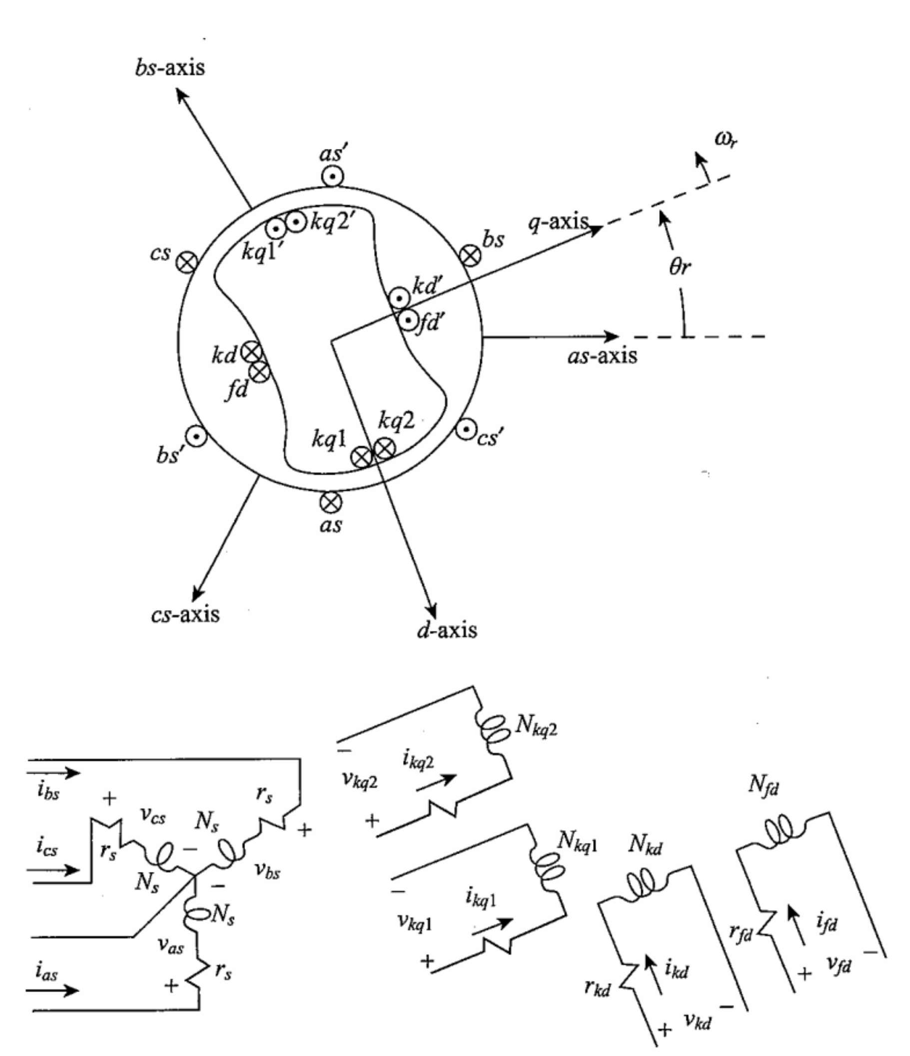
ECE 802, Electric Motor Control

Synchronous Machines

Synchronous Machines (Chapter 5)



voltage equations (stator)

$$v_{as} = r_s i_{as} + p\lambda_{as}$$

$$v_{bs} = r_s i_{bs} + p\lambda_{bs}$$

$$v_{cs} = r_s i_{cs} + p\lambda_{cs}$$

compress equations

$$\mathbf{v}_{abcs} = r_s \mathbf{i}_{abcs} + p\boldsymbol{\lambda}_{abcs}$$

voltage equations (rotor)

$$v_{kq1} = r_{rkq1} i_{kq1} + p\lambda_{kq1}$$

$$v_{kq2} = r_{rkq2} i_{kq2} + p\lambda_{kq2}$$

$$v_{fd} = r_{fd} i_{fd} + p\lambda_{fd}$$

$$v_{kd} = r_{rkd} i_{kd} + p\lambda_{kd}$$

compress equations

$$\mathbf{v}_{qdr} = \mathbf{r}_r \mathbf{i}_{qdr} + p\boldsymbol{\lambda}_{qdr}$$

Flux Linkage Equations

$$\begin{bmatrix} \boldsymbol{\lambda}_{abcs} \\ \boldsymbol{\lambda}_{qdr} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_s & \mathbf{L}_{sr} \\ (\mathbf{L}_{sr})^T & \mathbf{L}_r \end{bmatrix} \begin{bmatrix} \mathbf{i}_{abcs} \\ \mathbf{i}_{qdr} \end{bmatrix}$$

$$\mathbf{L}_s = \begin{bmatrix} L_{ls} + L_A - L_B \cos 2\theta_r & -\frac{1}{2} L_A - L_B \cos 2\left(\theta_r - \frac{\pi}{3}\right) & -\frac{1}{2} L_A - L_B \cos 2\left(\theta_r + \frac{\pi}{3}\right) \\ -\frac{1}{2} L_A - L_B \cos 2\left(\theta_r - \frac{\pi}{3}\right) & L_{ls} + L_A - L_B \cos 2\left(\theta_r - \frac{2\pi}{3}\right) & -\frac{1}{2} L_A - L_B \cos 2(\theta_r + \pi) \\ -\frac{1}{2} L_A - L_B \cos 2\left(\theta_r + \frac{\pi}{3}\right) & -\frac{1}{2} L_A - L_B \cos 2(\theta_r + \pi) & L_{ls} + L_A - L_B \cos 2\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix}$$

Stator-to-Rotor Inductance Matrix

$$\mathbf{L}_{sr} = \begin{bmatrix} L_{skq1}\cos\theta_r & L_{skq2}\cos\theta_r & L_{sfd}\sin\theta_r & L_{skd}\sin\theta_r \\ L_{skq1}\cos\left(\theta_r - \frac{2\pi}{3}\right) & L_{skq2}\cos\left(\theta_r - \frac{2\pi}{3}\right) & L_{sfd}\sin\left(\theta_r - \frac{2\pi}{3}\right) & L_{skd}\sin\left(\theta_r - \frac{2\pi}{3}\right) \\ L_{skq1}\cos\left(\theta_r + \frac{2\pi}{3}\right) & L_{skq2}\cos\left(\theta_r + \frac{2\pi}{3}\right) & L_{sfd}\sin\left(\theta_r + \frac{2\pi}{3}\right) & L_{skd}\sin\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix}$$

define inductances

$$L_{mq} = \tfrac{3}{2}(L_A - L_B)$$

$$L_{md} = \tfrac{3}{2}(L_A + L_B)$$

using chapter 1 procedures, magnetizing inductance can be written in terms of L_{mq} and L_{md} as

$$L_{skq1} = \left(\frac{N_{kq1}}{N_s}\right)\left(\frac{2}{3}\right)L_{mq} \qquad L_{sfd} = \left(\frac{N_{fd}}{N_s}\right)\left(\frac{2}{3}\right)L_{md}$$

$$L_{skq2} = \left(\frac{N_{kq2}}{N_s}\right)\left(\frac{2}{3}\right)L_{mq} \qquad L_{skd} = \left(\frac{N_{kd}}{N_s}\right)\left(\frac{2}{3}\right)L_{md}$$

Rotor Inductance Matrix

$$\mathbf{L}_r = \begin{bmatrix} L_{lkq1} + L_{mkq1} & L_{kq1kq2} & 0 & 0 \\ L_{kq1kq2} & L_{lkq2} + L_{mkq2} & 0 & 0 \\ 0 & 0 & L_{lfd} + L_{mfd} & L_{fdkd} \\ 0 & 0 & L_{fdkd} & L_{lkd} + L_{mkd} \end{bmatrix}$$

$$L_{mkq1} = \left(\frac{N_{kq1}}{N_s}\right)^2 \left(\frac{2}{3}\right) L_{mq}$$

$$L_{mfd} = \left(\frac{N_{fd}}{N_s}\right)^2 \left(\frac{2}{3}\right) L_{md}$$

$$L_{kq1kq2} = \left(\frac{N_{kq2}}{N_{kq1}}\right) L_{mkq1} = \left(\frac{N_{kq1}}{N_{kq2}}\right) L_{mkq2}$$

$$L_{fdkd} = \left(\frac{N_{kd}}{N_{fd}}\right) L_{mfd} = \left(\frac{N_{fd}}{N_{kd}}\right) L_{mkd}$$

$$L_{mkq2} = \left(\frac{N_{kq2}}{N_s}\right)^2 \left(\frac{2}{3}\right) L_{mq}$$

$$L_{mkd} = \left(\frac{N_{kd}}{N_s}\right)^2 \left(\frac{2}{3}\right) L_{md}$$

Refer Rotor to Stator

define new rotor variables

$$v'_j = \left(\frac{N_s}{N_j}\right) v_j \qquad \lambda'_j = \left(\frac{N_s}{N_j}\right) \lambda_j \qquad i'_j = \left(\frac{2}{3}\right)\left(\frac{N_j}{N_s}\right) i_j$$

where *j* can be used as *kq*1, *kq*2, *fd*, or *kd*

$$v_{fd}^r = r_{fd} i_{fd}^r + p\lambda_{fd}^r \qquad i_{fd}^r = \frac{3}{2}\frac{N_s}{N_j}\ i'^r_{fd}$$

$$r'_{fd} = \frac{3}{2}\left(\frac{N_s}{N_{fd}}\right)^2 r_{fd}$$

Refer Voltage Equations

rotor equations

$$\mathbf{r}'_r = \begin{bmatrix} r'_{kq1} & 0 & 0 & 0 \\ 0 & r'_{kq2} & 0 & 0 \\ 0 & 0 & r'_{fd} & 0 \\ 0 & 0 & 0 & r'_{kd} \end{bmatrix}$$

$$r'_j = \left(\frac{3}{2}\right)\left(\frac{N_s}{N_j}\right)^2 r_j$$

stator equations

$$\mathbf{v}_{abcs} = r_s \mathbf{i}_{abcs} + p\boldsymbol{\lambda}_{abcs}$$

Refer Stator Flux Linkage Equations

$$\mathbf{L}'_{sr} = \begin{bmatrix} L_{mq}\cos\theta_r & L_{mq}\cos\theta_r & L_{md}\sin\theta_r & L_{md}\sin\theta_r \\ L_{mq}\cos\left(\theta_r - \frac{2\pi}{3}\right) & L_{mq}\cos\left(\theta_r - \frac{2\pi}{3}\right) & L_{md}\sin\left(\theta_r - \frac{2\pi}{3}\right) & L_{md}\sin\left(\theta_r - \frac{2\pi}{3}\right) \\ L_{mq}\cos\left(\theta_r + \frac{2\pi}{3}\right) & L_{mq}\cos\left(\theta_r + \frac{2\pi}{3}\right) & L_{md}\sin\left(\theta_r + \frac{2\pi}{3}\right) & L_{md}\sin\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix}$$

Note: all magnetizing flux linkages are now in terms of L_{mq} and L_{md}

Refer Rotor Flux Linkages

$$\left(\frac{N_s}{N_j}\right)\boldsymbol{\lambda}_{qdr} = \boldsymbol{\lambda}'_{qdr} = \left(\frac{N_s}{N_j}\right)(\mathbf{L}_{sr})^T \mathbf{i}_{abcs} + \left(\frac{3}{2}\right)\left(\frac{N_s}{N_j}\right)^2 \mathbf{L}_r \mathbf{i}'_{qdr} = \left(\frac{2}{3}\right)(\mathbf{L}'_{sr})^T \mathbf{i}_{abcs} + \mathbf{L}'_r \mathbf{i}'_{qdr}$$

$$\mathbf{L}'_r = \begin{bmatrix} L'_{lkq1} + L_{mq} & L_{mq} & 0 & 0 \\ L_{mq} & L'_{lkq2} + L_{mq} & 0 & 0 \\ 0 & 0 & L'_{lfd} + L_{md} & L_{md} \\ 0 & 0 & L_{md} & L'_{lkd} + L_{md} \end{bmatrix}$$

where $$L'_{lj} = \left(\frac{3}{2}\right)\left(\frac{N_s}{N_j}\right)^2 L_{lj}$$

finally, we have

$$\begin{bmatrix} \boldsymbol{\lambda}_{abcs} \\ \boldsymbol{\lambda}'_{qdr} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_s & \mathbf{L}'_{sr} \\ \frac{2}{3}(\mathbf{L}'_{sr})^T & \mathbf{L}'_r \end{bmatrix} \begin{bmatrix} \mathbf{i}_{abcs} \\ \mathbf{i}'_{qdr} \end{bmatrix}$$

