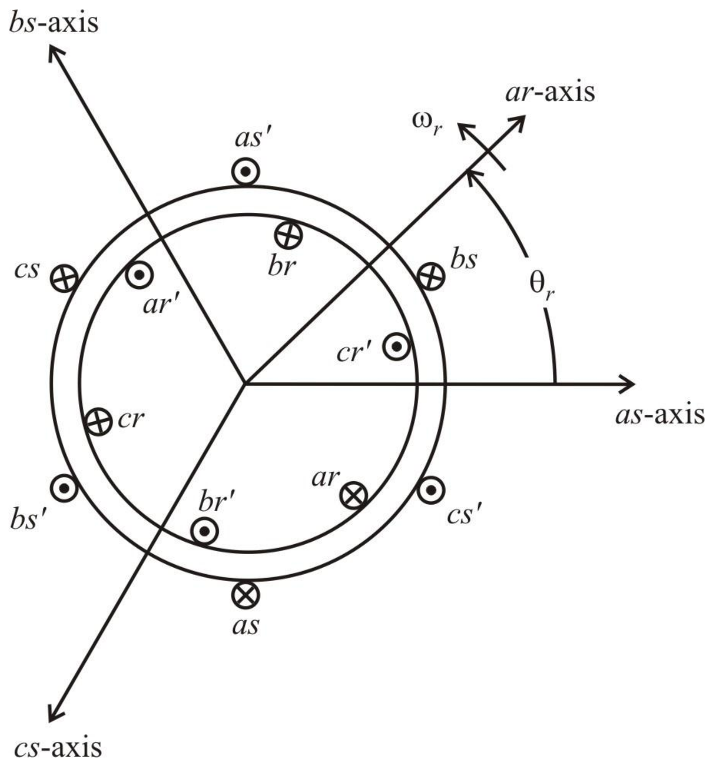
CLEMSON UNIVERSITY

ECE 802, Electric Motor Control

Induction Machines

Symmetrical Induction Machines (Chapter 6)



voltage equations

$$v_{abcs} = r_s i_{abcs} + p\lambda_{abcs}$$

$$v_{abcr} = r_r i_{abcr} + p\lambda_{abcr}$$

typically wye connected windings

Flux Linkage Equations

$$\begin{bmatrix} \lambda_{abcs} \\ \lambda_{abcr} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_s & \mathbf{L}_{sr} \\ \mathbf{L}_{rs} & \mathbf{L}_r \end{bmatrix} \begin{bmatrix} i_{abcs} \\ i_{abcr} \end{bmatrix}$$

$$\mathbf{L}_s = \begin{bmatrix} L_{ls} + L_{ms} & -\frac{1}{2} L_{ms} & -\frac{1}{2} L_{ms} \\ -\frac{1}{2} L_{ms} & L_{ls} + L_{ms} & -\frac{1}{2} L_{ms} \\ -\frac{1}{2} L_{ms} & -\frac{1}{2} L_{ms} & L_{ls} + L_{ms} \end{bmatrix}$$

$$L_{ls} = \frac{N_s^2}{\Re_{ls}}$$

$$L_{ms} = \frac{N_s^2}{\Re_m}$$

Rotor Inductances

$$\mathbf{L}_r = \begin{bmatrix} L_{lr} + L_{mr} & -\frac{1}{2}L_{mr} & -\frac{1}{2}L_{mr} \\ -\frac{1}{2}L_{mr} & L_{lr} + L_{mr} & -\frac{1}{2}L_{mr} \\ -\frac{1}{2}L_{mr} & -\frac{1}{2}L_{mr} & L_{lr} + L_{mr} \end{bmatrix}$$

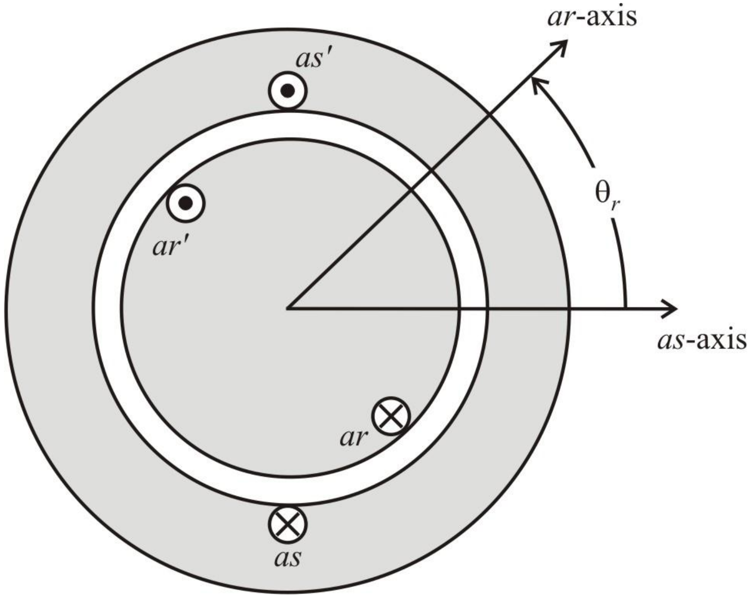
$$L_{lr} = \frac{N_r^2}{\Re_{lr}}$$

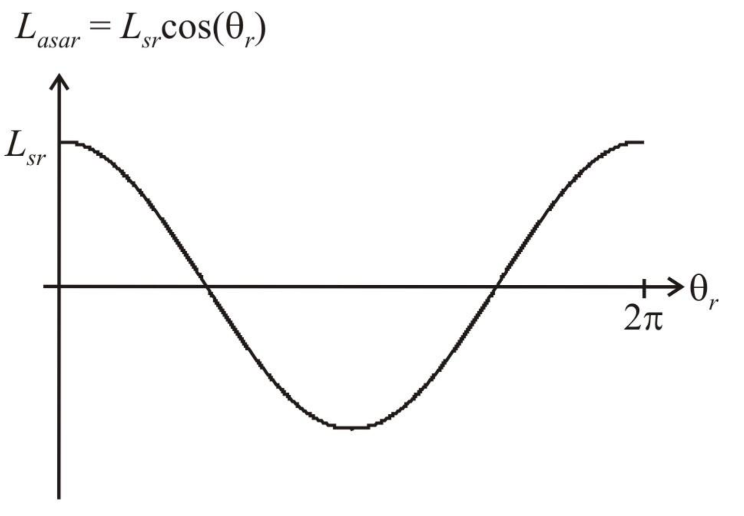
$$L_{mr} = \frac{N_r^2}{\Re_m}$$

Stator-to-Rotor Inductances

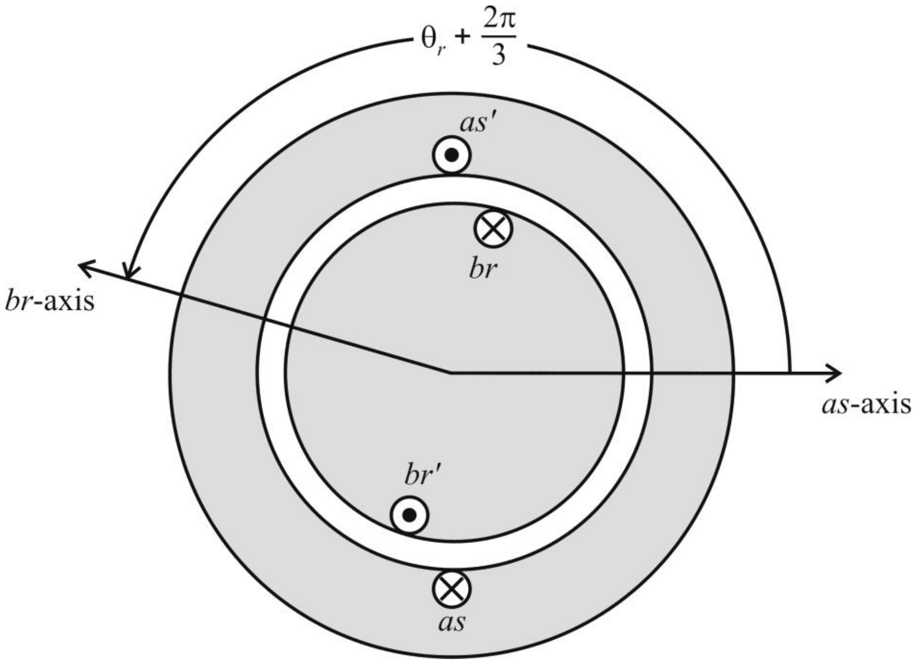
$$\mathbf{L}_{sr} = \begin{bmatrix} L_{asar} & L_{asbr} & L_{ascr} \\ L_{bsar} & L_{bsbr} & L_{bscr} \\ L_{csar} & L_{csbr} & L_{cscr} \end{bmatrix}$$

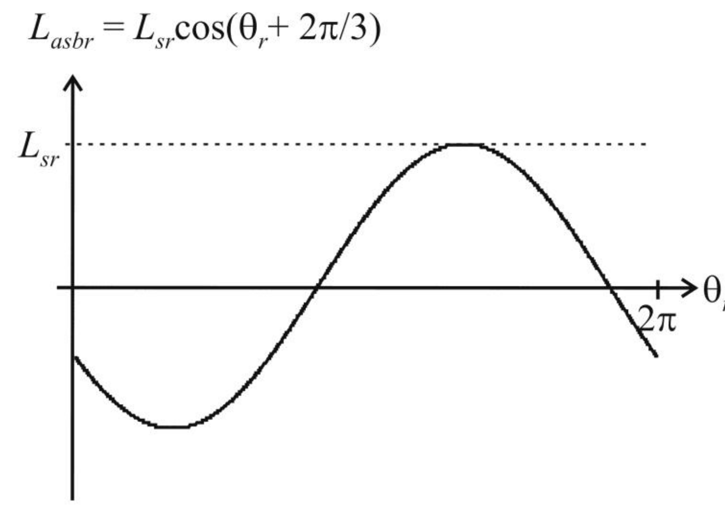
L_{asar} - component of λ_{as} due to i_{ar}





