

Lecture 22: Number Systems

Adapted from York University Intro to IT Slides.

Base 10 (Decimal numbers)

Why base 10?

We *happened* to use the current counting system, because we happened to have ten fingers.

Base 10(Decimal) uses 10 digits {0,1,2,3...,9}.

Each Digit in a Number Has a Weight

you multiply the weight times the digit, then add them up

decimal (base 10): each digit has a weight that is a power of 10

1000	100	10	1	weight
3	2	0	7	

3 x 1000 + 2 x 100 + 0 x 10 + 7 x 1

Or, $3000 + 200 + 0 + 7 = 3207$

Powers of 10

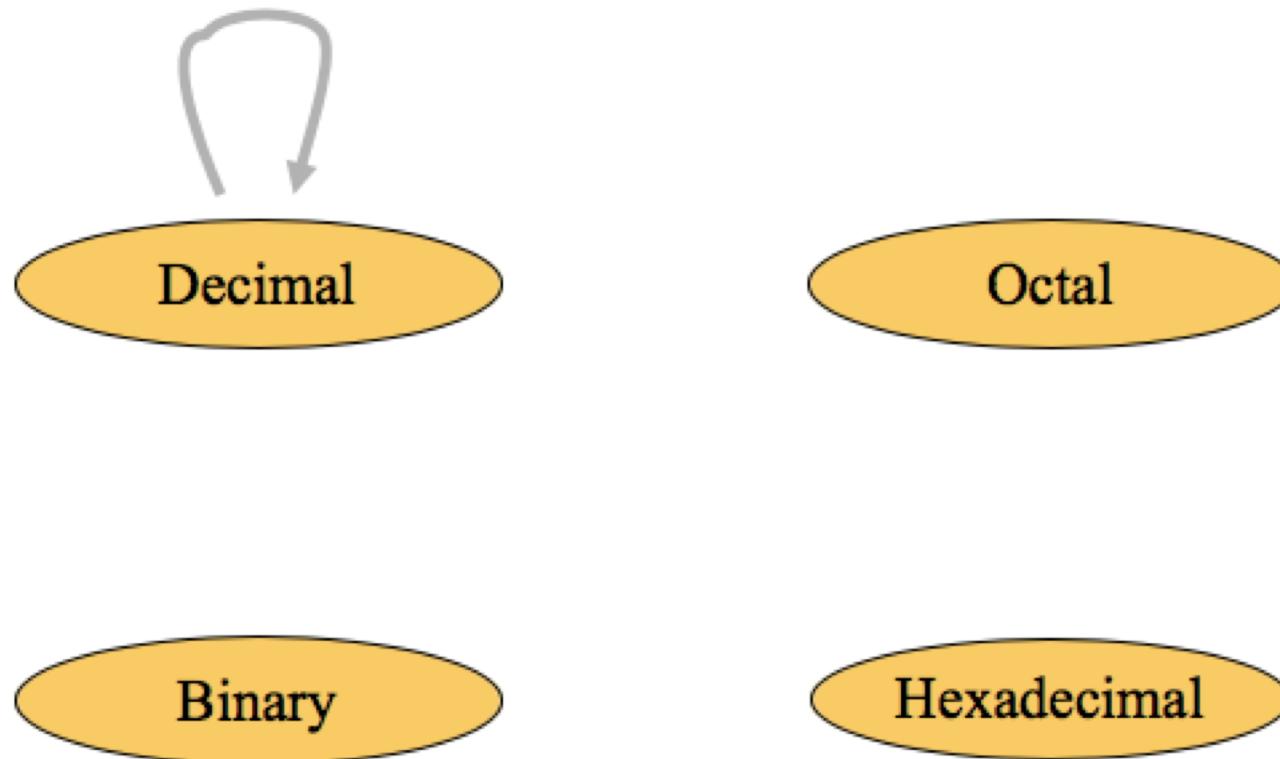
$$10^0 = 1$$

$$10^1 = 10$$

$$10^2 = 100$$

$$10^3 = 1000$$

Decimal to Decimal (just for fun)