Field Energy

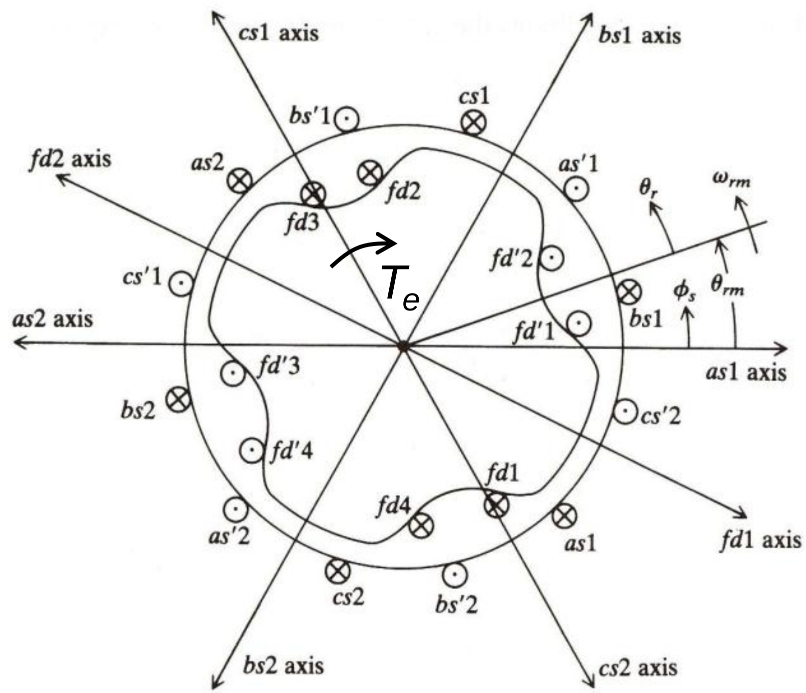
$$W_f = \tfrac{1}{2}(\mathbf{i}_{abcs})^T(\mathbf{L}_s - L_{ls}\mathbf{I})\mathbf{i}_{abcs}(\mathbf{i}_{abcs})^T\mathbf{L}'_{sr}\mathbf{i}'_{qdr} + \tfrac{1}{2}(\tfrac{3}{2})(\mathbf{i}'_{qdr})^T(\mathbf{L}'_r - \mathbf{L}'_{lr}\mathbf{I})\mathbf{i}'_{qdr}$$

where $$\mathbf{L}'_{lr} = \begin{bmatrix} L'_{lkq1} & 0 & 0 & 0 \\ 0 & L'_{lkq2} & 0 & 0 \\ 0 & 0 & L'_{lfd} & 0 \\ 0 & 0 & 0 & L'_{lkd} \end{bmatrix}$$

note: for a *P* pole machine,

$$\theta_r = \frac{P}{2}\theta_{rm}$$

also note positive torque opposes rotation in chapter 5



Torque Equation

$$W_c = W_f$$

$$\frac{\partial W_c}{\partial \theta_{rm}} = \frac{\partial W_c}{\frac{2}{P}\partial \theta_r} = \frac{P}{2}\frac{\partial W_c}{\partial \theta_r}$$

$$T_e = \left(\frac{P}{2}\right)\left\{\begin{aligned} &\frac{(L_{md} - L_{mq})}{3}\left[\begin{aligned}&\left(i_{as}^2 - \frac{1}{2}i_{bs}^2 - \frac{1}{2}i_{cs}^2 - i_{as}i_{bs} - i_{as}i_{cs} + 2i_{bs}i_{cs}\right)\sin\left(2\theta_r\right) \\ &+\frac{\sqrt{3}}{2}\left(i_{bs}^2 + i_{cs}^2 - 2i_{as}i_{bs} + 2i_{as}i_{cs}\right)\cos\left(2\theta_r\right)\end{aligned}\right] \\ &+L_{mq}\left(i'_{kq1} + i'_{kq2}\right)\left[\left(i_{as} - \frac{1}{2}i_{bs} - \frac{1}{2}i_{cs}\right)\sin\left(\theta_r\right) - \frac{\sqrt{3}}{2}\left(i_{bs} - i_{cs}\right)\cos\left(\theta_r\right)\right] \\ &+L_{md}\left(i'_{fd} + i'_{kd}\right)\left[\left(i_{as} - \frac{1}{2}i_{bs} - \frac{1}{2}i_{cs}\right)\cos\left(\theta_r\right) + \frac{\sqrt{3}}{2}\left(i_{bs} - i_{cs}\right)\sin\left(\theta_r\right)\right] \end{aligned}\right\}$$

Rotor Reference Frame Model

- Inductances do not vary with θ_r
- Inductance matrices are sparse
- There is a constant dc operating point (model linearization and classical control theory apply)

Transform to Rotor Reference Frame (Park's Equations)

stator voltage equation

$$\mathbf{v}_{abcs} = r_s \mathbf{i}_{abcs} + p\boldsymbol{\lambda}_{abcs}$$

$$K_s^r = \frac{2}{3}\begin{bmatrix} \cos(\theta_r) & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\ \sin(\theta_r) & \sin\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$K_s^r \mathbf{v}_{abcs} = r_s K_s^r \mathbf{i}_{abcs} + K_s^r p\boldsymbol{\lambda}_{abcs}$$

$$\mathbf{v}_{qd0s}^r = r_s \mathbf{i}_{qd0s}^r + K_s^r p\left\{\left(K_s^r\right)^{-1} \boldsymbol{\lambda}_{qd0s}^r\right\}$$

$$\mathbf{v}_{qd0s}^r = r_s \mathbf{i}_{qd0s}^r + K_s^r p\left\{\left(K_s^r\right)^{-1}\right\} \boldsymbol{\lambda}_{qd0s}^r + K_s^r \left(K_s^r\right)^{-1} p\,\boldsymbol{\lambda}_{qd0s}^r$$

$$K_s^r p\left\{\left(K_s^r\right)^{-1}\right\} = \omega_r \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{v}_{qd0s}^r = r_s \mathbf{i}_{qd0s}^r + \omega_r \boldsymbol{\lambda}_{dqs}^r + p\,\boldsymbol{\lambda}_{qd0s}^r$$

where $\boldsymbol{\lambda}_{dqs}^r = \begin{bmatrix} \lambda_{ds}^r \\ -\lambda_{qs}^r \\ 0 \end{bmatrix}$

Transform Flux Linkages

stator

$$\boldsymbol{\lambda}_{abcs} = \mathbf{L}_s \mathbf{i}_{abcs} + \mathbf{L}'_{sr} \mathbf{i}'^{r}_{qdr}$$

$$K_s^r \mathbf{L}_s (K_s^r)^{-1} = \begin{bmatrix} L_{ls} + L_{mq} & 0 & 0 \\ 0 & L_{ls} + L_{md} & 0 \\ 0 & 0 & L_{ls} \end{bmatrix}$$

$$K_s^r \mathbf{L}'_{sr} = \begin{bmatrix} L_{mq} & L_{mq} & 0 & 0 \\ 0 & 0 & L_{md} & L_{md} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

rotor

$$\boldsymbol{\lambda}'^{r}_{qdr} = \frac{2}{3}\left(\mathbf{L}_{sr}\right)^T \mathbf{i}_{abcs} + \mathbf{L}'_r \mathbf{i}'^{r}_{qdr}$$

$$\tfrac{2}{3}(\mathbf{L}'_{sr})^T (\mathbf{K}_s^r)^{-1} = \begin{bmatrix} L_{mq} & 0 & 0 \\ L_{mq} & 0 & 0 \\ 0 & L_{md} & 0 \\ 0 & L_{md} & 0 \end{bmatrix}$$

Final Set of Equations (Expanded)

voltage equations

$$v_{qs}^{r} = r_s i_{qs}^{r} + \omega_r \lambda_{ds}^{r} + p\lambda_{qs}^{r}$$

$$v_{ds}^{r} = r_s i_{ds}^{r} - \omega_r \lambda_{qs}^{r} + p\lambda_{ds}^{r}$$

$$v_{0s} = r_s i_{0s} + p\lambda_{0s}$$

$$v'^{r}_{kq1} = r'_{kq1} i'^{r}_{kq1} + p\lambda'^{r}_{kq1}$$

$$v'^{r}_{kq2} = r'_{kq2} i'^{r}_{kq2} + p\lambda'^{r}_{kq2}$$

$$v'^{r}_{fd} = r'_{fd} i'^{r}_{fd} + p\lambda'^{r}_{fd}$$

$$v'^{r}_{kd} = r'_{kd} i'^{r}_{kd} + p\lambda'^{r}_{kd}$$

flux linkage equations

$$\lambda_{qs}^{r} = L_{ls} i_{qs}^{r} + L_{mq}\left(i_{qs}^{r} + i'^{r}_{kq1} + i'^{r}_{kq2}\right)$$

$$\lambda_{ds}^{r} = L_{ls} i_{ds}^{r} + L_{md}\left(i_{ds}^{r} + i'^{r}_{fd} + i'^{r}_{kd}\right)$$

$$\lambda_{0s} = L_{ls} i_{0s}$$

$$\lambda'^{r}_{kq1} = L'_{lkq1} i'^{r}_{kq1} + L_{mq}\left(i_{qs}^{r} + i'^{r}_{kq1} + i'^{r}_{kq2}\right)$$

$$\lambda'^{r}_{kq2} = L'_{lkq2} i'^{r}_{kq2} + L_{mq}\left(i_{qs}^{r} + i'^{r}_{kq1} + i'^{r}_{kq2}\right)$$

$$\lambda'^{r}_{fd} = L'_{lfd} i'^{r}_{fd} + L_{md}\left(i_{ds}^{r} + i'^{r}_{fd} + i'^{r}_{kd}\right)$$

$$\lambda'^{r}_{kd} = L'_{lkd} i'^{r}_{kd} + L_{md}\left(i_{ds}^{r} + i'^{r}_{fd} + i'^{r}_{kd}\right)$$

