```
R version 3.5.0 (2018-04-23) -- "Joy in Playing"
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Platform: x86_64-w64-mingw32/x64 (64-bit)

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  Natural language support but running in an English locale

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[Previously saved workspace restored]

> #####portfolio c3 setting 1
> #####Consider porfolios on derivatives based on 10 underlying uncorrelated assets
> #####investigate the loss probability, which is critical to estimating VAR
> install.packages("rootSolve")
Installing package into ‘C:/Users/s1155058334/Documents/R/win-library/3.5’
(as ‘lib’ is unspecified)
--- Please select a CRAN mirror for use in this session ---
trying URL 'https://mirror-hk.koddos.net/CRAN/bin/windows/contrib/3.5/rootSolve_1.7.zip'
Content type 'application/zip' length 787735 bytes (769 KB)
downloaded 769 KB

package ‘rootSolve’ successfully unpacked and MD5 sums checked

The downloaded binary packages are in
        C:\Users\s1155058334\AppData\Local\Temp\RtmpWCuezf\downloaded_packages
> library(rootSolve)
Warning message:
package ‘rootSolve’ was built under R version 3.5.2
> install.packages("gtools")
Installing package into ‘C:/Users/s1155058334/Documents/R/win-library/3.5’
(as ‘lib’ is unspecified)
trying URL 'https://mirror-hk.koddos.net/CRAN/bin/windows/contrib/3.5/gtools_3.8.1.zip'
Content type 'application/zip' length 325812 bytes (318 KB)
downloaded 318 KB

package ‘gtools’ successfully unpacked and MD5 sums checked

The downloaded binary packages are in
        C:\Users\s1155058334\AppData\Local\Temp\RtmpWCuezf\downloaded_packages
> library(gtools)
Warning message:
package ‘gtools’ was built under R version 3.5.2
> install.packages("Matrix")
Installing package into ‘C:/Users/s1155058334/Documents/R/win-library/3.5’
(as ‘lib’ is unspecified)
trying URL 'https://mirror-hk.koddos.net/CRAN/bin/windows/contrib/3.5/Matrix_1.2-17.zip'
Content type 'application/zip' length 4475329 bytes (4.3 MB)
downloaded 4.3 MB

package ‘Matrix’ successfully unpacked and MD5 sums checked

The downloaded binary packages are in
        C:\Users\s1155058334\AppData\Local\Temp\RtmpWCuezf\downloaded_packages
> library(Matrix)
Warning message:
package ‘Matrix’ was built under R version 3.5.3
> rm(list=ls())
>
> set.seed(1000)
> ##consider exchange option that exchange the ith asset FOR the (i+5)th
> S0<-rep(100,10)
> ##the initial price of the five five assets
> U0<-S0[1:5]
```

```
> ##the initial price of the last five assets
> V0<-S0[6:10]
> T<-0.1
> sigma<-0.3
> r<-0.05
> K<-S0
> dt<-0.04
> ####################################################################################
> #since V0 is occupied herem, we name value of of the portfolio at time 0 as Value0#
> ####################################################################################
>
> #####define a function for calculating the value of a unit of exchange option (long position
), under risk neutral framework
> EXVU<-function(V,U,T,t,sigma,r){
+ d1<-(log(V/U)+(sigma^2)*(T-t))/sqrt(2)/sigma/sqrt(T-t)
+ d2<-d1-sqrt(2)*sigma*sqrt(T-t)
+ exvu<-V*pnorm(d1)-U*pnorm(d2)
+ return(exvu)
+ }
> #####All the partial derivatives, thus the greeks, are evaluated at t=0
