



Understanding Data

**Introduction to Digital Audio Processing
with Python**

Sound

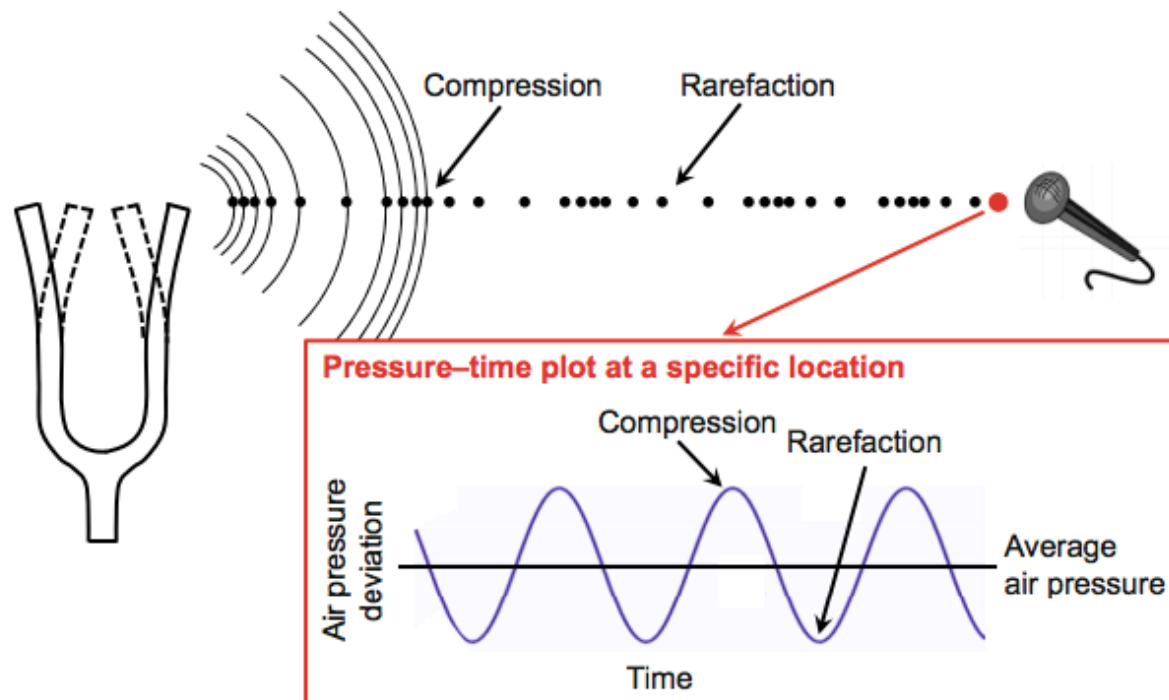
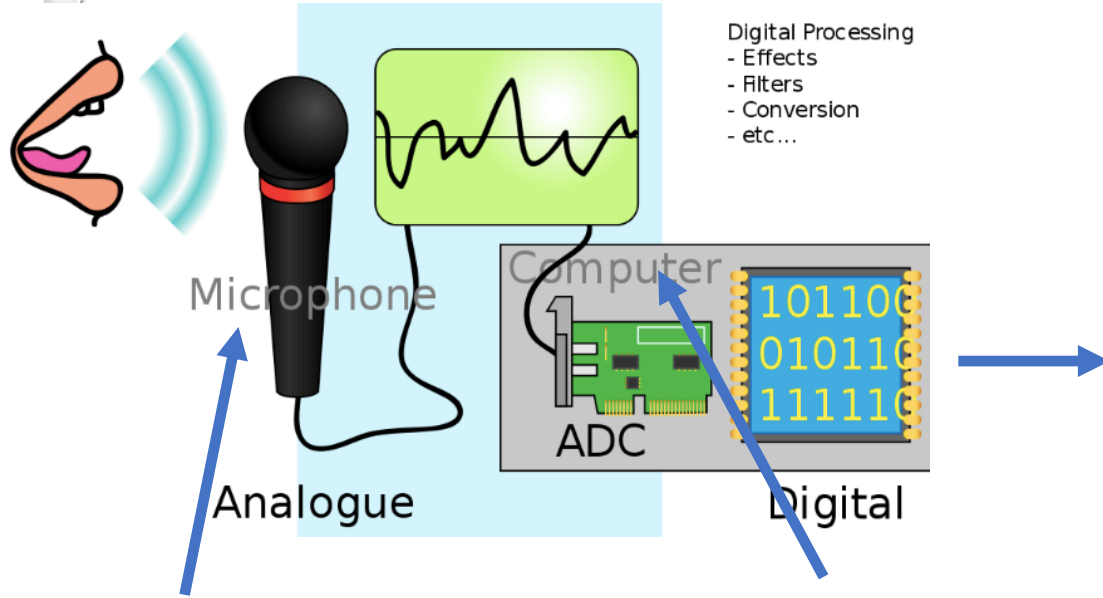


