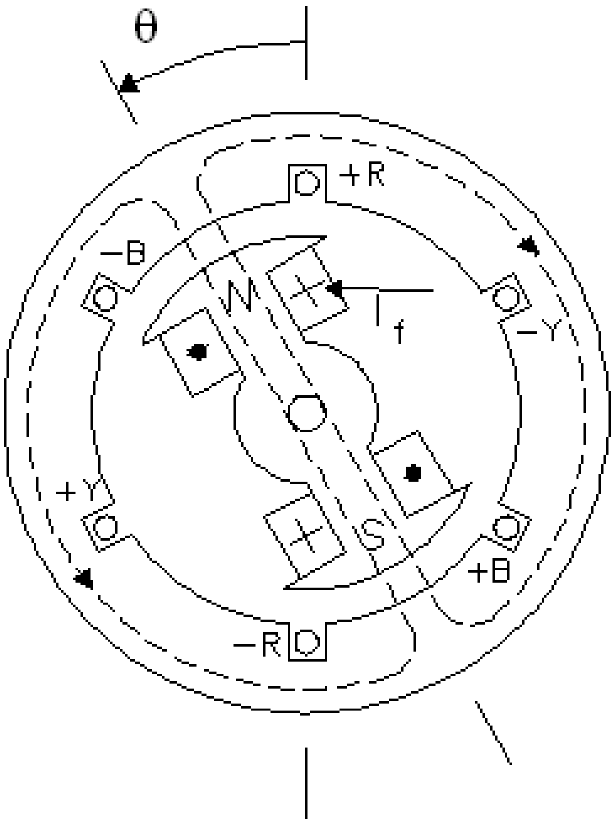
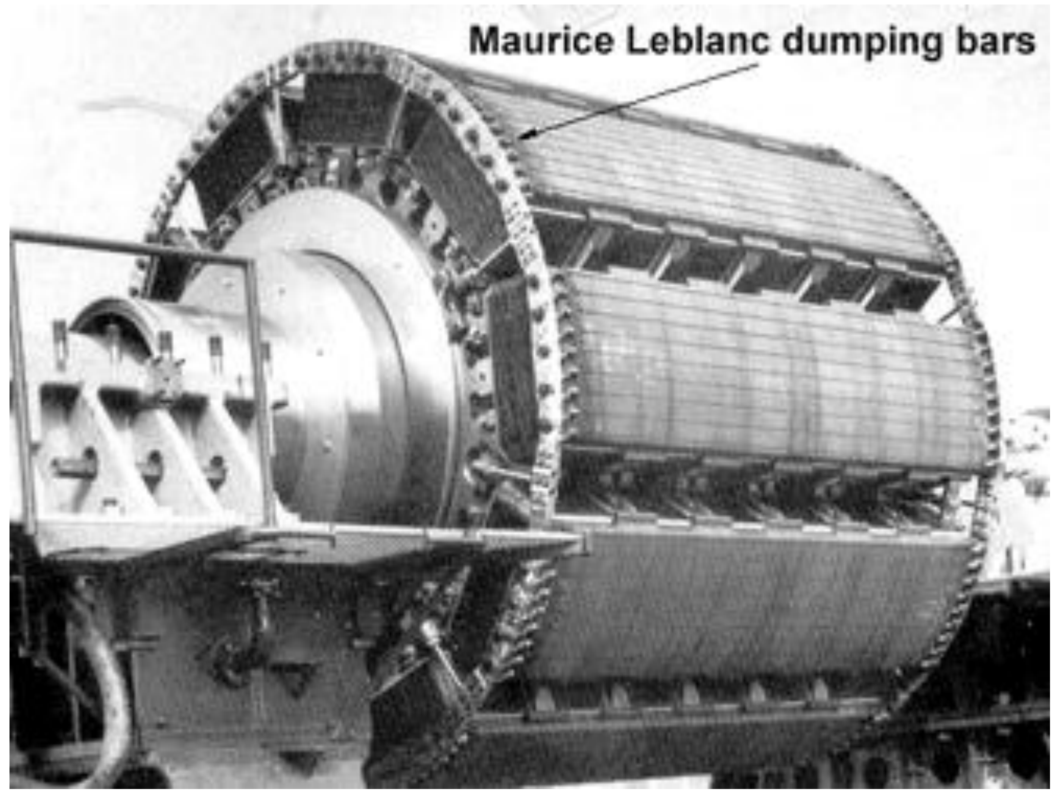
CLEMSON UNIVERSITY

# ECE 802, Electric Motor Control

# Reference Frame Theory

# Reference Frame Theory (Chapter 3)




Introduced by R.H. Park in 1929 to model synchronous machines

# Three-Phase Transformation to the Arbitrary Reference Frame

$$f_{qd0s} = K_s f_{abcs}$$

$$f_{qd0s} = \begin{bmatrix} f_{qs} \\ f_{ds} \\ f_{0s} \end{bmatrix} \qquad f_{abcs} = \begin{bmatrix} f_{as} \\ f_{bs} \\ f_{cs} \end{bmatrix}$$

*f* = voltage, current, or flux linkage

*q* = *q*-axis (quadrature axis)
*d* = *d*-axis (direct axis)
0 = zero sequence

*a* = *a*-phase
*b* = *b*-phase
*c* = *c*-phase

# The Reference Frame Transformation

$$K_s = \frac{2}{3}\begin{bmatrix} \cos(\theta) & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ \sin(\theta) & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\theta = \int_0^t \omega(\varsigma)\, d\varsigma + \theta(0)$$

$\omega$ = reference frame speed (rad/sec)

$\theta$ = reference frame position (rad)

# The Inverse Transformation

$$f_{abcs} = K_s^{-1} f_{qd0s}$$

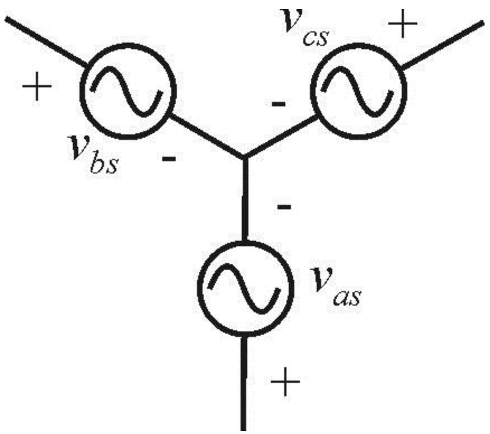
$$K_s^{-1} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 1 \\ \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\theta + \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) & 1 \end{bmatrix}$$

# Example: Three-Phase Set of Voltages

$$v_{as} = \sqrt{2} V_s \cos\left(\theta_e + \phi_v\right)$$

$$v_{bs} = \sqrt{2} V_s \cos\left(\theta_e + \phi_v - \frac{2\pi}{3}\right)$$

$$v_{cs} = \sqrt{2} V_s \cos\left(\theta_e + \phi_v + \frac{2\pi}{3}\right)$$




$$\theta_e = \omega_e t$$

$$\omega_e = 2\pi f$$

$f$ - electric frequency (Hz)

$\omega_e$ - electric radian frequency (rad/sec)

$\theta_e$ - electrical position (rad)

$V_s$ - rms Voltage (V)

$\phi_v$ - phase shift (rad)

# Transform to the Arbitrary Reference Frame

$$v_{qd0s} = K_s v_{abcs}$$

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ v_{0s} \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos(\theta) & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ \sin(\theta) & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \sqrt{2} V_s \cos(\theta_e + \phi_v) \\ \sqrt{2} V_s \cos\left(\theta_e + \phi_v - \frac{2\pi}{3}\right) \\ \sqrt{2} V_s \cos\left(\theta_e + \phi_v + \frac{2\pi}{3}\right) \end{bmatrix}$$

# Voltages in Arbitrary Reference Frame

**_q_-axis voltage**

$$v_{qs} = \frac{2}{3}\sqrt{2}V_s\left[\cos\left(\theta\right)\cos\left(\theta_e+\phi_v\right)+\cos\left(\theta-\tfrac{2\pi}{3}\right)\cos\left(\theta_e+\phi_v-\tfrac{2\pi}{3}\right)+\cos\left(\theta+\tfrac{2\pi}{3}\right)\cos\left(\theta_e+\phi_v+\tfrac{2\pi}{3}\right)\right]$$

using the identity,

$$\cos\left(x\right)\cos\left(y\right)+\cos\left(x-\tfrac{2\pi}{3}\right)\cos\left(y-\tfrac{2\pi}{3}\right)+\cos\left(x+\tfrac{2\pi}{3}\right)\cos\left(y+\tfrac{2\pi}{3}\right)=\frac{3}{2}\cos\left(x-y\right)$$

$$\boxed{v_{qs} = \sqrt{2}V_s\cos\left(\theta-\theta_e-\phi_v\right)}$$

**_d_-axis voltage**

$$v_{ds} = \frac{2}{3}\sqrt{2}V_s\left[\sin\left(\theta\right)\cos\left(\theta_e+\phi_v\right)+\sin\left(\theta-\tfrac{2\pi}{3}\right)\cos\left(\theta_e+\phi_v-\tfrac{2\pi}{3}\right)+\sin\left(\theta+\tfrac{2\pi}{3}\right)\cos\left(\theta_e+\phi_v+\tfrac{2\pi}{3}\right)\right]$$

using the identity,

$$\sin\left(x\right)\cos\left(y\right)+\sin\left(x-\tfrac{2\pi}{3}\right)\cos\left(y-\tfrac{2\pi}{3}\right)+\sin\left(x+\tfrac{2\pi}{3}\right)\cos\left(y+\tfrac{2\pi}{3}\right)=\frac{3}{2}\sin\left(x-y\right)$$

$$\boxed{v_{ds} = \sqrt{2}V_s\sin\left(\theta-\theta_e-\phi_v\right)}$$

**zero sequence voltage**

$$\boxed{v_{0s} = \frac{1}{3}\sqrt{2}V_s\left[\cos\left(\theta_e+\phi_v\right)+\cos\left(\theta_e+\phi_v-\tfrac{2\pi}{3}\right)+\cos\left(\theta_e+\phi_v+\tfrac{2\pi}{3}\right)\right]=0}$$

# Numerical Example

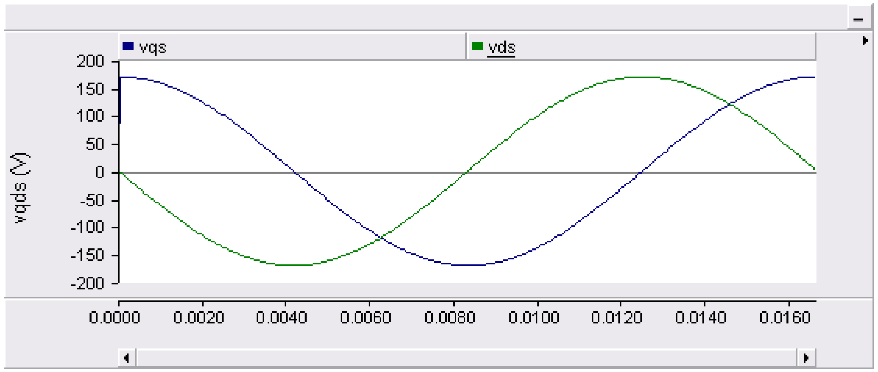
208 V, 3-phase, *f* = 60 Hz (208 V line-to-line rms)

$$V_s = \frac{208\ \text{V}}{\sqrt{3}} = 120\ \text{V} \qquad \phi_v = 0$$

1. Stationary reference frame $\theta = 0$

$$v_{qs}^s = \sqrt{2}V_s \cos\left(\theta_e\right)$$

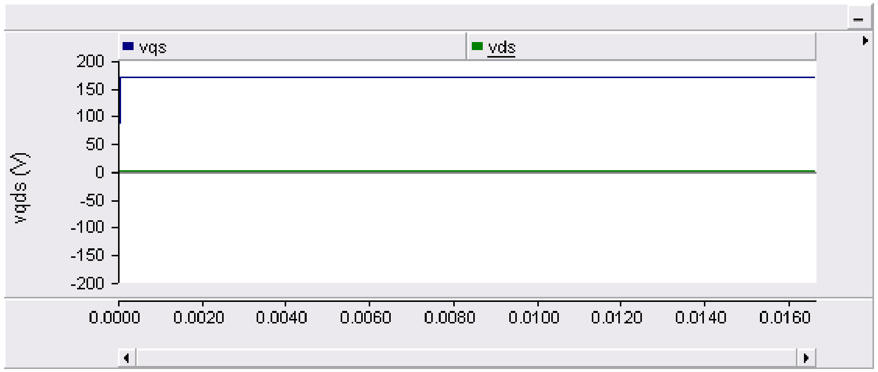
$$v_{ds}^s = -\sqrt{2}V_s \sin\left(\theta_e\right)$$



