Lecture 22: Number Systems

Adapted from York University Intro to IT Slides.

Base 10 (Decimal numbers)

Why base 10?

We *happened* to use the current counting system, because we happened to have ten fingers.

Base 10(Decimal) uses 10 digits {0,1,2,3…,9}.

Each Digit in a Number Has a Weight

you multiply the weight times the digit, then add them up

decimal (base 10): each digit has a weight that is a power of 10

1000	100	10	1	weight
3	**2**	**0**	**7**	

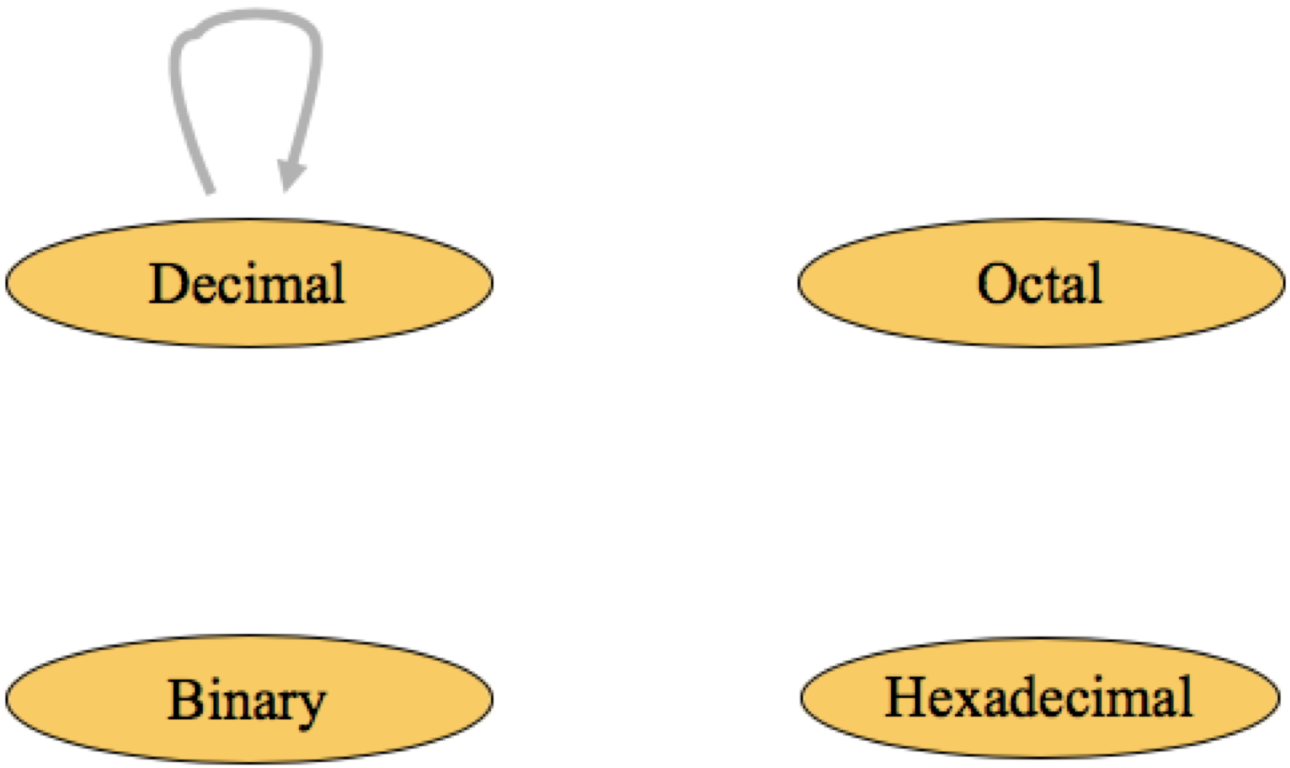
3 x1000 + 2x100 + 0x10 + 7x1

Or, 3000 + 200 + 0 + 7 = 3207

Powers of 10

$10^0 = 1$
$10^1 = 10$
$10^2 = 100$
$10^3 = 1000$

Decimal to Decimal (just for fun)

