An Analysis of Bitcoin with a Consumption and Savings Constraint

Jenna Christensen and Kai Jensen
Guanyi Yang, PhD
Department of Economics
St. Lawrence University

Abstract

Traditionally, studies that replicate the equity premium puzzle with a Lucas Asset Pricing Model examine the excess returns of a risky security or index relative to those of risk-free assets or treasury bonds. Until 2016, cryptocurrencies were largely unacknowledged by academics. Although the volatile behavior of cryptocurrency is now at the forefront of many financial economic works, our study is the only to examine the shock process of Bitcoin and the shock process of a relatively risk-free U.S. Treasury Bill, risk aversion parameters necessary to hold cryptocurrencies, and the consumer's decision to invest in cryptocurrencies over risk-free bonds. We conclude that the Bootstrap Model possesses an ability to describe the behavior of cryptocurrency that is superior to the ability of the Tauchen Algorithm. Furthermore, we find limited evidence that an increase in the agent's risk aversion parameter results in a decrease in risky investments and an increase in risk-less investments.

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Introduction

Nearly a decade ago, peer to peer payment networks and digital currencies were unknown to virtual communities and the general population. Cryptocurrencies including Bitcoin, LiteCoin, and DogeCoin became household names and grasped the attention of investors, analysts, and economists in 2016. Bitcoin's anonymous users and encrypted transactions made it the most prominent of virtual currencies. At its inception, Bitcoin traded at 0.63 U.S. dollars per Bitcoin, and by 2014 its value peaked at 1,101.65 U.S. dollars per Bitcoin. By November 20, 2017, its price was recorded at a record high of $8,237.45 and increased over 111.19% in the 15 preceding weeks. However, its price today is less than half of that record value. The fundamental determinants of Bitcoin's price include supply and demand interactions, individuals' expectations, and the development of technological instruments used for conducting Bitcoin transactions.

Unlike standard fiat money, Bitcoin is not within the domain of central governments, authorities, or individuals. The supply and demand for most monetary units are driven by macroeconomic variables which include interest rates, inflation, and the actions taken by central authorities. However, significant changes in the price of Bitcoin are attributable to specific factors relating to cryptocurrencies. Since Bitcoin's supply evolves according to a publicly known algorithm and is fairly inelastic, and the demand side of the market is mainly driven by the expectations of investors who plan on holding the currency and later selling it, Bitcoin has an exceptionally volatile behavior which makes it an extremely risky yet profitable investment. Its market performance includes steep increases and precipitous declines in value further suggesting the market is driven by the expectations of investors and spectators.

This empirical work seeks to analyze the shock process of Bitcoin as well as the shock process of relatively risk-free U.S. Treasury Bills (i.e. to answer the following question: What are the shock processes of Bitcoin and the shock processes of relatively risk-free U.S. Treasury Bills?). Furthermore, this study seeks to examine the risk aversion parameter necessary to generate the level of equity return observed in the historical price data of Bitcoin and U.S. Treasury Bills (i.e. to answer the following question: What is the risk aversion parameter necessary to generate the level of equity return observed in historical price data of Bitcoin and U.S. Treasury Bills?). To our knowledge, this study is the only work that examines the shock process of Bitcoin and the shock process of a relatively risk-free U.S. Treasury Bill, the risk aversion parameters necessary to hold cryptocurrencies, and the consumer's decision to invest in cryptocurrencies over risk-free bonds. Our results indicate that the Bootstrap Model possesses an ability to describe the behavior of cryptocurrency that is superior to the ability of the Tauchen Algorithm. In contrast to our expectations, we find merely suggestive evidence that an increase in the agent's risk aversion parameter results in a decrease in risky investments and an increase in risk-less investments.