> #####Calculating the greeks of exchange options on each assets pair (exchange the ith asset
FOR the (i+5)th)
> ##the initial price of the five five assets
> U0<-S0[1:5]
> ##the initial price of the last five assets
> V0<-S0[6:10]
> d1<-(log(V0/U0)+(sigma^2)*(T))/sqrt(2)/sigma/sqrt(T)
> d2<-d1-sqrt(2)*sigma*sqrt(T)
>   #####caluculating theta of the exchange options on each asset pair
>   (thetaexvu<-(-sigma)/sqrt(2)*V0*dnorm(d1)*T^(-1/2))
[1] -26.70172 -26.70172 -26.70172 -26.70172 -26.70172
>   #####caluculating delta of the exchange options on each asset pair
>   (deltav<-pnorm(d1))
[1] 0.5267418 0.5267418 0.5267418 0.5267418 0.5267418
>   (deltau<-(-pnorm(d2)))
[1] -0.4732582 -0.4732582 -0.4732582 -0.4732582 -0.4732582
>   #####caluculating gamma of the exchange options on each asset pair
>   (gammavv<-1/V0/sqrt(2)/sigma/sqrt(T)*dnorm(d1))
[1] 0.02966857 0.02966857 0.02966857 0.02966857 0.02966857
>   (gammauu<-1/U0/sqrt(2)/sigma/sqrt(T)*dnorm(d2))
[1] 0.02966857 0.02966857 0.02966857 0.02966857 0.02966857
>   ##gammavv and gammauu happens to be the same as V0=U0 in this portfolio
>   (gammavu<-(-1)/V0/U0/sqrt(2)/sigma/sqrt(T)*U0*dnorm(d2))
[1] -0.02966857 -0.02966857 -0.02966857 -0.02966857 -0.02966857
>   (gammauv<-(-1)/V0/U0/sqrt(2)/sigma/sqrt(T)*V0*dnorm(d1))
[1] -0.02966857 -0.02966857 -0.02966857 -0.02966857 -0.02966857
> #####FDM
> (EXVU(V0,U0,T-1/250,0,sigma,r)-EXVU(V0,U0,T,0,sigma,r))/(1/250)
[1] -26.97543 -26.97543 -26.97543 -26.97543 -26.97543
> (EXVU(V0+0.01,U0,T,0,sigma,r)-EXVU(V0,U0,T,0,sigma,r))/0.01
[1] 0.5268901 0.5268901 0.5268901 0.5268901 0.5268901
> (EXVU(V0,U0+0.01,T,0,sigma,r)-EXVU(V0,U0,T,0,sigma,r))/0.01
[1] -0.4731099 -0.4731099 -0.4731099 -0.4731099 -0.4731099
> (EXVU(V0+0.01,U0,T,0,sigma,r)-2*EXVU(V0,U0,T,0,sigma,r)+EXVU(V0-0.01,U0,T,0,sigma,r))/0.01/0
.01
[1] 0.02966857 0.02966857 0.02966857 0.02966857 0.02966857
> (EXVU(V0,U0+0.01,T,0,sigma,r)-2*EXVU(V0,U0,T,0,sigma,r)+EXVU(V0,U0-0.01,T,0,sigma,r))/0.01/0
.01
[1] 0.02966857 0.02966857 0.02966857 0.02966857 0.02966857
>
>
> #####define functions for the caluculating the value of a unit of the option(long position),
 under risk neutral framework
> Call<-function(S,T,t,sigma,r,K){
+ d1<-(log(S/K)+(r+0.5*sigma^2)*(T-t))/sigma/sqrt(T-t)
+ d2<-d1-sigma*sqrt(T-t)
+ c<-S*pnorm(d1)-K*exp(-r*(T-t))*pnorm(d2)
+ return(c)
+ }
> Put<-function(S,T,t,sigma,r,K){
+ d1<-(log(S/K)+(r+0.5*sigma^2)*(T-t))/sigma/sqrt(T-t)
+ d2<-d1-sigma*sqrt(T-t)
+ p<-(-S)*pnorm(-d1)+K*exp(-r*(T-t))*pnorm(-d2)
+ return(p)
```

```
+ }
> #####All the partial derivatives, thus the greeks, are evaluated at t=0
> #####Calculating the European call options' and European put options' greeks and thus, the p
ortfolio's greeks
> d1<-(log(S0/K)+(r+0.5*sigma^2)*T)/sigma/sqrt(T)
> d2<-d1-sigma*sqrt(T)
>   #####caluculating theta of the European Call option on the 10 assets
>   (thetac<-(-S0)*dnorm(d1)*sigma/2/sqrt(T)-r*K*exp(-r*T)*pnorm(d2))
 [1] -21.32684 -21.32684 -21.32684 -21.32684 -21.32684 -21.32684 -21.32684 -21.32684 -21.32684
 -21.32684
>   #####caluculating theta of the European Put options on the 10 assets
>   (thetap<-(-S0)*dnorm(d1)*sigma/2/sqrt(T)+r*K*exp(-r*T)*pnorm(-d2))
 [1] -16.35178 -16.35178 -16.35178 -16.35178 -16.35178 -16.35178 -16.35178 -16.35178 -16.35178
