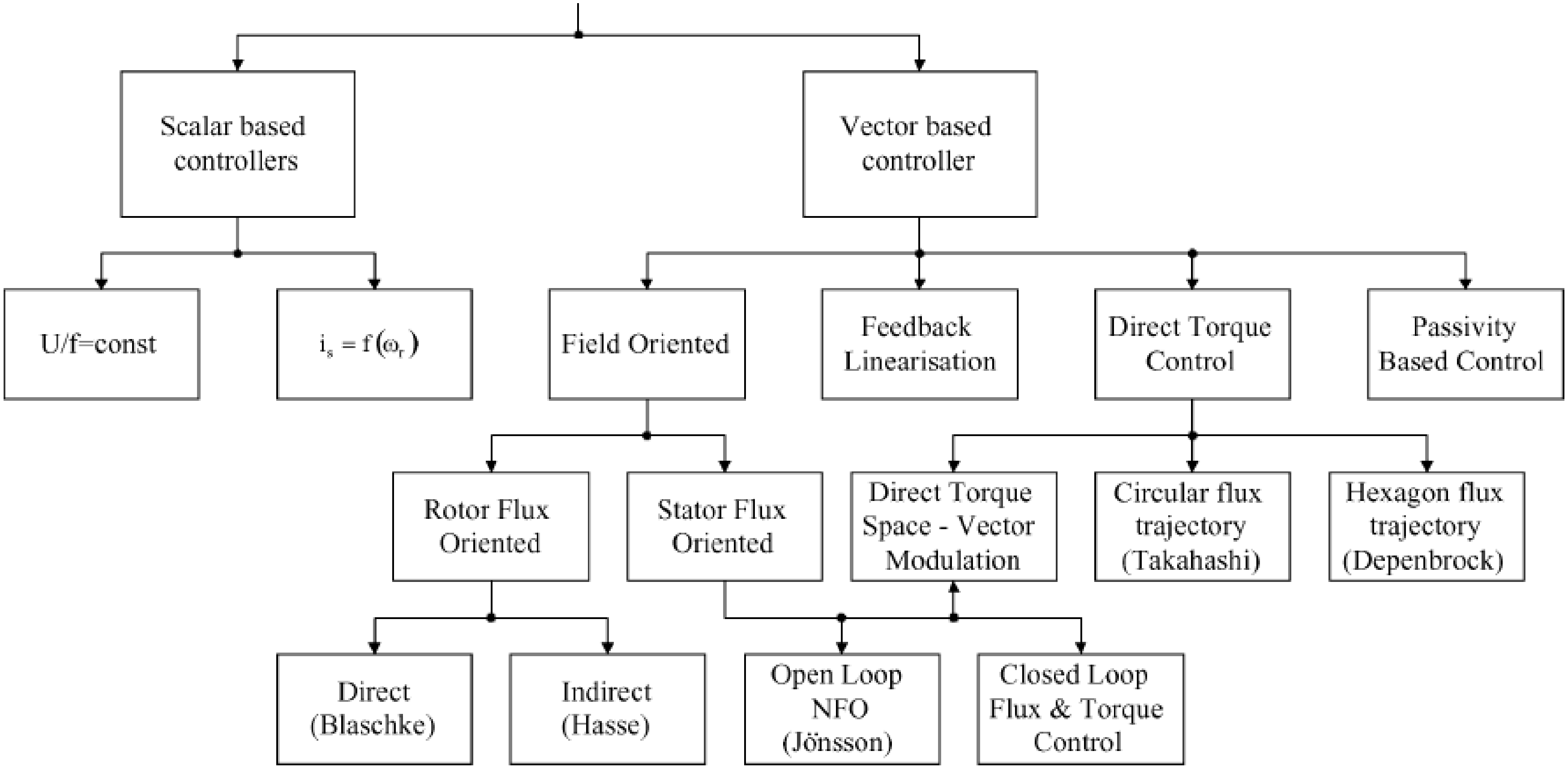
CLEMSON UNIVERSITY

ECE 891, Electric Motor Control

Direct Torque Control

Induction Motor Drive Controls



G.S. Buja and M.P. Kazmierkowski, "Direct Torque Control of PWM Inverter-Fed AC Motors - A Survey," *IEEE Transactions on Industrial Electronics*, volume 51, number 4, pages 744-757, August 2004.

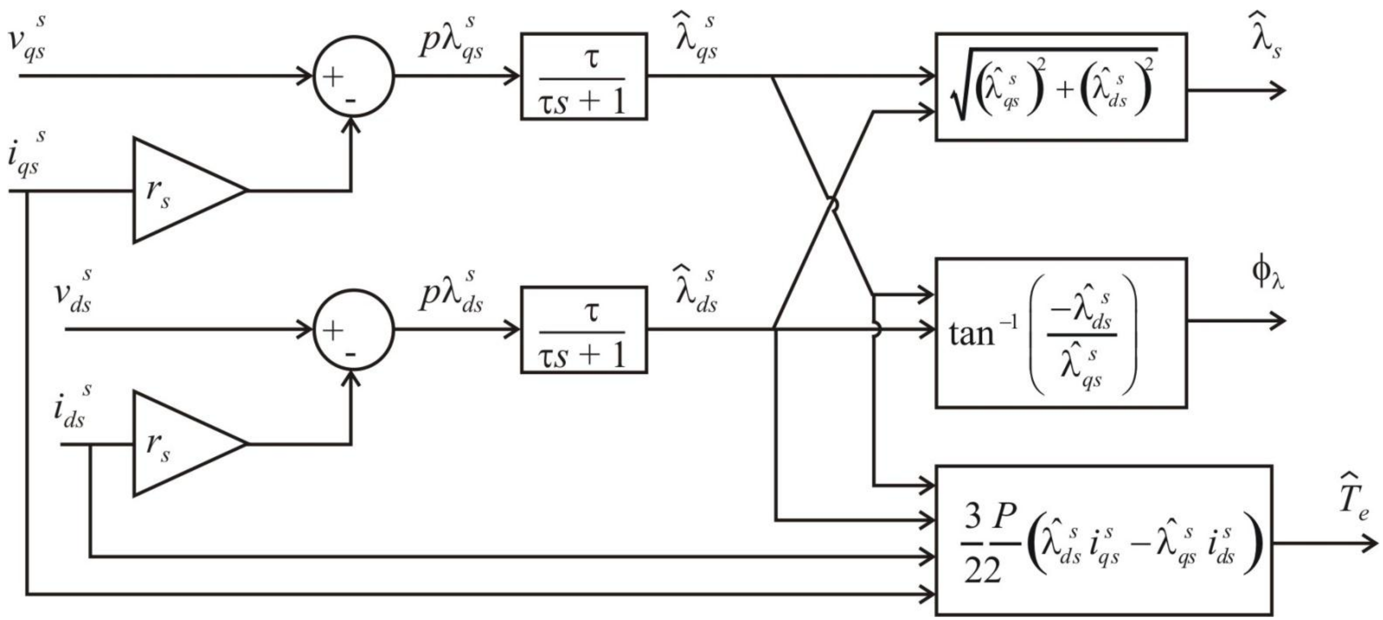
Flux and Torque Observer

stationary reference frame ($\omega = 0$)

$$v_{qs}^{s} = r_s i_{qs}^{s} + p\lambda_{qs}^{s}$$

$$v_{ds}^{s} = r_s i_{ds}^{s} + p\lambda_{ds}^{s}$$

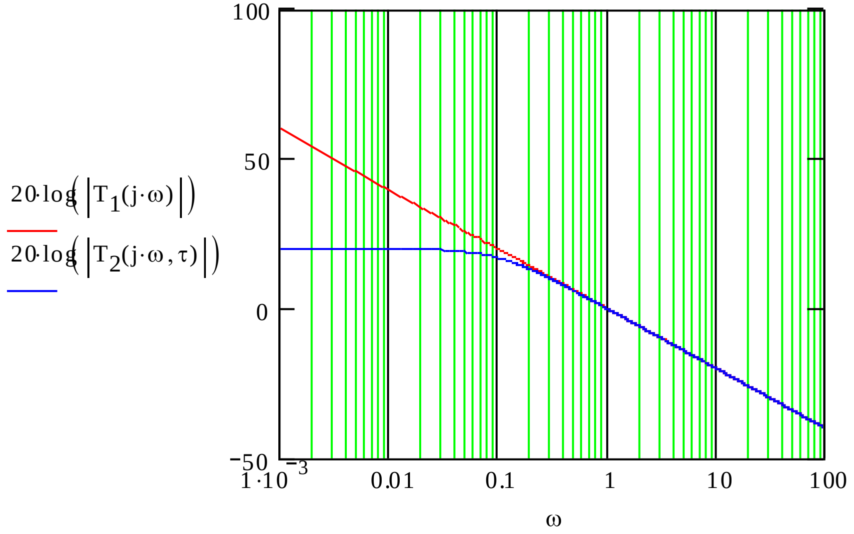
$$T_e = \frac{3}{2}\frac{P}{2}\left(\lambda_{ds}^{s} i_{qs}^{s} - \lambda_{qs}^{s} i_{ds}^{s}\right)$$



