Do Central Banks Respond to House Price Movements? A Bayesian DSGE Approach

by

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Abstract

This thesis explores whether monetary policy reacts to house price movements, by employing a New Keynesian model with a housing factor and estimated using Bayesian estimation techniques. The primary emphasis of this study centers on the United States, with supplementary analyses on central banks in several advanced economies. The principal finding of this thesis reveals that the Federal Reserve responded substantially but episodically to house prices. Furthermore, our investigation indicates that the Reserve Bank of Australia, the Bank of England, the Reserve Bank of New Zealand and the European Central Bank did not incorporate house prices into their policy framework, whereas the Bank of Canada did so episodically. The baseline result remains robust across a range of sensitivity checks.

Declaration

Unless explicitly acknowledged, this thesis is the product of my independent effort, articulated in my own words, and has not been previously submitted for assessment.

Furthermore, I affirm that it contains no material that has been employed to fulfill the requirements of any other academic degree or diploma bearing my name, whether conferred by a university or any other tertiary institution.

I also commit to refrain from incorporating any portion of this work, under my name, into submission for another degree or diploma at any university or tertiary institution without obtaining prior consent from the University of Adelaide and, where applicable, any partner institution involved in the joint awarding of this degree.

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1 Introduction

The United States subprime crisis and the 2008 financial crisis have heightened the importance of asset price trends, particularly within central banking circles. This heightened attention stems from the pivotal role of asset prices in the financial system, especially housing, which greatly affects wealth valuation, interest rate sensitivity, and collateral value in loan processes (Mishkin 2007). The interplay between monetary policy and the housing market has garnered significant interest. Many research studies have delved into how monetary policy affects house prices, indicating that house prices typically decline in response to contractionary monetary policy (Giuliodori 2005; Ahearne et al. 2005; Vargas-Silva 2008; Bjørnland and Jacobsen 2010).

In the US, house prices rose until 2008, with a significant surge from 2000 to 2005, exacerbating housing affordability issues. The US housing market dynamics suggest house prices influence business cycles, rather than merely reflecting macroeconomic trends (Iacoviello and Neri 2010). Hence, house prices might be crucial indicators in determining the position of monetary policy. This underscores the debate on whether monetary policy should react to house price movements to stabilize inflation and the output gap (Iacoviello 2005; Mishkin 2007; Jarociński and Smets 2008). Although the question remains open regarding the benefits of responding to house price movements in addition to inflation and output, dismissing the possibility that central banks have responded could introduce bias in the assessment of the interdependence between house prices and monetary policy. Exploring how monetary policy reacts to fluctuations in house prices and grasping the significance of house prices in the monetary policy transmission process is crucial for crafting an effective monetary policy strategy.

The very few studies that have examined house prices' role in monetary policy, and used the Vector Autoregression (VAR) method to explore interest rate's response to housing shocks (Giuliodori 2005; Iacoviello and Minetti 2008; Bjørnland and Jacobsen 2010). They find interest rates systematically respond to house price changes. However, to date, it is still uncertain if this is due to inflation's intermediary role or directly from housing. This thesis aims to contribute to the literature by empirically examining whether central banks have directly factored housing dynamics into their interest rate decisions. To achieve this objective, a more comprehensive model is necessary. A New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model with a housing factor appears to be a promising approach. To the best of my knowledge, none of the studies in the literature have examined monetary policy's time-varying response to house price movements using DSGE models with Bayesian estimation.

Unlike VAR models which are largely data-driven and might lack theoretical depth, DSGE models offer a solid theoretical framework. While VAR models might align closer with empirical data, DSGE models delve deeper into economic mechanics, capturing the stochastic behavior of economic variables and considering diverse sectors, and their interactions (An and Schorfheide 2007). They allow interpretations to be rooted in economic theories and streamline shock identification (Smets and Wouters 2003). Moreover, recent monetary policy research has emphasized the New Keynesian DSGE model, its theoretical coherence makes it well-suited for detailed theoretical and empirical policy examinations, as seen in works by Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007).

Within this framework, a backward-looking Taylor rule is examined. Following the foundational research by Taylor (1993), the Taylor rule has been acknowledged as a valuable tool in depicting the conduct of monetary policy in developed economies. I assume the central bank can respond directly to house prices by including a housing parameter in the Taylor rule. Then, I will utilize Bayesian techniques to estimate DSGE models with housing. Bayesian estimation utilizes data to understand unknowns using probability rules, it is particularly effective for DSGE models with numerous parameters, yielding a superior fit for fully solved DSGE models compared to other methods[1]. Crucially, Bayesian estimation provides a structured approach for evaluating the fitness of models.

The baseline study focuses on the United States. In order to assess whether the Federal Reserve (Fed) responded to fluctuations in house prices, I undertake two separate estimations of this New Keynesian DSGE model with housing. In the first estimation, the housing parameter in the Taylor rule is set to zero, signifying a model without a response to house price changes. In the second, it is set to a positive value, indicating a model that incorporates a response to such changes. A comparison between the log densities of the estimated models allows us to evaluate the overall fit of each to the available data, thereby determining if the central bank responded to shifts in house prices. The estimated parameters enable a thorough analysis of impulse response functions (IRFs), variance decomposition, and historical shock decomposition, enhancing the understanding of shock propagation and the monetary authority's reaction to housing price

1. DSGE models can face misspecification issues, leading the posterior distribution to align with inconsistent parameter regions. Bayesian techniques use priors as weights during estimation, ensuring the posterior distribution doesn't peak at irregular points. They also address model misspecifications by incorporating shocks, seen as observational errors, within the structural equations.

fluctuations. Subsequently, I will employ the rolling window method to determine if the central bank's approach to housing has shifted throughout the years. Furthermore, the same techniques will be used for extended analysis in Australia, New Zealand, the United Kingdom, the Euro Area, and Canada.

The empirical findings indicate that the Fed did react to house price movements. The further rolling window analysis suggests that this response was primarily between the years 1989 and 2004, coinciding with the recession of the 1990s and the housing bubble of the 2000s, periods characterized by significant house price volatility. Upon a thorough examination of the IRFs of these two models, my findings highlight the difference in the transmission of housing preference shocks, which is a disturbance to house prices. In the model that excludes housing from the Taylor rule, housing shocks only affect interest rates through their effects on inflation. Nevertheless, in the model that integrates housing, housing shocks exert a direct impact on interest rates. This is due to the significant housing parameter in the Taylor rule, establishing a direct and pronounced linkage between interest rates and house prices. In this way, the effects of inflation and housing on interest rates are separated. This allows us to identify how interest rates respond to house prices once it is established that monetary policy has factored in house prices in its interest rate decisions.

From forecast error variance decomposition analysis, monetary policy and inflation shocks turn out to contribute the most to the variation of interest rates, house prices, and the output gap forecasts in both models. While housing shocks appear to have a minimal impact on these variables, the proportion of variations in house prices attributed to housing shocks doubled when including housing in the Taylor rule. The historical shock decomposition further indicates that between 1990 and 2008, monetary policy and inflation played a significant role in influencing cyclical fluctuations in interest rates and output. Notably, housing preference shocks primarily explained the decline in house prices around 1990 and their subsequent rise in 2004. In the cross-country analysis, my findings suggest that, among the countries studied, only the Bank of Canada exhibits episodic responses to house prices, around housing market fluctuations in the 1980s and 1990s recession. Conversely, I do not find conclusive evidence of responses to house prices in Australia, New Zealand, the UK, and the Euro Area.

This thesis is structured as follows. I present a summary of the relevant literature in Section 2. Section 3 outlines the New Keynesian model setup. Section 4 presents the econometric strategy, while Section 5 documents the estimation results and quantitative analysis. Section 6 shows rolling window estimation for the US. In Section 7, I show cross-country analysis. Robustness checks are performed in Section 8. Finally, Section 9 concludes.

2 Literature Review

Numerous research efforts have delved into the relationship between the housing market and monetary policy. Iacoviello (2005) develops a New Keynesian model with a housing component to assess if central banks should adjust policies based on housing price changes. By adding a house price variable to the Taylor rule, he examines its effects on output and inflation volatility. Iacoviello (2005)'s results indicate that reacting to asset prices may have limited advantages for reducing output and inflation fluctuations. Mishkin (2007) discusses housing's role in the monetary system and its influence on monetary policy. Although housing is vital, he cautions against overly emphasizing house prices in policy decisions due to the challenge central banks face in identifying housing bubbles. Mishkin (2007) advocates for considering house prices in monetary policy only when they consistently affect inflation and employment. Iacoviello and Neri (2010) construct an estensive DSGE model, employing Bayesian estimation to explore US housing market dynamics and account for the persistent rise in real house prices over the past forty years. This trend is ascribed to the deceleration of technological advancement in the housing sector and the inclusion of land, an immutable factor, in the construction of new houses. Building on Iacoviello (2005) and Iacoviello and Neri (2010), Notarpietro and Siviero (2015) investigate if including house prices in a central bank's objectives can boost monetary policy efficacy, especially in economies with financial frictions. Their research reveals that the optimal monetary policy rule for maximizing welfare includes a responsive component to changes in house prices. Drawing from Iacoviello (2005)'s DSGE model, Sami and Alexander (2016) investigate the question of should monetary policy lean against housing market booms. Their investigation demonstrates that such proactive intervention can effectively mitigate the risks associated with the cyclical dynamics of debt. In this thesis, I will adopt a similar DSGE framework and restrictions on the Taylor rule as Iacoviello (2005), examining and contrasting models with and without the housing coefficient. However, my focus will shift from should the central banks respond to housing to exploring whether they indeed responded to housing.

In recent years, substantial empirical literature has emerged to examine central banks' reaction functions that incorporate asset prices. Lubik and Schorfheide (2007) employ Bayesian methods to a small-scale DSGE model of a small open economy. Their analysis involves a comparison of various Taylor rules. Through the application of posterior odds tests, they examine the question of whether central banks in Australia, New Zealand, Canada, and the UK have direct responses to exchange rates in their policy decisions. Remarkably, their findings reveal that the central banks of Australia and New Zealand indeed respond to exchange rates, whereas the Bank of England and Canada do not include exchange rates in their policy rule considerations. Kitney (2015) employs Bayesian methods to estimate a New Keynesian DSGE model featuring financial frictions. Kitney (2015) scrutinizes several monetary policy rules under the Henderson-McKibbin-Taylor framework to account for various aspects of the credit market, including credit growth, financial leverage, and credit spreads[2]. He indicates that the Fed responds to changes in credit spreads when determining the policy rate, as evidenced by a comparison of posterior odds. Additionally, Kitney (2015) carries out an analysis of IRFs and provides evidence that including credit spreads in the policy framework results in significant stabilization advantages. Existing studies have predominantly focused on analyzing the central bank's reactions to various assets, with limited attention directed toward house prices.

Deviating from the DSGE setting, there exists literature that investigates whether there are systematic interest rate adjustments in response to fluctuations in house prices, utilizing the VAR model. Bjørnland and Jacobsen (2010) explores the influence of house prices on the monetary policy transmission mechanism in Sweden, Norway, and the UK through the utilization of structural VARs. The study allows for simultaneous adjustments of interest rates and house prices in response to new information, revealing a notably enhanced role of house prices within the mechanism. Additionally, the researchers pinpoint a coherent and stable response of interest rates to house price variations. Nevertheless, it is crucial to highlight their acknowledgment that this systematic monetary policy response to house price shifts could partially originate from the impact of house prices on less contested objectives, like inflation. Aastveit, Furlanetto, and Loria (2023) utilize a Bayesian structural VAR model with stochastic volatility and time-varying parameters to examine the Fed's systematic reactions to variations in house and stock prices. Rather than strictly adhering to the conventional Taylor rule, they frame the interest rate equation in their VAR model—comprising of five variables: inflation, interest rate, output gap, stock price inflation, and house price—as an augmented monetary policy rule.

2. The Henderson-McKibbin Taylor rule is a Taylor rule variant introduced by Henderson and McKibbin (1993), includes more adaptable parameters and may account for additional financial variables.

This methodology allows them to track the Fed's consistent responses to changes in house and stock prices from 1975 to 2008. Aastveit, Furlanetto, and Loria (2023)'s study indicates that the Federal Reserve took into account real house price growth during 1991-1993, 1995-2000, and 2006-2008. My finding aligns with them for the first two intervals. However, I differ in the 2006-2008 period, which is noted for marking the end of the housing boom and the beginning of the subsequent downturn. This discrepancy might arise because Aastveit, Furlanetto, and Loria (2023)'s methodology estimates a time-varying coefficient, capturing accurate yearly shifts, whereas my rolling window approach can only depict a general trend due to its broader scope.

3 A New Keynesian Model with Housing

My theoretical foundation follows the work of Iacoviello (2005), who extends the New Keynesian setup of Bernanke, Gertler, and Gilchrist (1999) by introducing borrowing constraints linked to housing values and nominal debt. I analyze an infinite-horizon and discrete-time, economic structure inhabited by entrepreneurs, along with patient and impatient households. Patient households have a lower discount rate than both firms and impatient households. Entrepreneurs produce a homogeneous good by employing household labor and leveraging collateralizable real estate. Household activities encompass consumption, labor participation, and the requisition of real estate and money. The economic landscape includes retailers and a central bank. Retailers introduce nominal rigidity, while the central bank sets interest rates using a Taylor rule, responding to inflation, output, and house prices. To analyze the impact of asset ownership changes on the economy, this model considers housing investments by households and entrepreneurs. I assume constant real estate supply, leading to variable house prices. Given real-world financial constraints, both entrepreneurs and impatient households have limited borrowing capabilities. I denote patient households with "prime" and impatient households with "double prime"; unsignified characters signify entrepreneurs. This New Keynesian model's components and decisions are detailed below. The equilibrium conditions, steady state expressions, and log-linearized equations around zero inflation steady state are in Appendix B, Appendix C, and Appendix D, respectively.

3.1 Households

I employ a conventional household utility function and incorporate a housing parameter into it. The utility function maximized by patient households over their lifetime is expressed as follows:

$$\max_{c'_t, L'_t, b'_t, h'_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c'_t + j_t \ln h'_t - \frac{(L'_t)^\eta}{\eta} \right\}$$

Within this function, the operator E_0 denotes the expectation, β corresponds to the discount factor attributed to patient households, where β lies within the interval $(0, 1)$, j_t accommodates stochastic disruptions to the marginal utility of housing. Moreover, since j_t can directly impact housing demand, this approach provides a parsimonious means of evaluating the effect of an exogenous housing preference shock. At time t, c'_t signifies consumption, borrowing is B'_t, h'_t represents housing holdings, and L'_t stands for hours of labor.

Then the budget constraint of patient household (in real terms [3]) is

$$c'_t + q_t h'_t + \frac{R_{t-1} b'_{t-1}}{\pi_t} = w'_t L'_t + q_t h'_{t-1} + b'_t + F_t + T'_t \tag{1}$$

where the gross inflation rate is $\pi_t = P_t / P_{t-1}$, R_t signifies the gross nominal return on borrowing, F_t is lump-sum profits from firms. Then, Q_t is the nominal house price, the nominal wage is W_t, and P_t is the price of goods. T'_t is net transfers from the central bank.

Impatient households assign a higher discount to the future compared to patient households, as indicated by their discount factor $\beta'' < \beta$, which guarantees that impatient households are constrained around the steady states. Impatient household's optimization problem:

$$\max_{c''_t, h''_t, L''_t, b''_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta''^t \left\{ \ln c''_t + j_t \ln h''_t - \frac{(L''_t)^\eta}{\eta} \right\}$$

Impatient households' budget constraints is:

$$c''_t + \frac{R_{t-1} b''_{t-1}}{\pi_t} + q_t h''_t + \frac{R_{t-1} b''_{t-1}}{\pi_t} = w''_t L''_t + b''_t + q_t h''_{t-1} + T''_t \tag{2}$$

Durable assets such as housing serve a dual purpose, they function both as production factors and collateral for loans. The extent of credit limits is linked to the value of the housing used as collateral. Borrowing constraints are assumed for both impatient households and entrepreneurs,

3. Define $w_t = W_t / P_t$, $q_t = Q_t / P_t$, $b_t = B_t / P_t$ are the real wage, the real housing price, and the real borrowing respectively.

where the size of the nominal debt in the current period is restricted by the expected value of future housing, B''_t cannot exceed $m''E_t(Q_{t+1}h''_t/R_t)$. Writing this borrowing constraint in real terms[4] :

$$b''_t \leq m'' \mathbb{E}_t(\frac{q_{t+1}h''_t\pi_{t+1}}{R_t}) \tag{3}$$

This borrowing constraint conforms to the conventional lending standards employed in the mortgage market. These standards restrict the lending amount to a proportion of the asset's value.

3.2 Entrepreneurs

I use A to denote the technology parameter, K means capital (with a depreciation rate δ), h is the input of real estate, and L' and L'' represent patient and impatient labor respectively. The instantaneous conversion of output into consumption c_t, is infeasible. Entrepreneurs use capital, housing, and labor as inputs with a Cobb-Douglas constant returns-to-scale technology to manufacture an intermediate product $Y_{w,t}$:

$$Y_{w,t} = A_t K^{\mu}_{t-1} h^{\nu}_{t-1} (L'_t)^{\alpha(1-\nu-\mu)} (L''_t)^{(1-\alpha)(1-\nu-\mu)} \tag{4}$$

The assumption of this model is that retailers acquire the intermediate product $Y_{w,t}$ at the price of P^w_t from entrepreneurs. Subsequently, these retailers undertake a process of transformation, culminating in the creation of a composite final good with a corresponding price index of P_t. Then, the markup of final goods over intermediate goods can be denoted as $X_t = P_t/P^w_t$.

In equilibrium, entrepreneurial profits are assumed to surpass the interest rate, leading entrepreneurs to prefer borrowing to their limit, making the borrowing constraint binding. Moreover, to maintain the persistent binding nature of the borrowing constraint, I assume that entrepreneurs assign a higher discount rate to the future compared to patient households. This inclination prevents them from deferring consumption and quickly accumulating wealth. Entrepreneurs' problem:

$$\max_{c_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \gamma^t lnc_t$$

4. m'' is a parameter between zero and one signifying the loan-to-loan ratio. At zero, housing can't be used as collateral. If borrowers default, lenders can recover assets at a transaction cost of $(1 - m'')E_t(q_{t+1}h''_t)$.

where $\gamma < \beta$, and they are subject to the technology constraint, budget constraint, and borrowing constraint mentioned above(in real terms):

$$\frac{Y_{w,t}}{X_t} + q_t h_{t-1} + b_t - w_t' L_t' - w_t'' L_t'' = c_t + q_t h_t + \frac{R_{t-1} b_{t-1}}{\pi_t} + I_t + \xi_{k,t} \tag{5}$$

$$b_t \leq m\,\mathbb{E}_t\left(\frac{q_{t+1} h_t \pi_{t+1}}{R_t}\right) \tag{6}$$

where $\xi_{k,t}$ is capital adjustment cost:

$$\xi_{k,t} = \frac{\psi}{2}\Big(\frac{I_t}{K_{t-1}} - \delta\Big)^2 \frac{K_{t-1}}{\delta} \tag{7}$$

where the investment I_t follows:

$$I_t = K_t - (1-\delta) K_{t-1} \tag{8}$$

3.3 Retailers

Consider a continuum of retailers denoted by the index $z \in [0,1]$, who purchase intermediate products $Y_{w,t}$ at the cost of P_t^w. Subsequently, these retailers can costlessly transform the intermediate goods into retail outputs $Y_t(z)$ and vend them to a competitive final products firm at a price of $P_t(z)$. Final output produced by the final products firm ($\epsilon > 1$):

$$Y_t = \Big(\int_0^1 Y_t(z)^{\frac{\epsilon-1}{\epsilon}} dz\Big)^{\frac{\epsilon}{\epsilon-1}} \tag{9}$$

The retailer faces an individual demand and the price index is:

$$Y_t(z) = \Big(\frac{P_t(z)}{P_t}\Big)^{-\epsilon} Y_t \tag{10}$$

$$P_t^{1-\epsilon} = \int_0^1 P_t(z)^{1-\epsilon} dz \tag{11}$$

Retailers' profit function can be written as (in real terms[5]):

$$F_t(z) = P_t(z)^{1-\epsilon} P_t^{\epsilon-1} Y_t - X_t^{-1} P_t(z)^{-\epsilon} P_t(z)^{\epsilon} Y_t$$

During each time period, according to the Calvo assumption, the selling price has the potential to be adjusted with a probability of $1-\theta$. This implies that a proportion of $1-\theta$ among these

5. Nominal profit: $F_t(z)_{nominal} = P_t(z) Y_t(z) - P_t^w Y_{w,t}$. After replacing $Y_{w,t}$ by $Y_t(z)$, as $Y_{w,t}$ is transformed costlessly into $Y_t(z)$, plugging in demand function and dividing by P_t, we can get profit function in real terms

companies will change their price to a common reset price, denoted as P_t^*.

$$P_t^{1-\epsilon} = (1-\theta)P_t^{*,1-\epsilon} + \theta P_{t-1}^{1-\epsilon} \tag{12}$$

Additionally, retailers discount future profit by means of the patient households' stochastic discount factor, denoted as $\Lambda_{t,t+k} = \frac{c'_t}{c'_{t+k}}$. Then, they will have to pick the optimal $P_t(z)$ to maximize:

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \Big\{ \Lambda_{t,t+k} \Big[P_t(z)^{1-\epsilon} P_{t+k}^{\epsilon-1} Y_{t+k} - X_{t+k}^{-1} P_t(z)^{-\epsilon} P_{t+k}^{\epsilon} Y_{t+k} \Big] \Big\} \tag{13}$$

where, θ_k represents the probability that the price selected in time period t remains unchanged and remains effective until period $t+k$.

3.4 Monetary policy

I assume a backward-looking Taylor-type interest rate rule is employed by the Central Bank[6]:

$$R_t = (R_{t-1})^{r_R} (\bar{rr})^{1-r_R} \Big(\pi_{t-1}^{1+r_\pi} (\frac{Y_{t-1}}{Y})^{r_Y} \Big)^{1-r_R} e_{R,t} \tag{14}$$

In the given formula, $\bar{rr}$ symbolizes the steady-state real interest rate, while Y is the steady-state output. r_R serves as a parameter for smoothing, $1+r_\pi$ (where $r_\pi > 0$) is the coefficient of lagged inflation, and r_Y stands for the coefficient related to the output's deviation from the steady state, while $e_{R,t} \sim \mathrm{N}(0,\sigma^2)$ is a white noise shock (represents policy shock here).

Following the log-linearization process, it is assumed that the central bank can potentially respond to current fluctuations in house prices, incorporating a coefficient denoted as r_q:

$$\hat{R}_t = r_R \hat{R}_{t-1} + (1-r_R) \Big[r_q \hat{q}_t + r_Y \hat{Y}_{t-1} + (1+r_\pi)\hat{\pi}_{t-1} + r_Y \hat{Y}_{t-1} \Big] + \hat{e}_{R,t} \tag{15}$$

In the subsequent Bayesian estimation, I will estimate this model under two distinct scenarios. The first scenario is when $r_q = 0$, this signifies the original model without a direct response to housing by the central bank. The second scenario is when $r_q \neq 0$, represents the hypothesis that the Central Bank can indeed react to housing directly.

6. Backward-looking Taylor rules utilize readily available and accurate historical data, enhancing decision-making stability. Fuhrer (1997) provides empirical evidence that more robustly supports the adoption of a backward rule in comparison to a forward rule. McCallum (1999) highlights the delayed reporting of current-quarter output and inflation data, limiting real-time policymaker access.

3.5 Exogenous process

In the model, A_t, u_t, and j_t represent technology, inflation, and housing preference processes respectively. I assume these three variables follow an AR(1) process, where $e_{A,t} \sim \mathrm{N}(0, \sigma_A^2)$, $e_{j,t} \sim \mathrm{N}(0, \sigma_j^2)$ and $e_{u,t} \sim \mathrm{N}(0, \sigma_u^2)$ are white noise shocks, respectively representing technology shock, housing preference shock, and inflation shock. All of them are stationary (no unit root):

$$A_t = \rho_A A_{t-1} + e_{A,t} \tag{16}$$

$$u_t = \rho_u u_{t-1} + e_{u,t} \tag{17}$$

$$j_t = \rho_j j_{t-1} + e_{j,t} \tag{18}$$

4 Econometric Methodology

4.1 Bayesian Estimation with Metropolis-Hasting Algorithm

This thesis employs Bayesian techniques to estimate the parameters of the model, evaluate the goodness of fit for two different models using posterior model probabilities, and assess the significance of shocks. When addressing a rational expectations system, I employ the method outlined by Sims (2002). In this method, the log-linearized DSGE system can be expressed in the subsequent canonical form:

$$\Gamma_0 s_t = \Gamma_1 s_{t-1} + \Psi \epsilon_t + \Pi \eta_t, \tag{19}$$

where s_t represents a vector encompassing endogenous variables and expectations at time t, ϵ_t signifies a vector comprising shocks in the model, and a vector η_t captures one-period-ahead endogenous forecasting errors. The system matrices $[\Gamma_0, \Gamma_1, \Psi, \Pi]$ are functions determined by the parameters of the New Keynesian model $\theta_{\mathcal{M}}$. Throughout this thesis, I exclusively examine cases where the system possesses a unique stable solution. Consequently, in accordance with the standard algorithm introduced by Sims (2002), the solution of Equation (19) is a VAR in s_t:

$$s_t = \Phi_1(\theta_{\mathcal{M}}) s_{t-1} + \Phi_\epsilon(\theta_{\mathcal{M}}) \epsilon_t. \tag{20}$$

Equation (20) together with the observation equation

$$\mathcal{Y}_t = M + H s_t, \tag{21}$$

establishes a state-space representation for the New Keynesian model. In this representation, $\mathcal{Y}_t$ corresponds to the vector of observables at time t; M is a vector encompassing the means of the observed data; Vector H denotes the values that connect the model's definitions with the observed data. The detail of the observation equation is provided in Equation (29) later in the Section 4.2.