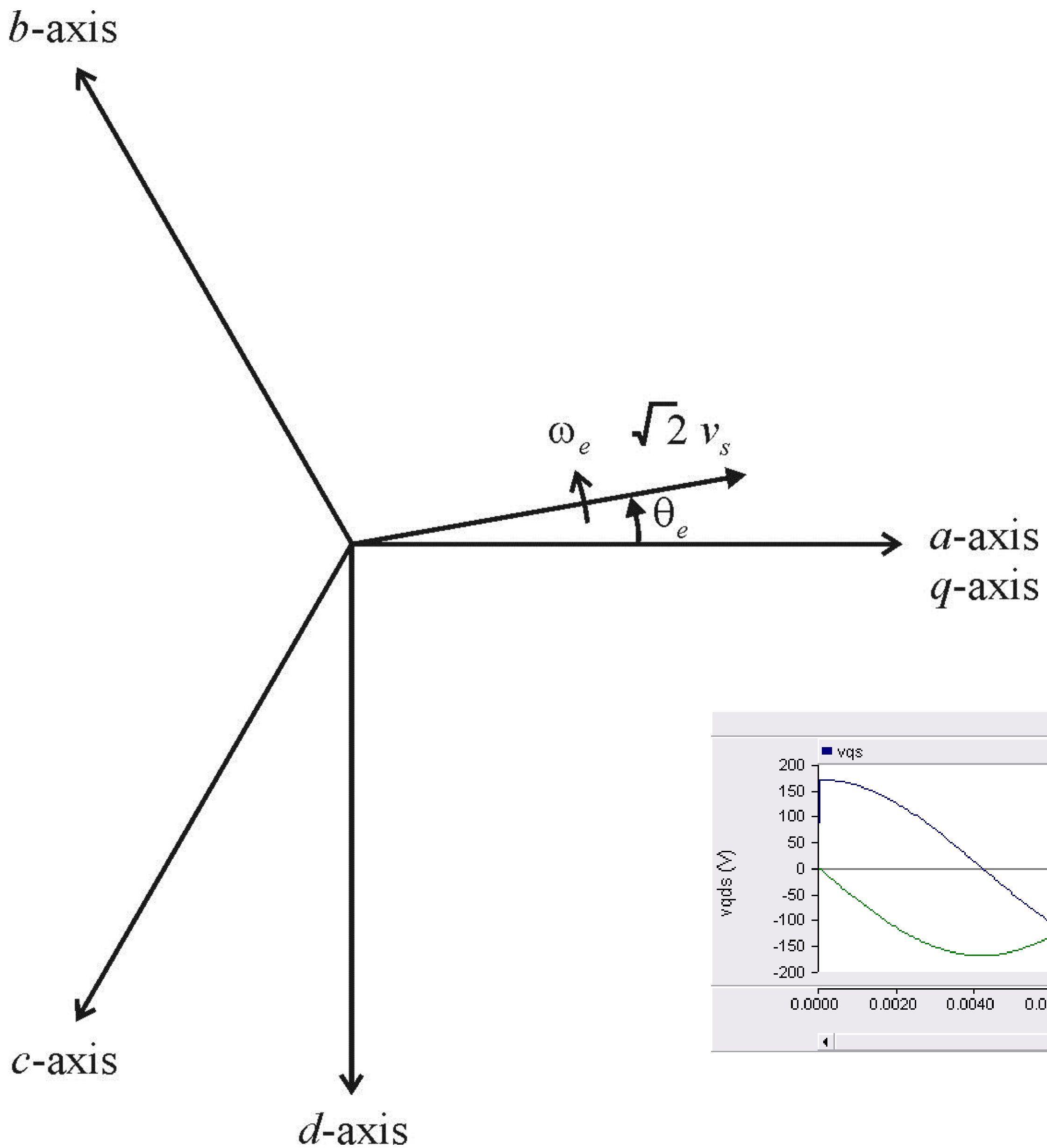

2. Synchronous reference frame $\theta = \theta_e$

$$v_{qs}^e = \sqrt{2}V_s \cos\left(\phi_v\right) = \sqrt{2}V_s$$

$$v_{ds}^e = -\sqrt{2}V_s \sin\left(\phi_v\right) = 0$$

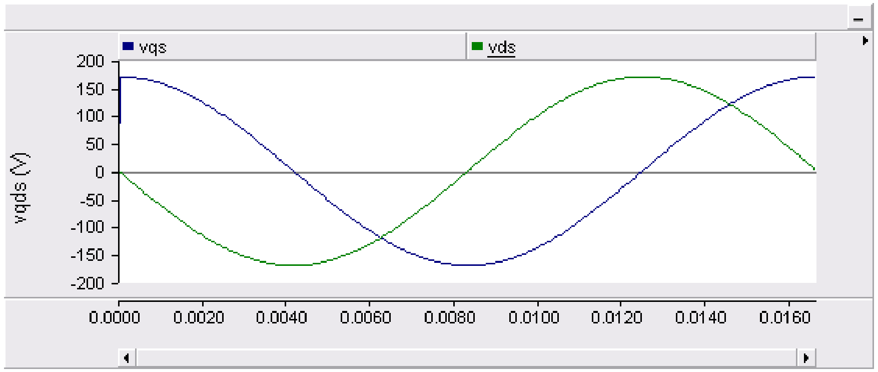


# Axis Sketch with θ=0

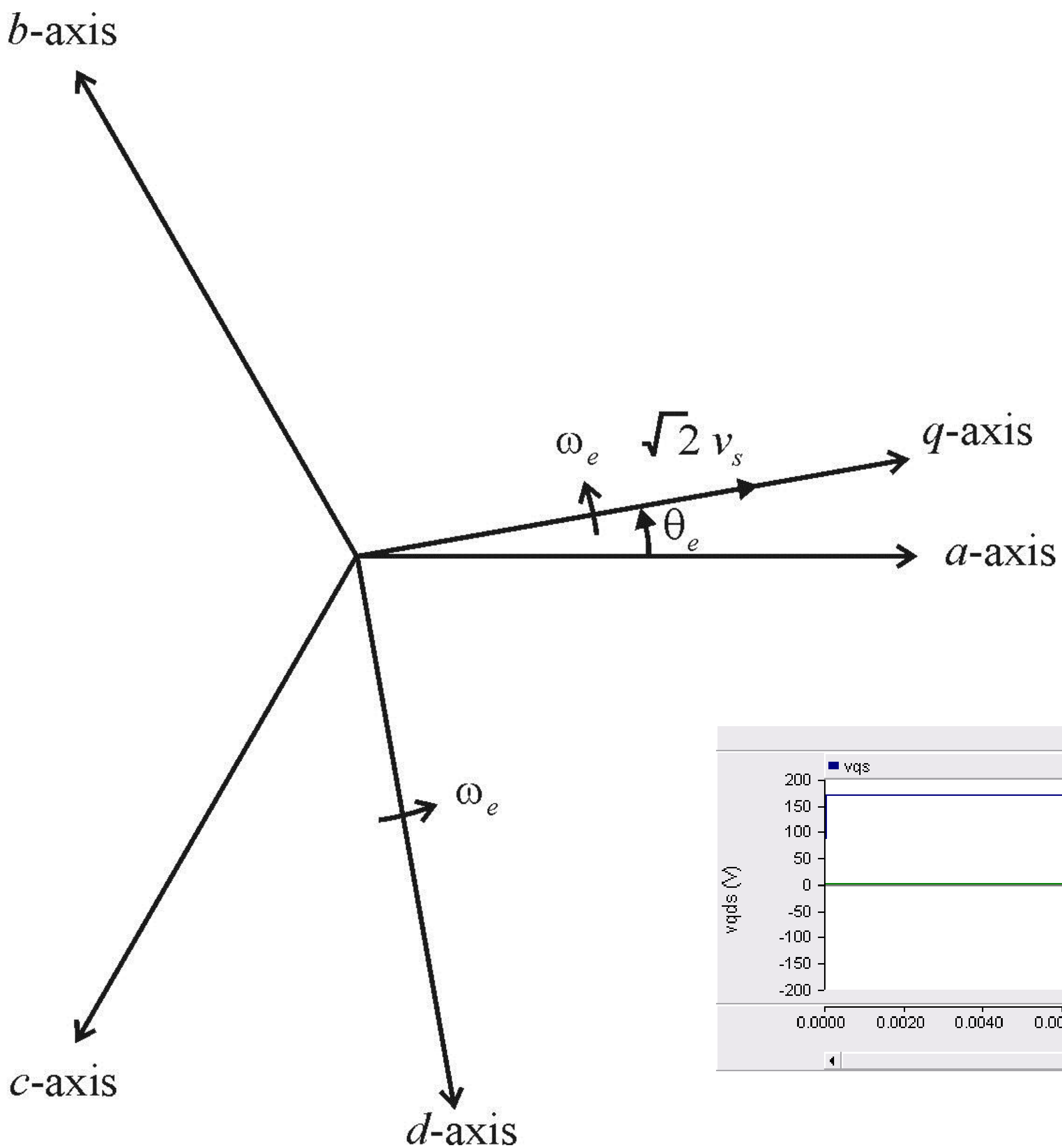



$$v_{qs}^{s} = \sqrt{2}V_s \cos(\theta_e)$$

$$v_{ds}^{s} = -\sqrt{2}V_s \sin(\theta_e)$$

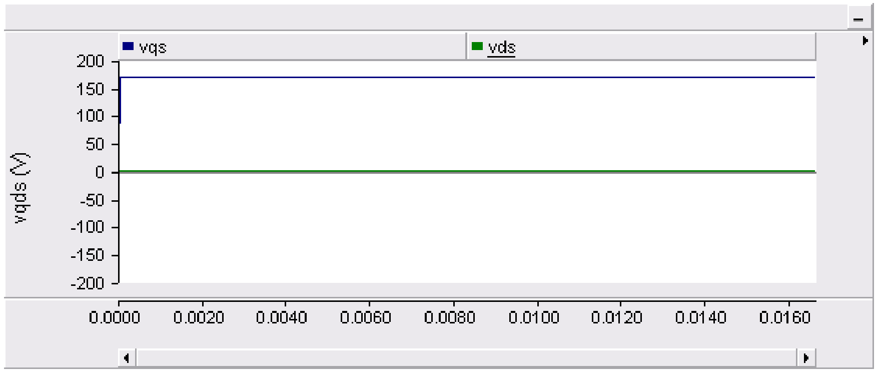


# Axis Sketch with $\theta = \theta_e$

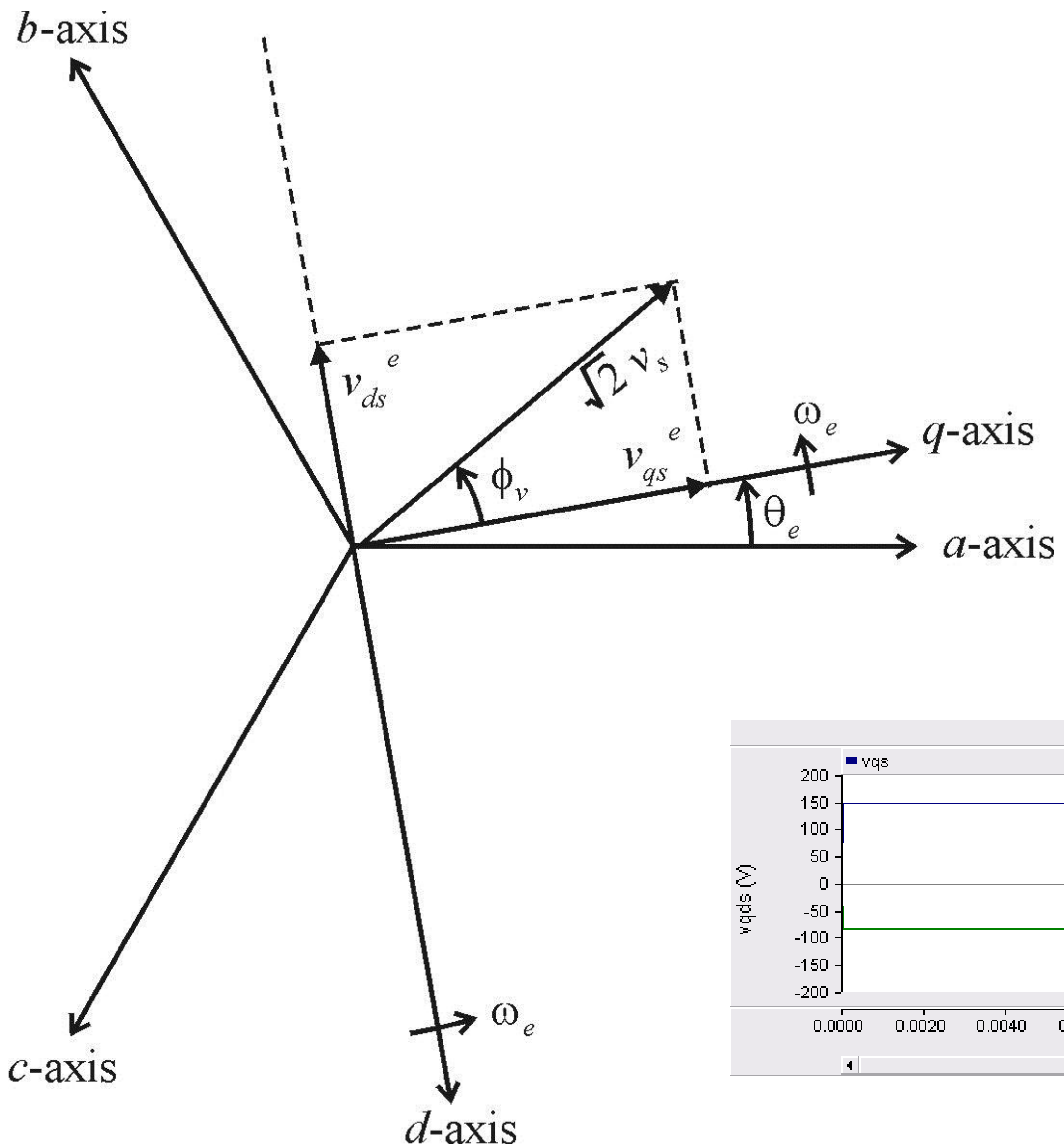



$$v_{qs}^{e} = \sqrt{2} V_s \cos(\phi_v) = 170\,\text{V}$$

$$v_{ds}^{e} = -\sqrt{2} V_s \sin(\phi_v) = 0$$

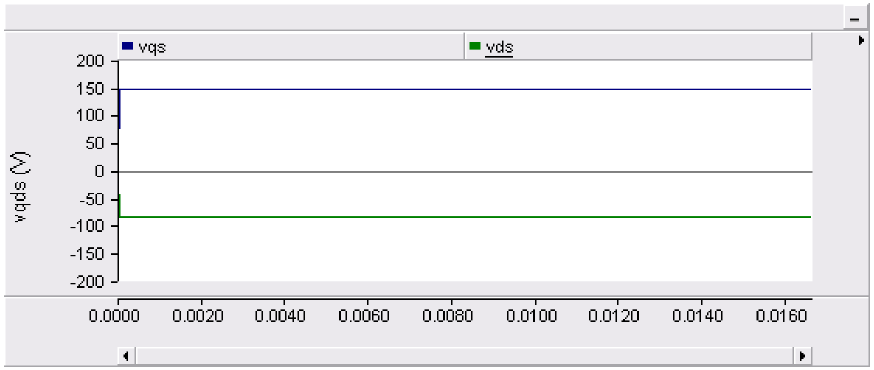


# Axis Sketch with $\theta=\theta_e$ and $\phi_v$=30º




$$v_{qs}^{e} = \sqrt{2}\,V_s \cos(\phi_v) = 147\,\mathrm{V}$$

$$v_{ds}^{e} = -\sqrt{2}\,V_s \sin(\phi_v) = -85\,\mathrm{V}$$



# Commonly Used Reference Frames

| | | | |
|---|---|---|---|
| Arbitrary | $\omega = \omega$ | $f_{qs}, f_{ds}, f_{0s}$ | $K_s$ |
| Stationary | $\omega = 0$ | $f_{qs}^s, f_{ds}^s, f_{0s}$ | $K_s^s$ |
| Synchronous | $\omega = \omega_e$ | $f_{qs}^e, f_{ds}^e, f_{0s}$ | $K_s^e$ |
| Rotor | $\omega = \omega_r$ | $f_{qs}^r, f_{ds}^r, f_{0s}$ | $K_s^r$ |

compact notation

$$v_{qd0s} = K_s v_{abcs}$$

$$v_{qd0s}^e = K_s^e v_{abcs}$$

Notes: In all reference frames $f_{0s} = \frac{1}{3}\left(f_{as} + f_{bs} + f_{cs}\right)$

The reference frame speed defines one reference frame. There are an infinite number since $\theta(0)$ can be set to any value.

# Frame-to-Frame Transformation

$$f_{qd0s}^{x} = K_s^x f_{abcs}$$

$${}^{x}K^{y} = \begin{bmatrix} \cos\left(\theta_y - \theta_x\right) & \sin\left(\theta_y - \theta_x\right) & 0 \\ \sin\left(\theta_y - \theta_x\right) & \cos\left(\theta_y - \theta_x\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