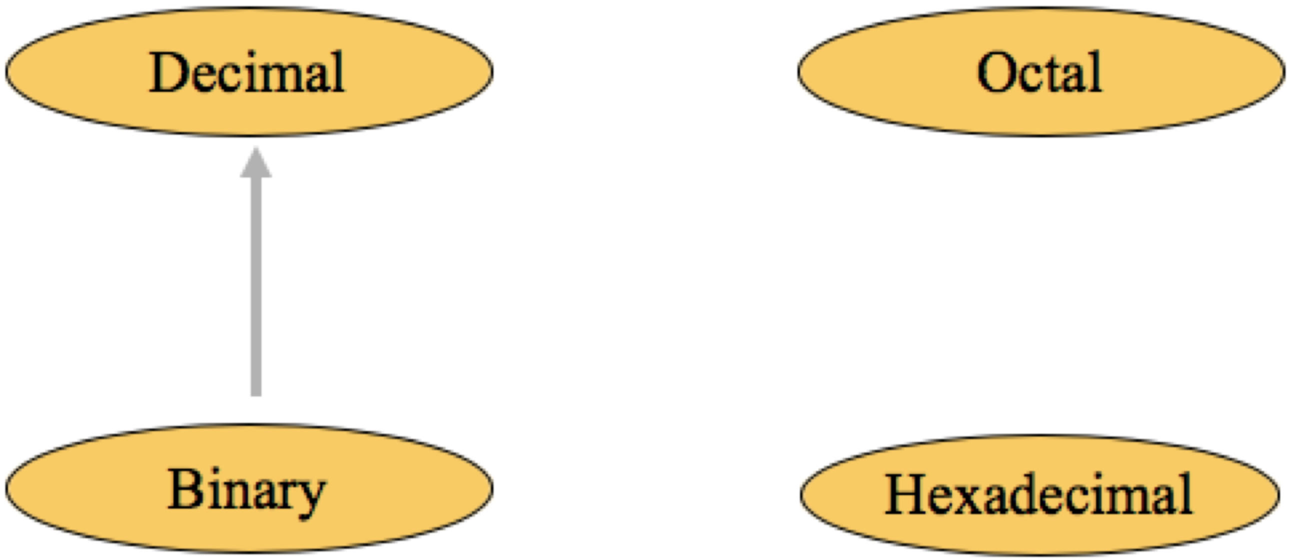


Weight

$$125_{10} \Rightarrow \begin{array}{rcr} 5 \times 10^{0} & = & 5 \\ 2 \times 10^{1} & = & 20 \\ 1 \times 10^{2} & = & \underline{100} \\ & & 125 \end{array}$$