Priors are characterized by a density function:

$$p(\theta_{\mathcal{M}}|\mathcal{M}), \tag{22}$$

it is independent of the data and incorporates all available non-data information regarding $\theta_{\mathcal{M}}$, which represents a vector of model $\mathcal{M}$'s parameters, where $\mathcal{M} \in \{H, O\}$, H denotes the model that incorporates housing response in the Taylor rule, and O denotes the original model without housing response in the Taylor rule. The likelihood function $p(Y_T|\theta_{\mathcal{M}}, \mathcal{M})$ denotes the probability density of the observed data, and Y_T represents observations up to time period T: $Y_T = \{\mathcal{Y}_1, ..., \mathcal{Y}_T\}$. It will be estimated using the Kalman filter recursion based on the state space form above. It takes the form:

$$\mathcal{L}(\theta_{\mathcal{M}}|Y_T, \mathcal{M}) = p(Y_T|\theta_{\mathcal{M}}, \mathcal{M}) = \prod_{t=1}^{T} p(\mathcal{Y}_t|Y_{t-1}, \theta_{\mathcal{M}}, \mathcal{M}) \tag{23}$$

Following Bayes' theorem, the posterior density is derived by combining the likelihood function with the prior density:

$$p(\theta_{\mathcal{M}}|Y_T, \mathcal{M}) = \frac{p(Y_T|\theta_{\mathcal{M}}, \mathcal{M})p(\theta_{\mathcal{M}}|\mathcal{M})}{p(Y_T|\mathcal{M})}. \tag{24}$$

In the Equation (24), the marginal data density conditional on the model is $p(Y_T|\mathcal{M})$, which is given by

$$p(Y_T|\mathcal{M}) = \int_{\theta_{\mathcal{M}}} p(Y_T|\theta_{\mathcal{M}}, \mathcal{M})p(\theta_{\mathcal{M}}|\mathcal{M})\, d\theta_{\mathcal{M}}. \tag{25}$$

Finally, we are interested in using data to get information about parameters $\theta_{\mathcal{M}}$. Given that the marginal density is a constant term, it does not have to be in the estimation. Using the identities above, the posterior kernel can be written as:

$$p(\theta_{\mathcal{M}}|Y_T, \mathcal{M}) \propto p(Y_T|\theta_{\mathcal{M}}, \mathcal{M})p(\theta_{\mathcal{M}}|\mathcal{M}) \equiv \mathcal{K}(\theta_{\mathcal{M}}|Y_T, \mathcal{M}) \tag{26}$$

In light of the proportionality relationship presented in Equation (26), maximizing the posterior distribution is effectively equivalent to maximizing the posterior kernel, since optimizers remain effective when subjected to positive monotonic transformations. Given the conditions above, the log posterior kernel can be written as

$$\ln \mathcal{K}(\theta_{\mathcal{M}}|Y_T, \mathcal{M}) = \ln \mathcal{L}(\theta_{\mathcal{M}}|Y_T, \mathcal{M}) + \ln p(\theta_{\mathcal{M}}|\mathcal{M}). \tag{27}$$

The first item can be derived following the implementation of the Kalman filter mentioned earlier, and the second item represents our priors, which are already known.

For estimating the posterior distribution, I will simulate the posterior kernel using the Metropolis-Hastings (MH) algorithm, which is a Markov Chain Monte Carlo (MCMC) optimization routine. To efficiently launch the MH algorithm, I will use a Monte Carlo optimization technique to identify an optimal posterior mode[7]. This mode not only serves as a starting point for the MH algorithm but also provides an initial estimate of the posterior covariance matrix for the parameters in question. In general, MH is an algorithm based on rejection sampling employed for producing a series of samples (Markov Chain) from an initially unknown distribution. Following the insights of An and Schorfheide (2007), the MH algorithm carries out the following procedures:

1. Begin with an initial starting value θ^0, from the posterior mode, then start to repeat step 2,3,4.
2. Then for simulations S=1,...,N_{sim}, draw a candidate vector θ^* from a jumping rule distribution [8]
$$J(\theta^*|\theta^{t-1}) = \mathcal{N}(\theta^{t-1}, c\Sigma_m)$$
where c is the scale parameter that controls the size of the jump, Σ_m denotes the inverse of the Hessian computed at the mode[9].
3. Then compute the acceptance ratio
$$r = \frac{p(\theta^*|Y_T)}{p(\theta^{t-1}|Y_T)} = \frac{\mathcal{K}(\theta^*|Y_T)}{\mathcal{K}(\theta^{t-1}|Y_T)}$$

7. The "posterior mode" refers to the parameter value $\theta_{\mathcal{M}}$ that maximizes the posterior distribution. In essence, it represents the most possible values for the parameters given the observed data

8. The jumping distribution suggests new candidates based on the current parameter value. It usually uses a Gaussian distribution centered at the posterior mode with a scaled asymptotic covariance matrix (An and Schorfheide 2007).

9. A small scale factor can lead to a high acceptance rate and slow-mixing Markov Chain of candidate parameters, requiring extended convergence time to the posterior distribution, as the chain may become trapped around a local maximum, and vice versa.

4. Decide to keep or discard the candidate θ^* :

$$\theta^t = \begin{cases} \theta^* & p = min(r, 1) \\ \theta^{t-1} & otherwise \end{cases}$$

If the acceptance rate is more than 1, the candidate is retained. If it's less than 1, the candidate reverts to its last period value, keeping a portion of it. Subsequently, an adjustment to the mean of the sampling distribution will be made. Following repeated iterations of this procedure sufficiently, the posterior distribution will be constructed based on the values that have been retained. In my analysis, I maintain an acceptance rate ranging from 25% to 30% across various model specifications (Herbst and Schorfheide 2016).

Furthermore, the Bayes factor from Kass and Raftery (1995) will be used to evaluate the model's fit and identify the most data-supported monetary policy rule. The Bayes factor represents the ratio of the marginal data densities:

$$BF_{HO} = \frac{p(Y_T|\theta_H)}{p(Y_T|\theta_O)} \tag{28}$$

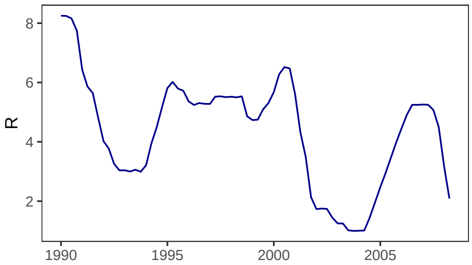
4.2 Data

The thesis uses four US quarterly time series: quarterly federal funds rate R_t, annualized quarterly growth rate of the GDP deflator $\Delta\pi_t$(seasonally adjusted), per capita real GDP growth rate ΔY_t, and demanded real house prices growth rate Δq_t[10]. The model is estimated over the period: 1990:Q1-2008:Q2, aligning with the era of the Great Moderation, characterized by notably decreased macroeconomic volatility. Figure 1 plots the time series (Details are in Appendix A).

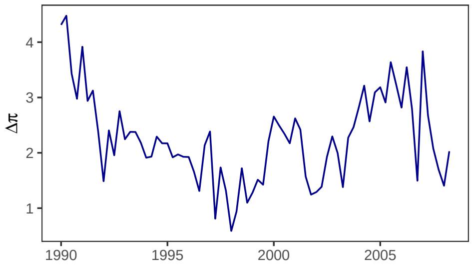
As aforementioned, the likelihood function can be estimated by the Kalman filter. However, we need to construct a state space representation that establishes a connection between the model variables s_t and a set of observables. While Equation (20) functions as the state equation, the observation equation is presented below

$$\begin{bmatrix} R_t \\ \Delta\pi_t \\ \Delta Y_t \\ \Delta q_t \end{bmatrix} = \begin{bmatrix} r^* + \pi^* \\ \pi^* \\ g^* \\ - \end{bmatrix} + \begin{bmatrix} 400\hat{R}_t \\ 400\hat{\pi}_t \\ 100(\hat{Y}_t - \hat{Y}_{t-1}) \\ 100(\hat{q}_t - \hat{q}_{t-1}), \end{bmatrix} \tag{29}$$

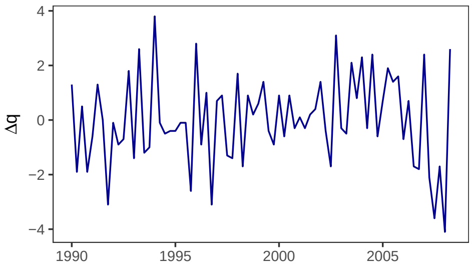
10. The model's primary measure is the Census Bureau's quality-adjusted index for new house prices, which is adjusted with the GDP deflator. I performed the long-to-wide data transformation manually, calculated the growth rates, and then manually computed the mean value for the purpose of demeaning the data.



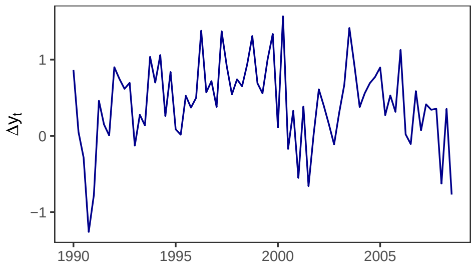
(a) Fereral Funds Rates



(b) Inflation Growth Rate



(c) Real House Prices Growth Rate



(d) Per Capita Real GDP Growth Rate

Figure 1. Time Series Plot for 1990:Q1-2008:Q2

Notes: Inflation growth rate is timed by 4. Real House Prices Growth Rate is demeaned.

where $g^* = 100(g-1)$, $\pi^* = 400(\pi - 1)$, and $r^* = 400(r-1)$, are steady state values.

4.3 Calibrated Parameters

In line with the applied DSGE literature, I calibrate fixed values for specific parameters that are challenging to precisely estimate from econometric models based on available data but can influence the state variables' steady-state values. The summary of calibration is shown in Table 1. I calibrate the standard parameter values by either directly adopting values from previous literature, primarily referring to Iacoviello (2005), or by choosing values within the typical range investigated in the real business cycle literature.

The Calvo price parameter θ is calibrated to a value of 0.75, which implies that the average duration intervening successive price adjustments is four quarters. I set labor disutility parameter η equal to 1.01[11]. I set X at 1.05, indicating a consistent 5 percent markup in the consumption-good sector. The capital depreciation rate δ is chosen to be equal to 0.03, consistent with Iacoviello and Neri (2010). For households' discount factors, I choose the patient household's discount factor β equal to 0.99 and set β'' at 0.95. This choice places β'' within

11. This suggests an almost flat labor supply curve, it may look weird, but it possesses the advantage of explaining the real wages' subtle changes in response to macroeconomic fluctuations.

the range of Lawrance (1991) and Samwick (1998)'s estimated values and keeps it lower than β[12]. This ensures that the impatient households are sufficiently impatient, making the collateral constraints bind near the steady state in the log-linearized model.

Table 1. Calibrated Parameters

Description	Parameter	Value
Entrepreneurs' discount factor	γ	0.98
Patient households' discount factor	β	0.99
Impatient households' discount factor	β''	0.95
Labor disutility	η	1.01
Housing services weight	j	0.1
Housing share	ν	0.03
Capital share	μ	0.3
Capital depreciation rate	δ	0.03
Markup	X	1.05
Wage share	α	0.64
Entrepreneurs' loan-to-value	m	0.89
Households' loan-to-value	m''	0.55
Probability unchanged price	θ	0.75

Following Iacoviello (2005) and Jin and Zeng (2007), I set the parameter γ to 0.98, which satisfies the model's assumption that $\beta'' < \gamma < \beta$. The elasticity of output with respect to entrepreneurial real estate ν is established at 0.03. Simultaneously, in conjunction with the elasticity of output concerning capital μ, which is set at 0.3, these parameters result in a 62% equilibrium value for the annual output of commercial real estate. The parameter j is assigned a value of 0.1. This selection results in a steady-state value of h/H (as shown in Appendix C), representing the entrepreneurial asset share, at approximately 20%. Additionally, j primarily governs the proportion of residential housing to annual output. When considered alongside μ and ν, this ratio remains relatively stable at around 150 percent.

Moreover, I determine the patient household wage share α to be 0.64 based on the estimated results of Iacoviello (2005). This choice strongly suggests the existence of households constrained by borrowing limitations, as their wage share $(1-\alpha)$ is approximately 36%. Lastly, in line with

12. Lawrance (1991) calculates discount factors for less wealthy households between 0.95 and 0.98 quarterly. In contrast, Samwick (1998) uses wealth data across age groups to determine discount factor distributions. He finds that about 75% of households have a mean discount factor of 0.99, whereas the other 25% have factors below 0.95.

Iacoviello (2005)'s methodology, I calibrate the loan-to-value ratios at 0.89 for entrepreneurs m and 0.55 for households m''.

4.4 Prior Distributions

Our prior distributions for the structural parameters are shown in Table 2 and Table 3. These priors align with findings from previous studies.

Table 2. Prior and Posterior Distribution When $r_q = 0$

	Prior			Posterior	
Parameter	Distribution	Mean	S.D	Mean	90% HPD interval
ψ	Gamma	2	1	4.66	[2.67,6.62]
ρ_j	Beta	0.5	0.2	0.91	[0.89,0.94]
ρ_A	Beta	0.5	0.2	0.31	[0.22,0.41]
ρ_u	Beta	0.5	0.2	0.05	[0.01,0.09]
r_R	Beta	0.75	0.1	0.5	[0.38,0.63]
r_π	Normal	1.5	0.1	1.51	[1.35,1.68]
r_Y	Normal	0.1	0.05	0.2	[0.14,0.27]
r^*	Gamma	2	0.5	1.99	[1.21,2.72]
π^*	Gamma	2.3	2	1.98	[0.09,3.80]
g^*	Normal	0.4	0.2	0.49	[0.28,0.59]
σ_R	Ive.gamma	0.5	0.1	0.066	[0.065,0.069]
σ_A	Ive.gamma	0.5	0.1	0.1	[0.09,0.12]
σ_j	Ive.gamma	0.5	0.1	0.17	[0.12,0.23]
σ_u	Ive.gamma	0.5	0.1	0.067	[0.065,0.069]

Following Iacoviello and Neri (2010), I employ inverse-gamma distributed priors for the standard errors of the innovations. These priors have a mean of 0.5 and a standard deviation of 0.1. Additionally, in line with Smets and Wouters (2007), I use a beta distribution with a mean of 0.5 and a standard deviation of 0.2 to AR(1) persistence parameters ρ_j, ρ_u, and ρ_A. The choice of the beta distribution is particularly optimal since it is defined on the interval [0,1), consistent with my assumptions of persistence parameters.

The prior for the capital adjustment cos ψ is defined by a gamma distribution with a mean of 2 and a standard error of 1, based on Iacoviello (2005)'s estimates. I ensure that ψ always exceeds 0, aligning with the gamma distribution's value range. For the monetary policy rule, initial assumptions are grounded in a Taylor rule with a primary and gradual response to inflation. When $r_\pi > 1$, a unique equilibrium is indicated. Following Iacoviello and Neri (2010), the prior distribution for the inflation coefficient r_π is a normal distribution around 1.50 with a 0.1 standard deviation. The interest rates smoothing parameter r_R has a mean of 0.75 and a 0.1 standard deviation. Given its requirement to be in the [0,1) interval, a beta distribution models it.

The response coefficients for the output gap and house prices have priors centered at 0.1 with a 0.05 standard deviation. This output prior aligns with Iacoviello (2005), who, using an OLS regression involving lagged interest rates, inflation, and detrended output, estimates r_Y to be around 0.1. Similarly, the housing prior mean resonates with Iacoviello (2005), suggesting an optimal r_q value between approximately 0.1 and 0.15. Finally, the prior distributions for the quarterly steady-state rates of interest r, inflation π, and output growth g^* are centered on their average values from the sample period.

4.5 Posterior Distributions

Table 2 displays the posterior mean and 90% highest posterior density (HPD) intervals for the structural parameters in the model that exclude response to housing[13]. Likewise, Table 3 presents the corresponding results for the model with response to housing.

Both tables show a higher AR(1) coefficient for the housing preference process, signifying its high persistence. In contrast, the coefficients for inflation and technological progress shocks are lower, indicating less persistence. None of the AR(1) coefficients reach the boundary of their 90% HPD interval, suggesting that they do not possess a unit root. In the subsequent section, I will delve into the implications of these findings with regard to IRFs. The posterior mean of the capital adjustment cost parameter ψ is twice its prior mean, indicating that in the presence of disturbances, capital adjustments are more sluggish than housing adjustments. The monetary policy rule estimates are consistent with those in Iacoviello and Neri (2010), with a reduced response to interest rates and a slight elevation in both inflation and output. The innovation variances for all four shocks have decreased, and these estimated innovation variances will be used subsequently to assess the response of endogenous variables to them.

When comparing the findings in Table 2 and Table 3, the addition of the housing element to the monetary policy rule results in minimal alterations in the estimated parameters. This

13. HPD interval represents a specific probability-based range within which an unobserved parameter value is likely to exist. This interval is situated within the domain of a posterior probability distribution.

trend will also be noticeable in the upcoming quantitative analysis of IRFs. Furthermore, the estimation yields a value of r_q equal to 0.11, indicating that the central bank's response to current-period house prices is somewhat similar in magnitude to its reaction to the lagged output gap. Nevertheless, it remains clear that inflation remains the primary focus of the monetary policy.

Table 3. Prior and Posterior Distribution When $r_q \neq 0$

	Prior			Posterior	
Parameter	Distribution	Mean	S.D	Mean	90% HPD interval
ψ	Gamma	2	1	4.40	[2.52,6.25]
ρ_j	Beta	0.5	0.2	0.93	[0.89,0.96]
ρ_A	Beta	0.5	0.2	0.44	[0.33,0.55]
ρ_u	Beta	0.5	0.2	0.04	[0.01,0.08]
r_R	Beta	0.75	0.1	0.49	[0.36,0.62]
r_π	Normal	1.5	0.1	1.53	[1.37,1.69]
r_Y	Normal	0.1	0.05	0.18	[0.11,0.25]
r_q	Normal	0.1	0.05	0.11	[0.06,0.16]
r^*	Gamma	2	0.5	1.95	[1.17,2.71]
π^*	Gamma	2.3	2	2.10	[0.02,4.14]
g^*	Normal	0.4	0.2	0.51	[0.41,0.6]
σ_R	Ive.gamma	0.5	0.1	0.066	[0.065,0.069]
σ_A	Ive.gamma	0.5	0.1	0.10	[0.08,0.10]
σ_j	Ive.gamma	0.5	0.1	0.20	[0.13,0.27]
σ_u	Ive.gamma	0.5	0.1	0.067	[0.065,0.069]

5 Properties of the Estimated Model

5.1 Model Comparison

Table 4 shows the empirical performance results for both the model includes and excludes housing in the Taylor rule. The log marginal data densities are shown in the table, which serve as indicators of the quality of each model's fit to the data. The Bayes factor or KR ratio is approximately 7.12, strongly suggesting the data favors including housing in the monetary policy model over its exclusion[14] . From this analysis, we infer that the Fed has responded to

14. Given Equation (28), according to Kass and Raftery (1995), the Bayes factor is computed as: 2(log data density H1 - log dara density H1)= $2(\ln p(Y_T|\theta_H) - \ln p(Y_T|\theta_O))$. The null hypothesis (H0) signifies the model without a housing response, which is less preferred. A K-R ratio between 2 and 6 shows "positive" evidence of

house price shifts during the sample period from 1990 I to 2008 II. In the following Section 6, a rolling window analysis will be used to broaden the sample period, enabling examination of the housing response's evolution and its implications over time.

Table 4. Bayesian log density for 1990 I-2008 II the United States

	Log data density
$r_q = 0 : \ln p(Y_T \| \theta_O)$	-917.72
$r_q \neq 0 : \ln p(Y_T \| \theta_H)$	-914.16
K-R ratio	7.12

5.2 Impulse Response Analysis

In this section, the objective is to examine the characteristics of the estimated DSGE model and discern the impact of introducing a housing variable into the Taylor rule. To achieve this, I employ the complete set of estimated parameters for two DSGE models. Figure 2 plots impulse responses using Bayesian posterior means of four key macroeconomics variables to four shocks, for these two models respectively. In the figure, the blue solid line represents the model without housing in the Taylor rule, while the red dashed line denotes the model with housing in the Taylor rule. Upon examining the IRFs of the two models, one can discern the differential response of interest rates to fluctuations in inflation and house prices. This analysis potentially evidences that house price movements have a direct effect on interest rates. This investigation can enhance our comprehension of how shocks operate within the monetary transmission framework, especially the role of housing. I initially examine the model excluding housing from the Taylor rule. Subsequently, I'll compare it to the model that incorporates housing into the Taylor rule.

5.2.1 Monetary Policy Shock

The impact of a contractionary monetary policy shock on the economy is illustrated in the first row of Figure 2. The shock size is set that there is an approximate 25 basis points increase in interest rates in response to this shock. Upon observation, it leads to a 5 basis points reduction in inflation, a 0.25 percent decline in real house prices, and a 0.5 percent decrease in the output gap. Inflation's response can be attributed to the fact that as interest rates elevate, individuals

the model reacting to house prices, while a 6 to 10 ratio indicates "strong" evidence.

tend to alter their preferences from spending to saving. Consequently, this shift in consumer behavior leads to a decrease in the demand for goods within the market, resulting in a downward pressure on prices and subsequently on inflation. Moreover, this response is relatively small, reflecting the price stickiness in the model. The negative reaction of inflation to higher interest rates is a well-documented phenomenon, observed in numerous research studies[15] (Iacoviello 2005; Bjørnland and Jacobsen 2010).

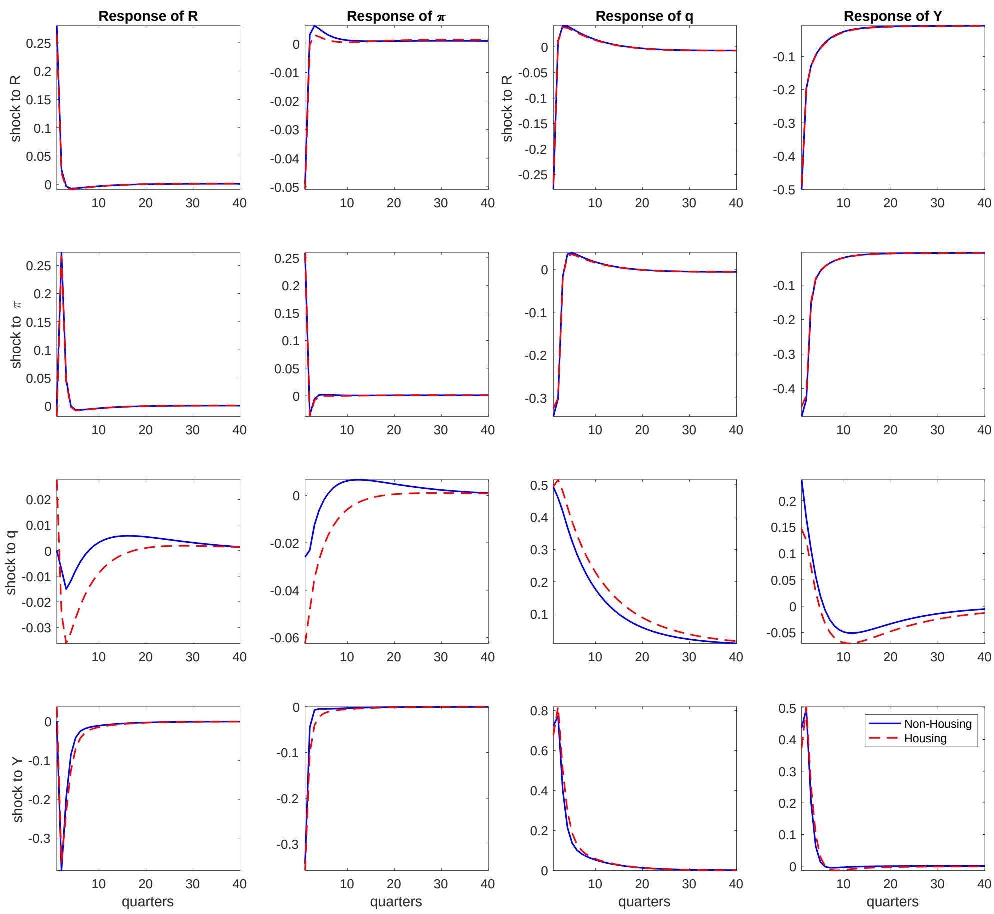


Figure 2. Impulse Response Functions for Model with and without Response to Housing

Concurring with Jarociński and Smets (2008), who employ a Bayesian VAR model to explore the impact of monetary policy shocks on US real house prices, an intermediate decline in house prices is observable. The influence of a monetary policy shock reverberates widely in the

15. However, Yuliya and Alejandro (2018) note a unique occurrence where inflation showed a positive response to a contractionary monetary policy shock during the pre-Volcker period, commonly referred to as the "price puzzle." Given that my sample begins in 1990, which falls within the post-Volcker era, I can confidently assert that there is no need to be concerned about the price puzzle problem.

housing market, particularly because the housing sector exhibits a greater sensitivity to interest rate changes compared to the broader economy. According to Mishkin (2007), an essential facet of the housing market's responsiveness to the monetary policy mechanism lies in the existence of both direct and indirect channels through which monetary effects are transmitted to the housing market. Monetary policy shifts directly affect housing user costs by influencing short and long-term interest rates. When short-term rates rise, they often drive up long-term rates due to anticipated future increases, subsequently elevating the average mortgage rate and thus increasing the user cost of housing. Moreover, Case and Shiller (2003) find that contractionary monetary policy reduces expected inflation and increases the after-tax user cost of housing. Elevated user costs decrease housing demand, thereby lowering house prices and output. Additionally, an indirect way through which effects are conveyed involves paths related to wealth and borrowing. When monetary policy tightens, the perceived value of individual housing wealth and actual house prices tend to fall due to a lessened demand for housing.

The response of output aligns with traditional theory, indicating that elevated interest rates may reduce aggregate demand and output. This observation is supported by numerous studies, including those by Mishkin (2007) and Goodhart and Hofmann (2008). Additionally, there are housing-dependent borrowing constraints in my model. Concurrent declines in house prices and inflation, alongside increased interest rates, restrict the borrowing capabilities of impatient households and entrepreneurs, as highlighted by Equation (6) and Equation (3). This, in turn, heightens the impact of external monetary policy disturbances, amplifying the adverse effects on the output gap[16].

5.2.2 Inflation Shock

The second row of Figure 2 reveals a 25 basis point transitory increase in inflation, according to the setting. It returns to the steady state after roughly three quarters, congruent with the autocorrelation parameter $\rho_u = 0.2$, signifying low persistence in inflation shocks. Consequently, output decreases by 0.5 percent, primarily due to heightened production costs from price increases, thereby reducing goods supply at existing price levels. Meanwhile, in my model, the debt is assumed to be nominal. Apart from the supply-side impact, it results in a shift of wealth from lenders to borrowers. Borrowers typically exhibit a higher propensity to consume, thereby

16. As mentioned by Iacoviello (2005), it is noted that monetary shocks' effects are heterogeneous, predominantly impacting debtors who bear the brunt of the monetary tightening, while lenders are only mildly affected.

increasing output to some extent. Eventually, the supply-side effects outweigh the borrower's demand-side influence, leading to a significant decline in output. This effect, known as debt deflation, has been discussed by Iacoviello (2005), and he also shows if the debt is indexed to inflation, the drop in output will be more substantial.

Moreover, an aggregate supply shock typically presents central bankers with a trade-off involving the balancing act of stabilizing inflation and stabilizing economic output[17]. In the figure, there is a sluggish 25 basis point increase in interest rates in response to a higher inflation rate. This sluggish adjustment can be attributed to the backward-looking nature of interest rates, and it suggests that the central bank's primary aim is to maintain stability in inflation, in line with the Taylor rule parameters in Table 2, where $r_\pi > r_Y$. The observed increase in interest rates aligns with the findings of several researchers who have explored the effects of an inflation shock, for instance, Iacoviello (2005), Goodhart and Hofmann (2008), and Demary (2010).