Transform Torque Equation

$$W_c = W_f = \tfrac{1}{2}(\mathbf{i}_{abcs})^T(\mathbf{L}_s - L_{ls}\mathbf{I})\mathbf{i}_{abcs}(\mathbf{i}_{abcs})^T\mathbf{L}'_{sr}\mathbf{i}'_{qdr} + \tfrac{1}{2}(\tfrac{3}{2})(\mathbf{i}'_{qdr})^T(\mathbf{L}'_r - \mathbf{L}'_{lr}\mathbf{I})\mathbf{i}'_{qdr}$$

$$T_e = -\frac{\partial W_c}{\partial \theta_{rm}} \qquad \theta_r = \frac{P}{2}\theta_{rm}$$

substituting equations of transformation

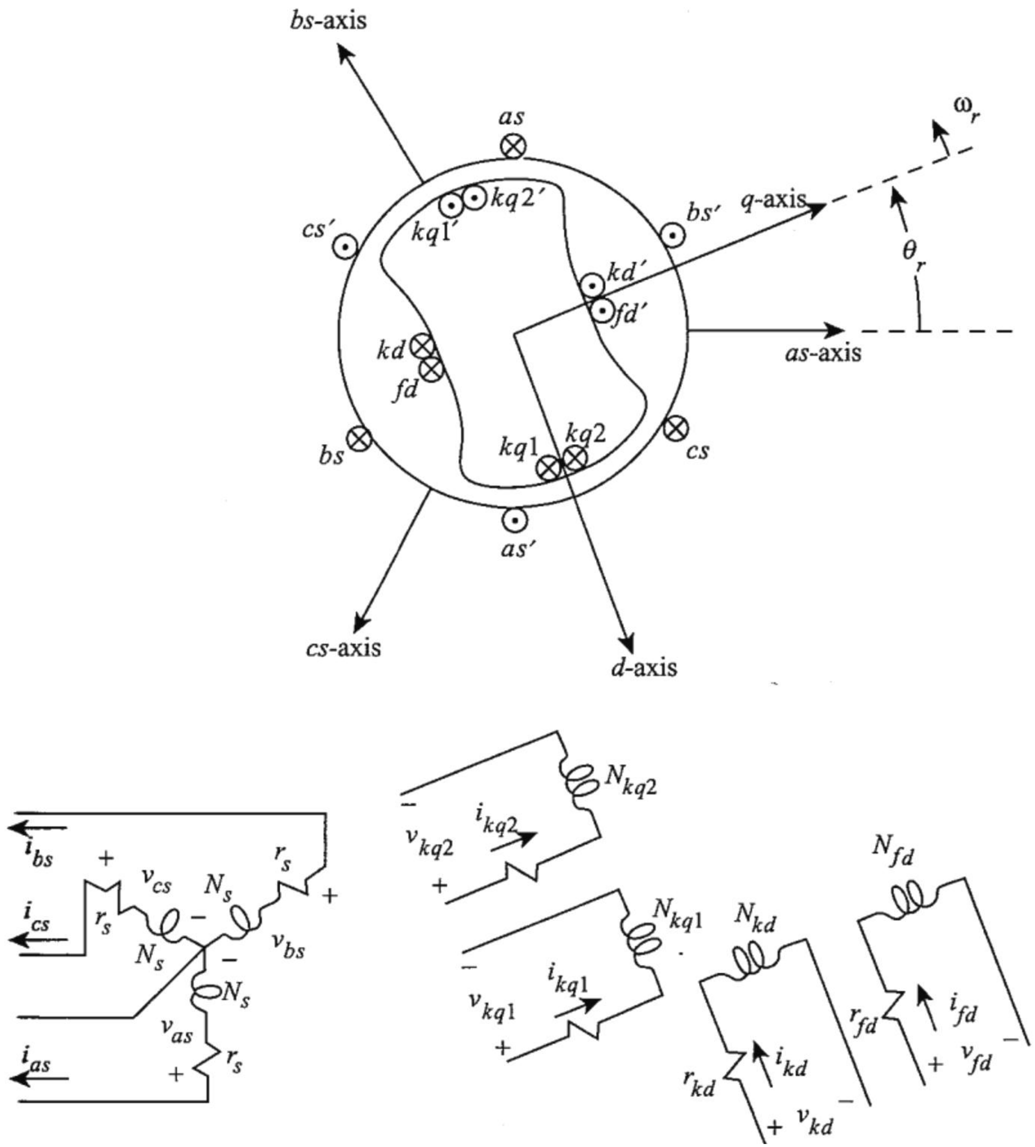
$$T_e = \frac{3}{2}\left(\frac{P}{2}\right)(\lambda_{qs}^r i_{ds}^r - \lambda_{ds}^r i_{qs}^r)$$

in terms of currents

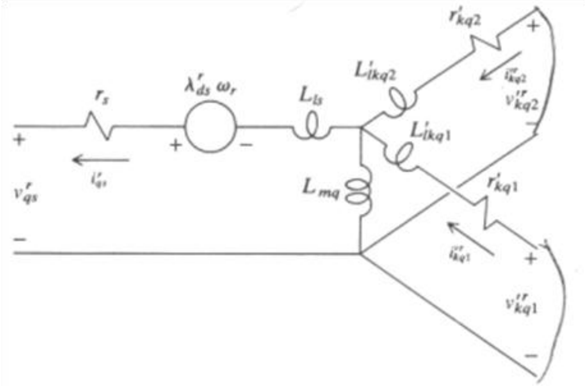
$$T_e = \frac{3}{2}\left(\frac{P}{2}\right)\left[L_{mq}\left(i_{qs}^r + i'^r_{kq1} + i'^r_{kq2}\right)i_{ds}^r - L_{md}\left(i_{ds}^r + i'^r_{fd} + i'^r_{kd}\right)i_{qs}^r\right]$$

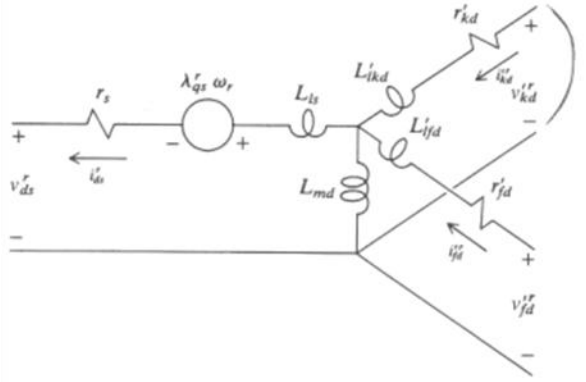
$$= \frac{3}{2}\left(\frac{P}{2}\right)\left[\left(L_{md} - L_{mq}\right)i_{qs}^r i_{ds}^r + L_{md}i'^r_{fd}i_{qs}^r + L_{md}i'^r_{kd}i_{qs}^r + L_{mq}\left(i'^r_{kq1} + i'^r_{kq2}\right)i_{ds}^r\right]$$

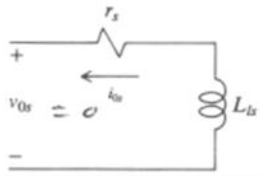
Stator Currents Positive Out of Machine



Synchronous Machine
q-d Equivalent Circuit







voltage equations

$$v_{qs}^r = -r_s i_{qs}^r + \omega_r \lambda_{ds}^r + p\lambda_{qs}^r$$

$$v_{ds}^r = -r_s i_{ds}^r - \omega_r \lambda_{qs}^r + p\lambda_{ds}^r$$

$$v_{0s} = -r_s i_{0s} + p\lambda_{0s}$$

$$v'^{r}_{kq1} = r'_{kq1} i'^{r}_{kq1} + p\lambda'^{r}_{kq1}$$

$$v'^{r}_{kq2} = r'_{kq2} i'^{r}_{kq2} + p\lambda'^{r}_{kq2}$$

$$v'^{r}_{fd} = r'_{fd} i'^{r}_{fd} + p\lambda'^{r}_{fd}$$

$$v'^{r}_{kd} = r'_{kd} i'^{r}_{kd} + p\lambda'^{r}_{kd}$$

flux linkage equations

$$\lambda_{qs}^r = -L_{ls} i_{qs}^r + L_{mq}\left(-i_{qs}^r + i'^{r}_{kq1} + i'^{r}_{kq2}\right)$$

$$\lambda_{ds}^r = -L_{ls} i_{ds}^r + L_{md}\left(-i_{ds}^r + i'^{r}_{fd} + i'^{r}_{kd}\right)$$

$$\lambda_{0s} = -L_{ls} i_{0s}$$

$$\lambda'^{r}_{kq1} = L'_{lkq1} i'^{r}_{kq1} + L_{mq}\left(-i_{qs}^r + i'^{r}_{kq1} + i'^{r}_{kq2}\right)$$

$$\lambda'^{r}_{kq2} = L'_{lkq2} i'^{r}_{kq2} + L_{mq}\left(-i_{qs}^r + i'^{r}_{kq1} + i'^{r}_{kq2}\right)$$

$$\lambda'^{r}_{fd} = L'_{lfd} i'^{r}_{fd} + L_{md}\left(-i_{ds}^r + i'^{r}_{fd} + i'^{r}_{kd}\right)$$

$$\lambda'^{r}_{kd} = L'_{lkd} i'^{r}_{kd} + L_{md}\left(-i_{ds}^r + i'^{r}_{fd} + i'^{r}_{kd}\right)$$

Torque equation

$$T_e = \frac{3}{2}\left(\frac{P}{2}\right)(\lambda_{qs}^r i_{ds}^r - \lambda_{ds}^r i_{qs}^r)$$

Mechanical equation

$$T_I - T_e = J\left(\frac{2}{P}\right)p\omega_r$$

Synchronous Machine Simulation

Solving for Currents

1. Invert the inductance matrix

$$\begin{bmatrix} -i_{qs}^{r} \\ i'^{r}_{kq1} \\ i'^{r}_{kq2} \end{bmatrix} = \begin{bmatrix} L_{ls}+L_{mq} & L_{mq} & L_{mq} \\ L_{mq} & L'_{lkq1}+L_{mq} & L_{mq} \\ L_{mq} & L_{mq} & L'_{lkq2}+L_{mq} \end{bmatrix}^{-1} \begin{bmatrix} \lambda_{qs}^{r} \\ \lambda'^{r}_{kq1} \\ \lambda'^{r}_{kq2} \end{bmatrix}$$

$$\begin{bmatrix} -i_{ds}^{r} \\ i'^{r}_{fd} \\ i'^{r}_{kd} \end{bmatrix} = \begin{bmatrix} L_{ls}+L_{md} & L_{md} & L_{md} \\ L_{md} & L'_{lfd}+L_{md} & L_{md} \\ L_{md} & L_{md} & L'_{lkd}+L_{md} \end{bmatrix}^{-1} \begin{bmatrix} \lambda_{ds}^{r} \\ \lambda'^{r}_{fd} \\ \lambda'^{r}_{kd} \end{bmatrix}$$

Note: inductance matrix is constant in the *q*-*d* model

Currents in Terms of Flux Linkages

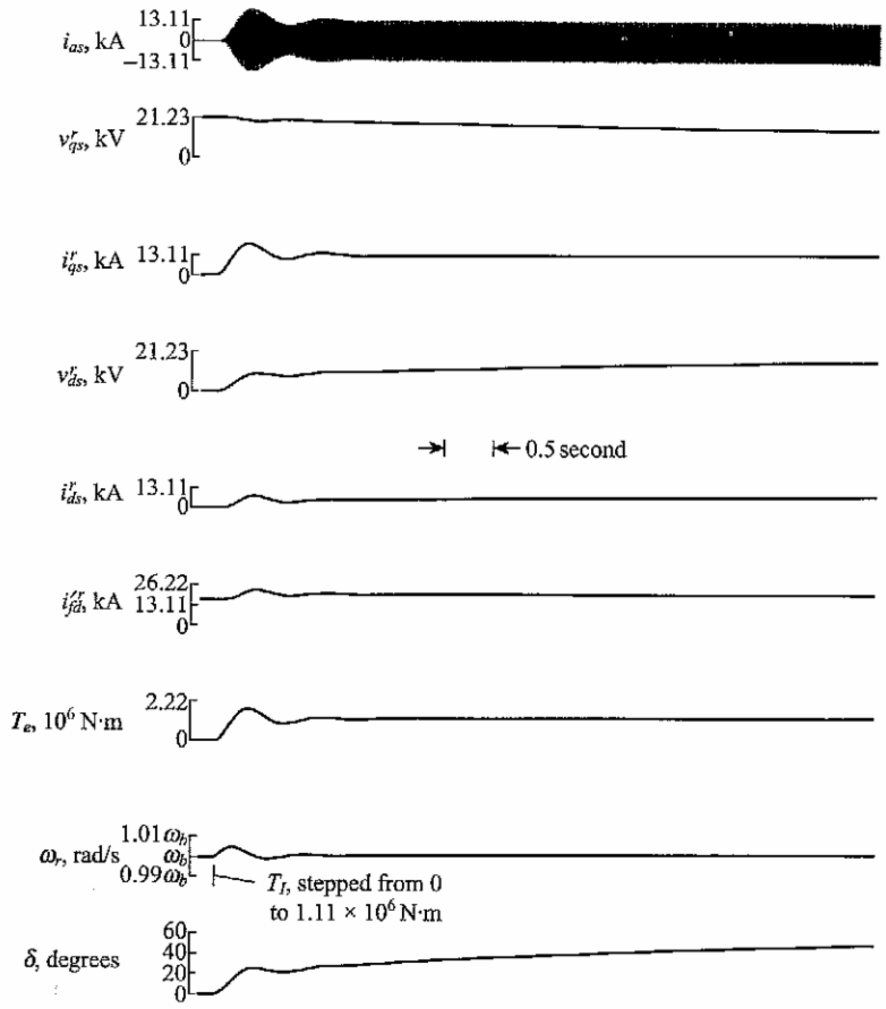
2. write in terms of magnetizing flux linkages

Book Simulation Example

TABLE 5.10-2. Steam Turbine Generator

Rating: 835 MVA
Line to line voltage: 26 kV
Power factor: 0.85
Poles: 2
Speed: 3600 r/min
Combined inertia of generator and turbine
$J = 0.0658 \times 10^6$ J · s², or $WR^2 = 1.56 \times 10^6$ lbm·ft² $H = 5.6$ seconds
Parameters in ohms and per unit
$r_s = 0.00243\,\Omega$, 0.003 pu
$X_{ls} = 0.1538\,\Omega$, 0.19 pu
$X_q = 1.457\,\Omega$, 1.8 pu $X_d = 1.457\,\Omega$, 1.8 pu
$r'_{kq1} = 0.00144\,\Omega$, 0.00178 pu $r'_{fd} = 0.00075\,\Omega$, 0.000929 pu
$X'_{lkq1} = 0.6578\,\Omega$, 0.8125 pu $X'_{lfd} = 0.1145\,\Omega$, 0.1414 pu
$r'_{kq2} = 0.00681\,\Omega$, 0.00841 pu $r'_{kd} = 0.01080\,\Omega$, 0.01334 pu
$X'_{lkq2} = 0.07602\,\Omega$, 0.0939 pu $X'_{lkd} = 0.06577\,\Omega$, 0.08125 pu

Book Simulation Example (Step Change in Torque)



5.10-6. Dynamic performance of a steam turbine generator during a step increase t torque from 0% to 50% rated.

