1] Hash Table Propability

1) (a): $\frac{m-1}{m}$

(b): $\frac{1}{m}$

2] (a) $\frac{m-1}{m} + \frac{m-2}{m} = \frac{2m-3}{m}$

(b) $\frac{m-1}{m}$

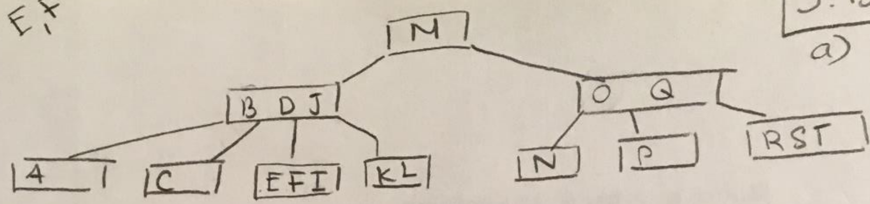
(c) $\frac{1}{m}$

2] Red Black Tree

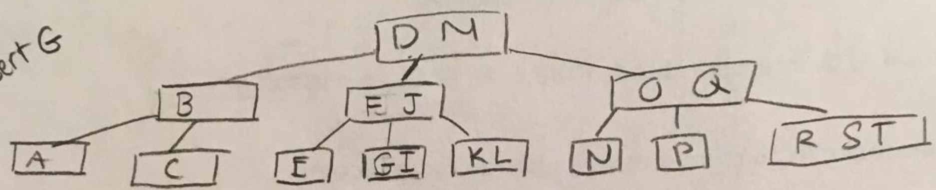
3. B Trees

a)

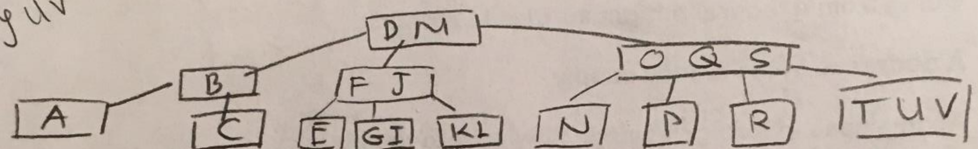
① Insert E, F



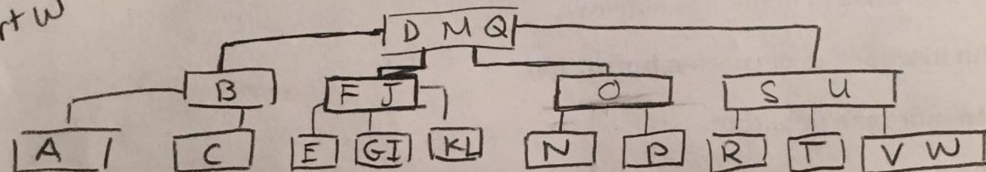
② Insert G



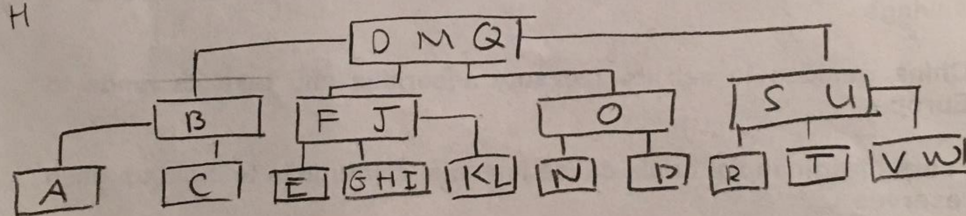
③ Inserting U V



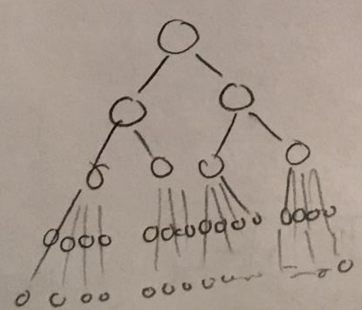
④ Insert W



⑤ Insert H

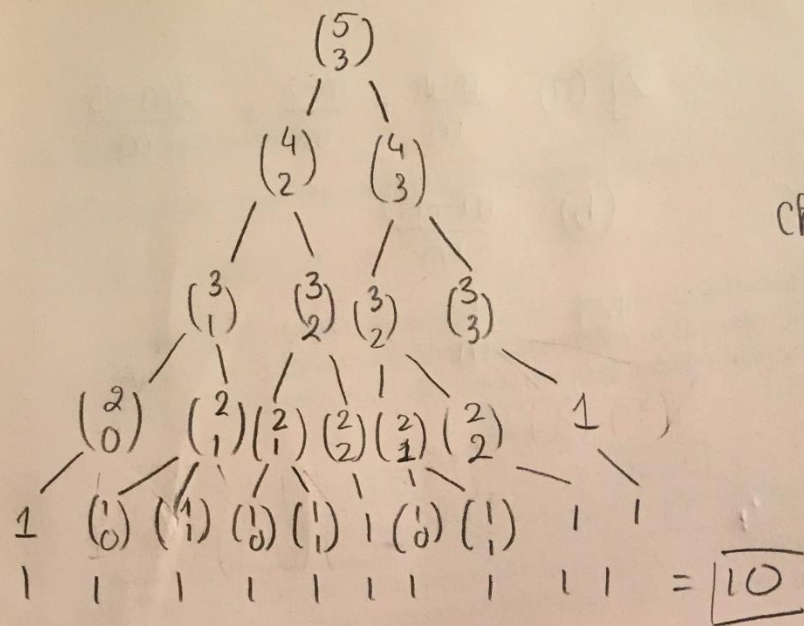


b)

lv		# node	# key
0		1	1
1		2	$2(k-1)$
2		$2k$	$2(k-1)(k)$
3		$2k^2$	$2(k-1)k^2$
⋮		⋮	⋮
h		$2k^{h-1}$	$2(k-1)k^{h-1}$

$\Rightarrow$ # num of key $\sum_{i=0}^{h-1} 2k^i(k-1) + 1$

4 Dynamic Programming



Check $\frac{5!}{2!\,3!} = \frac{5.4}{2!} = 10$ ✓

Pseudocode

- Create an array & initialize A[n][0] = 1 and A[n][k] = 1 where n=k

choose (n k)

Func

```
int choose (int ns , int ks)
{for  n  from 0 to ns
        for  k  from 0 to ks
             if (k==0) OR (n==k)          ⟸ Base Case
               | A[n][k] = 1
             else                          ⟸ Bottom up Case
               | A[n][k] =   A[n-1][k-1] + A[n-1][k]
             endif
        end for
  end for
  ret  A[ns][ks]
}
```

(3).

	0	1	2	3
0	1			
1	1	1		
2	1	2	1	
3	1	3	3	1
4	1	4	6	4
5	1	5	10	10