Transfer Function as an Integrator

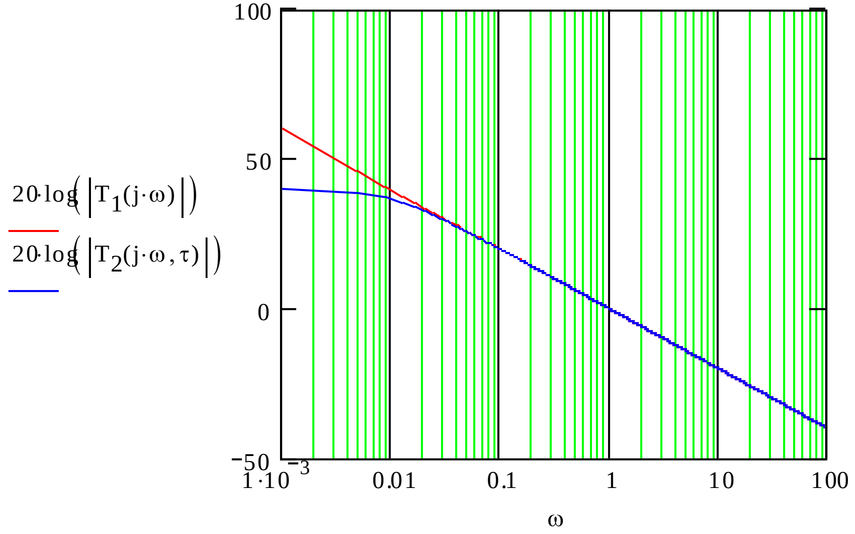
$T_1(s) := \frac{1}{s}$ $\qquad T_2(s,\tau) := \frac{\tau}{\tau \cdot s + 1}$

$\tau := 10$ $\qquad \left|T_2(j \cdot 0, \tau)\right| = 10$

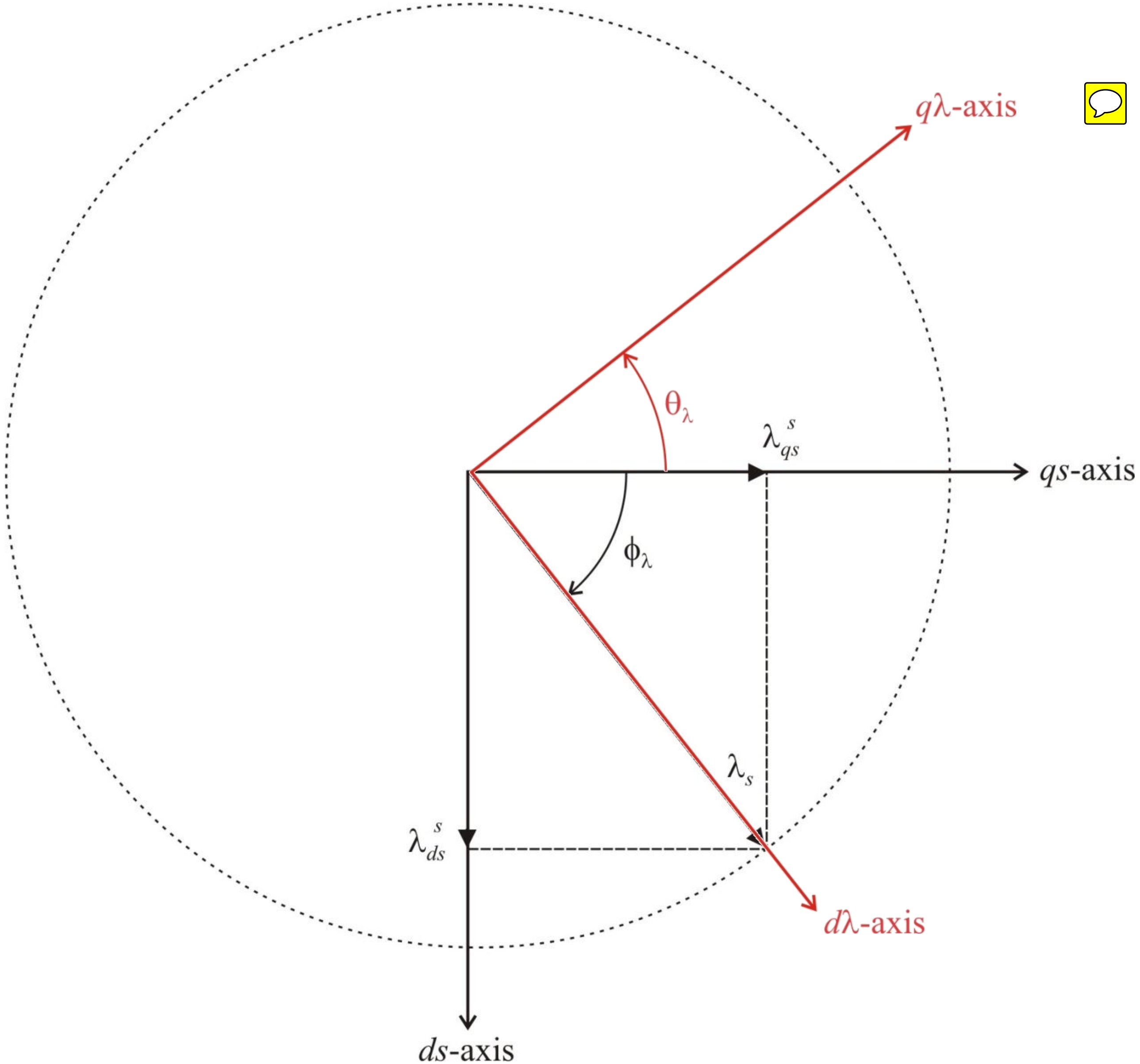


$T_1(s) := \frac{1}{s}$ $\qquad T_2(s,\tau) := \frac{\tau}{\tau \cdot s + 1}$

$\tau := 100$ $\qquad \left|T_2(j \cdot 0, \tau)\right| = 100$



Stator Flux Reference Frame



Motor Equations in the Stator Flux Linkage Reference Frame

flux reference frame $\theta = \phi_\lambda + \frac{\pi}{2}$ $\lambda_{qs}^{\lambda} = 0$ $\lambda_{ds}^{\lambda} = \lambda_s$

$$v_{qs}^{\lambda} = r_s i_{qs}^{\lambda} + \omega_e \lambda_{ds}^{\lambda} + p\lambda_{qs}^{\lambda}$$

$$v_{ds}^{\lambda} = r_s i_{ds}^{\lambda} - \omega_e \lambda_{qs}^{\lambda} + p\lambda_{ds}^{\lambda}$$

$$T_e = \frac{3}{2}\frac{P}{2}\left(\lambda_{ds}^{\lambda} i_{qs}^{\lambda} - \lambda_{qs}^{\lambda} i_{ds}^{\lambda}\right)$$

neglecting r_s terms

$$v_{tan} = v_{qs}^{\lambda} \approx \omega_e \lambda_s + p\lambda_{qs}^{\lambda}$$

$$v_{rad} = v_{ds}^{\lambda} \approx p\lambda_{ds}^{\lambda}$$

radial voltage used to control flux
tangential voltage used to control torque angle

IM Steady-State Example

$$RPM := \frac{2\cdot\pi\cdot rad}{60\cdot sec}$$

IM parameters

$r_s := 0.4\Omega$ $\quad P := 4$ $\quad r'_r := 0.2266\Omega$ $\quad lagging := 1$

$L_{ls} := 5.73mH$ $\quad L_M := 64.43mH$ $\quad L'_{lr} := 4.64mH$

operating conditions

$f_e := 60\cdot Hz$ $\quad \omega_e := 2\cdot\pi\cdot f_e$ $\quad \omega_e = 377\frac{rad}{sec}$

$\omega_{rm} := 1750RPM$ $\quad \omega_{rm} = 183.3\frac{rad}{sec}$

$v_s := \frac{220\cdot V}{\sqrt{3}}$ $\quad v_s = 127V$

synchronous speed (no-load speed)

$\omega_{em} := \left(\frac{2}{P}\right)\cdot\omega_e$ $\quad \omega_{em} = 188.5\frac{rad}{sec}$

$\omega_{em} = 1800RPM$

slip

$\omega_r := \frac{P}{2}\cdot\omega_{rm}$ $\quad \omega_r = 366.5\frac{rad}{sec}$

$sl := \frac{\omega_e - \omega_r}{\omega_e}$ $\quad sl = 2.78\%$

impedances

$Z_s := r_s + j \cdot \omega_e \cdot L_{ls}$ $\qquad |Z_s| = 2.2\Omega \qquad \arg(Z_s) = 79.5\text{deg}$

$Z_m := j \cdot \omega_e \cdot L_M$ $\qquad |Z_m| = 24.3\Omega \qquad \arg(Z_m) = 90\text{deg}$

$Z_r := \frac{r'_r}{sl} + j \cdot \omega_e \cdot L'_{lr}$ $\qquad |Z_r| = 8.34\Omega \qquad \arg(Z_r) = 12.1\text{deg}$

$Z_f := \frac{1}{\frac{1}{Z_m} + \frac{1}{Z_r}}$ $\qquad |Z_f| = 7.43\Omega \qquad \arg(Z_f) = 29.5\text{deg}$

