```
R version 3.5.0 (2018-04-23) -- "Joy in Playing"
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Platform: x86_64-w64-mingw32/x64 (64-bit)

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  Natural language support but running in an English locale

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[Previously saved workspace restored]

> #####portfolio b5
> #####Consider porfolios on derivatives based on 10 underlying uncorrelated assets
> #####investigate the loss probability, which is critical to estimating VAR
> install.packages("rootSolve")
Installing package into ‘C:/Users/s1155058334/Documents/R/win-library/3.5’
(as ‘lib’ is unspecified)
--- Please select a CRAN mirror for use in this session ---
trying URL 'https://mirror-hk.koddos.net/CRAN/bin/windows/contrib/3.5/rootSolve_1.7.zip'
Content type 'application/zip' length 787735 bytes (769 KB)
downloaded 769 KB

package ‘rootSolve’ successfully unpacked and MD5 sums checked

The downloaded binary packages are in
        C:\Users\s1155058334\AppData\Local\Temp\RtmpuyyVaY\downloaded_packages
> library(rootSolve)
Warning message:
package ‘rootSolve’ was built under R version 3.5.2
> install.packages("gtools")
Installing package into ‘C:/Users/s1155058334/Documents/R/win-library/3.5’
(as ‘lib’ is unspecified)
trying URL 'https://mirror-hk.koddos.net/CRAN/bin/windows/contrib/3.5/gtools_3.8.1.zip'
Content type 'application/zip' length 325812 bytes (318 KB)
downloaded 318 KB

package ‘gtools’ successfully unpacked and MD5 sums checked

The downloaded binary packages are in
        C:\Users\s1155058334\AppData\Local\Temp\RtmpuyyVaY\downloaded_packages
> library(gtools)
Warning message:
package ‘gtools’ was built under R version 3.5.2
> rm(list=ls())
>
> set.seed(3000)
> S0<-rep(100,10)
> T<-rep(0.1,10)
> sigma<-rep(0.3,10)
> r<-rep(0.05,10)
> K<-rep(100,10)
> H<-rep(95,10)
> dt<-0.04
> #####for the cash or nothing put options
> cash<-K
>
>
> #####define a function for calculating the value of a unit of down-and-out call option (long
 position), under risk neutral framework
> #####notice that H<K, refers to P.579 of Options, Futures, and Other Derivatives(8 ed.) for
the pricing formula of down-and-out call option.
> #####MUST take into account the situation that the call options are knocked out at time t+de
ltat
> Cdo<-function(S,T,t,sigma,r,K,H){
```

```
+ lam<-(r+0.5*sigma^2)/(sigma^2)
+ y<-log((H^2)/S/K)/sigma/sqrt(T-t)+lam*sigma*sqrt(T-t)
+ cdi<-S*(H/S)^(2*lam)*pnorm(y)-K*exp(-r*(T-t))*(H/S)^(2*lam-2)*pnorm(y-sigma*sqrt(T-t))
+
+ d1<-(log(S/K)+(r+0.5*sigma^2)*(T-t))/(sigma*sqrt(T-t))
+ d2<-d1-sigma*sqrt(T-t)
+ c<-S*pnorm(d1)-K*exp(-r*(T-t))*pnorm(d2)
+
+ value<-(c-cdi)*(S>H)
+ return(value)
+ }
> #####All the partial derivatives, thus the greeks, are evaluated at t=0
> #####Calculating the greeks of down-and-out call options on each assets
> #####Since the theoretical greeks is complecated to derive, we approximate them with the Fin
ite Difference Method at the moment
> dS<-0.01
>   #####approximating delta of the Cdo's on each asset using FDM
