Generating MMM diagram for defining the safety margin of self driving cars

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Generating MMM diagram for defining the safety margin of self driving cars

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Abstract. Autonomous driving, is considered as the future of mobility, and research on this topics growing exponentially. On the one hand, these vehicles must be controllable, making rapid maneuvers possible, in case of a sudden accident situation, while on the other hand their motion must be stable, to maintain directional stability under disturbances. It is important to specify the so called “safety margin”, the range of possible motions that the vehicle can produce with keeping its stability. The limits of the possible movement of a vehicle depend on its parameters. The aim of our study is to determine which parameters affect the controllability and stability of a vehicle. To find this, we used the "MRA moment method" to create the “Yaw Moment Diagram” in our simulation environment. This method is usually used to map the possible range of motion and judge the stability and controllability of a possible motion state. Here we will use a two track vehicle model, with nonlinear tire characteristics, and complete model of suspension linkage to create the MMM diagram of a vehicle with given parameters.

1. Introduction

This paper aims to understand how car parameters can effect to behaviour of an autonomous vehicle and determine which parameters affect the controllability and stability the car. It is important while a self-drive vehicle must avoid an accident because of an unexpected event, for example a pedestrian suddenly steps in front of the car.

By understanding how to control a vehicle at its limits, this knowledge can be applied to design better driver systems [1]. For instance, future systems could constantly and seamlessly provide combinations of steering, throttle, and brake to ensure that the autonomous driver remains in control of the vehicle at all times and can follow the designed path [2]. It is important to specify the so called “safety margin”, the range of possible motions that the vehicle can produce with keeping its stability [3]. To map the limit of the car MRA moment method [4] can be useful.

2. MRA moment method (MMM) [4]

The MRA Moment Method (MMM) is a quasi-static vehicle dynamical test method to get a picture of control and stability at constant speed. MMM can be used to evaluate the effect of a number of parameters, for example weight, center of gravity, downforce, weight distribution, etc.

To get the yaw moment diagram we have to maintain the velocity of the car at constant, while changing the body slip angle and steering angle to calculate the lateral force and yaw moment applied to the vehicle in each condition. The result is the yaw moment diagram which shows the received yaw moment - lateral force or yaw acceleration – lateral acceleration points. These related points are depicted on the diagram which draws the limit of the car, and also we can get information about its

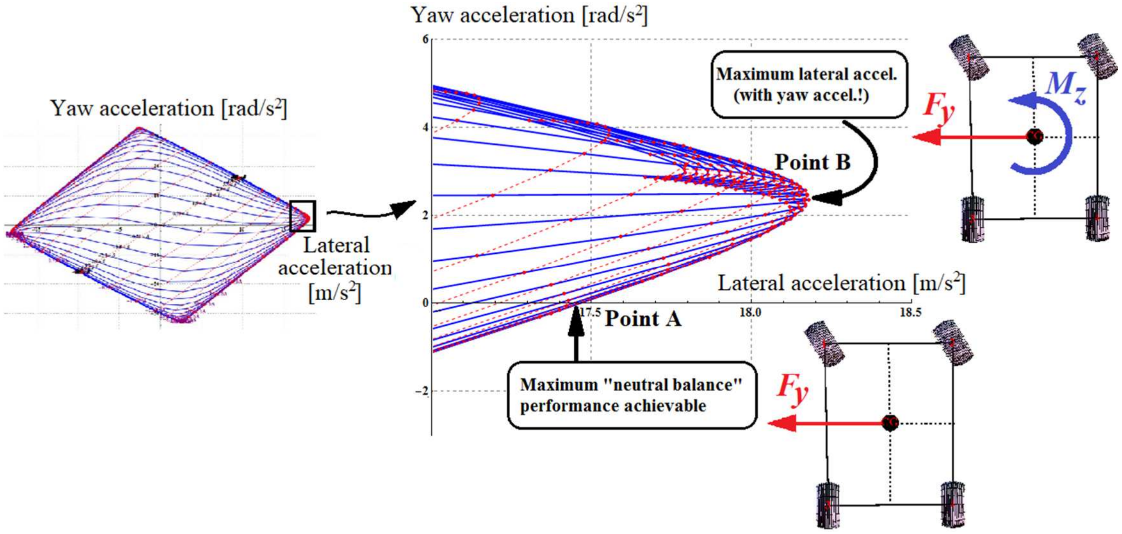

Figure 1. Exemplifying the oversteer behaviour in the yaw moment diagram.

under- or oversteer information. The limit of the car is the area of the diamond-like region shown in Figure 1. The information of under- or oversteering can be seen at the right (or left) corner of the diagram which is detailed in Figure 1.