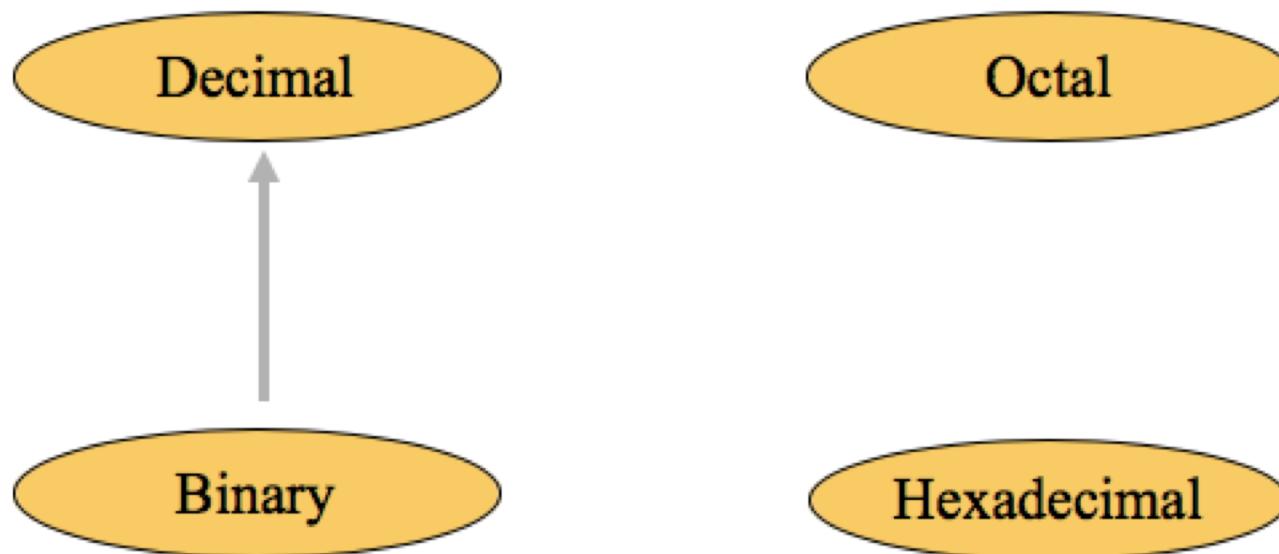


Weight

$$\begin{array}{rcl} 125_{10} \Rightarrow & 5 \times 10^0 & = 5 \\ & 2 \times 10^1 & = 20 \\ & 1 \times 10^2 & = \frac{100}{125} \end{array}$$

Base

Binary to Decimal



Binary to Decimal

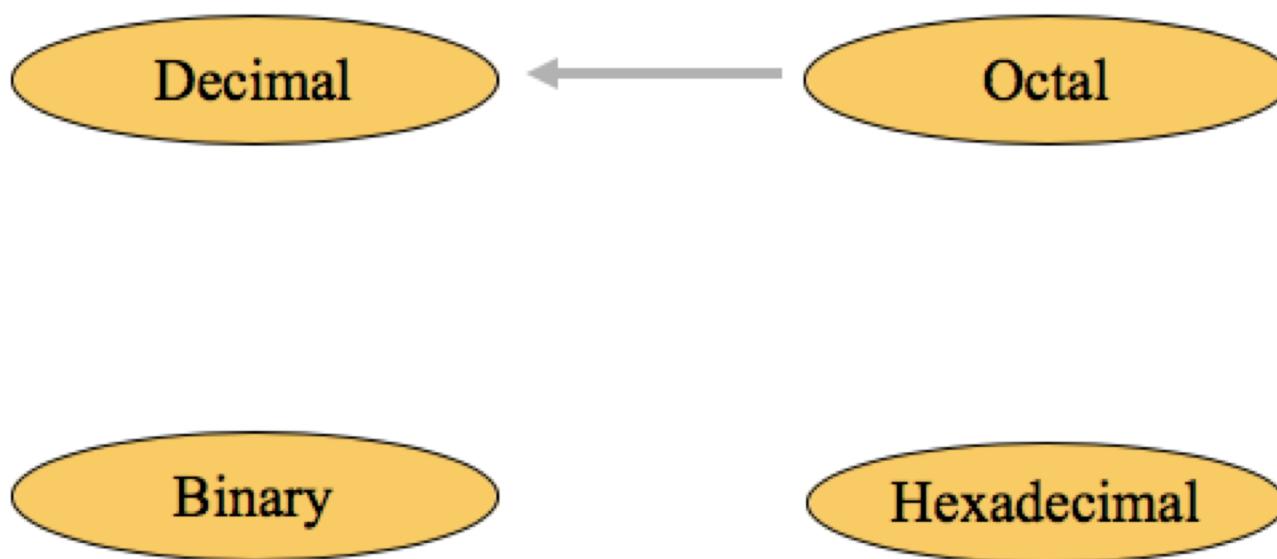
- Technique
 - Multiply each bit by 2^n , where n is the “weight” of the bit
 - The weight is the position of the bit, starting from 0 on the right
 - Add the results

