

An Analysis of Bitcoin with a Consumption and Savings Constraint

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Abstract

Traditionally, studies that replicate the equity premium puzzle with a Lucas Asset Pricing Model examine the excess returns of a risky security or index relative to those of risk-free assets or treasury bonds. Until 2016, cryptocurrencies were largely unacknowledged by academics. Although the volatile behavior of cryptocurrency is now at the forefront of many financial economic works, the risk premia necessary to hold cryptocurrencies are scantily studied. This empirical work seeks to analyze the shock process of Bitcoin as well as the shock process of relatively risk-free U.S. Treasury Bills (i.e. to answer of a relatively risk-free U.S. Treasury Bill?). Furthermore, this study seeks to examine the risk aversion parameter necessary to generate the level of equity return observed in the historical price data of Bitcoin and U.S. Treasury Bills (i.e. to generate the level of equity return observed in historical price data of Bitcoin and U.S. Treasury Bills?).

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To examine the shock process of Bitcoin, we employ a method of successive approximations with discrete state-space dynamic programming. During each discrete time period, the agent maximizes lifetime utility by choosing a level of consumption along with a number of bonds and an amount of Bitcoin to hold.

There are two financial assets in which the representative agent can invest: a risk-free Treasury Bill, TB_t , and cryptocurrency, BC_t . By saving with the asset TB_t , the agent subsequently earns a fixed risk-free rate of return, R_{TB} .

Alternatively, Bitcoin earns a stochastic return:

$$R_{BC} = (R_{TB} + \mu)z$$

that follows a Markov Chain formally described as:

$$z_i \in Z = \{z_1, z_2, z_3, \dots, z_{N_z}\}$$

The probability of landing in state $z_j \in z$ is defined as

$$\pi_{i,j} = P\{z' = z_j | z = z_i\}$$

Furthermore, the persistence of aggregate shock, ρ , follows:

$$\log(z) = \rho \log(z) + \epsilon$$

where the distribution of ϵ is defined as $N(0, \sigma_\epsilon^2)$.

Given financial investments BC_{t+1} and TB_{t+1} , we compute financial wealth during period $t + 1$ as

$$x_{t+1} = R_{TB}TB_{t+1} + R_{BC,t+1}BC_{t+1}$$

The representative agent must choose to save with risk-free Treasury Bonds or to invest in Bitcoin provided that the agent has a utility function following:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

with consumption, c , and a risk aversion parameter of γ , a partial equilibrium in the infinite horizon model is met when the following value function in time t is maximized. To solve for a partial equilibrium, we maximize the value function during time t by choosing how much to invest in Bitcoin and Treasury Bills during period $t + 1$.

$$V(TB, BC) = \max_{TB', BC'} \left\{ \frac{c^{1-\gamma}}{1-\gamma} + E(\beta V(TB', BC')) \right\}$$

subject to

$$\begin{aligned} C + TB' + BC' &= R_{TB}TB + R_{BC}BC \\ BC + TB &\leq 1 \end{aligned}$$