Base

Binary to Decimal



Binary to Decimal

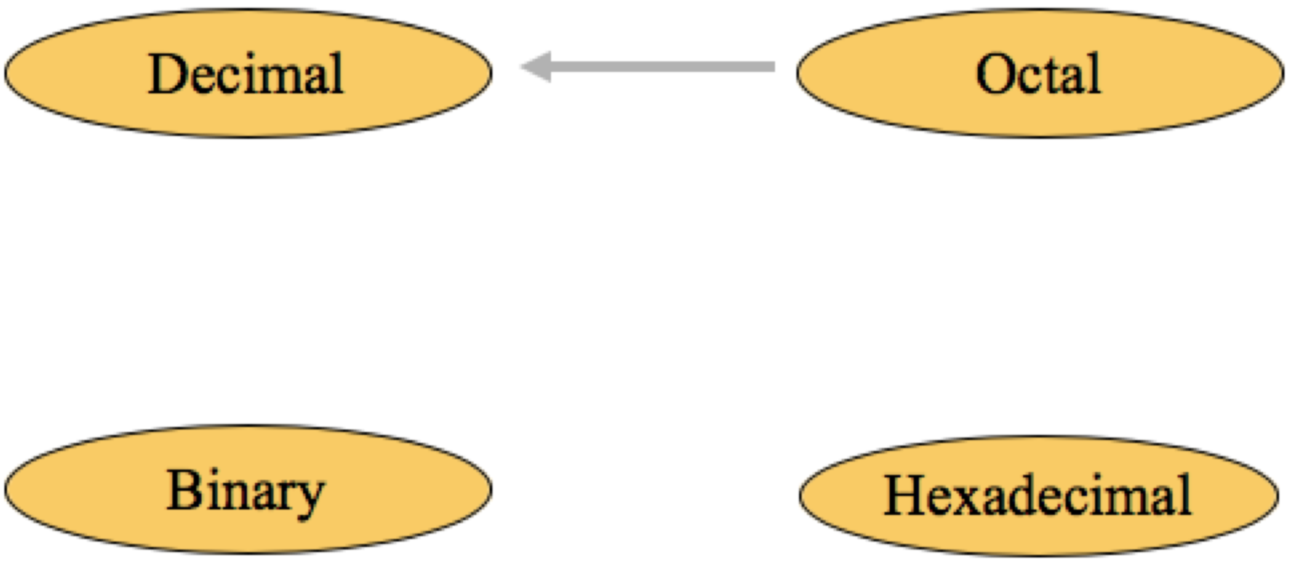
- Technique
  - Multiply each bit by 2^n, where n is the "weight" of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

Example

Bit "0"

$$101011_2 \Rightarrow \begin{array}{rcl} 1 \times 2^0 & = & 1 \\ 1 \times 2^1 & = & 2 \\ 0 \times 2^2 & = & 0 \\ 1 \times 2^3 & = & 8 \\ 0 \times 2^4 & = & 0 \\ 1 \times 2^5 & = & 32 \\ \hline & & 43_{10} \end{array}$$

Octal to Decimal



Octal to Decimal

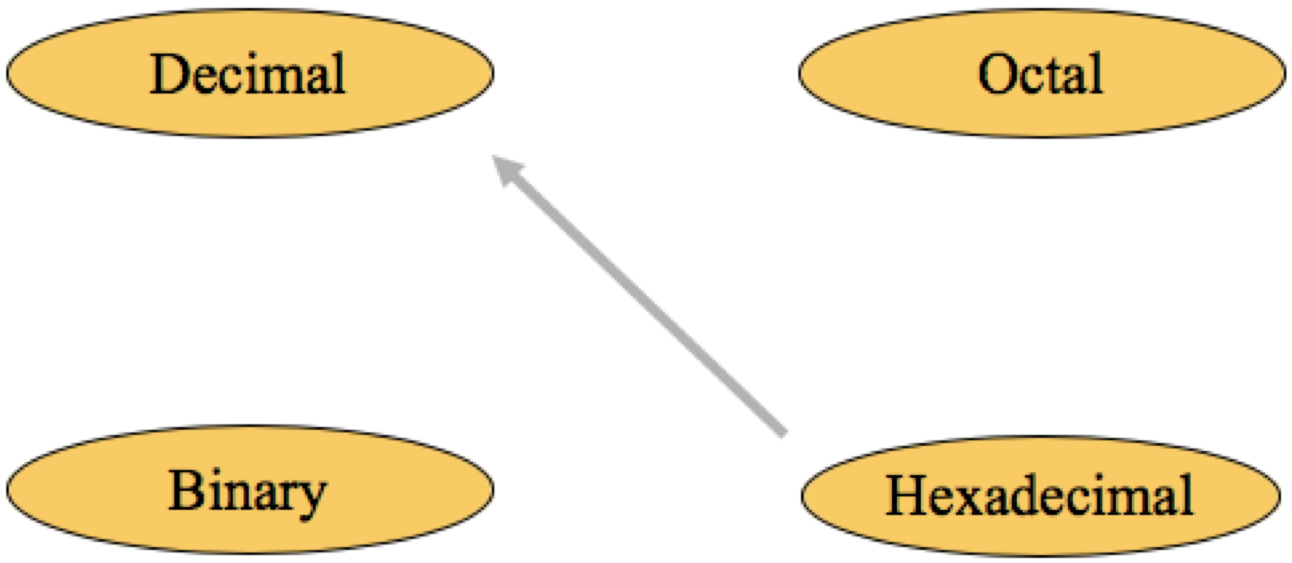
- Octal={0,1,2…,7}
- Technique
  - Multiply each bit by 8^n, where n is the "weight" of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