$Z_{in} := Z_s + Z_f$ $\qquad |Z_{in}| = 9.00\Omega \qquad \arg(Z_{in}) = 40.3\text{deg}$

currents

$V_{as} := v_s \cdot e^{j \cdot 0}$

$I_{as} := \frac{V_{as}}{Z_{in}}$ $\qquad |I_{as}| = 14.1\text{A} \qquad \arg(I_{as}) = -40.3\text{deg}$

$i_s := |I_{as}|$

$I_{ar} := -I_{as} \cdot \frac{Z_m}{Z_m + Z_r}$ $\qquad |I_{ar}| = 12.6\text{A} \qquad \arg(I_{ar}) = 157\text{deg}$

torque and power

$$L_{ss} := L_{ls} + L_M \qquad L_{ss} = 70.2\,\text{mH}$$

$$L'_{rr} := L'_{lr} + L_M \qquad L'_{rr} = 69.1\,\text{mH}$$

$$T_e := 3\cdot\frac{P}{2}\cdot\left(\left|I_{ar}\right|\right)^2\cdot\frac{r'_r}{sl\cdot\omega_e} \qquad T_e = 20.5\,\text{N}\cdot\text{m}$$

$$T_e := \frac{3\cdot\left(\frac{P}{2}\right)\cdot\omega_e\cdot L_M^{\,2}\cdot r'_r\cdot sl\cdot\left(\left|V_{as}\right|\right)^2}{\left[r_s\cdot r'_r + sl\cdot\omega_e^{\,2}\cdot\left(L_M^{\,2} - L_{ss}\cdot L'_{rr}\right)\right]^2 + \omega_e^{\,2}\cdot\left(r'_r\cdot L_{ss} + sl\cdot r_s\cdot L'_{rr}\right)^2} \qquad T_e = 20.5\,\text{N}\cdot\text{m}$$

$$\theta_z := \arg\left(Z_{in}\right) \qquad \theta_z = 40.3\,\text{deg}$$

$$pf := \cos\left(\theta_z\right) \qquad pf = 0.763\,\text{lagging}$$

$$P_{in} := 3\cdot\left|V_{as}\right|\cdot\left|I_{as}\right|\cdot pf \qquad P_{in} = 4.104\,\text{kW}$$

$$P_{out} := T_e\cdot\omega_{rm} \qquad P_{out} = 3.757\,\text{kW}$$

$$P_{out} = 5.04\,\text{hp}$$

$$eff := \frac{P_{out}}{P_{in}} \qquad eff = 91.6\%$$

synchronous reference frame with $\theta = \theta_e$

$$\theta := \theta_e$$

$$V_{qse} := \sqrt{2} \cdot v_s \cdot \cos(\theta - \theta_e)$$

$$V_{dse} := \sqrt{2} \cdot v_s \cdot \sin(\theta - \theta_e)$$

$$I_{qse} := \sqrt{2} \cdot i_s \cdot \cos(\theta - \theta_e + \theta_z)$$

$$I_{dse} := \sqrt{2} \cdot i_s \cdot \sin(\theta - \theta_e + \theta_z)$$

$$\lambda_{qse} := -\left(\frac{V_{dse} - r_s \cdot I_{dse}}{\omega_e}\right)$$

$$\lambda_{dse} := \frac{V_{qse} - r_s \cdot I_{qse}}{\omega_e}$$

$$\lambda_s := \sqrt{\lambda_{qse}^{\ 2} + \lambda_{dse}^{\ 2}}$$

$$\phi_{\lambda e} := \operatorname{atan}\left(\frac{-\lambda_{dse}}{\lambda_{qse}}\right)$$

$$I'_{qre} := \frac{\lambda_{qse} - L_{ss} \cdot I_{qse}}{L_M}$$

$$I'_{dre} := \frac{\lambda_{dse} - L_{ss} \cdot I_{dse}}{L_M}$$

$$\lambda'_{qre} := L'_{rr} \cdot I'_{qre} + L_M \cdot I_{qse}$$

$$\lambda'_{dre} := L'_{rr} \cdot I'_{dre} + L_M \cdot I_{dse}$$

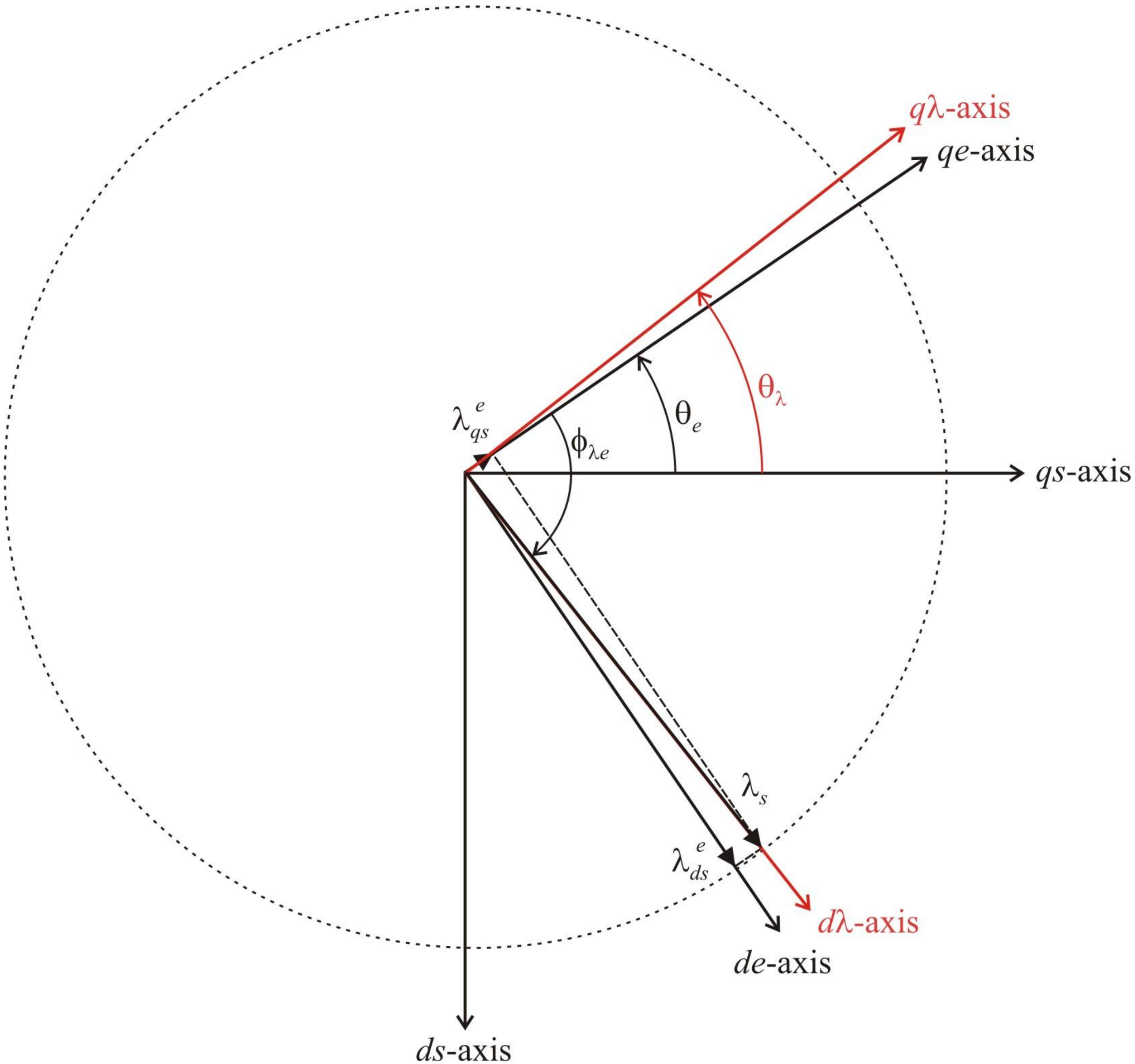
$$\lambda_r := \sqrt{\lambda'_{qre}{}^2 + \lambda'_{dre}{}^2}$$

$$\lambda_{qdse} := \begin{pmatrix} \lambda_{qse} \\ -\lambda_{dse} \end{pmatrix}$$

$$j\lambda'_{qdre} := \begin{pmatrix} \lambda'_{dre} \\ \lambda'_{qre} \end{pmatrix}$$

$$T_e := \frac{3}{2} \cdot \left(\frac{P}{2}\right) \cdot \left(\frac{L_M}{L_{ss} \cdot L'_{rr} - L_M{}^2}\right) \cdot \left(\lambda_{qdse} \cdot j\lambda'_{qdre}\right)$$