Example

Bit “0”

$$\begin{array}{r} 101011_2 \Rightarrow \\ 1 \times 2^0 = 1 \\ 1 \times 2^1 = 2 \\ 0 \times 2^2 = 0 \\ 1 \times 2^3 = 8 \\ 0 \times 2^4 = 0 \\ 1 \times 2^5 = 32 \\ \hline 43_{10} \end{array}$$

Octal to Decimal



Octal to Decimal

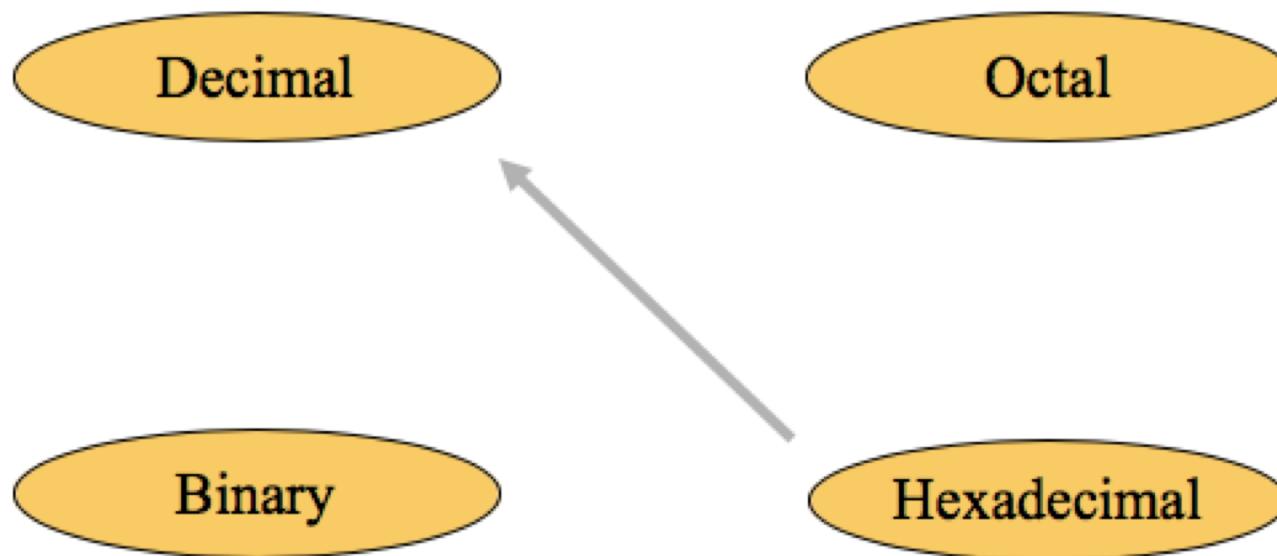
- Octal={0,1,2...,7}
- Technique
 - Multiply each bit by 8^n , where n is the “weight” of the bit
 - The weight is the position of the bit, starting from 0 on the right
 - Add the results