# Real Power in the *q-d* Reference Frame

$$P_{in} = v_{as}\, i_{as} + v_{bs}\, i_{bs} + v_{cs}\, i_{cs} = \begin{bmatrix} v_{as} & v_{bs} & v_{cs} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} = v_{abcs}^{T}\, i_{abcs}$$

$$P_{in} = \frac{3}{2}\left(v_{qs} i_{qs} + v_{ds} i_{ds}\right) + 3 v_{0s} i_{0s}$$

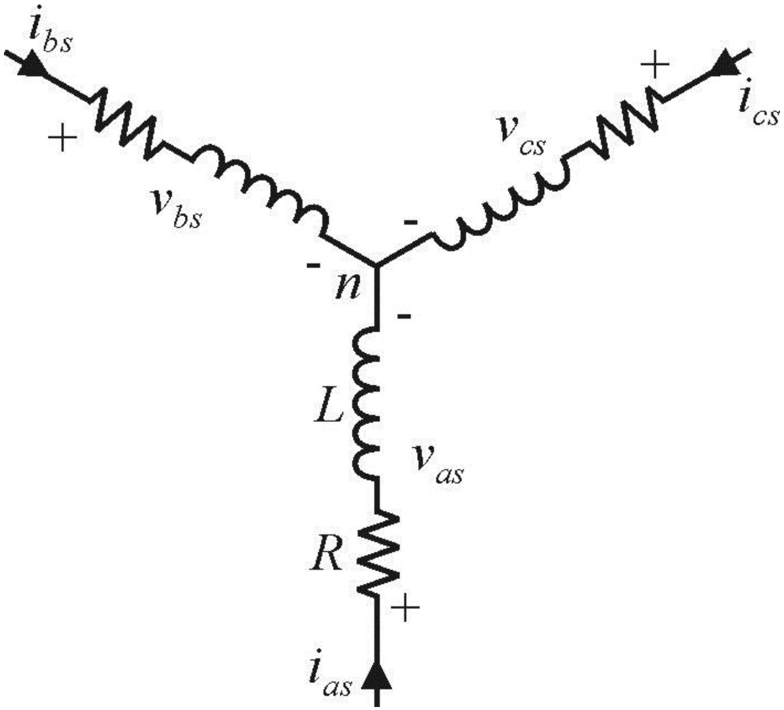
# Reference Frame Transformation

Developed by R.H. Park in 1929 for analysis of synchronous machines.

Allows treatment of balanced three-phase ac systems as two-phase dc systems.

- This leads to application of classical control theory
- Also simplifies control equations of some systems

# Transforming Circuit Elements: R-L Example




**voltage equations**

$$v_{as} = Ri_{as} + p\lambda_{as}$$

$$v_{bs} = Ri_{bs} + p\lambda_{bs}$$

$$v_{cs} = Ri_{cs} + p\lambda_{cs}$$

note: $p = \dfrac{d}{dt}$

**flux linkage equations**

$$\lambda_{as} = Li_{as}$$

$$\lambda_{bs} = Li_{bs}$$

$$\lambda_{cs} = Li_{cs}$$

# Transform Flux Linkage Equations

1. Compress equations

$$\lambda_{abcs} = Li_{abcs}$$

2. Transform equations

3. Expand equations

$$\lambda_{qs} = Li_{qs}$$

$$\lambda_{ds} = Li_{ds}$$

$$\lambda_{0s} = Li_{0s}$$

# Transform Voltage Equations

1. Compress equations

$$v_{abcs} = Ri_{abcs} + p\lambda_{abcs}$$

2. Transform equations

$$p\{K_s^{-1}\} = \omega \begin{bmatrix} -\sin(\theta) & \cos(\theta) & 0 \\ -\sin\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{2\pi}{3}\right) & 0 \\ -\sin\left(\theta + \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) & 0 \end{bmatrix}$$

$$K_s p\{K_s^{-1}\} = \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$v_{qd0s} = Ri_{qd0s} + p\lambda_{qd0s} + \omega\lambda_{dqs} \qquad \lambda_{dqs} = \begin{bmatrix} \lambda_{ds} \\ -\lambda_{qs} \\ 0 \end{bmatrix}$$