Synchronous Reference Frame Vectors



synchronous reference frame with $\theta = \theta_e + \phi_{\lambda e} + \pi/2$

$$\theta := \theta_e + \phi_{\lambda e} + \frac{\pi}{2}$$

$$V_{qs\,\lambda} := \sqrt{2}\cdot v_s\cdot\cos\left(\theta - \theta_e\right)$$

$$V_{ds\,\lambda} := \sqrt{2}\cdot v_s\cdot\sin\left(\theta - \theta_e\right)$$

$$I_{qs\,\lambda} := \sqrt{2}\cdot i_s\cdot\cos\left(\theta - \theta_e + \theta_z\right)$$

$$I_{ds\,\lambda} := \sqrt{2}\cdot i_s\cdot\sin\left(\theta - \theta_e + \theta_z\right)$$

$$\lambda_{qs\,\lambda} := -\left(\frac{V_{ds\,\lambda} - r_s\cdot I_{ds\,\lambda}}{\omega_e}\right)$$

$$\lambda_{ds\,\lambda} := \frac{V_{qs\,\lambda} - r_s\cdot I_{qs\,\lambda}}{\omega_e}$$

$$I'_{qr\lambda} := \frac{\lambda_{qs\,\lambda} - L_{ss} \cdot I_{qs\,\lambda}}{L_M}$$

$$I'_{dr\lambda} := \frac{\lambda_{ds\,\lambda} - L_{ss} \cdot I_{ds\,\lambda}}{L_M}$$

$$\lambda'_{qr\lambda} := L'_{rr} \cdot I'_{qr\lambda} + L_M \cdot I_{qs\,\lambda}$$

$$\lambda'_{dr\lambda} := L'_{rr} \cdot I'_{dr\lambda} + L_M \cdot I_{ds\,\lambda}$$

$$\lambda_{qds\,\lambda} := \begin{pmatrix} \lambda_{qs\,\lambda} \\ -\lambda_{ds\,\lambda} \end{pmatrix}$$

$$j\lambda'_{qdr\lambda} := \begin{pmatrix} \lambda'_{dr\lambda} \\ \lambda'_{qr\lambda} \end{pmatrix}$$

$$T_e := \frac{3}{2} \cdot \left(\frac{P}{2}\right) \cdot \left(\frac{L_M}{L_{ss} \cdot L'_{rr} - {L_M}^2}\right) \cdot \left(\lambda_{qds\,\lambda} \cdot j\lambda'_{qdr\lambda}\right)$$

Direct Torque Control

Flux observer determines magnitude and angle of stator flux linkages

Reference frame can be oriented so that the stator flux is entirely in the *d*-axis

Radial voltage (*d*-axis voltage) can be controlled to regulate the flux magnitude

Tangental voltage (*q*-axis voltage) can be controlled to regulate the torque

Table Lookup Implementation

Inverter switching based on *direct* control of flux and torque

Inverter Voltage Vectors

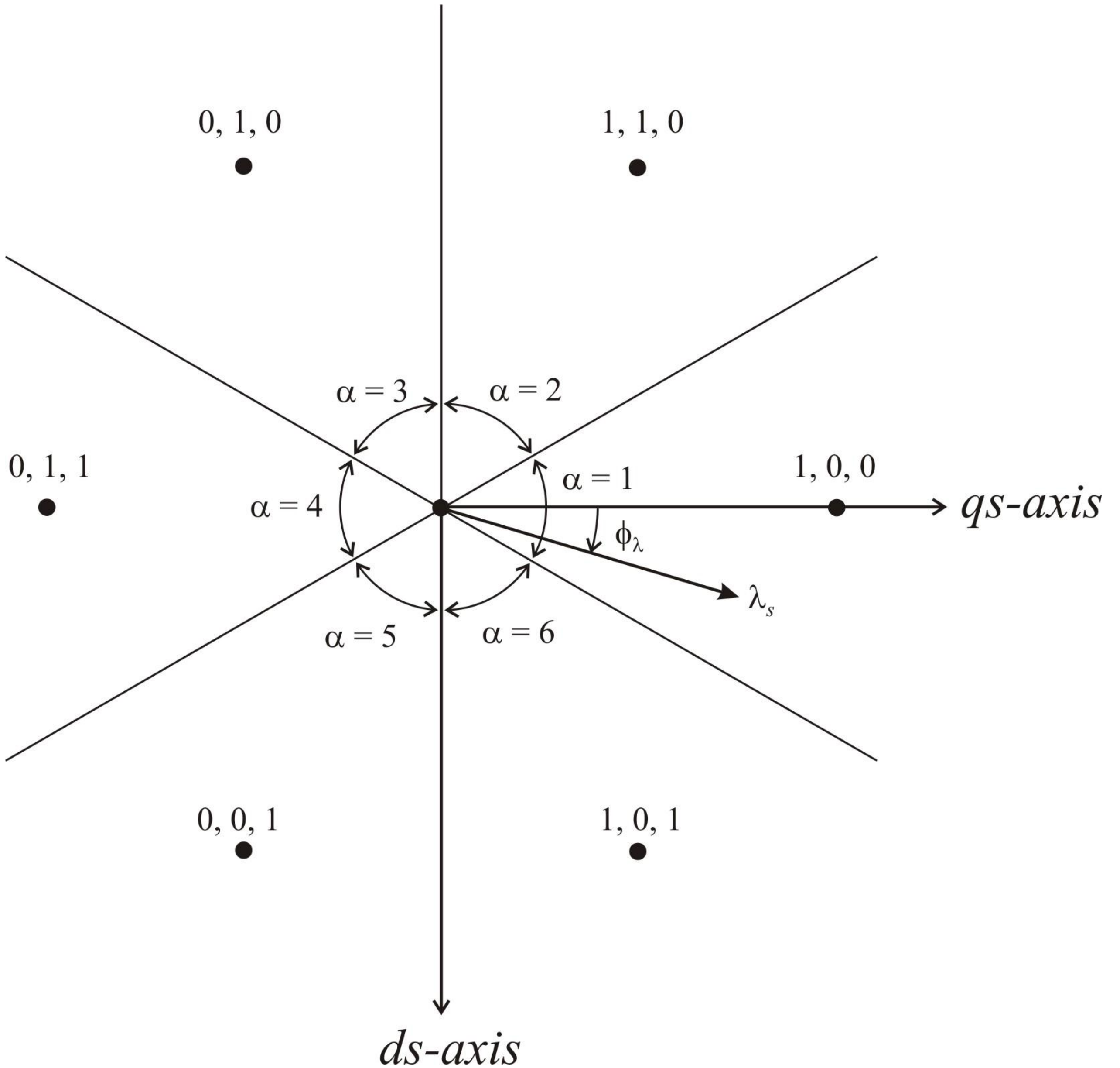
$$\begin{bmatrix} v_{ag} \\ v_{bg} \\ v_{cg} \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} v_{dc} \qquad \begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}\begin{bmatrix} v_{ag} \\ v_{bg} \\ v_{cg} \end{bmatrix}$$

Using the stationary reference frame transformation K_s^s

$$v_{qs}^s = v_{as} \qquad v_{ds}^s = \frac{v_{cs} - v_{bs}}{\sqrt{3}}$$

T_1	T_2	T_3	v_{qs}^s	v_{ds}^s
0	0	0	0	0
0	0	1	$-v_{dc}/3$	$v_{dc}/\sqrt{3}$
0	1	0	$-v_{dc}/3$	$-v_{dc}/\sqrt{3}$
0	1	1	$-2v_{dc}/3$	0
1	0	0	$2v_{dc}/3$	0
1	0	1	$v_{dc}/3$	$v_{dc}/\sqrt{3}$
1	1	0	$v_{dc}/3$	$-v_{dc}/\sqrt{3}$
1	1	1	0	0

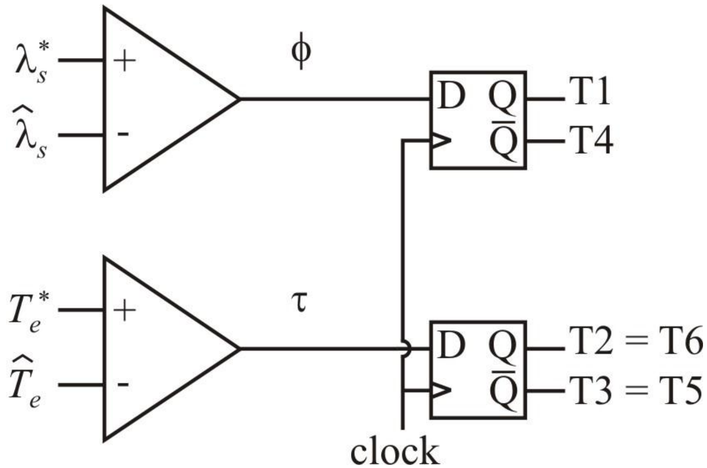
Voltage Vector Space



Switching Rules

switching based on *direct* control of flux and torque

switching rules for α =1



permutate outputs (T_1, T_2, T_3) based on α

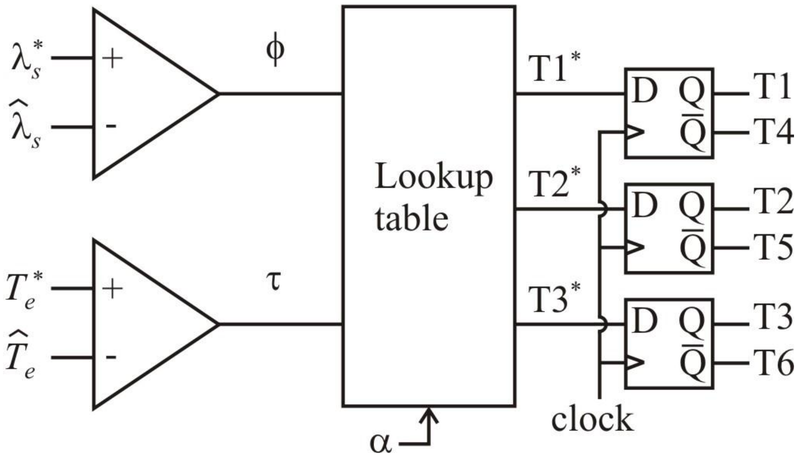
Other methods use different switching rules which make use of the 000 and 111 states to improve torque ripple