Furthermore, Demary (2010) applies a VAR model to investigate the impact of an inflationary shock on interest rates and housing prices across ten OECD nations. His findings reveal that in countries where the inflationary shock leads to a reduction in economic output, house prices tend to decrease. These results provide evidence of a significant connection between the output gap fluctuations and housing prices. This co-movement phenomenon is also observable in Figure 2. In the context of my model, when the output gap decreases and interest rates rise in response to an inflationary shock, there is a corresponding decline in house prices by 0.3 percent. One possible explanation for this decline in house prices following an inflationary shock is that the central bank may attempt to calm inflationary pressures by increasing interest rates. This, in turn, makes mortgage repayments more costly for households and consequently diminishes overall housing demand (Demary 2010).

5.2.3 Housing Preference Shock

The third row depicts the effects of a housing preference shock. A positive shock in housing preference increases households' demand for housing and thus leads to higher house prices. The housing shock size is set to lead to a 0.5 % increase in house prices. It takes approximately 40 quarters for house prices to return to their steady-state level. The sustained increase in house

17. An inflation shock can be viewed as a disturbance to aggregate supply, as it results in opposing movements in output and prices.

prices reflects the prolonged impact of the preference shock's dynamic process, consistent with the estimated autocorrelation $\rho_j = 0.91$. This aligns with the conclusions drawn from various studies that investigate housing dynamics following a preference shock within the DSGE framework. Notable examples include the works of Iacoviello and Neri (2010) and Ng (2015). Rising house prices initially reduce entrepreneurs' housing demand but also significantly loosen their borrowing constraints. As illustrated in Figure E.1, both entrepreneurs and impatient households experience enhanced collateral capacity, leading to increased borrowing and consumption, and then a subsequent 0.2 percent rise in the output. As time progresses, escalating housing prices deter entrepreneurs from housing-related investments, making the alleviation of their constraints temporary. Consequently, patient and impatient households predominantly consume housing, leading to an eventual decline in overall output. US-based research by Demary (2010) similarly reveals that a positive house price shock initially boosts the output gap for the first four quarters, then declines into negative territory over the ensuing 12 quarters.

With the relaxation of borrowing constraints, impatient households gain the capacity to persistently increase their borrowing. As housing currently offers relatively higher utility, impatient households shift their preferences away from consumption to boost their housing acquisitions. Consequently, the demand for non-housing consumption goods diminishes, resulting in a decline in overall non-housing production and inflation. This is demonstrated in the graph with a 3 basis point inflation reduction. This observation aligns with the findings of Sami and Alexander (2016), who assess the potential for monetary policy to counter the risk of a housing market crash using a DSGE model, they find that in response to a 0.5 percent increase in housing prices, inflation decreases by 2 basis points.

Notably, interest rates exhibit minimal responsiveness to this shock. Because a direct correlation between housing preference shocks and interest rates is currently absent, with any influence being mediated by the impact of housing shocks on inflation and output. This suggests that in a scenario when the model does not incorporate direct reactions to housing changes, inflation primarily drives the adjustments in interest rates. When faced with a downward shift in inflation caused by the housing shock, the interest rate exhibits a sluggish reduction of 1 basis point. Andrea and Morana (2014) employ a factor VAR model and the double Cholesky to identify structural shocks, finding that inflation initially reacts negatively to house price shocks, then increases before stabilizing. A similar pattern is observed in my model. The subsequent inflation increase may stem from the transient nature of relaxed borrowing constraints among impatient households, enduring for roughly five quarters. After this, demand for non-housing goods starts reverting, driving an inflationary uptick.

5.2.4 Productivity Shock

The final row of Figure 2 illustrates the reactions to an enhancement in productivity. The productivity shock size is set to increase output by 0.5%. An upsurge in productivity increases the efficiency of production processes, enabling firms to generate greater output utilizing the same resource base, and thus increase the output. It's noteworthy that the response of the output to a productivity shock is not as persistent as housing but exhibits more persistence than inflation's response to an inflation shock. This pattern is consistent with our estimated results for ρ_A, which equals 0.44. This parameter suggests that the output tends to exhibit moderate persistence in response to disturbances.

Meanwhile, an increase in productivity can be interpreted as a reduction in the cost of producing each unit, potentially leading to lower prices of products and services, thereby alleviating inflation pressures. This highlights the deflationary impact of a productivity shock, which can result in a 35 basis point reduction in inflation. The central bank, being highly sensitive to inflation shifts, subsequently reduces interest rates by 35 basis points. This explains why interest rates decrease despite an expanded output gap. The reactions of inflation and interest rates to productivity shocks have been thoroughly explored in academic research. For example, Zeno, Gernot, and Almuth (2011) conduct a VAR analysis on US time series data in their business cycle model to study the impact of a positive technology shock, finding a significant drop in both interest rates and inflation post-shock.

A productivity rise could lead market participants to experience stronger income growth, not due to working longer hours, but rather on a per-hour basis. As individuals become increasingly aware that this enhanced growth was a result of technological progress and could be sustainable, their optimism about future income grows, thus directly impacting their willingness to invest in housing. In the figure, there is a 1 percent increase in house prices in response to a productivity shock. As argued by Kahn (2009), a significant portion of price fluctuations from 1965 to 2007 can be attributed to shifts in productivity growth. These fluctuations in productivity have played a pivotal role in determining housing prices by influencing long-term income expectations and income growth. An interesting observation is that the peak of output and house prices experiences a time lag subsequent to productivity improvement. This is because housing price increases induce entrepreneurs to trim their housing inventory, which hampers the upsurge in output.

5.2.5 Comparison

In comparison, it is evident that upon incorporating housing into the Taylor rule, the responses of endogenous variables to monetary policy, inflation, and productivity shocks remain largely consistent. A noticeable divergence emerges when we analyze the IRFs of a housing preference shock. In Figure 2, in response to a 0.5% increase in house prices, the interest rate response exhibits marked differences when compared to the model that does not account for housing. In the absence of housing considerations, the interest rate response tends to be negative due to the exclusive response to inflation. By allowing the inclusion of housing in the Taylor rule, interest rates establish a direct relationship with house prices. When house prices rise, interest rates respond directly and immediately to housing shocks, resulting in a 3 basis point increase in interest rates within the initial two quarters. Interestingly, it's worth noting that interest rates exhibit a decline into negative territory after the initial two quarters. This phenomenon occurs because the central bank maintains a robust but sluggish response to inflation. As in the Taylor rule, the central bank can respond to lagged inflation but to current house prices. As time progresses, the central bank's reaction to negative inflation becomes more pronounced, essentially taking precedence over other factors. Consequently, this heightened response to inflation leads to the subsequent reduction in interest rates.

Meanwhile, inflation's negative response moves from 3 basis points to 6 basis points. This is reasonable given that the inclusion of housing results in a higher σ_j and ρ_j. It could lead to a higher and more persistent house price increase. This, in turn, leads to a higher increase in impatient household borrowing and a decrease in demand for non-housing goods, subsequently driving down inflation. The response to the output gap remains similar, with a slight reduction in magnitude from 0.2% to 0.15%. This adjustment is primarily due to the decrease in demand for non-housing goods we mentioned above.

5.3 Variance Decomposition

To identify which shocks account for forecast errors in chosen macroeconomic variables and evaluate the significance of individual shocks, I utilize conditional forecast error variance decomposition, which is derived using a Cholesky decomposition of the covariance matrix related to exogenous variables. All variance decompositions are calculated with respect to the posterior mean distribution. Time-varying variance decomposition is provided in Appendix E.

Table 5. Variance Decomposition of the Forecast Error when $r_q = 0$

	$\hat{R}$	$\hat{\pi}$	$\hat{q}$	$\hat{Y}$
$\hat{e}_R$	59.90	5.42	31.74	52.16
$\hat{e}_u$	37.06	89.47	49.74	45.91
$\hat{e}_j$	0.02	0.08	8.51	0.39
$\hat{e}_A$	3.02	5.03	10.01	1.53

Table 6. Variance Decomposition of the Forecast Error when $r_q \neq 0$

	$\hat{R}$	$\hat{\pi}$	$\hat{q}$	$\hat{Y}$
$\hat{e}_R$	57.74	5.56	28.16	53.25
$\hat{e}_u$	38.35	87.78	44.90	44.58
$\hat{e}_j$	0.12	0.48	15.26	0.48
$\hat{e}_A$	3.79	6.18	11.69	1.70

The outcomes from the model that exclude housing in the Taylor rule are displayed in Table 5. Interest rate fluctuations arise primarily from monetary policy and inflation shocks, contributing about 60% and 37% respectively. Housing shocks minimally influence interest rates, affecting them indirectly through inflation. This underscores the prominence of interest rates and inflation in interpreting US monetary policy. Inflation movements are mostly due to the inflation shock, accounting for a dominant 89%, with housing playing a negligible role in explaining inflation variations. The pattern of interest rate and inflation shock is common in the monetary economics literature (Iacoviello and Neri 2010; Demary 2010).

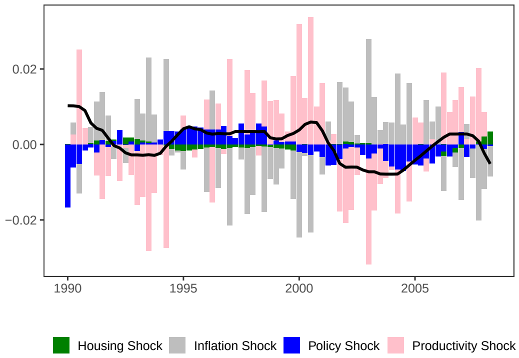
In Table 5's third column, house price fluctuations mainly result from interest rate and inflation shocks, while housing preference shock accounts for roughly 9%. This matches Tsatsaronis and Zhu (2004)' results, where these shocks majorly affect house prices, and housing accounts for about 8.9% of the variables. Inflation causes 50% of house price fluctuations, which is notable considering house prices are in real terms. A plausible explanation is housing's dual role as consumption and investment, making it a favored inflation hedge for households. In the fourth column, the output gap shifts mostly due to interest rate and inflation shocks, at 52% and 46% respectively, reflecting the output supply's sensitivity. In contrast, the productivity shock offers minimal understanding of output gap changes, consistent with Demary (2010)'s findings.

By incorporating housing into monetary policy, the influence of each shock on every variable remains largely the same, as depicted in Table 6. The housing preference shock's explanatory power increases. For example, its contribution to the variance in house prices grows from 8.51% to 15.26%. This change is primarily due to the addition of the housing factor in the interest rule decision, heightening its relevance and impact across the system. A similar trend is evident with the productivity shock, where its explanatory power for each variable also sees a marginal uptick. However, in both models, the share of output gap fluctuations attributed to a monetary policy shock seems overly large, contrasting with expectations from the post-estimation monetary policy rule. For context, Iacoviello and Neri (2010) find a monetary policy shock accounts for roughly 22.6% of output variance, while Demary (2010) notes it explains 14% of output changes. A potential reason is the limited range of shocks studied. My model only includes four general shocks, whereas Iacoviello and Neri (2010) examine ten, allowing a more nuanced analysis. It is important to mention that a monetary policy disturbance can affect other variables like investment, which then impacts output. Overlooking potential shocks leads to ascribing the influence of those shocks to the monetary policy shock.

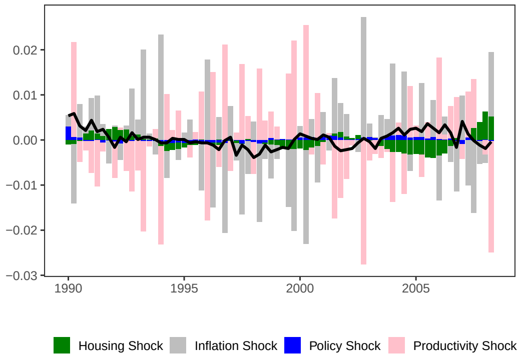
5.4 Historical Shock Decomposition

An intriguing question that emerges from the variance decomposition study we conducted earlier is the extent to which shocks have influenced the business cycles in the US. Figure 3 and Figure 4 provide visual representations of the historical shock decomposition for both $r_q = 0$ and $r_q \neq 0$ models respectively. The decomposition of historical shocks is computed upon the Kalman smoother, which dissects the historical divergences of endogenous variables from their steady-state values, attributing them to the influences of diverse shocks. The solid black line portrays the smoothed historical data in deviation from the mean, while the accompanying lines illustrate the four shocks based on the estimated parameters.

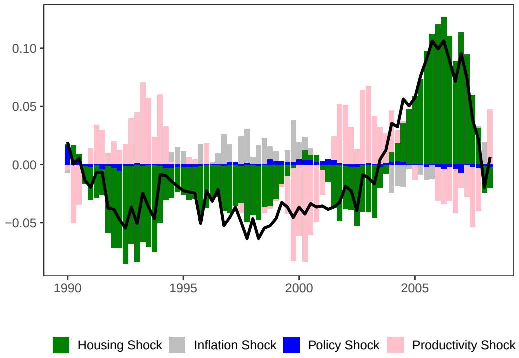
As shown in Figure 3, from 1990 to 2008 in the model without housing in the Taylor rule, there were modest fluctuations in interest rates and inflation. The most significant contributors to fluctuations were the shocks associated with inflation and productivity. Conversely, housing preference shocks exerted a comparatively minor influence. Furthermore, the movement in real house prices predominantly aligned with housing preference shocks. Apart from these,



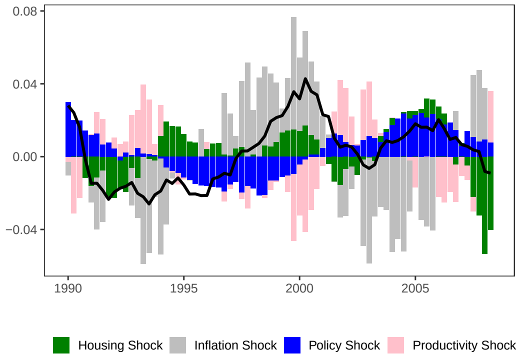
(a) Interest Rates



(b) Inflation



(c) Real House Prices

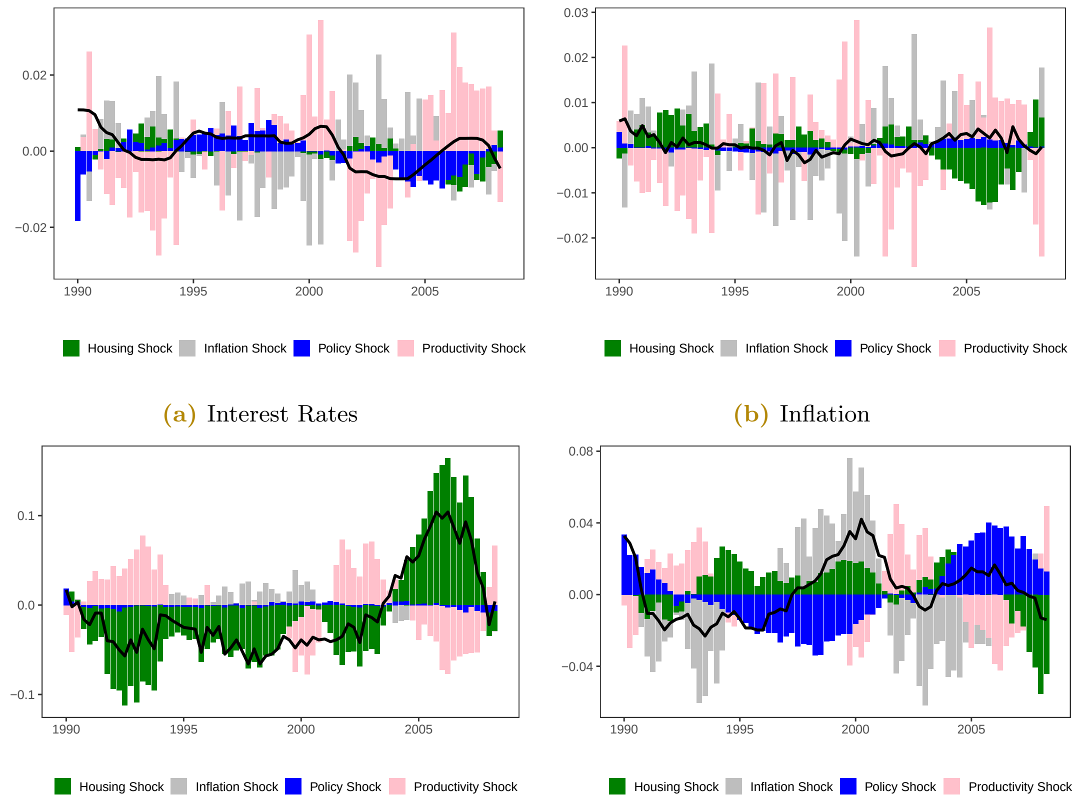


(d) Output

Figure 3. Historical Shock Decomposition when $r_q = 0$

productivity shocks provide some downward pressure on house prices, and they seem more substantial than inflation shocks. This is evident from the substantial increase in housing prices from 2002 to 2005; during this period, housing preference stood as the primary factor explaining the movement, followed by a contribution from productivity shocks. Figure 3d displays the historical shock decomposition of the output gap. Two output gap expansions are visible: 1997-2000 and 2003-2005. During these times, housing preference shocks had a significant positive effect. In the first expansion, inflation shocks had a greater impact than both monetary policy and housing shocks. However, the latest rise in the output gap shows inflation as a negative factor, with housing preference and monetary policy shocks becoming more influential.

In the model accounting for house prices, the overall contributions of each shock display a degree of similarity. However, incorporating housing into the monetary policy enhances the impact of housing preference shocks. As shown in Figure 4a, housing preference shocks now account for more interest rate variation, due to the central bank considering housing dynamics. Housing preference shocks also have a stronger influence on historical inflation patterns compared to Figure 3. Moreover, there has been a growing impact of housing preference shocks on the fluctuations in house prices. This trend is evident from the decline in house prices during the years 1990-1993, where the proportion of the drop attributable to housing preference shocks rose from 0.08 to 0.12. A comparable increase in the explanatory capacity of housing price shocks can also be witnessed in relation to the output gap. This aligns with our prior discourse on variance decomposition, where introducing the housing factor amplifies the role of housing preference shocks in accounting for variations in each endogenous variable.



(a) Interest Rates (b) Inflation

(c) Real House Prices (d) Output

Figure 4. Historical Shock Decomposition when $r_q \neq 0$

6 Rolling Window Analysis

Based on the results highlighted earlier, it would be intriguing to examine how the Fed's reaction to housing has evolved over time. To explore these potential shifts, I utilize the rolling window technique to analyze alterations in the response variable related to housing r_q. The sample encompasses 1969 I to 2019 I. The extended sample period commences in 1969, as this marks the inception of available house price data. I stopped in 2019, just before the onset of the

COVID-19 recession. The first window is 1969 I-1979 I. I adopt a rolling window size of 10 years and a step size of 5 years between consecutive rolling windows and re-estimate all parameters each time, then get the posterior mean in each sample. The detailed breakdown of windows is available in Table 7.

Table 7. Rolling Window Time Periods

	T1	T2	T3	T4	T5	T6	T7	T8	T9
Years	1969 I-1979 I	1974 I-1984 I	1979 I-1989 I	1984 I-1994 I	1989 I-1999 I	1994 I-2004 I	1999 I-2009 I	2004 I-2014 I	2009 I-2019 I

In the baseline study, the primary focus is on the period prior to the Great Recession. This choice is influenced by the circumstance that the federal funds rate is bounded by a zero lower threshold(ZLB) between 2009 and 2016. The ZLB rendered traditional monetary policy by limiting further interest rate reductions, prompting the Fed to employ unconventional tools like quantitative easing to influence long-term rates and stimulate the economy, it is a significant challenge in economic research. Wu and Xia (2016) introduce the shadow federal funds rate for the US economy analysis, representing the hypothetical federal funds rate without ZLB limitations, and incorporating negative values to account for unconventional policy use. Wu and Zhang (2019) further show that within a New Keynesian model, the shadow rate effectively conveys the impact of unconventional monetary policies similar to negative interest rates. In essence, the shadow rate can be regarded as a concise summary statistic that encapsulates the impact of unconventional monetary policies within the domain of interest rates. In my forthcoming rolling window estimation, the shadow rate is employed as a monetary policy instrument for the period from 2009: Q2 to 2015: Q4.

6.1 Housing Prices Parameter

Table 8 displays log data densities for models with and without housing in the Taylor rule over various time frames. The model with housing response outperforms solely in the periods 1989 I to 1999 I and 1994 I to 2004 I. A thorough analysis of the K-R ratio confirms positive evidence for the central bank's significant housing response only during these intervals, as also depicted in Figure 5. From 1969 to 1989, the central bank showed limited housing response, which then slightly decreased from 0.03 to 0.01 between 1974-1984, aligning with recessions in 1973, the 1980s, and 1981. Housing responsiveness increased in the periods 1989 I-1999 I and 1994 I-2004 I to values of 0.12 and 0.11. There was a further decline during T7 and T8, aligning with the

2008 Great Recession.

Table 8. Rolling Window Bayesian results for the US

	T1	T2	T3	T4	T5	T6	T7	T8	T9
Log-density									
$r_q = 0$	-611.05	-617.06	-590.48	-570.52	-573.13	-555.04	-597.28	-601.26	-578.21
$r_q \neq 0$	-613.00	-619.45	-591.64	-570.89	-570.72	-553.03	-598.12	-602.34	-578.23
K-R ratio	-3.9	-4.78	-2.32	-0.72	4.82	4.02	-1.68	-2.16	-0.02

In the early 1990s, the US faced an economic downturn due to the savings and loan crisis, largely sparked by the Tax Reform Act of 1986 which limited tax deductions on real estate investment losses. This led to declines in real estate and financial sectors, reduced credit availability, slowed economic activity, and a drop in house prices from 1991 (Leamer 2007; Bouchouicha and Ftiti 2012). To boost the economy, the central bank adopted an expansionary monetary policy. Our analysis suggests that house prices influenced this decision. Bouchouicha and Ftiti (2012) also find a correlation between housing indices and macroeconomic indicators during this period. This is consistent with Aastveit, Furlanetto, and Loria (2023) who, using a Bayesian structural VAR model, discover the Fed occasionally responded to house price changes, specifically from 1989 to 1994 and 1998 to 2001.

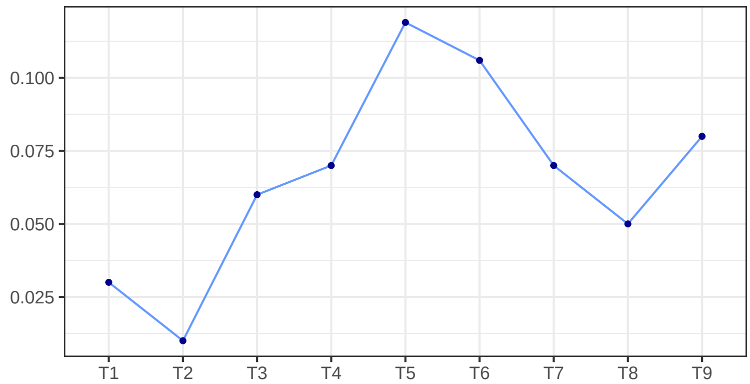
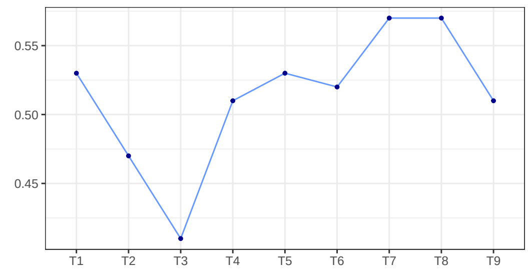


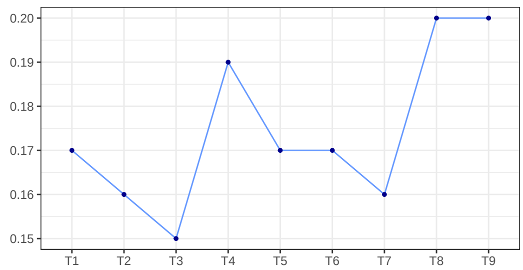
Figure 5. Time Varying Response to Housing r_q from 1969 to 2019

6.2 Other Monetary Parameters

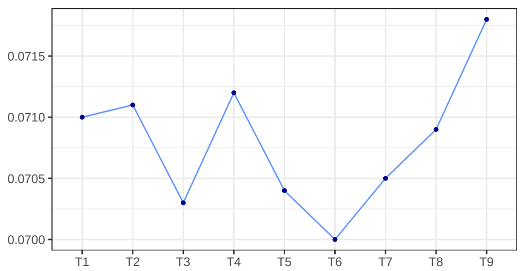
To ensure research comprehensiveness, I also conduct cross-temporal analyses of various monetary policy parameters. Figure 6d illustrates the response to inflation is stable across different years, with a marginal increase from T2 to T6, and a slight decline around the 2008 Great Recession. The estimated response to inflation from T1 to T6 aligns with findings by Fernández-Villaverde and Rubio-Ramírez (2008), who report an increased inflation response from the 1970s to the 2000s using a DSGE model with dynamically changing structural parameters. Similarly, Haque (2022) also notes a trend of increased responsiveness in the Taylor rule to the inflation gap. Conversely, the decrease observed from T6 to T8 aligns with Belongia and Ireland (2016)'s results. They employ a TVP-SV-VAR (Time-Varying Parameter Stochastic Volatility Vector Autoregression) model and find evidence of a slightly reduced response to inflation from 2000 to 2007. Nevertheless, this pattern underwent a reversal after 2009, as the shadow rate began to display a robust response to economic fundamentals (Aastveit, Furlanetto, and Loria 2023).



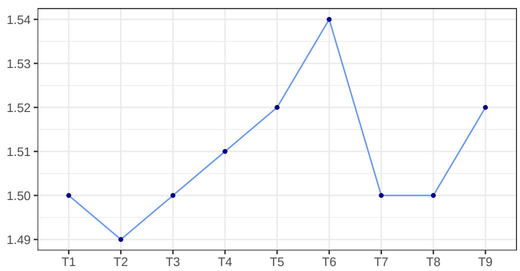
(a) Interest Rates Smoothing r_R



(b) Response to Output r_Y



(c) Standard Deviation of Policy shock σ_R



(d) Response to Inflation r_π

Figure 6. Central Bank Time-Varying Parameters from 1969 to 2019

Regarding the inertia in adjusting interest rates, as illustrated in Figure 6a, there has been an increasing trend in policy setting inertia over time as r_R grows. This observation aligns with the findings of Haque (2022), who finds there is an increase in the inertia of interest rate settings by comparing two sample periods: 1960-1979 and 1989-2008, which closely align with my rolling window sample periods. Figure 6c illustrates the sizes of the monetary policy shocks are stable, declining marginally from 0.071 to 0.07, before exhibiting a slight increase to 0.0716 by T9. This discovery aligns with the conclusions drawn by Cogley, Primiceri, and Sargent

(2010). They identify a marginal decrease in σ_R over two distinct datasets spanning the same timeframe as the current paper's selected range, T1-T6, albeit with a milder decline observed in this paper. In line with the research of Belongia and Ireland (2016) and Haque (2022), there has been a modest increase in the output gap coefficient, rising from 0.17 to 0.2 between 1969 and 2019.