 -16.35178
>   #####caluculating delta of the European Call option on the 10 assets
>   (deltac<-pnorm(d1))
 [1] 0.5398829 0.5398829 0.5398829 0.5398829 0.5398829 0.5398829 0.5398829 0.5398829 0.5398829
 0.5398829
>   #####caluculating delta of the European Put option on the 10 assets
>   (deltap<-pnorm(d1)-1)
 [1] -0.4601171 -0.4601171 -0.4601171 -0.4601171 -0.4601171 -0.4601171 -0.4601171 -0.4601171 -
0.4601171 -0.4601171
>   #####caluculating delta of the European Call option on the 10 assets
>   (gammac<-dnorm(d1)/S0/sigma/sqrt(T))
 [1] 0.04184189 0.04184189 0.04184189 0.04184189 0.04184189 0.04184189 0.04184189 0.04184189 0
.04184189 0.04184189
>   #####caluculating delta of the European Put option on the 10 assets
>   (gammap<-dnorm(d1)/S0/sigma/sqrt(T))
 [1] 0.04184189 0.04184189 0.04184189 0.04184189 0.04184189 0.04184189 0.04184189 0.04184189 0
.04184189 0.04184189
>
>
> #####characterize the portfolio(e.g. short 10 exchange options on each of the asset pair, sh
ort 10 ATM calls and short 5 ATM puts on each asset)
> weightexvu<-rep(-10,5)
> weightc<-rep(-10,10)
> weightp<-rep(-5,10)
> #####calculate the initial value of the portfolio (value at time 0)
> ##the initial price of the five five assets
> U0<-S0[1:5]
> ##the initial price of the last five assets
> V0<-S0[6:10]
> ##the initial price of the portfolio
> Value0<-sum(weightexvu*EXVU(V0,U0,T,0,sigma,r))+sum(weightc*Call(S0,T,0,sigma,r,K)+weightp*P
ut(S0,T,0,sigma,r,K))
> Value0
[1] -846.7491
> #####In order to suimulate the loss, we have to be able to samples the change in asset price
s(assumed to follow multivariate normal),
> #####Hence we need to approximate the SIGMA, given the asset price are uncorrelated, SIGMA i
s diagonal
> #####notice that SIGMA is about the underlying assets itself, not the derivatives based on t
hem
> sigmadum<-S0^2*exp(2*r*dt)*(exp(sigma^2*dt)-1)
> SIGMA<-diag(sigmadum,10)
>
>
> #####consider Delta-GAMMA approximation of the portfolio loss, calculating the greeks of the
 portfolio
> #####caluculating theta of the portfolio consisting of exchange options on 5 asset pairs
> THETA<-sum(weightexvu*thetaexvu)+sum(weightc*thetac+weightp*thetap)
> #####caluculating delta of the portfolio(by assets) consisting of exchange options on 5 asse
t pairs
> dumdelta<-c(weightexvu*deltau,weightexvu*deltav)+weightc*deltac+weightp*deltap
> delta<-matrix(dumdelta,10,1)
> #####caluculating gamma of the portfolio(by pairs of assets) consisting of exchange options
on 5 asset pairs, Given exchange option is multiasset, GAMMA is not diagonal
> GAMMA<-matrix(0,10,10)
> dumgamma<-c(weightexvu*gammauu,weightexvu*gammavv)+weightc*gammac+weightp*gammap
> diag(GAMMA)<-dumgamma
> for(i in 1:5){
+ GAMMA[i,i+5]<-weightexvu[i]*gammauv[i]
+ GAMMA[i+5,i]<-weightexvu[i]*gammavu[i]
+ }
```

```
> #####Caluculating parameters for the Delta-Gamma approximatino on the portfolio loss
> a0=-THETA*dt
> a=-delta
> A=-1/2*GAMMA
>
>
> #-----------------------------------------------------------------------------------------------
------------------------------------------#
>
>
> #####
> #####step1: Express Q in diagonal form
> Ct<-t(chol(SIGMA))
> ED<-eigen(t(Ct)%*%A%*%Ct)
> U<-ED$vectors
> LAMBDA<-diag(ED$values,10)
> C<-Ct%*%U
> b<-t(C)%*%a
> #define a function to calculate Q