The yaw moment diagram of oversteer car can be seen in Figure 1. There is two points at the right corner of the diagram. At "*Point A*" the yaw acceleration is zero, so in this point the car is neutral. There is a spare at front tires while the rear tires are at their limits. At "*Point B*" the car reaches the maximum lateral acceleration because the spare of the front tires are used, but the car has yaw acceleration with it. If we want to drive this car at maximum lateral acceleration which can available (*Point B*) we will have undesired yaw acceleration which turns more than we want, so the car is oversteer.

The under-, or oversteer behaviour depends on the position of the right tip compared to the yaw moment equal to zero line.

If the right tip of the diagram (*Point B*) is...

- above the yaw acceleration equal to zero line, the car is oversteer,
- at the yaw acceleration equal to zero line, the car is neutral,
- below the yaw acceleration equal to zero line, the car is understeer.

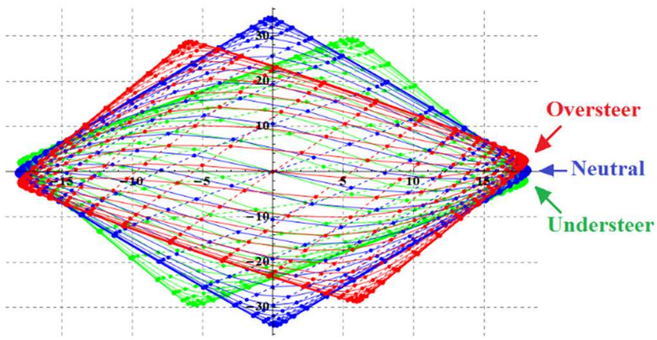

Figure 2. MMM diagram of an oversteer, neutral and understeer car.

3. Grip, balance, control and stability at MMM diagram [5]

The physical contents of the following introduced metrics are based on a controversial rather than an absolute correct theory. However, it is extremely useful if we want to tune the control, stability and balance of the car or raise the overall grip.

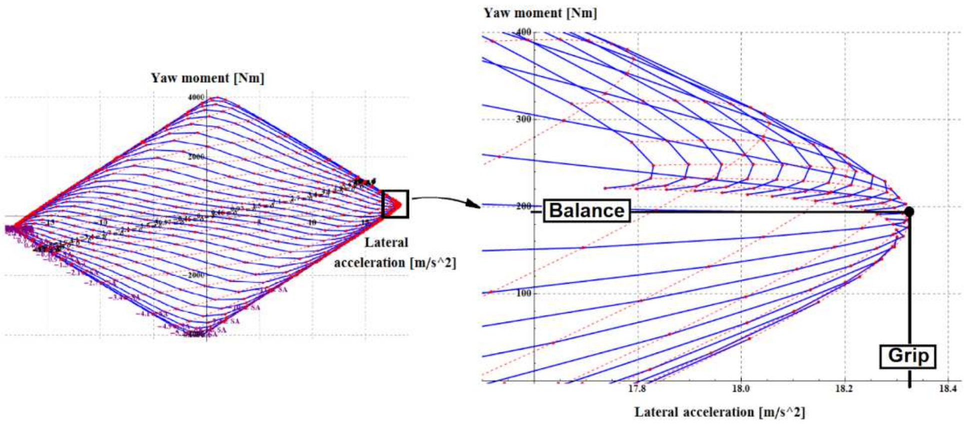

Figure 3. Definition of grip and balance.

With this theory we can also tell how the parameters can influence to the behaviour of the car. The degree of grip and balance is best seen at the tip of the right side of the MMM chart. It is shown at Figure 3.

The degree of grip is can be defined by the maximum lateral acceleration at the tip, while the balance of the car is characterized by its yaw moment of this point. The positive balance means that the car is oversteer (see Figure 1), the car is neutral if the balance is zero and understeer when the balance is negative. The control is the yaw moment which generated by 1 degree turning of the steering wheel (see Figure 4).