>   deltacdo<-(Cdo(S0+dS,T,0,sigma,r,K,H)-Cdo(S0,T,0,sigma,r,K,H))/dS
>   #####approximating gamma of the cdo's on each asset using FDM
>   gammacdo<-(Cdo(S0+dS,T,0,sigma,r,K,H)-2*Cdo(S0,T,0,sigma,r,K,H)+Cdo(S0-dS,T,0,sigma,r,K,H)
)/dS/dS
>   #####approximating theta of the cdo's on each asset by the relation among theta, delta and
 gamma of an derivative at time 0 (OFAOD ed.10 P.393)
>   thetacdo<-r*Cdo(S0,T,0,sigma,r,K,H)-r*S0*deltacdo-0.5*sigma^2*S0^2*gammacdo
>
>
> #####define a function for caluculating the value of a cash-or-nothing put option(long posit
ion), under risk neutral framework
> Pcon<-function(S,T,t,sigma,r,K,cash){
+ d1<-(log(S/K)+(r+0.5*sigma^2)*(T-t))/sigma/sqrt(T-t)
+ d2<-d1-sigma*sqrt(T-t)
+ pcon<-cash*exp(-r*(T-t))*pnorm(-d2)
+ return(pcon)
+ }
> #####All the partial derivatives, thus the greeks, are evaluated at t=0
> #####Calculating the greeks of cash-or-nothing put options on each assets
> d1<-(log(S0/K)+(r+0.5*sigma^2)*T)/sigma/sqrt(T)
> d2<-d1-sigma*sqrt(T)
>   #####caluculating theta of the cash or nothing put options on each asset
>   thetapcon<-r*cash*exp(-r*T)*pnorm(-d2)-cash*exp(-r*T)*dnorm(-d2)*(log(S0/K)/2/sigma*T^(-3/
2)-(r-0.5*sigma^2)/2/sigma*T^(-1/2))
>   #####caluculating delta of the cash or nothing put option on each asset
>   deltapcon<-(-cash)*exp(-r*T)/S0/sigma/sqrt(T)*dnorm(-d2)
>   #####caluculating gamma of the cash or nothing put option on each asset
>   gammapcon<-(r*Pcon(S0,T,0,sigma,r,K,cash)-thetapcon-r*S0*deltapcon)*2/(sigma^2)/(S0^2)
> ######Cross validate against approximated greeks using FDM
> ##dS<-0.01
> ##deltapconpi<-(Pcon(S0+dS,T,0,sigma,r,K,cash)-Pcon(S0,T,0,sigma,r,K,cash))/dS
> ##gammapconpi<-(Pcon(S0+dS,T,0,sigma,r,K,cash)-2*Pcon(S0,T,0,sigma,r,K,cash)+Pcon(S0-dS,T,0,
sigma,r,K,cash))/dS/dS
> ##thetapconpi<-r*Pcon(S0,T,0,sigma,r,K,cash)-r*S0*deltapconpi-0.5*sigma^2*S0^2*gammapconpi
> #####Checked
>
>
> #####characterize b5(short 10 down-and-out calls and short 5 cash-or-nothing put options on
each assets)
> weightcdo<-rep(-10,10)
> weightpcon<-rep(-5,10)
> #####calculate the initial value of the portfolio (value at time 0)
> V0<-sum(weightcdo*Cdo(S0,T,0,sigma,r,K,H)+weightpcon*Pcon(S0,T,0,sigma,r,K,cash))
> V0
[1] -2809.468
> #####In order to suimulate the loss, we have to be able to samples the change in asset price
s(assumed to follow multivariate normal),
> #####Hence we need to approximate the SIGMA, given the asset price are uncorrelated, SIGMA i
s diagonal
> sigmadum<-S0^2*exp(2*r*dt)*(exp(sigma^2*dt)-1)
> SIGMA<-diag(sigmadum,10)
>
>
> #####consider Delta-GAMMA approximation of the portfolio loss, calculating the greeks of the
 portfolio
> #####caluculating theta of the portfolio consisting of mix of the options on the 10 assets
> THETA<-sum(weightcdo*thetacdo+weightpcon*thetapcon)
```

```
> #####caluculating delta of the portfolio(by assets) consisting of mix of the options on the
10 assets
> delta<-matrix(weightcdo*deltacdo+weightpcon*deltapcon,10,1)
> #####caluculating gamma of the portfolio(by pairs of assets) consisting of mix of the option
s on the 10 assets.