Lookup Table

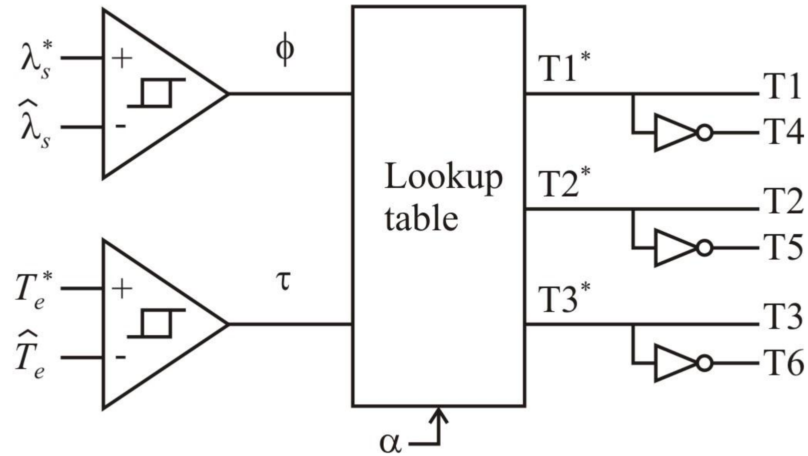
	$T1^*$	$T2^*$	$T3^*$
$\alpha = 1$	ϕ	τ	$\bar{\tau}$
$\alpha = 2$	$\bar{\tau}$	τ	$\bar{\phi}$
$\alpha = 3$	$\bar{\tau}$	ϕ	τ
$\alpha = 4$	$\bar{\phi}$	$\bar{\tau}$	τ
$\alpha = 5$	τ	$\bar{\tau}$	ϕ
$\alpha = 6$	τ	$\bar{\phi}$	$\bar{\tau}$