7 Cross Country Analysis

Addressing house price fluctuations is controversial, policymakers in different countries may hold unique concerns and approach this matter with varying responses (Selody and Wilkins 2004). Moreover, Ahearne et al. (2005) discover scant proof within central bank reports, minutes, and speeches indicating that foreign central banks responded to previous instances of increasing real house prices, apart from considering their impact on inflation and economic expansion. Considering the model's success in the baseline study for the US, it would be valuable to explore if central banks in other nations directly react to house prices using a Bayesian rolling window estimation akin to the baseline[18].

Country	Years
Australia	1973:Q2-2008:Q2
New Zealand	1990:Q2-2008:Q2
The United Kingdom	1987:Q2-2008:Q2
The Euro Area	1996:Q2-2008:Q2
Canada	1973:Q2-2008:Q2

Table 9. Sample Periods of Different Economies

Notes: The starting years are determined primarily by data availability and slightly adjusted to accommodate rolling windows. The analysis stops at 2008 for all regions due to the Global Financial Crisis, since unlike the US estimation, the availability of efficient shadow rates for each of these regions cannot be guaranteed.

Chosen economies and respective sample years are provided in Table 9. As previously noted, a K-R ratio above 2 suggests a significant response from the Central Bank to housing. In the upcoming study, I will present K-R ratios for various countries within specific windows. The

18. For these five regions, I use data similar to my baseline estimation, with one exception related to output. Since real GDP per capita data is unavailable for these specific regions, real GDP data from Fred is used and real GDP per capita is manually calculated by dividing the real GDP data by the corresponding population data.

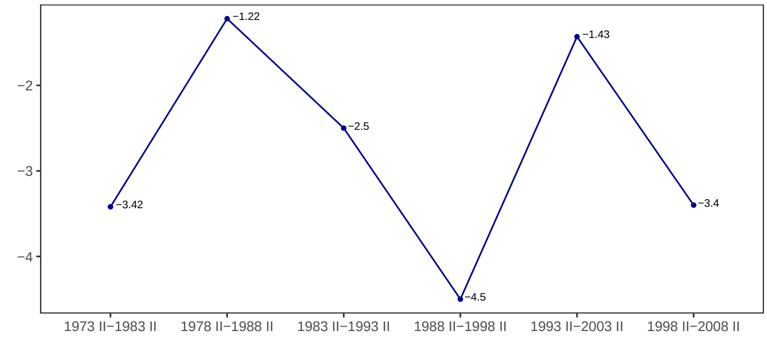
rolling window size for each country is determined by the data's characteristics. Detailed log densities are provided in Appendix F.

7.1 Australia and New Zealand

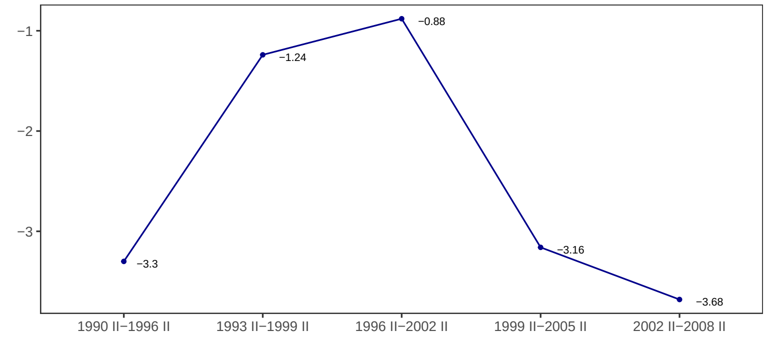
Figure 7a shows K-R ratios for various periods in Australia, revealing the model without housing in the Taylor rule consistently fits Australian data better, as all K-R ratios are below 2, indicating a preference for the model without housing in the Taylor rule. Consequently, I can infer that the Reserve Bank of Australia (RBA) did not incorporate a response to changes in house prices in previous years.

However, this conclusion contradicts the findings of Wadud, Bashar, and Ahmed (2012), who employ a structural VAR model to analyze the IRFs of interest rates in Australia. Wadud, Bashar, and Ahmed (2012) assert that a shock in the housing market leads to a short-term increase in interest rates, implying that the RBA does react to the housing dynamic. They suggest the RBA considers house prices in interest rate setting, in conjunction with the conventional targets of inflation and the output gap. Nonetheless, it's worth noting that the relationship between a housing shock and inflation is not examined by these researchers. This aspect is explored byBjørnland and Jacobsen (2010), who utilize a SVAR framework to analyze the IRFs of both interest rates and inflation to a house price shock. Their findings indicate that an unexpected house price surge can elevate both interest rates and inflation, suggesting that the monetary policy's response to house price shifts may actually stem from inflation's response. This interpretation is also consistent with previous observations in the US, where the model without housing in the Taylor rule shows that a housing shock influences inflation, consequently causing changes in interest rates.

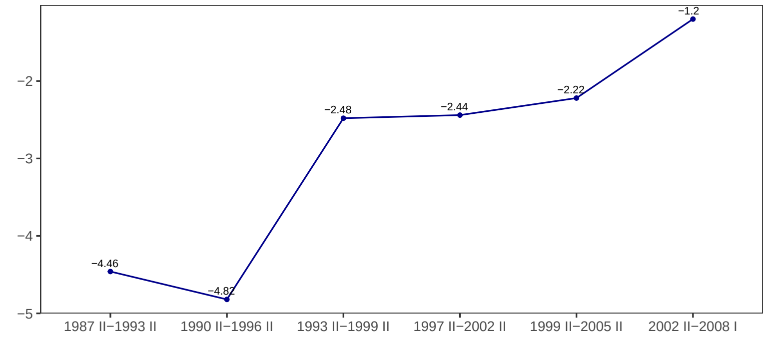
Similar to Australia, according to the K-R ratios outlined in Figure 7b, one can deduce that the Reserve Bank of New Zealand did not exhibit any response to changes in house prices between 1990 and 2008. This is consistent with findings by Kim and Lim (2022) who employ an identified VAR model to investigate the effects of house price shocks on interest rates in New Zealand, covering the period from 1974 I to 2021 IV. Their study indicates that the impact of interest rates in response to house price shocks was minimal, and the proportion of interest rate fluctuations attributed to house price inflation shocks was also insignificant.



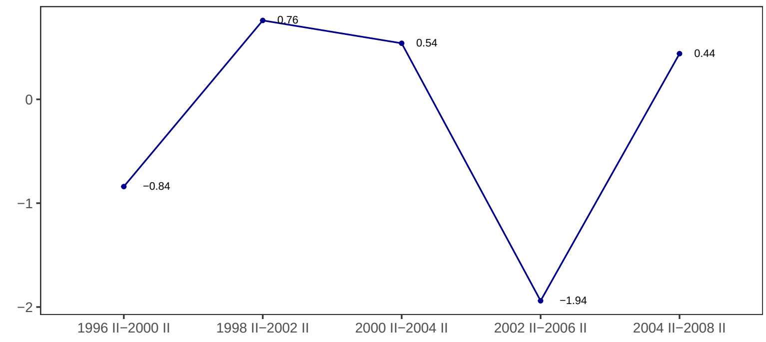
(a) Australia



(b) New Zealand



(c) The united Kingdom



(d) The Euro Area

Figure 7. Rolling Window K-R Ratios for Australia, New Zealand, The UK and The EA

7.2 The United Kingdom and Euro Area

In the United Kingdom, the model excludes house price reaction consistently shows higher log density across all examined time periods, with K-R ratios persistently below 2, as shown in Figure 7c. This suggests that the Bank of England did not consider housing a determinant in their interest rate decisions, even though some Monetary Policy Committee members expressed concerns about housing bubbles in 2004. Despite these concerns, Governor Mervyn King did not support raising interest rates in response to housing (Bank of England 2004). While no solid evidence indicates the Bank of England directly responding to house prices, it does not necessarily denote a total lack of interest rate responsiveness to housing shocks. Elbourne (2007) employ a SVAR model to study the UK economy, the results indicate interest rates immediately rise by 7 basis points in response to a 1% positive shock in real house prices. Nevertheless, the study does not dismiss the possibility that the interest rate's response to housing might be attributed to the impact of the house price shock on overall prices, given that the price level also displayed an increase of up to 0.06%. The discussion by Elbourne (2007) is consistent with previously discussed Bjørnland and Jacobsen (2010).

After 1999, the European Central Bank (ECB) assumed the responsibilities of national central banks, establishing a cohesive monetary policy for all EA countries. While examining the EA in general, no indications are apparent that the ECB integrated housing as a factor within its interest rate rule. However, it is crucial to acknowledge the potential variances in certain other EA nations. Barigozzi, Conti, and Luciani (2014) discover that despite the euro's introduction leading to a more uniform response in the monetary transmission mechanism across individual countries, there remains asymmetry among EA countries concerning the common monetary policy's impact on prices and unemployment. Meanwhile, as mentioned by Nocera and Roma (2017), housing markets in the Euro area exhibit heterogeneity, and aggregating or pooling data may result in biased inferences. These disparities stem from nation-specific structures rather than ECB policies. For instance, other than the UK discussed above, Bjørnland and Jacobsen (2010) find no direct evidence of Norges Bank responding to house price shocks. Conversely, Giavazzi and Mishkin (2006) discern that communications from the Riksbank conveyed to market participants the bank's adaptive approach towards its monetary policy, specifically aiming to mitigate the rapid escalation in house prices. Similarly, Ingves (2007) confirms that the Swedish central bank takes into account real estate prices when formulating its real interest rate policies, adjusting its monetary measures in response to a sudden surge in housing prices.

7.3 Canada

As in Figure 8, the Bank of Canada reacted significantly to house prices during two specific sample periods: 1983 II-1993 II and 1988 II-1998 II, the K-R ratios for these periods were found to be 7.4 and 5.4, respectively. These two periods captured the real estate dynamics in Canada during the 1980s and the subsequent housing crisis in the 1990s.

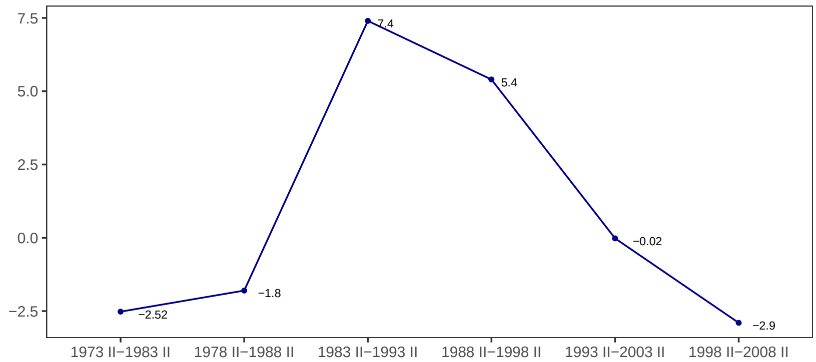


Figure 8. Rolling Window K-R Ratios for Canada

In the 1980s, Canada faced significant concerns due to soaring inflation rates. As Macdonald (2010) points out, enticing factors like low mortgage rates made it easier for potential buyers to enter a market they might otherwise find challenging. Consequently, Canadians pivoted their investment attention to the housing market, leading to a surge in house prices. This shift can also be seen as a proactive strategy to shield against possible future inflation. However, these housing price bubbles created a deceptive perception of the actual returns homeowners could anticipate on their investments, leading to speculative home buying and an excessive inflow of investments into the real estate sector. To address the inflation, the Bank of Canada asked the federal government to legislate a contractionary monetary policy, to raise the interest rates and mortgage rates (Macdonald 2010). This strategy aimed to rein in inflation but, combined with the accumulation of debt during the inflationary phase, led to a period of economic recession. The demand for homes waned, leading to a decline in housing prices throughout the 1990s[19] (Walks 2014). Declining house prices diminish the worth of collateral, subsequently diminishing the willingness of financial institutions to extend loans. This, in turn, results in

19. Vancouver experienced its first housing bubble burst in 1981, followed by a more gradual decline in its second bubble in 1994. Toronto encountered a single housing bubble that burst in 1989 (Macdonald 2010).

reduced expenditure on both investments and consumer goods, while also leading to a surge in bankruptcies. The aftermath of Canada's housing market boom and subsequent crash in the early 1990s brought about protracted and painful adjustments in both the real estate and financial sectors of the economy (Selody and Wilkins 2004). Goodhart and Hofmann (2008) use a panel VAR model for 17 countries (including Canada) from 1973 to 2006, and they identify there is a significant positive response of interest rates to a house price shock.

Given that Canada is the only country responding to fluctuations in house prices, a deeper exploration to scrutinize its quantitative characteristics is merited. In Appendix G, I provide estimation results, full sample model comparison, IRFs, variance decomposition, and historical shock decomposition.

8 Robustness Analysis

This thesis predominantly concentrates on the structural estimation of monetary policy rules. To ensure the robustness of the baseline results, I intend to introduce adjustments to the prior distribution of monetary policy parameters. Then, I intend to explore the sensitivity of the results to variations in the choice of alternative house price measures. Furthermore, I will re-estimate the model using alternative interest rate rules. These robustness checks are done for the baseline estimation for the US.

8.1 Alternative Prior Distributions

I increase the standard deviation of the smoothing parameter r_R from 0.1 to 0.2, in line with existing literature (e.g. Lubik and Schorfheide 2007; Li and Liu 2017). Next, I continue to maintain the belief that interest rates are more responsive to inflation but decrease the prior mean from 1.5 to 1.1, similar to Haque (2022). Recognizing that in the baseline estimation, the adjustment of r_π falls short in the posterior distribution, I also explore the possibility of increasing the standard deviation from 0.1 to 0.2. Additionally, the prior of house price coefficient r_q is relaxed by increasing its standard deviation from 0.05 to 0.1 and adjusting the prior mean from 0.1 to 0.2 to optimize parameter estimation flexibility.

In the first two scenarios, the response coefficients' estimates are reduced compared to the baseline, reflecting the decreased prior mean for these parameters. With an increased standard deviation, the inflation coefficient r_π shows notable fluctuations between its posterior and prior means, suggesting that available data effectively informs this parameter's determination. With the relaxation of the prior distribution of the house price coefficient r_q, there is a slight increase in the posterior estimates for r_R, r_π, and the posterior mean of r_q increased from 0.11 to 0.12. In all three scenarios, the structural parameters exhibit notable stability, indicating that the constraints linking the policy rule and the structural equations are comparatively weak. Under the alternative prior, the lag data densities and K-R ratios are displayed in Table 10. In all three alternations, the estimation results indicate that the data continue to support the model's responsiveness to house prices. It is plausible to conclude that the posterior estimates exhibit robustness to reasonable alterations in the prior assumptions.

Table 10. Robust checks: $r_q \neq 0$ versus $r_q = 0$

	$r_q = 0$	$r_q \neq 0$	
	Log density	Log density	K-R ratio
r_R: Beta(0.75,0.2)	-915.25	-911.09	8.32
r_π: Normal(1.1,0.2)	-918.20	-915.41	5.58
r_q: Normal(0.2,0.1)	-917.72	-916.16	3.12
FHFA House Price Index	-872.64	-871.25	2.78
Current-looking Taylor Rule	-906.43	-900.86	11.14
Forward-looking Taylor Rule	-933.90	-928.45	10.9

8.2 Alternative House Price Index

The baseline models are initially estimated using the US Census Bureau House Prices Index. To assess the sensitivity of results to the alternative house price measures, this thesis estimates the models using the FHFA Mortgage House Price Index[20] (HPI) while keeping all other assumptions the same, as inspired by Iacoviello and Neri (2010). The log densities are presented in Table 10, they indicate that both models exhibit a comparable fit during the sample period. With a K-R ratio of 2.78, there is a marginal but positive indication favoring the model that incorporates housing. In comparison to the log densities of the baseline, the models' overall fitness significantly improves, suggesting a higher degree of overall model fitness with the FHFA HPI

20. The Federal Housing Finance Agency HPI concisely tracks single-family house price changes using a weighted, repeat-sales index, measuring average price shifts in subsequent sales or refinancing of properties. It utilizes data from recurring mortgage transactions on properties securitized by Fannie Mae or Freddie Mac since January 1975. Details are provided in Appendix H.1.

data. The posterior estimates for most parameters remain consistent with the baseline results, with the exception of r_q, which decreases from 0.11 to 0.07. These results are quite reasonable, given that the two house price indexes closely track each other during the 1970–2006 period, except FHFA HPI has a stronger upward trend. In contrast, the Census HPI is characterized by high volatility, possibly attributable to measurement issues (Rappaport 2007).

8.3 Alternative Taylor Rules

In the baseline model, the Taylor rule is assumed to be backward-looking. A conventional perspective on the optimal interest rate rule suggests that to prevent real indeterminacy, the central bank should react to either expected inflation (forward-looking) or to present inflation (Kerr and King 1996; Bernanke and Woodford 1997). Contrarily, as highlighted by Carlstrom and Fuerst (2000), both forward-looking and current-looking interest rate rules invariably result in real indeterminacy. Carlstrom and Fuerst (2000) conclude that for real determinacy to be achieved, central bankers should respond to previous fluctuations in inflation. Given these discussions, it is pertinent to examine the robustness of my model when subjected to an alternative current-looking Taylor rule:

$$\widehat{R}_t = r_R \widehat{R}_{t-1} + (1 - r_R)\Big[r_q \widehat{q}_t + (1 + r_\pi)\widehat{\pi}_t + r_Y \widehat{Y}_t\Big] + \widehat{e}_{R,t} \tag{30}$$

, and an alternative forward-looking Taylor rule:

$$\widehat{R}_t = r_R \widehat{R}_{t-1} + (1 - r_R)\Big[r_q \widehat{q}_t + (1 + r_\pi)\widehat{\pi}_{t+1} + r_Y \widehat{Y}_{t+1}\Big] + \widehat{e}_{R,t} \tag{31}$$

The results of the alternative rules are shown in the last two rows of Table 10. When evaluating the log densities, models that incorporate housing in the Taylor rule remain more favorable. Notably, the K-R ratios stand at approximately 11, indicating "very strong" evidence supporting the model that includes housing in the Taylor rule. This reinforces our primary finding that during the sample period, the Fed responds to fluctuations in house prices.

9 Conclusion

This thesis employs the Bayesian estimation technique to estimate a New Keynesian model enriched with a housing factor. Within this framework, I add house prices in the Taylor-type interest rule to allow interest rates to directly respond to prevailing house prices. The primary contribution of this study lies in presenting empirical findings regarding the dynamic responsiveness of policy measures to changes in house prices. I find that the Federal Reserve's reaction to housing market fluctuations is generally positive but occurs sporadically, with significant responses primarily during the economic downturns of the 1990s and 2000s. Based on the estimated results, this study proceeds with various quantitative analyses.

Upon examination of IRFs using Bayesian posterior means, the model successfully isolates interest rate responses to inflation and housing, revealing the relationship between interest rates and house price movements. Different shocks' transmission mechanisms in my DSGE model become evident. When delving into variance decomposition, monetary policy and inflation shocks are the two most important contributors to macroeconomic variables fluctuations. It is observed that the integration of housing parameters into the Taylor rule leads to an increase in the proportion of variables' variances that can be explained by housing preference shocks.

Analyzing historical shock decomposition graphs, inflation shocks play an important role in explaining changes in all four variables, and the past fluctuations of house prices are largely driven by housing preference shocks. After including housing in the Taylor rule, one can visually discern housing preference shocks contribute more to previous fluctuations in chosen macroeconomic variables. Collectively, these quantitative explorations underscore the significance of housing within the monetary transmission system.

In a broader international context, in our cross-country analysis, I find central banks in Australia, New Zealand, England, and the European Central Bank appear not to react to shifts in house prices, whereas the Bank of Canada exhibits episodic responses, primarily during the recessions of the 1980s and 1990s. It is crucial to note that my findings do not imply that house prices are irrelevant to the decision-making processes of those central banks. Instead, there is just no compelling evidence suggesting that these central banks adjust their interest rates in response to house price fluctuations.

In future work, I plan to expand the existing model by incorporating elements that have demonstrated their ability in prior literature, to significantly enhance the accuracy and comprehensiveness of DSGE models in capturing the dynamics of business cycles. These enhancements may encompass features such as capital and housing accumulation and the incorporation of income and housing tax mechanisms. In the present thesis, a closed economy model is utilized to examine open economies. Going forward, there is an intention to expand the model to an open economy framework, incorporating external elements like exports. Additionally, it is possible to include more shocks in the model as Iacoviello and Neri (2010). Currently, the model is limited to only four shocks, which may introduce some bias into the quantitative analysis. Furthermore, to increase the comprehensiveness of the study, another improvement could be estimating more model parameters instead of calibration. It will necessitate the inclusion of additional observation equations and the utilization of relevant datasets.

References

Aastveit, Knut Are, Francesco Furlanetto, and Francesca Loria. 2023. "Has the Fed Responded to House and Stock Prices? A Time-Varying Analysis." *The Review of Economics and Statistics* 105, no. 5 (September): 1314–1324. ISSN: 0034-6535. https://doi.org/10.1162/rest_a_01120.

Ahearne, Alan G., John Ammer, Brian M. Doyle, Linda S. Kole, and Robert F. Martin. 2005. *Monetary Policy and House Prices: A Cross-Country Study.* International Finance Discussion Papers 841. Board of Governors of the Federal Reserve System.

An, Sungbae, and Frank Schorfheide. 2007. "Bayesian Analysis of DSGE Models." *Econometric Reviews* 26 (2-4): 113–172. https://doi.org/10.1080/07474930701220071.

Andrea, Beltratti, and Claudio. Morana. 2014. "International house prices and macroeconomic fluctuations." *Journal of Banking & Finance* 34 (3): 533–545. ISSN: 0378-4266. https://doi.org/10.1016/j.jbankfin.2009.08.020.

Bank of England. 2004. *MINUTES OF THE MONETARY POLICY COMMITTEE MEETING HELD ON 4-5 AUGUST 2004.* Government Publishing Website. https://www.bankofengland.co.uk/minutes/2004/monetary-policy-committee-august-2004.

Barigozzi, Matteo, Antonio M. Conti, and Matteo Luciani. 2014. "Do Euro Area Countries Respond Asymmetrically to the Common Monetary Policy?" *Oxford Bulletin of Economics and Statistics* 76 (5): 693–714. ISSN: 1468-0084. https://doi.org/10.1111/obes.12038.

Belongia, Michael T., and Peter N. Ireland. 2016. "The evolution of U.S. monetary policy: 2000–2007." *Journal of Economic Dynamics and Control* 73 (C): 78–93. https://doi.org/10.1016/j.jedc.2016.09.00.

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist. 1999. "The financial accelerator in a quantitative business cycle framework," 1:1341–1393. Handbook of Macroeconomics. Elsevier. https://doi.org/https://doi.org/10.1016/S1574-0048(99)10034-X.

Bernanke, Ben S., and Michael Woodford. 1997. "Inflation Forecasts and Monetary Policy." *Journal of Money, Credit and Banking* 29 (4): 653–684. ISSN: 00222879, 15384616, accessed October 5, 2023. http://www.jstor.org/stable/2953656.

Bjørnland, Hilde C., and Dag Henning Jacobsen. 2010. "The role of house prices in the monetary policy transmission mechanism in small open economies." *Journal of Financial Stability* 6, no. 4 (December): 218–229. https://ideas.repec.org/a/eee/finsta/v6y2010i4p218-229.html.

Bouchouicha, Ranoua, and Zied Ftiti. 2012. "Real estate markets and the macroeconomy: A dynamic coherence framework." *Economic Modelling* 29 (5): 1820–1829. ISSN: 0264-9993. https://doi.org/https://doi.org/10.1016/j.econmod.2012.05.034.

Carlstrom, Charles, and Timothy Fuerst. 2000. *Forward-looking versus backward-looking Taylor rules.* Working Papers (Old Series) 0009. Federal Reserve Bank of Cleveland. https://EconPapers.repec.org/RePEc:fip:fedcwp:0009.

Case, Karl, and Robert Shiller. 2003. "Is here a Bubble in the Housing Market?" *Brookings Papers on Economic Activity* 34 (2): 299–362.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy* 113 (1): 1–45. ISSN: 00223808, 1537534X, accessed September 21, 2023. http://www.jstor.org/stable/10.1086/426038.

Cogley, Timothy, Giorgio E Primiceri, and Thomas J Sargent. 2010. "Inflation-Gap Persistence in the US." *American Economic Journal: Macroeconomics* 2 (1): 43–69. ISSN: 1945-7707.

Demary, Markus. 2010. "The interplay between output, inflation, interest rates and house prices: international evidence." *Journal of Property Research* 27 (1): 1–17. ISSN: 0959-9916. https://doi.org/10.1080/09599916.2010.499015.

Elbourne, Adam. 2007. "The UK housing market and the monetary policy transmission mechanism: An SVAR approach." *Journal of Housing Economics* 17 (1): 65–87. ISSN: 1051-1377. https://doi.org/10.1016/j.jhe.2007.09.002.

Fernández-Villaverde, Jesús, and Juan F. Rubio-Ramírez. 2008. "How Structural Are Structural Parameters?" *NBER Macroeconomics Annual* 22:83–167. ISSN: 0889-3365, 1537-2642. https://doi.org/10.1086/ma.22.25554965.

Fuhrer, Jeffrey C. 1997. "Inflation/Output Variance Trade-Offs and Optimal Monetary Policy." *Journal of Money, Credit and Banking* 29 (2): 214–234. ISSN: 00222879, 15384616, accessed October 2, 2023.

Giavazzi, Francesco, and Frederic Mishkin. 2006. "An Evaluation of Swedish Monetary Policy between 1995 and 2005" (January).

Giuliodori, Massimo. 2005. "THE ROLE OF HOUSE PRICES IN THE MONETARY TRANSMISSION MECHANISM ACROSS EUROPEAN COUNTRIES." *Scottish Journal of Political Economy* 52 (4): 519–543. https://doi.org/https://doi.org/10.1111/j.1467-9485.2005.00354.x. eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1467-9485.2005.00354.x.

Goodhart, Charles, and Boris Hofmann. 2008. "House Prices, money, credit and the macroeconomy." *ECB Working paper,* no. 888.

Haque, Qazi. 2022. "Monetary Policy, Inflation Target and the Great Moderation: An Empirical Investigation." *International Journal of Central Banking* 18 (4). ISSN: 1556-5068. https://doi.org/10.2139/ssrn.3409539.

Henderson, Dale W., and Warwick J. McKibbin. 1993. "A comparison of some basic monetary policy regimes for open economies: implications of different degrees of instrument adjustment and wage persistence." *Carnegie-Rochester Conference Series on Public Policy* 39:221–317. ISSN: 0167-2231. https://doi.org/https://doi.org/10.1016/0167-2231(93)90011-K.

Herbst, Edward P., and Frank Schorfheide. 2016. *Bayesian Estimation of DSGE Models.* Princeton: Princeton University Press. ISBN: 9781400873739. https://doi.org/doi:10.1515/9781400873739.

Iacoviello, Matteo. 2005. "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle." *American Economic Review* 95 (3): 739–764. ISSN: 0002-8282. https://doi.org/10.1257/0002828054201477.

Iacoviello, Matteo, and Raoul Minetti. 2008. "The credit channel of monetary policy: Evidence from the housing market." *Journal of Macroeconomics* 30 (1): 69–96. ISSN: 0164-0704. https://doi.org/https://doi.org/10.1016/j.jmacro.2006.12.001.