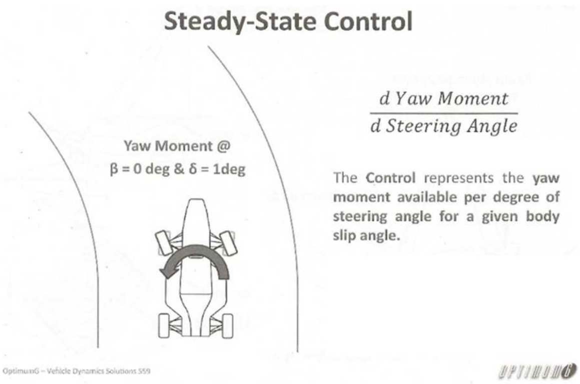

Figure 4. Defining of control [Claude Rouelle: Applied V.D. seminar].

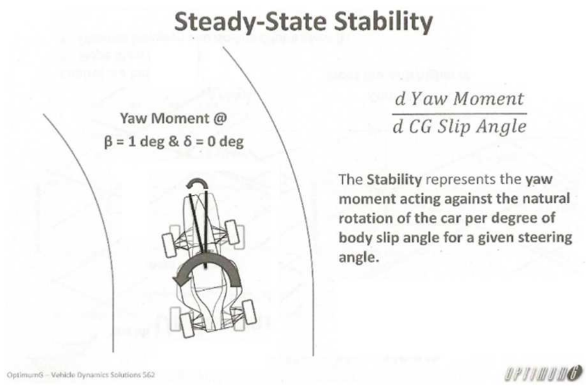

Figure 5. Defining of stability [Claude Rouelle: Applied V.D. seminar].

The stability of the car is the yaw moment generated during the 1 degree body slip. So, for example, if the momentum which generated by the body slip angle is positive, then this moment will generate additional body slip and the rear of the car "breaks out" (positive moment - Unstable). If the moment is negative then the resulting moment will prevent further body slip (ie rotation), so the car is will return to the straight position (negative torque - Stable).

The behavior of the car is most important at the moment, when the car is changing its state from straight to turn. Consequently, controllability and stability on the turn entrance can be defined by the following figure around the origin of MMM.

As can be seen in Figure 6 stability is a negative value, so less yaw moment means more stability.

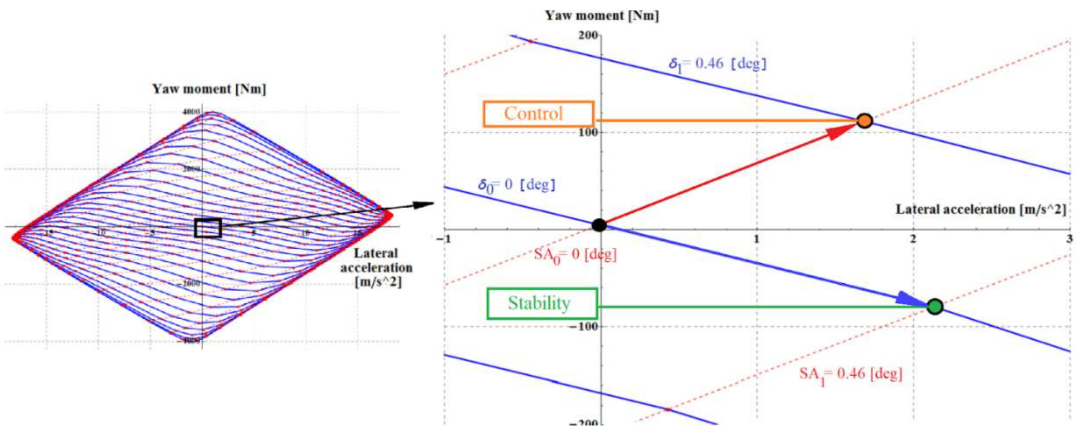

Figure 6. Control and stability at MMM diagram.

$$Control = \frac{d\,YawMoment}{d\,SteeringAngle} \cong \frac{YawMoment(SA = 0\,; \delta = \delta_1)}{\delta_1} \tag{3.1}$$

$$Stability = \frac{d\,YawMoment}{d\,CG\,SlipAngle} \cong \frac{YawMoment(SA = SA_1\,; \delta = 0)}{SA_1} \tag{3.2}$$

4. The mathematical model description to make the MMM diagram

To make the MMM diagram of a simulated car we have created a mathematical model with Wolfram Mathematica software. This is a 4 wheel model, so we can calculate with the effects of weight transfer, which is important for used PAC 2002 tire model [6].