> GAMMA<-diag(weightcdo*gammacdo+weightpcon*gammapcon,10)
> #####Caluculating parameters for the Delta-Gamma approximatino on the portfolio loss
> a0=-THETA*dt
> a=-delta
> A=-1/2*GAMMA
>
>
> #-----------------------------------------------------------------------------------------
------------------------------------------#
>
>
> #####
> #####step1: Express Q in diagonal form
> Ct<-t(chol(SIGMA))
> ED<-eigen(t(Ct)%*%A%*%Ct)
> U<-ED$vectors
> LAMBDA<-diag(ED$values,10)
> C<-Ct%*%U
> b<-t(C)%*%a
> #define a function to calculate Q
> Q<-function(Z){t(b)%*%Z+t(Z)%*%LAMBDA%*%Z}
>
>
> #####
> #####step2: Identify the IS distribution Z~N(thetax*B(thetax)%*%b,B(thetax)), B(thetax)=solv
e(I-2thetax*LAMBDA)
> ###Given x, find thetax that makes E[Q]=(x-a0) under the IS chagne of measure (assume D-G ap
proximation is exact)
> ###The x is adjusted so that the loss probability is close to 1.1%, xstd=2.75 under the orig
inal distribution of Z
> vecb<-as.vector(b)
> veclambda<-diag(LAMBDA)
> xstd<-2.75
> x<-(a0+sum(veclambda))+xstd*sqrt(sum(vecb^2)+2*sum(veclambda^2))
> ###To identify thetax, we numerically solve psipithetax=(x-a0), notice that E[Q]=psipitheta
for general theta
> psipithetax<-function(thetax){
+ (thetax*vecb[1]^2*(1-thetax*veclambda[1])/(1-2*thetax*veclambda[1])^2 + veclambda[1]/(1-2*th
etax*veclambda[1])
+ +thetax*vecb[2]^2*(1-thetax*veclambda[2])/(1-2*thetax*veclambda[2])^2 + veclambda[2]/(1-2*th
etax*veclambda[2])
+ +thetax*vecb[3]^2*(1-thetax*veclambda[3])/(1-2*thetax*veclambda[3])^2 + veclambda[3]/(1-2*th
etax*veclambda[3])
+ +thetax*vecb[4]^2*(1-thetax*veclambda[4])/(1-2*thetax*veclambda[4])^2 + veclambda[4]/(1-2*th
etax*veclambda[4])
+ +thetax*vecb[5]^2*(1-thetax*veclambda[5])/(1-2*thetax*veclambda[5])^2 + veclambda[5]/(1-2*th
etax*veclambda[5])
+ +thetax*vecb[6]^2*(1-thetax*veclambda[6])/(1-2*thetax*veclambda[6])^2 + veclambda[6]/(1-2*th
etax*veclambda[6])
+ +thetax*vecb[7]^2*(1-thetax*veclambda[7])/(1-2*thetax*veclambda[7])^2 + veclambda[7]/(1-2*th
etax*veclambda[7])
+ +thetax*vecb[8]^2*(1-thetax*veclambda[8])/(1-2*thetax*veclambda[8])^2 + veclambda[8]/(1-2*th
etax*veclambda[8])
+ +thetax*vecb[9]^2*(1-thetax*veclambda[9])/(1-2*thetax*veclambda[9])^2 + veclambda[9]/(1-2*th
etax*veclambda[9])
+ +thetax*vecb[10]^2*(1-thetax*veclambda[10])/(1-2*thetax*veclambda[10])^2 + veclambda[10]/(1-
2*thetax*veclambda[10]))-(x-a0)
+ }
> curve(psipithetax)
> abline(h=0,v=0)
> uni<-uniroot.all(psipithetax,c(0,0.5))
> uni
[1] 0.008497496 0.137959273
>
> ###choose the thetax that makes a valid change of measure
> k<-0
> for(i in 1:length(uni)){
+ if(sum(sign(1-2*uni[i]*veclambda))==length(veclambda)){
+ k<-i
```

```
+ break}
+ }
> (thetax<-uni[k])
[1] 0.008497496
> psipithetax(thetax)
[1] -0.4276395
> ax<-thetax-0.0001
> bx<-thetax+0.0001
> while(abs(psipithetax(thetax))>0.0000001){
+ thetax<-(ax+bx)/2
+ ifelse(sign(psipithetax(thetax))==sign(bx),bx<-(ax+bx)/2,ax<-(ax+bx)/2)
+ }
> thetax
[1] 0.008501598
> psipithetax(thetax)
[1] -2.281865e-08
>
> ###identify the IS distribution
> Bthetax<-solve(diag(10)-2*thetax*LAMBDA)
> muthetax<-thetax*Bthetax%*%b
>
> ###generate 5000000 samples of Q under IS change of measure, check whether E[Q] approximatel
y equals (x-a0)
> Qsamples<-rep(0,5000000)
> for(j in 1:5000000){
+ Z<-muthetax+chol(Bthetax)%*%matrix(rnorm(10),10,1)