Book Simulation Example (Three-Phase Fault)

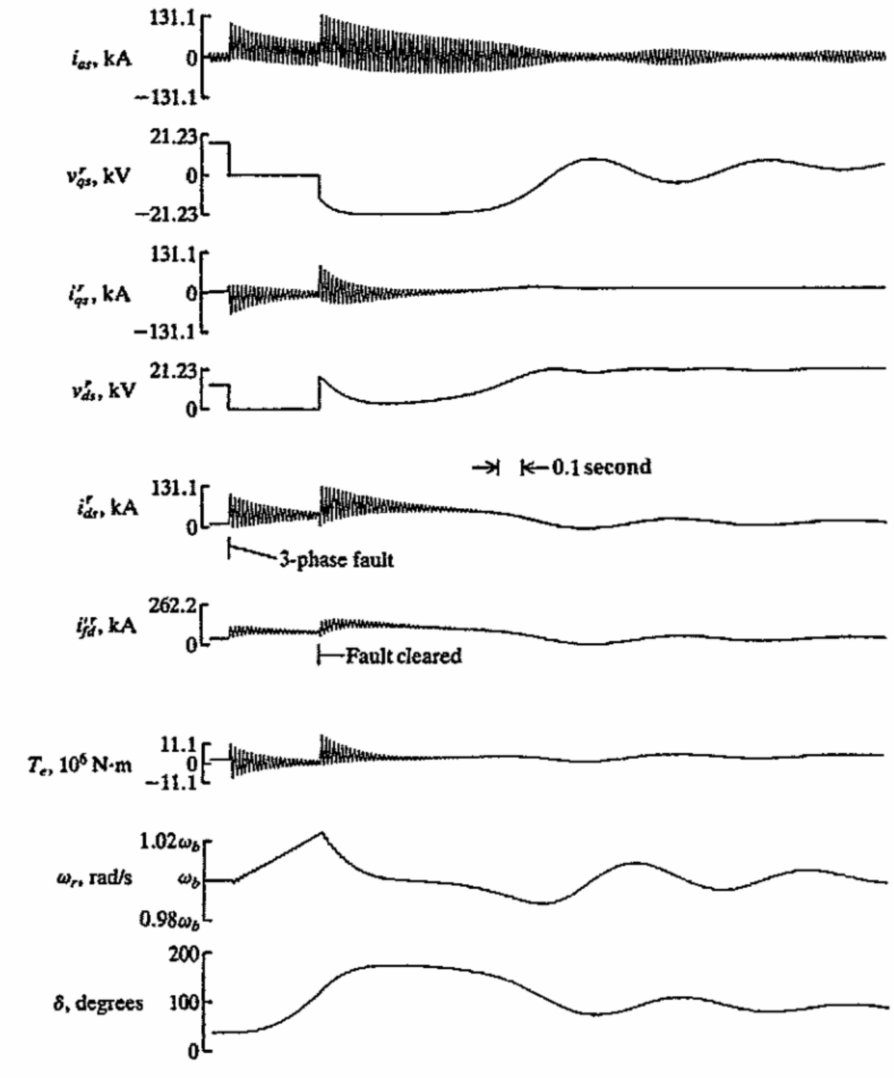


Figure 5.10-10. Dynamic performance of a steam turbine generator during a three-phase fault at the terminals.

Synchronous Machine Substitute Variables

Define

$$\Psi = \omega_b \lambda$$

$$X = \omega_b L$$

where ω_b is the base, or rated, electrical frequency

Machine Equations

$$v_{qs}^r = -r_s i_{qs}^r + \frac{\omega_r}{\omega_b}\psi_{ds}^r + \frac{p}{\omega_b}\psi_{qs}^r$$
$$v_{ds}^r = -r_s i_{ds}^r - \frac{\omega_r}{\omega_b}\psi_{qs}^r + \frac{p}{\omega_b}\psi_{ds}^r$$
$$v_{0s} = -r_s i_{0s} + \frac{p}{\omega_b}\psi_{0s}$$
$$v_{kq1}^{\prime r} = r_{kq1}^{\prime} i_{kq1}^{\prime r} + \frac{p}{\omega_b}\psi_{kq1}^{\prime r}$$
$$v_{kq2}^{\prime r} = r_{kq2}^{\prime} i_{kq2}^{\prime r} + \frac{p}{\omega_b}\psi_{kq2}^{\prime r}$$
$$v_{fd}^{\prime r} = r_{fd}^{\prime} i_{fd}^{\prime r} + \frac{p}{\omega_b}\psi_{fd}^{\prime r}$$
$$v_{kd}^{\prime r} = r_{kd}^{\prime} i_{kd}^{\prime r} + \frac{p}{\omega_b}\psi_{kd}^{\prime r}$$

$$\psi_{qs}^r = -X_{ls} i_{qs}^r + X_{mq}(-i_{qs}^r + i_{kq1}^{\prime r} + i_{kq2}^{\prime r})$$
$$\psi_{ds}^r = -X_{ls} i_{ds}^r + X_{md}(-i_{ds}^r + i_{fd}^{\prime r} + i_{kd}^{\prime r})$$
$$\psi_{0s} = -X_{ls} i_{0s}$$
$$\psi_{kq1}^{\prime r} = X_{lkq1}^{\prime} i_{kq1}^{\prime r} + X_{mq}(-i_{qs}^r + i_{kq1}^{\prime r} + i_{kq2}^{\prime r})$$
$$\psi_{kq2}^{\prime r} = X_{lkq2}^{\prime} i_{kq2}^{\prime r} + X_{mq}(-i_{qs}^r + i_{kq1}^{\prime r} + i_{kq2}^{\prime r})$$
$$\psi_{fd}^{\prime r} = X_{lfd}^{\prime} i_{fd}^{\prime r} + X_{md}(-i_{ds}^r + i_{fd}^{\prime r} + i_{kd}^{\prime r})$$
$$\psi_{kd}^{\prime r} = X_{lkd}^{\prime} i_{kd}^{\prime r} + X_{md}(-i_{ds}^r + i_{fd}^{\prime r} + i_{kd}^{\prime r})$$

Steady-State Calculations

constant V_s and constant $\omega_e = 2\pi f$

also, $\omega_r = \omega_e$ and $V_{0s} = I_{0s} = \Lambda_{0s} = 0$

$$v_{as} = \sqrt{2} V_s \cos\left(\omega_e t + \theta_{ev}(0)\right) \qquad i_{as} = \sqrt{2}\, I_s \cos\left(\omega_e t + \theta_{ei}(0)\right)$$

$$v_{bs} = \sqrt{2} V_s \cos\left(\omega_e t + \theta_{ev}(0) - \tfrac{2\pi}{3}\right) \qquad i_{bs} = \sqrt{2}\, I_s \cos\left(\omega_e t + \theta_{ei}(0) - \tfrac{2\pi}{3}\right)$$

$$v_{cs} = \sqrt{2} V_s \cos\left(\omega_e t + \theta_{ev}(0) + \tfrac{2\pi}{3}\right) \qquad i_{cs} = \sqrt{2}\, I_s \cos\left(\omega_e t + \theta_{ei}(0) + \tfrac{2\pi}{3}\right)$$

voltages and currents in the rotor reference frame

$$V_{qs}^{r} = \sqrt{2} V_s \cos(-\delta) \qquad \delta = \theta_r - \theta_{ev} \qquad \theta_{ev} = \omega_e t + \theta_{ev}(0)$$

$$V_{ds}^{r} = -\sqrt{2} V_s \sin(-\delta)$$

$$I_{qs}^{r} = \sqrt{2}\, I_s \cos\left(\theta_{ei}(0) - \theta_{ev}(0) - \delta\right)$$

$$I_{ds}^{r} = -\sqrt{2}\, I_s \sin\left(\theta_{ei}(0) - \theta_{ev}(0) - \delta\right)$$

Voltage and Current Phasors

$$\tilde{V}_{as} = V_s e^{j0} \qquad \tilde{I}_{as} = I_s e^{j\left[\theta_{ei}(0)-\theta_{ev}(0)\right]}$$

note

$$V_{qs}^{r} = \mathrm{Re}\left\{\sqrt{2}\,V_s e^{-j\delta}\right\} \qquad I_{qs}^{r} = \mathrm{Re}\left\{\sqrt{2}\,I_s e^{j\left[\theta_{ei}(0)-\theta_{ev}(0)\right]} e^{-j\delta}\right\}$$

$$V_{ds}^{r} = \mathrm{Re}\left\{j\sqrt{2}\,V_s e^{-j\delta}\right\} \qquad I_{ds}^{r} = \mathrm{Re}\left\{j\sqrt{2}\,I_s e^{j\left[\theta_{ei}(0)-\theta_{ev}(0)\right]} e^{-j\delta}\right\}$$

also

$$V_{qs}^{r} - jV_{ds}^{r} = \sqrt{2}\,V_s\left[\cos(-\delta) + j\sin(-\delta)\right] = \sqrt{2}V_s e^{-j\delta}$$

so

$$\sqrt{2}\,\tilde{V}_{as} e^{-j\delta} = V_{qs}^{r} - jV_{ds}^{r}$$

Steady-State Equations

steady-state *q-d* variables are dc so

$$p\Psi_{qs}^{r} = p\Psi_{ds}^{r} = p\Psi'^{r}_{kq1} = p\Psi'^{r}_{kq2} = p\Psi'^{r}_{fd} = p\Psi'^{r}_{kd} = 0$$

also $I'^{r}_{kq1} = I'^{r}_{kq2} = I'^{r}_{kd} = 0$ because $V'^{r}_{kq1} = V'^{r}_{kq2} = V'^{r}_{kd} = 0$

$$V_{qs}^{r} = -r_s\, I_{qs}^{r} - \frac{\omega_e}{\omega_b} X_d\, I_{ds}^{r} + \frac{\omega_e}{\omega_b} X_{md}\, I'^{r}_{fd}$$

$$X_d = X_{ls} + X_{md}$$

$$V_{ds}^{r} = -r_s\, I_{ds}^{r} + \frac{\omega_e}{\omega_b} X_q\, I_{qs}^{r}$$

$$X_q = X_{ls} + X_{mq}$$

$$V'^{r}_{fd} = r'_{fd}\, I'^{r}_{fd}$$

Phasor Diagram Development

Per-Phase Circuit Model

Steady-State Torque

$$P_{mech} = -\frac{3}{2}\left[\frac{\omega_e}{\omega_b}\left(X_d - X_q\right) I_{qs}^r I_{ds}^r - \frac{\omega_e}{\omega_b} X_{md} I'^r_{fd} I_{qs}^r\right] = T_{e,ss}\,\omega_{rm} = T_{e,ss}\left(\frac{2}{P}\right)\omega_r$$

$$\omega_r = \omega_e$$

$$T_{e,ss} = -\frac{3}{2}\left(\frac{P}{2}\right)\left(\frac{1}{\omega_b}\right)\left[\left(X_d - X_q\right) I_{qs}^r I_{ds}^r - X_{md} I'^r_{fd} I_{qs}^r\right]$$