> Q<-function(Z){t(b)%*%Z+t(Z)%*%LAMBDA%*%Z}
>
>
> #####
> #####step2: Identify the IS distribution Z~N(thetax*B(thetax)%*%b,B(thetax)), B(thetax)=solv
e(I-2thetax*LAMBDA)
> ###Given x, find thetax that makes E[Q]=(x-a0) under the IS chagne of measure (assume D-G ap
proximation is exact)
> ###The x is adjusted so that the loss probability is close to 1.1%, xstd=2.7 under the origi
nal distribution of Z
> vecb<-as.vector(b)
> veclambda<-diag(LAMBDA)
> xstd<-2.7
> x<-(a0+sum(veclambda))+xstd*sqrt(sum(vecb^2)+2*sum(veclambda^2))
> ###To identify thetax, we numerically solve psipithetax=(x-a0), notice that E[Q]=psipitheta
for general theta
> psipithetax<-function(thetax){
+ (thetax*vecb[1]^2*(1-thetax*veclambda[1])/(1-2*thetax*veclambda[1])^2 + veclambda[1]/(1-2*th
etax*veclambda[1])
+ +thetax*vecb[2]^2*(1-thetax*veclambda[2])/(1-2*thetax*veclambda[2])^2 + veclambda[2]/(1-2*th
etax*veclambda[2])
+ +thetax*vecb[3]^2*(1-thetax*veclambda[3])/(1-2*thetax*veclambda[3])^2 + veclambda[3]/(1-2*th
etax*veclambda[3])
+ +thetax*vecb[4]^2*(1-thetax*veclambda[4])/(1-2*thetax*veclambda[4])^2 + veclambda[4]/(1-2*th
etax*veclambda[4])
+ +thetax*vecb[5]^2*(1-thetax*veclambda[5])/(1-2*thetax*veclambda[5])^2 + veclambda[5]/(1-2*th
etax*veclambda[5])
+ +thetax*vecb[6]^2*(1-thetax*veclambda[6])/(1-2*thetax*veclambda[6])^2 + veclambda[6]/(1-2*th
etax*veclambda[6])
+ +thetax*vecb[7]^2*(1-thetax*veclambda[7])/(1-2*thetax*veclambda[7])^2 + veclambda[7]/(1-2*th
etax*veclambda[7])
+ +thetax*vecb[8]^2*(1-thetax*veclambda[8])/(1-2*thetax*veclambda[8])^2 + veclambda[8]/(1-2*th
etax*veclambda[8])
+ +thetax*vecb[9]^2*(1-thetax*veclambda[9])/(1-2*thetax*veclambda[9])^2 + veclambda[9]/(1-2*th
etax*veclambda[9])
+ +thetax*vecb[10]^2*(1-thetax*veclambda[10])/(1-2*thetax*veclambda[10])^2 + veclambda[10]/(1-
2*thetax*veclambda[10]))-(x-a0)
+ }
> curve(psipithetax)
> abline(h=0,v=0)
> uni<-uniroot.all(psipithetax,c(0,0.05))
> uni
[1] 0.009782488
>
> ###choose the thetax that makes a valid change of measure
> k<-0
> for(i in 1:length(uni)){
+ if(sum(sign(1-2*uni[i]*veclambda))==length(veclambda)){
+ k<-i
+ break}
+ }
> (thetax<-uni[k])
[1] 0.009782488
> psipithetax(thetax)
[1] -0.4530261
```

```
> ax<-thetax-0.0001
> bx<-thetax+0.0001
> while(abs(psipithetax(thetax))>0.0000001){
+ thetax<-(ax+bx)/2
+ ifelse(sign(psipithetax(thetax))==sign(bx),bx<-(ax+bx)/2,ax<-(ax+bx)/2)
+ }
> thetax
[1] 0.00978848
> psipithetax(thetax)
[1] -7.773531e-08
>
> ###identify the IS distribution
> Bthetax<-solve(diag(10)-2*thetax*LAMBDA)
> muthetax<-thetax*Bthetax%*%b
>
> ###generate 5000000 samples of Q under IS change of measure, check whether E[Q] approximatel
y equals (x-a0)
> Qsamples<-rep(0,5000000)
> for(j in 1:5000000){
+ Z<-muthetax+chol(Bthetax)%*%matrix(rnorm(10),10,1)
+ Qsamples[j]<-Q(Z)
+ }
> ###check whether E[Q] approximately equals (x-a0) under the importance sampling change of me
asure
> mean(Qsamples)
[1] 542.9082
> x-a0
[1] 542.7504
> a0
[1] -171.4144
> x
[1] 371.3361
> thetax
[1] 0.00978848
>
>
> ###display the parameters
>  SIGMA
          [,1]     [,2]     [,3]     [,4]     [,5]     [,6]     [,7]     [,8]     [,9]     [,10
]
 [1,] 36.20943  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.0000
0
