# Sensorless Control of PM Synchronous Motors Based on MRAS Method and Initial Position Estimation

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Abstract-This paper presents a scheme of Model Reference Adaptive System (MRAS) method for the sensorless control of PM synchronous motors. The stability of this scheme is guaranteed by the Popov Super Stability Theorem. Two-phase stator currents and DC bus voltage are used to estimate the rotor speed and position and it doesn't need much calculation. The experimental results show that the estimated values are in close agreement with those obtained from an encoder. The method of estimating the initial rotor position of PM synchronous motors is also discussed in this paper.

#### 1. INTRODUCTION

During recent years sensorless drives of PM synchronous motors have attracted much attention. Many techniques have been proposed in order to estimate the rotor speed and position, such as the open-loop estimators using stator voltages and currents, back e.m.f.—based position estimators, observer-based (e.g. extended Kalman filter) speed and position estimators and estimators based on inductance variation due to geometrical and saturation effects. In this paper, a scheme of MRAS method is described, which is very simple to be implemented. In this scheme, only the stator currents and the DC bus voltage are needed and satisfactory performance can be achieved. In the control of PM synchronous motors, getting the initial rotor position is an essential problem. This paper also discussed the method to estimate the initial rotor position value.

### II. PRINCIPLE OF MRAS CONTROL SCHEME

The stator current equations of the PM synchronous motor in the rotating d-q reference frame are given by:

$$\frac{di_d}{dt} = -\frac{R}{I}i_d + \omega i_q + \frac{u_d}{I} \tag{1}$$

$$\frac{di_q}{dt} = -\frac{R}{I}i_q - \omega i_d - \frac{\psi_r}{I}\omega + \frac{u_q}{I} \tag{2}$$

As the rotor speed  $\omega$  is included in these equations, we can choose the current model of the PM synchronous motor as the adjustable model, and the motor itself as the reference model. These two models both have the output  $i_a$  and  $i_q$ . According to the difference between the outputs of the two models, through a certain adaptive mechanism, we can get the estimated value of the rotor speed. Then the position can be obtained by integrating the speed.

The equations (1) and (2) can be rewritten into matrix form,

$$\frac{d}{dt} \begin{bmatrix} i_d + \frac{\psi_r}{L} \\ i_y \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \omega \\ -\omega & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} i_d + \frac{\psi_r}{L} \\ i_y \end{bmatrix} + \frac{1}{L} \begin{bmatrix} u_d + \frac{R\psi_r}{L} \\ u_g \end{bmatrix}$$
(3)

For the convenience of stability analysis, the speed  $\omega$  has been

confined to the system matrix 
$$A = \begin{bmatrix} -\frac{R}{L} & \omega \\ -\omega & -\frac{R}{L} \end{bmatrix}$$
. (4)

Let 
$$i_d' = i_d + \frac{\psi_r}{I}, \quad i_q' = i_q;$$
 (5)

$$u_{d}' = u_{d} + \frac{R\psi_{r}}{I}, \quad u_{q}' = u_{q};$$
 (6)

then (3) can be changed into

$$\frac{d}{dt} \begin{bmatrix} i_{d} \\ i_{q'} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \omega \\ -\omega & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} i_{d'} \\ i_{q'} \end{bmatrix} + \frac{1}{L} \begin{bmatrix} u_{d'} \\ u_{q'} \end{bmatrix}$$
 (7)

A simple form is 
$$\frac{d}{dt}i' = Ai' + Bu'$$
 (8)

The process of speed estimation can be described as follows:

$$\frac{d}{dt} \begin{bmatrix} \hat{i}_{d} \\ \hat{i}_{q'} \\ \hat{i}_{q'} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \hat{\omega} \\ -\hat{\omega} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} \hat{i}_{d'} \\ \hat{i}_{q'} \end{bmatrix} + \frac{1}{L} \begin{bmatrix} u_{d'} \\ u_{q'} \end{bmatrix}$$
(9)

The simple form of it is 
$$\frac{d}{dt}\hat{i}' = \hat{A}\hat{i}' + Bu'$$
 (10)

The error of the state variables is 
$$e = i' - \hat{i}'$$
 (11)

According to (8) (10) and (11), we can get:

$$\frac{d}{dt}e = Ae - Iw$$

$$v = De$$
(12)

where  $w = (\hat{A} - A)\hat{i}$ ,

if D = I, then v = Ie = e

According to the Popov super stability theory, if

(1)  $H(s) = D(sI - A)^{-1}$  is a strictly positive matrix,

(2)  $\eta(0, t_0) = \int_0^0 v^T w dt \ge -\gamma_0^2 \quad \forall t_0 \ge 0$ , where  $\gamma_0^2$  is a limited positive number, then  $\lim_{t \to \infty} e(t) = 0$ . The MRAS system will be stable.

Finally, the equation of  $\hat{\omega}$  can be achieved as

$$\hat{\omega} = \int_{0}^{1} k_{1} (i_{q}^{\dagger} i_{q}^{\dagger} - i_{q}^{\dagger} i_{d}^{\dagger}) d\tau + k_{2} (i_{d}^{\dagger} i_{q}^{\dagger} - i_{q}^{\dagger} i_{d}^{\dagger}) + \hat{\omega}(0)$$
(13)

where  $k_1, k_2 \ge 0$ 

Replacing  $i_d$ ,  $i_q$ , with  $i_d$ ,  $i_q$ , we can get

$$\hat{\omega} = \int_{S} k_{1} [i_{d} \hat{i_{q}} - i_{q} \hat{i_{d}} - \frac{\psi_{r}}{L} (i_{q} - \hat{i_{q}})] d\tau + k_{2} [i_{d} \hat{i_{q}} - i_{q} \hat{i_{d}} - \frac{\psi_{r}}{L} (i_{q} - \hat{i_{q}})] + \hat{\omega}(0)$$
(14)

In the above equation,  $\hat{i_d}$ ,  $\hat{i_q}$  can be calculated through the adjustable model,  $i_d$ ,  $i_q$  can be obtained by the transformation of the measured stator currents.

