$$G_{day} = 0 \tag{8.46}$$

where G_{day} is the daily soil heat flux density, MJ m^{-2} d^{-1}.

Over a monthly period, G for the soil profile can be significant, especially during spring and fall. Assuming a constant soil heat capacity of 2.0 MJ m^{-3} °C^{-1} and an effectively warmed soil depth of 2 m, G for monthly periods in MJ m^{-2} d^{-1} is estimated from the change in mean monthly air temperature as:

$$G_{month,i} = 0.07\,(T_{month,i+1} - T_{month,i-1}) \tag{8.47}$$

or, if $T_{month,i+1}$ is unknown,

$$G_{month,i} = 0.14\,(T_{month,i} - T_{month,i-1}) \tag{8.48}$$

where $T_{month,i}$ is mean air temperature of month i, $T_{month,i-1}$ is mean air temperature of the previous month, and $T_{month,i+1}$ is the mean air temperature of the next month (all °C).

For application to short-periods, e.g., when hourly data for G are required, other approaches must be used. One method common to research uses heat flux plates installed near the soil surface, usually at a depth of about 0.1 to 0.15 m. The total heat flux density is determined by performing a calorimetric balance of the soil layer above the plate. Because soil heat flux plates are not a common measurement at weather stations, the reader is referred to other references (Tanner, 1960; Brutsaert, 1982; and Allen et al., 1996) for application descriptions.

For hourly or shorter time periods, G, in the standardized calculation is expressed as a function of net radiation for the two reference types. For the standardized short reference ET_{os} :

$$G_{hr\ daytime} = 0.1\ R_n \tag{8.49a}$$

$$G_{hr\ nighttime} = 0.5\ R_n \tag{8.49b}$$

where G and R_n have the same measurement units (MJ m^{-2} h^{-1} for hourly or shorter time periods). For the standardized tall reference ET_{rs} :

$$G_{hr\ daytime} = 0.04\ R_n \tag{8.50a}$$

$$G_{hr\ nighttime} = 0.2\ R_n \tag{8.50b}$$

For standardization, nighttime is defined as when measured or calculated hourly net radiation R_n is < 0 (i.e., negative). The amount of energy consumed by G is subtracted from R_n when estimating ET_{os} or ET_{rs}. The coefficient 0.1 in Equation 8.49a represents the condition of only a small amount of dead thatch underneath the leaf canopy of the short (clipped grass) reference. Large amounts of thatch insulate the soil surface, reducing the daytime coefficient for grass to about 0.05. However, the 0.1 coefficient is part of the EWRI and FAO standardizations.

8.4.5 1985 Hargreaves Grass Reference Equation

The Hargreaves equation (Hargreaves and Samani, 1982, 1985; Hargreaves et al., 1985) is suggested as a means for estimating ET_o in situations where weather data are limited and only maximum and minimum air temperature data are available. The form of the 1985 Hargreaves equation is:

$$ET_o = 0.0023\,(T_{max} - T_{min})^{0.5}\,(T_{mean} + 17.8)\,(R_a) \tag{8.51}$$

where T_{max} and T_{min} = maximum and minimum daily air temperature, °C,

T_{mean} = mean daily air temperature, $(T_{max} + T_{min})/2$
R_a = average daily exoatmospheric radiation (Equation 8.29).

ET_o in Equation 8.51 has the same units as R_a and can be converted from MJ m^{-2} d^{-1} to mm d^{-1} by dividing by $\lambda = 2.45$ MJ kg^{-1}.

The Hargreaves equation was the highest-ranked temperature-based method in the ASCE Manual 70 analysis (Jensen et al., 1990). Droogers and Allen (2002) and Hargreaves and Allen (2003) reported Equation 8.51 to estimate well over a wide range of latitudes and climates for periods of 5 days or longer without significant error, provided mean wind speeds averaged between 1 and 3 m s^{-1}. Itenfisu et al. (2003) compared daily ET_o by Equation 8.51 and that by the standardized ASCE PM method for 49 sites in 16 states in U.S. and found mean ratios of Hargreaves ET_o to ASCE PM ET_{os} to range from 1.43 to 0.79, with a mean of 1.06 and a standard deviation of 0.13. The Hargreaves equation tended to estimate higher than the ASCE PM method when mean daily ET_o was low, and vice versa.

One advantage of an equation such as the Equation 8.51 relative to more complex equations such as the Penman or PM equation, which is often overlooked, is the reduced data requirement and therefore reduced chance for data error. This is advantageous in regions where solar radiation, humidity, and wind data are lacking or are of low or questionable quality (Droogers and Allen, 2002). Generally, air temperature can be measured with less error, with less sophisticated equipment, and by less trained individuals than can the other three parameters required by combination equations. Equation 8.51 can be calibrated against the PM equation (Equation 8.3) when data are available to produce a regionally calibrated temperature equation. Examples of this type of calibration in Spain include Martinez-Cob and Tejero-Juste (2004), Vanderlinden et al. (2004), and Gavilan et al. (2005). An alternative to using Equation 8.51 when data are lacking is to employ the PM equation using estimates for missing variables.