Figure taken from [Meinard Müller,
Fundamentals of Music Processing,
Figure 1.17, Springer 2015]

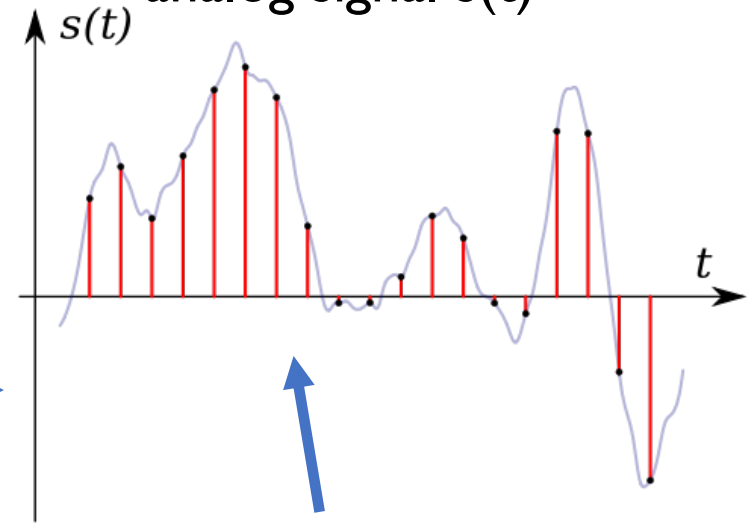
Analog to Digital Audio



The microphone converts acoustic energy to electrical energy.

Electrical analog signal is sampled and converted to an array of numbers via ADC (Analog to Digital Converter).

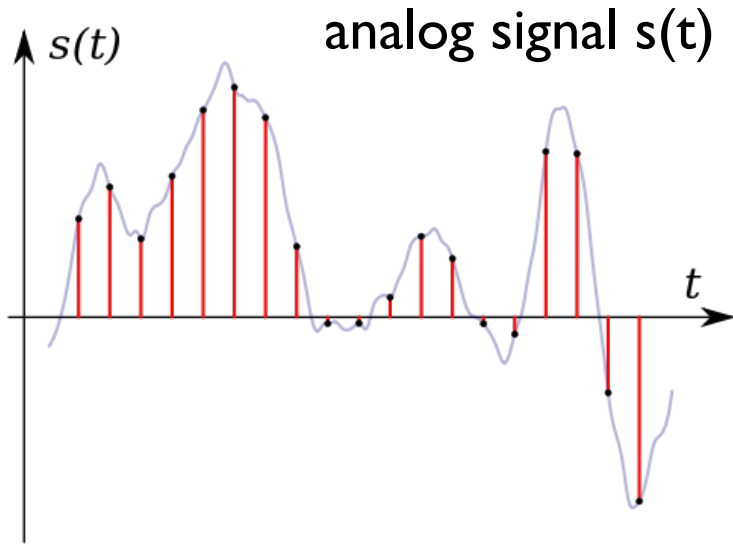
Sampling the analog signal $s(t)$



The array of sampled values from the electrical signal is **digital audio**.

Analog to Digital Audio

Sampling rate = 44,100 samples per second (CD quality)



analog signal $s(t)$



digital signal

$\{y_0, y_1, y_2, \dots, y_n\}$

array of 16-bit values (**bit depth**)
(1 bit for sign, 15 bits for value)