L_{asbr} - component of λ_{as} due to i_{br}





$$\mathbf{L}_{sr} = L_{sr}\begin{bmatrix} \cos(\theta_r) & \cos\left(\theta_r + \frac{2\pi}{3}\right) & \cos\left(\theta_r - \frac{2\pi}{3}\right) \\ \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos(\theta_r) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\ \cos\left(\theta_r + \frac{2\pi}{3}\right) & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos(\theta_r) \end{bmatrix}$$

$$L_{sr} = \frac{N_s N_r}{\Re_m}$$

$$\mathbf{L}_{rs} = \left(\mathbf{L}_{sr}\right)^{\mathrm{T}}$$

Refer Rotor Quantities to Stator

Put all magnetizing inductances in terms of L_{ms} (Steinmetz model)

$$\begin{bmatrix} \lambda_{abcs} \\ \dfrac{N_s}{N_r}\lambda_{abcr} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_s & \dfrac{N_s}{N_r}\mathbf{L}_{sr} \\ \dfrac{N_s}{N_r}\left(\mathbf{L}_{sr}\right)^T & \left(\dfrac{N_s}{N_r}\right)^2 \mathbf{L}_r \end{bmatrix} \begin{bmatrix} i_{abcs} \\ \dfrac{N_r}{N_s} i_{abcr} \end{bmatrix}$$

define $\lambda'_{abcr} = \dfrac{N_s}{N_r}\lambda_{abcr}$ $\quad i'_{abcr} = \dfrac{N_r}{N_s} i_{abcr}$ $\quad \mathbf{L}'_{sr} = \dfrac{N_s}{N_r}\mathbf{L}_{sr}$ $\quad \mathbf{L}'_r = \left(\dfrac{N_s}{N_r}\right)^2 \mathbf{L}_r$

$$\begin{bmatrix} \lambda_{abcs} \\ \lambda'_{abcr} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_s & \mathbf{L}'_{sr} \\ \left(\mathbf{L}'_{sr}\right)^{\mathrm{T}} & \mathbf{L}'_r \end{bmatrix} \begin{bmatrix} i_{abcs} \\ i'_{abcr} \end{bmatrix}$$

$$\mathbf{L}'_{sr} = L_{ms} \begin{bmatrix} \cos(\theta_r) & \cos\left(\theta_r + \frac{2\pi}{3}\right) & \cos\left(\theta_r - \frac{2\pi}{3}\right) \\ \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos(\theta_r) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\ \cos\left(\theta_r + \frac{2\pi}{3}\right) & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos(\theta_r) \end{bmatrix}$$

$$
\mathbf{L}'_r = \begin{bmatrix}
\left(\frac{N_s}{N_r}\right)^2 L_{lr} + L_{ms} & -\frac{1}{2} L_{ms} & -\frac{1}{2} L_{ms} \\
-\frac{1}{2} L_{ms} & \left(\frac{N_s}{N_r}\right)^2 L_{lr} + L_{ms} & -\frac{1}{2} L_{ms} \\
-\frac{1}{2} L_{ms} & -\frac{1}{2} L_{ms} & \left(\frac{N_s}{N_r}\right)^2 L_{lr} + L_{ms}
\end{bmatrix}
$$

define $L'_{lr} = \left(\frac{N_s}{N_r}\right)^2 L_{lr}$

Refer Voltage Equations

Stator

$$v_{abcs} = r_s i_{abcs} + p\lambda_{abcs}$$

Rotor

$$\frac{N_s}{N_r} v_{abcr} = \frac{N_s}{N_r} r_r i_{abcr} + p \frac{N_s}{N_r} \lambda_{abcr}$$

$$\frac{N_s}{N_r} v_{abcr} = \left(\frac{N_s}{N_r} \right)^2 r_r i'_{abcr} + p\lambda'_{abcr}$$

define $\quad v'_{abcr} = \frac{N_s}{N_r} v_{abcr} \qquad r'_r = \left(\frac{N_s}{N_r} \right)^2 r_r$

$$v'_{abcr} = r'_r i'_{abcr} + p\lambda'_{abcr}$$

Transform to the Arbitrary Reference Frame

stator variables, use $\theta \longrightarrow K_s$

rotor variables, use $\beta \longrightarrow K_r$

Transform Voltage Equations

Stator

$$v_{qd0s} = r_s i_{qd0s} + \omega \lambda_{dqs} + p\lambda_{qd0s}$$

$$\lambda_{dqs} = \begin{bmatrix} \lambda_{ds} \\ -\lambda_{qs} \\ 0 \end{bmatrix}$$

Rotor

$$v'_{qd0r} = r'_r i'_{qd0r} + \left(\omega - \omega_r\right)\lambda'_{dqr} + p\lambda'_{qd0r}$$

$$\lambda_{dqr} = \begin{bmatrix} \lambda'_{dr} \\ -\lambda'_{qr} \\ 0 \end{bmatrix}$$

Transform Flux Linkages

Stator

$$\lambda_{abcs} = \mathbf{L}_s i_{abcs} + \mathbf{L}'_{sr} i'_{abcr}$$

$$\lambda_{qd0s} = K_s \mathbf{L}_s \left(K_s\right)^{-1} i_{qd0s} + K_s \mathbf{L}'_{sr} \left(K_r\right)^{-1} i'_{qd0r}$$

Rotor

$$\lambda'_{abcr} = \left(\mathbf{L}'_{sr}\right)^{\mathrm{T}} i_{abcs} + \mathbf{L}'_r i'_{abcr}$$

$$\lambda'_{qd0r} = K_r \left(\mathbf{L}'_{sr}\right)^{\mathrm{T}} \left(K_s\right)^{-1} i_{qd0s} + K_r \mathbf{L}'_r \left(K_r\right)^{-1} i'_{qd0r}$$

$$\begin{bmatrix} \lambda_{qdos} \\ \lambda'_{qdor} \end{bmatrix} = \begin{bmatrix} K_s \mathbf{L}_s \left(K_s\right)^{-1} & K_s \mathbf{L}'_{sr} \left(K_r\right)^{-1} \\ K_r \left(\mathbf{L}'_{sr}\right)^{\mathrm{T}} \left(K_s\right)^{-1} & K_r \mathbf{L}'_r \left(K_r\right)^{-1} \end{bmatrix} \begin{bmatrix} i_{qdos} \\ i'_{qdor} \end{bmatrix}$$

$$K_s \mathbf{L}_s (K_s)^{-1} = \begin{bmatrix} L_{ls} + L_M & 0 & 0 \\ 0 & L_{ls} + L_M & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} \qquad L_M = \frac{3}{2} L_{ms}$$

$$K_r \mathbf{L}_r{}' (K_r)^{-1} = \begin{bmatrix} L'_{lr} + L_M & 0 & 0 \\ 0 & L'_{lr} + L_M & 0 \\ 0 & 0 & L'_{lr} \end{bmatrix}$$

$$K_s \mathbf{L}'_{sr} (K_r)^{-1} = K_r \left(\mathbf{L}'_{sr}\right)^{\mathrm{T}} (K_s)^{-1} = \begin{bmatrix} L_M & 0 & 0 \\ 0 & L_M & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

IM *q-d* Model Equations

$$v_{qs} = r_s i_{qs} + \omega\lambda_{ds} + p\lambda_{qs}$$

$$v_{ds} = r_s i_{ds} - \omega\lambda_{qs} + p\lambda_{ds}$$

$$v_{0s} = r_s i_{0s} + p\lambda_{0s}$$

$$v'_{qr} = r'_r i'_{qr} + (\omega - \omega_r)\lambda'_{dr} + p\lambda'_{qr}$$

$$v'_{dr} = r'_r i'_{dr} - (\omega - \omega_r)\lambda'_{qr} + p\lambda'_{dr}$$

$$v'_{0r} = r'_r i'_{0r} + p\lambda'_{0r}$$

$$\lambda_{qs} = L_{ls} i_{qs} + L_M \left(i_{qs} + i'_{qr}\right)$$

$$\lambda_{ds} = L_{ls} i_{ds} + L_M \left(i_{ds} + i'_{dr}\right)$$

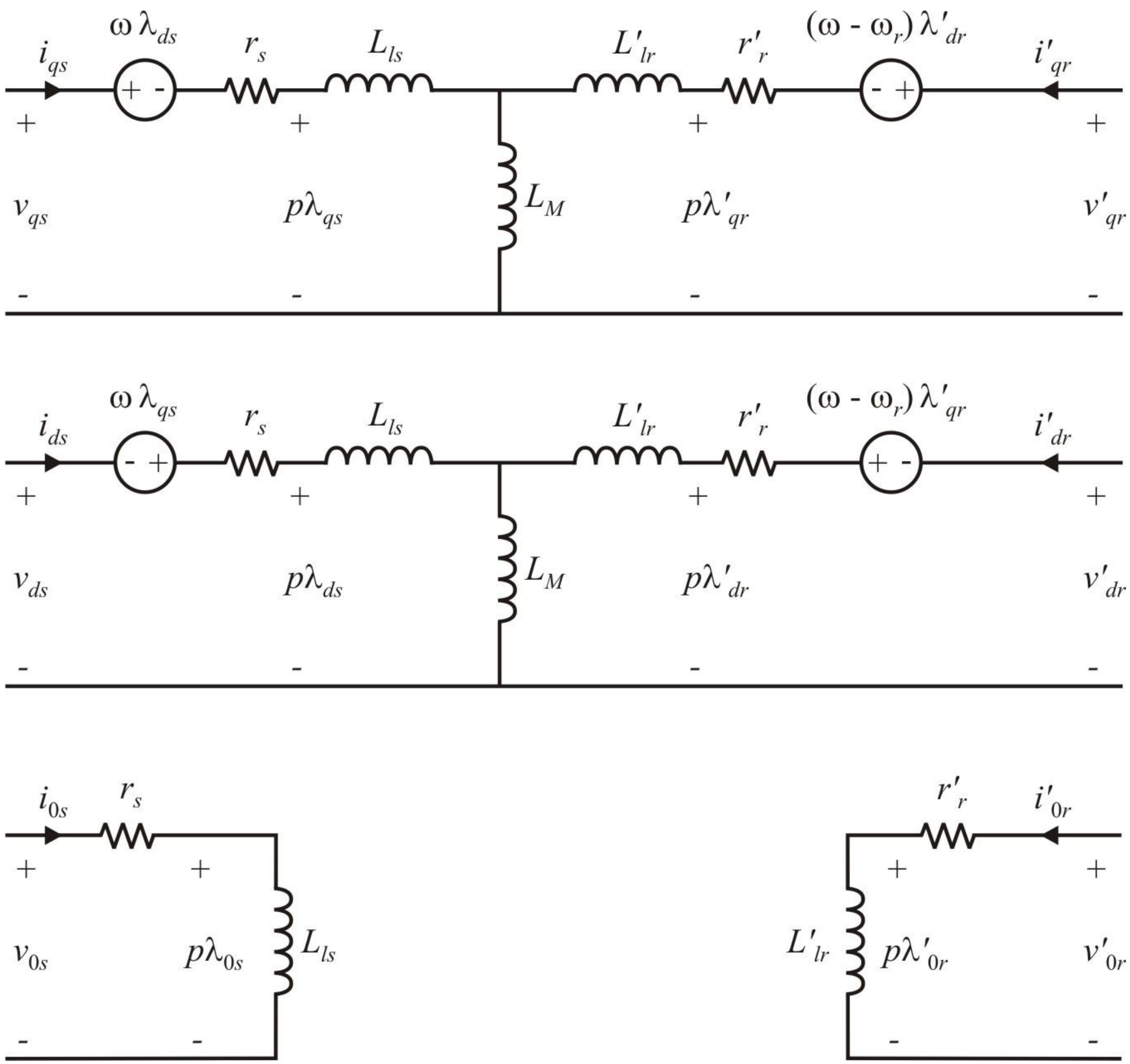
$$\lambda_{0s} = L_{ls} i_{0s}$$

$$\lambda'_{qr} = L'_{lr} i'_{qr} + L_M \left(i_{qs} + i'_{qr}\right)$$

$$\lambda'_{dr} = L'_{lr} i'_{dr} + L_M \left(i_{ds} + i'_{dr}\right)$$

$$\lambda_{or}' = L_{lr}' i_{or}'$$

IM *q-d* Equivalent Circuit



Torque Expressions

with $\omega = 0$

$$T_e = \frac{3}{2}\frac{P}{2}\frac{1}{\omega_r}\left(\omega_r \lambda'_{qr} i'_{dr} - \omega_r \lambda'_{dr} i'_{qr}\right)$$

$$T_e = \frac{3}{2}\frac{P}{2}\left(\lambda'_{qr} i'_{dr} - \lambda'_{dr} i'_{qr}\right)$$

with $\omega = \omega_r$

$$T_e = \frac{3}{2}\frac{P}{2}\frac{1}{\omega_r}\left(\omega_r \lambda_{ds} i_{qs} - \omega_r \lambda_{qs} i_{ds}\right)$$

$$T_e = \frac{3}{2}\frac{P}{2}\left(\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}\right)$$