DTC Implementation with Lookup Table

Comparator / latch circuit



Hysteresis comparator



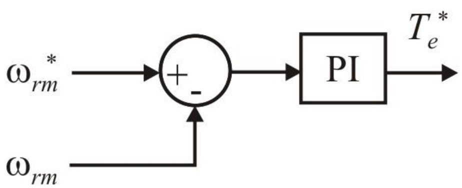
DTC Lookup Table IM Simulation

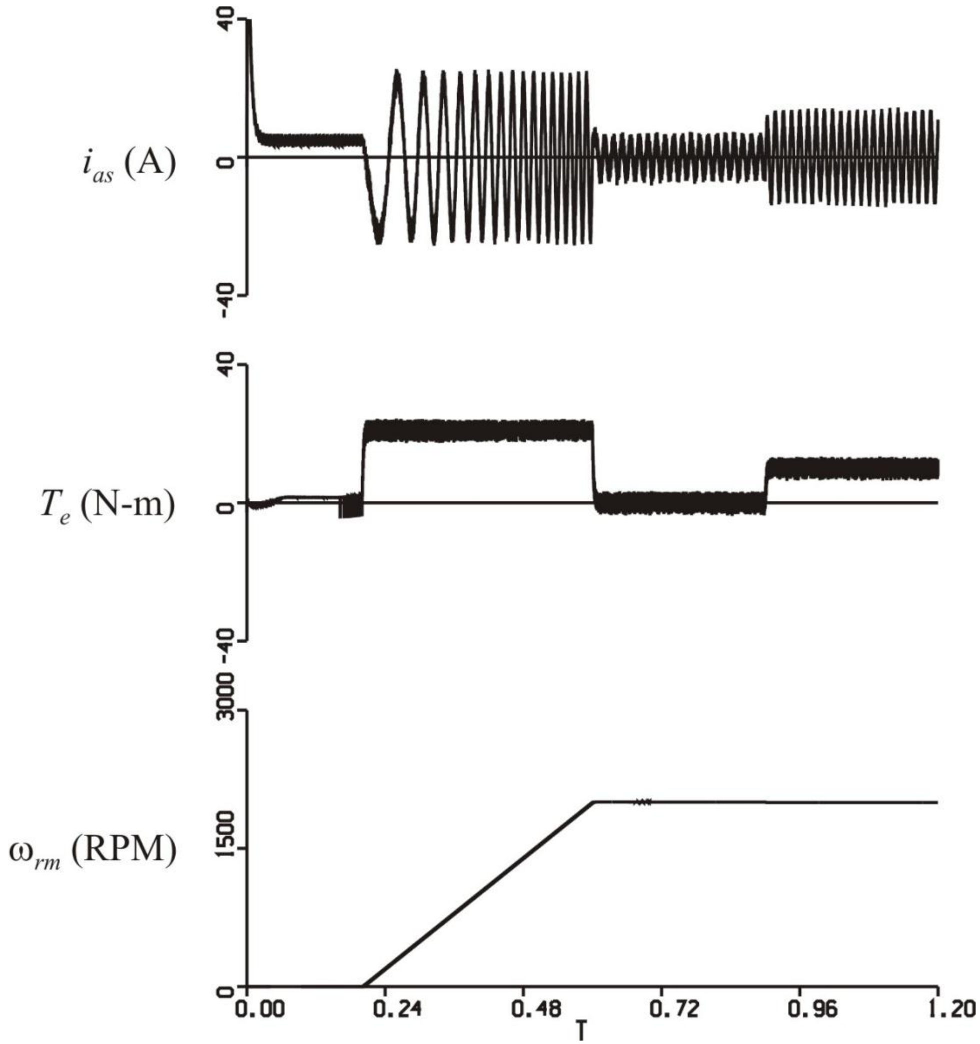
control settings

$$K_p = 20 \text{ N}\cdot\text{m}\cdot\text{s}$$

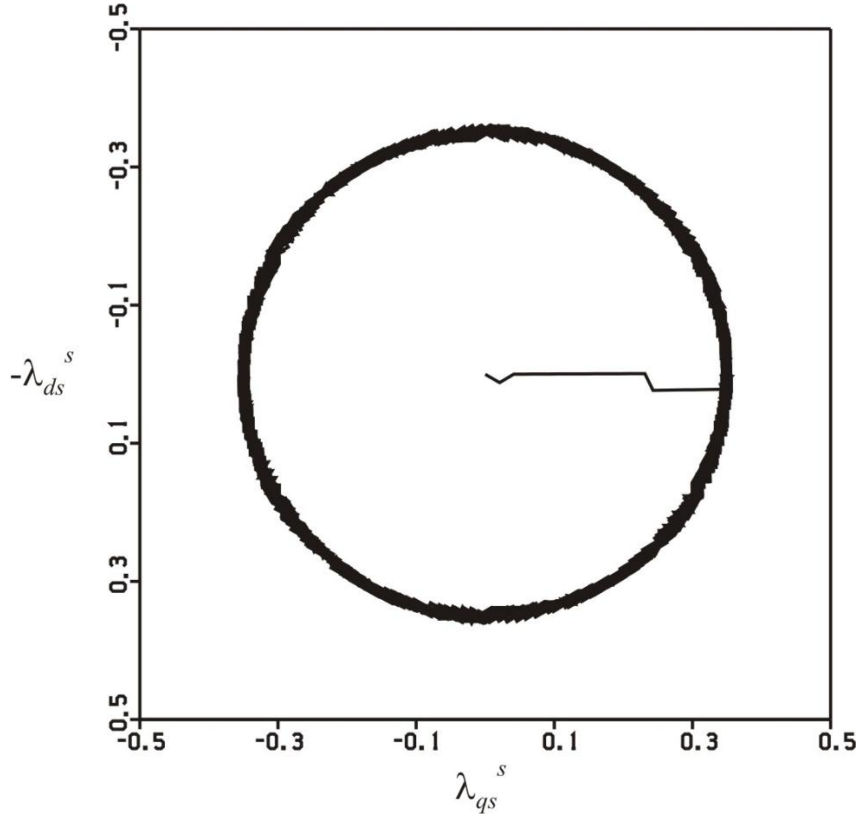
$$K_i = 30 \text{ N}\cdot\text{m}$$

$$\lambda_s^* = 0.35 \text{ V}\cdot\text{s}$$





Flux Linkage Vector Plot



Lookup Table DTC

Stator flux angle identifies the sector location of the flux linkage

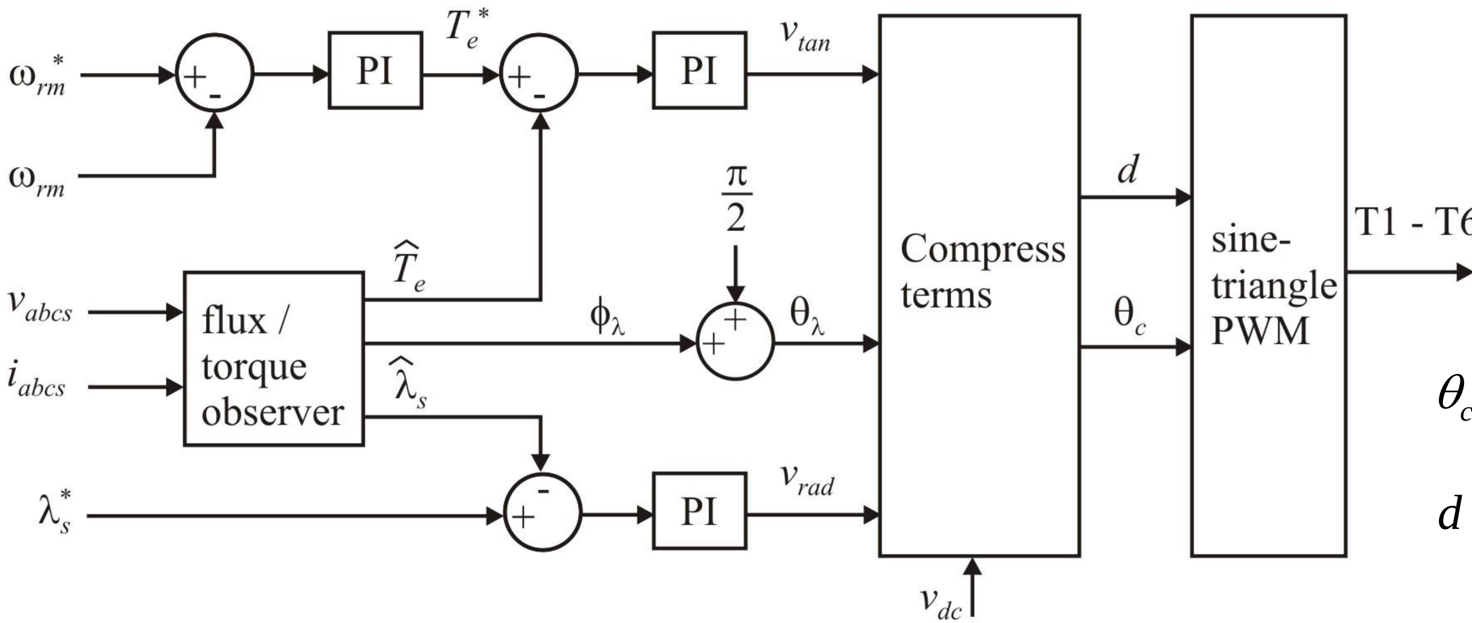
Inverter switching is obtained from a table based on sector angle so that the transistor switching controls radial and tangential voltage

Implementation can be carried out with a comparator/latch circuit or a hysteresis comparator

Direct Torque Control - Space Vector Modulation

Also called DTC-SVM or stator-oriented vector control

Control Diagram



$$\theta_c = \theta_\lambda$$

$$d = \frac{2\sqrt{2}V_s^*}{v_{dc}}$$

$$V_s^* = \frac{1}{\sqrt{2}}\sqrt{v_{\tan}^2 + v_{rad}^2}$$

command flux at rated steady-state value, or use field weakening (lower commanded flux at higher speeds)

DTC-SVM Induction Motor Simulation

control settings

$K_p = 20\ \mathrm{N \cdot m \cdot s}$

$K_i = 30\ \mathrm{N \cdot m}$

$\lambda_s^* = 0.35\ \mathrm{V \cdot s}$

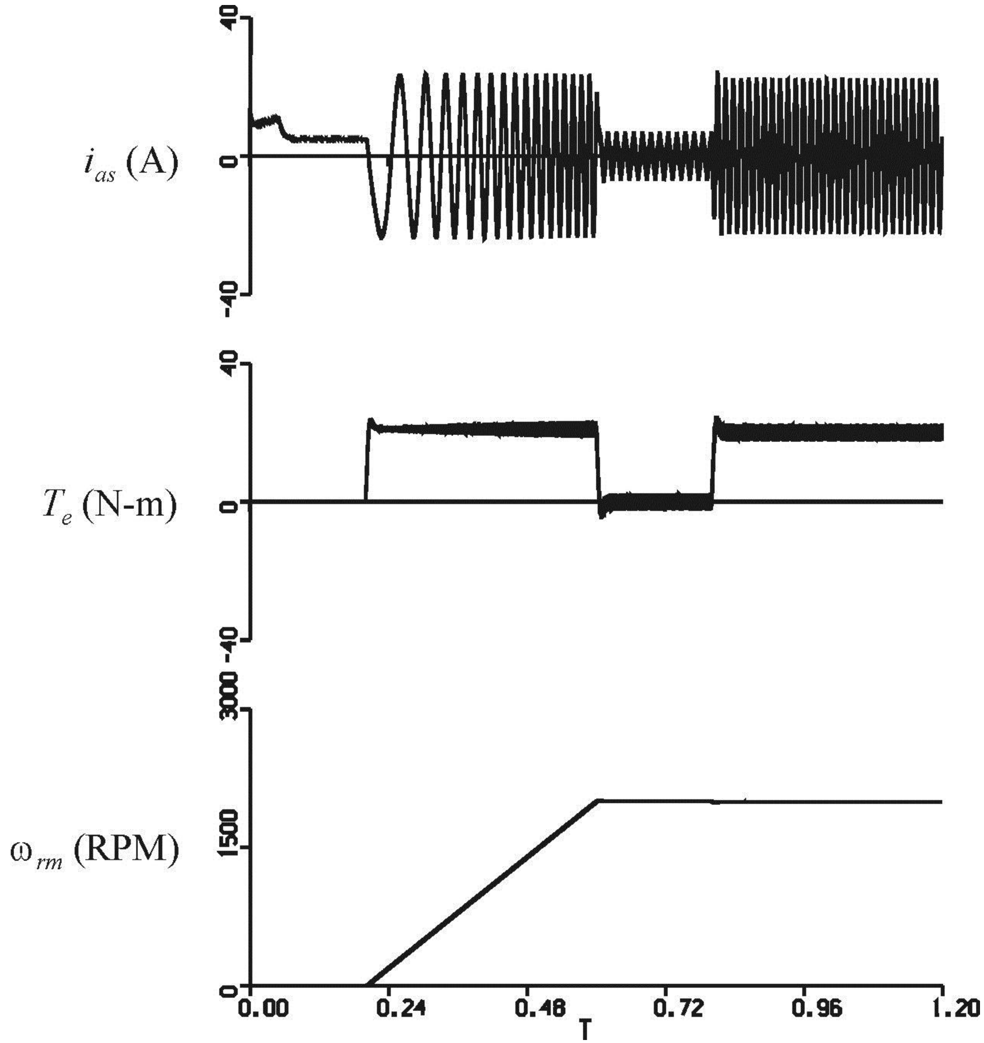
$K_{p,vtan} = 1.7\ \mathrm{A^{-1} \cdot s^{-1}}$