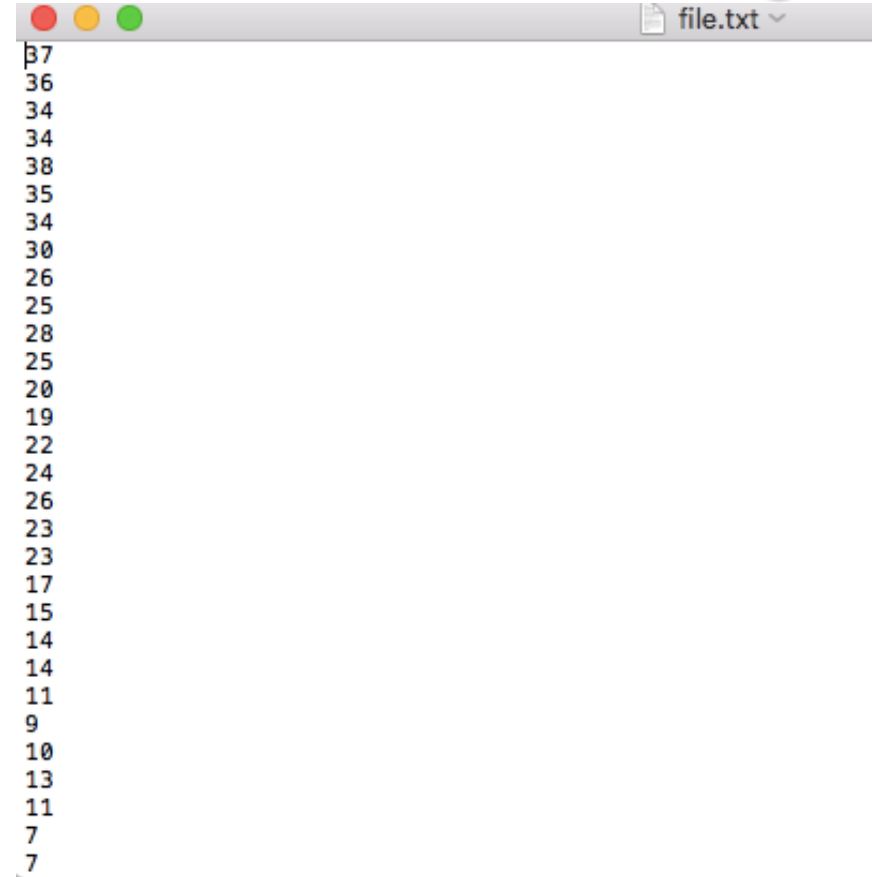
This allows for easy recording
(just stores a list of numbers) of
audio!

Play A List of Numbers as Audio

It is amazing that a digital sound file is simply an array or list of numbers!

The "file.txt" file on the right is a list of numbers sampled from an audio recording at the rate of 44,100 samples per second.

Let's load them using Python. Then send them to the speakers!



```
37
36
34
34
38
35
34
30
26
25
28
25
20
19
22
24
26
23
23
17
15
14
14
11
9
10
13
11
7
7
```

Play A List of Numbers as Audio

```
from IPython.display import Audio
import numpy as np
import matplotlib.pyplot as plt
ys = np.loadtxt("file.txt")
print(ys)
array([ 37.,  36.,  34., ..., 246., 262., 275.])

Audio(ys, rate = 44100)
```

The following code is
best run in a Jupyter
notebook.

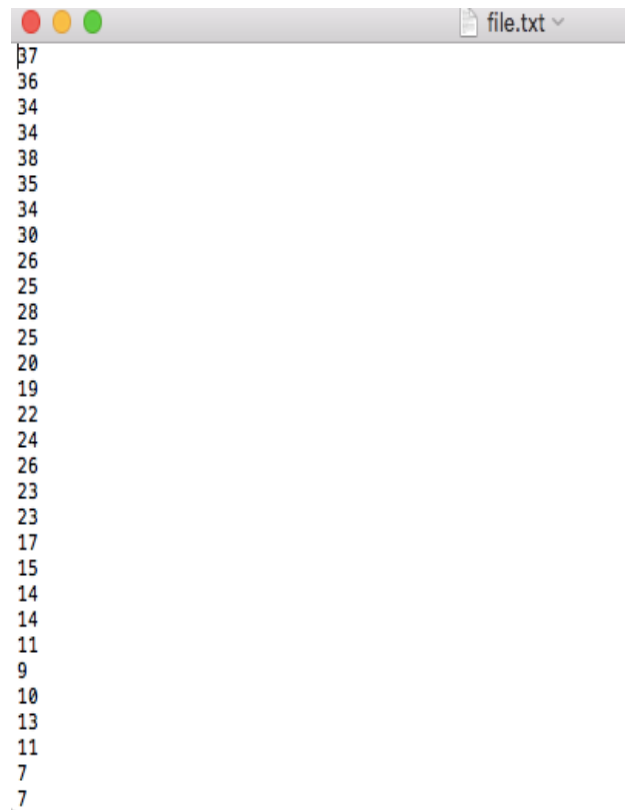
See the Jupyter Notebook Lab for this lecture to play the above
audio.

Play A List of Numbers as Audio

Physics behind the following explanation
is beyond the scope of this course.