The rotor position can be obtained by integrating the estimated speed:

$$\hat{\theta} = \int \hat{\omega} dt \tag{15}$$

The MRAS scheme is illustrated in Fig.1.

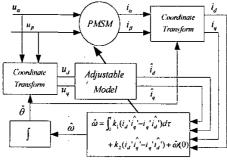


Fig.1. Block Schematic of MRAS

## III. EXPERIMENTAL RESULTS

Table.1 shows the parameters of the PM motor used in the experiments.

TABLE 1 PARAMETERS OF THE MOTOR	
Rated power	1000W
Rated torque	5.0Nm
Rated current	5.16A
Rated speed	2000r/min
Maximum speed	2500r/min
Number of pole pairs	4
R	1.82 Ω
L	15.06mH
J	$0.64 \times 10^{-3} \mathrm{kgm^2}$

The whole vector control system is shown in Fig. 2.

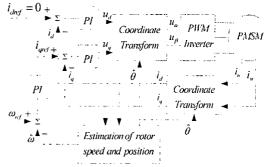


Fig.2. Block schematic of vector control system

Fig. 3 shows the estimated and measured speed in the start-up process when the reference speed is 419 rad/s. Fig. 4 shows the estimated and measured position at this speed. The position error is shown in Fig. 5. The same variables at the speed 62.8 rad/s are illustrated in Fig. 6 –8. We can see that the estimated speed and position are in close agreement with the values obtained from an encoder.

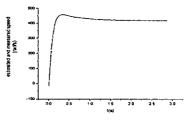


Fig. 3. Estimated and measured speed in the start-up process

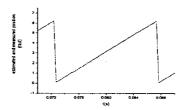


Fig. 4. Estimated and measured position

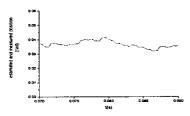


Fig. 5. Position error

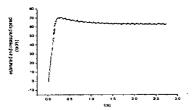


Fig. 6. Estimated and measured speed in the start-up process

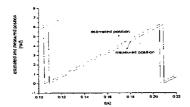


Fig. 7. Estimated and measured position

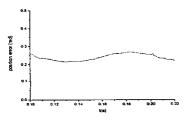


Fig. 8. Position error

## IV. INITIAL ROTOR POSITION ESTIMATION

In the sensorless vector control of PM synchronous motors, it is essential to get the accurate initial rotor position. If the value is inaccurately estimated, the rotor may temporarily rotate in the wrong direction or cause starting failure. The following method makes use of the magnetic saturation characteristic of the stator core. A series of voltage pulses of equal amplitudes and different directions are applied to the motor, by detecting and comparing the stator currents, the initial rotor position can be estimated.

Fig. 9 shows the model of the PM synchronous motor. The orthogonal two-phase  $\alpha - \beta$  frame is fixed to the stator windings.  $\theta_r$  represents the angle of the rotor position and  $\theta_r$  is the angle of the voltage vector. Fig. 10 shows the voltage vectors that

are provided to the motor. For the voltage vector whose angle is  $\theta_{\rm v}$ , its corresponding current can be obtained by the following equation:

$$i_n = i_\alpha * \cos \theta_v + i_\beta * \sin \theta_v \tag{16}$$

Therefore, the initial rotor position can be estimated by detecting the maximum current for the voltage vector.

This initial rotor position estimation method is applied to a motor whose rated current is 11A. Fig. 11 shows the current values of the 12 voltage vectors when the rotor position  $\theta_r = 90^\circ$ . The current corresponding to voltage vector 7, whose angle is  $90^\circ$ , is the maximum current. Therefore, the estimated rotor position is  $90^\circ$ .

In this method, it is very important to find the appropriate amplitude and providing time of the voltage vector to make sure that the differences of the currents are big enough to be detected and the rotor doesn't rotate during the estimation process. The amplitude and providing time of the voltage vector in this experiment is 110V and 200  $\mu s$ .

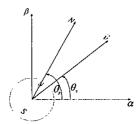


Fig. 9. Motor Model

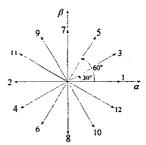


Fig. 10. Voltage Vectors

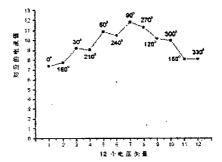


Fig. 11. Currents of Vectors 1-12

## V. CONCLUSION

In this paper, a scheme of MRAS method for the sensorless control of PM synchronous motors has been presented. The current model is used as the adjustable model, and the motor itself is used as the reference model. Using the output of these two models  $i_d$  and  $i_q$ , the rotor speed and position are estimated. The stability of the system is guaranteed by the Popov super stability theory. The experimental results provide evidence of the viability and performance of this scheme. Estimating the rotor position at standstill is an important problem in sensorless control of PM synchronous motors. The method in this paper can estimate the rotor position effectively and thus guarantees the start performance of the sensorless control system.

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