Iacoviello, Matteo, and Stefano Neri. 2010. "Housing Market Spillovers: Evidence from an Estimated DSGE Model." *American Economic Journal: Macroeconomics* 2 (2): 125–164. ISSN: 1945-7707, 1945-7715. https://doi.org/10.1257/mac.2.2.125.

Ingves, Stefan. 2007. "Housing and monetary policy: a view from an inflation-targeting central bank." *Proceedings - Economic Policy Symposium - Jackson Hole,* 433–443. https://ideas.repec.org/a/fip/fedkpr/y2007p433-443.html.

Jarociński, Marek, and Frank Smets. 2008. *House Prices and the Stance of Monetary Policy.* Working Paper Series 891. European Central Bank. https://ideas.repec.org/p/ecb/ecbwps/2008891.html.

Jin, Yi, and Zhixiong Zeng. 2007. "Real estate and optimal public policy in a credit-constrained economy." *Journal of Housing Economics* 16, no. 2 (June): 143–166. https://doi.org/https://doi.org/10.1016/j.jhe.2007.03.003..

Kahn, Jame A. 2009. "Productivity Swings and Housing Prices." *Federal Reserve Bank of New York:Current Issues in Economics and Fianace* 15 (3).

Kass, Robert E., and Adrian E. Raftery. 1995. "Bayes Factors." *Journal of the American Statistical Association* 90 (430): 773–795. ISSN: 01621459, accessed August 29, 2023. http://www.jstor.org/stable/2291091.

Kerr, William, and Robert G. King. 1996. "Limits on Interest Rate Rules in the IS Model." *Federal Reserve Bank of Richmond Economic Quarterly.*

Kim, Jan R., and Gieyoung Lim. 2022. "The Role of Property Prices in the Conduct of Monetary Policy: Evidence from Down Under." *Journal of Economics and Business* 5 (4): 124–132. ISSN: 2775-9237. https://doi.org/10.31014/aior.1992.05.04.464.

Kitney, Paul. 2015. *Does the central bank respond to credit market factors? A Bayesian DSGE approach.* CAMA Working Papers 2015-21. Centre for Applied Macroeconomic Analysis, Crawford School of Public Policy, The Australian National University. https://ideas.repec.org/p/een/camaaa/2015-21.html.

Lawrance, Emily C. 1991. "Poverty and the Rate of Time Preference: Evidence from Panel Data." *Journal of Political Economy* 99 (1): 54–77. ISSN: 00223808, 1537534X, accessed September 8, 2023. http://www.jstor.org/stable/2937712.

Leamer, Edward E. 2007. *Housing IS the Business Cycle.* Working Paper, Working Paper Series 13428. National Bureau of Economic Research, September. https://doi.org/10.3386/w13428.

Li, Bing, and Qing Liu. 2017. "On the choice of monetary policy rules for China: A Bayesian DSGE approach." *China Economic Review* 44:166–185. ISSN: 1043-951X. https://doi.org/https://doi.org/10.1016/j.chieco.2017.04.004.

Lubik, Thomas A., and Frank Schorfheide. 2007. "Do central banks respond to exchange rate movements? A structural investigation." *Journal of Monetary Economics* 54 (4): 1069–1087. ISSN: 0304-3932. https://doi.org/https://doi.org/10.1016/j.jmoneco.2006.01.009.

Macdonald, David. 2010. *Canada's Housing Bubble: An Accident Waiting to Happen.* CCPA National Office Report. Canadian Center for Policy Alternatives. https://policyalternatives.ca/newsroom/updates/canadas-housing-bubble.

McCallum, Bennett T. 1999. "Chapter 23 Issues in the design of monetary policy rules," 1:1483–1530. Handbook of Macroeconomics. Elsevier. https://doi.org/https://doi.org/10.1016/S1574-0048(99)10036-3.

Mishkin, Frederic S. 2007. "Housing and the Monetary Transmission Mechanism." *NBER Working Paper,* no. 13518.

Ng, Eric C.Y. 2015. "Housing market dynamics in China: Findings from an estimated DSGE model." *Journal of Housing Economics* 29:26–40. ISSN: 10511377. https://doi.org/10.1016/j.jhe.2015.05.003.

Nocera, Andrea, and Moreno Roma. 2017. "House prices and monetary policy in the euro area: evidence from structural VARs." *ECB Working Paper,* no. 2073.

Notarpietro, Alessandro, and Stefano Siviero. 2015. "Optimal Monetary Policy Rules and House Prices: The Role of Financial Frictions." *Journal of Money, Credit and Banking* 47 (S1): 383–410. https://doi.org/https://doi.org/10.1111/jmcb.12208. eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/jmcb.12208.

Rappaport, Jordan. 2007. "A guide to aggregate house price measures." *Economic Review* 92 (Q II): 41–71. https://ideas.repec.org/a/fip/fedker/y2007iqiip41-71nv.92no.2.html.

Sami, Alpanda, and Ueberfeldt Alexander. 2016. "Should monetary policy lean against housing market booms?" *Bank of Canada Staff Working Paper,* nos. 2016-19.

Samwick, Andrew A. 1998. "Discount rate heterogeneity and social security reform." *Journal of Development Economics* 1 (1): 117–146. ISSN: 0304-3878.

Selody, Jack, and Carolyn Wilkins. 2004. "Asset Prices and Monetary Policy: A Canadian Perspective on the Issues." *Bank of Canada Review* 2004 (Autumn): 3–14. https://EconPapers.repec.org/RePEc:bca:bcarev:v:2004:y:2004:i:autumn04:p:3-14.

Sims, Christopher. 2002. "Solving Linear Rational Expectations Models." *Computational Economics* 20 (1-2): 1–20. https://EconPapers.repec.org/RePEc:kap:compec:v:20:y:2002:i:1-2:p:1-20.

Smets, Frank, and Rafael Wouters. 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *The American Economic Review* 97 (3): 586–606. ISSN: 00028282, accessed September 8, 2023. http://www.jstor.org/stable/30035013.

Taylor, John B. 1993. "Discretion versus policy rules in practice." *Carnegie-Rochester Conference Series on Public Policy* 39:195–214. ISSN: 0167-2231. https://doi.org/https://doi.org/10.1016/0167-2231(93)90009-L.

Tsatsaronis, Kostas, and Haibin Zhu. 2004. "What drives housing price dynamics: cross-country evidence." *BIS Quarterly Review* (March). https://ideas.repec.org/a/bis/bisqtr/0403f.html.

Vargas-Silva, Carlos. 2008. "Monetary policy and the US housing market: A VAR analysis imposing sign restrictions." *Journal of Macroeconomics* 30 (3): 977–990. ISSN: 0164-0704. https://doi.org/https://doi.org/10.1016/j.jmacro.2007.07.004.

Wadud, I.K.M. Mokhtarul, Omar H.M.N. Bashar, and Huson Joher Ali Ahmed. 2012. "Monetary policy and the housing market in Australia." *Journal of Policy Modeling* 34 (6): 849–863. ISSN: 01618938, accessed August 29, 2023. https://doi.org/10.1016/j.jpolmod.2012.06.002.

Walks, Alan. 2014. "Canada's Housing Bubble Story: Mortgage Securitization, the State, and the Global Financial Crisis." *International Journal of Urban and Regional Research* 38 (1): 256–284. https://doi.org/https://doi.org/10.1111/j.1468-2427.2012.01184.x.

Wu, Jing Cynthia, and Fan Dora Xia. 2016. "Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound." *Journal of Money, Credit and Banking* 48 (2-3): 253–291. https://doi.org/https://doi.org/10.1111/jmcb.12300.

Wu, Jing Cynthia, and Ji Zhang. 2019. "A shadow rate New Keynesian model." *Journal of Economic Dynamics and Control* 107:103728. ISSN: 0165-1889. https://doi.org/10.1016/j.jedc.2019.103728.

Yuliya, Lovcha, and Perez-Laborda Alejandro. 2018. "Monetary policy shocks, inflation persistence, and long memory." *Journal of Macroeconomics* 55:117–127. ISSN: 0164-0704. https://doi.org/10.1016/j.jmacro.2017.10.006.

Zeno, Enders, J. Müller Gernot, and Scholl Almuth. 2011. "How do fiscal and technology shocks affect real exchange rates?: New evidence for the United States." *Journal of International Economics* 83 (1): 53–69. ISSN: 0022-1996. https://doi.org/10.1016/j.jinteco.2010.08.005.

Appendices

A Data

A.1 Baseline: US

- **Federal Funds Effective Rate (FEDFUNDS):** Percent,Quarterly,Not Seasonally Adjusted, https://fred.stlouisfed.org/series/FEDFUNDS#0.
- **Gross Domestic Product: Implicit Price Deflator (GDPDEF):** Percent Change, Quarterly, Seasonally Adjusted, https://fred.stlouisfed.org/series/GDPDEF.
- **Real Gross Domestic Product per Capita (A939RX0Q048SBEA):** Percent Change, Quarterly, Seasonally Adjusted Annual Rate, https://fred.stlouisfed.org/series/A939RX0Q048SBEA.
- **Real House Prices:** Census Bureau House Price Index, Quarterly, Deflated using the GDP Deflator for the nonfarm business sector, Demeaned, https://www.census.gov/construction/nrs/xls/price_sold_cust.xls. A description of this price index is available: https://www.census.gov/construction/cpi/pdf/descpi_sold.pdf
- **Alternative Real House Price:** FHFA House Price Index, Quarterly, Deflated using the GDP Deflator, Demeaned, https://www.fhfa.gov/DataTools/Downloads/Pages/House-Price-Index-Datasets.aspx#qpo

A.2 Australia

- **Interest Rates: 3-Month or 90-Day Rates and Yields: Bank Bills: Total for Australia (IR3TBB01AUM156N):** Percent, Quarterly, Not Seasonally Adjusted, https://fred.stlouisfed.org/series/IR3TBB01AUM156N.
- **National Accounts: National Accounts Deflators: Gross Domestic Product: GDP Deflator for Australia (AUSGDPDEFQISMEI):** Percent Change, Quarterly, Seasonally Adjusted, https://fred.stlouisfed.org/series/AUSGDPDEFQISMEI.
- **National Accounts: GDP by Expenditure: Constant Prices: Gross Domestic Product - Total for Australia:**Australian Dollar, Quarterly, Seasonally Adjusted, https://fred.stlouisfed.org/series/AUSGDPRQDSMEI.
- **Real Residential Property Prices for Australia (QAUR628BIS):** Percent change, Quarterly,Not Seasonally Adjusted, https://fred.stlouisfed.org/series/QAUR628BIS.

- **Australia Population:**https://datacommons.org/place/country/AUS/?utm_medium=explore&mprop=count&popt=Person&hl=en.

A.3 New Zealand

- **3-Month or 90-day Rates and Yields: Bank Bills for New Zealand (IR3TBB01NZM156N):** Percent, Quarterly, Not Seasonally Adjusted, https://fred.stlouisfed.org/series/IR3TBB01NZM156N.
- **National Accounts: National Accounts Deflators: Gross Domestic Product: GDP Deflator for Australia (AUSGDPDEFQISMEI):** Percent Change, Quarterly, Seasonally Adjusted, https://fred.stlouisfed.org/series/AUSGDPDEFQISMEI.
- **Gross Domestic Product by Expenditure in Constant Prices: Total Gross Domestic Product for New Zealand (NAEXKP01NZA189S):** New Zealand Dollar, Quarterly, Seasonally Adjusted, https://fred.stlouisfed.org/series/NAEXKP01NZA189S.
- **Real Residential Property Prices for New Zealand (QNZR628BIS):** Percent change, Quarterly,Not Seasonally Adjusted , https://fred.stlouisfed.org/series/QNZR628BIS.
- **New Zealand Population:**https://datacommons.org/place/country/NZL/?utm_medium=explore&mprop=count&popt=Person&hl=en.

A.4 UK

- **3-Month or 90-day Rates and Yields: Interbank Rates for the United Kingdom (IR3TIB01GBM156N):** Percent, Quarterly, Not Seasonally Adjusted, https://fred.stlouisfed.org/series/IR3TIB01GBM156N.
- **GDP Implicit Price Deflator in United Kingdom (GBRGDPDEFQISMEI):** Percent Change, Quarterly, Seasonally Adjusted, https://fred.stlouisfed.org/series/GBRGDPDEFQISMEI
- **Real Gross Domestic Product for United Kingdom (DISCONTINUED) (CLVMNACSCAB1GQUK):**Millions of Chained 2010 British Pounds, Quarterly, Seasonally Adjusted, https://fred.stlouisfed.org/series/CLVMNACSCAB1GQUK.
- **Real Residential Property Prices for United Kingdom (QGBR628BIS):** Percent change, Quarterly,Not Seasonally Adjusted , https://fred.stlouisfed.org/series/QGBR628BIS.
- **The United Kingdom Population:** https://datacommons.org/place/country/GBR/?utm_medium=explore&mprop=count&popt=Person&hl=en.

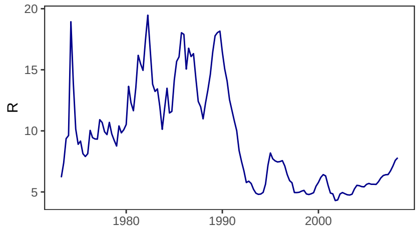
A.5 Europe Area

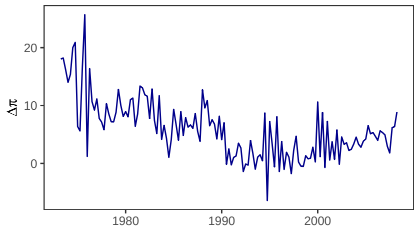
- **Interest Rates: 3-Month or 90-Day Rates and Yields: Interbank Rates: Total for the Euro Area (19 Countries):** Percent, Quarterly, Not Seasonally Adjusted, https://fred.stlouisfed.org/series/IR3TIB01EZM156N.
- **National Accounts: National Accounts Deflators: Gross Domestic Product: GDP Deflator for the Euro Area (19 Countries) (NAGIGP01EZQ661S):** Percent Change, Quarterly, Seasonally Adjusted, https://fred.stlouisfed.org/series/NAGIGP01EZQ661S.
- **Real Gross Domestic Product (Euro/ECU Series) for Euro Area (19 Countries) (CLVMEURSCAB1GQEA19):** Millions of Chained 2010 Euros, Quarterly, Seasonally Adjusted, https://fred.stlouisfed.org/series/CLVMEURSCAB1GQEA19.
- **Real Residential Property Prices for Euro area (QXMR628BIS):** Percent change, Quarterly,Not Seasonally Adjusted , https://fred.stlouisfed.org/series/QXMR628BIS.
- **Europe Area Population:** https://data.worldbank.org/indicator/SP.POP.TOTL?end=2008&locations=EU&start=1990&view=chart.

A.6 Canada

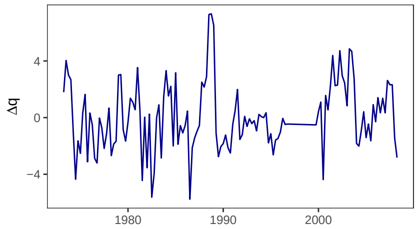
- **Interest Rates: 3-Month or 90-Day Rates and Yields: Interbank Rates: Total for Canada (IR3TIB01CAM156N) :** Percent, Quarterly, Not Seasonally Adjusted, https://fred.stlouisfed.org/series/IR3TIB01CAM156N.
- **National Accounts: National Accounts Deflators: Gross Domestic Product: GDP Deflator for Canada (CANGDPDEFQISMEI):** Percent Change, Quarterly, Seasonally Adjusted, https://fred.stlouisfed.org/series/CANGDPDEFQISMEI.
- **National Accounts: GDP by Expenditure: Constant Prices: Gross Domestic Product - Total for Canada:** Canadian Dollar, Quarterly, Seasonally Adjusted, https://fred.stlouisfed.org/series/NAEXKP01CAQ189S.
- **Real Residential Property Prices for Canada (QCAR368BIS):** Percent change, Quarterly, Not Seasonally Adjusted , https://fred.stlouisfed.org/series/QCAR368BIS.
- **Canada Population:**https://datacommons.org/place/country/CAN/?utm_medium=explore&mprop=count&popt=Person&hl=en.

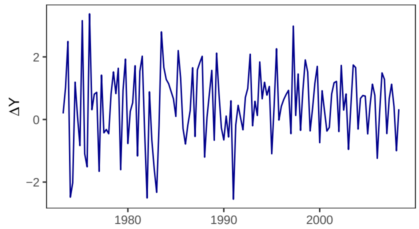
A.7 Time Series Plots





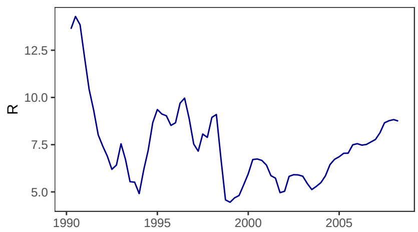
(a) Interest Rates **(b)** Inflation Growth Rate

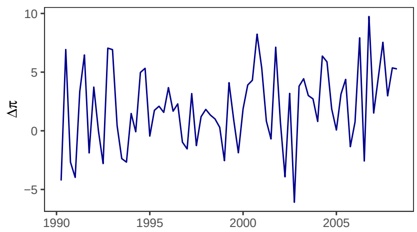




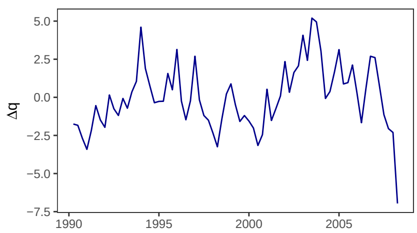
(c) Real House Prices Growth Rate **(d)** Per Capita Real GDP Growth Rate

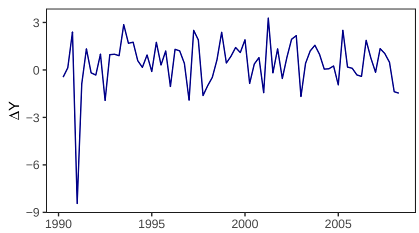
Figure A.1. Australia: Time Series Plot for 1973:Q2-2008:Q2





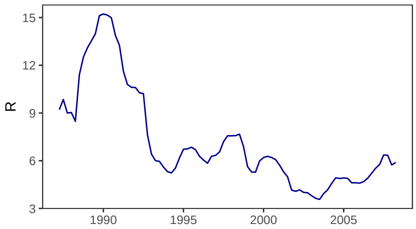
(a) Interest Rates **(b)** Inflation Growth Rate



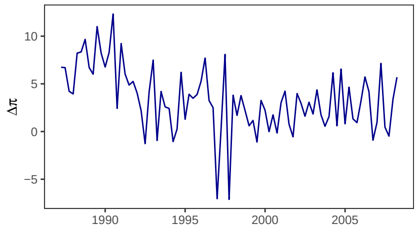


(c) Real House Prices Growth Rate **(d)** Per Capita Real GDP Growth Rate

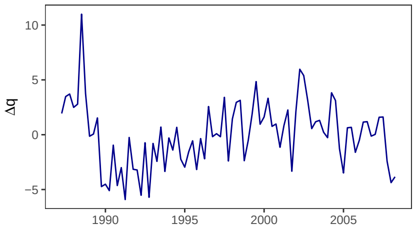
Figure A.2. New Zealand: Time Series Plot for 1990:Q2-2008:Q2



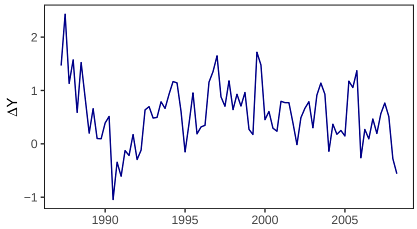
(a) Interest Rates



(b) Inflation Growth Rate

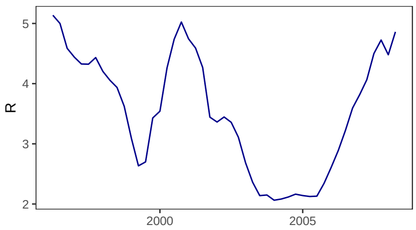


(c) Real House Prices Growth Rate

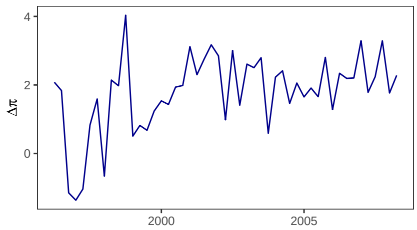


(d) Per Capita Real GDP Growth Rate

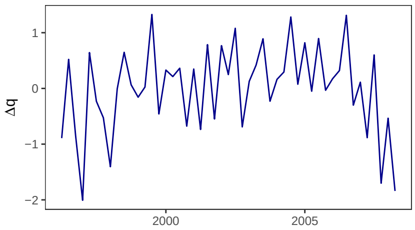
Figure A.3. The United Kingdom: Time Series Plot for 1987:Q2-2008:Q2



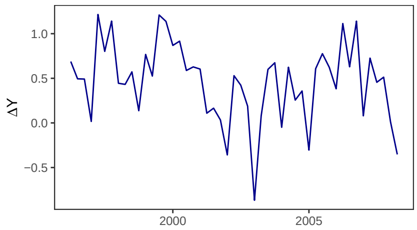
(a) Interest Rates



(b) Inflation Growth Rate

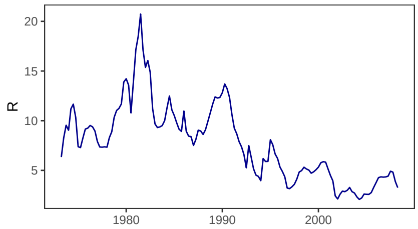


(c) Real House Prices Growth Rate

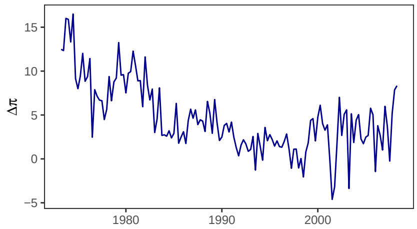


(d) Per Capita Real GDP Growth Rate

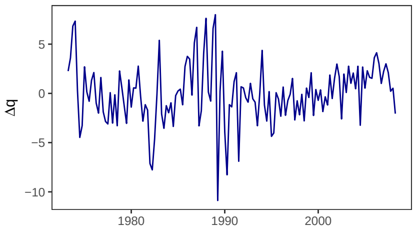
Figure A.4. The Europe Area: Time Series Plot for 1986:Q2-2008:Q2



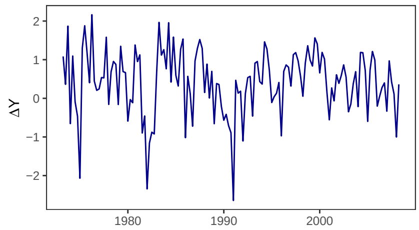
(a) Interest Rates



(b) Inflation Growth Rate



(c) Real House Prices Growth Rate



(d) Per Capita Real GDP Growth Rate

Figure A.5. Canada: Time Series Plot for 1973:Q2-2008:Q2

B Equilibrium Conditions

B.1 Entrepreneurs

Using all constraints of entrepreneurs, a Lagrangian can be written as:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \gamma^t \Big\{ lnc_t + \lambda_t(m\,\mathbb{E}_t\, q_{t+1}h_t\pi_{t+1} - b_tR_t) + \mu_{2,t}\Big(I_t + (1-\delta)K_{t-1} - K_t\Big)$$
$$+ \mu_{1,t}\Bigg[\frac{A_tK_{t-1}^{\mu}h_{t-1}^{\nu}(L_t')^{\alpha(1-\mu-\nu)}(L_t'')^{(1-\alpha)(1-\mu-\nu)}}{X_t} + b_t + q_th_{t+1} - w_t'L_t' - w_t''L_t'' - c_t - q_th_t$$
$$- \frac{R_{t-1}b_{t-1}}{\pi_t} - I_t - \frac{\psi}{2\delta}\Big(\frac{I_t}{K_{t-1}} - \delta\Big)^2 K_{t-1}\Bigg]\Big\}$$

The first-order conditions with respect to c_t, L_t',L_t'', I_t, K_t, b_t, h_t are:

$$\frac{1}{c_t} = \mu_{1,t} \tag{32}$$

$$w_t' = \frac{\alpha(1-\mu-\nu)Y_t}{L_t'X_t} \tag{33}$$

$$w_t'' = \frac{(1-\alpha)(1-\mu-\nu)Y_t}{L_t''X_t} \tag{34}$$

$$\mu_{2,t} = \mu_{1,t}\Big(1 + \frac{\psi}{\delta}(\frac{I_t}{K_{t-1}} - \delta)\Big) \tag{35}$$

$$\mu_{2,t} = \gamma(1-\delta)\,\mathbb{E}_t\,\mu_{2,t+1} + \gamma\,\mathbb{E}_t\,\mu_{1,t+1}\Bigg[\frac{\mu Y_{t+1}}{K_tX_{t+1}} - \frac{\psi}{2\delta}\Big(\frac{I_{t+1}}{K_t} - \delta\Big)^2 + \frac{\psi}{\delta}\Big(\frac{I_{t+1}}{K_t} - \delta\Big)\frac{I_{t+1}}{K_t}\Bigg] \tag{36}$$

$$\mu_{1,t} = \lambda_tR_t + \gamma\,\mathbb{E}_t\,\frac{\mu_{1,t+1}R_t}{\pi_{t+1}} \tag{37}$$

$$\mu_{1,t}q_t = \lambda_tm\,\mathbb{E}_t\,q_{t+1}\pi_{t+1} + \gamma\,\mathbb{E}_t\,\mu_{1,t+1}\Bigg[\frac{\nu Y_{t+1}}{h_tX_{t+1}} + q_{t+1}\Bigg] \tag{38}$$

Set $\mu_{2,t} = v_t$ for convenience, the first-order conditions (32) and (35) can be combined to get:

$$v_t = \frac{1}{c_t}\Big(1 + \frac{\psi}{\delta}(\frac{I_t}{K_{t-1}} - \delta)\Big) \tag{39}$$

Then combine (39) with (36) to get:

$$v_t = \gamma\,\mathbb{E}_t\Bigg[\frac{\mu Y_{t+1}}{c_{t+1}K_tX_{t+1}} + (1-\delta)v_{t+1}\Bigg] + \gamma\,\mathbb{E}_t\,\frac{1}{c_{t+1}}\Bigg[\frac{\psi}{\delta}\Big(\frac{I_{t+1}}{K_t} - \delta\Big)\frac{I_{t+1}}{K_t} - \frac{\psi}{2\delta}\Big(\frac{I_{t+1}}{K_t} - \delta\Big)^2\Bigg] \tag{40}$$

Similarly, combine (32) and (38) to get:

$$\frac{q_t}{c_t} = \lambda_tm\,\mathbb{E}_t\,q_{t+1}\pi_{t+1} + \gamma\,\mathbb{E}_t\,\frac{1}{c_{t+1}}\Bigg[\frac{\nu Y_{t+1}}{h_tX_{t+1}} + q_{t+1}\Bigg] \tag{41}$$