The numbers here are proportional to the
instantaneous velocities of the speaker cone.
(Smith)

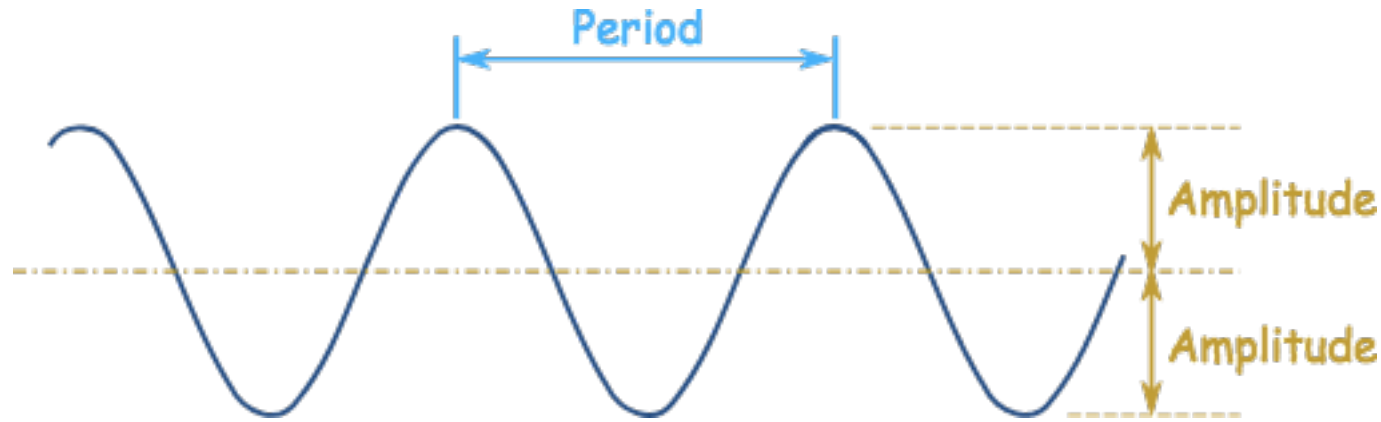
This means that your speaker has to move 44,100
times a second to faithfully reproduce this
piano note sound!



```
file.txt
37
36
34
34
38
35
34
30
26
25
28
25
20
19
22
24
26
23
23
17
15
14
14
11
9
10
13
11
7
7
```

Period, Amplitude and Frequency

- A signal that exhibits the same repeated pattern(cycle) is **periodic**.