Example

$$\begin{array}{rcl} 724_8 &=>& 4 \times 8^0 = 4 \\ && 2 \times 8^1 = 16 \\ && 7 \times 8^2 = \frac{448}{468_{10}} \end{array}$$

Example

Hexadecimal to Decimal



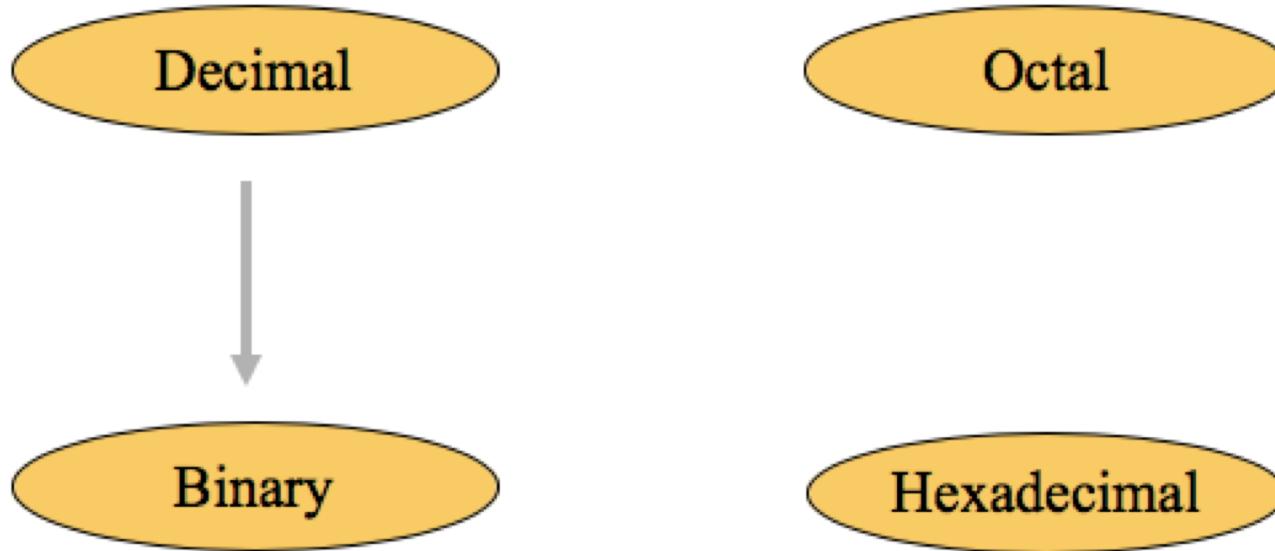
Hexadecimal to Decimal

- Hexadecimal = {0,...,9,A=10,B=11,...F=15}
- Technique
 - Multiply each bit by 16^n , where n is the “weight” of the bit
 - The weight is the position of the bit, starting from 0 on the right
 - Add the results

Example

$$\begin{array}{rcl} \text{ABC}_{16} \Rightarrow & C \times 16^0 = 12 \times 1 = 12 \\ & B \times 16^1 = 11 \times 16 = 176 \\ & A \times 16^2 = 10 \times 256 = \underline{2560} \\ & & 2748_{10} \end{array}$$