More Torque Expressions

in terms of currents

$$\lambda_{qs} = \left(L_{ls} + L_M\right) i_{qs} + L_M i'_{qr}$$

$$\lambda_{ds} = \left(L_{ls} + L_M\right) i_{ds} + L_M i'_{dr}$$

$$T_e = \frac{3}{2}\frac{P}{2}\left[\left(L_{ls} + L_M\right) i_{ds} i_{qs} + L_M i_{qs} i'_{dr} - \left(L_{ls} + L_M\right) i_{qs} i_{ds} - L_M i_{ds} i'_{qr}\right]$$

$$T_e = \frac{3}{2}\frac{P}{2} L_M \left(i_{qs} i'_{dr} - i_{ds} i'_{qr}\right)$$

in terms of flux linkages

$$T_e = \frac{3}{2}\frac{P}{2}\left(\frac{L_M}{L_{ss} L'_{rr} - L_M^2}\right)\left(\lambda_{qs}\lambda'_{dr} - \lambda_{ds}\lambda'_{qr}\right)$$

$$L_{ss} = L_{ls} + L_M$$

$$L'_{rr} = L'_{lr} + L_M$$

Torque in *a*-*b*-*c* Variables

$$T_e = -\frac{P}{2} L_{ms} \left\{ \begin{bmatrix} i_{as}\left(i'_{ar} - \frac{1}{2} i'_{br} - \frac{1}{2} i'_{cr} \right) + \\ i_{bs}\left(i'_{br} - \frac{1}{2} i'_{ar} - \frac{1}{2} i'_{cr} \right) + \\ i_{cs}\left(i'_{cr} - \frac{1}{2} i'_{ar} - \frac{1}{2} i'_{br} \right) \end{bmatrix} \sin(\theta_r) + \atop \frac{\sqrt{3}}{2} \left[i_{as}\left(i'_{br} - i'_{cr} \right) + i_{bs}\left(i'_{cr} - i'_{ar} \right) + i_{cs}\left(i'_{ar} - i'_{br} \right) \right] \cos(\theta_r) \right\}$$

Torque in *q*-*d* variables

$$T_e = \frac{3}{2}\frac{P}{2}\left(\lambda'_{qr} i'_{dr} - \lambda'_{dr} i'_{qr} \right) \qquad T_e = \frac{3}{2}\frac{P}{2} L_M \left(i_{qs} i'_{dr} - i_{ds} i'_{qr} \right)$$

$$T_e = \frac{3}{2}\frac{P}{2}\left(\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds} \right) \qquad T_e = \frac{3}{2}\frac{P}{2}\left(\frac{L_M}{L_{ss} L'_{rr} - L_M^2} \right)\left(\lambda_{qs} \lambda'_{dr} - \lambda_{ds} \lambda'_{qr} \right)$$

Steady-State Calculations (*q-d* Model)

$RPM := \frac{2 \cdot \pi \cdot rad}{60 \cdot s}$

IM parameters

$r_s := 0.4\Omega$ $\quad P := 4$ $\quad r'_r := 0.2266\Omega$ $\quad lagging := 1$

$L_{ls} := 5.73\,mH$ $\quad L_M := 64.4\,mH$ $\quad L'_{lr} := 4.64\,mH$

$L_{ss} := L_{ls} + L_M$ $\quad L_{ss} = 70.1\,mH$

$L'_{rr} := L'_{lr} + L_M$ $\quad L'_{rr} = 69\,mH$

operating conditions

$f_e := 60 \cdot Hz$ $\quad \omega_e := 2 \cdot \pi \cdot f_e$ $\quad \omega_e = 377\frac{rad}{s}$

$\omega_{rm} := 1750\,RPM$ $\quad \omega_{rm} = 183.3\frac{rad}{s}$

$V_{LL} := 220 \cdot V$ $\quad V_s := \frac{V_{LL}}{\sqrt{3}}$ $\quad V_s = 127V$

synchronous speed (no-load speed)

$\omega_{em} := \left(\frac{2}{P}\right) \cdot \omega_e$ $\quad \omega_{em} = 188.5\frac{rad}{s}$

$\omega_{em} = 1800RPM$

slip

$\omega_r := \frac{P}{2} \cdot \omega_{rm}$ $\quad \omega_r = 366.5\frac{rad}{s}$

$s := \frac{\omega_e - \omega_r}{\omega_e}$ $\quad s = 0.0278$

Steady-State Calculations (*q*-*d* Model)

steady-state calculations (synchronous reference frame)

$$V_{qs\_e} := \sqrt{2}\cdot V_s \qquad V_{ds\_e} := 0\cdot V \qquad V'_{qr\_e} := 0\cdot V \qquad V'_{dr\_e} := 0\cdot V$$

$$\begin{pmatrix} I_{qs\_e} \\ I_{ds\_e} \\ I'_{qr\_e} \\ I'_{dr\_e} \end{pmatrix} := \begin{bmatrix} r_s & \omega_e\cdot L_{ss} & 0 & \omega_e\cdot L_M \\ -\omega_e\cdot L_{ss} & r_s & -\omega_e\cdot L_M & 0 \\ 0 & (\omega_e - \omega_r)\cdot L_M & r'_r & (\omega_e - \omega_r)\cdot L'_{rr} \\ -(\omega_e - \omega_r)\cdot L_M & 0 & -(\omega_e - \omega_r)\cdot L'_{rr} & r'_r \end{bmatrix}^{-1} \cdot \begin{pmatrix} V_{qs\_e} \\ V_{ds\_e} \\ V'_{qr\_e} \\ V'_{dr\_e} \end{pmatrix}$$

Steady-State Calculations (*q*-*d* Model)

current and torque

$$I_s := \frac{1}{\sqrt{2}}\cdot\sqrt{I_{qs\_e}{}^2 + I_{ds\_e}{}^2}$$

$$\lambda_{qs\_e} := L_{ss}\cdot I_{qs\_e} + L_M\cdot I'_{qr\_e}$$

$$\lambda_{ds\_e} := L_{ss}\cdot I_{ds\_e} + L_M\cdot I'_{dr\_e}$$

$$T_e := \frac{3}{2}\cdot\frac{P}{2}\cdot\left(\lambda_{ds\_e}\cdot I_{qs\_e} - \lambda_{qs\_e}\cdot I_{ds\_e}\right)$$

Steady-State Calculations (*q-d* Model)

magnetizing and rotor flux linkages

$$\lambda_{qM\_e} := L_M \cdot \left(I_{qs\_e} + I'_{qr\_e}\right)$$

$$\lambda_{dM\_e} := L_M \cdot \left(I_{ds\_e} + I'_{dr\_e}\right)$$

$$\Lambda_M := \frac{1}{\sqrt{2}} \cdot \sqrt{\lambda_{qM\_e}{}^2 + \lambda_{dM\_e}{}^2}$$

$$\lambda'_{qr\_e} := L'_{rr} \cdot I'_{qr\_e} + L_M \cdot I_{qs\_e}$$

$$\lambda'_{dr\_e} := L'_{rr} \cdot I'_{dr\_e} + L_M \cdot I_{ds\_e}$$

$$\lambda'_r := \frac{1}{\sqrt{2}} \cdot \sqrt{\lambda'_{qr\_e}{}^2 + \lambda'_{dr\_e}{}^2}$$

Induction Machine *q-d* Model

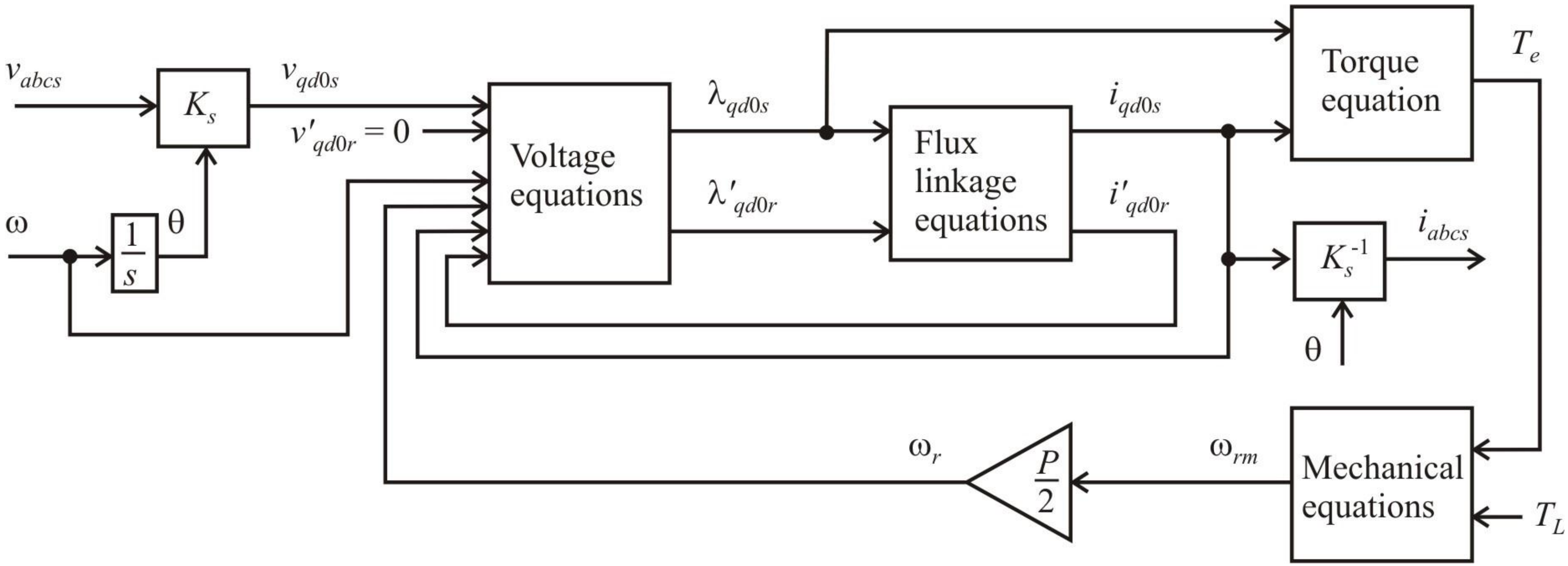
Rotor referred to stator

Formulated in the arbitrary reference frame (could later be set to synchronous, stationary, rotor)

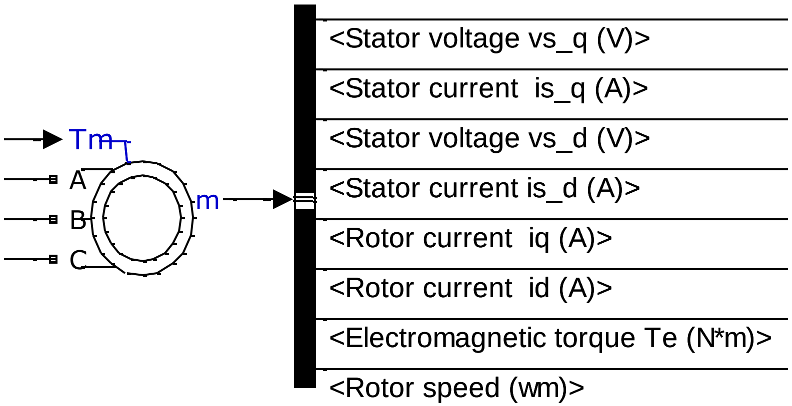
Steady-state variables are constant in the synchronous reference frame

Torque can be expressed in terms of stator quantities, rotor quantities, currents, or flux linkages

