I. Longest Increasing Sub-sequence

Long Trac

1) The longest sub-sequence for the array above is 5: The array [100, 112, 113, 115, 120]

2) Induction

a) Base Case:

longest increasing subsequence

$K = 0$

$LIS(A) \leq n - k \leq n$

b) Inductive Steps:

Assume: $LIS(A) \leq n - K_0$

Prove: $LIS(A) \leq n - k_n$ $\quad K_n = K_0 + 1$

" Because the inner loop checked all possible solutions in the itteration of K_{old} and it can't find LIS(A). Therefore, it has to increment K_0 into $K_n = K_0 + 1$ so that the LIS(A) can not be longer then $n - K_{new}$

c) Termination step

return $n-k$ as LIS(A) when the loop find the most suitable choice. The outer loop never terminates until it found the right k ~~becau~~ and we know it is the right k because all the smaller k (longer subsequence length) has been tested & proved to be incorrect

best case scenario

when the whole array is ascending

runtime (n)

worst case

when the array is descending

run time (2^n)

2) Asymptotic Notation

1) $2n^3+n^2+4 \in \Theta(n^3)$

* $2n^3+n^2+4 \in O(n^3)$

$2n^3+n^2+4 \leq 2n^3+n^2\cdot 1+4\cdot 1\cdot 1$

$\leq 2n^3+n^3+4n^3 \leq 7n^3$ $\quad c=7, n=1 \quad (1)$

* $2n^3+n^2+4 \in \Omega(n^3)$

$2n^3+n^2+4 \geq 2n^3+n^2 \geq 2n^3$ $\quad c=2, n=1 \quad (2)$

$(1) \,\&\, (2) \Rightarrow 2n^3+n^2+4 \in \Theta(n^3)$

2) $3n^4-9n^2+4n \in \Theta(n^4)$

* $3n^4-9n^2+4n \in O(n^4)$

$3n^4-9n^2+4n \leq 3n^4-9n+4n^4$

$\leq 7n^4-9n \leq 7n^4$ $\quad c=7, n=1 \quad (1)$

* $3n^4-9n^2+4n \in \Omega(n^4)$

$3n^4-9n^2+4n \geq 3n^4-9n^2 \geq 3n^4-n^4$

$\geq 2n^4$ $\quad c=2, n=3 \quad (9n^2=n^4 \Rightarrow n=3) \quad (2)$

$(1)(2) \Rightarrow 3n^4-9n^2+4n \in \Theta(n^4)$

3 a) $f(n) \in O(5g_1(n)+100)$

we have $f(n) \in O(g_1(n))$

$\Leftrightarrow f(n) \leq c_1 g_1(n) \quad \exists c_1>0\ \exists n_1>0\ \forall n>n_1$

when $c_1 = 5 > 0$

$f(n) \leq 5g_1(n) \leq 5g_1(n)+100 \Rightarrow \boxed{a: \text{TRUE}}$

b) $f(n) \in O(g_1(n)+g_2(n))$

$\Leftrightarrow f(n) \in O(g_1(n))$

$\rightarrow f(n) \leq c_1 g_1(n) \quad \exists c_1>0\ \exists n_1>0\ \forall n>n_1 \quad (1)$

$f(n) \in O(g_2(n))$

$\rightarrow f(n) \leq c_2 g_2(n) \quad \exists c_2>0\ \exists n_2>0\ \forall n>n_2 \quad (2)$

$f(n) \leq 2f(n) \leq (c_1+c_2)g_1(n)+(c_1+c_2)g_2(n) \leq (c_1+c_2)(g_1(n)+g_2(n)) \leq c_3(g_1(n)+g_2(n))$

$\Rightarrow f(n) \in O(g_1(n)+g_2(n))$ TRUE

c) $f(n) \in O\left(\frac{g_1(n)}{g_2(n)}\right)$

$(1)\&(2) \Rightarrow (1) \Rightarrow f(n) \leq c_1 g_1(n) \leq (c_1+c_2)g_1(n) \quad (3)$

$(2) \rightarrow f(n) \leq c_2 g_2(n) \leq (c_1+c_2)g_2(n) \quad (4)$

$\frac{(3)}{(4)}: \frac{f(n)}{f(n)} \leq \frac{(c_1+c_2)g_1(n)}{(c_1+c_2)g_2(n)} \leq \frac{g_1(n)}{g_2(n)}$

$\Rightarrow f(n) \leq \frac{g_1(n)}{g_2(n)} f(n) \Rightarrow \boxed{c: \text{FALSE}}$

d) $f(n) \in O(\max(g_1(n), g_2(n)))$

$(1) \,\&\, (2)$

$\Rightarrow f(n) \leq \max(c_1 g_1(n), c_2 g_2(n)) \leq$

$\max((c_1+c_2)g_1(n), (c_1+c_2)g_2(n)) \leq$

$\max(c\, g_1(n), c\, g_2(n))$ with $c=c_1+c_2$

$c \cdot \max(g_1(n), g_2(n))$

$\Rightarrow f(n) \in \Theta(\max(g_1(n), g_2(n))) \Rightarrow \boxed{\text{TRUE}}$

3 Summations

a) $\sum_{i=5}^{n} (4i+1)$ (arithmetic S)

$4\sum_{i=5}^{n} i + \sum_{i=5}^{n} 1$

$4\left(\sum_{i=1}^{n} i + \sum_{i=1}^{4} i\right) + \sum_{i=5}^{n} 1$

$4\left(\frac{n(n+1)}{2} - 10\right) + (n-5+1)$

$2n^2+2n-40+n-5+1$

$2n^2-3n-44 \Rightarrow \boxed{\sum_{i=5}^{n}(4i+1) \in O(n^2)}$

b) $\sum_{i=0}^{\log_2(n)} 2^i$ (geometric S)

$= \frac{2^{\log_2(n)}-1}{2-1} = n-1$

$\Rightarrow \boxed{\sum_{i=0}^{\log_2(n)} 2^i \in O(n)}$