+ Qsamples[j]<-Q(Z)
+ }
> ###check whether E[Q] approximately equals (x-a0) under the importance sampling change of me
asure
> mean(Qsamples)
[1] 807.5491
> x-a0
[1] 807.4484
> a0
[1] -33.50115
> x
[1] 773.9472
> thetax
[1] 0.008501598
> #####By trial and error, a more accurate thetax would be 0.008501598
>
> ###display the parameters
>  SIGMA
          [,1]     [,2]     [,3]     [,4]     [,5]     [,6]     [,7]     [,8]     [,9]    [,10
]
 [1,] 36.20943  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.0000
0
 [2,]  0.00000 36.20943  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.0000
0
 [3,]  0.00000  0.00000 36.20943  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.0000
0
 [4,]  0.00000  0.00000  0.00000 36.20943  0.00000  0.00000  0.00000  0.00000  0.00000  0.0000
0
 [5,]  0.00000  0.00000  0.00000  0.00000 36.20943  0.00000  0.00000  0.00000  0.00000  0.0000
0
 [6,]  0.00000  0.00000  0.00000  0.00000  0.00000 36.20943  0.00000  0.00000  0.00000  0.0000
0
 [7,]  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000 36.20943  0.00000  0.00000  0.0000
0
 [8,]  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000 36.20943  0.00000  0.0000
0
 [9,]  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000 36.20943  0.0000
0
[10,]  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000 36.2094
3
>  THETA
[1] 837.5286
>  a0
[1] -33.50115
>  delta
          [,1]
 [1,] 14.04863
 [2,] 14.04863
```

```
 [3,] 14.04863
 [4,] 14.04863
 [5,] 14.04863
 [6,] 14.04863
 [7,] 14.04863
 [8,] 14.04863
 [9,] 14.04863
[10,] 14.04863
>  a
           [,1]
 [1,] -14.04863
 [2,] -14.04863
 [3,] -14.04863
 [4,] -14.04863
 [5,] -14.04863
 [6,] -14.04863
 [7,] -14.04863
 [8,] -14.04863
 [9,] -14.04863
[10,] -14.04863
>  GAMMA
            [,1]       [,2]       [,3]       [,4]       [,5]       [,6]       [,7]       [,8]
      [,9]      [,10]
 [1,] -0.3734297  0.0000000  0.0000000  0.0000000  0.0000000  0.0000000  0.0000000  0.0000000
 0.0000000  0.0000000
 [2,]  0.0000000 -0.3734297  0.0000000  0.0000000  0.0000000  0.0000000  0.0000000  0.0000000
 0.0000000  0.0000000
 [3,]  0.0000000  0.0000000 -0.3734297  0.0000000  0.0000000  0.0000000  0.0000000  0.0000000
 0.0000000  0.0000000
 [4,]  0.0000000  0.0000000  0.0000000 -0.3734297  0.0000000  0.0000000  0.0000000  0.0000000
 0.0000000  0.0000000
 [5,]  0.0000000  0.0000000  0.0000000  0.0000000 -0.3734297  0.0000000  0.0000000  0.0000000
 0.0000000  0.0000000
 [6,]  0.0000000  0.0000000  0.0000000  0.0000000  0.0000000 -0.3734297  0.0000000  0.0000000
 0.0000000  0.0000000
 [7,]  0.0000000  0.0000000  0.0000000  0.0000000  0.0000000  0.0000000 -0.3734297  0.0000000
 0.0000000  0.0000000
 [8,]  0.0000000  0.0000000  0.0000000  0.0000000  0.0000000  0.0000000  0.0000000 -0.3734297
 0.0000000  0.0000000
 [9,]  0.0000000  0.0000000  0.0000000  0.0000000  0.0000000  0.0000000  0.0000000  0.0000000
-0.3734297  0.0000000
[10,]  0.0000000  0.0000000  0.0000000  0.0000000  0.0000000  0.0000000  0.0000000  0.0000000
 0.0000000 -0.3734297
>  A
           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]      [,8]      [,9
]     [,10]
 [1,] 0.1867149 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.000000
0 0.0000000
 [2,] 0.0000000 0.1867149 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.000000
0 0.0000000