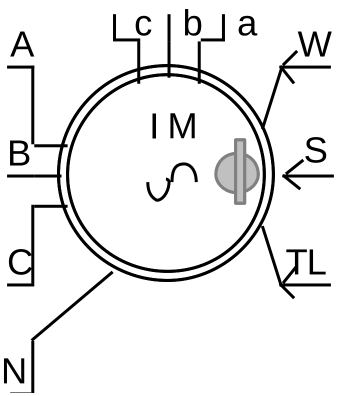
Induction Machine Simulation



Predefined IM Models



Matlab Simulink SimPowerSystems

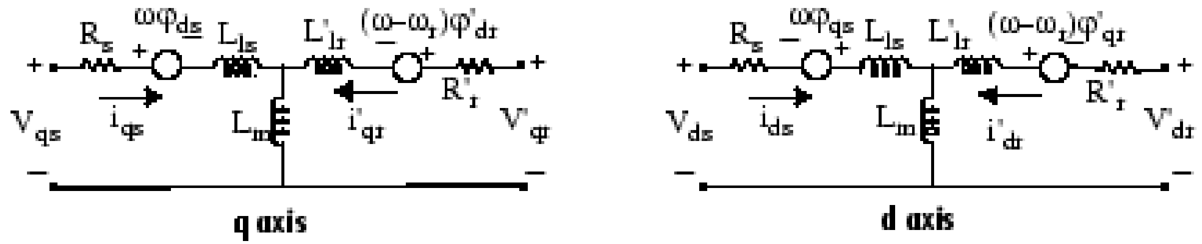


PSCAD

- Notation and model mostly matches the book
- Constant speed and free rotor modes
- Per unit parameters (SI or pu in Simulink)

SimPowerSystems IM Model

Electrical System



$$V_{qs} = R_s i_{qs} + \frac{d}{dt}\varphi_{qs} + \omega\varphi_{ds}$$

$$V_{ds} = R_s i_{ds} + \frac{d}{dt}\varphi_{ds} - \omega\varphi_{qs}$$

$$V'_{qr} = R'_r i'_{qr} + \frac{d}{dt}\varphi'_{qr} + (\omega - \omega_r)\varphi'_{dr}$$

$$V'_{dr} = R'_r i'_{dr} + \frac{d}{dt}\varphi'_{dr} - (\omega - \omega_r)\varphi'_{qr}$$

$$T_e = 1.5\, p(\varphi_{ds} i_{qs} - \varphi_{qs} i_{ds})$$

where

$$\varphi_{qs} = L_s i_{qs} + L_m i'_{qr}$$

$$\varphi_{ds} = L_s i_{ds} + L_m i'_{dr}$$

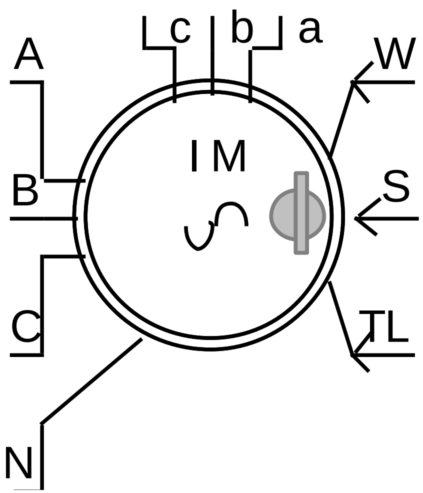
$$\varphi'_{qr} = L'_r i'_{qr} + L_m i_{qs}$$

$$\varphi'_{dr} = L'_r i'_{dr} + L_m i_{ds}$$

$$L_s = L_{ls} + L_m$$

$$L'_r = L'_{lr} + L_m$$

PSCAD IM Model



A, B, C, N - stator electrical terminals

a b c - rotor electrical terminals

S = 1 - constant speed mode
S = 0 - mechanical equations

W - speed when S = 1
initial speed when S = 0
expressed in per unit as ω_r / ω_e

TL - load torque when S = 0

IM Per-Unit Example

Induction machine ratings/parameters

$\omega_b := 2\cdot\pi\cdot 60\cdot\mathrm{Hz}$ $\qquad \omega_b = 377\,\frac{\mathrm{rad}}{\mathrm{s}}$

$P_B := 10\cdot\mathrm{hp}$ $\qquad P_B = 7.457\,\mathrm{kW}$

$V_B := \frac{220\cdot\mathrm{V}}{\sqrt{3}}$ $\qquad V_B = 127\,\mathrm{V}$

$P := 6$

$r_{s\_pu} := 0.0453$

$X_{ls\_pu} := 0.0775$

$X_{M\_pu} := 2.042$

$r'_{r\_pu} := 0.0222$

$X'_{lr\_pu} := 0.0322$

$H := 0.5\,\mathrm{s}$ $\qquad 2\cdot H = 1\,\mathrm{s}$

per unit quantities

$I_B := \frac{P_B}{3\cdot V_B}$ $\qquad I_B = 19.57\,\mathrm{A}$

$Z_B := \frac{V_B}{I_B}$ $\qquad Z_B = 6.491\,\Omega$

$T_B := \frac{P_B}{\left(\frac{2}{P}\right)\cdot\omega_b}$ $\qquad T_B = 59.3\,\mathrm{N\cdot m}$

Induction Machine SI Quantities

machine parameters

$$r_s := r_{s\_pu} \cdot Z_B \qquad r_s = 0.294\,\Omega$$

$$L_{ls} := \frac{X_{ls\_pu} \cdot Z_B}{\omega_b} \qquad L_{ls} = 1.33\,mH$$

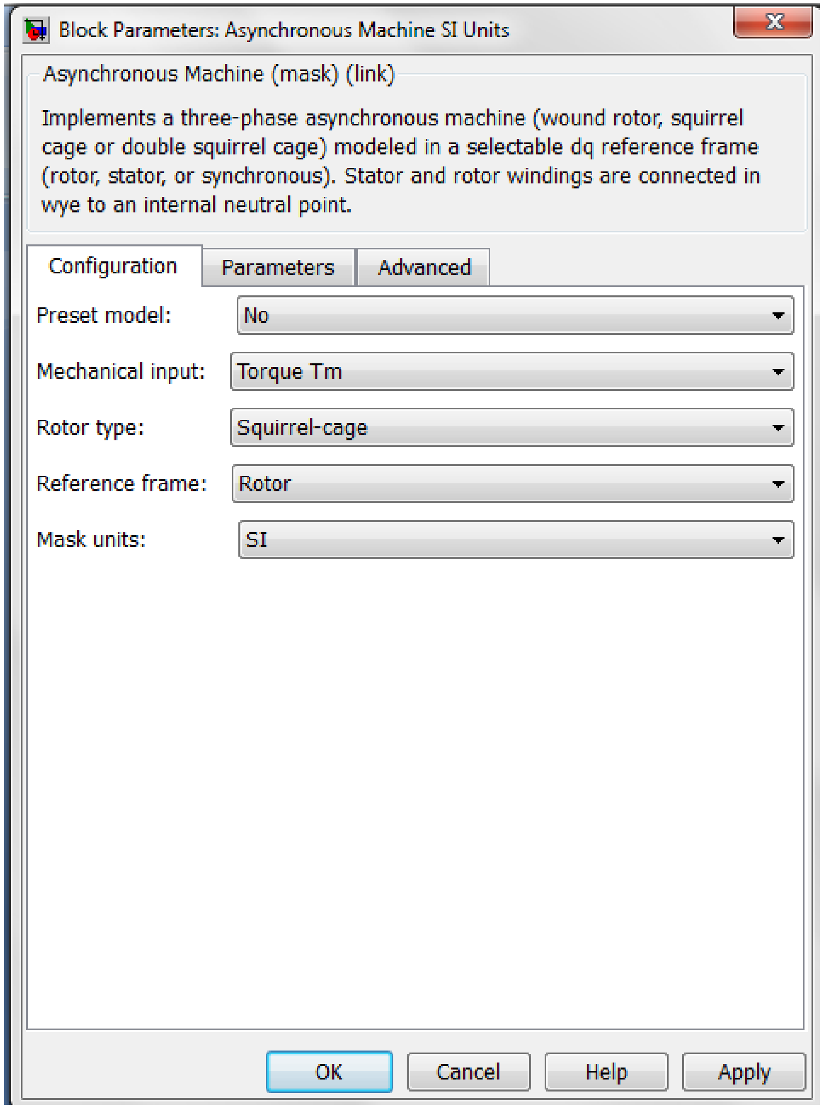
$$L_M := \frac{X_{M\_pu} \cdot Z_B}{\omega_b} \qquad L_M = 35.16\,mH$$

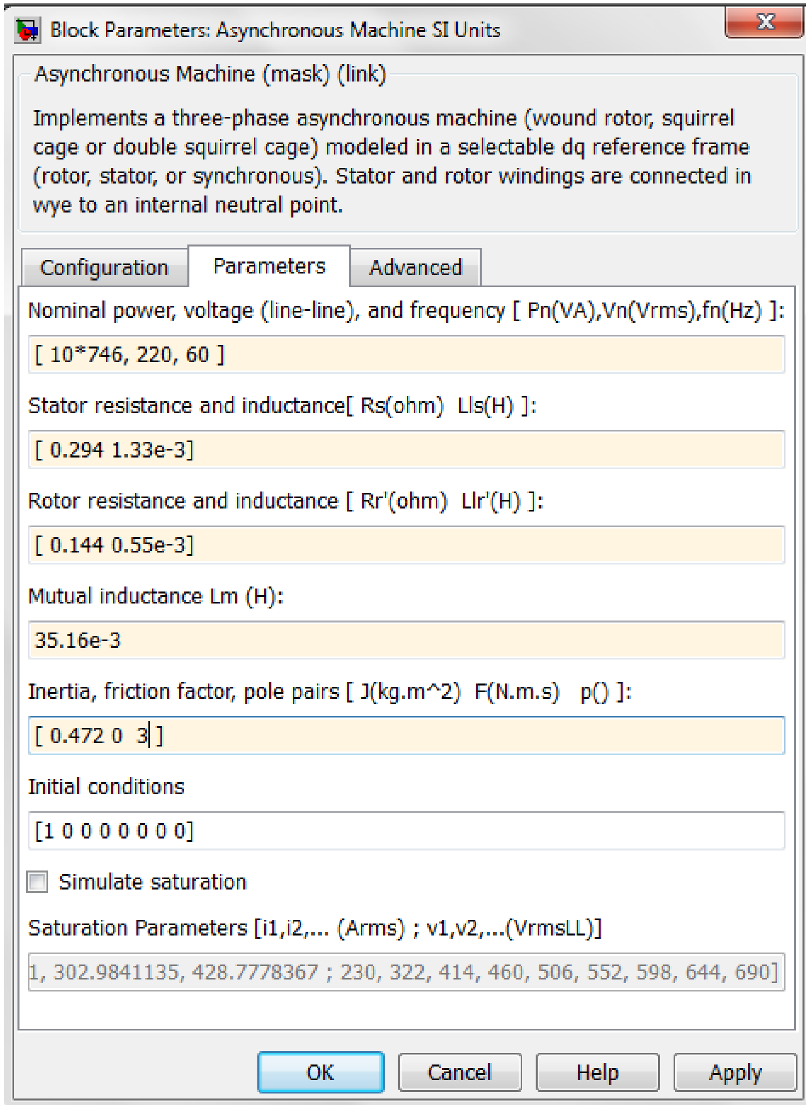
$$r'_r := r'_{r\_pu} \cdot Z_B \qquad r'_r = 0.144\,\Omega$$

$$L'_{lr} := \frac{X'_{lr\_pu} \cdot Z_B}{\omega_b} \qquad L'_{lr} = 0.55\,mH$$

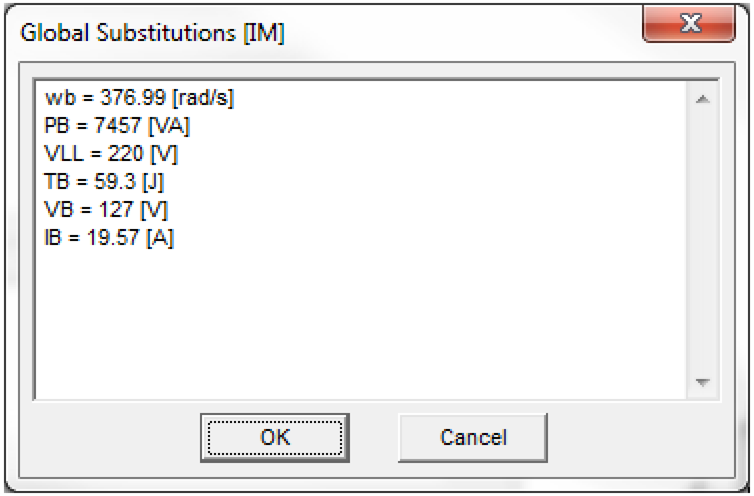
$$J := 2 \cdot H \cdot \frac{T_B}{\left(\frac{2}{P}\right) \cdot \omega_b} \qquad J = 0.472\,kg\,m^2$$

