An Analysis of Risky Assets with an Investment and Savings Constraint

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Abstract

This empirical work seeks to analyze the shock process of Bitcoin as well as the shock process of relatively risk free U.S. Treasury Bills (i.e. to answer the following question: What is the shock process of Bitcoin and the shock process of a relatively risk free U.S. Treasury Bill?). Furthermore, this study seeks to examine the risk aversion parameter necessary to generate the level of equity return observed in the historical price data of Bitcoin and U.S. Treasury Bills (i.e. to answer the following question: What is the risk aversion parameter necessary to generate the level of equity return observed in historical price data of Bitcoin and U.S. Treasury Bills?). Traditionally, studies that replicate the equity premium puzzle with a Lucas Asset Pricing Model examine the excess returns of a risky security or index relative to those of risk free assets or treasury bonds. Until 2016, cryptocurrencies were largely unacknowledged by academics. Although the volatile behavior of cryptocurrency is now at the forefront of many financial economic works, the risk premia necessary to hold cryptocurrencies are scantly studied.

There are two financial assets in which the representative agent can invest: a risk free Treasury Bill, b_t, and cryptocurrency s_t. The agent chooses to borrow or save using the asset b_t. Subsequently, he earns the risk free rate or return, R_f. We suppose that the borrowing rate R_b exceed the savings rate so that $R_b = R_f + \omega$ and let debt be non-defaultable with a borrowing limit $b > 0$.

Alternatively, the returns of cryptocurrency R_s are calculated at time t by $R_t = ln(P_t/P_{t-1})$ with P_t equal to the price of the Bitcoin price on day t. Bitcoin earns a stochastic return $R_{s,t+1} - R_f = \mu + \eta_{t+1}$ during period $t+1$ where η_{t+1} is assumed to be the innovation to excess returns distributed as $N(0, \sigma_\eta^2)$. The stochastic return of Bitcoin follows a Markov Chain formally described as $z_i \in Z = \{z_1, z_2, z_3, ...z_{N_z}\}$. We define the probability of landing in state $z_j \in z$ as $\pi_{i,j} = P\{z' = z_j | z = z_i\}$ Furthermore, the persistence of aggregate shock, ρ, follows $log(z) = \rho log(z) + \epsilon$ where the distribution of ϵ is defined as $N(0, \sigma_\epsilon^2)$.

Given financial investments during time t, b_{t+1}, and s_{t+1}, we compute financial wealth during period $t+1$ as $x_{t+1} = R_j b_{t+1} + R_{s,t+1} s_{t+1}$. The representative agent must choose to invest in risk free Treasury Bonds, invest in Bitcoin, or save their earnings. Provided that the representative agent has a utility function following $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ with consumption, c, and a risk aversion parameter of γ, a partial equilibrium in the infinite horizon model is met when the following value function in time t is maximized.

$$V^R(t, b, s) = max_{b',s'}\{\frac{c^{1-\gamma}}{1-\gamma} + \beta V^R(t+1, b', s')\}$$

subject to

$$c + b' + s' = R_j b + R_s s$$
$$b' \geq b$$
$$s' > 0$$

To solve for a partial equilibrium, the value function in the last period is *set to*