 [2,]  0.00000 36.20943  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.0000
0
 [3,]  0.00000  0.00000 36.20943  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.0000
0
 [4,]  0.00000  0.00000  0.00000 36.20943  0.00000  0.00000  0.00000  0.00000  0.00000  0.0000
0
 [5,]  0.00000  0.00000  0.00000  0.00000 36.20943  0.00000  0.00000  0.00000  0.00000  0.0000
0
 [6,]  0.00000  0.00000  0.00000  0.00000  0.00000 36.20943  0.00000  0.00000  0.00000  0.0000
0
 [7,]  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000 36.20943  0.00000  0.00000  0.0000
0
 [8,]  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000 36.20943  0.00000  0.0000
0
 [9,]  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000 36.20943  0.0000
0
[10,]  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000 36.2094
3
>  THETA
[1] 4285.359
>  a0
[1] -171.4144
>  delta
           [,1]
 [1,]  1.634338
 [2,]  1.634338
 [3,]  1.634338
 [4,]  1.634338
 [5,]  1.634338
 [6,] -8.365662
 [7,] -8.365662
 [8,] -8.365662
```

```
 [9,] -8.365662
[10,] -8.365662
>  a
          [,1]
 [1,] -1.634338
 [2,] -1.634338
 [3,] -1.634338
 [4,] -1.634338
 [5,] -1.634338
 [6,]  8.365662
 [7,]  8.365662
 [8,]  8.365662
 [9,]  8.365662
[10,]  8.365662
>  GAMMA
            [,1]       [,2]       [,3]       [,4]       [,5]       [,6]       [,7]       [,8]
      [,9]      [,10]
 [1,] -0.9243141  0.0000000  0.0000000  0.0000000  0.0000000  0.2966857  0.0000000  0.0000000
 0.0000000  0.0000000
 [2,]  0.0000000 -0.9243141  0.0000000  0.0000000  0.0000000  0.0000000  0.2966857  0.0000000
 0.0000000  0.0000000
 [3,]  0.0000000  0.0000000 -0.9243141  0.0000000  0.0000000  0.0000000  0.0000000  0.2966857
 0.0000000  0.0000000
 [4,]  0.0000000  0.0000000  0.0000000 -0.9243141  0.0000000  0.0000000  0.0000000  0.0000000
 0.2966857  0.0000000
 [5,]  0.0000000  0.0000000  0.0000000  0.0000000 -0.9243141  0.0000000  0.0000000  0.0000000
 0.0000000  0.2966857
 [6,]  0.2966857  0.0000000  0.0000000  0.0000000  0.0000000 -0.9243141  0.0000000  0.0000000
 0.0000000  0.0000000
 [7,]  0.0000000  0.2966857  0.0000000  0.0000000  0.0000000  0.0000000 -0.9243141  0.0000000
 0.0000000  0.0000000
 [8,]  0.0000000  0.0000000  0.2966857  0.0000000  0.0000000  0.0000000  0.0000000 -0.9243141
 0.0000000  0.0000000
 [9,]  0.0000000  0.0000000  0.0000000  0.2966857  0.0000000  0.0000000  0.0000000  0.0000000
-0.9243141  0.0000000
[10,]  0.0000000  0.0000000  0.0000000  0.0000000  0.2966857  0.0000000  0.0000000  0.0000000
 0.0000000 -0.9243141
>  A
            [,1]       [,2]       [,3]       [,4]       [,5]       [,6]       [,7]       [,8]
      [,9]      [,10]
 [1,]  0.4621570  0.0000000  0.0000000  0.0000000  0.0000000 -0.1483429  0.0000000  0.0000000
 0.0000000  0.0000000
 [2,]  0.0000000  0.4621570  0.0000000  0.0000000  0.0000000  0.0000000 -0.1483429  0.0000000
 0.0000000  0.0000000
 [3,]  0.0000000  0.0000000  0.4621570  0.0000000  0.0000000  0.0000000  0.0000000 -0.1483429
 0.0000000  0.0000000
 [4,]  0.0000000  0.0000000  0.0000000  0.4621570  0.0000000  0.0000000  0.0000000  0.0000000
-0.1483429  0.0000000
 [5,]  0.0000000  0.0000000  0.0000000  0.0000000  0.4621570  0.0000000  0.0000000  0.0000000
 0.0000000 -0.1483429
 [6,] -0.1483429  0.0000000  0.0000000  0.0000000  0.0000000  0.4621570  0.0000000  0.0000000