 [3,] 0.0000000 0.0000000 0.1867149 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.000000
0 0.0000000
 [4,] 0.0000000 0.0000000 0.0000000 0.1867149 0.0000000 0.0000000 0.0000000 0.0000000 0.000000
0 0.0000000
 [5,] 0.0000000 0.0000000 0.0000000 0.0000000 0.1867149 0.0000000 0.0000000 0.0000000 0.000000
0 0.0000000
 [6,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.1867149 0.0000000 0.0000000 0.000000
0 0.0000000
 [7,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.1867149 0.0000000 0.000000
0 0.0000000
 [8,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.1867149 0.000000
0 0.0000000
 [9,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.186714
9 0.0000000
[10,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.000000
0 0.1867149
>
>  b
           [,1]
 [1,] -84.53663
 [2,] -84.53663
 [3,] -84.53663
 [4,] -84.53663
 [5,] -84.53663
```

```
 [6,] -84.53663
 [7,] -84.53663
 [8,] -84.53663
 [9,] -84.53663
[10,] -84.53663
>  LAMBDA
          [,1]     [,2]     [,3]     [,4]     [,5]     [,6]     [,7]     [,8]     [,9]    [,10
]
 [1,] 6.760838 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.00000
0
 [2,] 0.000000 6.760838 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.00000
0
 [3,] 0.000000 0.000000 6.760838 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.00000
0
 [4,] 0.000000 0.000000 0.000000 6.760838 0.000000 0.000000 0.000000 0.000000 0.000000 0.00000
0
 [5,] 0.000000 0.000000 0.000000 0.000000 6.760838 0.000000 0.000000 0.000000 0.000000 0.00000
0
 [6,] 0.000000 0.000000 0.000000 0.000000 0.000000 6.760838 0.000000 0.000000 0.000000 0.00000
0
 [7,] 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 6.760838 0.000000 0.000000 0.00000
0
 [8,] 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 6.760838 0.000000 0.00000
0
 [9,] 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 6.760838 0.00000
0
[10,] 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 6.76083
8
>  muthetax
            [,1]
 [1,] -0.8120459
 [2,] -0.8120459
 [3,] -0.8120459
 [4,] -0.8120459
 [5,] -0.8120459
 [6,] -0.8120459
 [7,] -0.8120459
 [8,] -0.8120459
 [9,] -0.8120459
[10,] -0.8120459
>  Bthetax
          [,1]     [,2]     [,3]     [,4]     [,5]     [,6]     [,7]     [,8]     [,9]    [,10
]
 [1,] 1.129887 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.00000
0
 [2,] 0.000000 1.129887 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.00000
0
 [3,] 0.000000 0.000000 1.129887 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.00000
0
 [4,] 0.000000 0.000000 0.000000 1.129887 0.000000 0.000000 0.000000 0.000000 0.000000 0.00000
0
 [5,] 0.000000 0.000000 0.000000 0.000000 1.129887 0.000000 0.000000 0.000000 0.000000 0.00000
0
 [6,] 0.000000 0.000000 0.000000 0.000000 0.000000 1.129887 0.000000 0.000000 0.000000 0.00000
0
 [7,] 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.129887 0.000000 0.000000 0.00000
0
 [8,] 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.129887 0.000000 0.00000
0
 [9,] 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.129887 0.00000
0
[10,] 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.12988
7
>
>
> #---------------------------------------------------------------------------------------------
----#
>
> ###define a function that calculate the Loss for a generated ds (ds=C%*%Z)