Decimal to Binary



Decimal to Binary

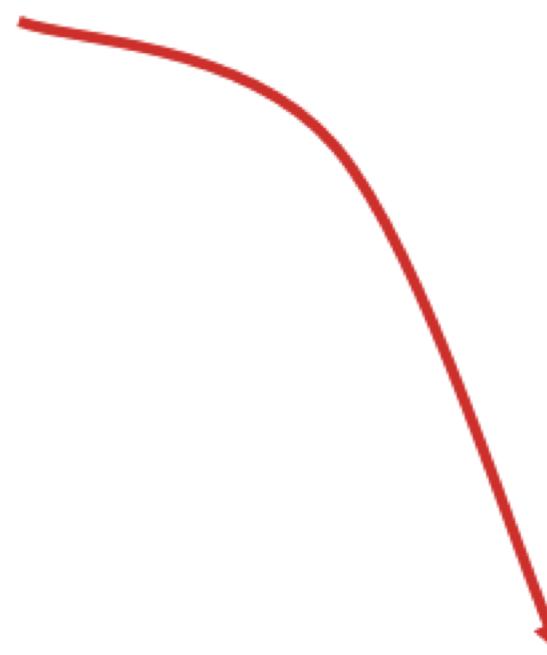
- Technique
 - Divide by two, keep track of the remainder
 - First remainder is bit 0 (LSB, least-significant bit)
 - Second remainder is bit 1
 - Etc.

Example

$$125_{10} = ?_2$$

$$\begin{array}{r} 2 \mid 125 \\ 2 \mid 62 \quad 1 \\ 2 \mid 31 \quad 0 \\ 2 \mid 15 \quad 1 \\ 2 \mid 7 \quad 1 \\ 2 \mid 3 \quad 1 \\ 2 \mid 1 \quad 1 \\ \hline 0 \quad 1 \end{array}$$

$$125_{10} = 1111101_2$$



Decimal to Hexadecimal

- Technique
 - Divide by 16
 - Keep track of the remainder

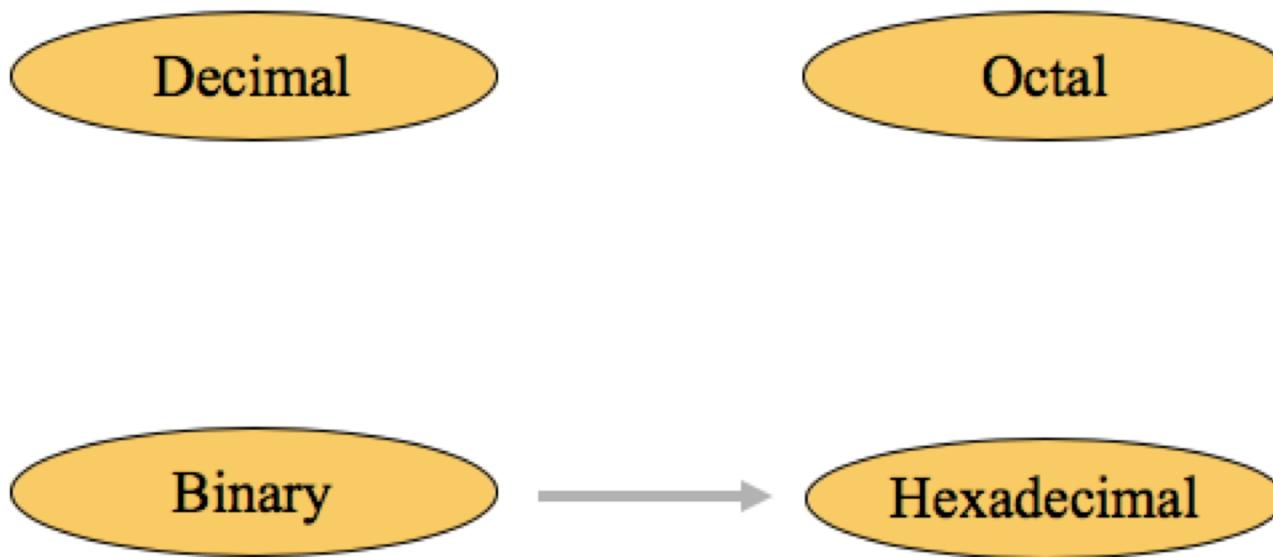
Example

$$1234_{10} = ?_{16}$$

$$\begin{array}{r} 16 \longdiv{1234} \\ 16 \longdiv{77} \quad 2 \\ 16 \longdiv{4} \quad 13 = D \\ \hline 0 \quad 4 \end{array}$$

$$1234_{10} = 4D2_{16}$$

Binary to Hexadecimal



Binary to Hexadecimal

- Technique
 - Group bits in fours, starting on right
 - Convert to hexadecimal digits