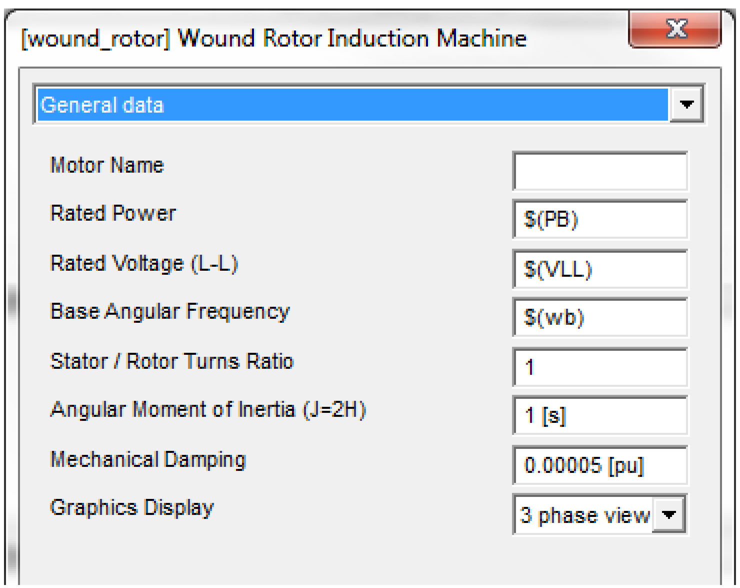
SimPowerSystems Inputs

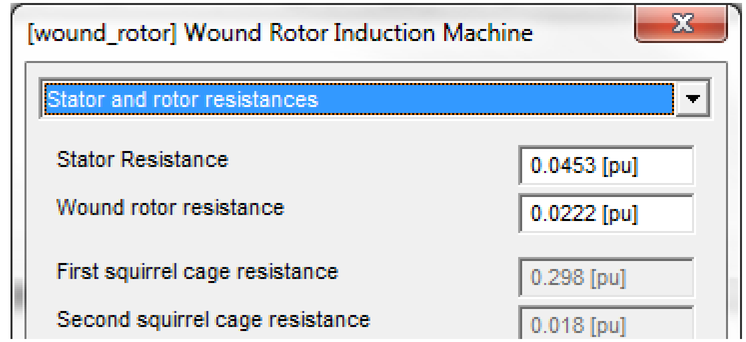


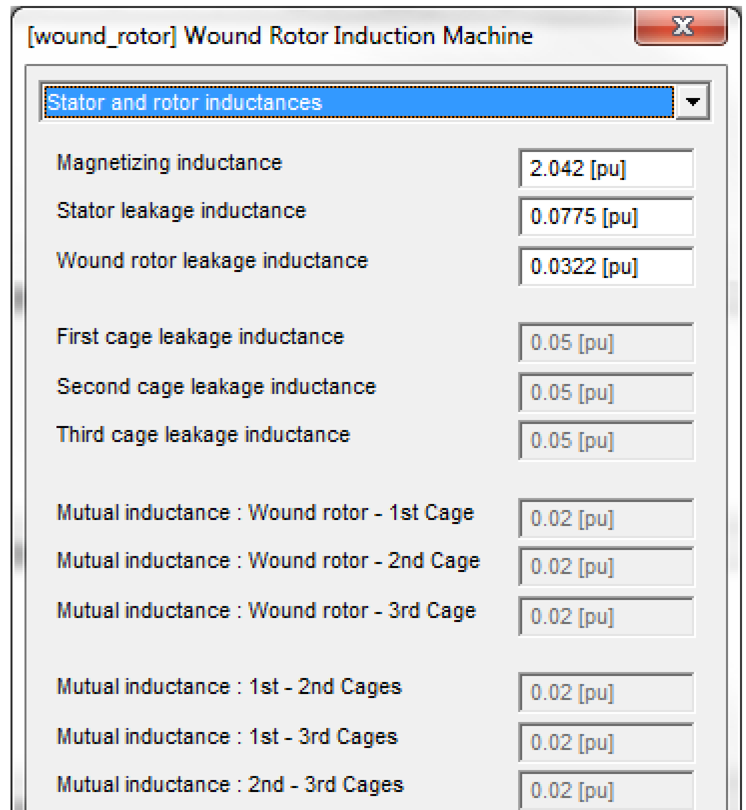


PSCAD Model Inputs

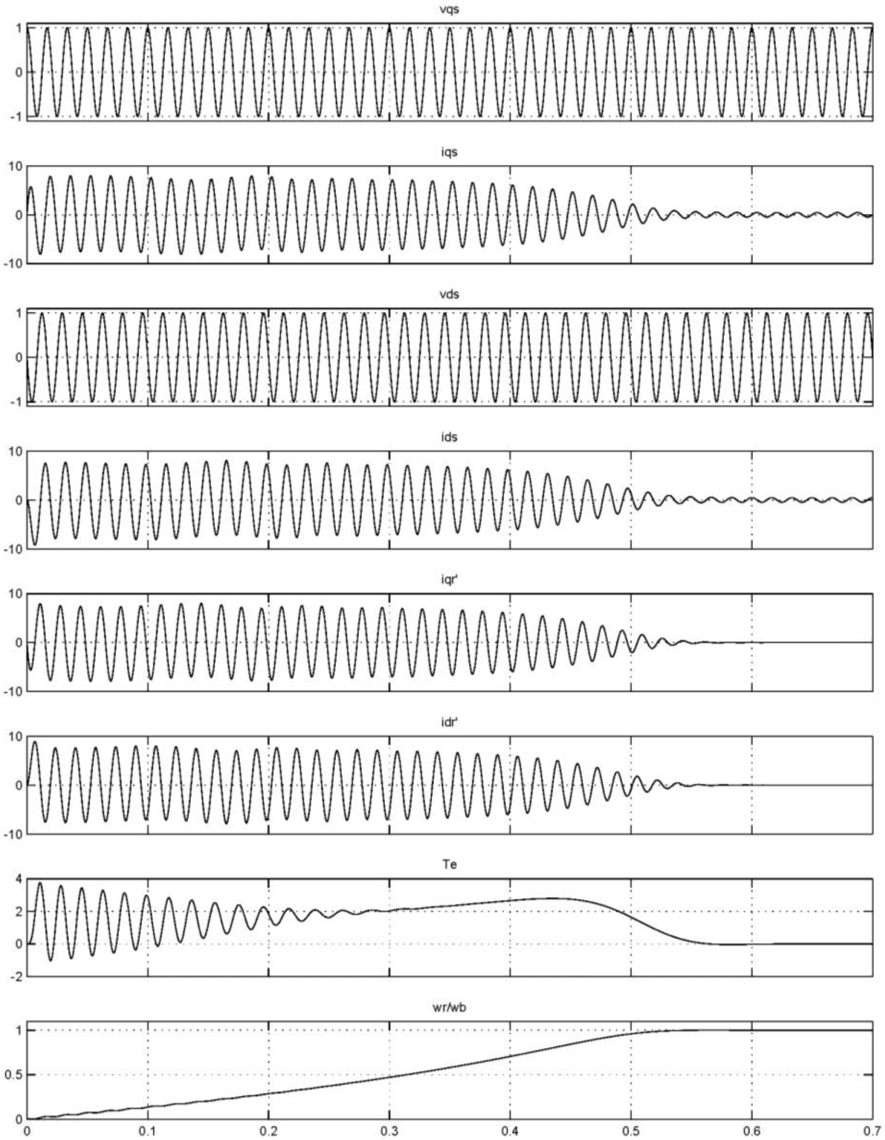




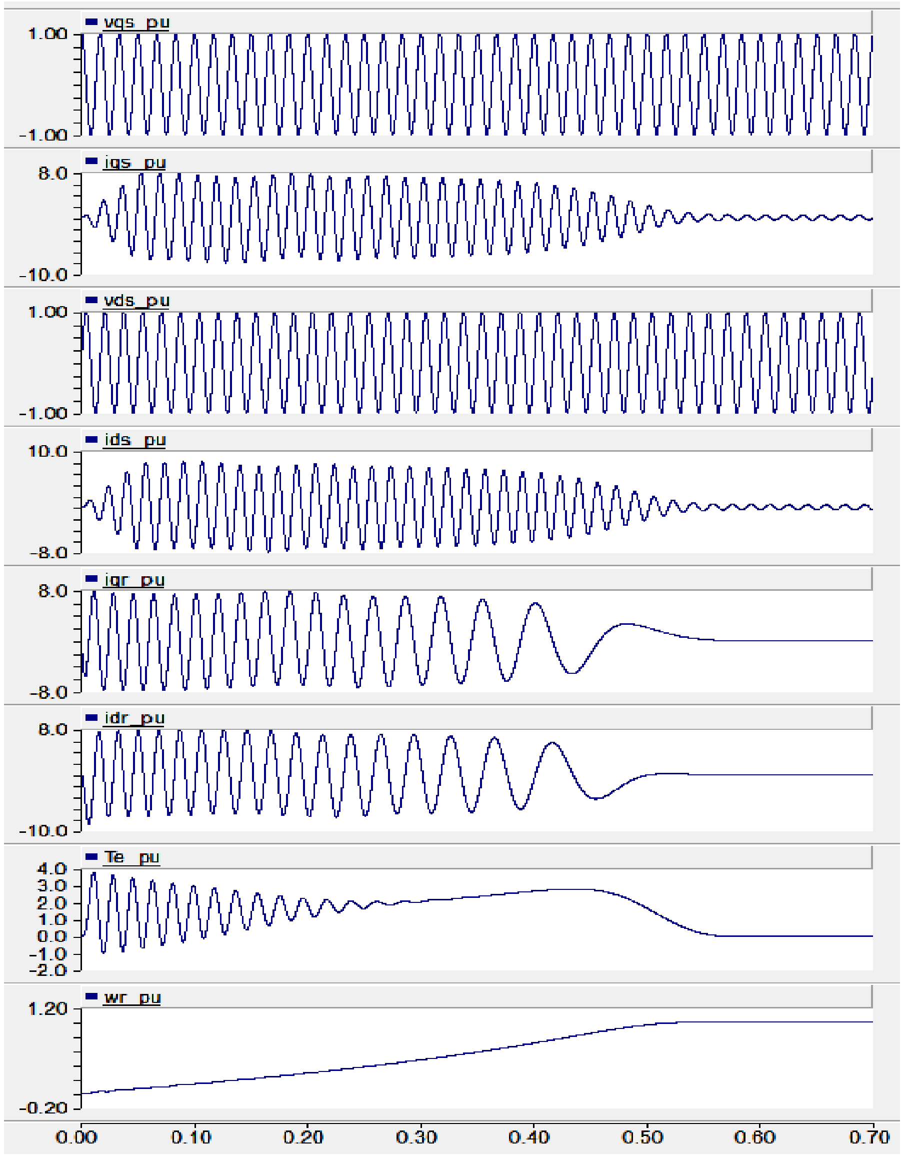




Stationary Reference Frame (Book Figure 6.11-2)