Example

$$
\begin{array}{lrcl}
724_8 \Rightarrow & 4 \times 8^0 & = & 4 \\
& 2 \times 8^1 & = & 16 \\
& 7 \times 8^2 & = & \underline{448} \\
& & & 468_{10}
\end{array}
$$

Example

Hexadecimal to Decimal



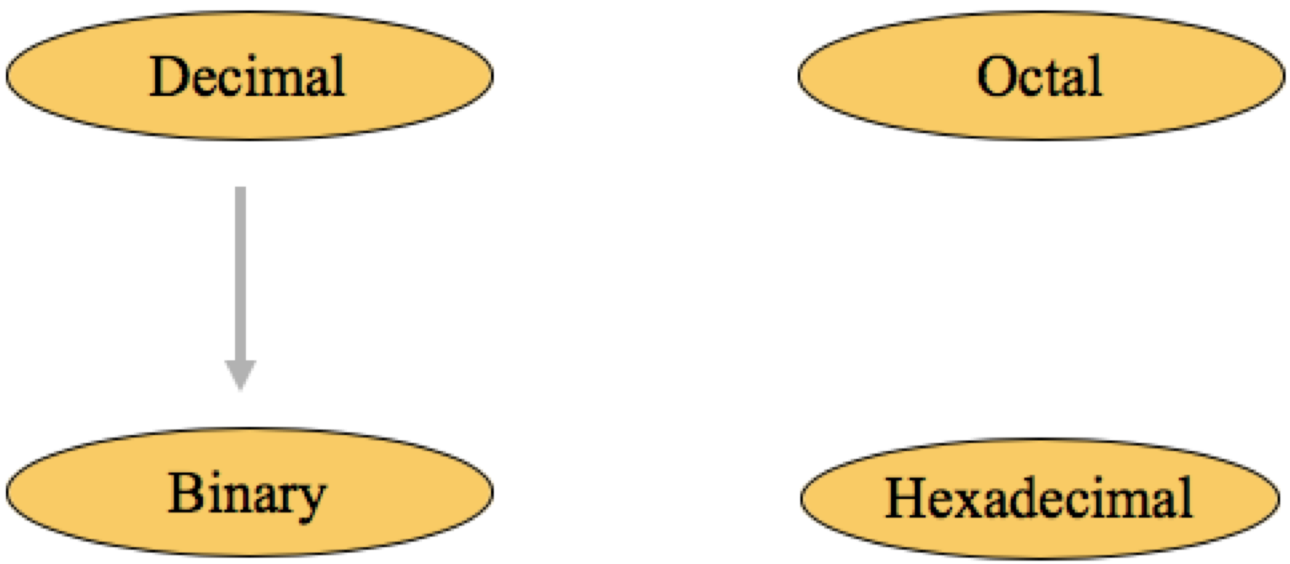
Hexadecimal to Decimal

- Hexadecimal={0,…,9,A=10,B=11,…F=15}
- Technique
  - Multiply each bit by 16^n, where n is the "weight" of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

Example

$$
\begin{array}{llll}
ABC_{16} \Rightarrow & C \times 16^0 = 12 \times 1 = & 12 \\
 & B \times 16^1 = 11 \times 16 = & 176 \\
 & A \times 16^2 = 10 \times 256 = & 2560 \\
\hline
 & & 2748_{10}
\end{array}
$$

Decimal to Binary



Decimal to Binary

- Technique
  - Divide by two, keep track of the remainder
  - First remainder is bit 0 (LSB, least-significant bit)
  - Second remainder is bit 1
  - Etc.