Example

$$1010111011_2 = ?_{16}$$

10 1011 1011
↓ ↓ ↓
2 B B

$$1010111011_2 = 2BB_{16}$$

Bits and Bytes

- 1 **bit** is a single bit of information, a 1 or 0
 - Only two possible values
- 1 **byte** is 8 bits, an 8 bit word
 - 256 possible values from 0-255 **base 10** or 00000000 to 11111111 **base 2**
- 10100110 is a single byte

Bits and Bytes

- Java integer type int uses 32 bits.
- Its maximum value is $2^{31} - 1$.
- Its minimum value is -2^{31} .
- We can access these values through the Integer class.

```
public static void main(String[] arg) {  
    System.out.println(Integer.MAX_VALUE);  
    System.out.println(Integer.MIN_VALUE);  
}
```

Output:

2147483647
-2147483648

Counting

Let's count in Base 10.

0,1,2,3,4,5,6,7,8,9,_____,_____,...,18,19,_____,_____.

Answer: 10,11,20,21. Impressive...

Let's count in Base 5.

0,1,2,3,4,_____,_____,_____,_____,_____,_____.

Answer:10,11,12,13,14,20.

Let's keep going!

40,41,42,43,44,_____,_____,_____.

Answer:100,101,102.

Adding Binary

- $0+0=0$
- $1+0=1$
- $1+1=10$

$$\begin{array}{r} 11001011 \\ + 11100110 \\ \hline 110110001 \end{array}$$

Quick Quiz

What's the answer in base 8?

$$6_{10} + 3_{10} = 11_8$$

What's the answer in base 5?

$$2_{10} + 6_{10} = 13_5$$

What's the answer in base 12?

$$9_{10} + 15_{10} = 20_{12}$$

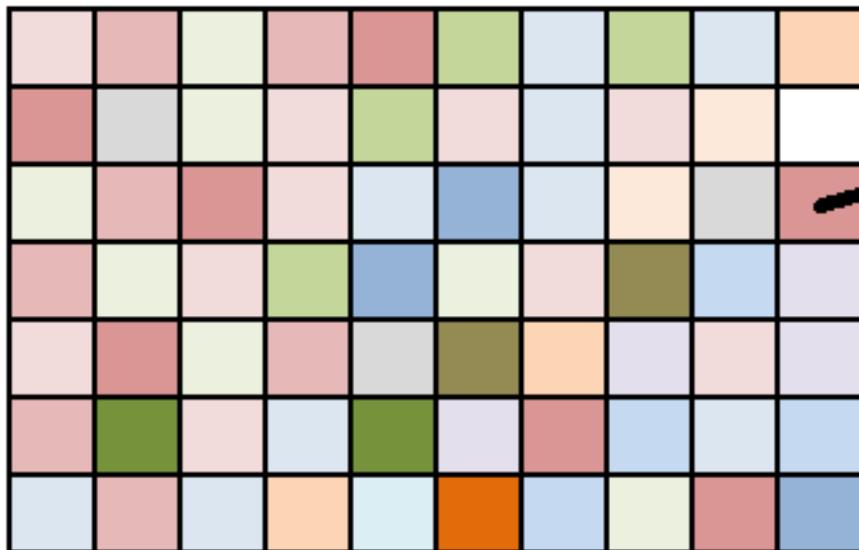
People

There are 10 types of people in the world; those who understand binary and those who don't.

There are 10 types of people in the world; those who understand binary and those who have friends.

Encoding Data in Binary

24-bit image



RGB (218, 150, 149)

R = 11011010
G = 10010110
B = 10010101

What is an image? an audio file? a movie?
It is simply an array of 0s and 1s!