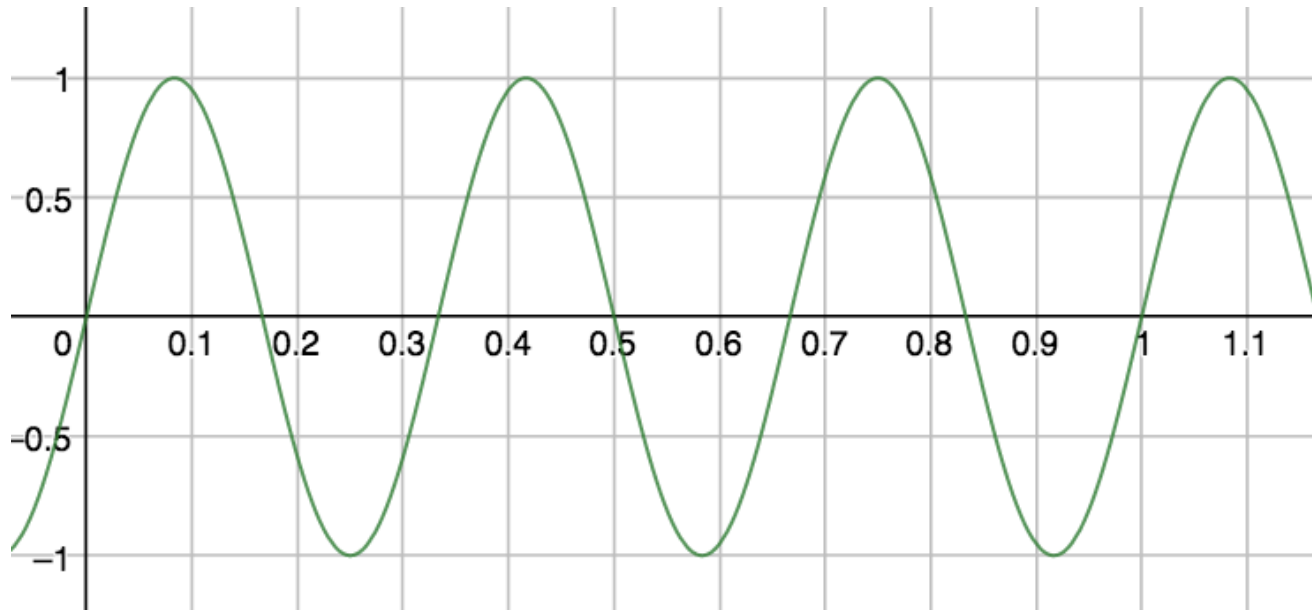


$$\text{period} = \frac{1}{\text{frequency}}$$

An Example

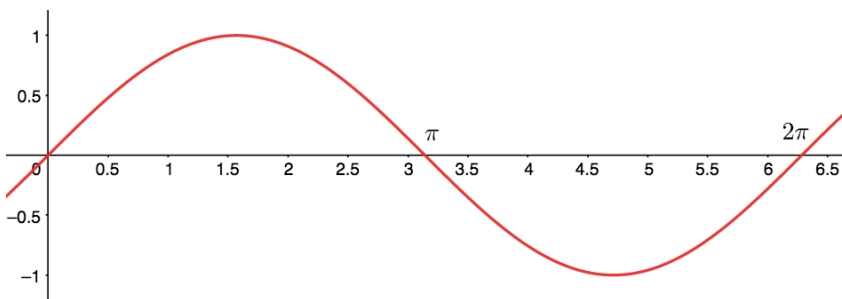
Period = $1/3$ sec per cycle

Frequency = 3 cycles per second

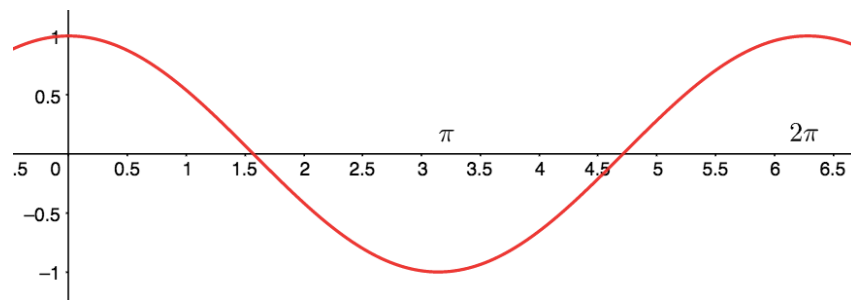


Sine and Cosine Wave

$$y = \sin(t).$$



$$y = \cos(t).$$



Sinusoids

- Both the $y = \sin(t)$ and $y = \cos(t)$ completes one cycle in 2π seconds:

$$y = \sin(t), t \in [0, 2\pi] \text{ (one cycle in } 2\pi \text{ seconds)}$$

$$y = \sin(2\pi t), t \in [0, 1] \text{ (one cycle in 1 second = 1 Hz)}$$

$$y = \sin(2\pi ft), t \in [0, 1] \text{ (f cycles in 1 second = f Hz)}$$

Real Sinusoids

In general, the sinusoids $y = a \sin(2\pi ft)$ and $y = a \cos(2\pi ft)$

has amplitude $= a$, frequency $= f$ in Hz,

period $= \frac{1}{f}$ in seconds,

Note: For simplicity, we'll ignore the phase of the sinusoids although the phase is an important attribute in digital audio/signal processing. Here's the full sinusoid with phase "phi":

$$y = a \sin(2\pi ft + \phi)$$

An Example

Suppose you have the signal $y = 3 \sin(2\pi 4t)$.

Find the amplitude, frequency and period.

amplitude = 3

frequency = 4 Hz

period = 1/4 sec

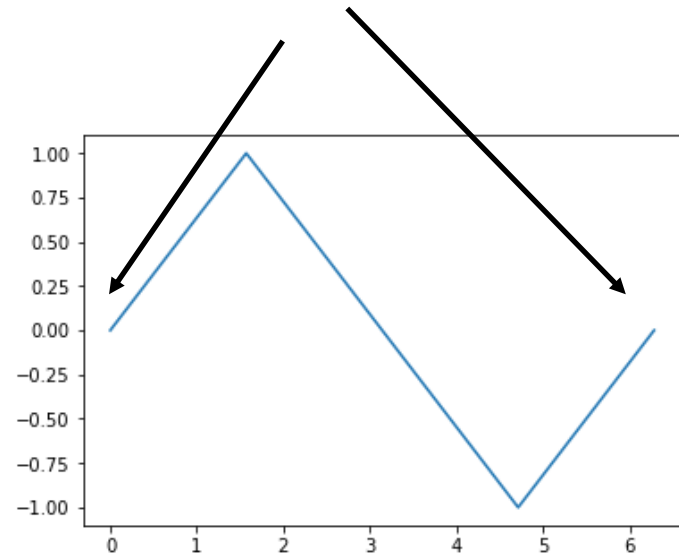
Plotting Functions

In the previous lectures, we used the Python and the matplotlib library to plot images. Now we will use it to plot functions.