$K_{i,vtan} = 200\ \mathrm{A^{-1} \cdot s^{-2}}$

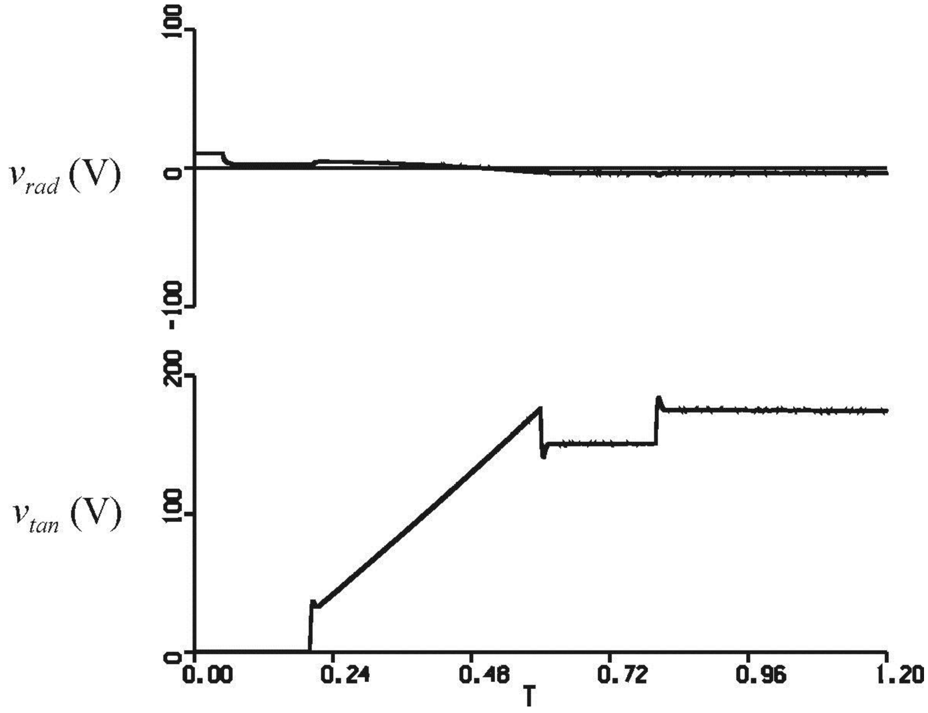
$K_{p,vrad} = 500\ \mathrm{s^{-1}}$

$K_{i,vrad} = 20\ \mathrm{s^{-2}}$

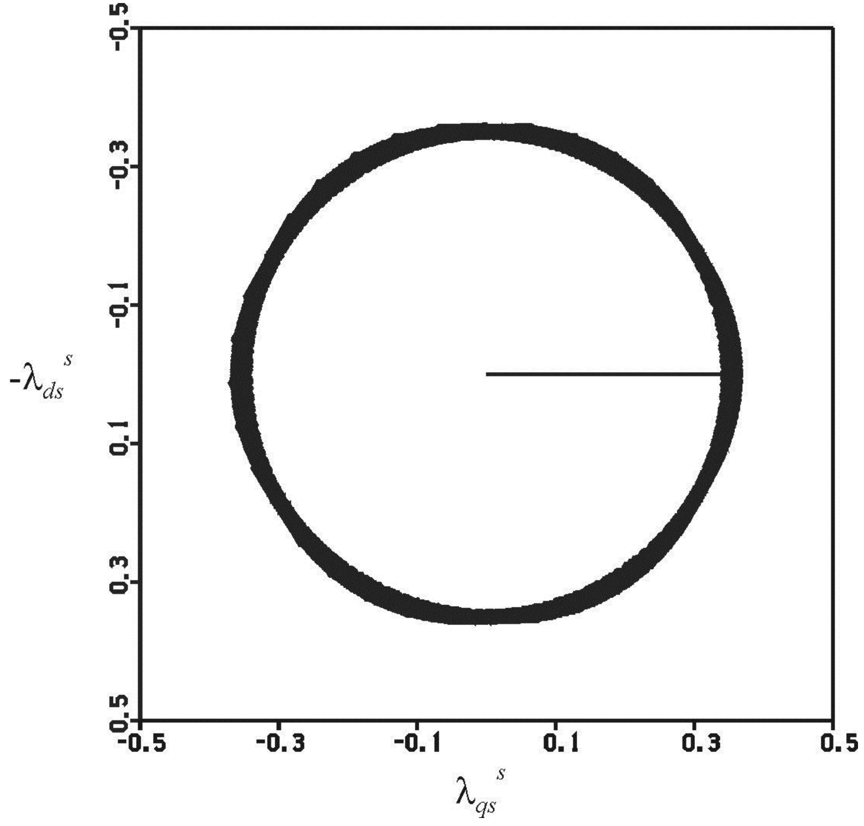
v_{rad} limited to ±10V



Radial and Tangential Voltage



Flux Linkage Vector Plot



DTC-SVM

PI controls are used to turn commanded torque and flux into commanded tangential and radial voltages

With the commanded voltages and flux angle, standard sine-triangle modulation or space-vector modulation (SVM) can be applied to switch the inverter

Brushless Dc Drives

rotor reference frame ($\theta = \theta_r$)

$$\lambda_{qs}^r = L_s i_{qs}^r$$

$$\lambda_{ds}^r = L_s i_{ds}^r + \lambda'_m$$

$$v_{qs}^r = r_s i_{qs}^r + \omega_r \lambda_{ds}^r + p\lambda_{qs}^r$$

$$v_{ds}^r = r_s i_{ds}^r - \omega_r \lambda_{qs}^r + p\lambda_{ds}^r$$

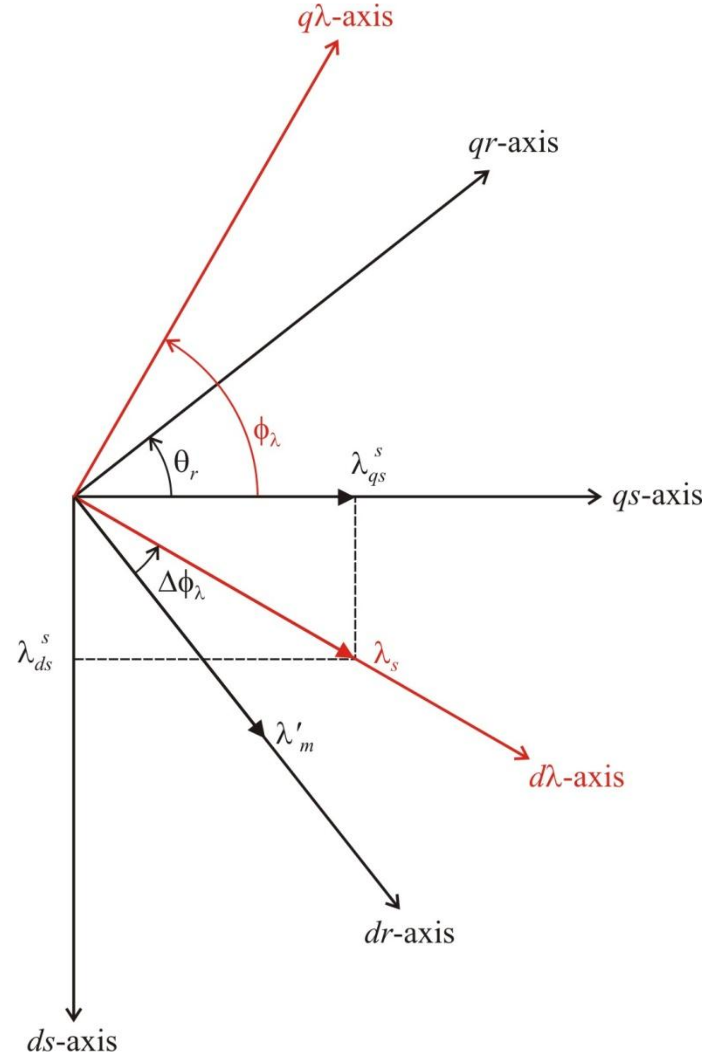
flux reference frame ($\theta = \phi_\lambda$)

$$\lambda_{qs}^\lambda = 0$$

$$\lambda_{ds}^\lambda = \lambda_s$$

$$v_{qs}^\lambda = r_s i_{qs}^\lambda + \omega_r \lambda_s + p\lambda_{qs}^\lambda = v_{tan}$$

$$v_{ds}^\lambda = r_s i_{ds}^\lambda + p\lambda_{ds}^\lambda = v_{rad}$$



ignoring r_s, v_{rad} sets $p\lambda^\lambda{}_{ds}$ determining the change in flux λ_s
and v_{tan} sets $p\lambda^\lambda{}_{qs}$ determining torque (angle between λ_s and λ'_m)

Brushless Dc Drive System

- Lookup table method used
- Flux / torque estimator and switching rules are generalized concepts (apply to IM and PMSM)
- Note: no need for position encoder

Machine equations

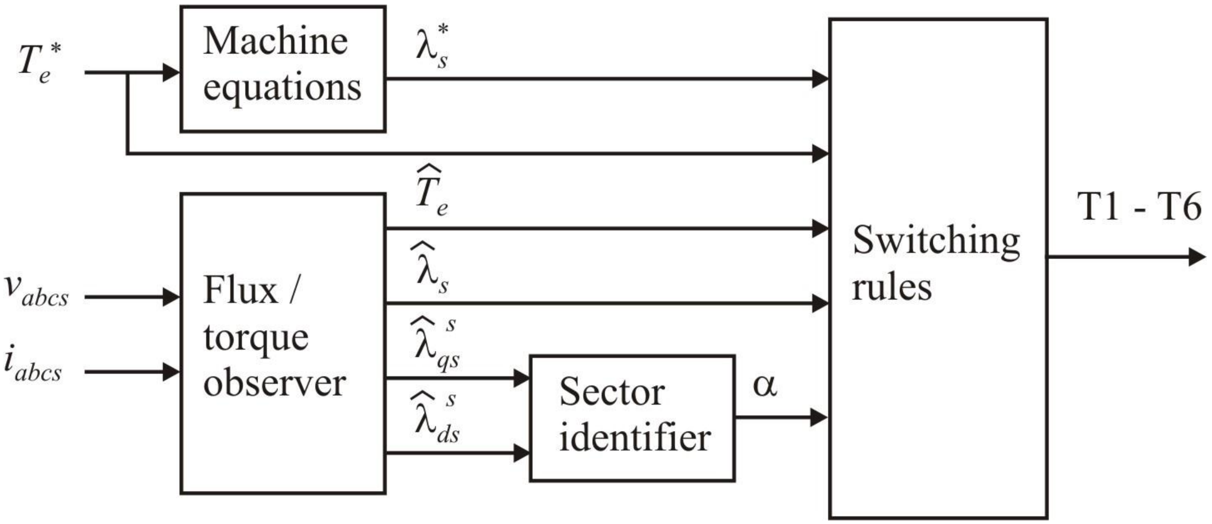
$$i_{qs}^{r*} = \frac{2}{3}\frac{2}{P}\frac{T_e^*}{\lambda'_m}$$