Literature Review

Robert Lucas (1978) examines the stochastic behavior of equilibrium prices in a representative, pure exchange, single good economy with identical consumers. His paper first examines the behavior of asset prices in a one-good pure exchange economy with identicalconsumers and introduces a method of constructing equilibrium prices. Lucas later defines the general equilibrium as a pair of functions: a price function and an optimum value function. To reach a competitive equilibrium, all output must be consumed, all asset shares must be held, and all asset prices must solve the dynamic program. Thus, the general equilibrium and market clearing price for trees at time t must satisfy the following: $s_t^*, a_t^* = p_t$, and $c_t^* = d_t$. Hence, the equilibrium price must satisfy $p_t = \mathbb{E}\{\sum_{n=1}^{\infty} \beta^n \frac{u'(d_{t+n})}{u'(d_t)}\}$. Lusas' paper was the first of its kind to model risky asset ownership decisions and determine how risk premiums are incorporated in the price of an asset.

Subsequent to the publication of the Lucas Asset Pricing Model, Mehra and Prescott (1985) present the equity premium puzzle. They find that in a competitive pure exchange economy, the average annual yield of equity is, at most, four-tenths of a percent higher than that of short-term debt. In stark contrast, the historical yield observed by Mehra and Prescott has a premium of six percent when accounting for U.S. business cycle fluctuations and reasonable risk aversion levels. They conclude that the historical U.S. equity premium, the return earned by a risky security in excess of that earned by a relatively risk-free U.S. Treasury Bill, is not only irrational but also inexplicable. According to Nada (2013), the economies used in Mehra and Prescott's study have a "stationary equilibrium for growth rate process on consumption as well as returns". Nada maintains that the elasticity of substitution between consumption in time period t and time period $t + 1$ is sufficiently small to yield a six percent average premium, but the magnitude of the covariance between the marginal utility of consumption and equity returns is not sufficiently large enough to justify the equity premium observed. Mehra and Prescott's equity premium puzzle ignited an extensive research effort within the fields of macroeconomics and finance. A plethora of theoretical speculations and plausible explanations for this anomaly have been presented, but no single solution has been widely accepted by economists.

Traditionally, studies that replicate the equity premium puzzle with a Lucas Asset Pricing Model examine the excess returns of a risky security or index relative to those of risk-free assets or treasury bonds. Although virtual currencies resemble the role of money and create an alternative environment for conducting business, it was not until 2016 that cryptocurrencies were unacknowledged by academics. Cryptocurrencies are commonly used as methods of payments, but it is heavily debated whether they truly function as currencies. Since the role cryptocurrency plays is unclear to many, how cryptocurrency is regulated by financial institutions is controversial. Vandezande (2017) claims that it is increasingly important to analyze the behavior of cryptocurrencies as financial tools because there are few explanations for the current behavior of cryptocurrencies as investment tools. He analyzes the extent to which virtual currencies are regulated within the European Union and ascertains that cryptocurrencies have the highest risk among all types of virtual currencies. Although Vandezande (2017) does not include empirical tests, he further maintains that investors are

not fully informed about the risk relating to cryptocurrency investments due to the absence of regulatory bodies and the enforcement of protection mechanisms. He lastly suggests that legal frameworks used for traditional currenciesand financial investments are applicable to the various types of virtual currencies and cryptocurrency service providers.

Much of the financial literature contains ambiguous results concerning the behavior of cryptocurrencies. Thus, the debate about whether cryptocurrencies are a speculative investment asset or a currency remains ongoing. Corbet, Meegan, et al. (2018) examine the relationships between cryptocurrencies and other financial assets with the Diebold and Yilmaz methodology, Barunik and Krehlik methodology, and a standard Multivariate Generalized Autoregressive Conditional Heteroskedasticity model with dynamic conditional correlations (MVGARCH-DCC) model. They hypothesize that, "cryptocurrency markets, i.e. Bitcoin, Ripple, and Litecoin, are strongly interconnected and demonstrate similar patterns of return and volatility transmission with other assets." To study the return and volatility transmission among Bitcoin, Ripple, and Litecoin and research the excess return and volatility transmission to gold, bond, equities, and the global volatility index (VIX), they measure changes in the correlations of the aforementioned assets' volatilities and returns. Their findings demonstrate that cryptocurrencies are relatively isolated from market shocks and decoupled from popular financial assets, despite the fact that the performance of each cryptocurrency is correlated to the performance of other cryptocurrencies. Corbet, Meegan, et al. (2018) also find that Bitcoin, Ripple, and Litecoin are highly sensitive to industry regulations and technological malfunctions. Ergo, the interconnectedness among cryptocurrencies indicates that substantial changes in cryptocurrency prices are attributable to speculative activity. These results suggest that cryptocurrencies can be effective tools for portfolio diversification.

