

## F Visualization on Size Distribution

A better-estimated distribution brings a better compression rate. Here, we visualize the distribution on  $|S|$  to verify that our method is a better estimator.

The distribution of our model is just the  $P(S; r)$  defined in section 5.1. For the distribution of single-item-based method, let  $q_i = \Pr(i \in S)$ , then the  $Q(S) = \prod_{i \in S} q_i \cdot \prod_{j \in V-S} (1 - q_j)$ .

The exact size distribution of  $P(S; r)$  and  $Q(S)$  can be calculated by ordinary generating function efficiently.

### F.1 Calculation of the size distribution

*F.1.1 Ordinary Generating Function.* We define the ordinary generating function(OGF) of a size distribution as follows.

$$F(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots$$

where the  $a_i$  is the  $\Pr(|S| = i)$ , for  $i \in \mathbb{N}$ .

*F.1.2 Calculation of the Size Distribution of Single-item-based Model.* For  $i \in V$ , let  $q_i = \Pr(i \in S)$ . The probability of a  $S \subseteq V$  for the single-item-based model is

$$Q(S) = \prod_{i \in S} q_i \cdot \prod_{j \in V-S} (1 - q_j)$$

We can calculate the OGF of  $Q(S)$  by the following steps.

- (1)  $F(x)=1$
- (2) For  $i = 1, \dots, K$ ,

$$F(x) := F(x) \times ((1 - q_i) + q_i x)$$

With  $O(K)$  steps, we can obtain the size distribution of  $Q(S)$ .

*F.1.3 Calculation of the Size Distribution of binary tree Model.* For nodes  $l$  in the binary tree, we can recursively calculate the OGF  $F_v(x)$  of  $P(S; v)$ . The notations are the same as section 5.1.

For a leaf node  $l$ , the OGF  $F_l(x) = x$ . For an internal node  $v$ , let  $v$ 's left and right child be  $u$  and  $w$ , we have

$$F_v(x) = p(0; v)F_u(x) + p(1; v)F_w(x) + p(2; v)F_u(x)F_w(x)$$

With  $O(2K + 1)$  steps, we can obtain the OGF  $F_r(x)$  of the root node  $r$ .

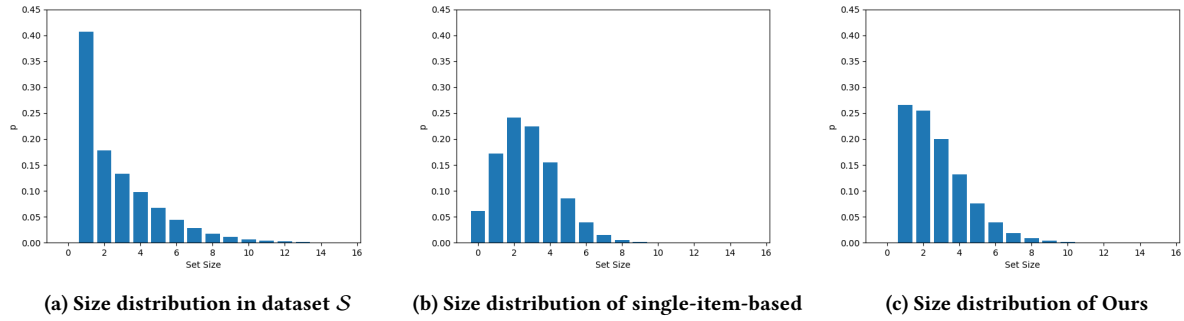
### F.2 Results

The figure 10 and figure 11 visualizes the size distribution of ground truth distribution  $P^*(S)$ ,  $Q(S)$  and  $P(S; r)$  on the Tmall dataset. As illustrated in the figures, the  $Q(S)$  has a positive value on  $|S| = 0$ , while there is zero probability for  $|S| = 0$  in  $P^*(S)$  and  $P(S; r)$ .

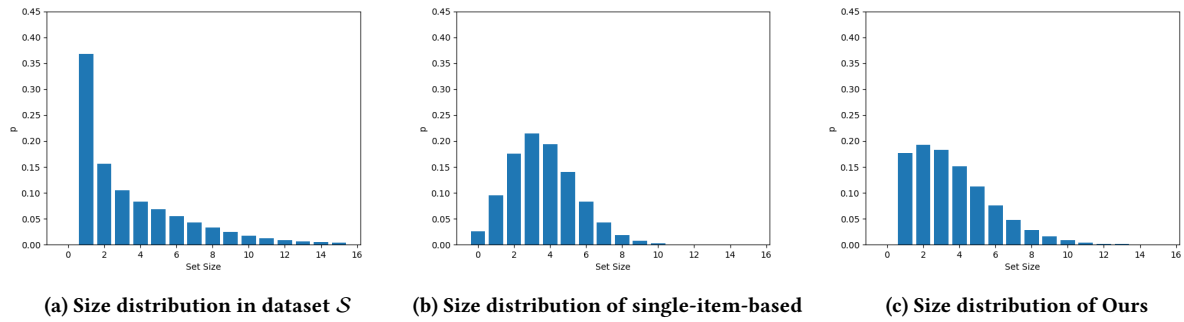
Both the ground truth size distribution and our model's size distribution have a similar fast-decaying shape. While the size distribution of the single-item-based model has a different bell shape like Gaussian.

The visualization of the size distribution further confirmed that our binary tree model matches the pattern of real-world data better and thus brings us a lower compression rate.

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**Figure 10: The size distribution on the Tmall dataset. We only present the proportion of  $|S| \leq 16$ . Figure (10a): The empirical size distribution in the Tmall dataset. Figure (10b): The size distribution of the single-item-based baseline model learned by the Tmall dataset. Figure (10c): The size distribution of the binary tree model learned by the Tmall dataset.**



**Figure 11: The size distribution on the HKTVmall dataset. We only present the proportion of  $|S| \leq 16$ . Figure (11a): The empirical size distribution in the HKTVmall dataset. Figure (11b): The size distribution of the single-item-based baseline model learned by the HKTVmall dataset. Figure (11c): The size distribution of the binary tree model learned by the HKTVmall dataset.**