An Analysis of Bitcoin with a Consumption and Savings Constraint

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To examine the shock process of Bitcoin, we employ a method of successive approximations with discrete state-space dynamic programming. During each discrete time period, the agent maximizes llifetime utility by choosing a level of consumption along with a number of bonds and an amount of Bitcoin to hold.

There are two financial assets in which the representative agent can invest: a risk-free Treasury Bill, TB_t, and cryptocurrency, BC_t. By saving with the asset TB_t, the agent subsequently earns a fixed risk-free rate of return, R_{TB}.

Alternatively, Bitcoin earns a stochastic return:

$$R_{BC} = (R_{TB} + \mu)z$$

that follows a Markov Chain formally described as:

$$z_i \in Z = \{z_1, z_2, z_3, ...z_{N_z}\}$$

The probability of landing in state $z_j \in z$ is defined as:

$$\pi_{i,j} = P\{z' = z_j | z = z_i\}$$

Furthermore, the persistence of aggregate shock, ρ, follows:

$$log(z) = \rho log(z) + \epsilon$$

where the distribution of ϵ is defined as $N(0, \sigma_\epsilon^2)$.

Given financial investments BC_{t+1} and TB_{t+1}, we compute financial wealth during period $t+1$ as:

$$x_{t+1} = R_{TB}TB_{t+1} + R_{BC,t+1}BC_{t+1} + Y$$

where Y is an initial endowment. The representative agent must choose to save with risk-free Treasury Bonds or to invest in Bitcoin provided that the agent has a utility function following:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

with consumption, c, and a risk aversion parameter of γ, a partial equilibrium in the infinite horizon model is met when the following value function in time t is maximized. To solve for a partial equilibrium, we maximize the value function during time t by choosing how much to invest in Bitcoin and Treasury Bills during period $t+1$.

$$V(TB, BC) = max_{TB', BC'}\{\frac{c^{1-\gamma}}{1-\gamma} + E(\beta V(TB', BC'))\}$$

subject to

$$C + \frac{1}{R_{TB}}TB' + \frac{1}{R_{BC}}BC' = TB + BC + Y$$

$$C \geq 0$$