Although cryptocurrencies may serve as useful portfolio diversifiers their returns do not behave similarly to standard asset classes. Liu and Tsyvinski (2018) investigate whether the cryptocurrency market behaves similarly to the stock market. They do so by determining whether or not the returns of cryptocurrency are compensated by risk factors derived from the stock market and analyzing CAPM alphas, CAPM betas, and Eugene Fama and Kenneth French's five risk factors. Thereafter, they study the exposure of cryptocurrency returns to the Australian Dollar, Canadian Dollar, Euro, Singaporean Dollar, and UK Pound. Although major national currencies strongly comove with one another, the exposures of all cryptocurrencies to major currencies are not statistically significant. Hence, Liu and Tsyvinski (2018) fail to reject the null hypothesis that cryptocurrency serves as another medium of exchange. They also examine the exposure of cryptocurrency returns to precious metal commodities and test whether or not cryptocurrencies serve as a store of value. Again, they find that the exposure of cryptocurrencies is insignificant. Traditional currencies fulfill three objectives: a unit of account, a store of value, and a medium of exchange. However, the implementation of empirical asset pricing models and the analysis of co-movements among Bitcoin, Ripple, Ethereum, stocks, currencies, commodities, macroeconomic factors, and cryptocurrency market specific factors show that cryptocurrencies can be assessed using simple financial tools, but they behave in a radically different manner than traditional assets. Liu and Tsyvinski (2018) lastly conclude that only cryptocurrency market specific factors including momentum and investor attention consistently explain market returns.

Model Environment and Financial Model

To examine the shock process of Bitcoin, we employ a method of successive approximations with discrete state-space dynamic programming. During each discrete time period, a single trading day, an individual agent maximizes lifetime utility by choosing a level of consumption along with a number of bonds and an amount of Bitcoin to hold.

There are two financial assets in which the representative agent can invest: a risk-free Treasury Bill, TB_t, and cryptocurrency, BC_t. By saving with the asset TB_t, the agent subsequently earns a fixed risk-free rate of return, R_{TB}.

Alternatively, Bitcoin earns a stochastic return:

$$R_{BC} = (R_{TB} + \mu)z$$

that follows a Markov Chain formally described as:

$$z_i \in Z = \{z_1, z_2, z_3, ...z_{N_z}\}$$

The probability of landing in state $z_j \in z$ is defined as:

$$\pi_{i,j} = P\{z' = z_j | z = z_i\}$$

Furthermore, the persistence of aggregate shock, ρ, follows:

$$log(z) = \rho log(z) + \epsilon$$

where the distribution of ϵ is defined as $N(0, \sigma_\epsilon^2)$.

Given financial investments BC_{t+1} and TB_{t+1}, we compute financial wealth during period $t + 1$ as:

$$x_{t+1} = R_{TB}TB_{t+1} + R_{BC,t+1}BC_{t+1}$$

The representative agent must choose to save with risk-free Treasury Bonds or to invest in Bitcoin provided that the agent has a utility function following:

$$u(c) = \frac{c^{1-\gamma}}{1 - \gamma}$$

with consumption, c, and a risk aversion parameter of γ. A partial equilibrium in the infinite horizon model is met when the following value function in time t is maximized. To solve for a partial equilibrium, we maximize the value function during time t by choosing how much to invest in Bitcoin and Treasury Bills during period $t + 1$. The agent must maximize the following:

$$V(TB, BC) = max_{TB',BC'}\{\frac{c^{1-\gamma}}{1 - \gamma} + E(\beta V(TB', BC'))\}$$

subject to

$$C + TB' + BC' = R_{TB}TB + R_{BC}BC$$

$$C \geq 0$$

Data Description

We assume the risk-free rate of return is four percent based on historical data collected from the U.S. Department of the Treasury. The Bitcoin price data that we include was freely gathered from the Federal Reserve Bank of St. Louis. We use the daily closing prices of Bitcoin from January 1, 2017 to March 29, 2019. The returns of cryptocurrency, R_{BC}, are calculated at time t by:

$$R_t = ln(\frac{P_t}{P_{t-1}})$$

with P_t equal to the price of the Bitcoin price on day t.

To calculate the persistence of returns, we include two lags in an autoregressive (AR) model. We consider an AR 1 model and an AR 3 model based on the results of the autocorrelation function (ACF) and partial autocorrelation function (PACF). The ACF describes the autocorrelation between each observation and an observation one period before that. In contrast, the PACF describes the direct relationship between an observation and its lag. The partial autocorrelation functions suggest that there exist significant correlations for one lag and three lags. Hence, the autoregressive regression assumes the form:

$$R_t = \alpha + \beta_1 R_{t-1} + \beta_2 R_{t-3} + \epsilon$$

During the examined period, the returns of Bitcoin ranged from -97.119% to 39,663.05%, and the mean return during this holding period was 238.228%. With 90% confidence, we conclude that the correlation coefficients of both lags included in the regression are statistically significant.

Table 1: Summary Statistics for Bitcoin Returns

Statistic	Value
Minimum	0.0288
Maximum	396.6305
Mean	2.3823
Median	1.0035
Standard Deviation	15.5196
AR 1 Correlation Coefficient	0.0618
AR 3 Correlation Coefficient	0.0740
Number of Observations	819.0000

Results

The Tauchen Algorithm

We approach the simulation of Bitcoin returns in two ways: The Tauchen Algorithm and The Bootstrap Method. The employment of the Tauchen Algorithm provides discretized aggregate shock states accompanied by a transition probability matrix for each state. We allow a total number of 60 exogenous varying shocks to occur with a maximum loss of 99.88% and a maximum gain of 1,004.9%. The coefficient of relative risk aversion and the subjective discount factor are among the most important parameters in a risky intertemporal environment. Therefore, we assume the agent has a subjective discount value of $\beta = .85$ and a risk aversion parameter $\gamma = 2$. To examine the shock process of Bitcoin, we adjust the persistence, ρ, the mean, μ, and the variance of the error terms, σ_e, included in the algorithm and simulate the returns of Bitcoin over 819 periods. This model fails to accurately simulate returns with summary statistics equal to those observed in the data. Calibrating the model so that $\mu = -6.5, \rho = -.98$, and $\sigma_e = .0085$ provides the summary statistics seen below. However, variations of the aforementioned parameters in the model do not provide summary statistics that converge to those observed in the data. Furthermore, averaging the summary statistics of multiple simulations does not result in the convergence of summary statistics.

The Bootstrap Model

The Bootstrap Method defines $X_1, X_2, ..., X_n$ as independent, identically distributed random variables with the distribution function F. Suppose that $R(X, F)$ is a random variable of both the observed returns Bitcoin with the distribution F. The Bootstrap method estimates the sampling distribution of $R(X, F)$ on the basis of observed returns. We employ the Monte Carlo approximation of the bootstrap distribution. In turn, repeated realizations of returns, X^{*^i}, are generated by taking random samples of size n from $\hat{F}_n$. Furthermore, the histogram of corresponding values $F(X^{*^i}, \hat{F}_n)$ is taken as an approximation of the actual bootstrap

