LONG TRAC

I Probability

(1) a) Sample space: $S_1 = \{1, 2, 3, 4, 5, 6\}$

b) Describe out come:

$X_1(1) = X_1(6) = +\$30$

$X_1(2) = X_1(3) = X_1(4) = X_1(5) = -\$3$

c) Expected value for $n=1$

$EX_1 = \sum_{s \in S} p(s) X_n(s)$

$= \frac{1}{6} \cdot 30 - \frac{1}{6} 3(1) - \frac{1}{6} 3(1) - \frac{1}{6} 3(1) - \frac{1}{6} 3(1) + \frac{1}{6} 30$

$= 8$

(2) a) Sample space : $S_7 = S_1^7$

b) Describe random var to map an outcome

$X_{total} = \sum_{i=1}^{7} X_i$

c) Compute Expected Value using the linearity:

* General formula for Expected value : $\underline{10-2n}$

$\sum_{i=1}^{7} E(X_i) = \sum_{i=1}^{7} 10-2n$

$= \sum_{i=1}^{7} 10 - 2\sum_{i=1}^{7} n \qquad n=7$

$= 70 - 2.49 = -\$28$

Base on this expected value I'll not play this game

(3) The largest value of n so you are not loosing this game is $n=0$

$\sum_{i=1}^{5} 10 - 2\sum_{i=1}^{5} 5 = 0$

but if you want to win then $n=2$

$\sum_{i=1}^{2} 10 - 2\sum_{i=1}^{2} 2 = 12\$$

II Variant of quickSort

(1) Variant 1 : $T(n) = T(\frac{1}{x} n) + T(\frac{x-1}{x} n) + n$

cost: n $\quad * \frac{1}{x} \quad T(n) \quad * \frac{x-1}{x}$

n $\quad T(\frac{1}{x} n) \quad T(\frac{x-1}{x} n)$

h_{low} n $\quad T(\frac{1}{x^2} n) \quad T(\frac{x-1}{x^2}) \quad T(\frac{x-1}{x^2} n) \quad T((\frac{x-1}{x})^2 n) \quad h_{high}$

n $\quad T(1) \quad \dots \quad T(1)$

$x^{h_{low}} = n$

$\Rightarrow h_{low} = \log_x n \qquad \left(\frac{x}{x-1}\right)^{h_{high}} = n$

$\Rightarrow h_{high} = \log_{(\frac{x}{x-1})} n$

total cost $\Rightarrow \boxed{n \log_x n \le T(n) \le n \log_{\frac{x}{x-1}} n}$ (cost per lv is: n)

(2) Variant 2 : $T(n) = 2T(n/2) + n^{1.1}$

Master Theorem:

$\left.\begin{matrix} a=2 \\ b=2 \end{matrix}\right\} \Rightarrow n^{\log_2 2} = n \quad$ vs $f(n) = n^{1.1}$

$\Rightarrow f(n) \in \Omega(n^{1+\varepsilon}) \qquad \varepsilon = 0.1$

and $af(\frac{n}{b}) \le c$

$2\left(\frac{n}{2}\right)^{1.1} = \frac{2}{2^{1.1}} f(n) \Rightarrow c = 2^{-\frac{1}{10}} < 1$ ✓

$\Rightarrow \boxed{T(n) \in \Theta(n^{1.1})}$

(3) Variant 3 : $T(n) = T(n-1) + \sqrt{n}$

sub stitution

$= T(n-2) + \sqrt{n-1} + \sqrt{n}$

$= T(n-3) + \sqrt{n-2} + \sqrt{n-1} + \sqrt{n}$

$\vdots$

$= T(1) + \sum_{n=1}^{n} \sqrt{i}$

We have

$\sum_{n=1}^{n} 1 < \sum_{i=1}^{n} \sqrt{i} < \sum_{i=1}^{n} i$

$\Theta(n) < \sum_{i=1}^{n} \sqrt{i} < \Theta(n^2)$

$\Rightarrow \Theta(n) < T(n) < \Theta(n^2)$

$\Rightarrow$ The first case will be the best case for bigger array

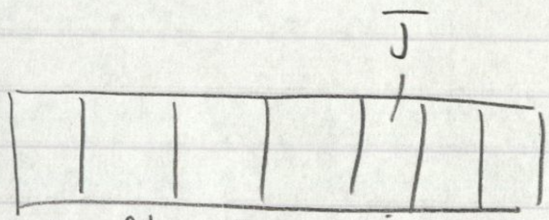
III. Linear Search

(1) Best Case: Appear in the 1^{th} element $O(1)$

(2) Worst Case: It is not even inside the array

$$O(n)$$

(3)



check	$\frac{n}{n}$	$\frac{n-1}{n}$	$\frac{n-2}{n}$	$\frac{n-j}{n}$
uncheck	$\frac{0}{n}$	$\frac{1}{n}$	$\frac{2}{n}$	$\frac{j}{n}$

$$\Rightarrow \boxed{X_j = \frac{n-j}{n}}$$

(4) Define X using n copies of random variable defined in previous step

$$X_{(s)} = \sum_{i=1}^{n} X_i(s)$$

(5) Expected value

$$E X_{(s)} = \sum_{i=1}^{n} E X_i(s)$$