3. Expand equations

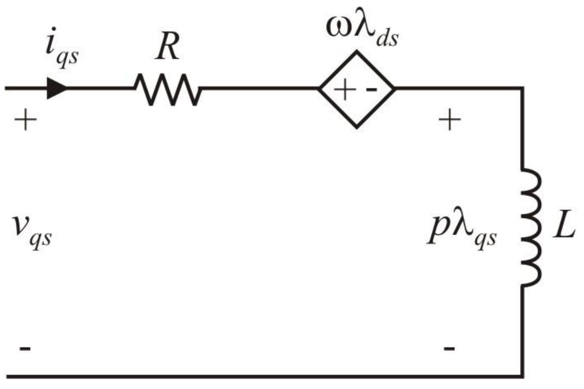
$$v_{qs} = Ri_{qs} + p\lambda_{qs} + \omega\lambda_{ds}$$

$$v_{ds} = Ri_{ds} + p\lambda_{ds} - \omega\lambda_{qs}$$

$$v_{0s} = Ri_{0s} + p\lambda_{0s}$$

# Equivalent Circuit

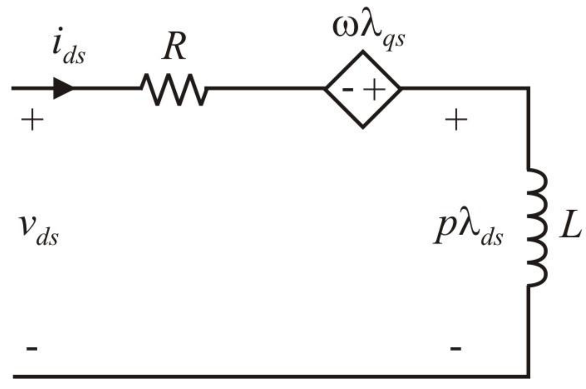
*q*-axis




$$v_{qs} = Ri_{qs} + p\lambda_{qs} + \omega\lambda_{ds}$$

$$\lambda_{qs} = Li_{qs}$$

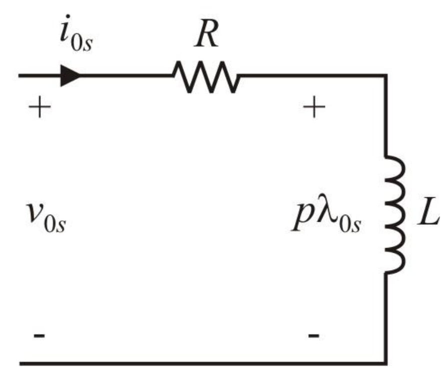
*d*-axis




$$v_{ds} = Ri_{ds} + p\lambda_{ds} - \omega\lambda_{qs}$$

$$\lambda_{ds} = Li_{ds}$$

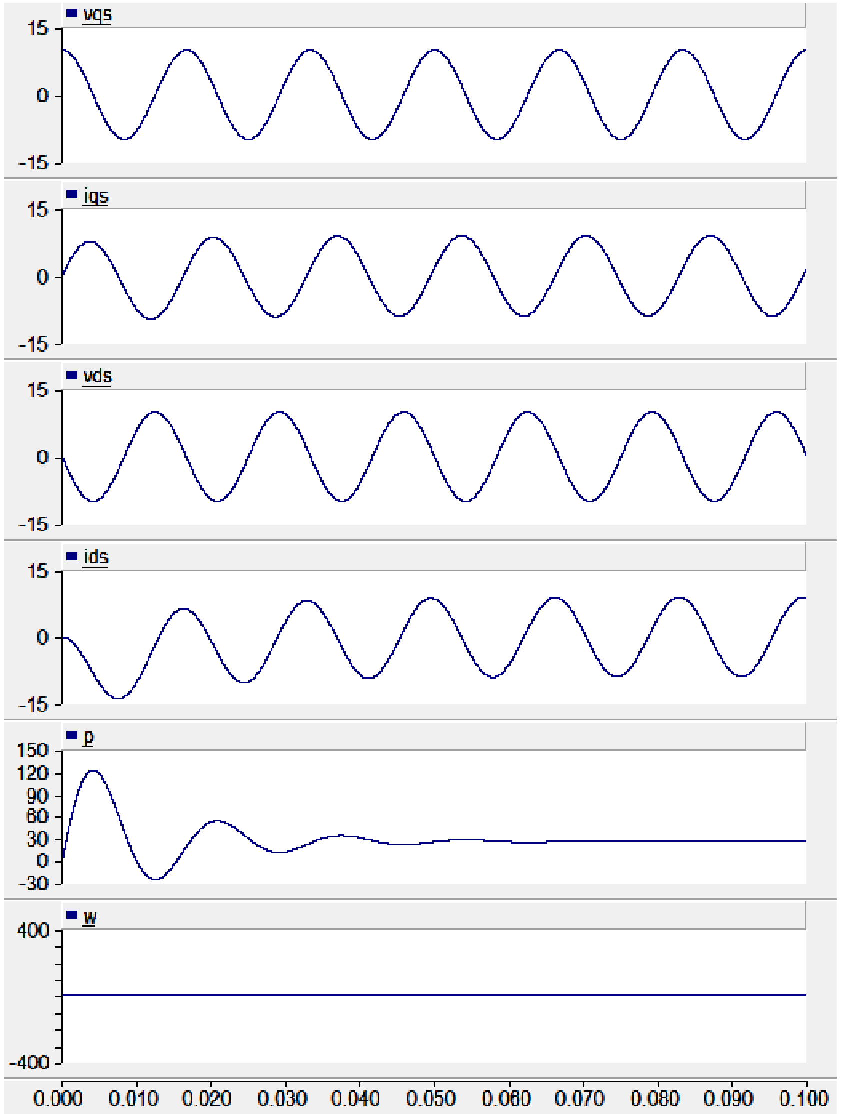
zero sequence




$$v_{0s} = Ri_{0s} + p\lambda_{0s}$$

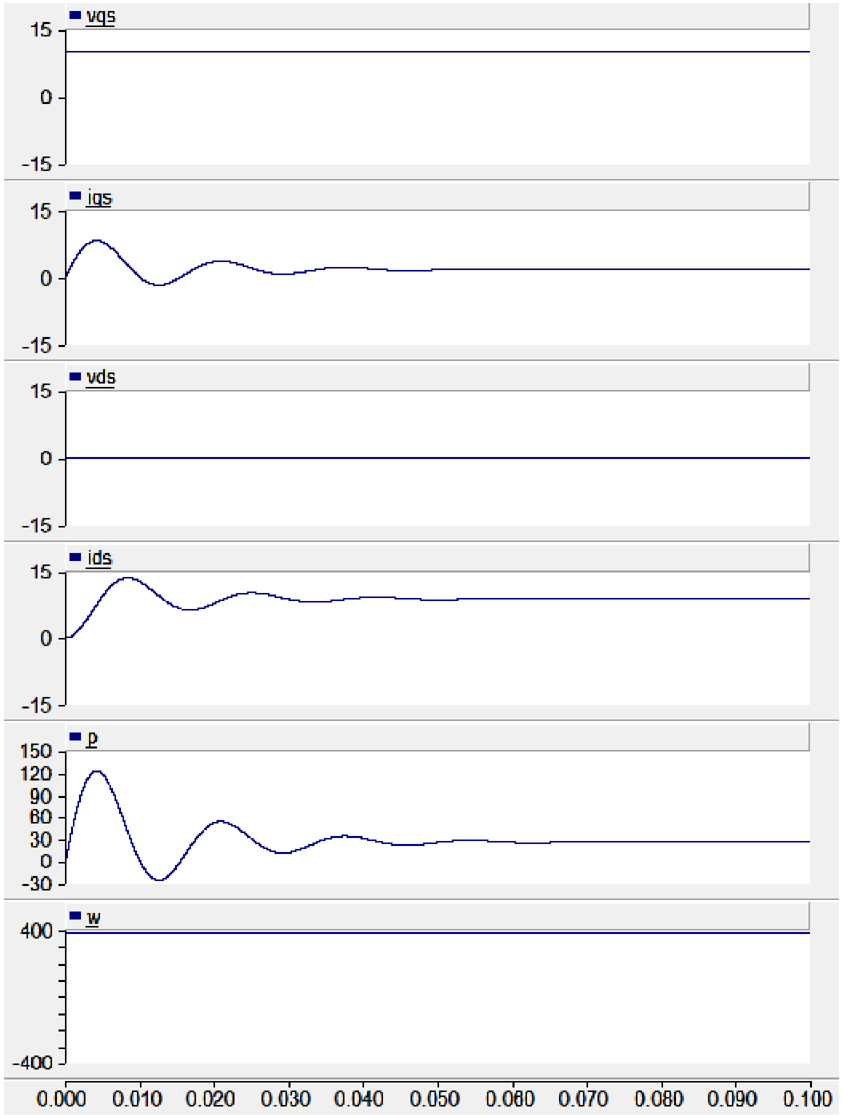
$$\lambda_{0s} = Li_{0s}$$

# R-L Example from Book Stationary Reference Frame



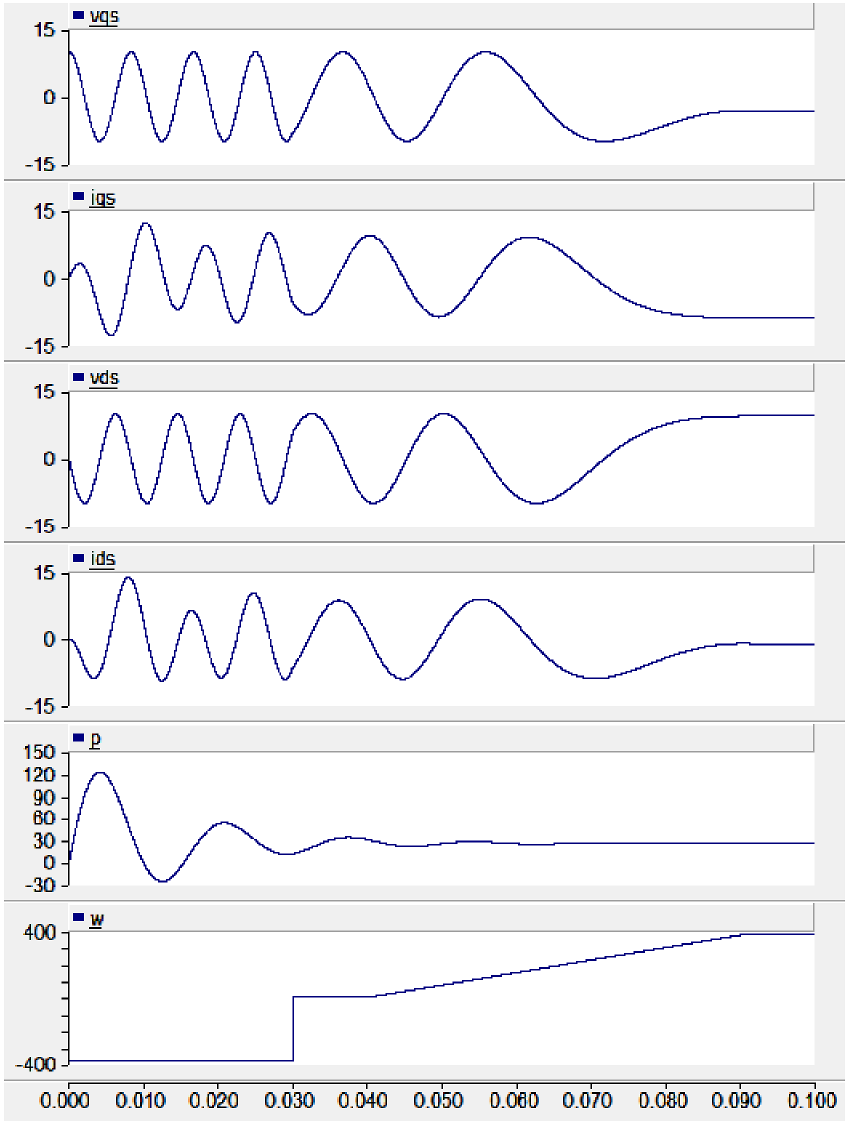

matches Figure 3.10-1

# R-L Example from Book
# Synchronous Reference Frame



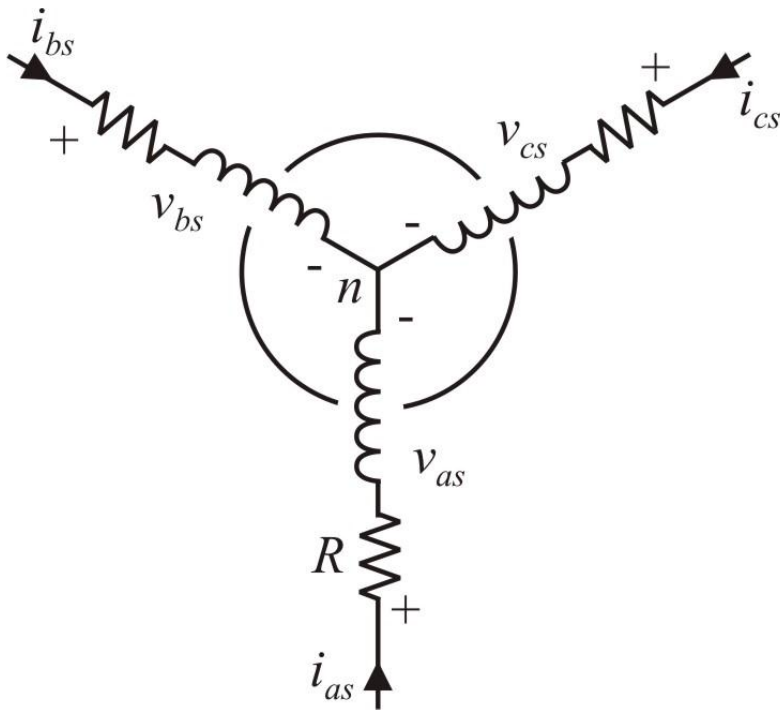

matches Figure 3.10-2

# R-L Example from Book Varying Reference Frame




matches Figure 3.10-3

# Coupled Inductors




voltage equations

$$v_{abcs} = Ri_{abcs} + p\lambda_{abcs}$$

$$v_{qd0s} = Ri_{qd0s} + p\lambda_{qd0s} + \omega\lambda_{dqs} \qquad \lambda_{dqs} = \begin{bmatrix} \lambda_{ds} \\ -\lambda_{qs} \\ 0 \end{bmatrix}$$

