de-mining, this point is not important. However, in the case of snow removal, a snow removing robot may have to plan optimal paths to transfer the snow from the coverage site. This suggests the use of multiple robots: one robot to remove the snow from the ground, and another robot to transfer the snow to a central dumping zone.

The long-term goal of this work is in sensor based floor coverage, which is the determination of a coverage path from solely line of sight sensor information. Sensor based floor coverage is useful even when full knowledge of the worlds is available to the robot, but is too cumbersome to input into the robot. For example, an automatic de-mining robot would not be useful if each user had to program a model (if it existed) into the robot. The sensor based approach will be based on Rimon and Canny's roadmap work which uses critical points to guarantee the connectivity of a roadmap.

Acknowledgments

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