> L<-function(dS){
+ Sdt<-S0+as.vector(dS)
+         Vdt<-sum(weightcdo*Cdo(Sdt,T,dt,sigma,r,K,H)+weightpcon*Pcon(Sdt,T,dt,sigma,r,K,cash
))
+ return(V0-Vdt)
```

```
+ }
> ###define a function that calculate the likelihood ratio for a generated Z
> likelihood<-function(Z){
+ p1<-sum((1/2)*((thetax*vecb)^2/(1-2*thetax*veclambda)-log(1-2*thetax*veclambda)))
+ p2<-thetax*Q(Z)
+ return(exp(p1-p2))
+ }
>
> #-----------------------------------------------------------------------------------------------
----#
>
>
> #####
> #####step3: Define k strata
> ###plot the empirical CDF
> Qsamples<-sort(Qsamples,decreasing=FALSE)
> ECDFb5<-ecdf(Qsamples)
> #plot.ecdf(Qsamples)
> ###mimics the quantiles of Q using the 5000000 samples of Q generated in the previous step
> stratabyQ<-rep(0,40-1)
> for(i in 1:39){stratabyQ[i]<-quantile(Qsamples,0.025*i)}
> stratabyQ
 [1]  196.8038  290.0317  351.3856  399.0340  438.9165  473.7160  505.0928  533.6544  560.4106
  585.7383  609.7575  632.8191
[13]  655.0433  676.7334  697.8152  718.5402  739.0852  759.4083  779.6488  799.8870  820.1476
  840.5805  861.2837  882.2493
[25]  903.5576  925.4474  948.0464  971.3481  995.5572 1021.0183 1048.0145 1076.8296 1107.9985
 1142.3368 1181.0642 1225.8950
[37] 1280.2102 1351.3315 1461.9436
> ECDFb5(stratabyQ)
 [1] 0.025 0.050 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250 0.275 0.300 0.325 0.350 0.375
 0.400 0.425 0.450 0.475 0.500
[21] 0.525 0.550 0.575 0.600 0.625 0.650 0.675 0.700 0.725 0.750 0.775 0.800 0.825 0.850 0.875
 0.900 0.925 0.950 0.975
>
> ###calculate the optimal alocation of samples size for each strata
> options(warn=-1)
> bins<-c(stratabyQ,.Machine$double.xmax)
> vars<-matrix(0,40,10000)
> counts<-rep(0,40)
> while(sum(counts)!=400000){
+ Z<-muthetax+chol(Bthetax)%*%matrix(rnorm(10),10,1)
+ k<-tail(binsearch(function(y) bins[y]-(Q(Z)), range=c(1, length(bins)))$where,1)
+ if(counts[k]<10000){
+ counts[k]<-counts[k]+1
+ vars[k,counts[k]]<-ifelse(L(C%*%Z)>x,1,0)*likelihood(Z)
+ }
+ else{}
+ }
> (dumvar<-apply(vars,1,var))
 [1] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+0
0 9.840296e-05 1.022001e-04
[10] 2.165604e-04 3.699105e-04 1.079643e-03 2.055780e-03 2.261316e-03 2.140477e-03 1.681862e-0
3 1.204805e-03 8.161134e-04
[19] 5.284948e-04 3.334967e-04 2.118773e-04 1.242773e-04 7.435835e-05 4.115145e-05 2.422504e-0
5 1.251744e-05 7.459260e-06
[28] 3.739792e-06 1.990309e-06 1.023459e-06 5.759874e-07 2.432131e-07 1.230038e-07 7.039274e-0
8 3.634879e-08 1.993011e-08
[37] 1.248842e-08 7.131167e-09 3.830684e-09 1.549497e-09
> #Since we assume equiprobable strata, pj=1/k, where k is the number of strata, which is 40 i
n the case
> (qj<-sqrt(dumvar)/sum(sqrt(dumvar)))
 [1] 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+0
0 2.305211e-02 2.349267e-02
[10] 3.419765e-02 4.469460e-02 7.635664e-02 1.053647e-01 1.105064e-01 1.075132e-01 9.530198e-0
2 8.066129e-02 6.638683e-02
[19] 5.342285e-02 4.243776e-02 3.382586e-02 2.590614e-02 2.003879e-02 1.490731e-02 1.143771e-0
2 8.221754e-03 6.346796e-03
[28] 4.493973e-03 3.278438e-03 2.350942e-03 1.763653e-03 1.146041e-03 8.150158e-04 6.165533e-0
4 4.430489e-04 3.280663e-04
[37] 2.596931e-04 1.962398e-04 1.438285e-04 9.147492e-05
>
>
> #####
```

```
> #####step4: Perform the simulation
