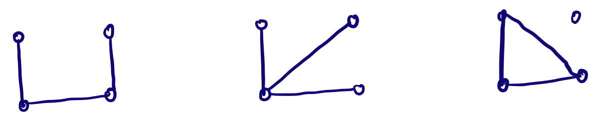
图同构：

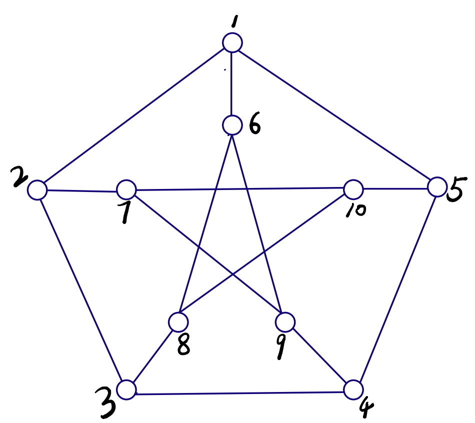
$G \cong H$，$G=(V,E)$，$H=(V',E')$，$\exists f$ (双射)

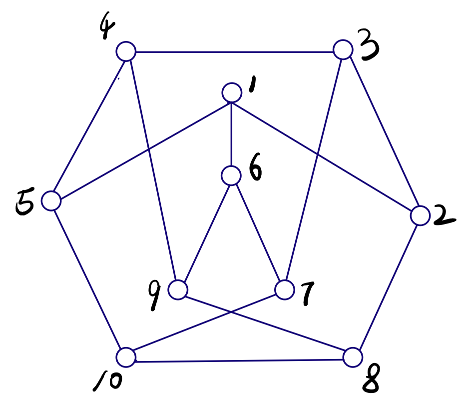
s.t. $\forall u,v \in V$，$(u,v)\in E \Leftrightarrow (f(u),f(v)) \in E'$

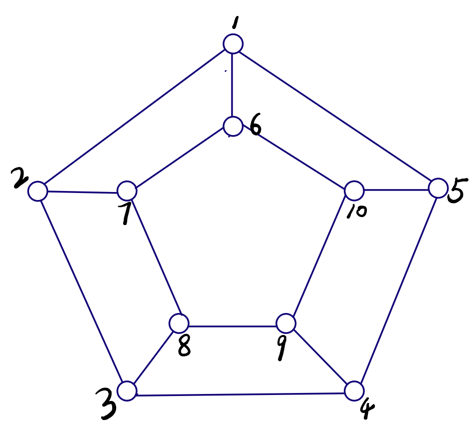
例，4个顶点3条边的同构下面3个图中一个.



对于彼特森图.



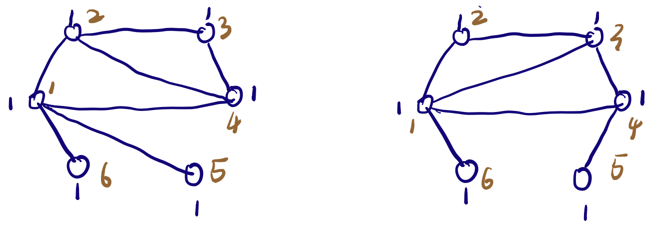




只根据度序列无法判断两个图是否同构

color_refinement 算法：迭代两次实例

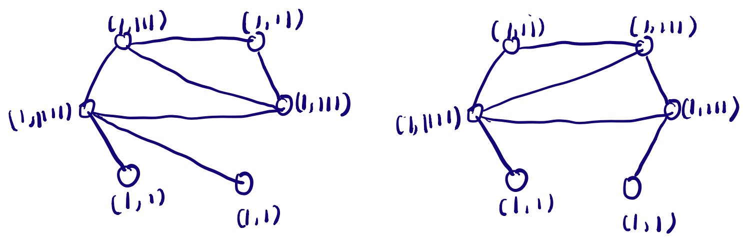
1.初始化每个节点的颜色



G1　　　　G2

第一次迭代：

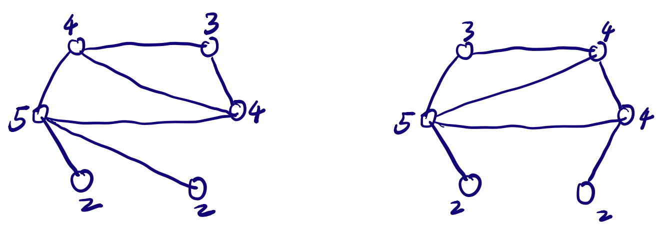
2.聚合邻居颜色.