 0.0000000  0.0000000
 [7,]  0.0000000 -0.1483429  0.0000000  0.0000000  0.0000000  0.0000000  0.4621570  0.0000000
 0.0000000  0.0000000
 [8,]  0.0000000  0.0000000 -0.1483429  0.0000000  0.0000000  0.0000000  0.0000000  0.4621570
 0.0000000  0.0000000
 [9,]  0.0000000  0.0000000  0.0000000 -0.1483429  0.0000000  0.0000000  0.0000000  0.0000000
 0.4621570  0.0000000
[10,]  0.0000000  0.0000000  0.0000000  0.0000000 -0.1483429  0.0000000  0.0000000  0.0000000
 0.0000000  0.4621570
>
>  b
           [,1]
 [1,]  42.549634
 [2,]   3.862199
 [3,]  60.050196
 [4,] -60.050196
 [5,]   3.862199
 [6,]  28.641537
 [7,]   6.434621
 [8,] -39.990886
 [9,]  39.990886
[10,]   6.434621
>  LAMBDA
```

```
          [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]      [,8]      [,9]     [,10
]
 [1,] 22.10585  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.0000
0
 [2,]  0.00000 22.10585  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.0000
0
 [3,]  0.00000  0.00000 22.10585  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.0000
0
 [4,]  0.00000  0.00000  0.00000 22.10585  0.00000  0.00000  0.00000  0.00000  0.00000  0.0000
0
 [5,]  0.00000  0.00000  0.00000  0.00000 22.10585  0.00000  0.00000  0.00000  0.00000  0.0000
0
 [6,]  0.00000  0.00000  0.00000  0.00000  0.00000 11.36303  0.00000  0.00000  0.00000  0.0000
0
 [7,]  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000 11.36303  0.00000  0.00000  0.0000
0
 [8,]  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000 11.36303  0.00000  0.0000
0
 [9,]  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000 11.36303  0.0000
0
[10,]  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000 11.3630
3
>  muthetax
             [,1]
 [1,]  0.73425739
 [2,]  0.06664801
 [3,]  1.03625570
 [4,] -1.03625570
 [5,]  0.06664801
 [6,]  0.36056640
 [7,]  0.08100501
 [8,] -0.50344260
 [9,]  0.50344260
[10,]  0.08100501
>  Bthetax
          [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]      [,8]      [,9]     [,10
]
 [1,] 1.762939 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.00000
0
 [2,] 0.000000 1.762939 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.00000
0
 [3,] 0.000000 0.000000 1.762939 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.00000
0
 [4,] 0.000000 0.000000 0.000000 1.762939 0.000000 0.000000 0.000000 0.000000 0.000000 0.00000
0
 [5,] 0.000000 0.000000 0.000000 0.000000 1.762939 0.000000 0.000000 0.000000 0.000000 0.00000
0
 [6,] 0.000000 0.000000 0.000000 0.000000 0.000000 1.286097 0.000000 0.000000 0.000000 0.00000
0
 [7,] 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.286097 0.000000 0.000000 0.00000
0
 [8,] 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.286097 0.000000 0.00000
0
 [9,] 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.286097 0.00000
0
[10,] 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.28609
7
>
>
> #-----------------------------------------------------------------------------------------
----#
>
> ###define a function that calculate the Loss for a generated ds (ds=C%*%Z)
> L<-function(dS){
+ Sdt<-S0+as.vector(dS)
+ Udt<-Sdt[1:5]
+ Vdt<-Sdt[6:10]
+         Valuedt<-sum(weightexvu*EXVU(Vdt,Udt,T,dt,sigma,r))+sum(weightc*Call(Sdt,T,dt,sigma,