Let's plot $y = \sin(t), t \in [0, 2\pi]$.

```
import matplotlib.pyplot as plt
# generate 5 equal-spaced
# samples in interval [0, 2*pi]
# more samples = better graph
ts = np.linspace(0, 2*np.pi, 5)
# apply the sin function to samples
ys = np.sin(ts)
# plot it
fig, ax = plt.subplots()
ax.plot(ts, ys)
```

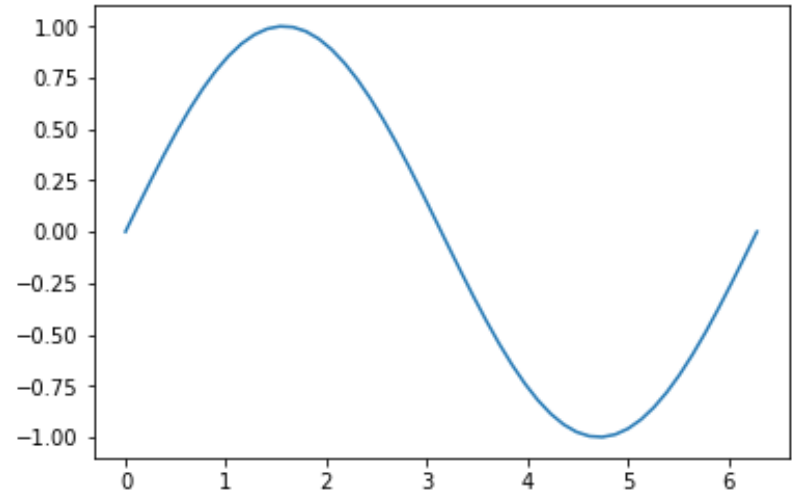
The five points include
the two endpoints.



More Samples

Increasing the number of samples generate a smoother graph. Below uses 50 equally-spaced samples.

```
import matplotlib.pyplot as plt  
ts = np.linspace(0, 2*np.pi, 50)  
ys = np.sin(ts)  
fig, ax = plt.subplots()  
ax.plot(ts, ys)
```

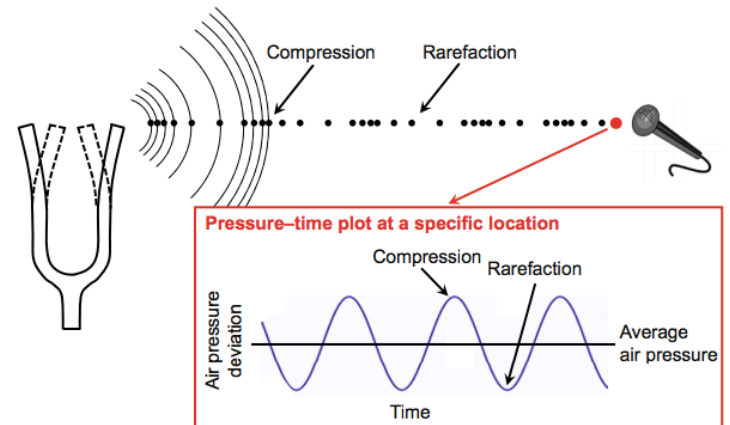


Middle C

- Middle C (or C4) has the frequency 261.6 Hz.

$$y(t) = \sin(2\pi \cdot 261.6 \cdot t), t \in [0, 2].$$

We'll simulate the above sound using Python in the lab for this lecture and will hear that it simulates the sound generated by a tuning fork.



C Major

The C major chord C-E-G has frequencies 261.6 Hz, 329.6 Hz, 392 Hz.

The analog signal for this chord is

$$y(t) = \sin(2\pi \cdot 261.6 \cdot t) + \sin(2\pi \cdot 329.6 \cdot t) + \sin(2\pi \cdot 392 \cdot t).$$

We will simulate these sounds in Python in the next lecture.

Loudness vs Pitch

- **Loudness** is our brain's perception of amplitude.

Pitch is our brain's perception of frequency.

- Our perception of pitch is based on the logarithm of frequency.
- The **interval** between two notes is the perceived difference between the two pitches.
- As a result, the interval we hear from two notes depends on the ratio of their frequencies, not the difference.

Fundamental Frequency and Overtones

pure tone = one frequency

Most sounds are more complex.

E4 piano note = supposition of many frequencies: 330 Hz, 660 Hz, 990 Hz, 1320 Hz and 1650 Hz.

Fundamental frequency = 330 Hz.

Overtones or **Harmonics** = 660 Hz, 990 Hz, 1320 Hz, 1650 Hz.