> ###define a function to generate estimates of P{L>xp} using three methods: SMC, IS, ISSQ, IS
SQO
> options(warn=-1)
> run<-function(n,strata){
+
+ results<-rep(0,4)
+ SMC<-0
+ IS<-0
+ ISSQ<-0
+ ISSQO<-0
+
+ bins<-c(stratabyQ,.Machine$double.xmax)
+ binscount<-rep(0,strata)
+ binscountpi<-rep(0,strata)
+
+ nj<-round(n*qj)
+ nj[match(max(nj),nj)]<-nj[match(max(nj),nj)]+(n-sum(nj))
+
+
+ for(i in 1:n){
+ Z1<-matrix(rnorm(10),10,1)
+ #Standard Monte Carlo
+ dS1<-C%*%Z1
+ L1<-L(dS1)
+ SMC<-SMC+(ifelse(L1>x,1,0)*(1/n))
+
+ Z2<-muthetax+chol(Bthetax)%*%Z1
+ #Monte Carlo (IS)
+ dS2<-C%*%Z2
+ L2<-L(dS2)
+ IS<-IS+(ifelse(L2>x,1,0)*likelihood(Z2)*(1/n))
+ kthbins<-tail(binsearch(function(y) bins[y]-(Q(Z2)), range=c(1, length(bins)))$where,1)
+ #Monte Carlo (IS and Stratification)
+ if(binscount[kthbins]<(n/strata)){
+ binscount[kthbins]<-binscount[kthbins]+1
+ ISSQ<-ISSQ+(ifelse(L2>x,1,0)*likelihood(Z2)*(1/n))
+ }
+ else{
+ }
+ #Monte Carlo (IS and Stratification with optimized smaple size for each strata)
+ if(binscountpi[kthbins]<nj[kthbins]){
+ binscountpi[kthbins]<-binscountpi[kthbins]+1
+ ISSQO<-ISSQO+(ifelse(L2>x,1,0)*likelihood(Z2)*(1/nj[kthbins])*(1/strata))
+ }
+ else{
+ }
+ }
+ results[1]<-SMC
+ results[2]<-IS
+
+
+ while(sum(binscount)<n){
+ Z2<-muthetax+chol(Bthetax)%*%matrix(rnorm(10),10,1)
+ kthbins<-tail(binsearch(function(y) bins[y]-(Q(Z2)), range=c(1, length(bins)))$where,1)
+ #Monte Carlo (IS and Stratification) continue...
+ if(binscount[kthbins]<(n/strata)){
+ binscount[kthbins]<-binscount[kthbins]+1
+ ISSQ<-ISSQ+(ifelse(L(C%*%Z2)>x,1,0)*likelihood(Z2)*(1/n))
+ }
+ else{
+ }
+ #Monte Carlo (IS and Stratification with optimized sample size for each strata) continue...
+ if(binscountpi[kthbins]<nj[kthbins]){
+ binscountpi[kthbins]<-binscountpi[kthbins]+1
+ ISSQO<-ISSQO+(ifelse(L(C%*%Z2)>x,1,0)*likelihood(Z2)*(1/nj[kthbins])*(1/strata))
+ }
+ else{
+ }
+ }
+ results[3]<-ISSQ
+
+ while(sum(binscountpi)<n){
+ Z2<-muthetax+chol(Bthetax)%*%matrix(rnorm(10),10,1)
```

```
+ kthbins<-tail(binsearch(function(y) bins[y]-(Q(Z2)), range=c(1, length(bins)))$where,1)
+ #Monte Carlo (IS and Stratification with optimized sample size for each strata) continue...
+ if(binscountpi[kthbins]<nj[kthbins]){
+ binscountpi[kthbins]<-binscountpi[kthbins]+1
+ ISSQO<-ISSQO+(ifelse(L(C%*%Z2)>x,1,0)*likelihood(Z2)*(1/nj[kthbins])*(1/strata))
+ }
+ else{
+ }
+ }
+ results[4]<-ISSQO
+
+ return(results)
+ }
> run(1000,40)
[1] 0.00800000 0.01137408 0.01144932 0.01080505
> run(10000,40)
[1] 0.00950000 0.01061067 0.01067615 0.01055742
>
> ###define a function to generate the replications
> replication<-function(N,n,strata){
+ dum<-c(0,0,0,0)
+ for(i in 1:N){
+ dum<-rbind(dum,run(n,strata))
+ }
+ return(tail(dum,-1))
+ }
>
>
> #####
> #####Step5:evaluate the performance of the algorithm
> SAMPLES<-replication(10000,10000,40)
> (ISratio<-var(SAMPLES[,1])/var(SAMPLES[,2]))
[1] 21.64032
> (ISSQratio<-var(SAMPLES[,1])/var(SAMPLES[,3]))
[1] 31.45442
> (ISSQOratio<-var(SAMPLES[,1])/var(SAMPLES[,4]))
[1] 94.41758
>
> n<-10000
> strata<-40
> var(SAMPLES[,1])
[1] 1.076677e-06
> (sum(sqrt(dumvar)*(1/strata)))^2/n
[1] 1.157355e-08
> (theoreticalISSQOratio<-var(SAMPLES[,1])/((sum(sqrt(dumvar)*(1/strata)))^2/n))
[1] 93.02914
>
> save.image("C:\\Users\\s1155058334\\Desktop\\portfolio b5  (5.3) os5 pending\\b5os5workspace
")
>
```