Example

$125_{10} = ?_2$

2	125	
2	62	1
2	31	0
2	15	1
2	7	1
2	3	1
2	1	1
	0	1

$125_{10} = 1111101_2$

Decimal to Hexadecimal

- Technique
  - Divide by 16
  - Keep track of the remainder

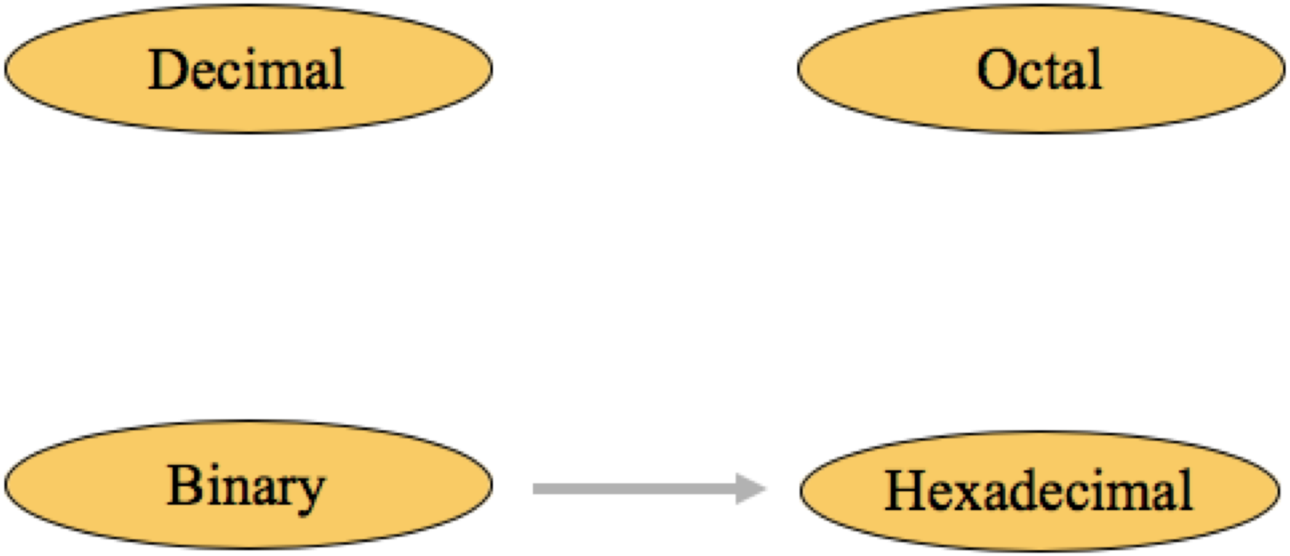
Example

$1234_{10} = ?_{16}$

16	1234	
16	77	2
16	4	13 = D
	0	4

$1234_{10} = 4D2_{16}$

Binary to Hexadecimal



Binary to Hexadecimal

- Technique
  - Group bits in fours, starting on right
  - Convert to hexadecimal digits

Example

$1010111011_2 = ?_{16}$

10 1011 1011

2 B B

$1010111011_2 = 2BB_{16}$

Bits and Bytes

- 1 **bit** is a single bit of information, a 1 or 0
  - Only two possible values

- 1 **byte** is 8 bits, an 8 bit word
  - 256 possible values from 0-255 **base 10** or 00000000 to 11111111 **base 2**

- 10100110 is a single byte

Bits and Bytes

- Java integer type int uses 32 bits.
- Its maximum value is 2^31 – 1.
- Its minimum value is -2^31.
- We can access these values through the Integer class.

```
public static void main(String[] arg) {
      System.out.println(Integer.MAX_VALUE);
      System.out.println(Integer.MIN_VALUE);
}
```

Output:

2147483647

-2147483648

Counting

Let’s count in Base 10.

0,1,2,3,4,5,6,7,8,9,__,___,…,18,19,__,__.

