ECE 419/619, Electric Machines and Drives, Spring 2013
Quiz #1, February 21st, 4:30pm-4:45pm

For the following problems, assume $B_m = 0$.

1. Write the armature voltage equation, torque equation, and mechanical equation for the PMDC machine.

2. The table below has PMDC motor manufacturer data for three different 24V motors.

	$\omega_{r,no-load}$ (rad/s)	r_a (Ω)	L_{AA} (μH)	J ($\mathrm{kg \cdot m^2}$)
Motor 1	746	0.75	332	$2.91 \cdot 10^{-5}$
Motor 2	503	1.33	439	$4.29 \cdot 10^{-5}$
Motor 3	448	1.09	550	$5.46 \cdot 10^{-5}$

Determine the back-emf constant k_v for each. Calculate the stall current and stall torque for each motor.

3. Which motor above is adequate for supplying 0.5N·m of torque at 380rad/s? For this motor, compute the natural frequency and damping ratio based on the transfer function given below. Is the motor over-damped or under-damped in response to a step change in load?

$$\frac{\Delta\omega_r}{\Delta T_L} = \frac{-\left(\dfrac{r_a}{JL_{AA}} + \dfrac{1}{J}s\right)}{s^2 + \left(\dfrac{r_a J + L_{AA} B_m}{J\,L_{AA}}\right)s + \left(\dfrac{r_a\,B_m + k_v^2}{J\,L_{AA}}\right)} = \frac{-\dfrac{1}{J}\left(s + \dfrac{1}{\tau_a}\right)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

PMDC machine equations

$$v_a = r_a \cdot i_a + L_{AA} \cdot \frac{d}{dt} i_a + k_v \cdot \omega_r$$

$$T_e = k_v \cdot i_a$$

$$T_e = B_m \cdot \omega_r + J \cdot \frac{d}{dt} \omega_r$$

motor data

$i := 1, 2 .. 3$

$V_a := 24 \cdot V$

$T_{e\_rated_i} :=$

$0.134 \cdot N \cdot m$
$0.134 \cdot N \cdot m$
$0.225 \cdot N \cdot m$

$I_{a\_rated_i} :=$

$5.3 \cdot A$
$4.6 \cdot A$
$5.2 \cdot A$

$I_{a\_peak_i} :=$

$32 \cdot A$
$18 \cdot A$
$22 \cdot A$

$\omega_{r\_rated_i} :=$

$649 \cdot \frac{rad}{s}$
$398 \cdot \frac{rad}{s}$
$355 \cdot \frac{rad}{s}$

$\omega_{r\_no\_load_i} :=$

$746 \cdot \frac{rad}{s}$
$503 \cdot \frac{rad}{s}$
$448 \cdot \frac{rad}{s}$

$r_{a_i} :=$

$0.75 \cdot \Omega$
$1.33 \cdot \Omega$
$1.09 \cdot \Omega$

$L_{AA_i} :=$

$332 \cdot \mu H$
$439 \cdot \mu H$
$550 \cdot \mu H$

$J_i :=$

$2.91 \cdot 10^{-5} \cdot kg \cdot m^2$
$4.29 \cdot 10^{-5} \cdot kg \cdot m^2$
$5.46 \cdot 10^{-5} \cdot kg \cdot m^2$

back-emf constants

$$k_{v_i} := \frac{V_a}{\omega_{r\_no\_load_i}}$$

$k_{v_i} =$

0.032
0.048
0.054

$\frac{V \cdot s}{rad}$

stall current and stall torque

$$I_{a\_stall_i} := \frac{V_a}{r_{a_i}}$$

$I_{a\_stall_i} =$

32	A
18	
22	

$$T_{e\_stall_i} := k_{v_i} \cdot I_{a\_stall_i}$$

$T_{e\_stall_i} =$

1.029	N·m
0.861	
1.18	

operating point

$$\omega_{ro} := 380 \cdot \frac{rad}{s}$$

$$I_{ao_i} := \frac{V_a - k_{v_i} \cdot \omega_{ro}}{r_{a_i}}$$

$I_{ao_i} =$

15.7	A
4.4	
3.3	

$$T_{eo_i} := k_{v_i} \cdot I_{ao_i}$$

$T_{eo_i} =$

0.505	N·m
0.211	
0.179	

motor 1 provides 0.5 N-m of torque at 380 rad/s

$k := 1$

natural frequency and damping ratio

$$\frac{\Delta\omega_r}{\Delta T_L} = \frac{-\left(\frac{r_a}{J \cdot L_{AA}} + \frac{1}{J} \cdot s\right)}{s^2 + \left(\frac{r_a \cdot J + L_{AA} \cdot B_m}{J \cdot L_{AA}}\right) \cdot s + \left(\frac{r_a \cdot B_m + k_v^{\,2}}{J \cdot L_{AA}}\right)}$$

$$B_m := 0 \cdot \frac{kg \cdot m^2}{s}$$

$$\alpha := \frac{1}{2} \cdot \left(\frac{r_{a_k} \cdot J_k + L_{AA_k} \cdot B_m}{J_k \cdot L_{AA_k}}\right)$$

$$\alpha = 1130 \, \frac{rad}{s}$$

$$\omega_n := \sqrt{\frac{r_{a_k} \cdot B_m + \left(k_{v_k}\right)^2}{J_k \cdot L_{AA_k}}}$$

$$\boxed{\omega_n = 327 \, \frac{rad}{s}}$$

$$\zeta := \frac{\alpha}{\omega_n}$$

$$\boxed{\zeta = 3.451}$$

overdamped