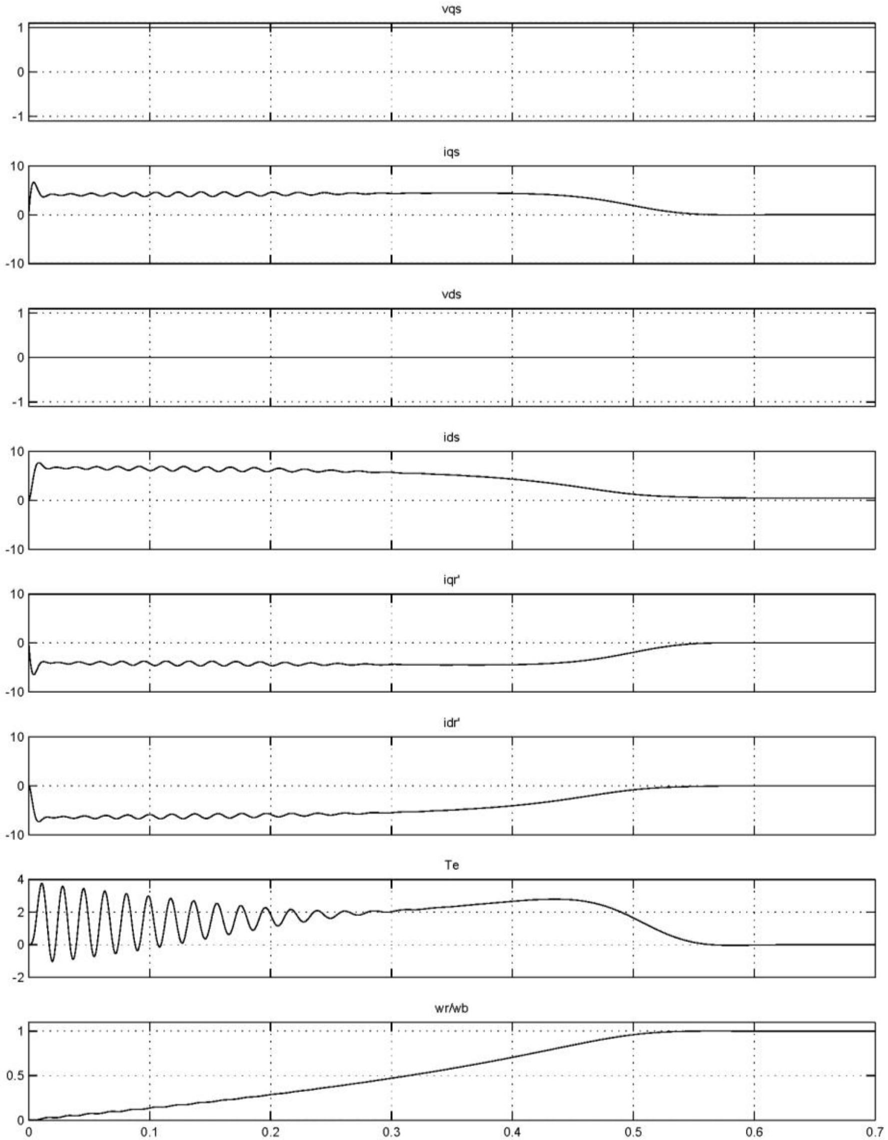


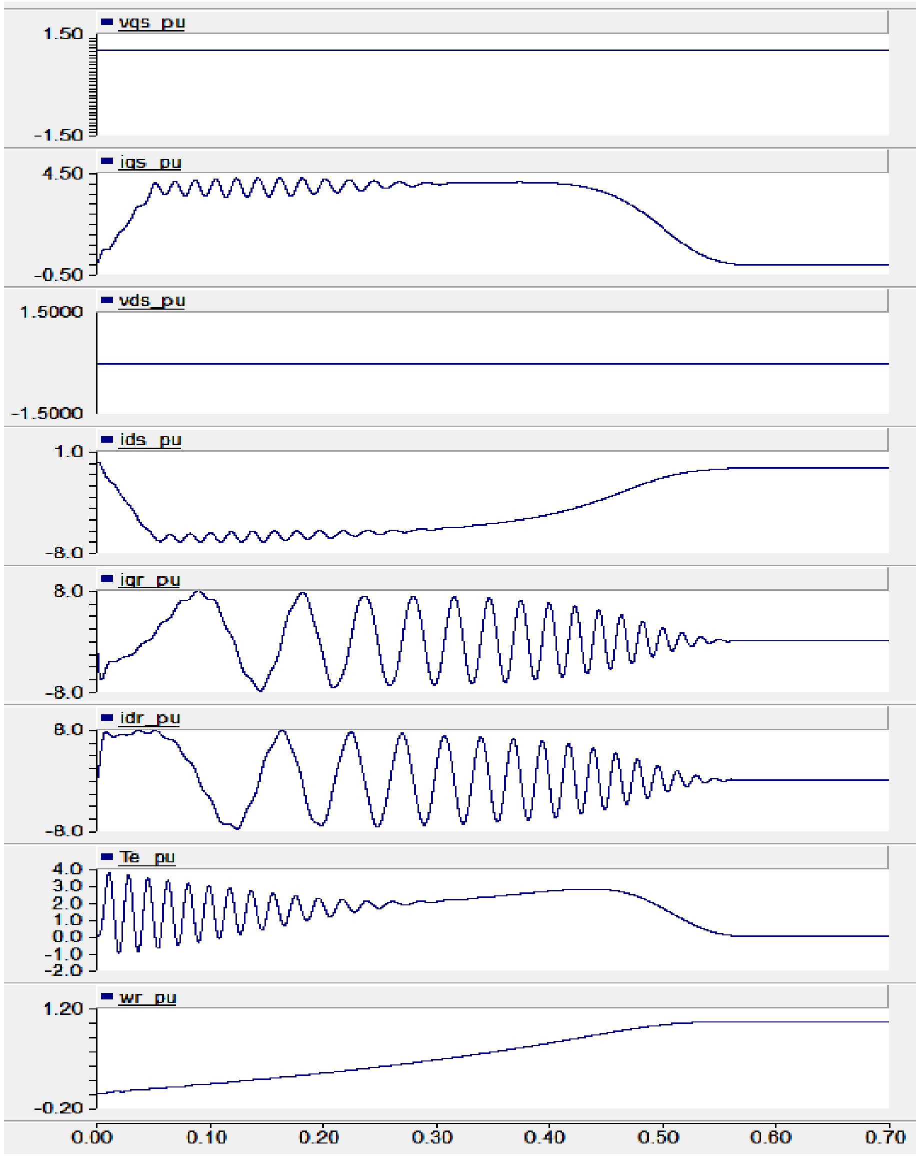
Simulink



PSCAD

Synchronous Reference Frame (Book Figure 6.11-4)