Answer: 10,11,20,21. Impressive…

Let’s count in Base 5.

0,1,2,3,4,__,__,__,__,__,___.

Answer:10,11,12,13,14,20.

Let’s keep going!

40,41,42,43,44,___,___,___.

Answer:100,101,102.

Adding Binary

- 0+0=0
- 1+0=1
- 1+1=10

```
   11001011
+  11100110
-----------
  110110001
```

Quick Quiz

What's the answer in base 8?

$$6_{10} + 3_{10} = 11_8$$

What's the answer in base 5?

$$2_{10} + 6_{10} = 13_5$$

What's the answer in base 12?

$$9_{10} + 15_{10} = 20_{12}$$

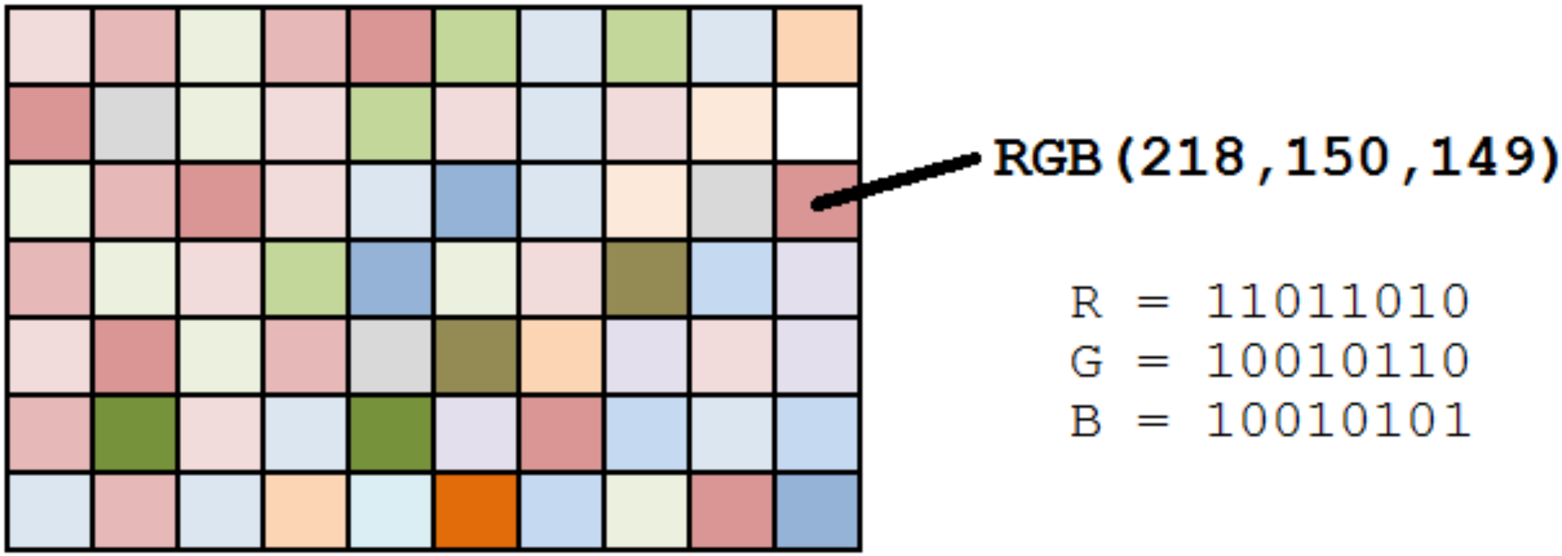
People

There are 10 types of people in the world; those who understand binary and those who don’t.

There are 10 types of people in the world; those who understand binary and those who have friends.

Encoding Data in Binary

24-bit image



What is an image? an audio file? a movie?
It is simply an array of 0s and 1s!

Homework

1) Complete the number system worksheet.

2) Write the method decimalToBinary which accepts an integer and returns the string representation of the decimal integer in binary.

```
public static String decimalToBinary(int n){



}
```