# Coupled Inductors

flux linkage equations

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix} = \begin{bmatrix} L_{ls} + L_{ms} & -\frac{1}{2} L_{ms} & -\frac{1}{2} L_{ms} \\ -\frac{1}{2} L_{ms} & L_{ls} + L_{ms} & -\frac{1}{2} L_{ms} \\ -\frac{1}{2} L_{ms} & -\frac{1}{2} L_{ms} & L_{ls} + L_{ms} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}$$

$$K_s \mathbf{L}_s K_s^{-1} = \begin{bmatrix} L_{ls} + \frac{3}{2} L_{ms} & 0 & 0 \\ 0 & L_{ls} + \frac{3}{2} L_{ms} & 0 \\ 0 & 0 & L_{ls} \end{bmatrix}$$

expanded form

$$\lambda_{qs} = \left( L_{ls} + \frac{3}{2} L_{ms} \right) i_{qs}$$

$$\lambda_{ds} = \left( L_{ls} + \frac{3}{2} L_{ms} \right) i_{ds}$$

$$\lambda_{0s} = L_{ls} i_{0s}$$

# Transformation of Circuit Elements

Balanced three-phase ac circuits transformed to two-phase dc circuits (neglecting the zero sequence and assuming analysis in the synchronous reference frame)

Flux linkage equations for inductive circuits were used for generality to other circuits; including electric machines

Coupling terms between the *q*- and *d*-axes result from the transformation.  These will later be viewed as back-emf terms when observing electric machinery in the synchronous reference frame.

# Balanced Steady-State Calculations

$$\theta_e = \int_0^t \omega_e(\varsigma)\,d\varsigma + \theta_e(0)$$

define $\phi_v = \theta_e(0)$

$$= \omega_e t + \phi_v$$

$$\theta = \int_0^t \omega(\varsigma)\,d\varsigma + \theta(0)$$

$$= \omega t + \theta(0)$$

# Balanced Steady-State Voltages

$$v_{as} = \sqrt{2}V_s \cos\left(\omega_e t + \phi_v\right)$$

$$v_{bs} = \sqrt{2}V_s \cos\left(\omega_e t + \phi_v - \frac{2\pi}{3}\right)$$

$$v_{cs} = \sqrt{2}V_s \cos\left(\omega_e t + \phi_v + \frac{2\pi}{3}\right)$$

$$\tilde{V}_{as} = V_s e^{j\phi_v} \longrightarrow \tilde{V}_{as} = V_s \angle \phi_v$$

$$\tilde{V}_{bs} = \tilde{V}_s e^{-j\frac{2\pi}{3} + j\phi_v}$$

$$\tilde{V}_{cs} = \tilde{V}_s e^{j\frac{2\pi}{3} + j\phi_v}$$

# Synchronous Reference Frame *q-d* Voltages

Synchronous: $\omega = \omega_e$ and $\theta(0) = 0$

$$V_{qs}^{e} = \sqrt{2}V_s \cos\left(\phi_v\right)$$

$$V_{ds}^{e} = -\sqrt{2}V_s \sin\left(\phi_v\right)$$

note:

$$\sqrt{2}\,V_s = \sqrt{\left(V_{qs}^{e}\right)^2 + \left(V_{ds}^{e}\right)^2}$$

$$\phi_v = \tan^{-1}\left(\frac{-V_{ds}^{e}}{V_{qs}^{e}}\right)$$

# R-L Circuit Example

$R := 1\cdot\Omega$ $\quad L := 10\cdot mH$ $\quad f := 60\cdot Hz$ $\quad V_s := 120\cdot V$ $\quad \phi_v := 30\cdot deg$

**load impedance**

$\omega_e := 2\cdot\pi\cdot f$

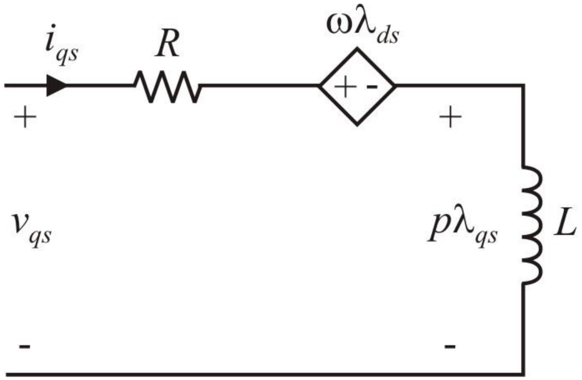
$Z := R + j\cdot\omega_e\cdot L$

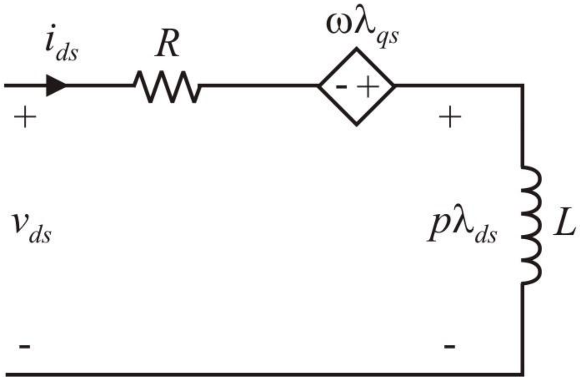
**solution in a-b-c**

$V_{as} := V_s\cdot e^{j\cdot\phi_v}$

$I_{as} := \frac{V_{as}}{Z}$

# Steady-State Calculation ($\omega = \omega_e$)




**solution in q-d**

$$V_{qse} := \sqrt{2} \cdot V_s \cdot \cos(\phi_V)$$

$$V_{dse} := -\sqrt{2} \cdot V_s \cdot \sin(\phi_V)$$

steady-state equations

$$V_{qs}^e = RI_{qs}^e + \omega_e LI_{ds}^e$$

$$V_{ds}^e = RI_{ds}^e - \omega_e LI_{qs}^e$$

steady-state equations in matrix form

$$\begin{bmatrix} V_{qs}^e \\ V_{ds}^e \end{bmatrix} = \begin{bmatrix} R & \omega_e L \\ -\omega_e L & R \end{bmatrix} \begin{bmatrix} I_{qs}^e \\ I_{ds}^e \end{bmatrix}$$

solve for currents

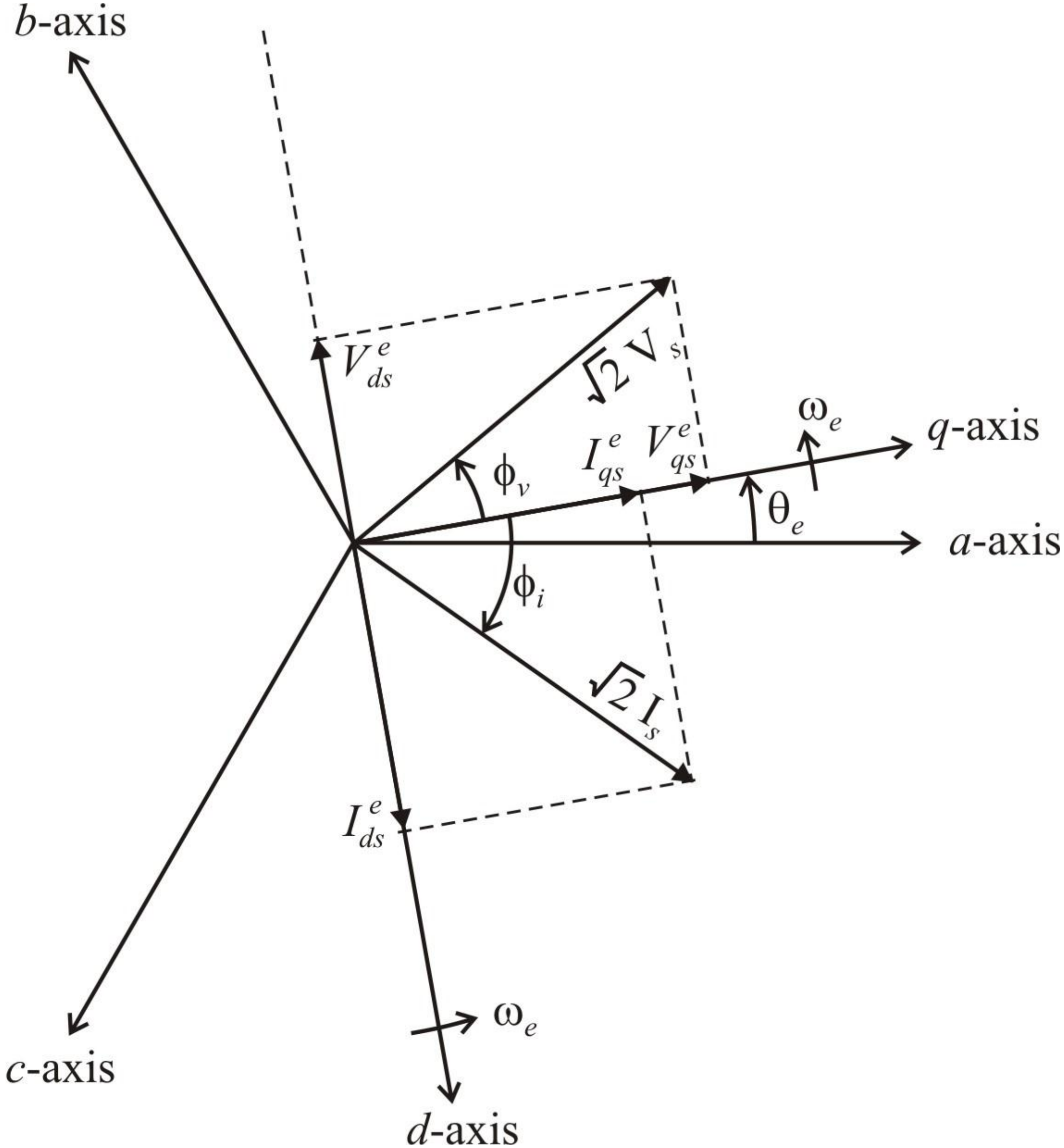
$$\begin{bmatrix} I_{qs}^e \\ I_{ds}^e \end{bmatrix} = \frac{1}{R^2 + \omega_e{}^2 L^2} \begin{bmatrix} R & -\omega_e L \\ \omega_e L & R \end{bmatrix} \begin{bmatrix} V_{qs}^e \\ V_{ds}^e \end{bmatrix}$$

$$\begin{pmatrix} I_{qse} \\ I_{dse} \end{pmatrix} := \begin{pmatrix} R & \omega_e \cdot L \\ -\omega_e \cdot L & R \end{pmatrix}^{-1} \cdot \begin{pmatrix} V_{qse} \\ V_{dse} \end{pmatrix}$$