r,K)+weightp*Put(Sdt,T,dt,sigma,r,K))
+ return(Value0-Valuedt)
+ }
> ###define a function that calculate the likelihood ratio for a generated Z
> likelihood<-function(Z){
+ p1<-sum((1/2)*((thetax*vecb)^2/(1-2*thetax*veclambda)-log(1-2*thetax*veclambda)))
```

```
+ p2<-thetax*Q(Z)
+ return(exp(p1-p2))
+ }
>
> #----------------------------------------------------------------------------------------------
----#
>
>
> #####
> #####step3: Define k strata
> ###plot the empirical CDF
> Qsamples<-sort(Qsamples,decreasing=FALSE)
> ECDFc3s1<-ecdf(Qsamples)
> #plot.ecdf(Qsamples)
> ###mimics the quantiles of Q using the 5000000 samples of Q generated in the previous step
> stratabyQ<-rep(0,40-1)
> for(i in 1:39){stratabyQ[i]<-quantile(Qsamples,0.025*i)}
> stratabyQ
 [1]  102.5813  153.1045  188.8467  218.0805  243.3537  265.9742  286.9621  306.7757  325.5634
  343.5319  361.0846  378.3198
[13]  395.0942  411.6255  428.0417  444.3530  460.6776  477.0278  493.5303  510.2158  527.1748
  544.4369  562.1158  580.2800
[25]  599.1239  618.6745  639.0469  660.3981  682.8643  706.7577  732.4204  760.2443  790.7688
  825.1715  864.2665  910.3725
[37]  967.6654 1044.2789 1168.0314
> ECDFc3s1(stratabyQ)
 [1] 0.025 0.050 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375
 0.400 0.425 0.450 0.475 0.500
[21] 0.525 0.550 0.575 0.600 0.625 0.650 0.675 0.700 0.725 0.750 0.775 0.800 0.825 0.850 0.875
 0.900 0.925 0.950 0.975
>
> ###calculate the optimal alocation of samples size for each strata
> options(warn=-1)
> bins<-c(stratabyQ,.Machine$double.xmax)
> vars<-matrix(0,40,10000)
> counts<-rep(0,40)
> while(sum(counts)!=400000){
+ Z<-muthetax+chol(Bthetax)%*%matrix(rnorm(10),10,1)
+ k<-tail(binsearch(function(y) bins[y]-(Q(Z)), range=c(1, length(bins)))$where,1)
+ if(counts[k]<10000){
+ counts[k]<-counts[k]+1
+ vars[k,counts[k]]<-ifelse(L(C%*%Z)>x,1,0)*likelihood(Z)
+ }
+ else{}
+ }
> (dumvar<-apply(vars,1,var))
 [1] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+0
0 0.000000e+00 0.000000e+00
[10] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+0
0 0.000000e+00 0.000000e+00
[19] 0.000000e+00 0.000000e+00 1.797536e-04 1.535283e-03 2.233554e-03 1.117822e-03 3.441629e-0
4 9.869974e-05 3.328336e-05
[28] 1.194368e-05 5.206408e-06 2.934131e-06 1.934876e-06 1.277275e-06 8.824212e-07 6.008634e-0
7 3.733452e-07 2.253588e-07
[37] 1.291846e-07 6.341139e-08 2.467260e-08 4.843366e-09
> #Since we assume equiprobable strata, pj=1/k, where k is the number of strata, which is 40 i
n the case
> (qj<-sqrt(dumvar)/sum(sqrt(dumvar)))
 [1] 0.0000000000 0.0000000000 0.0000000000 0.0000000000 0.0000000000 0.0000000000 0.000000000
0 0.0000000000 0.0000000000
[10] 0.0000000000 0.0000000000 0.0000000000 0.0000000000 0.0000000000 0.0000000000 0.000000000
0 0.0000000000 0.0000000000
[19] 0.0000000000 0.0000000000 0.0740117949 0.2162999204 0.2608917650 0.1845645867 0.102410424
9 0.0548428657 0.0318475282
[28] 0.0190779249 0.0125959583 0.0094558780 0.0076787171 0.0062388475 0.0051856129 0.004279076
1 0.0033730084 0.0026205920
[37] 0.0019841182 0.0013900991 0.0008671008 0.0003841809
>
>
> #####
> #####step4: Perform the simulation
> ###define a function to generate estimates of P{L>xp} using three methods: SMC, IS, ISSQ, IS
SQO