8.4.6 Effect of Timestep Size on Calculations

The Penman and Penman-Monteith equations can be applied to hourly and 24-h timesteps. The 24-h timesteps can use daily, weekly, 10-d, and monthly averages for weather data. Under many climatic conditions, calculating ET_o or ET_r using hourly timesteps and then summing over 24 hours provides estimates that closely equal ET_o or ET_r calculated using 24-h average data with 24-h calculation timesteps, especially when applying the standardized ASCE-EWRI PM method (Itenfisu et al., 2003; ASCE-EWRI, 2005). Generally, 24-h ET_o and ET_r have potential for higher accuracy when computed using hourly or shorter timesteps and then summed to 24-hour totals. Hourly calculation is better able to consider impacts of abrupt and gradual changes in weather parameters during the course of a day on ET (Irmak et al., 2005; Allen et al., 2006). Examples of this are high wind conditions during afternoons with low humidity, overpass of cloud fronts and rain events, wintertime, and nighttime calm.

8.4.7 Limited Data Availability

Many historical weather data sets include only measurements of daily air temperature. When ET_o estimates are desired, one of three approaches is recommended:

1. When approximate calculations of ET_o are suitable, apply Equation 8.51.
2. When more accuracy is required, calibrate Equation 8.51 at a regional station or stations that have R_s and u_2 data. If T_d or other humidity data are not available, T_d can be estimated from T_{min} as described in Equation 8.16. The desired standard reference method can be used as the calibration basis (e.g., Equation 8.3) at

8.4.4.9 Exoatmospheric radiation. Exoatmospheric radiation, R_a, (also known as extraterrestrial radiation) is defined as the short-wave solar radiation in the absence of an atmosphere, and is a well-behaved function of the day of the year, time of day, and latitude. R_a is needed for calculating R_{so}, which is in turn used in calculating R_n. For daily (24-hour) periods, R_a is estimated from the solar constant, the solar declination, and the day of the year:

$$R_a = \frac{24}{\pi} G_{sc}\, d_r \left[\omega_s \sin(\varphi)\sin(\delta) + \cos(\varphi)\cos(\delta)\sin(\omega_s)\right] \tag{8.29}$$

where R_a = exoatmospheric radiation, MJ m^{-2} d^{-1}

G_{sc} = solar constant, 4.92 MJ m^{-2} h^{-1}

d_r = inverse relative distance factor (squared) for the earth-sun, unitless

ω_s = sunset hour angle, radians

φ = station latitude, radians, positive for the northern hemisphere and negative for the southern hemisphere

δ = solar declination, radians.

Parameters d_r and δ are calculated as:

$$d_r = 1 + 0.033\cos\left(\frac{2\pi}{365} J\right) \tag{8.30}$$

math domain error
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does not work

$$\delta = 0.409\sin\left(\frac{2\pi}{365} J - 1.39\right) \tag{8.31}$$

where J is the number of the day in the year between 1 (1 January) and 365 or 366 (31 December). The constant 365 in Equations 8.23 and 8.24 is held at 365 even during a leap year. J can be calculated as:

$$J = D_M - 32 + \mathrm{Int}\left(275\frac{M}{9}\right) + 2\,\mathrm{Int}\left(\frac{3}{M+1}\right) + \mathrm{Int}\left(\frac{M}{100} - \frac{\mathrm{Mod}(Y,4)}{4} + 0.975\right) \tag{8.32a}$$

where D_M is the day of the month (1-31), M is the number of the month (1-12), and Y is the number of the year (for example 1996 or 96). The "Int" function in Equation 8.32 finds the integer number of the argument in parentheses by rounding downward. The "Mod(Y,4)" function finds the modulus (remainder) of the quotient $Y/4$.

For monthly periods, the day of the year at the middle of the month (J_{month}) is approximately:

$$J_{month} = \mathrm{Int}(30.4\,M - 15) \tag{8.32b}$$

The sunset hour angle, ω_s, is given by:

$$\omega_s = \arccos\left[-\tan(\varphi)\tan(\delta)\right] \tag{8.33}$$

The "arccos" function is the arc-cosine function and represents the inverse of the cosine. This function is not available in all computer languages, so that ω_s can alternatively be computed using the arc-tangent (inverse tangent) function:

$$\omega_s = \frac{\pi}{2} - \arctan\left[\frac{-\tan(\varphi)\tan(\delta)}{X^{0.5}}\right] \tag{8.34}$$

where

$$X = 1 - \left[\tan(\varphi)\right]^2\left[\tan(\delta)\right]^2 \tag{8.35}$$