Model Outline

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Lucas Model

The firm's output with decreasing returns to scale: $y = F(n) = zn^{\alpha}$

The aggregate total factor of productivity z is found with a Markov chain rule following:

$z_i \in Z = \{\ z_1, z_2, ...z_{N_z}\}$

$log(z) = \rho(z) + \epsilon$

The probability of moving from state z to a particular state in Z is:

$\pi_{ij} = Pr\{z' = z_j | z = z_i\}$

The firm's problem: $d(z) = max_n\{zn^{\alpha} - wn\}$ where $d(z)$ represents the dividends.

The household is defined by:

$V(s, z_i) = max\{u(c) + \beta \sum_{j=1}^{N_z} \pi_{ij} V(s', z_j)\}$
subject to

$c + p(z_i)s' = (d(z_i) + p(z_i))s + w(z_i)$

We define the utility function as: $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$

Lucas Model During a Lifecycle

The Value Function:

$V_T = max\{u(c, l)\}$

subject to

$c = s + w(z_i)n \; n + l = 1$

The Value Function During the Last Age: (we will start with this and then work backwards)

$V_t(s, z_i) = max\{u(c, l) + \beta \sum_{j=1}^{N_z} \pi_{ij} V_{t+1}(s', z_j)\}$

subject to

$c + s' = (1 + r)s + w(z_i)n$

$n + l = 1$

General Equilibrium is defined as a pair of functions: the price function and the value function. All output must be consumed, all asset shares must be held, and all asset prices solve the dynamic problem.

The value function is solves so that: $s' = g(s, z_i)$

Prices satisfy the folloiwng:

$w(z_i) = az_i$

$d(z_i) = (1 - a)z_i$

The decision rule satisfies the following:

$g(1, z_i) = 1$

Key Variables:

α: Labor share of output

γ: Risk aversion parameter

β: Discounting factor

ρ: persistence of aggregate shock

ω_e^2: variance of the iid part of the shock

We will gather price data for T-Bills from the U.S. Treasury's website. Historical price data for Bitcoin can easily be gathered from Yahoo Finance and similar sites. We will compute daily returns as $R_{it} = \frac{ln(P_t)}{ln(P_{t-1})}$. Thereafter, we will compute the equity premium as $\frac{ln(P_{it})}{ln(P_{i,t-1})} - \frac{ln(P_j t)}{ln(P_{j,t-1})}$.