> options(warn=-1)
```

```
> run<-function(n,strata){
+
+ results<-rep(0,4)
+ SMC<-0
+ IS<-0
+ ISSQ<-0
+ ISSQO<-0
+
+ bins<-c(stratabyQ,.Machine$double.xmax)
+ binscount<-rep(0,strata)
+ binscountpi<-rep(0,strata)
+
+ nj<-round(n*qj)
+ nj[match(max(nj),nj)]<-nj[match(max(nj),nj)]+(n-sum(nj))
+
+
+ for(i in 1:n){
+ Z1<-matrix(rnorm(10),10,1)
+ #Standard Monte Carlo
+ dS1<-C%*%Z1
+ L1<-L(dS1)
+ SMC<-SMC+(ifelse(L1>x,1,0)*(1/n))
+
+ Z2<-muthetax+chol(Bthetax)%*%Z1
+ #Monte Carlo (IS)
+ dS2<-C%*%Z2
+ L2<-L(dS2)
+ IS<-IS+(ifelse(L2>x,1,0)*likelihood(Z2)*(1/n))
+ kthbins<-tail(binsearch(function(y) bins[y]-(Q(Z2)), range=c(1, length(bins)))$where,1)
+ #Monte Carlo (IS and Stratification)
+ if(binscount[kthbins]<(n/strata)){
+ binscount[kthbins]<-binscount[kthbins]+1
+ ISSQ<-ISSQ+(ifelse(L2>x,1,0)*likelihood(Z2)*(1/n))
+ }
+ else{
+ }
+ #Monte Carlo (IS and Stratification with optimized smaple size for each strata)
+ if(binscountpi[kthbins]<nj[kthbins]){
+ binscountpi[kthbins]<-binscountpi[kthbins]+1
+ ISSQO<-ISSQO+(ifelse(L2>x,1,0)*likelihood(Z2)*(1/nj[kthbins])*(1/strata))
+ }
+ else{
+ }
+ }
+ results[1]<-SMC
+ results[2]<-IS
+
+
+ while(sum(binscount)<n){
+ Z2<-muthetax+chol(Bthetax)%*%matrix(rnorm(10),10,1)
+ kthbins<-tail(binsearch(function(y) bins[y]-(Q(Z2)), range=c(1, length(bins)))$where,1)
+ #Monte Carlo (IS and Stratification) continue...
+ if(binscount[kthbins]<(n/strata)){
+ binscount[kthbins]<-binscount[kthbins]+1
+ ISSQ<-ISSQ+(ifelse(L(C%*%Z2)>x,1,0)*likelihood(Z2)*(1/n))
+ }
+ else{
+ }
+ #Monte Carlo (IS and Stratification with optimized sample size for each strata) continue...
+ if(binscountpi[kthbins]<nj[kthbins]){
+ binscountpi[kthbins]<-binscountpi[kthbins]+1
+ ISSQO<-ISSQO+(ifelse(L(C%*%Z2)>x,1,0)*likelihood(Z2)*(1/nj[kthbins])*(1/strata))
+ }
+ else{
+ }
+ }
+ results[3]<-ISSQ
+
+ while(sum(binscountpi)<n){
+ Z2<-muthetax+chol(Bthetax)%*%matrix(rnorm(10),10,1)
+ kthbins<-tail(binsearch(function(y) bins[y]-(Q(Z2)), range=c(1, length(bins)))$where,1)
+ #Monte Carlo (IS and Stratification with optimized sample size for each strata) continue...
+ if(binscountpi[kthbins]<nj[kthbins]){
+ binscountpi[kthbins]<-binscountpi[kthbins]+1
```

```
+ ISSQO<-ISSQO+(ifelse(L(C%*%Z2)>x,1,0)*likelihood(Z2)*(1/nj[kthbins])*(1/strata))
+ }
+ else{
+ }
+ }
+ results[4]<-ISSQO
+
+ return(results)
+ }
> run(1000,40)
[1] 0.00700000 0.01076615 0.01102326 0.01126839
> run(10000,40)
[1] 0.01190000 0.01093086 0.01099433 0.01105049
>
> ###define a function to generate the replications
> replication<-function(N,n,strata){
+ dum<-c(0,0,0,0)
+ for(i in 1:N){
+ dum<-rbind(dum,run(n,strata))
+ }
+ return(tail(dum,-1))
+ }
>
>
> #####
> #####Step5:evaluate the performance of the algorithm
> SAMPLES<-replication(10000,10000,40)
> (ISratio<-var(SAMPLES[,1])/var(SAMPLES[,2]))
[1] 23.0198
> (ISSQratio<-var(SAMPLES[,1])/var(SAMPLES[,3]))
[1] 81.34989
> (ISSQOratio<-var(SAMPLES[,1])/var(SAMPLES[,4]))
[1] 548.4376
>
> n<-10000
> strata<-40
> var(SAMPLES[,1])
[1] 1.112437e-06
> (sum(sqrt(dumvar)*(1/strata)))^2/n
[1] 2.050953e-09
> (theoreticalISSQOratio<-var(SAMPLES[,1])/((sum(sqrt(dumvar)*(1/strata)))^2/n))
[1] 542.4001
>
> save.image("C:\\Users\\s1155058334\\Desktop\\portfolio c3 setting 1 (5.3) os5 pending\\c31os
5workspace")
>
```