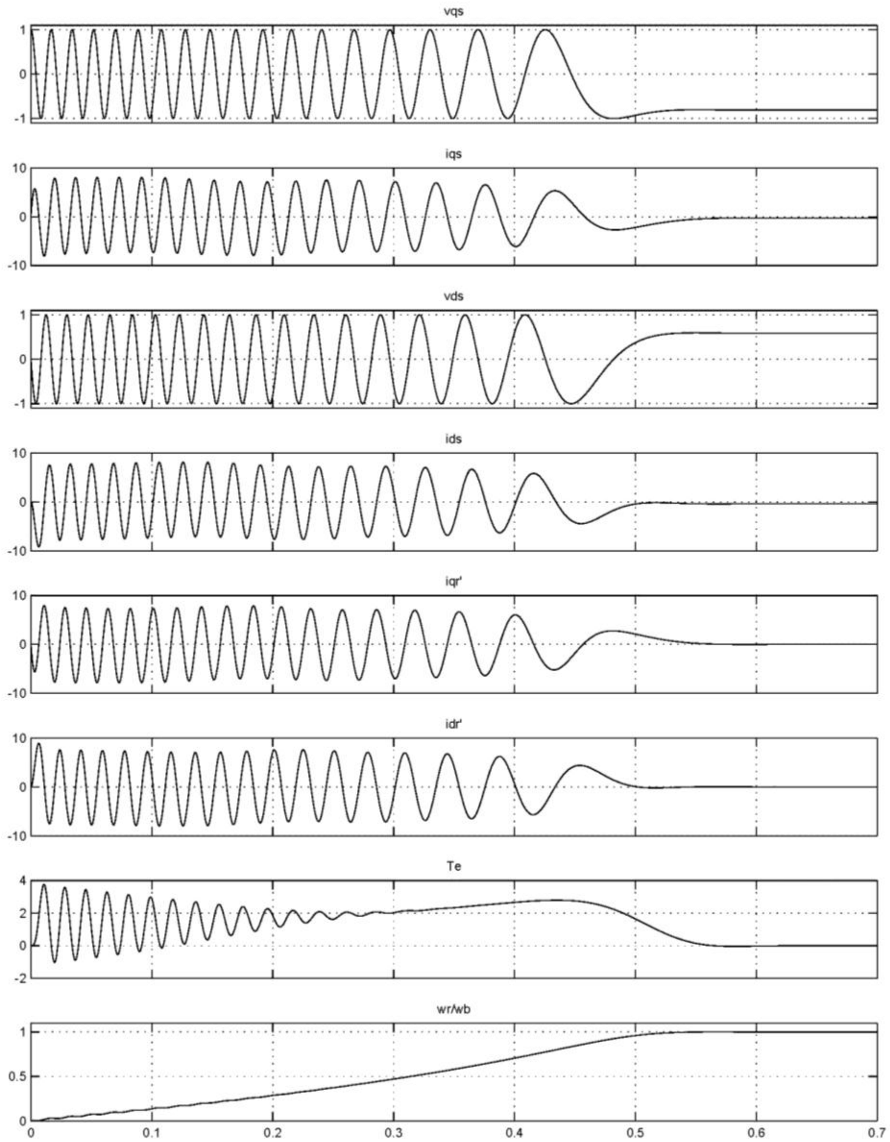


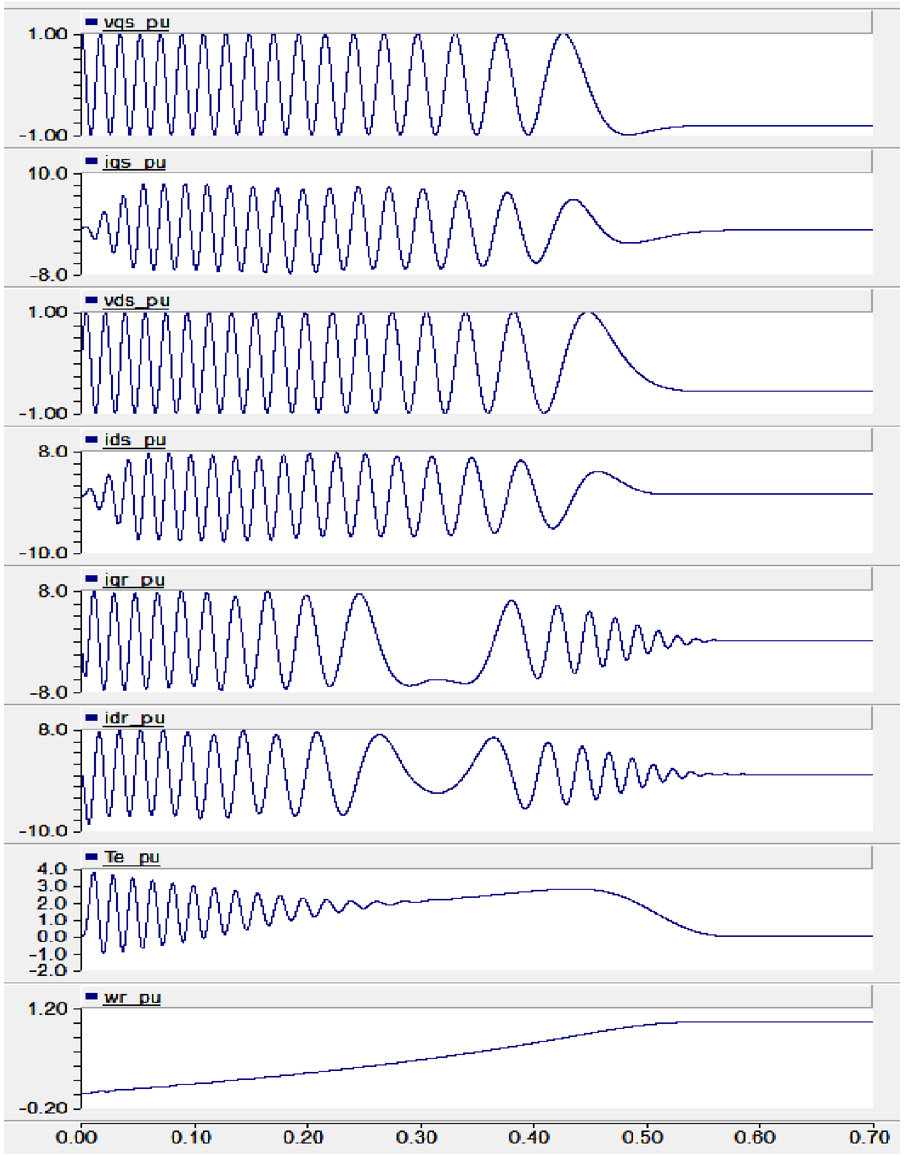


Simulink

PSCAD

Rotor Reference Frame (Book Figure 6.11-3)





Simulink

PSCAD

Induction Machine Simulation

Transient and steady-state performance prediction

Flux linkages or currents can be state variables

Simulink and PSCAD models are based on the standard *q-d* machine model with some additional features such as saturation

Simulink: parameters in SI units (or per-unit) using the same *q-d* model as in the book.

PSCAD: parameters entered in per-unit. Internal output variables in per-unit.

IM Steady-State Calculations

Could use *q*-*d* equations or *a*-*b*-*c* equations

Instantaneous voltages

$$v_{as} = \sqrt{2} V_s \cos(\theta_e)$$

$$v_{bs} = \sqrt{2} V_s \cos\left(\theta_e - \frac{2\pi}{3}\right)$$

$$v_{cs} = \sqrt{2} V_s \cos\left(\theta_e + \frac{2\pi}{3}\right)$$

$$\theta_e = \omega_e t \qquad \omega_e = 2\pi f$$

voltage phasors

$$\tilde{V}_{as} = V_s \angle 0$$

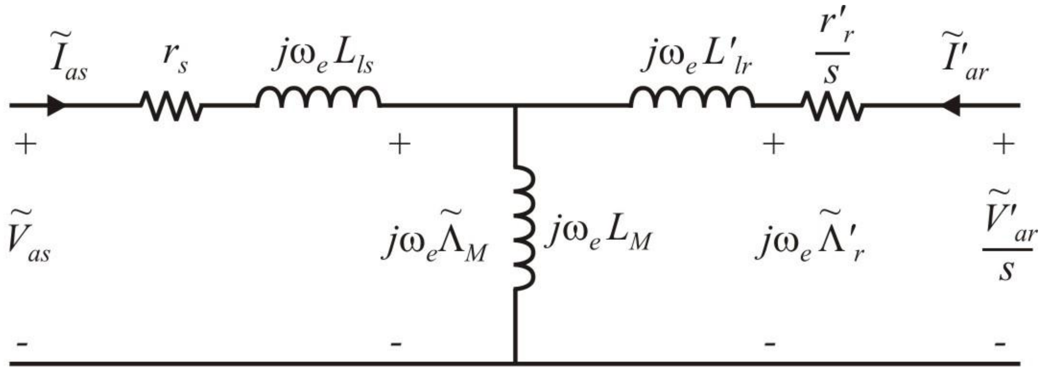
$$\tilde{V}_{bs} = V_s \angle -120^o$$

$$\tilde{V}_{cs} = V_s \angle 120^o$$

constant ω_{rm} $\qquad \omega_r = \frac{P}{2} \omega_{rm}$

define slip $\quad s = \dfrac{\omega_e - \omega_r}{\omega_e}$

Per-Phase Steady-State Circuit



squirrel cage $\tilde{V}'_{ar} = 0$

Steady-State (Average) Torque

$$P_{out} = 3r'_r \left(\frac{1-s}{s} \right) \left| \tilde{I}'_{ar} \right|^2$$

$$P_{out} = T_e \omega_{rm}$$

$$\omega_{rm} = \frac{2}{P} \omega_r = \frac{2}{P} (1-s) \omega_e$$

$$T_e = \frac{P}{2} \frac{1}{(1-s)\omega_e} P_{out}$$

$$T_e = 3 \frac{P}{2} \frac{r'_r}{s\omega_e} \left| \tilde{I}'_{ar} \right|^2$$

Torque in Terms of Voltage

input impedance $$Z_{in} = Z_s + \left(Z_m \| Z'_r \right)$$

$$Z_s = r_s + j\omega_e L_{ls} \qquad Z'_r = \frac{r'_r}{s} + j\omega_e L'_{lr} \qquad Z_m = j\omega_e L_M$$

currents

$$\tilde{I}_{as} = \frac{\tilde{V}_{as}}{Z_{in}} \qquad \tilde{I}'_{ar} = -\left(\frac{Z_m}{Z_m + Z'_r} \right) \tilde{I}_{as} = -\left(\frac{Z_m}{Z_m + Z'_r} \right) \frac{\tilde{V}_{as}}{Z_{in}}$$

torque

$$T_e = \frac{3 \frac{P}{2} \omega_e L_M{}^2 r'_r s \left| \tilde{V}_{as} \right|^2}{\left[r_s r'_r + s\omega_e^2 \left(L_M{}^2 - L_{ss} L'_{rr} \right) \right]^2 + \omega_e^2 \left(r'_r L_{ss} + s r_s L'_{rr} \right)^2}$$

$$L_{ss} = L_{ls} + L_M$$

$$L'_{rr} = L'_{lr} + L_M$$

Steady-State Calculations (Per-Phase Model)

$$RPM := \frac{2\cdot\pi\cdot rad}{60\cdot s}$$

IM parameters

$r_s := 0.4\Omega$ $\quad P := 4$ $\quad r'_r := 0.2266\Omega$ $\quad lagging := 1$

$L_{ls} := 5.73mH$ $\quad L_M := 64.4mH$ $\quad L'_{lr} := 4.64mH$

$L_{ss} := L_{ls} + L_M$ $\quad L_{ss} = 70.1mH$

$L'_{rr} := L'_{lr} + L_M$ $\quad L'_{rr} = 69mH$

operating conditions

$f_e := 60\cdot Hz$ $\quad \omega_e := 2\cdot\pi\cdot f_e$ $\quad \omega_e = 377\frac{rad}{s}$

$\omega_{rm} := 1750RPM$ $\quad \omega_{rm} = 183.3\frac{rad}{s}$

$V_{LL} := 220\cdot V$ $\quad V_s := \frac{V_{LL}}{\sqrt{3}}$ $\quad V_s = 127V$

synchronous speed (no-load speed)

$\omega_{em} := \left(\frac{2}{P}\right)\cdot\omega_e$ $\quad \omega_{em} = 188.5\frac{rad}{s}$

$\omega_{em} = 1800RPM$

slip

$\omega_r := \frac{P}{2}\cdot\omega_{rm}$ $\quad \omega_r = 366.5\frac{rad}{s}$

$s := \frac{\omega_e - \omega_r}{\omega_e}$ $\quad s = 0.0278$

Steady-State Calculations (Per-Phase Model)

impedances

$$Z_s := r_s + j \cdot \omega_e \cdot L_{ls}$$

$$Z_m := j \cdot \omega_e \cdot L_M$$

$$Z'_r := \frac{r'_r}{s} + j \cdot \omega_e \cdot L'_{lr}$$

$$Z_f := \frac{1}{\frac{1}{Z_m} + \frac{1}{Z'_r}}$$

$$Z_{in} := Z_s + Z_f$$

currents

$$V_{as} := V_s \cdot e^{j \cdot 0}$$

$$I_{as} := \frac{V_{as}}{Z_{in}}$$

$$I'_{ar} := -I_{as} \cdot \frac{Z_m}{Z_m + Z'_r}$$

Steady-State Calculations (Per-Phase Model)

torque and power

$$T_e := 3 \cdot \frac{P}{2} \cdot \left(\left| I'_{ar} \right| \right)^2 \cdot \frac{r'_r}{s \cdot \omega_e}$$

$$T_e := \frac{3 \cdot \left(\frac{P}{2} \right) \cdot \omega_e \cdot L_M{}^2 \cdot r'_r \cdot s \cdot \left(\left| V_{as} \right| \right)^2}{\left[r_s \cdot r'_r + s \cdot \omega_e{}^2 \cdot \left(L_M{}^2 - L_{ss} \cdot L'_{rr} \right) \right]^2 + \omega_e{}^2 \cdot \left(r'_r \cdot L_{ss} + s \cdot r_s \cdot L'_{rr} \right)^2}$$

$$\theta := \arg\left(Z_{in} \right)$$

$$pf := \cos(\theta)$$

$$P_{in} := 3 \cdot \left| V_{as} \right| \cdot \left| I_{as} \right| \cdot pf$$

$$P_{out} := T_e \cdot \omega_{rm}$$

$$eff := \frac{P_{out}}{P_{in}}$$

Steady-State Calculations (Per-Phase Model)

rotor flux linkage

$$\Lambda'_r := \frac{-I'_{ar} \cdot \frac{r'_r}{s}}{j \cdot \omega_e}$$

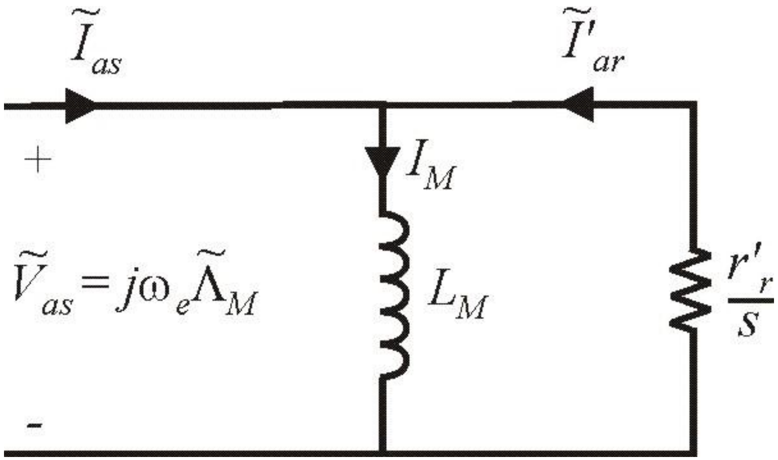
IM Steady-State Model

Straightforward and quick calculation of steady-state operation

Model provides insight into motor operation

Can be used to predict peak torque, stall torque, and starting current

Simplified IM Steady-State Model



$$T_e = 3\frac{P}{2}\frac{r'_r}{s\omega_e}\left|\tilde{I}'_{ar}\right|^2$$

$$\left|\tilde{I}'_{ar}\right|^2 = \frac{\left|\tilde{V}_{as}\right|^2 s^2}{r'^2_r}$$

$$T_e = 3\frac{P}{2}\frac{s\left|\tilde{V}_{as}\right|^2}{r'_r\omega_e} = 3\frac{P}{2}\left(\frac{\left|\tilde{V}_{as}\right|}{\omega_e}\right)^2\frac{\omega_e - \omega_r}{r'_r}$$