Time vs Frequency Domains

In the **time domain**, the voltage/current signal is a function of time.

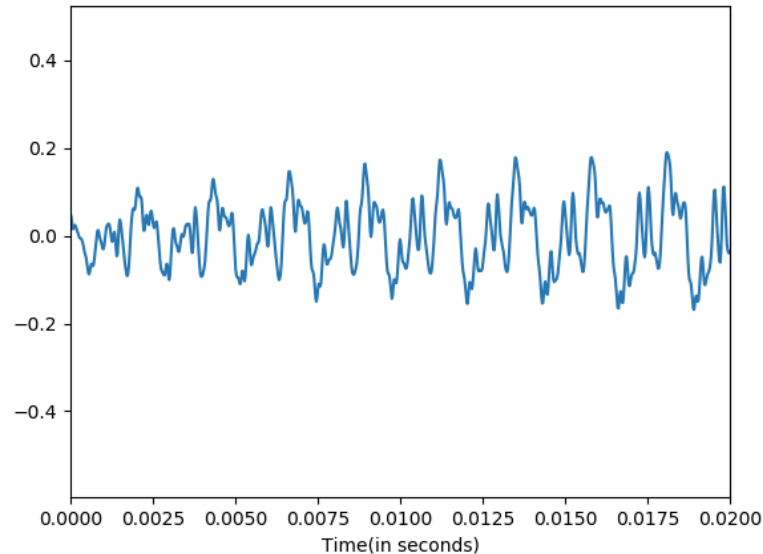
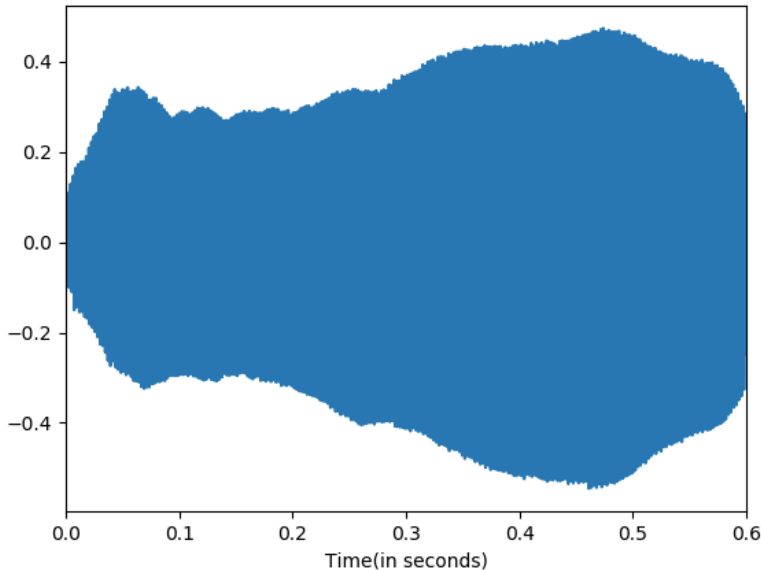
In the **frequency domain**, the signal is represented as a function of frequencies that are present in the signal.

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Time Domain

The signal in the time domain is very difficult to analyze. It is hard to know what sound is generated by the plot below.

The right plot is the zoomed in version of the left.



Frequency Domain

Converting the signal into its frequency domain allows us to understand its frequency content.

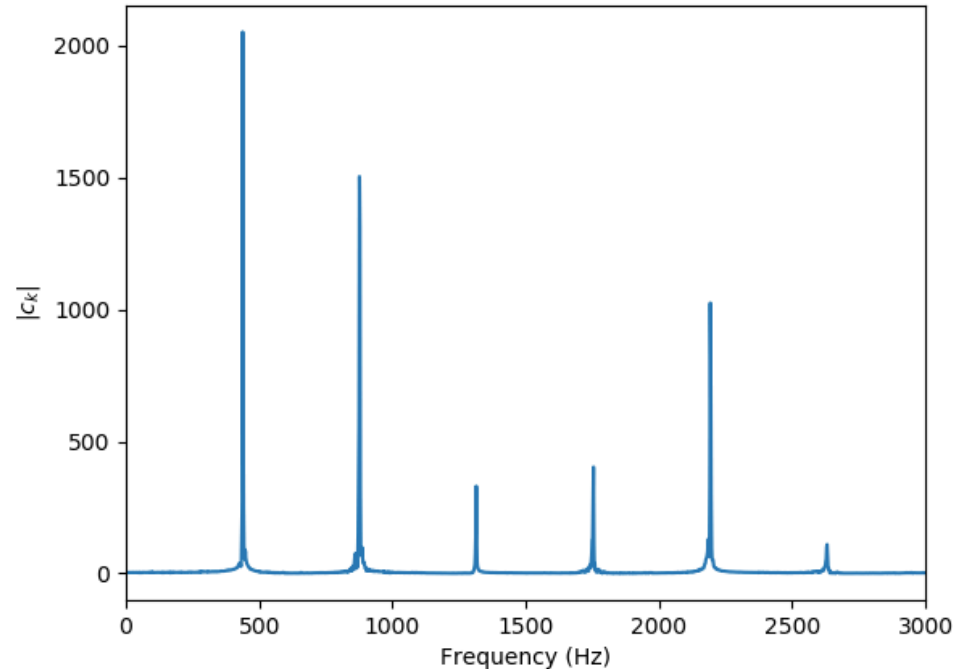
What chord is this?

