

An Analysis of Bitcoin with a Consumption and Savings Constraint

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To examine the shock process of Bitcoin, we employ a method of successive approximations with discrete state-space dynamic programming. During each discrete time period, the agent maximizes lifetime utility by choosing a level of consumption along with a number of bonds and an amount of Bitcoin to hold.

There are two financial assets in which the representative agent can invest: a risk-free Treasury Bill, TB_t , and cryptocurrency, BC_t . By saving with the asset TB_t , the agent subsequently earns a fixed risk-free rate of return, R_{TB} .

Alternatively, Bitcoin earns a stochastic return:

$$R_{BC} = (R_{TB} + \mu)z$$

that follows a Markov Chain formally described as:

$$z_i \in Z = \{z_1, z_2, z_3, \dots, z_{N_z}\}$$

The probability of landing in state $z_j \in z$ is defined as:

$$\pi_{i,j} = P\{z' = z_j | z = z_i\}$$

Furthermore, the persistence of aggregate shock, ρ , follows:

$$\log(z) = \rho \log(z) + \epsilon$$

where the distribution of ϵ is defined as $N(0, \sigma_\epsilon^2)$.

Given financial investments BC_{t+1} and TB_{t+1} , we compute financial wealth during period $t + 1$ as:

$$x_{t+1} = R_{TB}TB_{t+1} + R_{BC,t+1}BC_{t+1}$$

The representative agent must choose to save with risk-free Treasury Bonds or to invest in Bitcoin provided that the agent has a utility function following:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

with consumption, c , and a risk aversion parameter of γ , a partial equilibrium in the infinite horizon model is met when the following value function in time t is maximized. To solve for a partial equilibrium, we maximize the value function during time t by choosing how much to invest in Bitcoin and Treasury Bills during period $t + 1$.

$$V(TB, BC) = \max_{TB', BC'} \left\{ \frac{c^{1-\gamma}}{1-\gamma} + E(\beta V(TB', BC')) \right\}$$

subject to

$$\begin{aligned} C + TB' + BC' &= R_{TB}TB + R_{BC}BC \\ BC + TB &\leq 1 \end{aligned}$$