Note: model suggest
- Torque proportional to slip
- Torque inversely proportional to rotor resistance
- Torque proportional to stator voltage squared

Calculations Using the Simplified Model

Table 4.10-1 Induction Machine Parameters

Machine Rating										
hp	Volts	rpm	T_B (N · m)	$I_{B(abc)}$ (amps)	r_s (ohms)	X_{ls} (ohms)	X_M (ohms)	X'_{lr} (ohms)	r'_r (ohms)	J (kg · m^2)
3	220	1710	11.9	5.8	0.435	0.754	26.13	0.754	0.816	0.089
50	460	1705	198	46.8	0.087	0.302	13.08	0.302	0.228	1.662
500	2300	1773	1.98×10^3	93.6	0.262	1.206	54.02	1.206	0.187	11.06
2250	2300	1786	8.9×10^3	421.2	0.029	0.226	13.04	0.226	0.022	63.87

Note
- Inertia is proportional to power rating
- Rotor resistance is small for large machines

3 horsepower machine

$f_e := 60 \cdot Hz$

$V_{as} := \frac{220 \cdot V}{\sqrt{3}}$ $\qquad V_{as} = 127V$

$\omega_e := 2 \cdot \pi \cdot f_e$ $\qquad \omega_e = 377 \frac{rad}{s}$

$P := 4$

$r_s := 0.435\Omega$

$L_{ls} := \frac{0.754\Omega}{\omega_e}$ $\qquad L_{ls} = 2\,mH$

$L_M := \frac{26.13\Omega}{\omega_e}$ $\qquad L_M = 69.3mH$

$L'_{lr} := \frac{0.754\Omega}{\omega_e}$ $\qquad L'_{lr} = 2\,mH$

$r'_r := 0.816\Omega$

$L_{ss} := L_{ls} + L_M$ $\qquad L_{ss} = 71.3mH$

$L'_{rr} := L'_{lr} + L_M$ $\qquad L'_{rr} = 71.3mH$

$\omega_{rm} := 1710\,RPM$ $\qquad \omega_{rm} = 179 \frac{rad}{s}$

$\omega_r := \frac{P}{2} \cdot \omega_{rm}$ $\qquad \omega_r = 358 \frac{rad}{s}$

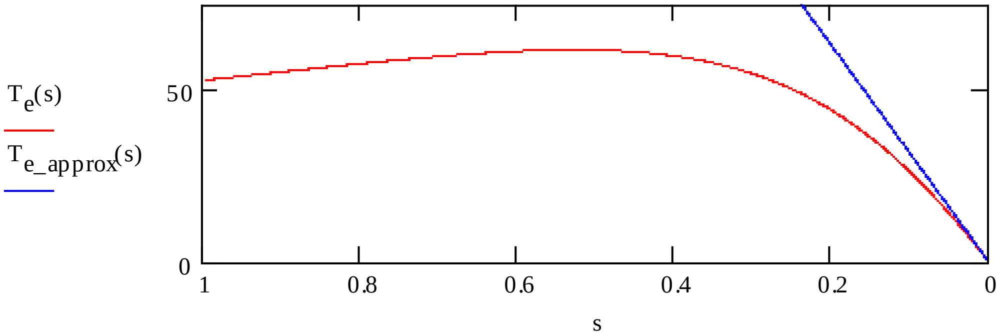
$s_{rated} := \frac{\omega_e - \omega_r}{\omega_e}$ $\qquad s_{rated} = 0.05$

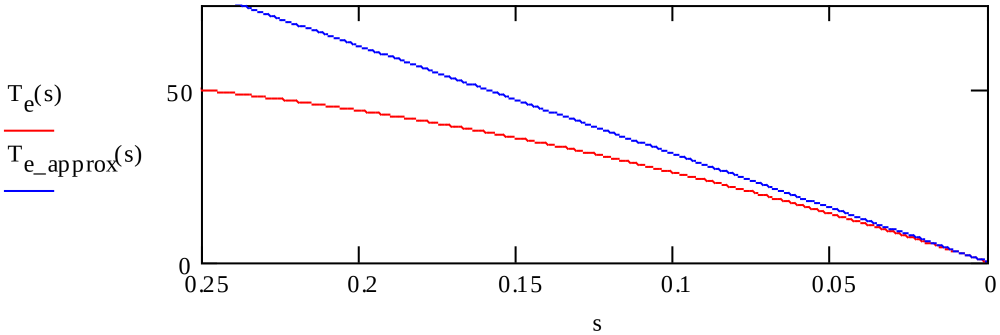
torque equations and plots

$$T_e(s) := \frac{3\cdot\frac{P}{2}\cdot\omega_e\cdot L_M{}^2\cdot r'_r\cdot s\cdot\left(\left|V_{as}\right|\right)^2}{\left[r_s\cdot r'_r + s\cdot\omega_e{}^2\cdot\left(L_M{}^2 - L_{ss}\cdot L'_{rr}\right)\right]^2 + \omega_e{}^2\cdot\left(r'_r\cdot L_{ss} + s\cdot r_s\cdot L'_{rr}\right)^2}$$

$$T_{e\_approx}(s) := 3\cdot\frac{P}{2}\cdot\frac{s\cdot\left(\left|V_{as}\right|\right)^2}{r'_r\cdot\omega_e}$$

$s := 0.0001, 0.001 .. 1$





2250 horsepower machine

$f_e := 60 \cdot Hz$

$V_{as} := \frac{2250\,V}{\sqrt{3}}$ $\quad V_{as} = 1299V$

$\omega_e := 2\cdot\pi\cdot f_e$ $\quad \omega_e = 377\frac{rad}{s}$

$P := 4$

$r_s := 0.029\Omega$

$L_{ls} := \frac{0.226\Omega}{\omega_e}$ $\quad L_{ls} = 0.599mH$

$L_M := \frac{13.04\Omega}{\omega_e}$ $\quad L_M = 34.6mH$

$L'_{lr} := \frac{0.226\Omega}{\omega_e}$ $\quad L'_{lr} = 0.599mH$

$r'_r := 0.022\Omega$

$L_{ss} := L_{ls} + L_M$ $\quad L_{ss} = 35.2mH$

$L'_{rr} := L'_{lr} + L_M$ $\quad L'_{rr} = 35.2mH$

$\omega_{rm} := 1786RPM$ $\quad \omega_{rm} = 187\frac{rad}{s}$

$\omega_r := \frac{P}{2}\cdot\omega_{rm}$ $\quad \omega_r = 374\frac{rad}{s}$

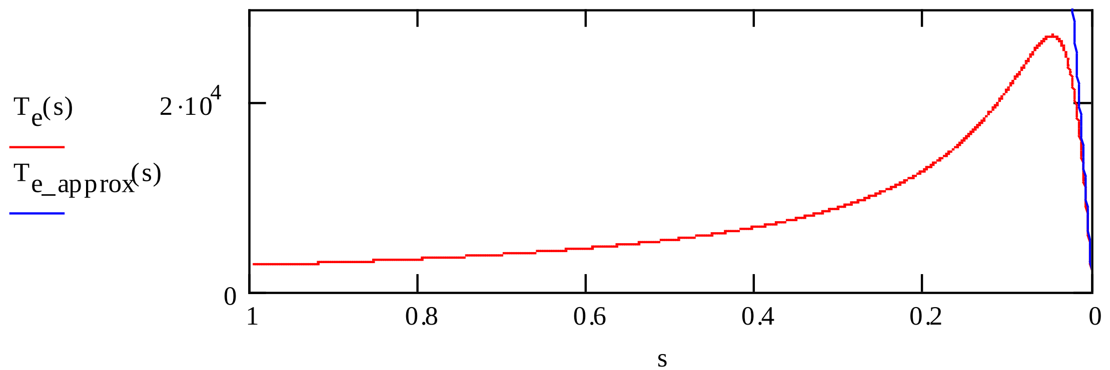
$s_{rated} := \frac{\omega_e - \omega_r}{\omega_e}$ $\quad s_{rated} = 0.0078$

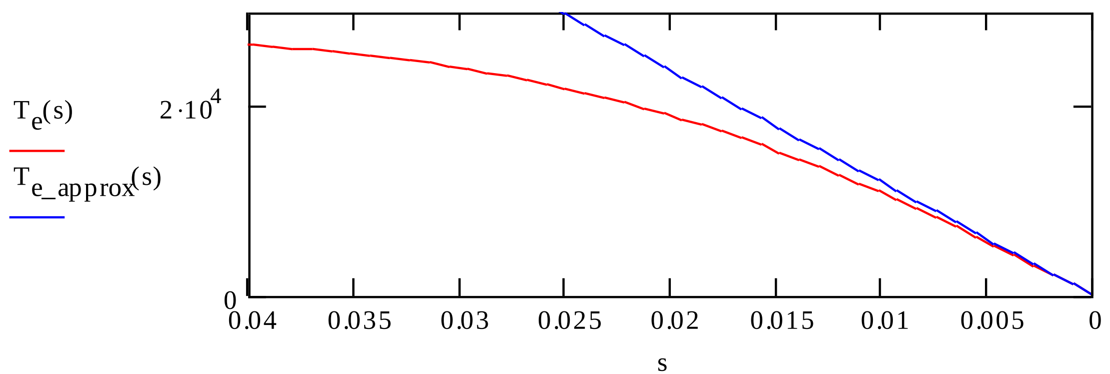
torque equations and plots

$$T_e(s) := \frac{3\cdot\frac{P}{2}\cdot\omega_e\cdot L_M^{\,2}\cdot r'_r\cdot s\cdot\left(\left|V_{as}\right|\right)^2}{\left[r_s\cdot r'_r + s\cdot\omega_e^{\,2}\cdot\left(L_M^{\,2} - L_{ss}\cdot L'_{rr}\right)\right]^2 + \omega_e^{\,2}\cdot\left(r'_r\cdot L_{ss} + s\cdot r_s\cdot L'_{rr}\right)^2}$$

$$T_{e\_approx}(s) := 3\cdot\frac{P}{2}\cdot\frac{s\cdot\left(\left|V_{as}\right|\right)^2}{r'_r\cdot\omega_e}$$

$$s := 0.0001, 0.001 .. 1$$





Induction Machine Simplified Model

Simple determination of steady-state operation

Accurate in normal operation close to no-load speed