$$I_s := \frac{1}{\sqrt{2}} \cdot \sqrt{I_{qse}{}^2 + I_{dse}{}^2}$$

$$\phi_i := \operatorname{atan}\left( \frac{-I_{dse}}{I_{qse}} \right)$$

# Voltage and Current Vectors



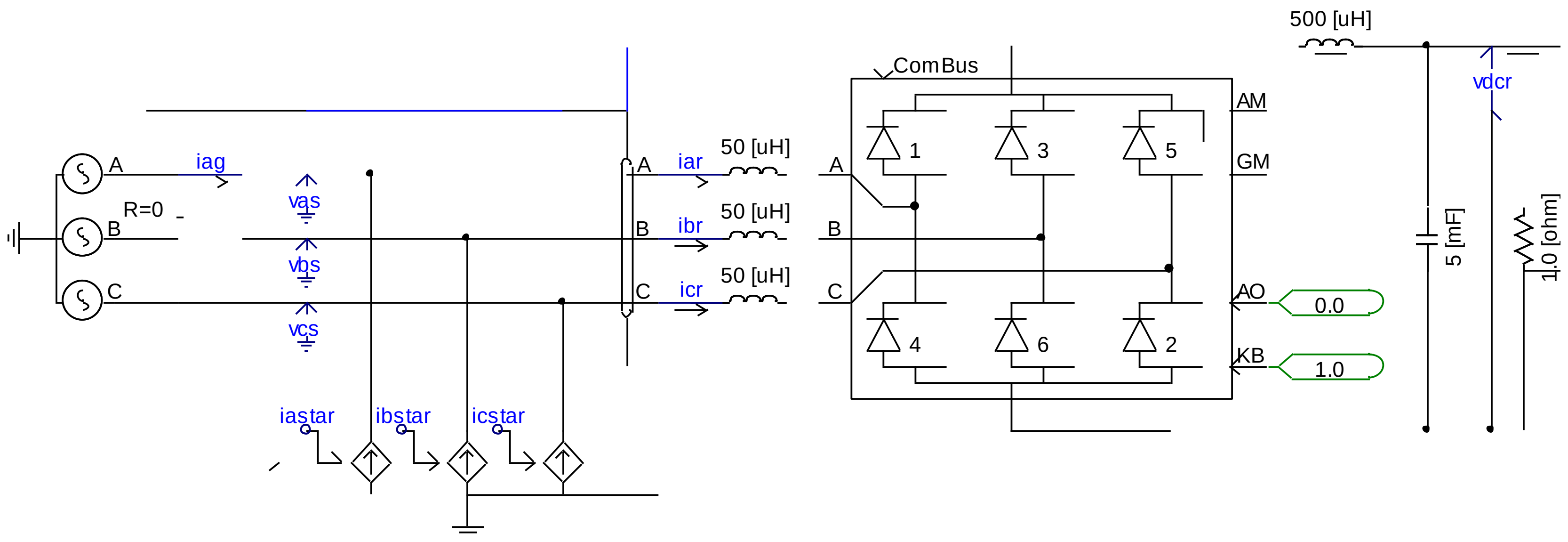
# Steady-State *q-d* Calculations

In the synchronous reference frame, the *q-d* circuits are supplied from dc and the corresponding dc solution is steady-state (inductors treated as short-circuit, capacitors treated as open-circuit)

Can be used to analyze steady-state operation of electric machines

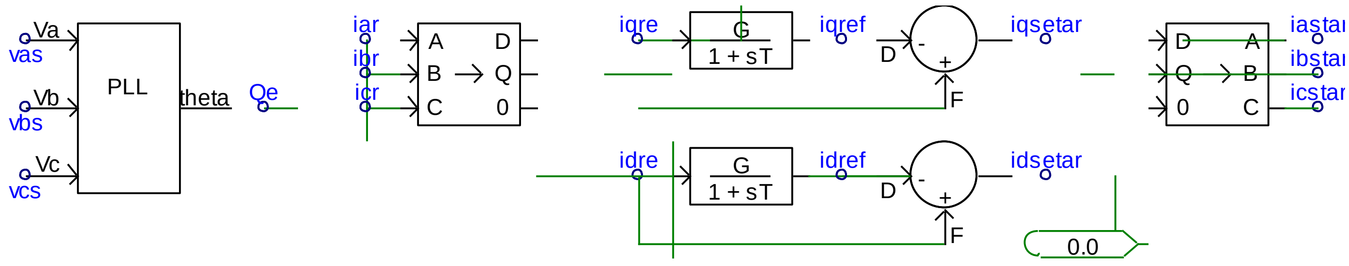
Equations can be linearized about the dc operating point for application of control theory

# Active Filter Example

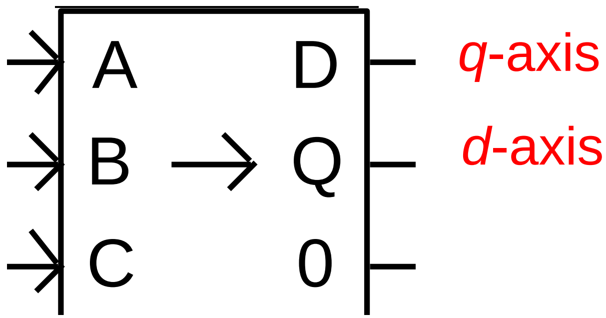



Ref_ActiveFilter.psc

# Active Filter Control



# PSCAD Transformation Block





abc to dq0:

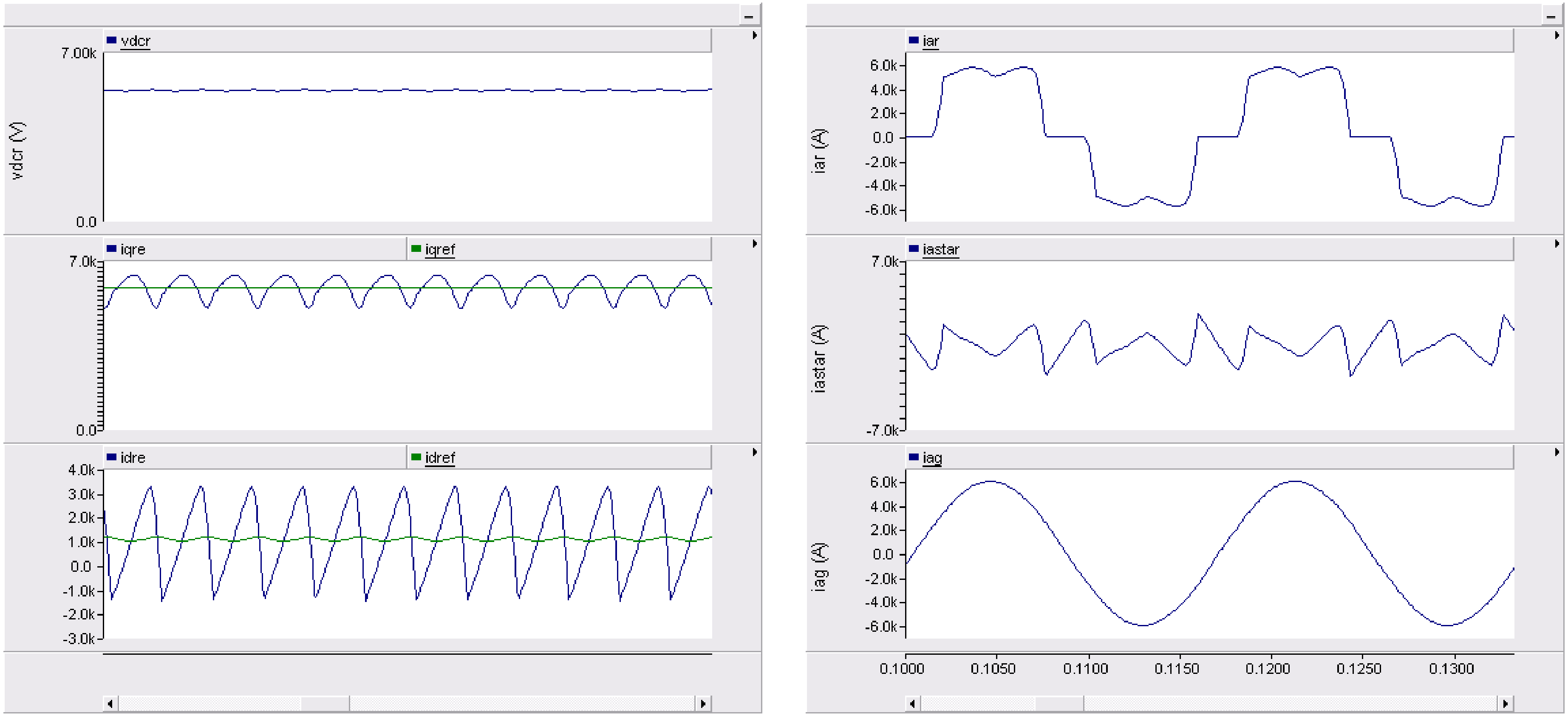
$$\begin{bmatrix} d \\ q \\ 0 \end{bmatrix} = \frac{2}{3} \cdot \begin{bmatrix} \cos(\theta) & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ \sin(\theta) & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

dq0 to abc:

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 1 \\ \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\theta + \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) & 1 \end{bmatrix} \cdot \begin{bmatrix} d \\ q \\ 0 \end{bmatrix}$$

- *q*- and *d*-axis swapped as compared to the notation in Krause's book
- double-click to set reference frame angle, which may also be a variable

# Active Filter Waveforms



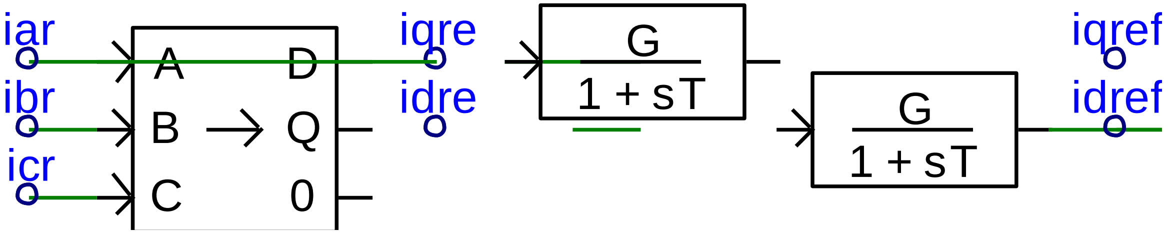
# Active Filter Example

Common in higher power applications where the main rectifier contains significant harmonics

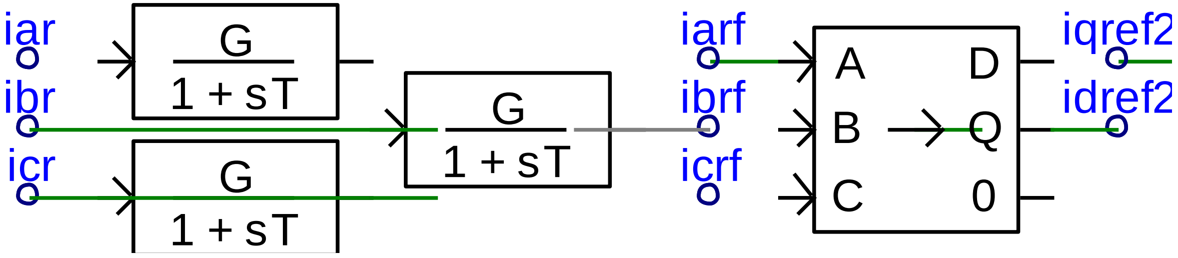
Demonstrates an effective control method using the synchronous reference frame

# Passive Filter Example

transform, then filter




filter, then transform




Is there a difference?