Combine (32) and (37) to get:

$$\frac{1}{c_t} = \lambda_tR_t + \gamma\,\mathbb{E}_t\,\frac{R_t}{c_{t+1}\pi_{t+1}} \tag{42}$$

B.2 Impatient Households

I will begin by discussing impatient households. These households face a borrowing constraint. Apart from this constraint, impatient households share similarities with patient households. By including all constraints, and adjustment costs, a Lagrangian can be written as:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta'' \Bigg\{ \ln c_t'' + j_t ln h_t'' - \frac{(L_t'')^\eta}{\eta} + \lambda_t'' \Big[m'' \, \mathbb{E}_t \, q_{t+1} h_t'' \pi_{t+1} - b_t'' R_t \Big]$$
$$+ \mu_t'' \Bigg[b_t'' + w_t'' L_t'' + q_t h_{t-1}'' + T_t'' - \xi_{h,t}'' - c_t'' - q_t h_t'' - \frac{R_{t-1} b_{t-1}''}{\pi_t} \Bigg] \Bigg\}$$

Then, first-order conditions with respect to c_t'', b_t'', L_t'', h_t'' are:

$$\frac{1}{c_t''} = \mu_t'' \tag{43}$$

$$\mu_t'' = R_t \lambda_t'' + \beta'' \, \mathbb{E}_t \, \frac{\mu_{t+1}'' R_t}{\pi_{t+1}} \tag{44}$$

$$\mu_t'' w_t'' = (L_t'')^{\eta - 1} \tag{45}$$

$$\frac{j_t}{h_t''} + m'' \lambda_t'' \, \mathbb{E}_t \, q_{t+1} \pi_{t+1} - \mu_t'' q_t + \beta'' \, \mathbb{E}_t \, \mu_{t+1}'' q_{t+1} = 0 \tag{46}$$

Then by combining these conditions, we can easily get:

$$\frac{1}{c_t''} = R_t \lambda_t'' + \beta'' \, \mathbb{E}_t \, \frac{R_t}{c_{t+1}'' \pi_{t+1}} \tag{47}$$

$$\frac{w_t''}{c_t''} = (L_t'')^{\eta - 1} \tag{48}$$

$$\frac{q_t}{c_t''} = \frac{j_t}{h_t''} + \beta'' m'' \lambda_t'' \, \mathbb{E}_t \, q_{t+1} \pi_{t+1} + \mathbb{E}_t \, \frac{\beta'' q_{t+1}}{c_{t+1}''} \tag{49}$$

B.3 Patient Households

The problem for patient households is similar to impatient households, with the sole distinction being the absence of borrowing constraints in the former. To avoid duplicating the Lagrangian expression, the combined first-order conditions are:

$$\frac{w_t'}{c_t'} = (L_t')^{\eta - 1} \tag{50}$$

$$\frac{1}{c_t'} = \beta \, \mathbb{E}_t \, \frac{R_t}{c_{t+1}' \pi_{t+1}} \tag{51}$$

$$\frac{q_t}{c_t'} = \frac{j_t}{h_t'} + \beta \, \mathbb{E}_t \, \frac{q_{t+1}}{c_{t+1}'} \tag{52}$$

B.4 Retailers

Given

$$\max_{P_t(z)} \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \Big\{ \Lambda_{t,t+k} \Big[P_t(z)^{1-\epsilon} P_{t+k}^{\epsilon-1} Y_{t+k} - X_{t+k}^{-1} P_t(z)^{-\epsilon} P_{t+k}^{\epsilon} Y_{t+k} \Big] \Big\}$$

The first order condition for retailers' problem is:

$$(\epsilon - 1) P_t(z)^{-\epsilon}\, \mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k \Lambda_{t,t+k} P_{t+k}^{\epsilon-1} Y_{t+k} - \epsilon P_t(z)^{-\epsilon-1}\, \mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k \Lambda_{t,t+k} X_{t+k}^{-1} P_{t+k}^{\epsilon} Y_{t+k} = 0$$

For convienience, I denote $P_t(z)$ as P_t^*, rearrange equation we can get:

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \Lambda_{t,t+k} X_{t+k}^{-1} P_{t+k}^{\epsilon} Y_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \Lambda_{t,t+k} P_{t+k}^{\epsilon-1} Y_{t+k}} \tag{53}$$

Then we can use $S_{1,t}$ and $S_{2,t}$ to represent the numerator and denominator respectively:

$$S_{1,t} = X_t^{-1} P_t^{\epsilon} Y_t + \theta\beta\, \mathbb{E}_t\, \Lambda_{t,t+1} S_{1,t+1}$$

$$S_{2,t} = P_t^{\epsilon-1} Y_t + \theta\beta\, \mathbb{E}_t\, \Lambda_{t,t+1} S_{2,t+1}$$

For rewriting them in real terms, define $s_{1,t} = S_{1,t}/P_t^{\epsilon}$, $s_{2,t} = S_{2,t}/P_t^{\epsilon-1}$, and define the relative reset prices $\pi_t^* = P_t^*/P_t$, as $S_{1,t}/S_{2,t} = s_{1,t}/s_{2,t} * P_t$. Then we can transform the above three equations:

$$\pi_t^* = \frac{\epsilon}{\epsilon - 1} \frac{s_{1,t}}{s_{2,t}} \tag{54}$$

$$s_{1,t} = X_t^{-1} Y_t + \theta\beta\, \mathbb{E}_t\, \Lambda_{t,t+1} \pi_{t+1}^{\epsilon} s_{1,t+1} \tag{55}$$

$$s_{2,t} = Y_t + \theta\beta\, \mathbb{E}_t\, \Lambda_{t,t+1} \pi_{t+1}^{\epsilon-1} s_{1,t+1} \tag{56}$$

For getting equation (13), we start with:

$$P_t^{1-\epsilon} = \int_0^{1-\theta} P_t^{*,1-\epsilon} dz + \int_{1-\theta}^{1} P_{t-1}(z)^{1-\epsilon} dz$$

As the order of these firms does not matter for the study, we can transform it to:

$$P_t^{1-\epsilon} = (1-\theta) P_t^{*,1-\epsilon} + \int_{1-\theta}^{1} P_{t-1}(z)^{1-\epsilon} dz$$

Then according to the Calvo assumption, given the stochastic nature of the firm selection process for updates and the presence of a continuum of firms, it follows that the sum of individual prices within a designated subset of the unit interval exhibits a proportional linkage to the integral over the complete unit interval. This proportionality is contingent upon the relative extent of the subset within the unit interval over which the integral is taken:

$$\int_{1-\theta}^{1} P_{t-1}(z)^{1-\epsilon} dz = \theta \int_{0}^{1} P_{t-1}(z)^{1-\epsilon} dz = \theta P_{t-1}^{1-\epsilon}$$

Collectively, we reach equation (13), which implies that by integrating the influence of heterogeneity, we can focus on the collective behavior of aggregates without the need to monitor each firm's specific actions:

$$P_t^{1-\epsilon} = (1-\theta) P_t^{*,1-\epsilon} + \theta P_{t-1}^{1-\epsilon}$$

This equation signifies that the aggregate price level constitutes a convex combination of the revised price and the prior price level (all raised to $1-\epsilon$).Then divide both sides by $P_t^{1-\epsilon}$ to express it in terms of inflation (define $P_t^*/P_t = \pi_t^*$, $P_{t-1}/P_t = \pi_t$):

$$(1-\theta)(\pi_t^*)^{1-\epsilon} + \theta \pi_t^{\epsilon-1} = 1 \tag{57}$$

where $1-\theta$ of firm will update their price, while θ will keep the same price.

B.5 Aggregation

Integrate Equation (10) across z, with production function Equation (4), and define the price dispersion $v_t^p = \int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\epsilon} dz$, we can get

$$Y_t v_t^p = A_t K_{t-1}^{\mu} h_{t-1}^{\nu} (L_t')^{\alpha(1-\mu-\nu)} (L_t'')^{(1-\alpha)(1-\mu-\nu)} \tag{58}$$

On the demand side, aggregation involves summing up the budget constraints of the three agents:

$$c_t' + q_t h_t' + \frac{R_{t-1} b_{t-1}'}{\pi_t} + c_t'' + q_t h_t'' + \frac{R_{t-1} b_{t-1}''}{\pi_t} + c_t + q_t h_t + \frac{R_{t-1} b_{t-1}}{\pi_t} + I_t + \xi_{k,t} =$$
$$b_t' + w_t' L_t' + q_t h_{t-1}' + F_t + T_t' + b_t'' + w_t'' L_t'' + q_t h_{t-1}'' + T_t'' + \frac{Y_{w,t}}{X_t} + b_t + q_t h_{t-1} - w_t' L_t' - w_t'' L_t''$$

Based on the assumption, we have excluded the consideration of money, so T_t' and T_t'' are equal to 0. Bonds market clearing condition:

$$b_t + b_t' + b_t'' = 0$$

Housing's aggregate stock is fixed at H:

$$h_t + h_t' + h_t'' = H$$

We already derived $F_t(z)$:

$$F_t(z) = P_t(z)^{1-\epsilon} P_t^{\epsilon-1} Y_t - X_t^{-1} P_t(z)^{-\epsilon} P_t^{\epsilon} Y_t$$

Then aggregate profits F_t is:

$$\begin{aligned} F_t = \int_0^1 F_t(z) &= \int_0^1 P_t(z)^{1-\epsilon} P_t^{\epsilon-1} Y_t dz - \int_0^1 X_t^{-1} P_t(z)^{-\epsilon} P_t^{\epsilon} dz \\ &= P_t^{\epsilon} Y_t \int_0^1 P_t(z)^{\epsilon-1} dz - \frac{Y_t}{X_t} \int_0^1 \Big(\frac{P_t(z)}{P_t}\Big)^{-\epsilon} dz \\ &= Y_t - \frac{Y_t v_t^p}{X_t} \end{aligned}$$

By plugging these equations back into the aggregate budget constraint, we can finally get:

$$Y_t = c_t + c_t' + c_t'' + I_t + \xi_{k,t} \tag{59}$$

B.6 Full Set of Equilibrium Conditions

$$\frac{1}{c_t''} = R_t \lambda_t'' + \beta'' \mathbb{E}_t \frac{R_t}{c_{t+1}'' \pi_{t+1}} \tag{60}$$

$$\frac{w_t''}{c_t''} = (L_t'')^{\eta-1} \tag{61}$$

$$\frac{q_t}{c_t''} = \frac{j_t}{h_t''} + m'' \lambda_t'' \mathbb{E}_t q_{t+1} \pi_{t+1} + \mathbb{E}_t \frac{\beta'' q_{t+1}}{c_{t+1}''} \tag{62}$$

$$b_t'' = m'' \mathbb{E}_t [q_{t+1} h_t'' \pi_{t+1} / R_t] \tag{63}$$

$$b_t'' = c_t'' + q_t (h_t'' - h_{t-1}'') + \frac{R_{t-1} b_{t-1}''}{\pi_t} - w_t'' L_t'' \tag{64}$$

$$\frac{w_t'}{c_t'} = (L_t')^{\eta-1} \tag{65}$$

$$\frac{1}{c_t'} = \beta \mathbb{E}_t \frac{R_t}{c_{t+1}' \pi_{t+1}} \tag{66}$$

$$\frac{q_t}{c_t'} = \frac{j_t}{h_t'} + \mathbb{E}_t \frac{\beta q_{t+1}}{c_{t+1}'} \tag{67}$$

$$w_t' = \frac{\alpha (1 - \mu - \nu) Y_t}{L_t' X_t} \tag{68}$$

$$w_t'' = \frac{(1-\alpha)(1 - \mu - \nu) Y_t}{L_t'' X_t} \tag{69}$$

$$b_t = m \mathbb{E}_t [q_{t+1} h_t \pi_{t+1} / R_t] \tag{70}$$

$$b_t = q_t(h_t - h_{t-1}) + w'_t L'_t + w''_t L''_t + c_t + \frac{R_{t-1} b_{t-1}}{\pi_t} + I_t + \xi_{k,t} - \frac{Y_t v_t^p}{X_t} \tag{71}$$

$$\frac{1}{c_t} = \lambda_t R_t + \gamma \, \mathbb{E}_t \, \frac{R_t}{c_{t+1} \pi_{t+1}} \tag{72}$$

$$v_t = \frac{1}{c_t} \Big(1 + \frac{\psi}{\delta} \big(\frac{I_t}{K_{t-1}} - \delta \big) \Big) \tag{73}$$

$$v_t = \gamma \, \mathbb{E}_t \Big[\frac{\mu Y_{t+1}}{c_{t+1} K_t X_{t+1}} + (1 - \delta) v_{t+1} \Big] + \gamma \, \mathbb{E}_t \, \frac{1}{c_{t+1}} \Big[\frac{\psi}{\delta} \Big(\frac{I_{t+1}}{K_t} - \delta \Big) \frac{I_{t+1}}{K_t} - \frac{\psi}{2\delta} \Big(\frac{I_{t+1}}{K_t} - \delta \Big)^2 \Big] \tag{74}$$

$$\frac{q_t}{c_t} = \lambda_t m \, \mathbb{E}_t \, q_{t+1} \pi_{t+1} + \gamma \, \mathbb{E}_t \, \frac{1}{c_{t+1}} \Big[\frac{\nu Y_{t+1}}{h_t X_{t+1}} + q_{t+1} \Big] \tag{75}$$

$$\pi_t^* = \frac{\epsilon}{\epsilon - 1} \frac{s_{1,t}}{s_{2,t}} \tag{76}$$

$$s_{1,t} = X_t^{-1} Y_t + \theta \beta \, \mathbb{E}_t \, \Lambda_{t,t+1} \pi_{t+1}^{\epsilon} s_{1,t+1} \tag{77}$$

$$s_{2,t} = Y_t + \theta \beta \, \mathbb{E}_t \, \Lambda_{t,t+1} \pi_{t+1}^{\epsilon - 1} s_{1,t+1} \tag{78}$$

$$(1 - \theta)(\pi_t^*)^{1 - \epsilon} + \theta \pi_t^{\epsilon - 1} = 1 \tag{79}$$

$$R_t = (R_{t-1})^{r_R} (\bar{rr})^{1 - r_R} \Big(\pi_{t-1}^{1 + r_\pi} \big(\frac{Y_{t-1}}{Y} \big)^{r_Y} \Big)^{1 - r_R} e_{R,t} \tag{80}$$

$$Y_t v_t^p = A_t K_{t-1}^{\mu} h_{t-1}^{\nu} (L'_t)^{\alpha(1 - \mu - \nu)} (L''_t)^{(1 - \alpha)(1 - \mu - \nu)} \tag{81}$$

$$Y_t = c_t + c'_t + c''_t + I_t + \xi_{k,t} \tag{82}$$

$$h_t + h'_t + h''_t = H \tag{83}$$

$$\xi_{k,t} = \frac{\psi}{2} \Big(\frac{I_t}{K_{t-1}} - \delta \Big)^2 \frac{K_{t-1}}{\delta} \tag{84}$$

C The Steady State

C.1 Basic Steady State Equations

In the steady state, we can see all adjustment costs are equal to 0. The steady state will be represented by variables without a time subscript. We have $I = \delta K$ in the steady state. Assuming zero inflation in the steady state, which means $\pi = \pi^* = 1$, $v_p = 1$, then by Equation (53), we can get:

$$X = \frac{\epsilon}{\epsilon - 1} \tag{85}$$

By Equation (66), we have:

$$R = \frac{1}{\beta} = \bar{rr} \tag{86}$$

By Equation (72), we have:

$$\lambda c = \beta - \gamma \tag{87}$$

By Equation (73), we have $v = \frac{1}{c}$, the Euler equation for capital (Equation (74)) in the steady state can be transformed as :

$$1 = \left(\frac{\mu Y}{KX} + 1 - \delta \right)$$

Then we can get the steady state capital stock:

$$K = \frac{\mu}{X} \frac{1}{\left[\frac{1}{\gamma} - 1 + \delta \right]} \tag{88}$$

Then using the conditions above, we can first transform equations in the Appendix B.6:

- Patient households:

$$\frac{w'}{c'} = (L')^{\eta - 1} \tag{89}$$

$$\frac{j}{h'} = (1 - \beta) \frac{q}{c'} \tag{90}$$

- Impatient households:

$$\frac{w''}{c''} = (L'')^{\eta - 1} \tag{91}$$

$$\frac{1}{c''}(1 - \beta'' R) = R\lambda'' \tag{92}$$

$$\frac{j}{h''} + m''\lambda'' q = (1 - \beta) \frac{q}{c''} \tag{93}$$

$$b'' = \frac{m'' q h''}{R} \tag{94}$$

- Entrepreneurs:

$$\frac{1}{c}(1-\gamma R) = R\lambda \tag{95}$$

$$\frac{q}{c} = mq\lambda + \frac{\gamma}{c}(\frac{\nu Y}{hX} + q) \tag{96}$$

$$b = \frac{mqh}{R} \tag{97}$$

$$Y = AK^{\mu}h^{\nu}(L')^{\alpha(1-\mu-\nu)}(L'')^{(1-\alpha)(1-\mu-\nu)} \tag{98}$$

- Wage:

$$w'L' = \frac{\alpha(1-\mu-\nu)Y}{X} \tag{99}$$

$$w''L'' = \frac{(1-\alpha)(1-\mu-\nu)Y}{X} \tag{100}$$

- Aggregation:

$$H = h + h' + h'' \tag{101}$$

$$Y = c + c' + c'' + \delta K \tag{102}$$

$$b'' = c'' + Rb'' - w''L'' \tag{103}$$

$$b = w'L' + w''L'' + c + Rb + \delta K - \frac{Y}{X} \tag{104}$$

C.2 Expressions for Estimation

Next, we use the conditions above to eliminate and rearrange things. Additionally, as I have tried, it is hard to eliminate Y directly, so I will express everything as a ratio over Y. By doing so, I could implement my estimation efficiently. We first start to eliminate wage terms, combine Equation (89) with Equation (99), Equation (91) with Equation (100), we can get:

$$c'(L')^{\eta} = \frac{\alpha(1-\mu-\nu)Y}{X} \tag{105}$$

$$c''(L'')^{\eta} = \frac{(1-\alpha)(1-\mu-\nu)Y}{X} \tag{106}$$

Combine Equation (103) with Equation (100), Equation (104) with Equation (99) we can get:

$$b''(1-R) = c'' - \frac{(1-\alpha)(1-\mu-\nu)Y}{X} \tag{107}$$

$$b(1-R) = c + \frac{(1-\mu-\nu)Y}{X} + \delta K - \frac{Y}{X} \tag{108}$$

Then combine Equation (86) with Equation (90) and Equation (93), yield:

$$\lambda = \frac{1}{c}(\beta - \gamma) \tag{109}$$

$$\lambda'' = \frac{1}{c''}(\beta - \beta'') \tag{110}$$

Then, we put Equation (109) and Equation (110) in Equation (93) and Equation (96) to get the housing Euler equations:

$$\frac{q}{c} = \frac{1}{c}(\beta - \gamma) + \frac{\gamma}{c}\Big(\frac{\nu Y}{hX} + q\Big) \tag{111}$$

$$\frac{q}{c''} = \frac{j}{h''} + \beta''\frac{q}{c''} + \frac{m''q}{c''}(\beta - \beta'') \tag{112}$$

$$\frac{q}{c'} = \beta\frac{q}{c'} + \frac{j}{h'} \tag{113}$$

Then, look at Equation (111), we can see c could cancel out, and we can multiply both sides by h, we can get:

$$qh = \frac{\gamma\nu Y}{X} + \gamma qh + mqh(\beta - \gamma)$$

This can be simplified to:

$$\frac{qh}{Y} = \frac{\gamma\nu}{1 - m\beta - (1-m)\gamma}\frac{1}{X}$$

We define $\gamma_e = m\beta + (1-m)\gamma$, so we can finally have:

$$\frac{qh}{Y} = \frac{\gamma\nu}{X(1-\gamma_e)} \tag{114}$$

Similarly, we can get:

$$qh'' = \frac{jc''}{1 - \beta'' - m''(\beta - \beta'')} \tag{115}$$

$$qh' = \frac{jc'}{1-\beta} \tag{116}$$

We will go back later to replace c'' and c' and express them as a ratio over Y.

By Equation (104), we can have :

$$c = b\Big(\frac{\beta - 1}{\beta}\Big) + \frac{(\mu+\nu)Y}{X} - \delta K$$

With Equation (97), we then get:

$$c = mqh(\beta - 1) + \frac{(\mu+\nu)Y}{X} - \delta K$$

With known $\frac{qh}{Y}$ above, we finally get:

$$\frac{c}{Y} = \frac{(\mu+\nu)}{X} - m(1-\beta)\frac{qh}{Y} - \frac{\delta K}{Y} \tag{117}$$

Similarly, with Equation (107), we can solve for c'':

$$c'' = m''qh''(\beta - 1) + \frac{(1-\alpha)(1-\mu-\nu)Y}{X}$$

With Equation (115), we can transform it as:

$$c'' = m''(\beta-1)\frac{jc''}{1-\beta''-m''(\beta-\beta'')} + \frac{(1-\alpha)(1-\mu-\nu)Y}{X}$$

Then:

$$\frac{c''}{Y} = \frac{[1-\beta''-m''(\beta-\beta'')]}{1-\beta''-m''(\beta-\beta'')+jm''(1-\beta)}\frac{(1-\alpha)(1-\mu-\nu)}{X} \tag{118}$$

Lastly, with resource constraint, we can solve for c':

$$c' = Y - c - c'' - \delta K$$

We already have c and c'' above, plug them in, and we can get:

$$\frac{c'}{Y} = 1 - \frac{1}{X} + \frac{\alpha(1-\mu-\nu)}{X} + (1-\beta)\left[m\frac{qh}{Y} + m''\frac{qh''}{Y}\right] \tag{119}$$

Now we go back to qh' and qh''. Combine Equation (115) and Equation (118), we can get:

$$\frac{qh''}{Y} = \frac{j}{1-\beta''-m''(\beta-\beta'')+m''j(1-\beta)}\frac{\alpha(1-\mu-\nu)}{X} \tag{120}$$

Similarly, combine Equation (116) and Equation (119), we get:

$$\frac{qh'}{Y} = \frac{j}{1-\beta}\left[1-\frac{1}{X}+\frac{\alpha(1-\mu-\nu)}{X}\right] + jm\frac{qh}{Y} + jm''\frac{qh''}{Y} \tag{121}$$

Then, as we know $b = \beta mqh$, so

$$\begin{aligned}\frac{b}{Y} &= \beta m\frac{qh}{Y}\\ &= \frac{\gamma\nu}{1-m\beta-(1-m)\gamma}\frac{\beta m}{X}\end{aligned} \tag{122}$$

Similarly for b'':

$$\frac{b''}{Y} = \frac{j\beta m''}{1-\beta''-m''(\beta-\beta'')+jm''(1-\beta)s''} \tag{123}$$

Some other expressions:

$$\frac{h}{h'} = \frac{qh}{Y} / \frac{qh'}{Y} \tag{124}$$

$$\frac{h''}{h''} = \frac{qh''}{Y} / \frac{qh'}{Y} \tag{125}$$

$$\frac{I}{Y} = 1 - \frac{c}{Y} - \frac{c'}{Y} - \frac{c''}{Y} \tag{126}$$

D The Log-linearized Model

In this section, I log-linearize the aforementioned conditions. Variables with a "hat" will indicate log deviations. I will present a detailed step-by-step process of log-linearization for deriving the Phillips Curve, as it is the only one that needs a lot of substitutions. Subsequent derivations will adhere to the same logical approach, just following the standard log-linearization process.

D.1 Log-linearization process

- **New Keynesian Phillips Curve:** Take logs of the reset price expression Equation (76):

  $$\ln \pi_t^* = \ln\Big(\frac{s_{1,t}}{s_{2,t}}\Big)$$

  Do the first-order Taylor series expansion:

  $$\ln \pi_t^* + \frac{1}{\pi^*}(\pi_t^* - \pi*) = \ln s_1 + \frac{1}{s_1}(s_{1,t} - s_1) - \ln s_2 - \frac{1}{s_1}(s_{1,t} - s_1)$$

  Note that $\ln \pi^* = \ln s_1 - \ln s_2$, they can cancel out. So we can have:

  $$\widehat{\pi_t}^* = \widehat{s}_{1,t} - \widehat{s}_{2,t}$$

  Log-linearize Equation (77):

  $$\ln s_{1,t} = \ln\Big(X_t^{-1} Y_t + \theta\beta\, \mathbb{E}_t\, \Lambda_{t,t+1} \pi_{t+1}^{\epsilon} s_{1,t+1}\Big)$$

  Then around zero inflation steady state, and we know in steady state $\Lambda = c'/c' = 1$, we can get:

  $$\widehat{s}_{1,t} = \frac{1}{s_1}\frac{Y}{X}\Big(\widehat{Y} - \widehat{X}\Big) + \theta\beta\, \mathbb{E}_t\, \widehat{\Lambda}_{t,t+1} + \epsilon\theta\beta\, \mathbb{E}_t\, \widehat{\pi}_{t+1} + \theta\beta\, \mathbb{E}_t\, \widehat{s}_{1,t+1}$$

In steady state, we have:

$$
\begin{aligned}
s_1 &= \frac{Y}{X} + \theta\beta s_1 \\
&= \frac{Y}{(1-\theta\beta)X}
\end{aligned}
$$

This simplifies the equation above to:

$$\widehat{s}_{1,t} = (1-\theta\beta)(\widehat{Y} - \widehat{X}) + \theta\beta\widehat{\mathbb{E}}_t\Lambda_{t,t+1} + \epsilon\theta\beta\,\mathbb{E}_t\,\widehat{\pi}_{t+1} + \theta\beta\,\mathbb{E}_t\,\widehat{s}_{1,t+1}$$

Same procedure could be used for $s_{2,t}$:

$$\widehat{s}_{2,t} = \frac{Y}{s_2}\widehat{Y} + \theta\beta\,\mathbb{E}_t\,\widehat{\Lambda}_{t,t+1} + (\epsilon-1)\theta\beta\,\mathbb{E}_t\,\widehat{\pi}_{t+1} + \theta\beta\,\mathbb{E}_t\,\widehat{s}_{1,t+1}$$

And in steady state, similarly, we have:$s_2 = \frac{Y}{(1-\theta\beta)}$. We can then get:

$$\widehat{s}_{2,t} = (1-\theta\beta)\widehat{Y} + \theta\beta\,\mathbb{E}_t\,\widehat{\Lambda}_{t,t+1} + (\epsilon-1)\theta\beta\,\mathbb{E}_t\,\widehat{\pi}_{t+1} + \theta\beta\,\mathbb{E}_t\,\widehat{s}_{1,t+1}$$

Combine these equations, we can have:

$$\widehat{\pi}^*_t = \widehat{s}_{1,t} - \widehat{s}_{2,t} = -(1-\theta\beta)\widehat{X}_t + \theta\beta\,\mathbb{E}_t\,\widehat{\pi}_{t+1} + \theta\beta\,\mathbb{E}_t\Big(\widehat{s}_{1,t+1} - \widehat{s}_{2,t+1}\Big)$$

Now we need to find a way to replace $\hat{\pi}^*_t$, we can log-linearize Equation (79):

$$0 = \ln\Big((1-\theta)(\pi^*_t)^{1-\epsilon} + \theta\pi_t^{\epsilon-1}\Big)$$

$$0 = (\epsilon-1)\theta\Big(\pi_t - \pi\Big) + (1-\epsilon)(1-\theta)\Big(\pi^*_t - \pi\Big)$$

Then we have:

$$\widehat{\pi}^*_t = \frac{\theta}{1-\theta}\widehat{\pi}_t$$

Make this substitution above, we have:

$$\widehat{\pi}_t = -\frac{(1-\theta)(1-\theta\beta)}{\theta}\widehat{X}_t + \beta\,\mathbb{E}_t\,\widehat{\pi}_{t+1} \tag{127}$$

Furthermore, we also add $\widehat{u}_t$ in the equation:

$$\widehat{\pi}_t = -\frac{(1-\theta)(1-\theta\beta)}{\theta}\widehat{X}_t + \beta\,\mathbb{E}_t\,\widehat{\pi}_{t+1} + \widehat{u}_t \tag{128}$$

Here, u_t follows an AR(1) process, it represents a residual term that encompasses external factors beyond the scope of the model, which could potentially influence the marginal cost.. In this way, we can examine inflation shocks to this model.