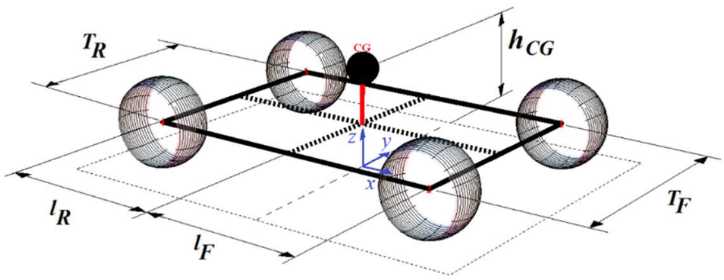

Figure 7. The mathematical model.

Table 1a. The notation and indexes in equations.

Notations		Indexes	
W	Wheelbase [m]	F	Front
T	Track [m]	R	Rear
l	Length [m]	*long* or x	Longitudinal direction
$distr_F$	Weight distribution at front [%]	*lat* or y	Lateral direction
h	Height [m]	z	Vertical direction
m	Mass of the car [kg]	CG	Center of Gravity
I	Yaw inertia of the car [kgm^2]	Aero	Aerodynamic
v	Velocity of the car [m/s]	SM	Suspended mass
A	Forehead surface [m^2]	NSM	Non-suspended mass

Table 1b. The notation and indexes in equations.

Notations		Indexes	
C_f	Downforce coefficient [-]	RC	Roll center
$aerodistr_F$	Downforce distribution at front [%]	1	Front Left wheel
rearsteer	Steering intensity of rear wheels [-]	2	Front Right wheel
ackerman	Ackerman steering intensity of wheels [-]	3	Rear Left wheel
δ	Steering angle [rad]	4	Rear Right wheel
F	Force [N]		
R	Turning radius [m]		
Ω	Yaw rate [rad/s]		
ω	Yaw rate vector [rad/s]		

4.1. The geometry and steering system of the model

Let's define the distance between wheels and center of gravity (CoG).

$$l_F = W \cdot (1 - distrF), \tag{4.1}$$

$$l_R = W \cdot distrF \tag{4.2}$$

Modelling the steering system at front and rear wheels we took into account the adjustment of the toe angle and the ackerman steering characteristic. We have chosen the left-hand turn of the wheel as a positive direction, so as the value of the toe is positive in the case of toe in, this angle is negative on the left and positive on the right. The ackerman steering can be well modelled when the rotation of each wheel is approximated by the quadratic function of the steering angle (δ).

$$\delta_1 = -\delta_{toe,F} + (\delta + ackerman_F \cdot \delta^2) \tag{4.3}$$

$$\delta_2 = \delta_{toe,F} + (\delta - ackerman_F \cdot \delta^2) \tag{4.4}$$

We have modelled the steerable rear wheels in proportion to the front wheels with the value of the *rearsteer* factor. The *rearsteer* value is positive when steering outboard increasing the turnability of the car.

$$\delta_3 = -\delta_{toe,R} - rearsteer \cdot (\delta - ackerman_R \cdot \delta^2) \tag{4.5}$$

$$\delta_4 = \delta_{toe,R} - rearsteer \cdot (\delta + ackerman_R \cdot \delta^2) \tag{4.6}$$

Besides the steering system it is important to define the point coordinate where the tire forces are actuated. These points are the contact patches.

$$\mathbf{r_1} = \begin{bmatrix} L_F \\ T_F/2 \\ 0 \end{bmatrix}; \quad \mathbf{r_2} = \begin{bmatrix} L_F \\ -T_F/2 \\ 0 \end{bmatrix}; \quad \mathbf{r_3} = \begin{bmatrix} -L_R \\ T_R/2 \\ 0 \end{bmatrix}; \quad \mathbf{r_4} = \begin{bmatrix} -L_R \\ -T_R/2 \\ 0 \end{bmatrix}; \tag{4.7}$$

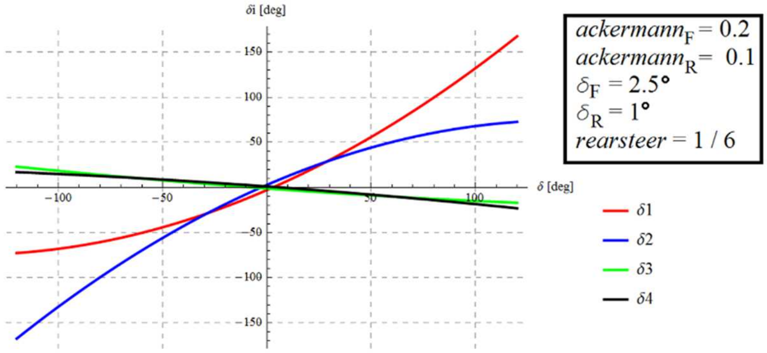

Figure 8. The steering system of the wheels.