G1　　　　G2

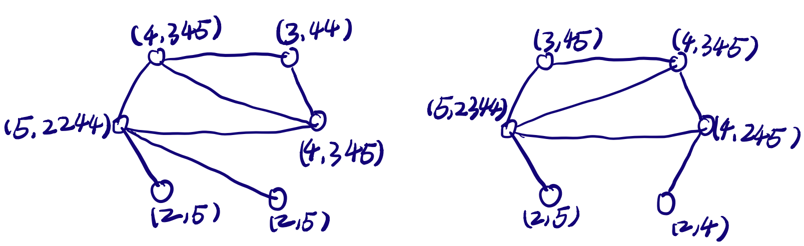
3.哈希映射

1,1 → 2
1,11 → 3
1,111 → 4
1,1111 → 5
⇒



G1　　　　G2

第二次迭代：4.聚合邻居信息.

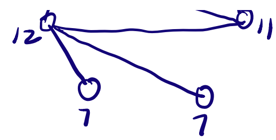


G1　　　　G2

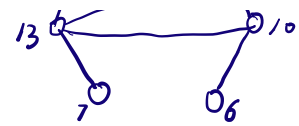
5.哈希映射

(2,4) → 6
(2,5) → 7
(3,44) → 8

(3,145) → 9
(4,245) → 10
(4,345) → 11
(5,2244) → 12
(5,2344) → 13



G_1



G_2

6. 两次迭代结束后，计算两个图的颜色向量内积

$$\begin{array}{cccccccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ \phi(G_1) = & [6, & 2, & 1, & 2, & 1, & 0, & 2, & 1, & 0, & 0, & 2, & 1, & 0] \end{array} \quad \text{color}$$

$$\phi(G_2) = [6, 2, 1, 2, 1, 1, 1, 0, 1, 1, 1, 0, 1]$$

则 $\phi(G_1)^T \cdot \phi(G_2) = 36+4+1+4+1+2+2 = 50$

图同构是一个NP-hard问题，如果 $d \le \bar{d}$（度有界）则 $\exists\, O(n\bar{d}^{k-1})$ 算法找到 k大小的图同构数量.

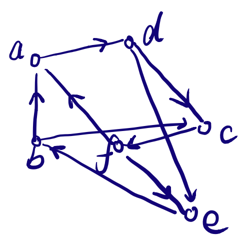
Isomorphism

图的自同构群:

对于图 $G=(V,E)$ 的自同构是顶点集的置换 σ，使得对 $(u,v) \in E$，当且仅当 $(\sigma(u),\sigma(v)) \in E$ 则 σ 为G的自身图同构.

给定一个图，其所有自同构的集合在复合运算下构成群，称为这个图的自同构群.

例:



将图分解成 $\{a, c, e\}$, $\{b, d, f\}$

(1) 单位映射 ϕ_1:

ϕ_1: $a \to a$; $b \to b$; $c \to c$; $d \to d$; $e \to e$; $f \to f$

(2) ϕ_2: $a \to a$; $b \to f$; $c \to e$; $d \to d$; $e \to c$; $f \to b$

(3) ϕ_3: $a \to c$; $b \to d$; $c \to e$; $d \to f$; $e \to a$; $f \to b$

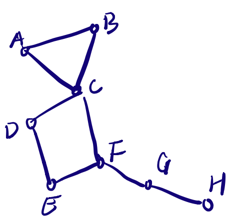
(4) ϕ_4: $a \to c$; $b \to b$; $c \to a$; $d \to f$; $e \to e$; $f \to d$

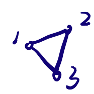
(5) ϕ_5: $a\to e; b\to f; c\to a; d\to b; e\to c; f\to d$

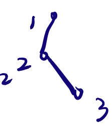
(6) ϕ_6: $a\to e; b\to d; c\to c; d\to b; e\to a; f\to f$

$$\varphi: G \longrightarrow Aut(G)$$
$$g \to Gg$$

图同构：点同构



H_1 

H_2 

$\Rightarrow$

$\varphi_1: 1\to 1,\ 2\to 2,\ 3\to 3$
$\varphi_2: 1\to 1,\ 2\to 3,\ 3\to 2$
$\varphi_3: 1\to 2,\ 2\to 1,\ 3\to 3$
$\varphi_4: 1\to 2,\ 2\to 3,\ 3\to 1$
$\varphi_5: 1\to 3,\ 2\to 1,\ 3\to 2$
$\varphi_6: 1\to 3,\ 2\to 2,\ 3\to 1$