- **Entrepreneurs' investment schedule:** Plug Equation (73) into Equation (74), we can get:

$$\frac{1}{c_t}\left(1+\frac{\psi}{\delta}\left(\frac{I_t}{K_{t-1}}-\delta\right)\right)=\gamma\,\mathbb{E}_t\Big[\frac{\mu Y_{t+1}}{c_{t+1}K_tX_{t+1}}+\frac{(1-\delta)}{c_{t+1}}\left(1+\frac{\psi}{\delta}\left(\frac{I_{t+1}}{K_t}-\delta\right)\right)\Big]$$
$$+\gamma\,\mathbb{E}_t\,\frac{1}{c_{t+1}}\Big[\frac{\psi}{\delta}\Big(\frac{I_{t+1}}{K_t}-\delta\Big)\frac{I_{t+1}}{K_t}-\frac{\psi}{2\delta}\Big(\frac{I_{t+1}}{K_t}-\delta\Big)^2\Big]$$

Take logs:

$$-\ln c_t+\ln\left(1+\frac{\psi}{\delta}\left(\frac{I_t}{K_{t-1}}-\delta\right)\right)=\ln\gamma+\mathbb{E}_t\ln\left[\frac{\mu Y_{t+1}}{c_{t+1}K_tX_{t+1}}+\frac{(1-\delta)}{c_{t+1}}\left(1+\frac{\psi}{\delta}\left(\frac{I_{t+1}}{K_t}-\delta\right)\right)\right.$$
$$\left.+\frac{1}{c_{t+1}}\left(\frac{\psi}{\delta}\left(\frac{I_{t+1}}{K_t}-\delta\right)\frac{I_{t+1}}{K_t}-\frac{\psi}{2\delta}\left(\frac{I_{t+1}}{K_t}-\delta\right)^2\right)\right]$$

This equation is too long to linearize directly, we first linearize the left-hand side:

$$-\ln c+\widehat{c}_t+\psi\widehat{I}_t-\psi\widehat{K}_{t-1}$$

Then the right-hand side, we can get:

$$-\ln c+\gamma\,\mathbb{E}_t\left[-\left(\frac{\mu Y}{KX}+1-\delta\right)\widehat{c}_{t+1}+\psi\hat{I}_{t+1}-\psi\widehat{K}_t+\frac{\mu Y}{KX}\left(\widehat{Y}_{t+1}-\widehat{X}_{t+1}-\widehat{K}_{t+1}\right)\right]$$

Use Equation (88), we can simplify to:

$$-\ln c+\gamma\,\mathbb{E}_t\left[\gamma\psi(\widehat{I}_{t+1}-\widehat{K}_t)+\widehat{c}_{t+1}+[1-\gamma\,(1-\delta)]\left(\widehat{Y}_{t+1}-\widehat{X}_{t+1}-\widehat{K}_{t+1}\right)\right]$$

Then we combine the left-hand side and right-hand side, to get:

$$\widehat{I}_t=\widehat{K}_{t-1}+\gamma(\mathbb{E}_t\,\widehat{I}_{t+1}-\widehat{K}_t)+\frac{1-\gamma(1-\delta)}{\psi}\,\mathbb{E}_t\left(\widehat{Y}_{t+1}-\widehat{X}_{t+1}-\widehat{K}_{t+1}\right)+\frac{\widehat{c}_t-\mathbb{E}_t\,\widehat{c}_{t+1}}{\psi} \quad (129)$$

- **Household Euler equation:** Log-linearize patient condition Equation (66):

$$\widehat{c}'_t=\mathbb{E}_t\,\widehat{c}'_{t+1}-\widehat{rr}_t \quad (130)$$

where $\hat{rr}_t$ is the expression of real interest rate:

$$\widehat{rr}_t=\hat{R}_t-\mathbb{E}_t\,\widehat{\pi}_{t+1} \quad (131)$$

Log impatient condition Equation (60)

$$-\ln c_t=\ln\Big[\frac{\gamma R}{c_{t+1}\pi_{t+1}}+\lambda_t R_t\Big]$$

Totally differentiate, with $R=\frac{1}{\beta}$, and rearrange to get

$$-\widehat{c}_t=-\frac{\gamma}{\beta}\,\mathbb{E}_t\,\hat{c}_{t+1}+\frac{\gamma}{\beta}\widehat{rr}_t+\frac{\lambda c}{\beta}(\widehat{\lambda}_t+\widehat{R}_t) \quad (132)$$

Combine Equation (131) and Equation (132), with the condition $\lambda c = \beta - \gamma$ in the steady state, we can thus have:

$$\begin{aligned} \mathbb{E}_t\,\widehat{\pi}_{t+1} &= \widehat{R}_t - \widehat{rr}_t \\ &= -\frac{\beta}{\beta-\gamma}\widehat{c}_t - \widehat{\lambda}_t + \frac{\beta c}{\beta-\gamma}\,\mathbb{E}_t\,\widehat{c}_{t+1} - \Big(\frac{\beta c}{\beta-\gamma}+1\Big)\widehat{rr}_t \end{aligned} \tag{133}$$

- **Houisng:**Equation (75) can written as:

$$\frac{q_t}{c_t} = \lambda_t m\,\mathbb{E}_t\,q_{t+1}\pi_{t+1} + \gamma\,\mathbb{E}_t\,\frac{1}{c_{t+1}}\left[\frac{\nu Y_{t+1}}{h_t X_{t+1}} + q_{t+1}\right]$$

Then log-linearize it:

$$\begin{aligned} \widehat{q}_t - \widehat{c}_t = \frac{c}{q}\,\mathbb{E}_t\Big[&-\frac{\gamma}{c^2}(\frac{\nu Y}{hX}+q)(c_{t+1}-c) + \frac{\gamma\nu}{Xhc}(Y_{t+1}-Y) - \frac{\gamma\nu Y}{X^2hc}(X_{t+1}-X) \\ &- \frac{\gamma\nu Y}{Xh^2c}(h_t-h) + \frac{\gamma}{c}(q_{t+1}-q) + m\lambda(q_{t+1}-q) + qm(\lambda_t-\lambda) + qm\lambda(\pi_{t+1}-\pi)\Big] \end{aligned}$$

Then from Equation (114), we have $1-\gamma_e = \frac{\gamma\nu Y}{qXh}$, plug them in the above equation, we can transform the right-hand side to:

$$\gamma_e\,\mathbb{E}_t\,\widehat{q}_{t+1} + (1-\gamma_e)\,\mathbb{E}_t(\widehat{Y}_{t+1} - \widehat{h}_t - \widehat{X}_{t+1} + m(\beta-\gamma)\,\mathbb{E}_t(\widehat{\lambda}_t + \widehat{\pi}_{t+1}) - (\gamma_e - 1 - c)\,\mathbb{E}_t\,\widehat{c}_{t+1}$$

Then we combine both side, and substitute Equation (133) in the equation to eliminate inflation, and simplify to:

$$\widehat{q}_t = (1-\gamma_e)\,\mathbb{E}_t(\widehat{Y}_{t+1} - \widehat{h}_t - \hat{X}_{t+1}) - (1-m\beta)\,\mathbb{E}_t(\widehat{c}_{t+1} - \widehat{c}_t) - m\widehat{\beta}\widehat{rr}_t + \gamma_e\,\mathbb{E}_t\,\hat{q}_{t+1} \tag{134}$$

Then using same approach, we can solve for Equation (62) and Equation (67):

$$\widehat{q}_t = \gamma_h\,\mathbb{E}_t\,\widehat{q}_{t+1} + m''\beta\widehat{rr}_t + (1-\gamma_h)(\widehat{j}_t - \widehat{h}''_t) - (1-m''\beta)\Big(\widehat{c}''_t - \Omega\,\mathbb{E}_t\,\widehat{c}''_{t+1}\Big) \tag{135}$$

And

$$\widehat{q}_t = (1-\beta)\widehat{j}_t + \beta\,\mathbb{E}_t\,\widehat{q}_{t+1} + \iota\widehat{h}_t + \iota''\widehat{h}''_t + \widehat{c}'_t - \beta\,\mathbb{E}_t\,\widehat{c}'_{t+1} \tag{136}$$

with

$$\Omega = \frac{\beta'' - m''\beta''}{1-m''\beta} \tag{137}$$

$$\iota = (1-\beta)\frac{h}{h'} \tag{138}$$

$$\iota'' = (1-\beta)\frac{h''}{h'} \tag{139}$$

Equation (134), Equation (135), and Equation (136) delineate the optimal consumption and housing conditions for entrepreneurs, and impatient and patient households, respectively. The prevailing house price is determined by two primary factors: its linear relationship with the discounted future housing price and its responsiveness to variations in consumption levels.

- **Wage and labor:** By Equation (61) and Equation (65), the log-linearized labor demand conditions are:

$$\widehat{w}'_t = (\eta - 1)\widehat{L}'_t + \widehat{c}'_t \tag{140}$$

$$\widehat{w}''_t = (\eta - 1)\widehat{L}''_t + \widehat{c}''_t \tag{141}$$

By Equation (69) and Equation (70), we can have:

$$\widehat{w}'_t = \widehat{Y}_t - \widehat{X}_t - \widehat{L}'_t \tag{142}$$

$$\widehat{w}''_t = \widehat{Y}_t - \widehat{X}_t - \widehat{L}''_t \tag{143}$$

Combine the four equations above, we can get the expression for labor demand:

$$\widehat{L}'_t = \frac{\widehat{Y}_t - \widehat{X}_t - \widehat{c}'_t}{\eta} \tag{144}$$

$$\widehat{L}''_t = \frac{\widehat{Y}_t - \widehat{X}_t - \widehat{c}''_t}{\eta} \tag{145}$$

- **Production function:** Log-linearize Equation (81):

$$\widehat{Y}_t = \widehat{A}_t + \mu\widehat{K}_{t-1} + \nu\widehat{h}_{t-1} + \alpha(1-\mu-\nu)\widehat{L}'_t + (1-\alpha)(1-\mu-\nu)\widehat{L}''_t \tag{146}$$

Substitute Equation (167) and Equation (168) into the equation above, then rearrange, we can then get:

$$\widehat{Y}_t = \frac{\eta}{\eta-(1-\mu-\nu)}\left(\widehat{A}_t + \mu\widehat{K}_{t-1} + \nu\widehat{h}_{t-1}\right) - \frac{1-\mu-\nu}{\eta-(1-\mu-\nu)}\left[\widehat{X}_t + \alpha\widehat{c}'_t + (1-\alpha)\widehat{c}''_t\right] \tag{147}$$

The production function is given under the labor market's clearing conditions. The term $\widehat{A}t + \mu\widehat{K}t - 1 + \nu\widehat{h}_{t-1}$ indicates the inputs of technology, capital, and housing. Meanwhile, $\widehat{X}_t + \alpha\widehat{c}'_t + (1-\alpha)\widehat{c}''_t$ denotes the entrepreneurs' production used by retailers and both patient and impatient households.

- **Flow of funds and bonds:** First, we know the motion of capital:

$$\widehat{K}_t = \delta\widehat{I}_t + (1-\delta)\widehat{K}_{t-1} \tag{148}$$

Next, we plug Equation (63) into Equation (69), to get:

$$b'' = q_t(h''_t - h''_{t-1}) + c''_t + \frac{R_{t-1}b''_{t-1}}{\pi_t} - \frac{(1-\alpha)(1-\mu-\nu)}{X_t}$$

This one just follows the standard log0linearization process. Log-linearize it we can get:

$$\widehat{Y}_t - \widehat{X}_t = \frac{X}{Y}\left[(c''_t - c) + q(h''_t - h)q(h''_{t_1} - h) + b(R_{t-1} - R) + R(b_{t-1} - b) - Rb(\pi_t - \pi) - (b''_t - b)\right]$$

Rearrange, we finally get:

$$\frac{b''}{Y}\widehat{b}''_t = \frac{qh''}{Y}(\widehat{h}''_t - \widehat{h}''_{t-1}) + \frac{c''}{Y}\widehat{c}''_t + \frac{Rb''}{Y}\left(\widehat{b}''_{t-1} + \widehat{R}_{t-1} - \widehat{\pi}_t\right) - s''\widehat{Y}_t + s''\widehat{X}_t \tag{149}$$

Similarly, combine Equation (68),Equation (69), and Equation (71) and take log-linearization, we can get:

$$\frac{b}{Y}\widehat{b}_t = \frac{c}{Y}\widehat{c}_t + \frac{qh}{Y}(\widehat{h}_t - \widehat{h}_{t-1}) + \frac{Rb}{Y}\left(\widehat{b}_{t-1} + \widehat{R}_{t-1} - \widehat{\pi}_t\right) - (1 - s' - s'')(\widehat{Y}_t - \widehat{X}_t) + \frac{I}{Y}\widehat{I}_t \tag{150}$$

- **Resource Constraint:** Log-linearize Equation (82):

$$\widehat{Y}_t = \frac{c}{Y}\widehat{c}_t + \frac{c'}{Y}\widehat{c}_t' + \frac{c''}{Y}\widehat{c}_t'' + \frac{I}{Y}\widehat{I}_t \tag{151}$$

- **Taylor type monetary policy rule:** Log-linearize Equation (80) we get:

$$\widehat{R}_t = r_R\hat{R}_{t-1} + (1 - r_R)\Big[(1 + r_\pi)\widehat{\pi}_{t-1} + r_Y\widehat{Y}_{t-1}\Big] + \widehat{e}_{R,t} \tag{152}$$

- **Shock process:**

$$\widehat{j}_t = \rho_j\widehat{j}_{t-1} + \widehat{e}_{j,t} \tag{153}$$

$$\widehat{A}_t = \rho_A\widehat{A}_{t-1} + \widehat{e}_{A,t} \tag{154}$$

$$\widehat{u}_t = \rho_u\widehat{u}_{t-1} + \widehat{e}_{u,t} \tag{155}$$

- **Borrowing constraint:**

$$\widehat{b}_t = \mathbb{E}_t\,\widehat{q}_{t+1} + \widehat{h}_t - \widehat{rr}_t \tag{156}$$

$$\widehat{b}''_t = \mathbb{E}_t\,\widehat{q}_{t+1} + \widehat{h}''_t - \widehat{rr}_t \tag{157}$$

D.2 Full Set of Log-linearized Equations

$$\widehat{\pi}_t = -\frac{(1-\theta)(1-\theta\beta)}{\theta}\widehat{X}_t + \beta\,\mathbb{E}_t\,\widehat{\pi}_{t+1} + \widehat{u}_t \tag{158}$$

$$\widehat{I}_t = \widehat{K}_{t-1} + \gamma(\mathbb{E}_t\,\widehat{I}_{t+1} - \widehat{K}_t) + \frac{1-\gamma(1-\delta)}{\psi}\,\mathbb{E}_t\left(\widehat{Y}_{t+1} - \widehat{X}_{t+1} - \widehat{K}_{t+1}\right) + \frac{\widehat{c}_t - \mathbb{E}_t\,\widehat{c}_{t+1}}{\psi} \tag{159}$$

$$\widehat{K}_t = \delta\widehat{I}_t + (1-\delta)\widehat{K}_{t-1} \tag{160}$$

$$\widehat{c}'_t = \mathbb{E}_t\,\widehat{c}'_{t+1} - \widehat{rr}_t \tag{161}$$

$$\widehat{rr}_t = \hat{R}_t - \mathbb{E}_t\,\widehat{\pi}_{t+1} \tag{162}$$

$$\mathbb{E}_t\,\widehat{\pi}_{t+1} = -\frac{\beta}{\beta-\gamma}\widehat{c}_t - \widehat{\lambda}_t + \frac{\beta c}{\beta-\gamma}\,\mathbb{E}_t\,\widehat{c}_{t+1} - \Big(\frac{\beta c}{\beta-\gamma}+1\Big)\widehat{rr}_t \tag{163}$$

$$\widehat{q}_t = \gamma_e\,\mathbb{E}_t\,\hat{q}_{t+1} + (1-\gamma_e)\,\mathbb{E}_t(\widehat{Y}_{t+1} - \widehat{h}_t - \hat{X}_{t+1}) - (1-m\beta)\,\mathbb{E}_t(\widehat{c}_{t+1} - \widehat{c}_t) - m\widehat{\beta rr}_t \tag{164}$$

$$\widehat{q}_t = \gamma_h\,\mathbb{E}_t\,\widehat{q}_{t+1} + m''\beta\widehat{rr}_t + (1-\gamma_h)(\widehat{j}_t - \widehat{h}''_t) - (1-m''\beta)\Big(\widehat{c}''_t - \Omega\,\mathbb{E}_t\,\widehat{c}''_{t+1}\Big) \tag{165}$$

$$\widehat{q}_t = \beta\,\mathbb{E}_t\,\widehat{q}_{t+1} + (1-\beta)\widehat{j}_t + \iota\widehat{h}_t + \iota''\widehat{h}''_t + \widehat{c}'_t - \beta\,\mathbb{E}_t\,\widehat{c}'_{t+1} \tag{166}$$

$$\widehat{L}'_t = \frac{\widehat{Y}_t - \widehat{X}_t - \widehat{c}'_t}{\eta} \tag{167}$$

$$\widehat{L}''_t = \frac{\widehat{Y}_t - \widehat{X}_t - \widehat{c}''_t}{\eta} \tag{168}$$

$$\widehat{Y}_t = \frac{\eta}{\eta-(1-\mu-\nu)}\Big(\widehat{A}_t + \mu\widehat{K}_{t-1} + \nu\widehat{h}_{t-1}\Big) - \frac{1-\mu-\nu}{\eta-(1-\mu-\nu)}\Big[\widehat{X}_t + \alpha\widehat{c}'_t + (1-\alpha)\widehat{c}''_t\Big] \tag{169}$$

$$\frac{b''}{Y}\widehat{b}''_t = \frac{c''}{Y}\widehat{c}''_t + \frac{qh''}{Y}(\widehat{h}''_t - \widehat{h}''_{t-1}) + \frac{Rb''}{Y}\Big(\widehat{b}''_{t-1} + \widehat{R}_{t-1} - \widehat{\pi}_t\Big) - s''(\widehat{Y}_t - \widehat{X}_t) \tag{170}$$

$$\frac{b}{Y}\widehat{b}_t = \frac{c}{Y}\widehat{c}_t + \frac{qh}{Y}(\widehat{h}_t - \widehat{h}_{t-1}) + \frac{Rb}{Y}\Big(\widehat{b}_{t-1} + \widehat{R}_{t-1} - \widehat{\pi}_t\Big) - (1-s'-s'')(\widehat{Y}_t - \widehat{X}_t) + \frac{I}{Y}\widehat{I}_t \tag{171}$$

$$\widehat{Y}_t = \frac{c}{Y}\widehat{c}_t + \frac{c'}{Y}\widehat{c}_t{}' + \frac{c''}{Y}\widehat{c}_t{}'' + \frac{I}{Y}\widehat{I}_t \tag{172}$$

$$\widehat{R}_t = r_R\hat{R}_{t-1} + (1-r_R)\Big[(1+r_\pi)\widehat{\pi}_{t-1} + r_Y\hat{Y}_{t-1}\Big] + \widehat{e}_{R,t} \tag{173}$$

$$\widehat{j}_t = \rho_j\widehat{j}_{t-1} + \widehat{e}_{j,t} \tag{174}$$

$$\widehat{A}_t = \rho_A\widehat{A}_{t-1} + \widehat{e}_{A,t} \tag{175}$$

$$\widehat{u}_t = \rho_u\widehat{u}_{t-1} + \widehat{e}_{u,t} \tag{176}$$

$$\widehat{b}_t = \mathbb{E}_t\,\widehat{q}_{t+1} + \widehat{h}_t - \widehat{rr}_t \tag{177}$$

$$\widehat{b}_t'' = \mathbb{E}_t \, \widehat{q}_{t+1} + \widehat{h}_t'' - \widehat{r}r_t \tag{178}$$

E Additional Quantitative Results for Baseline Analysis

E.1 Additional Baseline Impulse Response Functions

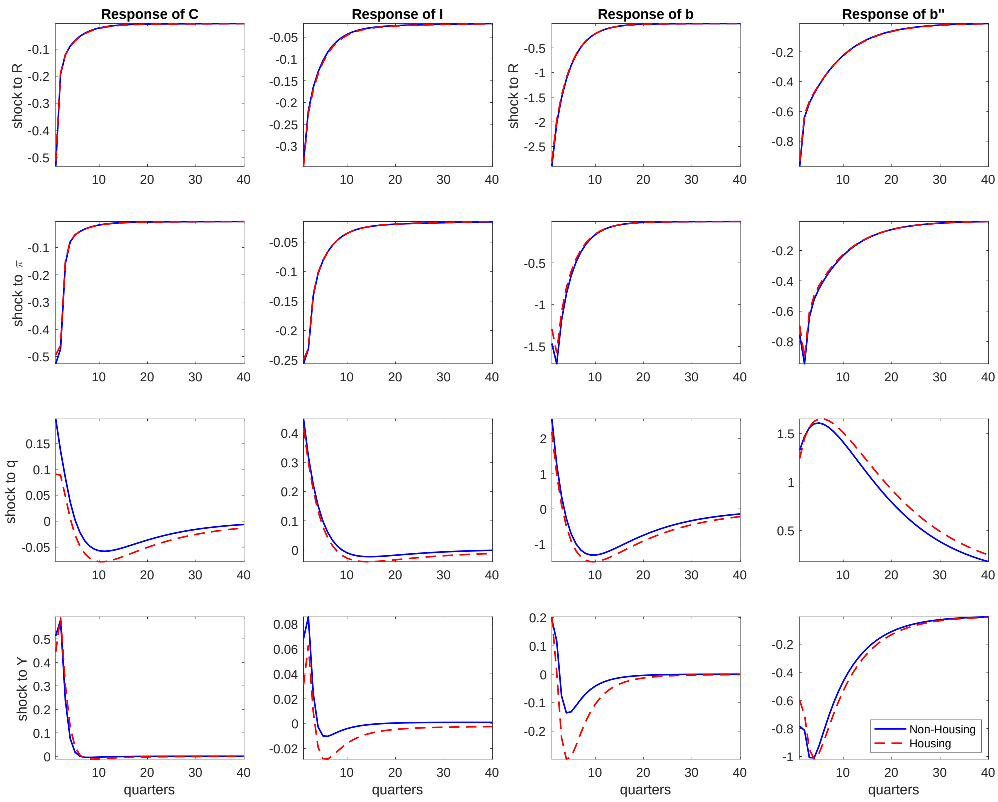


Figure E.1. US Impulse Response Functions of Consumption, Investment and Borrowing Constraints

E.2 Time-Varying Variance Decomposition

Table E.1. Time Varying Conditional Variance Decomposition (in percent)

	$r_q = 0$				$r_q \neq 0$			
	$\hat{e}_R$	$\hat{e}_j$	$\hat{e}_A$	$\hat{e}_u$	$\hat{e}_R$	$\hat{e}_j$	$\hat{e}_A$	$\hat{e}_u$
Period 1:								
$\hat{R}$	100.00	0.00	0.00	0.00	99.63	0.03	0.04	0.30
$\hat{\pi}$	5.08	0.03	5.06	89.83	5.35	0.21	5.80	88.64
$\hat{q}$	46.20	2.75	6.32	44.72	46.05	4.03	6.45	43.47
$\hat{Y}$	62.10	0.27	0.97	36.66	63.81	0.15	0.83	35.22
Period 4:								
$\hat{R}$	59.94	0.01	2.98	37.08	57.83	0.08	3.66	38.43
$\hat{\pi}$	5.15	0.06	5.05	89.75	5.24	0.42	6.18	88.15
$\hat{q}$	31.54	5.64	10.49	52.33	29.02	9.24	12.52	49.22
$\hat{Y}$	51.33	0.31	1.60	46.76	52.39	0.19	1.79	45.63
Period 8:								
$\hat{R}$	59.91	0.01	3.02	37.07	57.73	0.11	3.79	38.37
$\hat{\pi}$	5.22	0.06	5.05	89.68	5.25	0.47	6.20	88.08
$\hat{q}$	31.29	7.50	10.32	50.89	28.15	12.78	12.26	46.81
$\hat{Y}$	51.87	0.31	1.56	46.26	53.01	0.22	1.73	45.04
Period 12:								
$\hat{R}$	59.90	0.01	3.02	37.07	57.72	0.12	3.80	38.37
$\hat{\pi}$	5.23	0.06	5.05	89.66	5.25	0.48	6.20	88.07
$\hat{q}$	31.12	8.19	10.25	50.43	27.72	14.26	12.07	45.95
$\hat{Y}$	51.96	0.34	1.55	46.15	53.08	0.30	1.72	44.89
Period 16:								
$\hat{R}$	59.90	0.01	3.02	37.07	57.71	0.12	3.80	38.37
$\hat{\pi}$	5.23	0.07	5.05	89.65	5.26	0.48	6.20	88.06
$\hat{q}$	31.02	8.47	10.23	50.27	27.50	14.93	12.00	45.57
$\hat{Y}$	51.98	0.36	1.55	46.11	53.09	0.37	1.72	44.83
Period 20:								
$\hat{R}$	59.90	0.01	3.02	37.07	57.71	0.12	3.80	38.37
$\hat{\pi}$	5.24	0.07	5.05	89.64	5.27	0.48	6.20	88.05
$\hat{q}$	30.96	8.61	10.21	50.22	27.40	15.25	11.96	45.39
$\hat{Y}$	51.99	0.37	1.54	46.09	53.08	0.41	1.72	44.78

F Rolling Window Estimation Results

Table F.1. Rolling Window Bayesian results for Australia

	1973 II-1983 II	1978 II-1988 II	1983 II-1993 II	1988 II-1998 II	1993 II-2003 II	1998 II-2008 II
Log-density						
$r_q = 0$	-608.91	-590.74	-593.96	-590.97	-574.80	-573.50
$r_q \neq 0$	-610.62	-591.35	-595.21	-593.22	-576.23	-575.20
K-R ratio	-3.42	-1.22	-2.5	-4.5	-1.43	-3.4