Solving Steady-State Equations

$$\begin{bmatrix} V_{qs}^{r} \\ V_{ds}^{r} \end{bmatrix} = \begin{bmatrix} -r_s & -\frac{\omega_e}{\omega_b} X_d \\ \frac{\omega_e}{\omega_b} X_q & -r_s \end{bmatrix} \begin{bmatrix} I_{qs}^{r} \\ I_{ds}^{r} \end{bmatrix} + \begin{bmatrix} \frac{\omega_e}{\omega_b} X_{md} I'^{r}_{fd} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} I_{qs}^{r} \\ I_{ds}^{r} \end{bmatrix} = \frac{\begin{bmatrix} -r_s & \frac{\omega_e}{\omega_b} X_d \\ -\frac{\omega_e}{\omega_b} X_q & -r_s \end{bmatrix} \begin{bmatrix} V_{qs}^{r} - \frac{\omega_e}{\omega_b} X_{md} I'^{r}_{fd} \\ V_{ds}^{r} \end{bmatrix}}{r_s^2 + \left(\frac{\omega_e}{\omega_b}\right)^2 X_q X_d}$$

Steady-State Torque in Terms of Voltage

$$T_{e,ss} = \frac{3}{2}\left(\frac{P}{2}\right)\left(\frac{1}{\omega_b}\right)\left\{\begin{array}{l} \dfrac{r_s X_{md} I'^{r}_{fd}}{r_s^2 + \left(\dfrac{\omega_e}{\omega_b}\right)^2 X_q X_d}\left(V_{qs}^r - \dfrac{\omega_e}{\omega_b} X_{md} I'^{r}_{fd} - \dfrac{\omega_e}{\omega_b}\dfrac{X_d}{r_s} V_{ds}^r\right) \\ + \dfrac{X_d - X_q}{\left[r_s^2 + \left(\dfrac{\omega_e}{\omega_b}\right)^2 X_q X_d\right]^2}\left[r_s \dfrac{\omega_e}{\omega_b} X_q \left(V_{qs}^r - \dfrac{\omega_e}{\omega_b} X_{md} I'^{r}_{fd}\right)^2 + \right. \\ \left.\left(r_s^2 - \left(\dfrac{\omega_e}{\omega_b}\right)^2 X_q X_d\right) V_{ds}^r \left(V_{qs}^r - \dfrac{\omega_e}{\omega_b} X_{md} I'^{r}_{fd}\right) - r_s \dfrac{\omega_e}{\omega_b} X_d V_{ds}^{r\,2}\right] \end{array}\right\}$$

Torque in Terms of Delta Angle (Neglecting Stator Resistance)

$$T_{e,ss} = \frac{3}{2}\left(\frac{P}{2}\right)\left(\frac{1}{\omega_b}\right)\left[\frac{E'^{r}_{xfd}\sqrt{2}V_s}{\left(\frac{\omega_e}{\omega_b}\right)X_d}\sin(\delta) + \left(\frac{\omega_b}{\omega_e}\right)^2\left(\frac{1}{X_q} - \frac{1}{X_d}\right)V_s^{\,2}\sin(2\delta)\right]$$

where $E'^{r}_{xfd} = X_{md} I'^{r}_{fd}$

Example Machine

$\omega_b := 2\cdot\pi\cdot 60\cdot \text{Hz}$

$\text{kVA} := \text{kV}\cdot\text{A}$

$\text{lagging} := 1$

$\text{RPM} := \dfrac{2\cdot\pi\cdot\text{rad}}{\text{min}}$

given parameters

$P := 24$ $r_s := 7.8\,\text{m}\Omega$ $X_{ls} := 0.02\cdot\Omega$ $r_{fd} := 3.8\,\Omega$

open-circuit test data

$$V_{s\_oc} := \frac{208}{\sqrt{3}}\cdot V$$

$$V_{s\_oc} = 120V$$

$$I_{fd\_oc} := 30\cdot A$$

$$\omega_{e\_oc} := \omega_b$$

open-circuit test calculations

$$V_{qsr\_oc} := \sqrt{2}\cdot V_{s\_oc}$$

$$V_{dsr\_oc} := 0\cdot V$$

$$\delta_{oc} := \text{atan}\left(\frac{-V_{dsr\_oc}}{V_{qsr\_oc}}\right)$$

$$\delta_{oc} = 0\,\text{deg}$$

$$E'_{xfd} := \frac{\omega_b}{\omega_{e\_oc}}\cdot V_{qsr\_oc}$$

$$E'_{xfd} = 170V$$

Inductive Load Test

inductive load test (same *I_{fd}* as the open-circuit test)

$$V_{s\_L} := \frac{158}{\sqrt{3}} \cdot V \qquad V_{s\_L} = 91V$$

$$I_{s\_L} := 100 \cdot A$$

$$\omega_{e\_L} := \omega_b$$

$$\delta_L := 0 \cdot deg$$

$$\theta_{ev0\_L} := 0 \cdot deg$$

$$\theta_{ei0\_L} := -90 \cdot deg$$

Inductive Load Calculations

inductive load calculations

$$V_{qsr\_L} := \sqrt{2} \cdot V_{s\_L} \cdot \cos\left(-\delta_L\right)$$

$$V_{dsr\_L} := -\sqrt{2} \cdot V_{s\_L} \cdot \sin\left(-\delta_L\right)$$

$$I_{qsr\_L} := \sqrt{2} \cdot I_{s\_L} \cdot \cos\left(\theta_{ei0\_L} - \theta_{ev0\_L} - \delta_L\right)$$

$$I_{dsr\_L} := -\sqrt{2} \cdot I_{s\_L} \cdot \sin\left(\theta_{ei0\_L} - \theta_{ev0\_L} - \delta_L\right)$$

$$X_d := \frac{\left(\dfrac{\omega_{e\_oc}}{\omega_b}\right) \cdot E'_{xfd} - V_{qsr\_L}}{\left(\dfrac{\omega_{e\_oc}}{\omega_b}\right) I_{dsr\_L}}$$

$$X_{md} := X_d - X_{ls}$$

$$X_{mq} := \frac{2}{3} \cdot X_{md}$$

$$X_q := X_{ls} + X_{mq}$$

Open-Circuit Test Calculations with Magnetizing Reactance

return to open-circuit test with magnetizing reactances

$$I'_{fd\_oc} := \frac{E'_{xfd}}{X_{md}}$$

$$N_{fd} := \frac{3}{2} \cdot \frac{I'_{fd\_oc}}{I_{fd\_oc}} \qquad N_s := 1$$

$$r'_{fd} := \frac{3}{2} \cdot \left(\frac{N_s}{N_{fd}}\right)^2 \cdot r_{fd}$$

Rated Operation

rated generator operation

$$V_s := \frac{208}{\sqrt{3}} \cdot V \qquad V_s = 120V$$

$$S := 90 \cdot kVA$$

$$pf := 0.8 \text{lagging}$$

$$f_e := 60 \cdot Hz$$

$$I_{fd} := 60 \cdot A$$

$$\omega_e := 2 \cdot \pi \cdot f_e \qquad \omega_e = 377 \frac{rad}{s}$$

$$I_s := \frac{S}{3 \cdot V_s} \qquad I_s = 250A$$

$$\theta_{ev0} := 0 \cdot deg$$

$$\theta_{ei0} := \theta_{ev0} - acos(pf) \qquad \theta_{ei0} = -36.9deg$$

$$V_{as} := V_s \cdot e^{j \cdot 0 \cdot deg}$$

$$I_{as} := I_s \cdot e^{j \cdot \left(\theta_{ei0} - \theta_{ev0}\right)}$$

Rated Operation Calculations

back-emf and torque angle

$$E_a := V_{as} + \left(r_s + j \cdot \frac{\omega_e}{\omega_b} \cdot X_q \right) \cdot I_{as}$$

$$r_s := 0 \cdot \Omega$$

$$E_a := V_{as} + \left(r_s + j \cdot \frac{\omega_e}{\omega_b} \cdot X_q \right) \cdot I_{as}$$

$$\delta := \arg\left(E_a\right)$$

$$V_{qsr} := \sqrt{2} \cdot V_s \cdot \cos(-\delta)$$

$$V_{dsr} := -\sqrt{2} \cdot V_s \cdot \sin(-\delta)$$

$$I_{qsr} := \sqrt{2} \cdot I_s \cdot \cos\left(\theta_{ei0} - \theta_{ev0} - \delta\right)$$

$$I_{dsr} := -\sqrt{2} \cdot I_s \cdot \sin\left(\theta_{ei0} - \theta_{ev0} - \delta\right)$$

Phasor Diagram

Rated Operation Torque

torque

$$I'_{fd} := \frac{2}{3}\cdot\left(\frac{N_{fd}}{N_s}\right)I_{fd}$$

$$E'_{xfd} := X_{md}\cdot I'_{fd}$$

$$T_e := \frac{3}{2}\cdot\left(\frac{P}{2}\right)\cdot\left(\frac{1}{\omega_b}\right)\cdot\left[\frac{E'_{xfd}\cdot\sqrt{2}\cdot V_s}{\left(\frac{\omega_e}{\omega_b}\right)\cdot X_d}\cdot\sin(\delta) + \left(\frac{\omega_b}{\omega_e}\right)^2\cdot\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\cdot V_s^{\,2}\cdot\sin(2\cdot\delta)\right]$$

Synchronous Machine Impedances and Time Constants (Chapter 7)

Stator Flux Linkages

Write the flux linkages in terms of general transfer functions

$$\Psi_{qs}^{r} = -X_q(s)\, i_{qs}^{r}$$

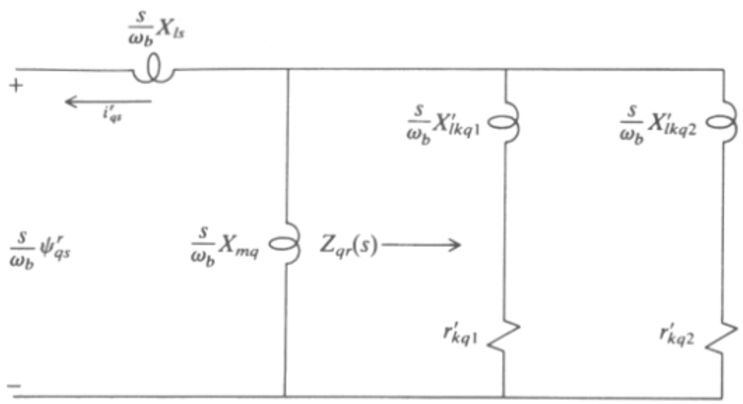
$$\Psi_{ds}^{r} = -X_d(s)\, i_{ds}^{r} + G(s)\, v'^{r}_{fd}$$

note: $X_q(s)$ and $X_d(s)$ are not the synchronous reactances X_q and X_d

The transfer functions do not restrict the model to a specific form. They can be found by:

$$X_q(s) = -\frac{\Psi_{qs}^{r}}{i_{qs}^{r}} \qquad X_d(s) = -\left.\frac{\Psi_{ds}^{r}}{i_{ds}^{r}}\right|_{v'^{r}_{fd}=0} \qquad G(s) = \left.\frac{\Psi_{ds}^{r}}{v'^{r}_{fd}}\right|_{i_{ds}^{r}=0}$$