# The Low-Pass Filter (LPF)

$$x \rightarrow \boxed{\frac{G}{1 + sT}} \rightarrow y$$

$$G = 1 \qquad T = \tau$$

transfer function

$$\frac{y}{x} = \frac{1}{\tau s + 1}$$

time constant $\tau = \dfrac{1}{\omega_c}$

cut-off frequency $\omega_c = 2\pi f_c$

LPF gain

$$\left|\frac{y}{x}\right| = \frac{1}{\sqrt{\tau^2 \omega^2 + 1}} = \frac{1}{\sqrt{\left(\dfrac{f}{f_c}\right)^2 + 1}}$$

LPF phase delay

$$\angle y - \angle x = -\tan^{-1}(\tau\,\omega) = -\tan^{-1}\left(\frac{f}{f_c}\right)$$

# Transform, Then Filter

filter output $\tilde{i}_{qd0r}^{e}$

$$\frac{\tilde{i}_{qd0r}^{e}}{i_{qd0r}^{e}} = \frac{1}{\tau s + 1}$$

$$\tilde{i}_{qd0r}^{e} + \tau p \tilde{i}_{qd0r}^{e} = i_{qd0r}^{e}$$

steady-state

$$\tilde{I}_{qd0r}^{e} = I_{qd0r}^{e}$$

# Filter, Then Transform

$$\frac{\tilde{i}_{abcr}}{i_{abcr}} = \frac{1}{\tau s + 1}$$

$$\tilde{i}_{abcr} + \tau p \tilde{i}_{abcr} = i_{abcr}$$

$$K_s^e \tilde{i}_{abcr} + K_s^e \tau p \tilde{i}_{abcr} = K_s^e i_{abcr}$$

$$\downarrow$$

$$\tilde{i}_{qd0r}^e + \tau K_s^e p \left\{ \left( K_s^e \right)^{-1} \tilde{i}_{qd0r}^e \right\} = i_{qd0r}^e$$

$$\tilde{i}_{qd0r}^e + \tau K_s^e \left( K_s^e \right)^{-1} p \tilde{i}_{qd0r}^e + \tau K_s^e p \left\{ \left( K_s^e \right)^{-1} \right\} \tilde{i}_{qd0r}^e = i_{qd0r}^e$$

$$\tilde{i}_{qd0r}^e + \tau p \tilde{i}_{qd0r}^e + \omega_e \tau \tilde{i}_{dqr}^e = i_{qd0r}^e \qquad \tilde{i}_{dqr}^e = \begin{bmatrix} \tilde{i}_{dr}^e \\ -\tilde{i}_{qr}^e \\ 0 \end{bmatrix}$$

# Filter, Then Transform (Steady-State)

steady-state

$$\tilde{I}^{e}_{qd0r} + \omega_e \tau \, \tilde{I}^{e}_{dqr} = I^{e}_{qd0r}$$

$$\tilde{I}^{e}_{dqr} = \begin{bmatrix} \tilde{I}^{e}_{dr} \\ -\tilde{I}^{e}_{qr} \\ 0 \end{bmatrix}$$

expand equations

$$\tilde{I}^{e}_{qr} + \omega_e \tau \tilde{I}^{e}_{dr} = I^{e}_{qr}$$

$$\tilde{I}^{e}_{dr} - \omega_e \tau \tilde{I}^{e}_{qr} = I^{e}_{dr}$$

in matrix form

$$\begin{bmatrix} 1 & \omega_e \tau \\ -\omega_e \tau & 1 \end{bmatrix} \begin{bmatrix} \tilde{I}^{e}_{qr} \\ \tilde{I}^{e}_{dr} \end{bmatrix} = \begin{bmatrix} I^{e}_{qr} \\ I^{e}_{dr} \end{bmatrix}$$

$$\begin{bmatrix} \tilde{I}^{e}_{qr} \\ \tilde{I}^{e}_{dr} \end{bmatrix} = \begin{bmatrix} 1 & \omega_e \tau \\ -\omega_e \tau & 1 \end{bmatrix}^{-1} \begin{bmatrix} I^{e}_{qr} \\ I^{e}_{dr} \end{bmatrix} = \frac{\begin{bmatrix} 1 & -\omega_e \tau \\ \omega_e \tau & 1 \end{bmatrix} \begin{bmatrix} I^{e}_{qr} \\ I^{e}_{dr} \end{bmatrix}}{1 + \omega_e^2 \tau^2}$$

Note:

$$\omega_e \tau = \frac{f}{f_c}$$

# Passive Filter Example

$f := 60 \cdot Hz$

$\omega_e := 2 \cdot \pi \cdot f$

$\tau := 10 \cdot ms$

$\omega_c := \frac{1}{\tau}$

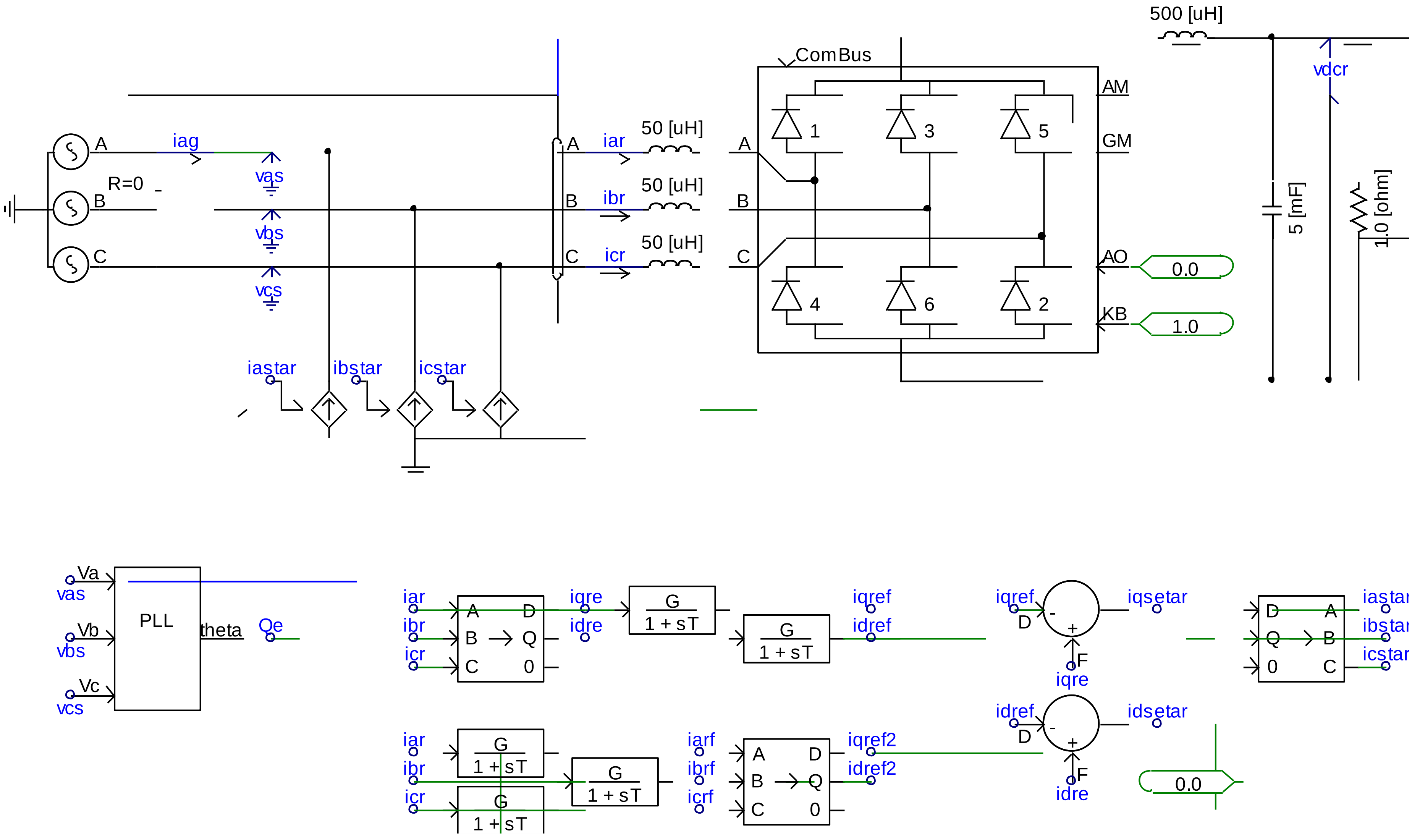
$f_c := \frac{\omega_c}{2 \cdot \pi}$

**transform, then filter (from PSCAD)**

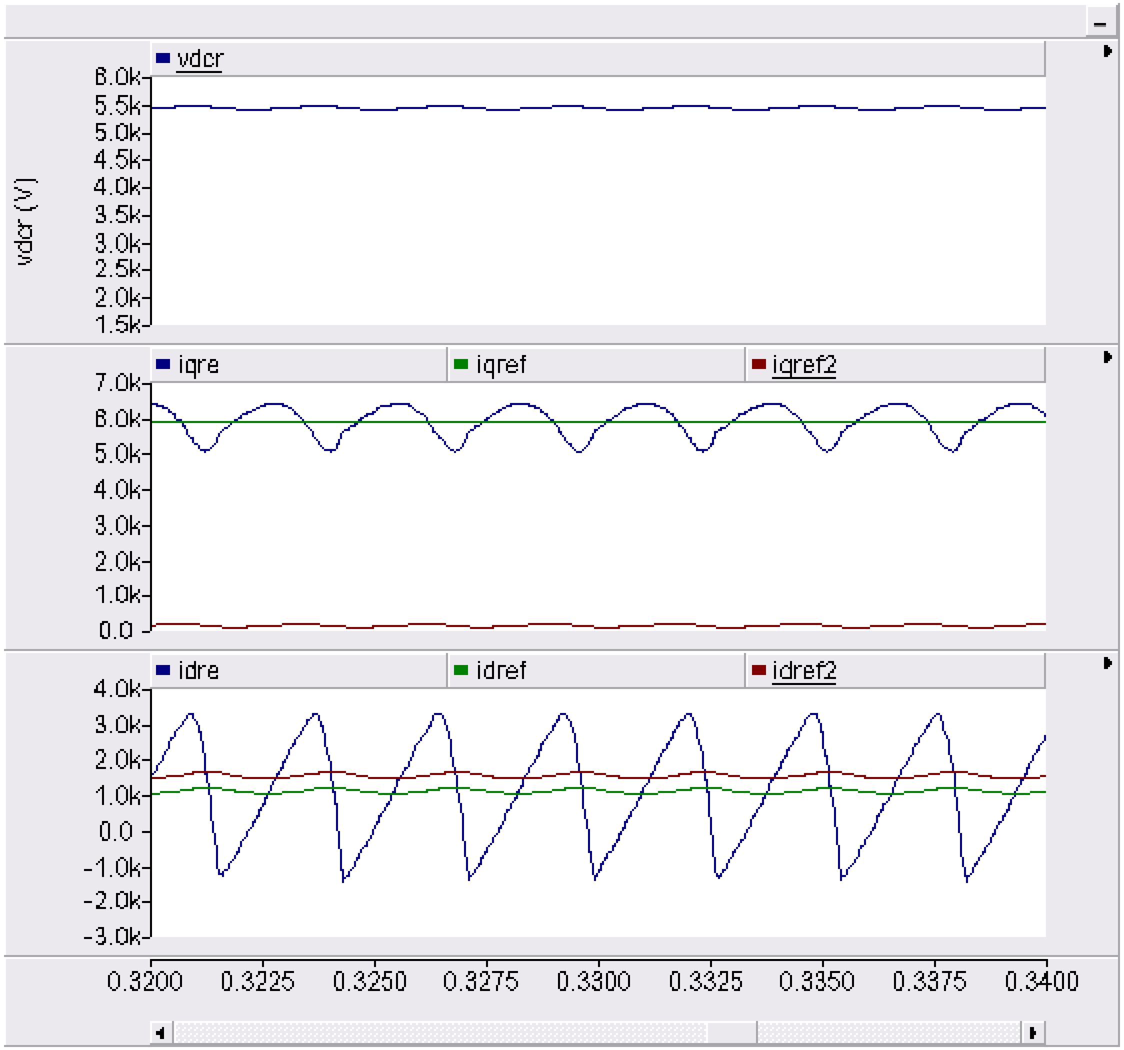
**filter, then transform**

$$\begin{pmatrix} I_{qsef} \\ I_{dsef} \end{pmatrix} := \begin{pmatrix} 1 & \omega_e \cdot \tau \\ -\omega_e \cdot \tau & 1 \end{pmatrix}^{-1} \cdot \begin{pmatrix} I_{qse} \\ I_{dse} \end{pmatrix}$$