The tire forces at contact points. During the MRA test, the speed of the vehicle is maintained at constant, so the longitudinal forces acting on the car need not be addressed. Thus, the force vectors on the contact patch of the wheels can be written in the car coordinate system by the equation (4.8).

$$\mathbf{F}_i = \begin{bmatrix} -Fy_i \cdot \sin[\delta_i] \\ Fy_i \cdot \cos[\delta_i] \\ Fz_i \end{bmatrix} \qquad i = 1,2,3,4 \tag{4.8}$$

The Fy_i lateral forces are the functions of slip angles - which I got from the tire model - and Fz_i normal forces were taken into account by the weight transfer because of lateral acceleration. The car moves along a circular path at constant longitudinal speed, meaning that the lateral acceleration can be written as follows:

$$a_{lat} = \frac{v_{long}^2 + v_{lat}^2}{R_{CG}} \tag{4.9}$$

4.1.1. Normal forces

Normal forces consist of 3 parts. The first part is static normal force, when the car is not moving. The second part is from lateral weight transfer as in equation (4.10),(4.11) and the third part is from aerodynamic force as in equation (4.12).

$$\Delta Fz_{lat,F} = \Delta Fz_{lat,F,NSM} + \Delta Fz_{lat,F,SM,geometric} + \Delta Fz_{lat,F,SM,elastic} \tag{4.10}$$

$$\Delta Fz_{lat,R} = \Delta Fz_{lat,R,NSM} + \Delta Fz_{latR,SM,geometric} + \Delta Fz_{lat,R,SM,elastic} \tag{4.11}$$

$$F_{aero} = 0.5 \cdot \rho \cdot A \cdot v_{long}^2 \cdot C_f \tag{4.12}$$

Lateral weight transfer consists of 3 parts:

- From non-suspended mass (NSM)
- From suspended mass (SM), which goes to the other side in suspension rods (geometric)
- From suspended mass (SM), which goes to the other side in elastic parts, such as springs and anti-roll bars (elastic)

Non-suspended mass weight transfer:

$$\Delta Fz_{lat,F,NSM} = \frac{2\ m_{NSM}\ a_{lat}\ h_{CG,NSM}}{T_F} \tag{4.13}$$

$$\Delta Fz_{lat,R,NSM} = \frac{2\ m_{NSM}\ a_{lat}\ h_{CG,NSM}}{T_R} \tag{4.14}$$

Geometric weight transfer:

$$\Delta Fz_{lat,F,SM,geometric} = \frac{m_{SM}\ distrF\ a_{lat}\ h_{RC,F}}{T_F} \tag{4.15}$$

$$\Delta Fz_{latR,SM,geometric} = \frac{m_{SM}\ (1 - distrF)\ a_{lat}\ h_{RC.R}}{T_R} \tag{4.16}$$

Elastic weight transfer:

$$\Delta Fz_{lat,F,SM,elastic} = \frac{m_{SM}\ a_{lat}\ (h_{CG,SM} - h_{RC})}{T_F} \cdot ARStfDistr \tag{4.17}$$

$$\Delta Fz_{lat,R,SM,elastic} = \frac{m_{SM}\ a_{lat}\ (h_{CG,SM} - h_{RC})}{T_R} \cdot (1 - ARStfDistr) \tag{4.18}$$

where $ARStfDistr$ is the roll stiffness distribution percent at front [5].

Thus, for example the normal force at front left tire in case of positive steering (to the left):

$$Fz_1 = 0.5 \cdot m \cdot g \cdot distrF - \Delta Fz_{lat,F} + 0.5 \cdot F_{aero} \cdot aerodistr_F \tag{4.19}$$

The other normal forces can be written the same thinking. The positive direction of lateral acceleration is left, so if the car is turning left, it is accelerating left, so the weight transfer part will be positive at right wheels - numbered 2 and 4 - and negative at left wheels - numbered 1 and 3.

4.1.2. Slip angles

The tire lateral forces are the function of normal forces and slip angles too. Slip angles can be calculated from speed condition. The car CoG velocity, yaw rate vector, each velocity of wheels and each slip angle of tires can be given by Figure 9.