Standard Q-Axis Model



For this structure, the impedance $X_q(s)$ is

$$X_q(s) = X_q \frac{1 + (\tau_{q4} + \tau_{q5})s + \tau_{q4}\tau_{q6}s^2}{1 + (\tau_{q1} + \tau_{q2})s + \tau_{q1}\tau_{q3}s^2}$$

where

$$\tau_{q1} = \frac{1}{\omega_b r'_{kq1}}(X'_{lkq1} + X_{mq})$$

$$\tau_{q2} = \frac{1}{\omega_b r'_{kq2}}(X'_{lkq2} + X_{mq})$$

$$\tau_{q3} = \frac{1}{\omega_b r'_{kq2}}\left(X'_{lkq2} + \frac{X_{mq}X'_{lkq1}}{X'_{lkq1} + X_{mq}}\right)$$

$$\tau_{q4} = \frac{1}{\omega_b r'_{kq1}}\left(X'_{lkq1} + \frac{X_{mq}X_{ls}}{X_{ls} + X_{mq}}\right)$$

$$\tau_{q5} = \frac{1}{\omega_b r'_{kq2}}\left(X'_{lkq2} + \frac{X_{mq}X_{ls}}{X_{ls} + X_{mq}}\right)$$

$$\tau_{q6} = \frac{1}{\omega_b r'_{kq2}}\left(X'_{lkq2} + \frac{X_{mq}X_{ls}X'_{lkq1}}{X_{mq}X_{ls} + X_{mq}X'_{lkq1} + X_{ls}X'_{lkq1}}\right)$$

The standard *q*-axis impedance can be seen as a collection of time constants and reactances

By a similar procedure, $X_d(s)$ can be written in terms of reactances and time constants

Standard Reactances

synchronous reactances

$$X_q = X_{ls} + X_{mq}$$
$$X_d = X_{ls} + X_{md}$$

note:

$$X_q(0) = X_q$$
$$X_d(0) = X_d$$

transient reactances

$$X_q' = X_{ls} + \frac{X_{mq}X_{lkq1}'}{X_{lkq1}' + X_{mq}}$$
$$X_d' = X_{ls} + \frac{X_{md}X_{lfd}'}{X_{lfd}' + X_{md}}$$

sub-transient reactances

$$X_q'' = X_{ls} + \frac{X_{mq}X_{lkq1}'X_{lkq2}'}{X_{mq}X_{lkq1}' + X_{mq}X_{lkq2}' + X_{lkq1}'X_{lkq2}'}$$
$$X_d'' = X_{ls} + \frac{X_{md}X_{lfd}'X_{lkd}'}{X_{md}X_{lfd}' + X_{md}X_{lkd}' + X_{lfd}'X_{lkd}'}$$

note:

$$X_q(\infty) = X_q''$$
$$X_d(\infty) = X_d''$$

Notes About Time Constants

- In the *d*-axis, the field acts in the transient time scale and the damper winding acts in the sub-transient period.
- In the *q*-axis, the *kq*1 winding acts in the transient period and the *kq*2 in the sub-transient period.
- Open-circuit time constants represent the time for transients to decay with the stator windings open-circuited.
- Short-circuit time constants represent the time for transients to decay with the stator short-circuited.

$$X_q(s) = X_q \frac{\left(1+\tau'_q s\right)\left(1+\tau''_q s\right)}{\left(1+\tau'_{qo}\right)\left(1+\tau''_{qo} s\right)}$$

$$X_d(s) = X_d \frac{\left(1+\tau'_d s\right)\left(1+\tau''_d s\right)}{\left(1+\tau'_{do}\right)\left(1+\tau''_{do} s\right)}$$

Standard Time Constants (Approximate)

Table 7.5-1 Standard Synchronous Machine Time Constants

Open-Circuit Time Constants

$$\tau'_{qo} = \frac{1}{\omega_b r'_{kq1}}(X'_{lkq1} + X_{mq})$$

$$\tau'_{do} = \frac{1}{\omega_b r'_{fd}}(X'_{lfd} + X_{md})$$

$$\tau''_{qo} = \frac{1}{\omega_b r'_{kq2}}\left(X'_{lkq2} + \frac{X_{mq}X'_{lkq1}}{X_{mq} + X'_{lkq1}}\right)$$

$$\tau''_{do} = \frac{1}{\omega_b r'_{kd}}\left(X'_{lkd} + \frac{X_{md}X'_{lfd}}{X_{md} + X'_{lfd}}\right)$$

Short-Circuit Time Constants

$$\tau'_{q} = \frac{1}{\omega_b r'_{kq1}}\left(X'_{lkq1} + \frac{X_{mq}X_{ls}}{X_{mq} + X_{ls}}\right)$$

$$\tau'_{d} = \frac{1}{\omega_b r'_{fd}}\left(X'_{lfd} + \frac{X_{md}X_{ls}}{X_{md} + X_{ls}}\right)$$

$$\tau''_{q} = \frac{1}{\omega_b r'_{kq2}}\left(X'_{lkq2} + \frac{X_{mq}X_{ls}X'_{lkq1}}{X_{mq}X_{ls} + X_{mq}X'_{lkq1} + X_{ls}X'_{lkq1}}\right)$$

$$\tau''_{d} = \frac{1}{\omega_b r'_{kd}}\left(X'_{lkd} + \frac{X_{md}X_{ls}X'_{lfd}}{X_{md}X_{ls} + X_{md}X'_{lfd} + X_{ls}X'_{lfd}}\right)$$

obtained from simple R-L time constant relationships

Derived Time Constants

Table 7.6-1 Derived Synchronous Machine Time Constants

Open-Circuit Time Constants

$$\tau'_{qo} = \frac{1}{\omega_b r'_{kq1}}(X'_{lkq1} + X_{mq}) + \frac{1}{\omega_b r'_{kq2}}(X'_{lkq2} + X_{mq})$$

$$\tau'_{do} = \frac{1}{\omega_b r'_{fd}}(X'_{lfd} + X_{md}) + \frac{1}{\omega_b r'_{kd}}(X'_{lkd} + X_{md})$$

$$\tau''_{qo} = \frac{(1/\omega_b r'_{kq2})[X'_{lkq2} + (X_{mq}X'_{lkq1}/X'_{lkq1} + X_{mq})]}{1 + [(1/\omega_b r'_{kq2})(X'_{lkq2} + X_{mq})/(1/\omega_b r'_{kq1})(X'_{lkq1} + X_{mq})]}$$

$$\tau''_{do} = \frac{(1/\omega_b r'_{kd})[X'_{lkd} + (X_{md}X'_{lfd}/X'_{lfd} + X_{md})]}{1 + [(1/\omega_b r'_{kd})(X'_{lkd} + X_{md})/(1/\omega_b r'_{fd})(X'_{lfd} + X_{md})]}$$

Short-Circuit Time Constants

$$\tau'_{q} = \frac{1}{\omega_b r'_{kq1}}\left(X'_{lkq1} + \frac{X_{mq}X_{ls}}{X_{ls} + X_{mq}}\right) + \frac{1}{\omega_b r'_{kq2}}\left(X'_{lkq2} + \frac{X_{mq}X_{ls}}{X_{ls} + X_{mq}}\right)$$

$$\tau'_{d} = \frac{1}{\omega_b r'_{fd}}\left(X'_{lfd} + \frac{X_{md}X_{ls}}{X_{ls} + X_{md}}\right) + \frac{1}{\omega_b r'_{kd}}\left(X'_{lkd} + \frac{X_{md}X_{ls}}{X_{ls} + X_{md}}\right)$$

$$\tau''_{q} = \frac{(1/\omega_b r'_{lkq2})[X'_{lkq2} + (X_{mq}X_{ls}X'_{lkq1}/X_{mq}X_{ls} + X_{mq}X'_{lkq1} + X_{ls}X'_{lkq1})]}{1 + \{(1/\omega_b r'_{kq2})[X'_{lkq2} + (X_{mq}X_{ls}/X_{ls} + X_{mq})]/(1/\omega_b r'_{kq1})[X'_{lkq1} + (X_{mq}X_{ls}/X_{ls} + X_{mq})]\}}$$

$$\tau''_{d} = \frac{(1/\omega_b r'_{kd})[X'_{lkd} + (X_{md}X_{ls}X'_{lfd}/X_{md}X_{ls} + X_{md}X'_{lfd} + X_{ls}X'_{lfd})]}{1 + \{(1/\omega_b r'_{kd})[X'_{lkd} + (X_{md}X_{ls}/X_{ls} + X_{md})]/(1/\omega_b r'_{fd})[X'_{lfd} + (X_{md}X_{ls}/X_{ls} + X_{md})]\}}$$

derived from the inverse of the roots of the characteristic equations of X_q(s) and X_d(s)

Synchronous Machine Example: Extract parameters from given reactances and time constants

$kVA := kV \cdot A$

$m\Omega := 10^{-3} \cdot \Omega$

number of poles $P := 24$

rated operation

$S_{out} := 100 \cdot kVA$ $P_{out} := 75 \cdot kW$ $f_e := 400 \cdot Hz$ $V_{LL} := 208 \cdot V$ $I_{fd} := 10 \cdot A$

base impedance

$\omega_b := 2 \cdot \pi \cdot f_e$ $\omega_b = 2513 \frac{rad}{s}$

$V_b := \frac{V_{LL}}{\sqrt{3}}$ $V_b = 120V$

$I_b := \frac{S_{out}}{3 \cdot V_b}$ $I_b = 278A$

$Z_b := \frac{V_b}{I_b}$ $Z_b = 0.433\Omega$

resistances

$r_s := 0.005\Omega$ $r_{fd} := 2 \cdot \Omega$

synchronous reactances

$X_d := 70\% \cdot Z_b$ $\qquad$ $X_d = 0.303\,\Omega$

$X_q := 50\% \cdot Z_b$ $\qquad$ $X_q = 0.216\,\Omega$

stator leakage reactance

$X_{ls} := 5\% \cdot Z_b$ $\qquad$ $X_{ls} = 0.022\,\Omega$

transient reactances and time constants

$X'_d := 15\% \cdot Z_b$ $\qquad$ $\tau'_d := 0.05\,s$ $\qquad$ $\tau'_{do} := 0.150\,s$

$X'_q := 25\% \cdot Z_b$ $\qquad$ $\tau'_q := 0.03\,s$ $\qquad$ $\tau'_{qo} := 0.035\,s$

subtransient reactances

$X''_d := 12\% \cdot Z_b$ $\qquad$ $\tau''_d := 0.001\,s$ $\qquad$ $\tau''_{do} := 0.002\,s$

$X''_q := 13\% \cdot Z_b$ $\qquad$ $\tau''_q := 0.0005\,s$ $\qquad$ $\tau''_{qo} := 0.001\,s$

calculate magnitizing reactances

$X_{md} := X_d - X_{ls}$ $\qquad$ $X_{md} = 0.281\Omega$

$X_{mq} := X_q - X_{ls}$ $\qquad$ $X_{mq} = 0.1947\Omega$

leakage reactances from transient and sub-transient reactances

guess values --->

$X'_{lkq1} := X_{ls}$ $\qquad$ $X'_{lfd} := X_{ls}$

$X'_{lkq2} := X_{ls}$ $\qquad$ $X'_{lkd} := X_{ls}$

Given

$$X'_q = X_{ls} + \frac{X_{mq} \cdot X'_{lkq1}}{X'_{lkq1} + X_{mq}}$$

$$X'_d = X_{ls} + \frac{X_{md} \cdot X'_{lfd}}{X'_{lfd} + X_{md}}$$

$$X''_q = X_{ls} + \frac{X_{mq} \cdot X'_{lkq1} \cdot X'_{lkq2}}{X_{mq} \cdot X'_{lkq1} + X_{mq} \cdot X'_{lkq2} + X'_{lkq1} \cdot X'_{lkq2}}$$

$$X''_d = X_{ls} + \frac{X_{md} \cdot X'_{lfd} \cdot X'_{lkd}}{X_{md} \cdot X'_{lfd} + X_{md} \cdot X'_{lkd} + X'_{lfd} \cdot X'_{lkd}}$$

$$\begin{pmatrix} X'_{lkq1} \\ X'_{lkq2} \\ X'_{lfd} \\ X'_{lkd} \end{pmatrix} := \text{Find}\left(X'_{lkq1}, X'_{lkq2}, X'_{lfd}, X'_{lkd}\right)$$