$$i_{ds}^{r*} = 0$$

$$\lambda_{qs}^{r*} = L_s i_{qs}^{r*}$$

$$\lambda_{ds}^{r*} = L_s i_{ds}^{r*} + \lambda'_m$$

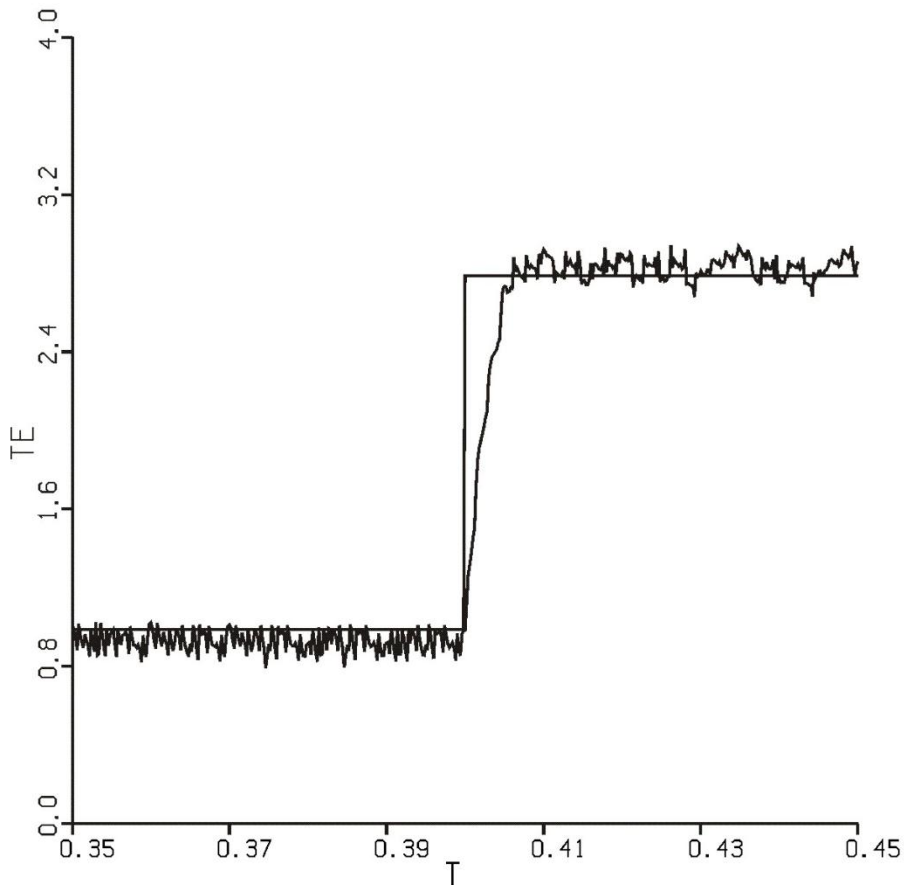
$$\lambda_s^* = \sqrt{\left(\lambda_{qs}^{r*}\right)^2 + \left(\lambda_{ds}^{r*}\right)^2}$$



Torque Response

$P := 4$ $r_s := 2.9 \cdot \Omega$ $L_s := 11.35 \cdot mH$ $\lambda'_m := 0.156 \cdot V \cdot s$

$\omega_{rm} := 314.2 \cdot \frac{rad}{s}$ $T_{e\_star\_1} := 0.99 \cdot N \cdot m$ $T_{e\_star\_2} := 2.79 \cdot N \cdot m$



L. Zhong, M.F. Rahman, W.Y. Hu, and K.W. Lim, "Analysis of direct torque control in permanent magnet synchronous motor drives," *IEEE Transactions on Power Electronics*, volume 12, number 3, pages 528-536, May 1997.

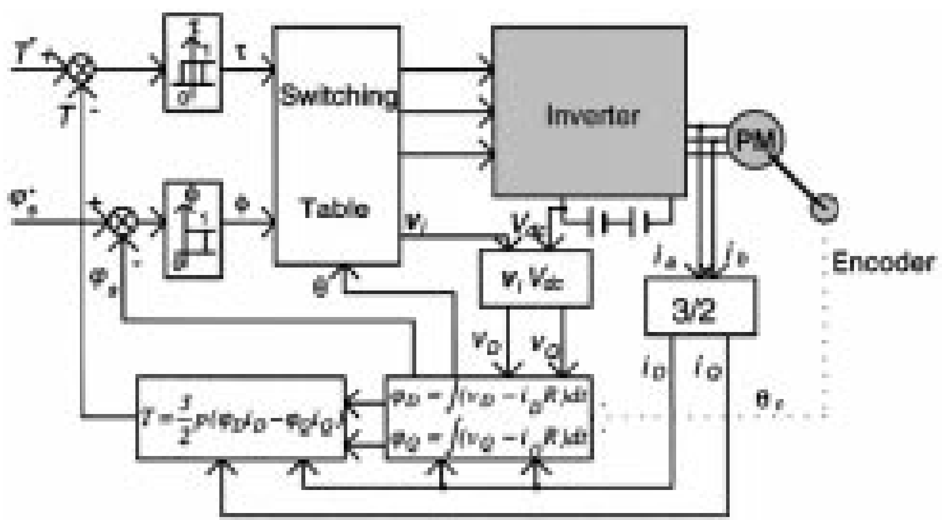
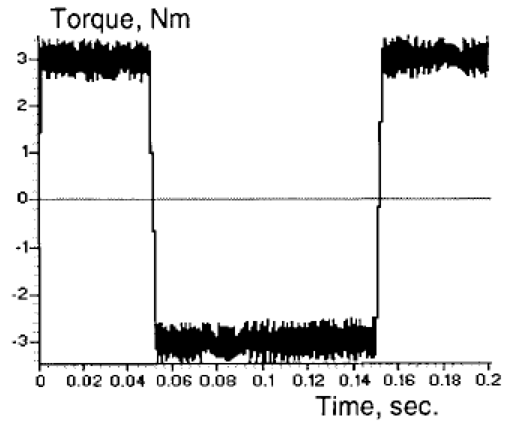
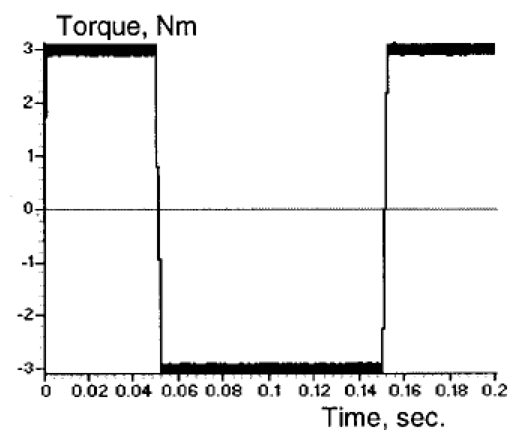


Fig. 9. The block diagram of a PMSM drive with DTC.



$T_{sw} = 100\ \mu s$



$T_{sw} = 10\ \mu s$

Y. Hu, G Tian, Y. Gu, Z. You, L.X. Tang, and M.F. Rahman, "In-depth research on direct torque control of permanent magnet synchronous motor," *Proceedings of the IEEE Industrial Electronics Society Conference*, volume 2, pages 1060-1065, November 2002.

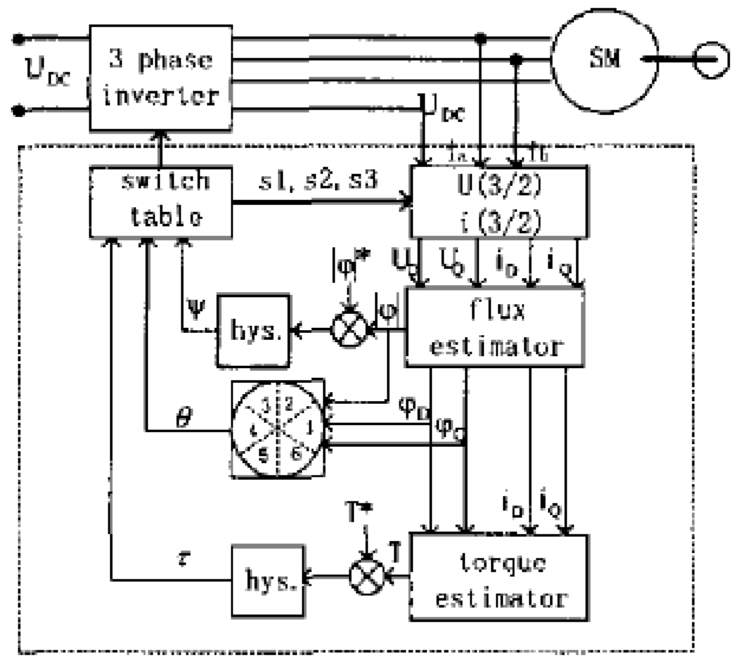
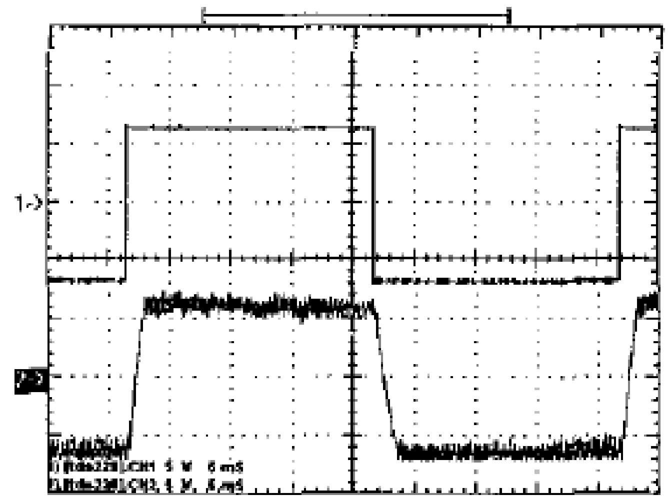


Fig. 1. The block diagram of a PMSM drive with DTC



Ch1: torque reference

Ch2: torque response 1div=4N.m

torque response (-5N.m~+5N.m)

Fig. 4. The experimental results