# PSCAD Simulation



# PSCAD Waveforms



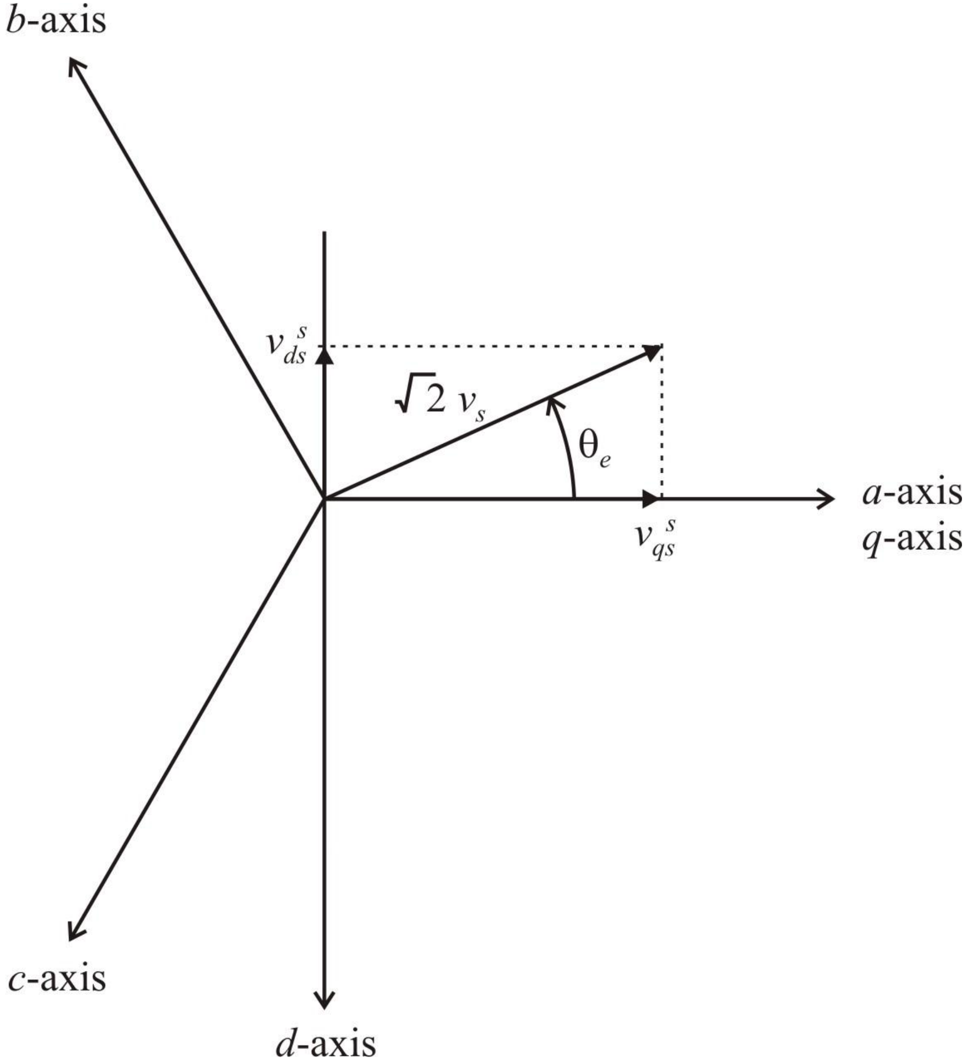
# Passive Filter Example

Illustrates the low-pass filter (LPF)

Shows difference between placing a passive filter before and after a transformation to the synchronous reference frame

For accurate steady-state numbers, the LPF must be placed after the transformation to the synchronous reference frame

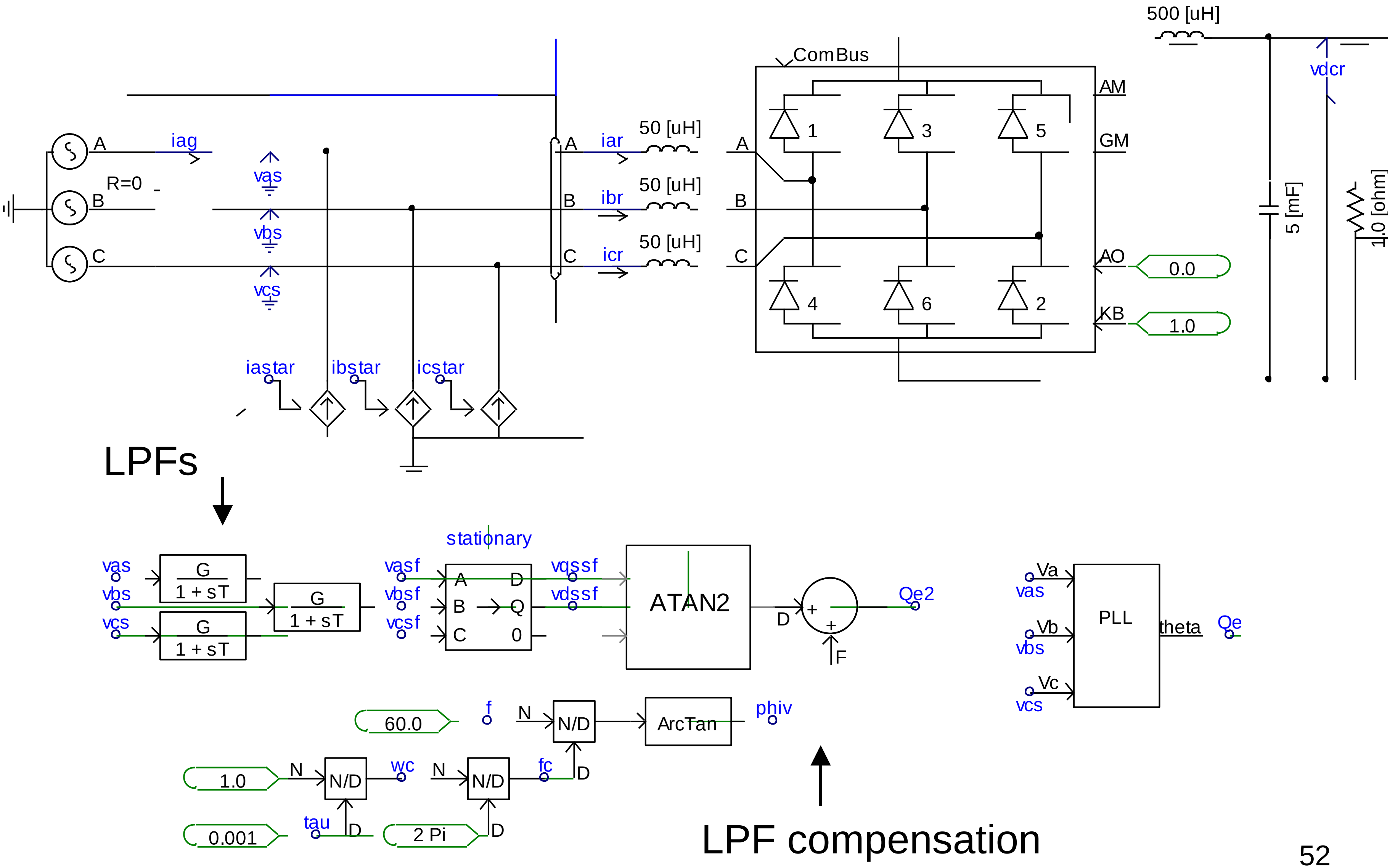
# Phase-Locked Loop (PLL) Example



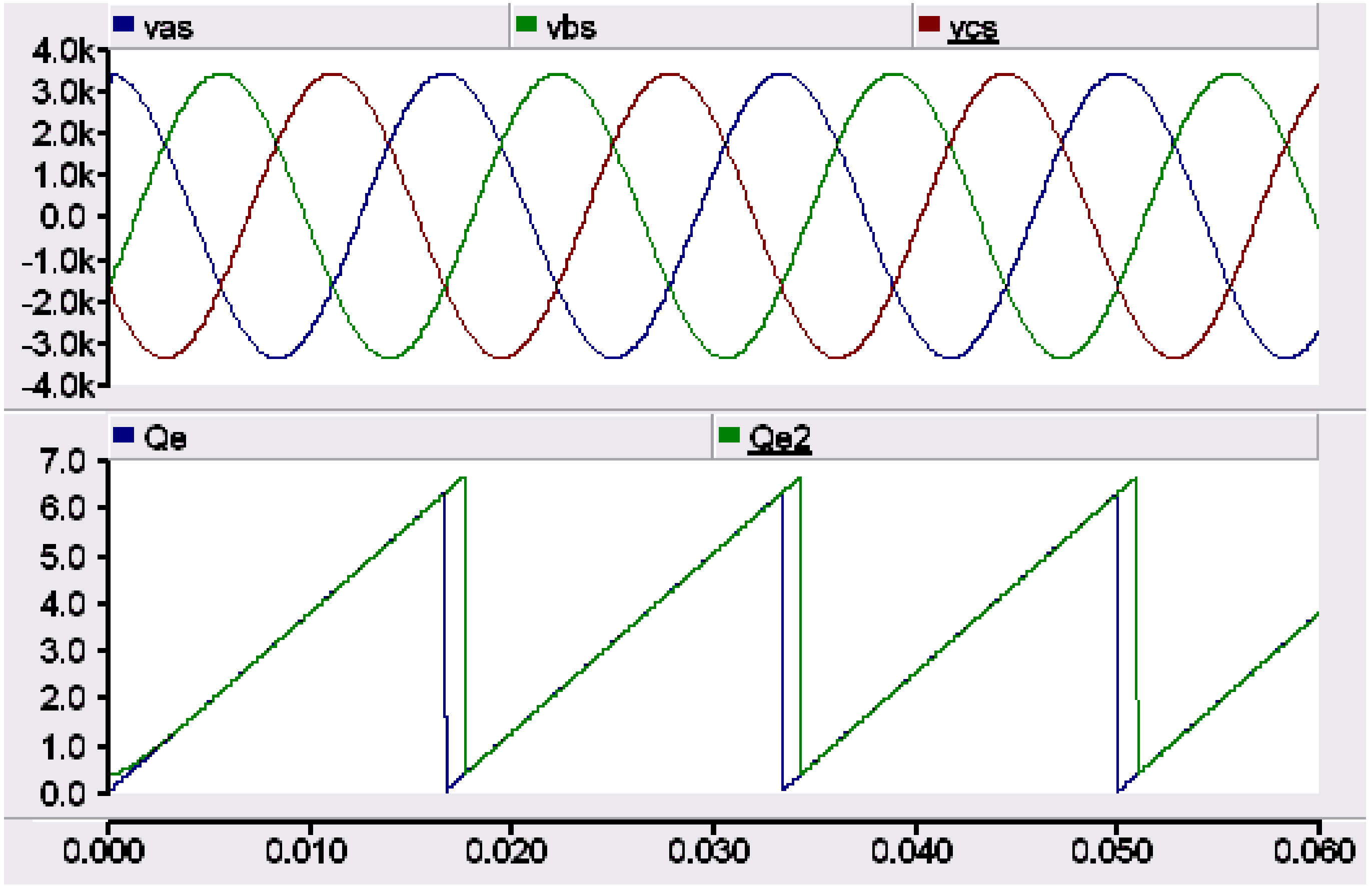

The electrical angle can be observed by using the stationary reference frame voltages

$$\theta_e = \tan^{-1}\left(\frac{-v_{ds}^s}{v_{qs}^s}\right)$$

# PLL Simulation



# PLL Simulation Waveforms



# PLL Example

PLL obtains electrical angle from processing the measured voltages (without a PI control loop)

Using this method, filtering of the line voltages may be required. The PLL angle can be adjusted to compensate for filter delay.