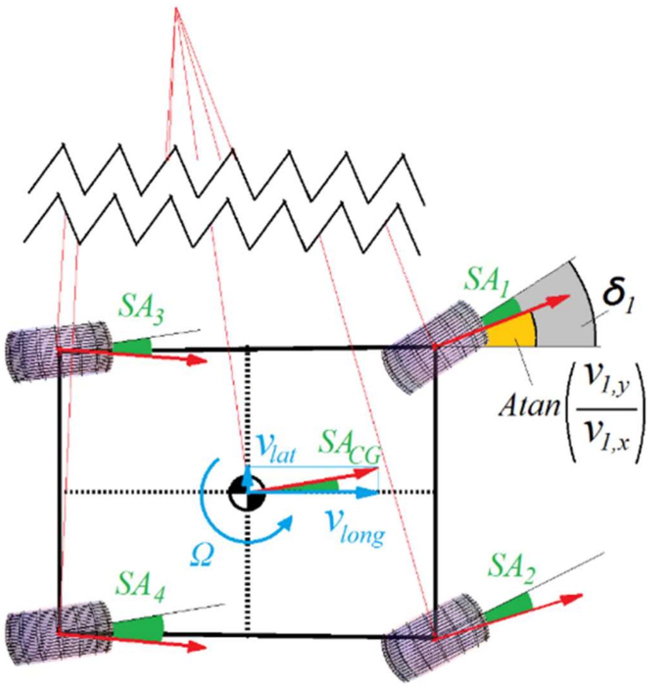

Figure 9. Slip angles.

$$\mathbf{v_{CG}} = \begin{bmatrix} v_{long} \\ v_{lat} \\ 0 \end{bmatrix} \quad \boldsymbol{\omega} = \begin{bmatrix} 0 \\ 0 \\ \Omega \end{bmatrix} \tag{4.20}$$

$$v_{lat} = \tan(SA_{CG}) \cdot v_{long} \tag{4.21}$$

$$\Omega = \frac{\sqrt{v_{long}^2 + v_{lat}^2}}{R_{CG}} \tag{4.22}$$

$$\mathbf{v_i} = \mathbf{v_{CG}} + \boldsymbol{\omega} \times \mathbf{r_i} \tag{4.23}$$

$$SA_i = \arctan\left(\frac{\mathbf{v}_{i,y}}{\mathbf{v}_{i,x}}\right) - \delta_i \qquad i = 1,2,3,4 \tag{4.23}$$

4.2. Lateral acceleration and yaw acceleration

The unknown described parameters so far are the steering angle δ, the turning radius R_{CG}, and the body slip angle. To write these equations (4.9 - 4.24) above to Fy_i, these will be the only function of the δ, SA_{CG} and R_{CG}. Thus, the lateral acceleration and yaw acceleration are:

$$lateral\ acceleration = \frac{\sum_{i=1}^{4}(\mathbf{F}_i)_y}{m} \tag{4.25}$$

$$yaw\ acceleration = \frac{\sum_{i=1}^{4}(\mathbf{r}_i \times \mathbf{F}_i)_z}{I_{CG}} \tag{4.26}$$

4.3. Calculation

The *lateral acceleration* and *yaw acceleration* are the function of δ, SA_{CG} and R_{CG}. We give the δ and SA_{CG} parameters, while R_{CG} parameter can be calculated by equation (4.9, 4.21 and 4.25). To write this R_{CG} parameter into *lateral acceleration* and *yaw acceleration* we will get 1 point in the MMM diagram, so we have to calculate it many δ and SA_{CG} parameters to map the total MMM.

5. How can car parameters effect to MMM

As a result of the MRA study, the effects of a number of parameters can be examined. Which parameter influences maximum grip and which parameters cause under- or oversteer behaviour. This method has the potential to evaluate the effects of countless parameters, only a few of them are shown below.

The first parameter is weight distribution. As you can see the right tip of MMM diagram az Figure 10 with more weight at front results understeer behaviour. Moreover, to look at the origin, the control of car is reduced as stability is increased. Performing the simulations and querying the values for different weight distributions, the following chart is obtained:

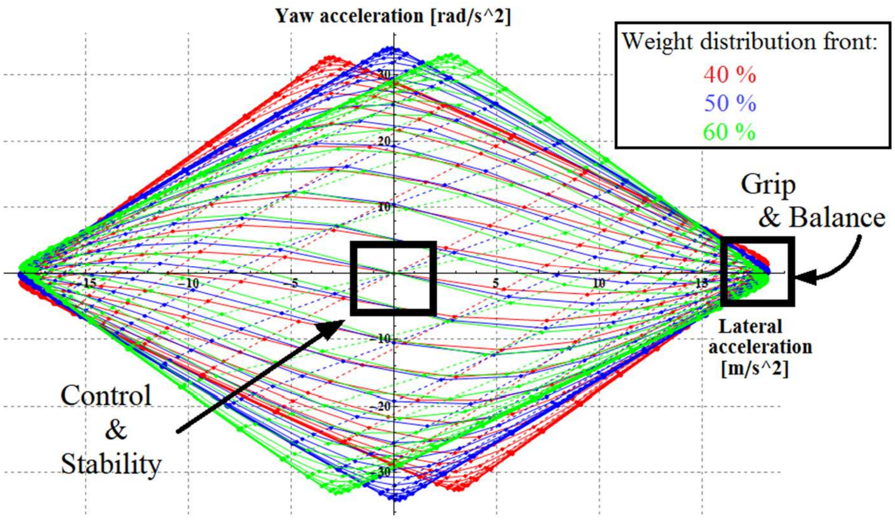

Figure 10. The effect of weight distribution at front.

In this case, the 50-50% weight car generates the highest grip. As expected, moving the center of gravity to front, the car goes towards the understeer behaviour, and therefore its control is reduced while stability is increased. The second parameter which we show is toe angle.

As can be seen, the rear wheel assembly has huge effect to stability and control. Therefore, in the case of a badly designed, the car which has high bumpsteer may be nervous on the braking zone, so it can easily lose its stability and become uncontrollable. Toe out (negative toe) can result in better maneuverability (increased control) but it can easily make the car nervous (reduce stability).

The detailed calculation above can also be made for other parameters. Figure 13 summarizes the effect of each parameter on control and stability. As can be seen, speed, toe angles and weight distribution effect most to vehicle stability and control, as much as that the other parameters are not significant. These other parameters are detailed on the right side of the diagram.

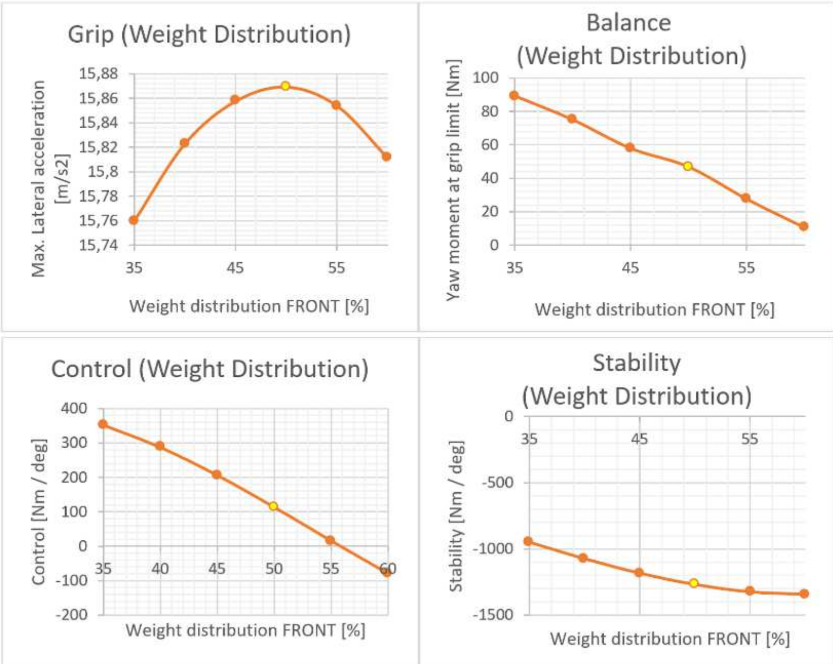

Figure 11. The effect of weight distribution at front.