distribution (Fuh, 1989). To utilize the Bootstrap Method, we adjust the collected returns of Bitcoin and discretize the data with 102 equally spaced bins. Thereafter, we fit the bootstrap method with 1,000 boots. The results of our simulations closely align with the observed data. To assess the fit of each model, we examine 100 replicates of the Tauchen Algorithm and the Bootstrap Model. Subsequently, we average the summary statistics of each model and compare them to the summary statistics of the observed data. Repeated simulations of the Bootstrap Model replicate returns with summary statistics similar to those observed in the data. The results of a single simulation of the Tauchen Algorithm and the Bootstrap Model are displayed below. Averaging the summary statistics of multiple simulations of the Bootstrap Model results in values that approach those observed in the data. Hence, the Bootstrap Model provides a more accurate representation of the behavior of Bitcoin relative to the Tauchen Algorithm.

Table 2: Summary Statistics for Simulated Bitcoin Returns

Statistic	Tauchen Algorithm Value	Bootstrap Algorithm Method
Minimum	0.0012	0.0288
Maximum	11.0490	436.2894
Mean	1.1212	3.4927
Median	0.6942	1.1849
Standard Deviation	15.5196	22.1173
AR 1 Correlation Coefficient	0.0021	0.0329
Number of Observations	819.0000	819.0000

The Decision Rule

The amount of Bitcoin and the number of Treasury Bills the agent decides to hold depends on his risk aversion parameter. Thus, as the risk aversion parameter, γ, increases, the agent is expected to hold fewer amounts of Bitcoin and more Treasury bonds. To verify this hypothesis, we run 100 simulations for each of the following three scenarios: $\gamma = .5$, $\gamma = 2$, and $\gamma = 3$. The average holdings of Bitcoin and Treasury Bonds are shown below.

Table 3: Effects of Risk Aversion on Asset Allocation with the Tauchen Algorithm

Asset	Gamma = .5	Gamma = 2	Gamma = 3
Bitcoin	0.5203	0.2356	0.3557
Treasury Bond	0.2868	0.3341	0.2672

Table 4: Effects of Risk Aversion on Asset Allocation with the Bootstrap Method

Asset	Gamma = .5	Gamma = 2	Gamma = 3
Bitcoin	0.4671	0.4578	0.7913
Treasury Bond	0.3355	0.2697	0.4544

The results of our simulations do not provide sufficient evidence to conclude that an increase in the risk aversion parameter results in a decrease in an individual's investment in Bitcoin. We observe that an increase in the risk aversion parameter from .5 to 2 results in a decrease in the amount of Bitcoin held by the investor and co-occurs with an increase in the agent's investment in Bitcoin. This suggests that the individual's choice is not to solely decrease the amount of Bitcoin he holds. Rather, his choice is to minimize losses by diversifying his portfolio. However, in both the Tauchen Algorithm simulations as well as the Bootstrap Method simulations, an increase in the risk aversion parameter from 2 to 3, results in an increase in the agent's investment in Bitcoin. This decision choice starkly contrasts the decision choice observed when the risk aversion parameter increases from .5 to 2. We also find that the agent's investment in Treasury Bonds varies based on the model employed. Ergo, we can not strictly conclude that an increase in the risk aversion parameter decreases an individual's investment in a risky asset.

Discussion and Conclusion

The results of this study suggest that individuals' investment choices vary based on the risk aversion parameter and exogenous shocks to returns. Although the Tauchen Algorithm and the Bootstrap Model possess the ability to describe the behavior of cryptocurrency, the Bootstrap Model permits relatively more accurate replications. The discrepancies between the returns of each model are attributable to the Bootstrap Model's inclusion of random samples from the observed data.

It is atypical that an increase in an agent's risk aversion does not result in decreased investments in risky assets along with increased investments in risk-less assets. These findings suggest there exists a threshold for investments in risk-free bonds as the risk aversion parameter increases. If the decision rule assumes a concave shape due to an investment threshold, there may be further economic implications for the agent's investment decisions. Both the Tauchen Algorithm and Bootstrap Model include market shocks that significantly vary throughout each simulation. In turn, the observed decision rule may be determined by the effect exogenous shocks have on an individual's investment decision in addition to adjustments made to the risk aversion parameter.

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