440 Hz = A4, 880 Hz = A5,

1320 Hz = E6, 1760 Hz = A6,

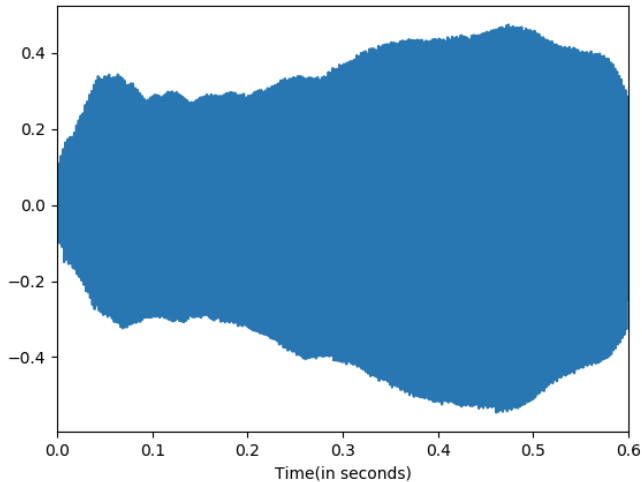
2200 Hz = C#7

The set of frequencies in a signal and their magnitudes is called the **spectrum** of the signal.



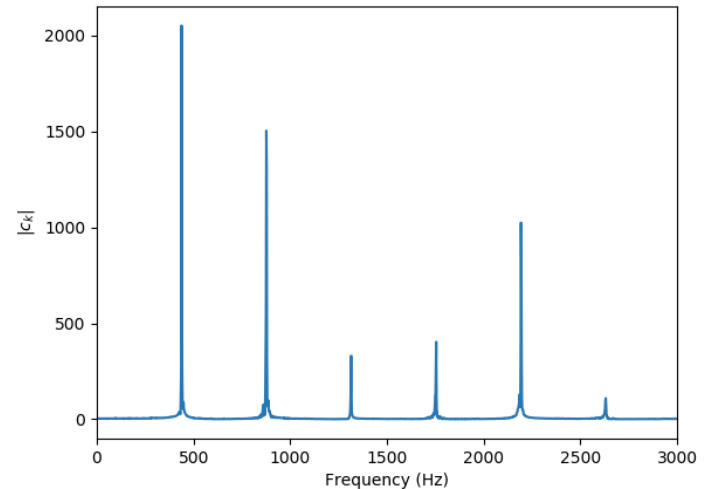
Discrete Fourier Transform (DFT)

The Discrete Fourier Transform is an equation that converts the time domain signal into its frequency domain equivalence. The Inverse Discrete Fourier Transform is its inverse.



Time Domain

DFT
→
←
IDFT



Frequency Domain

Spectrum Analyzers



Important Take Aways

- 1) Digital audio is simply sampling from a continuous, analog function at some sampling rate producing an array or list of numbers.
- 2) Sinusoids take on the form $y = a \sin(2\pi ft + \phi)$
- 3) Pitch is our brain's perception of frequency. (logarithm scale)
 - fundamental frequency vs harmonics
- 4) Time and Frequency domains are two equivalent representations of a signal.
- 5) The Discrete Fourier Transform and the Inverse Discrete Fourier Transform allows us to go back and forth between representations.

References

- 1) Müller, Meinard, Fundamentals of Music Processing, Springer 2015.
- 2) Downey, Allen, ThinkDSP, Green Tea Press 2012.
- 3) Smith, Julius, The Mathematics of the Discrete Fourier Transform, W3K Publishing 2007.
- 4) Loy, Gareth, Musimathics, Volumes 1 and 2. The MIT Press 2011.
- 5) Newman, Mark, Computational Physics, Createspace Independent Publishing Platform 2012.
- 6) Soklaski, Ryan. MIT Lincoln Lab Researcher. Beaver Works Summer Institute.