$X'_{lfd} = 0.051\Omega$

$X'_{lkd} = 0.101\Omega$

$X'_{lkq1} = 0.156\Omega$

$X'_{lkq2} = 0.058\Omega$

guess values for resistances
based on standard open-circuit equations

$$r'_{kq1o} := \frac{1}{\omega_b \cdot \tau'_{qo}} \cdot \left(X'_{lkq1} + X_{mq}\right) \qquad r'_{kq1o} = 3.98\,m\Omega$$

$$r'_{fdo} := \frac{1}{\omega_b \cdot \tau'_{do}} \cdot \left(X'_{lfd} + X_{md}\right) \qquad r'_{fdo} = 0.882\,m\Omega$$

$$r'_{kq2o} := \frac{1}{\omega_b \cdot \tau''_{qo}} \cdot \left(X'_{lkq2} + \frac{X_{mq} \cdot X'_{lkq1}}{X_{mq} + X'_{lkq1}}\right) \qquad r'_{kq2o} = 57.38\,m\Omega$$

$$r'_{kdo} := \frac{1}{\omega_b \cdot \tau''_{do}} \cdot \left(X'_{lkd} + \frac{X_{md} \cdot X'_{lfd}}{X_{md} + X'_{lfd}}\right) \qquad r'_{kdo} = 28.69\,m\Omega$$

resistances from derived synchrnous machine open-circuit time constants

Given

$$\tau'_{qo} = \frac{1}{\omega_b \cdot r'_{kq1o}} \cdot \left(X'_{lkq1} + X_{mq}\right) + \frac{1}{\omega_b \cdot r'_{kq2o}} \cdot \left(X'_{lkq2} + X_{mq}\right)$$

$$\tau'_{do} = \frac{1}{\omega_b \cdot r'_{fdo}} \cdot \left(X'_{lfd} + X_{md}\right) + \frac{1}{\omega_b \cdot r'_{kdo}} \cdot \left(X'_{lkd} + X_{md}\right)$$

$$\tau''_{qo} = \frac{\dfrac{1}{\omega_b \cdot r'_{kq2o}} \cdot \left(X'_{lkq2} + \dfrac{X_{mq} \cdot X'_{lkq1}}{X'_{lkq1} + X_{mq}}\right)}{1 + \dfrac{\dfrac{1}{\omega_b \cdot r'_{kq2o}} \cdot \left(X'_{lkq2} + X_{mq}\right)}{\dfrac{1}{\omega_b \cdot r'_{kq1o}} \cdot \left(X'_{lkq1} + X_{mq}\right)}}$$

$$\tau''_{do} = \frac{\dfrac{1}{\omega_b \cdot r'_{kdo}} \cdot \left(X'_{lkd} + \dfrac{X_{md} \cdot X'_{lfd}}{X'_{lfd} + X_{md}}\right)}{1 + \dfrac{\dfrac{1}{\omega_b \cdot r'_{kdo}} \cdot \left(X'_{lkd} + X_{md}\right)}{\dfrac{1}{\omega_b \cdot r'_{fdo}} \cdot \left(X'_{lfd} + X_{md}\right)}}$$

$$\begin{pmatrix} r'_{kq1o} \\ r'_{kq2o} \\ r'_{fdo} \\ r'_{kdo} \end{pmatrix} := \mathrm{Find}\left(r'_{kq1o}, r'_{kq2o}, r'_{fdo}, r'_{kdo}\right)$$

$r'_{kq1o} = 4.21\,\mathrm{m}\Omega$

$r'_{fdo} = 0.915\,\mathrm{m}\Omega$

$r'_{kq2o} = 54.35\,\mathrm{m}\Omega$

$r'_{kdo} = 27.64\,\mathrm{m}\Omega$

guess resistance values
based on standard short-circuit equations

$$r'_{kq1s} := \frac{1}{\omega_b \cdot \tau'_q} \cdot \left(X'_{lkq1} + \frac{X_{mq} \cdot X_{ls}}{X_{mq} + X_{ls}} \right) \qquad r'_{kq1s} = 2.32\,m\Omega$$

$$r'_{fds} := \frac{1}{\omega_b \cdot \tau'_d} \cdot \left(X'_{lfd} + \frac{X_{md} \cdot X_{ls}}{X_{md} + X_{ls}} \right) \qquad r'_{fds} = 0.567\,m\Omega$$

$$r'_{kq2s} := \frac{1}{\omega_b \cdot \tau''_q} \cdot \left(X'_{lkq2} + \frac{X_{mq} \cdot X_{ls} \cdot X'_{lkq1}}{X_{mq} \cdot X_{ls} + X_{mq} \cdot X'_{lkq1} + X_{ls} \cdot X'_{lkq1}} \right) \qquad r'_{kq2s} = 59.68\,m\Omega$$

$$r'_{kds} := \frac{1}{\omega_b \cdot \tau''_d} \cdot \left(X'_{lkd} + \frac{X_{md} \cdot X_{ls} \cdot X'_{lfd}}{X_{md} \cdot X_{ls} + X_{md} \cdot X'_{lfd} + X_{ls} \cdot X'_{lfd}} \right) \qquad r'_{kds} = 45.9\,m\Omega$$

resistances from derived synchrnous machine short-circuit time constants

Given

$$\tau'_q = \frac{1}{\omega_b \cdot r'_{kq1s}} \cdot \left(X'_{lkq1} + \frac{X_{mq} \cdot X_{ls}}{X_{ls} + X_{mq}} \right) + \frac{1}{\omega_b \cdot r'_{kq2o}} \cdot \left(X'_{lkq2} + \frac{X_{mq} \cdot X_{ls}}{X_{ls} + X_{mq}} \right)$$

$$\tau'_d = \frac{1}{\omega_b \cdot r'_{fds}} \cdot \left(X'_{lfd} + \frac{X_{md} \cdot X_{ls}}{X_{ls} + X_{md}} \right) + \frac{1}{\omega_b \cdot r'_{kdo}} \cdot \left(X'_{lkd} + \frac{X_{md} \cdot X_{ls}}{X_{ls} + X_{md}} \right)$$

$$\tau''_q = \frac{\frac{1}{\omega_b \cdot r'_{kq2s}} \cdot \left(X'_{lkq2} + \frac{X_{mq} \cdot X_{ls} \cdot X'_{lkq1}}{X_{mq} \cdot X_{ls} + X_{mq} \cdot X'_{lkq1} + X_{ls} \cdot X'_{lkq1}} \right)}{1 + \frac{\frac{1}{\omega_b \cdot r'_{kq2s}} \cdot \left(X'_{lkq2} + \frac{X_{mq} \cdot X_{ls}}{X_{ls} + X_{mq}} \right)}{\frac{1}{\omega_b \cdot r'_{kq1s}} \cdot \left(X'_{lkq1} + \frac{X_{mq} \cdot X_{ls}}{X_{ls} + X_{mq}} \right)}}$$

$$\tau''_d = \frac{\frac{1}{\omega_b \cdot r'_{kds}} \cdot \left(X'_{lkd} + \frac{X_{md} \cdot X_{ls} \cdot X'_{lfd}}{X_{md} \cdot X_{ls} + X_{md} \cdot X'_{lfd} + X_{ls} \cdot X'_{lfd}} \right)}{1 + \frac{\frac{1}{\omega_b \cdot r'_{kds}} \cdot \left(X'_{lkd} + \frac{X_{md} \cdot X_{ls}}{X_{ls} + X_{md}} \right)}{\frac{1}{\omega_b \cdot r'_{fds}} \cdot \left(X'_{lfd} + \frac{X_{md} \cdot X_{ls}}{X_{ls} + X_{md}} \right)}}$$

$$\begin{pmatrix} r'_{kq1s} \\ r'_{kq2s} \\ r'_{fds} \\ r'_{kds} \end{pmatrix} := \mathrm{Find}\left(r'_{kq1s}, r'_{kq2s}, r'_{fds}, r'_{kds}\right)$$

$r'_{kq1s} = 2.37\,m\Omega$

$r'_{fds} = 0.587\,m\Omega$

$r'_{kq2s} = 58.55\,m\Omega$

$r'_{kds} = 44.91\,m\Omega$

final resistances

$$r'_{kq1} := \frac{1}{2}\cdot\left(r'_{kq1o} + r'_{kq1s}\right)$$

$$r'_{fd} := \frac{1}{2}\cdot\left(r'_{fdo} + r'_{fds}\right)$$

$$r'_{kq2} := \frac{1}{2}\cdot\left(r'_{kq2o} + r'_{kq2s}\right)$$

$$r'_{kd} := \frac{1}{2}\cdot\left(r'_{kdo} + r'_{kds}\right)$$

Synchronous Machine Linearized Equations (Chapter 8)

Applications:
- Eigenvalue analysis
- Transfer functions
- Stability analysis
- Classical control

Taylor Series

single variable

$$g(f) = g(f_0) + \left.\frac{dg(f)}{df}\right|_{f_o} \Delta f + \left.\frac{d^2 g(f)}{df^2}\right|_{f_o} \Delta f^2 + \left.\frac{d^3 g(f)}{df^3}\right|_{f_o} \Delta f^3 + \cdots$$

where $f = f_o + \Delta f$
f_o - operating point
Δf - change in f about operating point

linear approximation

$$g(f) \approx g(f_o) + \left.\frac{dg(f)}{df}\right|_{f_o} \Delta f = g_o + \Delta g$$

express equations in delta variables

$$\Delta g(\Delta f) = \left.\frac{dg(f)}{df}\right|_{f_o} \Delta f$$

Approximate Function of Two Variables

$$g\left(f_1, f_2\right) \approx g\left(f_{1o}, f_{2o}\right) + \left.\frac{\partial g\left(f_1, f_2\right)}{df_1}\right|_{f_{1o}, f_{2o}} \Delta f_1 + \left.\frac{\partial g\left(f_1, f_2\right)}{df_2}\right|_{f_{1o}, f_{2o}} \Delta f_2$$

Per Unit System

base speed: ω_b - rated ω_e

base power: $$P_B = 3V_{B(abc)} I_{B(abc)} = \frac{3}{2} V_{B(qd0)} I_{B(qd0)}$$

base voltage: $V_{B(abc)}$ - rated rms value of v_{as}

base current: $I_{B(abc)}$ - rated rms value of i_{as}

base torque: $$T_B = \frac{P_B}{\left(\frac{2}{P}\right)\omega_b} = \frac{\left(\frac{3}{2}\right) V_{B(qd0)} I_{B(qd0)}}{\left(\frac{2}{P}\right)\omega_b}$$

Torque in Per Unit

using substitute variables

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(\frac{1}{\omega_b}\right)\left(\Psi_{ds}^r i_{qs}^r - \Psi_{qs}^r i_{ds}^r\right) = \frac{\frac{3}{2}\Psi_{ds}^r i_{qs}^r - \frac{3}{2}\Psi_{qs}^r i_{ds}^r}{\frac{2}{P}\omega_b}$$

in per-unit

$$T_e = \Psi_{ds}^r i_{qs}^r - \Psi_{qs}^r i_{ds}^r = X_{md}\left(-i_{ds}^r + i'^r_{fd} + i'^r_{kd}\right) i_{qs}^r - X_{mq}\left(-i_{qs}^r + i'^r_{kq1} + i'^r_{kq2}\right) i_{ds}^r$$ (in p.u.)

mechanical equations

$$T_I - T_e = \left(\frac{2}{P}\right) J\, p\omega_r$$

in per-unit

$$T_I - T_e = \left(\frac{2}{P}\right)\frac{J\omega_b}{T_B}\, p\frac{\omega_r}{\omega_b} = 2H\, p\frac{\omega_r}{\omega_b}$$ (in p.u.)

where $H = \left(\frac{1}{2}\right)\left(\frac{2}{P}\right)\frac{J\omega_b}{T_B}$

Linearized Torque Equation

$$T_I = T_e + 2H\, p \frac{\omega_r}{\omega_b} = T_{Io} + \Delta T_I$$

$$\Delta T_I = \left.\frac{\partial T_I}{\partial i_{qs}^r}\right|_o \Delta i_{qs}^r + \left.\frac{\partial T_I}{\partial i_{ds}^r}\right|_o \Delta i_{ds}^r + \cdots$$

where *o* represents the operating point

$$i_{qso}^r, i_{dso}^r, i'^r_{fdo}, i'^r_{kdo}, i'^r_{kq1o}, i'^r_{kq2o}, \omega_{ro}$$

see (8.3-19) for full equation

Torque Angle Equation

$$\delta = \theta_r - \theta_{ev}$$

$$p\delta = \omega_r - \omega_e$$

$$p\Delta\delta = \Delta\omega_r - \Delta\omega_e$$

$$p\Delta\delta = \omega_b \left(\frac{\Delta\omega_r}{\omega_b} \right)$$ if $\Delta\omega_e = 0$