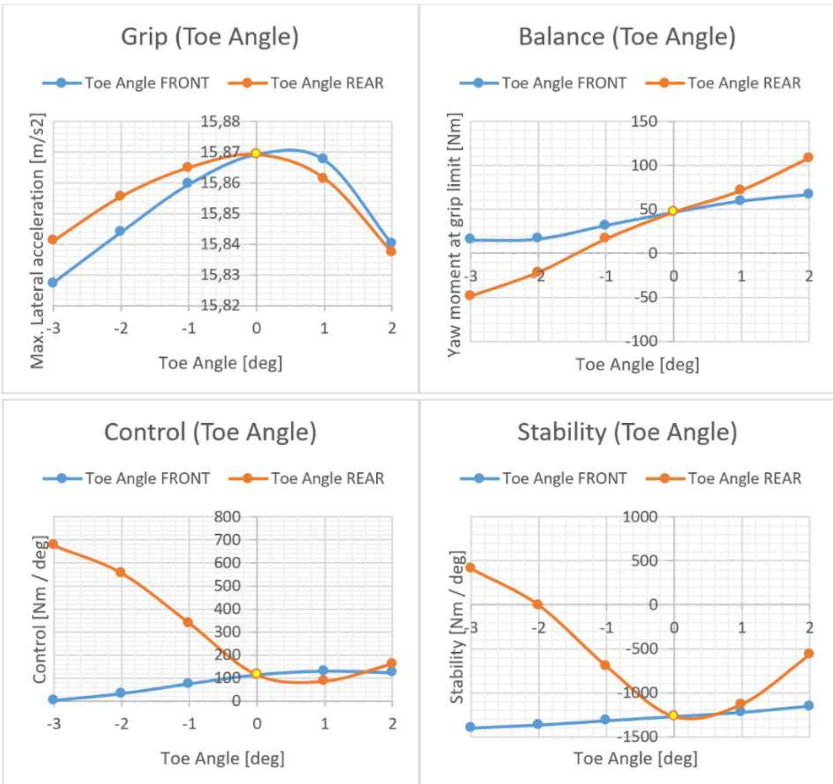

Figure 12. The effect of weight distribution at front

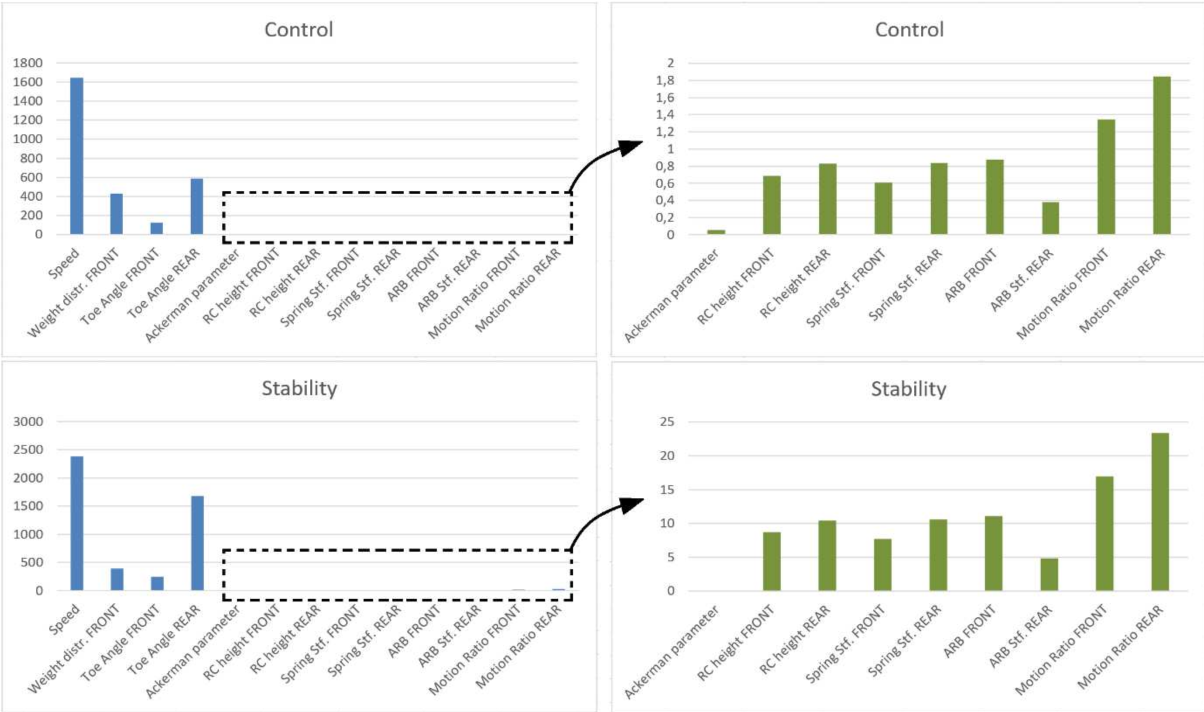

Figure 13. Which parameters effect most.

6. Conclusion

Stability and control are very important in self-driving cars. In order to make a good, maneuverable car, it needs good balance of control, and stability which is important for the car. By adjusting the various parameters of the car, we can get suitable vehicle. The most important of these parameters are the weight distribution of the car and the alignment of the suspension, especially to toe angles.

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