Table F.2. Rolling Window Bayesian results for New Zealand

	1990 II-1996 II	1993 II-1999 II	1996 II-2002 II	1999 II-2005 II	2002 II-2008 II
Log-density					
$r_q = 0$	-381.18	-364.44	-366.92	-374.19	-379.06
$r_q \neq 0$	-382.83	-365.06	-367.36	-375.77	-380.90
K-R ratio	-3.3	-1.24	-0.88	-3.16	-3.68

Table F.3. Rolling Window Bayesian results for the United Kingdom

	1987 II-1993 II	1990 II-1996 II	1993 II-1999 II	1997 II-2002 II	1999 II-2005 II	2002 II-2008 II
Log-density						
$r_q = 0$	-386.35	-378.73	-368.37	-374.72	-370.64	-385.24
$r_q \neq 0$	-388.58	-381.14	-369.61	-375.93	-371.78	-385.84
K-R ratio	-4.46	-4.82	-2.48	-2.44	-2.22	-1.2

Table F.4. Rolling Window Bayesian results for the Euro Area

	1996 II-2000 II	1998 II-2002 II	2000 II-2004 II	2002 II-2006 II	2004 II-2008 II
Log-density					
$r_q = 0$	-239.26	-234.84	-230.91	-234.12	-237.19
$r_q \neq 0$	-239.68	-234.46	-230.64	-235.09	-237.41
K-R ratio	-0.84	0.76	0.54	-1.94	-0.44

Table F.5. Rolling Window Bayesian results for Canada

	1973 II-1983 II	1978 II-1988 II	1983 II-1993 II	1988 II-1998 II	1993 II-2003 II	1998 II -2008 II
Log-density						
$r_q = 0$	-601.49	-592.72	-595.70	-591.42	-563.67	-568.88
$r_q \neq 0$	-602.75	-593.62	-592.00	-588.72	-563.68	-570.33
K-R ratio	-2.52	-1.8	7.4	5.4	-0.02	-2.9

G Canada Quantitative Results

G.1 Canada Full Sample Estimation Results and Comparison

Table G.1. Canada Full Sample Prior and Posterior Distribution

	Prior			$r_q \neq 0$ Posterior		$r_q = 0$ Posterior	
Parameter	Distribution	Mean	S.D	Mean	90% HPD interval	Mean	90% HPD interval
ψ	Gamma	2	1	4.4703	[2.3536,6.2327]	4.3977	[2.4024,6.3194]
ρ_j	Beta	0.5	0.2	0.9580	[0.9389,0.9779]	0.9544	[0.9366,0.9726]
ρ_A	Beta	0.5	0.2	0.4021	[0.2415,0.5447]	0.2755	[0.1483,0.3926]
ρ_u	Beta	0.5	0.2	0.0425	[0.0060,0.0776]	0.0441	[0.0064,0.0847]
r_R	Beta	0.75	0.1	0.6299	[0.5307,0.7386]	0.6280	[0.5249,0.7383]
r_π	Normal	1.5	0.1	1.4170	[1.2522,1.5843]	1.4297	[1.2743,1.6013]
r_Y	Normal	0.1	0.05	0.2707	[0.2075,0.3376]	0.2746	[0.2130,0.3424]
r_q	Normal	0.1	0.05	0.0952	[0.0555,0.1339]	—	—
r^*	Gamma	2	0.5	2.1166	[1.3057,2.9514]	2.1222	[1.2345,2.9118]
π^*	Gamma	2.3	2	1.6172	[0.0039,3.3026]	1.5269	[0.0139,3.0846]
g^*	Normal	0.4	0.2	0.2117	[0.1004,0.3170]	0.2721	[0.1692,0.3788]
σ_R	Ive.gamma	0.5	1	0.0661	[0.0645,0.0680]	0.0661	[0.0645,0.0681]
σ_A	Ive.gamma	0.5	1	0.1426	[0.1180,0.1682]	0.1710	[0.1481,0.1957]
σ_j	Ive.gamma	0.5	1	0.2339	[0.1470,0.3161]	0.1855	[0.1267,0.2424]
σ_u	Ive.gamma	0.5	1	0.0665	[0.0645,0.0691]	0.0667	[0.0645,0.0695]

Table G.2. Bayesian log density for Full Sample in Canada

	Log data density
$r_q = 0 : \ln p(Y_T \mid \theta_O)$	-1094.98
$r_q \neq 0 : \ln p(Y_T \mid \theta_H)$	-1091.22
K-R ratio	7.52

G.2 Canada Impulse Responses

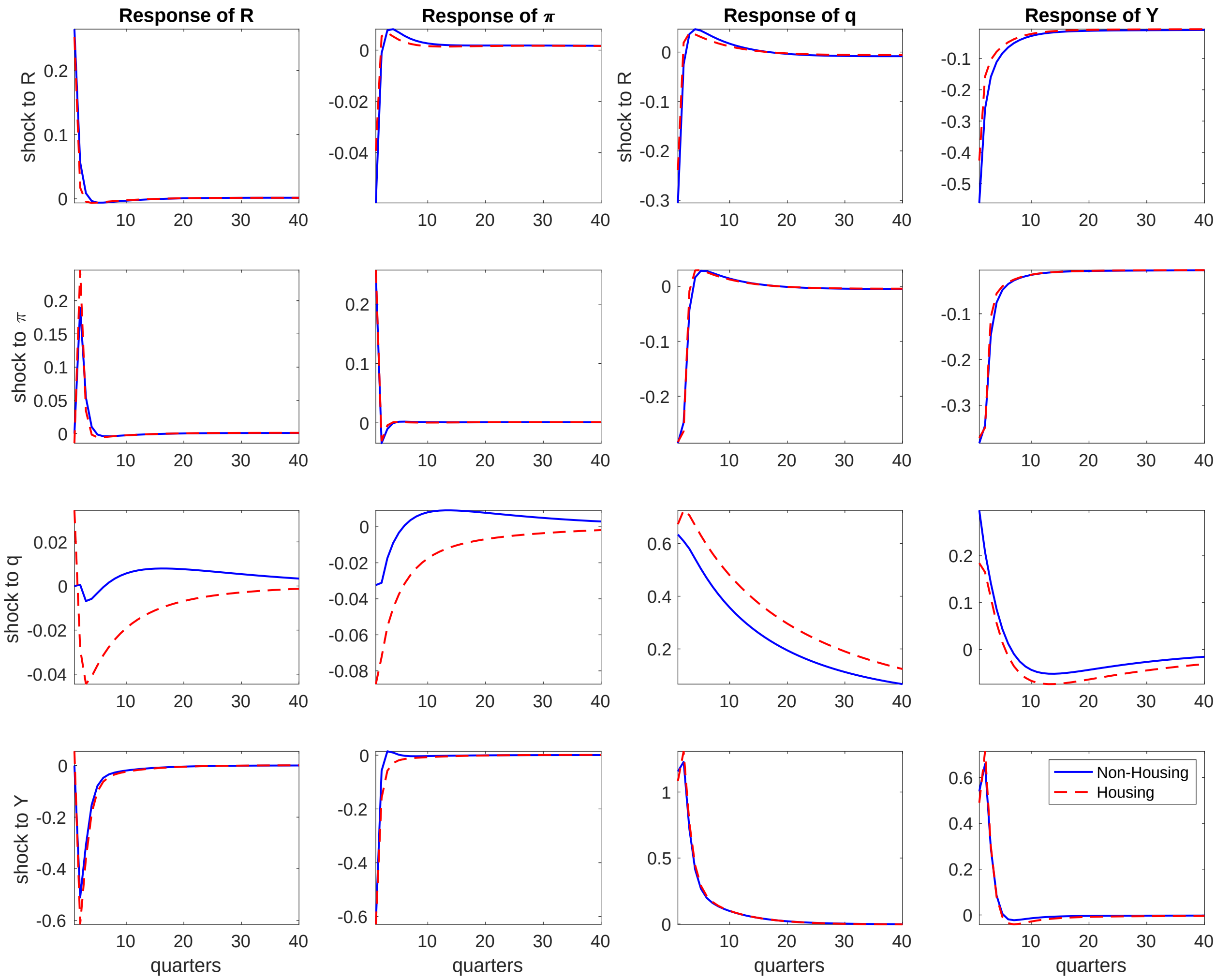


Figure G.1. Canada Bayesian Posterior Mean Impulse Response Functions for Model with and without Response to Housing

G.3 Canada Variance Decomposition

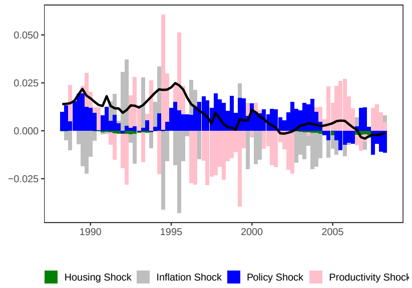
Table G.3. Canada Variance Decomposition of the Forecast Error when $r_q = 0$

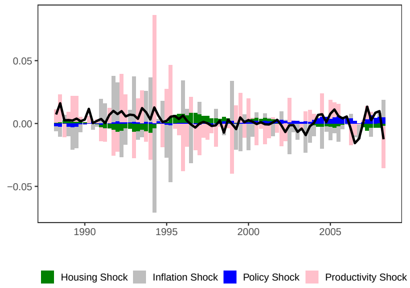
	$\hat{R}$	$\hat{\pi}$	$\hat{q}$	$\hat{Y}$
$\hat{e}_R$	69.73	7.64	30.07	67.86
$\hat{e}_j$	0.03	0.19	24.96	0.80
$\hat{e}_A$	6.60	13.36	18.39	2.21
$\hat{e}_u$	23.63	78.81	26.57	29.13

Table G.4. Canada Variance Decomposition of the Forecast Error when $r_q \neq 0$

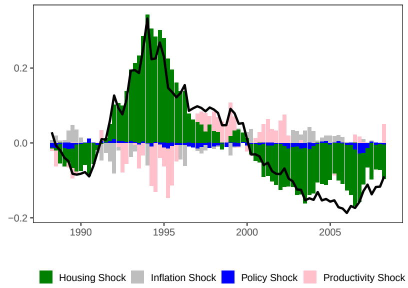
	$\hat{R}$	$\hat{\pi}$	$\hat{q}$	$\hat{Y}$
$\hat{e}_R$	56.22	3.59	13.78	54.12
$\hat{e}_j$	0.41	1.79	51.09	1.79
$\hat{e}_A$	8.80	14.23	14.49	3.47
$\hat{e}_u$	34.57	80.39	20.63	40.63

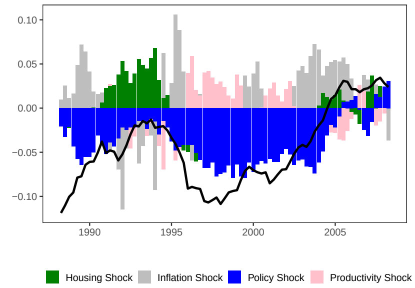
G.4 Canada Historical Shock Decomposition





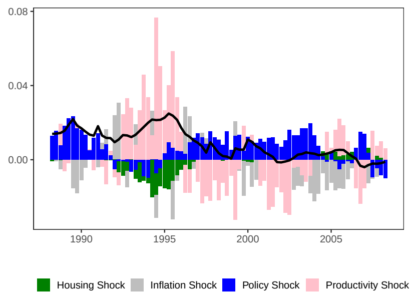
(a) Interest Rates **(b)** Inflation

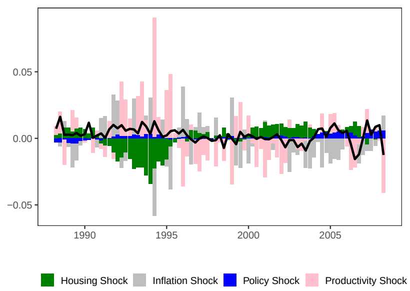




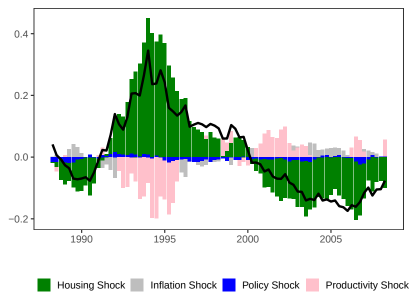
(c) Real House Prices **(d)** Output

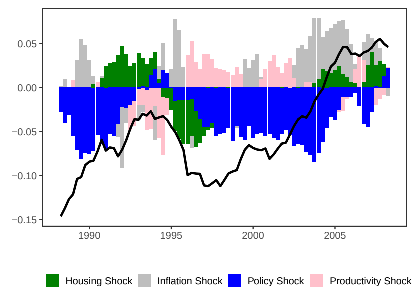
Figure G.2. Canada Historical Shock Decomposition when $r_q = 0$





(a) Interest Rates **(b)** Inflation



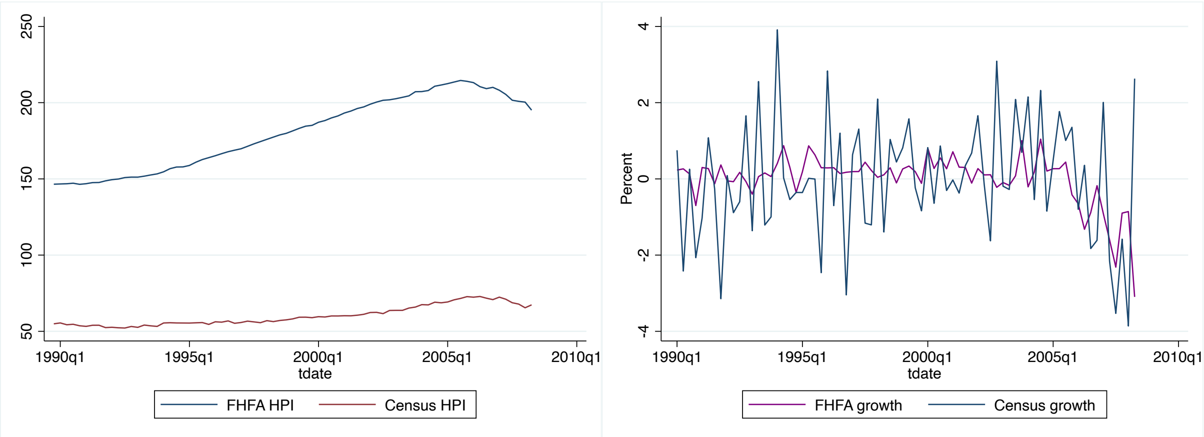


(c) Real House Prices **(d)** Output

Figure G.3. Canada Historical Shock Decomposition when $r_q \neq 0$

H Robustness Check Results

H.1 FHFA HPI Data Comparison



(a) Real House Price Index (b) Real House Price Index Growth

Figure H.1. FHFA HPI and Census HPI Comparison

H.2 Robustness Bayesian Results

Table H.1. Alternative r_R Prior and Posterior Distribution

	Prior			$r_q \neq 0$ Posterior		$r_q = 0$ Posterior	
Parameter	Distribution	Mean	S.D	Mean	90% HPD interval	Mean	90% HPD interval
ψ	Gamma	2	1	4.3118	[2.3030,6.1901]	4.5227	[2.5577,6.5913]
ρ_j	Beta	0.5	0.2	0.9305	[0.9028,0.9608]	0.9152	[0.8855,0.9439]
ρ_A	Beta	0.5	0.2	0.3876	[0.2836,0.4999]	0.2758	[0.1852,0.3631]
ρ_u	Beta	0.5	0.2	0.0425	[0.0040,0.0798]	0.0445	[0.0045,0.0822]
r_R	Beta	0.75	0.2	0.2825	[0.0932,0.4658]	0.3113	[0.1136,0.5100]
r_π	Normal	1.5	0.1	1.5106	[1.3573,1.6710]	1.4995	[1.3323,1.6652]
r_Y	Normal	0.1	0.05	0.1779	[0.1063,0.2527]	0.2031	[0.1341,0.2711]
r_q	Normal	0.1	0.05	0.1163	[0.0706,0.1605]	—	—
r^*	Gamma	2	0.5	1.9456	[1.1174,2.7045]	1.9705	[1.1375,2.7315]
π^*	Gamma	2.3	2	2.0259	[0.0356,4.0336]	2.0911	[0.0420,4.0865]
g^*	Normal	0.4	0.2	0.5148	[0.4246,0.6005]	0.5041	[0.4270,0.5796]
σ_R	Ive.gamma	0.5	1	0.0663	[0.0645,0.0687]	0.0664	[0.0645,0.0689]
σ_A	Ive.gamma	0.5	1	0.0947	[0.0796,0.1095]	0.1065	[0.0891,0.1220]
σ_j	Ive.gamma	0.5	1	0.1982	[0.1297,0.2621]	0.1706	[0.1136,0.2212]
σ_u	Ive.gamma	0.5	1	0.0666	[0.0645,0.0693]	0.0666	[0.0645,0.0693]

Table H.2. Alternative r_π Prior and Posterior Distribution

	Prior			$r_q \neq 0$ Posterior		$r_q = 0$ Posterior	
Parameter	Distribution	Mean	S.D	Mean	90% HPD interval	Mean	90% HPD interval
ψ	Gamma	2	1	4.5105	[2.4541,6.5025]	4.6102	[2.7887,6.5466]
ρ_j	Beta	0.5	0.2	0.9262	[0.8953,0.9596]	0.9123	[0.8825,0.9441]
ρ_A	Beta	0.5	0.2	0.4792	[0.3420,0.6118]	0.3627	[0.2317,0.4951]
ρ_u	Beta	0.5	0.2	0.0402	[0.0050,0.0758]	0.0432	[0.0058,0.0792]
r_R	Beta	0.75	0.1	0.4724	[0.3490,0.5981]	0.4926	[0.3602,0.6164]
r_π	Normal	1.1	0.2	1.2844	[1.0075,1.5573]	1.1679	[0.8792,1.4774]
r_Y	Normal	0.1	0.05	0.1814	[0.1120,0.2494]	0.2125	[0.1417,0.2781]
r_q	Normal	0.1	0.05	0.1000	[0.0534,0.1455]	—	—
r^*	Gamma	2	0.5	1.9789	[1.1342,2.7180]	1.9900	[1.1581,2.7804]
π^*	Gamma	2.3	2	1.9706	[0.0302,3.8840]	2.0473	[0.0191,4.0565]
g^*	Normal	0.4	0.2	0.5074	[0.4077,0.5997]	0.4970	[0.4141,0.5822]
σ_R	Ive.gamma	0.5	1	0.0664	[0.0645,0.0688]	0.0663	[0.0645,0.0689]
σ_A	Ive.gamma	0.5	1	0.0898	[0.0751,0.1050]	0.1008	[0.0858,0.1163]
σ_j	Ive.gamma	0.5	1	0.2087	[0.1305,0.2826]	0.1799	[0.1220,0.2379]
σ_u	Ive.gamma	0.5	1	0.0668	[0.0645,0.0696]	0.0665	[0.0645,0.0692]

Table H.3. Alternative r_q Prior and Posterior Distribution

	Prior			$r_q \neq 0$ Posterior		$r_q = 0$ Posterior	
Parameter	Distribution	Mean	S.D	Mean	90% HPD interval	Mean	90% HPD interval
ψ	Gamma	2	1	4.5637	[2.5974,6.4927]	4.6563	[2.6723,6.6208]
ρ_j	Beta	0.5	0.2	0.9277	[0.8954,0.9595]	0.9152	[0.8864,0.9435]
ρ_A	Beta	0.5	0.2	0.4912	[0.3819,0.6060]	0.3140	[0.2152,0.4097]
ρ_u	Beta	0.5	0.2	0.0414	[0.0068,0.0789]	0.0466	[0.0057,0.0883]
r_R	Beta	0.75	0.1	0.6020	[0.5768,0.6338]	0.5032	[0.3773,0.6266]
r_π	Normal	1.5	0.1	1.5480	[1.3765,1.7063]	1.5103	[1.3503,1.6752]
r_Y	Normal	0.1	0.05	0.1810	[0.1100,0.2495]	0.2050	[0.1371,0.2740]
r_q	Normal	0.2	0.1	0.1173	[0.0658,0.1664]	—	—
r^*	Gamma	2	0.5	1.9507	[1.1779,2.6968]	1.9893	[1.2148,2.7245]
π^*	Gamma	2.3	2	2.1540	[0.0013,4.3514]	2.1561	[0.0159,4.3195]
g^*	Normal	0.4	0.2	0.4979	[0.3926,0.6006]	0.4933	[0.4078,0.5767]
σ_R	Ive.gamma	0.5	1	0.0665	[0.0645,0.0692]	0.0664	[0.0645,0.0689]
σ_A	Ive.gamma	0.5	1	0.0898	[0.0761,0.1037]	0.1044	[0.0895,0.1201]
σ_j	Ive.gamma	0.5	1	0.2141	[0.1353,0.2966]	0.1721	[0.1200,0.2257]
σ_u	Ive.gamma	0.5	1	0.0666	[0.0645,0.0692]	0.0666	[0.0645,0.0693]

Table H.4. Alternative FHFA House Price Measure Prior and Posterior Distributions

	Prior			$r_q \neq 0$ Posterior		$r_q = 0$ Posterior	
Parameter	Distribution	Mean	S.D	Mean	90% HPD interval	Mean	90% HPD interval
ψ	Gamma	2	1	3.4984	[1.7099,5.2498]	3.5175	[1.8054,5.2267]
ρ_j	Beta	0.5	0.2	0.9274	[0.9044,0.9489]	0.9218	[0.9023,0.9426]
ρ_A	Beta	0.5	0.2	0.3588	[0.2269,0.4756]	0.3103	[0.1973,0.4181]
ρ_u	Beta	0.5	0.2	0.0359	[0.0040,0.0649]	0.0358	[0.0036,0.0665]
r_R	Beta	0.75	0.1	0.4794	[0.3563,0.6040]	0.4886	[0.3709,0.6116]
r_π	Normal	1.5	0.1	1.5251	[1.3576,1.6855]	1.5166	[1.3447,1.6705]
r_Y	Normal	0.1	0.05	0.1415	[0.0681,0.2105]	0.1488	[0.0785,0.2204]
r_q	Normal	0.1	0.05	0.0746	[0.0130,0.1286]	—	—
r^*	Gamma	2	0.5	2.0130	[1.1616,2.8358]	1.9967	[1.1928,2.7945]
π^*	Gamma	2.3	2	2.1078	[0.0188,4.0359]	1.9836	[0.0092,3.7410]
g^*	Normal	0.4	0.2	0.5447	[0.4506,0.6382]	0.5440	[0.4667,0.6226]
σ_R	Ive.gamma	0.5	1	0.0665	[0.0645,0.0691]	0.0663	[0.0645,0.0687]
σ_A	Ive.gamma	0.5	1	0.0764	[0.0655,0.0854]	0.0767	[0.0656,0.0861]
σ_j	Ive.gamma	0.5	1	0.1256	[0.0905,0.1612]	0.1169	[0.0874,0.1443]
σ_u	Ive.gamma	0.5	1	0.0665	[0.0645,0.0692]	0.0664	[0.0645,0.0688]

Table H.5. Alternative Contemporaneous Taylor Rule Prior and Posterior Distributions

	Prior			$r_q \neq 0$ Posterior		$r_q = 0$ Posterior	
Parameter	Distribution	Mean	S.D	Mean	90% HPD interval	Mean	90% HPD interval
ψ	Gamma	2	1	4.6367	[2.5653,6.6014]	4.5000	[2.3958,6.5972]
ρ_j	Beta	0.5	0.2	0.9284	[0.9031,0.9546]	0.9150	[0.8887,0.9438]
ρ_A	Beta	0.5	0.2	0.3377	[0.2412,0.4337]	0.2421	[0.1682,0.3130]
ρ_u	Beta	0.5	0.2	0.0449	[0.0048,0.0840]	0.0500	[0.0064,0.0915]
r_R	Beta	0.75	0.1	0.4447	[0.3157,0.5791]	0.4628	[0.3318,0.5856]
r_π	Normal	1.5	0.1	1.5118	[1.3464,1.6860]	1.5191	[1.3523,1.6860]
r_Y	Normal	0.1	0.05	0.2601	[0.1871,0.3362]	0.2767	[0.2018,0.3494]
r_q	Normal	0.1	0.05	0.1788	[0.1102,0.2478]	—	—
r^*	Gamma	2	0.5	1.9659	[1.1941,2.7345]	1.9483	[1.1307,2.6828]
π^*	Gamma	2.3	2	2.0594	[0.0078,4.1183]	2.0704	[0.0194,4.0499]
g^*	Normal	0.4	0.2	0.5111	[0.4271,0.5977]	0.5011	[0.4271,0.5830]
σ_R	Ive.gamma	0.5	1	0.0663	[0.0645,0.0685]	0.0663	[0.0645,0.0686]
σ_A	Ive.gamma	0.5	1	0.0997	[0.0845,0.1156]	0.1129	[0.0965,0.1301]
σ_j	Ive.gamma	0.5	1	0.1748	[0.1229,0.2296]	0.1506	[0.1052,0.1923]
σ_u	Ive.gamma	0.5	1	0.0666	[0.0645,0.0692]	0.0666	[0.0645,0.0695]

Table H.6. Alternative Forward-looking Taylor Rule Prior and Posterior Distributions

	Prior			$r_q \neq 0$ Posterior		$r_q = 0$ Posterior	
Parameter	Distribution	Mean	S.D	Mean	90% HPD interval	Mean	90% HPD interval
ψ	Gamma	2	1	4.9157	[2.9121,6.8140]	4.8334	[2.8776,6.7999]
ρ_j	Beta	0.5	0.2	0.9278	[0.9015,0.9556]	0.9096	[0.8817,0.9385]
ρ_A	Beta	0.5	0.2	0.6834	[0.6181,0.7498]	0.6102	[0.5418,0.6799]
ρ_u	Beta	0.5	0.2	0.0433	[0.0046,0.0780]	0.0440	[0.0077,0.0836]
r_R	Beta	0.75	0.1	0.5060	[0.3884,0.6172]	0.5441	[0.4375,0.6437]
r_π	Normal	1.5	0.1	1.5628	[1.4110,1.7193]	1.5397	[1.3821,1.6970]
r_Y	Normal	0.1	0.05	0.2156	[0.1412,0.2859]	0.2497	[0.1792,0.3191]
r_q	Normal	0.1	0.05	0.1612	[0.1014,0.2334]	—	—
r^*	Gamma	2	0.5	1.9735	[1.1613,2.7256]	1.9926	[1.1845,2.8157]
π^*	Gamma	2.3	2	2.0023	[0.0052,4.1539]	1.9595	[0.0180,4.0375]
g^*	Normal	0.4	0.2	0.5013	[0.3962,0.6038]	0.4930	[0.3953,0.5894]
σ_R	Ive.gamma	0.5	1	0.0665	[0.0645,0.0689]	0.0664	[0.0645,0.0688]
σ_A	Ive.gamma	0.5	1	0.0811	[0.0685,0.0939]	0.0889	[0.0744,0.1028]
σ_j	Ive.gamma	0.5	1	0.1988	[0.1297,0.2665]	0.1739	[0.1187,0.2307]
σ_u	Ive.gamma	0.5	1	0.0669	[0.0645,0.0699]	0.0666	[0.0645,0.0692]