$\psi_1: 1\to 1,\ 2\to 2,\ 3\to 3$
$\psi_2: 1\to 3,\ 2\to 2,\ 3\to 1$

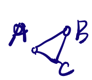
$v=c$.

$$X_{H_1}^{V}(v) = |\{ G_s \simeq H \mid v \in V_{G_s},\ f(v) \in O_{H_{bi}}^{v} \}|$$

H_1 下 $\quad orb(v) = \{ u \in V_H \mid \exists g \in Aut(H),\ s.t.\ g(u) = v \}$

$orb(1) = \{1,2,3\}$
$orb(2) = \{1,2,3\}$
$orb(3) = \{1,2,3\}$

$$\{O_{H_1}^{V}, \cdots, O_{H,d_H}^{V}\} = \{ orb(v) \mid v \in V_H \}$$
$$= \{1,2,3\}$$

$X_{H_1}^{V}(C) = \{$  $\simeq H \} = 1$

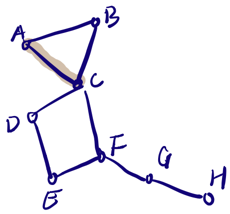
H_2 下

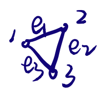
$\{1,3\}$, $\{2\}$

$\{2\}$ 下 $\quad$ ACD, ACF, BCD, BCF, $\quad 4 \quad \Rightarrow \quad$ 1 4 3

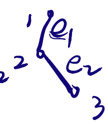
$\{1,3\}$ 下. $\quad$ CDE, CFE, CFG. $\quad 3$

图同构: 边同构



H_1 

$\Rightarrow$

H_2 

φ_1: $e_1 \to e_1$, $e_2 \to e_2$, $e_3 \to e_3$

φ_2: $e_1 \to e_1$, $e_2 \to e_3$, $e_3 \to e_2$

φ_3: $e_1 \to e_2$, $e_2 \to e_1$, $e_3 \to e_3$

φ_4: $e_1 \to e_2$, $e_2 \to e_3$, $e_3 \to e_1$

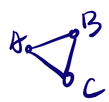
φ_5: $e_1 \to e_3$, $e_2 \to e_2$, $e_3 \to e_1$

φ_6: $e_1 \to e_3$, $e_2 \to e_1$, $e_3 \to e_3$

ψ_1: $e_1 \to e_1$ $e_2 \to e_2$

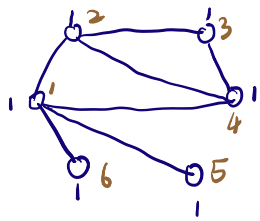
ψ_2: $e_1 \to e_2$, $e_2 \to e_1$

H_1

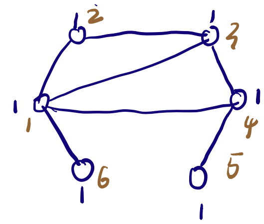
$\{e_1, e, e_3\}$:  : 1

H_2 $\{e_1 e_2\}$: ACD, ACF : 2

46
+3



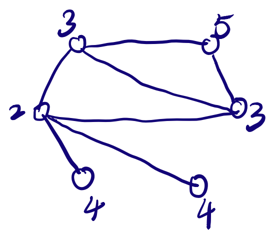
G



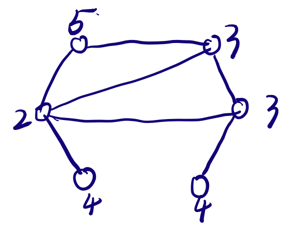
G_2

↓

2: 11111
3: 1111
4: 11
5: 111

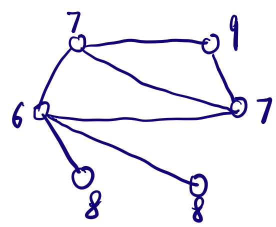


G

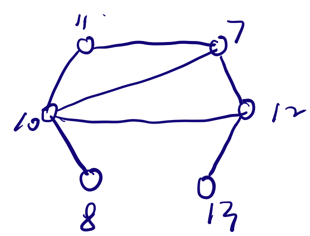


G_2

6: 2 33 44
7: 3 2 5 3
8: 4 2
9: 5 3 3
10: 2 5 3 3 4
11: 5 2 3
12: 3 2 3 4
13: 4 3



G



G_2

2 + 2