Synchronous Machine Linearized Equations

$$
\begin{bmatrix} \Delta v_{qs}^{r} \\ \Delta v_{ds}^{r} \\ \Delta v_{kq1}^{\prime r} \\ \Delta v_{kq2}^{\prime r} \\ \Delta e_{xfd}^{\prime r} \\ \Delta v_{kd}^{\prime r} \\ \Delta T_I \\ 0 \end{bmatrix}
=
\begin{bmatrix}
-r_s - \frac{p}{\omega_b} X_q & -\frac{\omega_e}{\omega_b} X_d & \frac{p}{\omega_b} X_{mq} & \frac{p}{\omega_b} X_{mq} & \frac{\omega_e}{\omega_b} X_{md} & \frac{\omega_e}{\omega_b} X_{md} & -X_d i_{dso}^{r} + X_{md} i_{fdo}^{\prime r} & 0 \\
\frac{\omega_e}{\omega_b} X_q & -r_s - \frac{p}{\omega_b} X_d & -\frac{\omega_e}{\omega_b} X_{mq} & -\frac{\omega_e}{\omega_b} X_{mq} & \frac{p}{\omega_b} X_{md} & \frac{p}{\omega_b} X_{md} & X_q i_{qso}^{r} & 0 \\
-\frac{p}{\omega_b} X_{mq} & 0 & r_{kq1}^{\prime} + \frac{p}{\omega_b} X_{kq1}^{\prime} & \frac{p}{\omega_b} X_q & 0 & 0 & 0 & 0 \\
-\frac{p}{\omega_b} X_{mq} & 0 & \frac{p}{\omega_b} X_{mq} & r_{kq2}^{\prime} + \frac{p}{\omega_b} X_{kq2}^{\prime} & 0 & 0 & 0 & 0 \\
0 & -\frac{X_{md}}{r_{fd}^{\prime}} \left(\frac{p}{\omega_b} X_{md} \right) & 0 & 0 & \frac{X_{md}}{r_{fd}^{\prime}} \left(r_{fd}^{\prime} + \frac{p}{\omega_b} X_{fd}^{\prime} \right) & \frac{X_{md}}{r_{fd}^{\prime}} \left(\frac{p}{\omega_b} X_{md} \right) & 0 & 0 \\
0 & -\frac{p}{\omega_b} X_{md} & 0 & 0 & \frac{p}{\omega_b} X_{md} & r_{kd}^{\prime} + \frac{p}{\omega_b} X_{kd}^{\prime} & 0 & 0 \\
X_{mq} i_{dso}^{r} - X_{md} (i_{dso}^{r} - i_{fdo}^{\prime r}) & -X_{md} i_{qso}^{r} + X_{mq} i_{qso}^{r} & -X_{mq} i_{dso}^{r} & -X_{mq} i_{dso}^{r} & X_{md} i_{qso}^{r} & X_{md} i_{qso}^{r} & 2Hp & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\omega_b & p
\end{bmatrix}
\begin{bmatrix} \Delta i_{qs}^{r} \\ \Delta i_{ds}^{r} \\ \Delta i_{kq1}^{\prime r} \\ \Delta i_{kq2}^{\prime r} \\ \Delta i_{fd}^{\prime r} \\ \Delta i_{kd}^{\prime r} \\ \frac{\Delta \omega_r}{\omega_b} \\ \Delta \delta \end{bmatrix}
$$

Source Voltage

$$v_{as} = \sqrt{2} v_s \cos\left(\theta_{ev}\right)$$

$$v_{bs} = \sqrt{2} v_s \cos\left(\theta_{ev} - \tfrac{2\pi}{3}\right)$$

$$v_{cs} = \sqrt{2} v_s \cos\left(\theta_{ev} + \tfrac{2\pi}{3}\right)$$

synchronous reference frame

$$v_{qs}^{e} = \sqrt{2} v_s \qquad v_{qso}^{e} = \sqrt{2} v_s \qquad \Delta v_{qs}^{e} = 0$$

$$v_{ds}^{e} = 0 \qquad v_{dso}^{e} = 0 \qquad \Delta v_{ds}^{e} = 0$$

rotor reference frame

$$\begin{bmatrix} v_{qs}^{r} \\ v_{ds}^{r} \end{bmatrix} = \begin{bmatrix} \cos\left(\theta_r - \theta_{ev}\right) & -\sin\left(\theta_r - \theta_{ev}\right) \\ \sin\left(\theta_r - \theta_{ev}\right) & \cos\left(\theta_r - \theta_{ev}\right) \end{bmatrix} \begin{bmatrix} v_{qs}^{e} \\ v_{ds}^{e} \end{bmatrix} = \begin{bmatrix} \cos\left(\delta\right) & -\sin\left(\delta\right) \\ \sin\left(\delta\right) & \cos\left(\delta\right) \end{bmatrix} \begin{bmatrix} v_{qs}^{e} \\ v_{ds}^{e} \end{bmatrix}$$

Linearized Source Voltage Equations

$$v_{qs}^{r} = v_{qs}^{e}\cos\left(\delta\right) - v_{ds}^{e}\sin\left(\delta\right) = v_{qso}^{r} + \Delta v_{qs}^{r}$$

$$\Delta v_{qs}^{r} = \cos\left(\delta_{o}\right)\Delta v_{qs}^{e} - \sin\left(\delta_{o}\right)\Delta v_{ds}^{e} - \left[\sin\left(\delta_{o}\right)v_{qso}^{e} + \cos\left(\delta_{o}\right)v_{dso}^{e}\right]\Delta\delta$$

note: $v_{dso}^{r} = \sin\left(\delta_{o}\right)v_{qso}^{e} + \cos\left(\delta_{o}\right)v_{dso}^{e}$

$$\Delta v_{qs}^{r} = \cos\left(\delta_{o}\right)\Delta v_{qs}^{e} - \sin\left(\delta_{o}\right)\Delta v_{ds}^{e} - v_{dso}^{r}\Delta\delta$$

$$\begin{bmatrix} \Delta v_{qs}^{r} \\ \Delta v_{ds}^{r} \end{bmatrix} = \begin{bmatrix} \cos\left(\delta_{o}\right) & -\sin\left(\delta_{o}\right) \\ \sin\left(\delta_{o}\right) & \cos\left(\delta_{o}\right) \end{bmatrix} \begin{bmatrix} \Delta v_{qs}^{e} \\ \Delta v_{ds}^{e} \end{bmatrix} + \begin{bmatrix} -v_{dso}^{r} \\ v_{qso}^{r} \end{bmatrix}\Delta\delta$$

Transformations

transformation to rotor

$$\begin{bmatrix} \Delta f_{qs}^{r} \\ \Delta f_{ds}^{r} \end{bmatrix} = \begin{bmatrix} \cos(\delta_o) & -\sin(\delta_o) \\ \sin(\delta_o) & \cos(\delta_o) \end{bmatrix} \begin{bmatrix} \Delta f_{qs}^{e} \\ \Delta f_{ds}^{e} \end{bmatrix} + \begin{bmatrix} -f_{dso}^{r} \\ f_{qso}^{r} \end{bmatrix} \Delta\delta$$

where *f* can be *v*, *i*, or λ

inverse transformation

$$\begin{bmatrix} \Delta f_{qs}^{e} \\ \Delta f_{ds}^{e} \end{bmatrix} = \begin{bmatrix} \cos(\delta_o) & \sin(\delta_o) \\ -\sin(\delta_o) & \cos(\delta_o) \end{bmatrix} \begin{bmatrix} \Delta f_{qs}^{r} \\ \Delta f_{ds}^{r} \end{bmatrix} + \begin{bmatrix} f_{dso}^{e} \\ -f_{qso}^{e} \end{bmatrix} \Delta\delta$$

compressed equations

$$\Delta \mathbf{f}_{qds}^{r} = \mathbf{T}\Delta \mathbf{f}_{qds}^{e} + \mathbf{F}^{r}\Delta\delta$$

$$\Delta \mathbf{f}_{qds}^{e} = (\mathbf{T})^{-1}\Delta \mathbf{f}_{qds}^{r} + \mathbf{F}^{e}\Delta\delta$$

Equations in Functional Form

$$
\begin{bmatrix}
\cos\delta_o \Delta v^e_{qs} - \sin\delta_o \Delta v^e_{ds} \\
\sin\delta_o \Delta v^e_{qs} + \cos\delta_o \Delta v^e_{ds} \\
\Delta v'^r_{kq1} \\
\Delta v'^r_{kq2} \\
\Delta e'^r_{xfd} \\
\Delta v'^r_{kd} \\
\Delta T_I \\
0
\end{bmatrix}
=
\left[\begin{array}{cc}
 & \left(r_s + \frac{p}{\omega_b}X_q\right) i^r_{dso} + \frac{\omega_e}{\omega_b} X_d i^r_{qso} + v^r_{dso} \\
 & \left(-r_s - \frac{p}{\omega_b}X_d\right) i^r_{qso} + \frac{\omega_e}{\omega_b} X_q i^r_{dso} - v^r_{qso} \\
\text{First} & \frac{p}{\omega_b} X_{mq} i^r_{dso} \\
\text{seven} & \frac{p}{\omega_b} X_{mq} i^r_{dso} \\
\text{columns} & -\frac{X_{md}}{r'_{fd}}\left(\frac{p}{\omega_b} X_{md}\right) i^r_{qso} \\
\text{same as} & -\frac{p}{\omega_b} X_{md} i^r_{qso} \\
(8.3.19) & -i^r_{dso}\left[X_{mq} i^r_{dso} - X_{md}\left(i^r_{dso} - i'^r_{fdo}\right)\right] \\
 & -i^r_{qso}\left(X_{md} i^r_{qso} - X_{mq} i^r_{qso}\right) \\
 & p
\end{array}\right]
\begin{bmatrix}
\cos\delta_o \Delta i^e_{qs} - \sin\delta_o \Delta i^e_{ds} \\
\sin\delta_o \Delta i^e_{qs} + \cos\delta_o \Delta i^e_{ds} \\
\Delta i'^r_{kq1} \\
\Delta i'^r_{kq2} \\
\Delta i'^r_{fd} \\
\Delta i'^r_{kd} \\
\frac{\Delta\omega_r}{\omega_b} \\
\Delta\delta
\end{bmatrix}
$$

$\downarrow$ rearrange to form

$$p\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

$$p\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

$$\mathbf{A} = (\mathbf{E})^{-1}\mathbf{F}$$

$$\mathbf{B} = (\mathbf{E})^{-1}$$

$$\mathbf{E} = \begin{bmatrix} (\mathbf{T})^{-1}\mathbf{W}_p\mathbf{T} & (\mathbf{T})^{-1}\mathbf{Y}_p \\ \mathbf{Q}_p\mathbf{T} & \mathbf{S}_p \end{bmatrix}$$

$$\mathbf{F} = -\begin{bmatrix} (\mathbf{T})^{-1}\mathbf{W}_k\mathbf{T} & (\mathbf{T})^{-1}\mathbf{Y}_k \\ \mathbf{Q}_k\mathbf{T} & \mathbf{S}_k \end{bmatrix}$$

$$x = \begin{bmatrix} \Delta i_{qs}^{e} \\ \Delta i_{ds}^{e} \\ \Delta i'^{r}_{kq1} \\ \Delta i'^{r}_{kq2} \\ \Delta i'^{r}_{fd} \\ \Delta i'^{r}_{kd} \\ \Delta \frac{\omega_r}{\omega_b} \\ \Delta\delta \end{bmatrix} \qquad u = \begin{bmatrix} \Delta v_{qs}^{e} \\ \Delta v_{ds}^{e} \\ \Delta v'^{r}_{kq1} \\ \Delta v'^{r}_{kq2} \\ \Delta e'^{r}_{xfd} \\ \Delta v'^{r}_{kd} \\ \Delta T_I \\ 0 \end{bmatrix}$$

Eigenvalue Analysis

Find Eigenvalues from $\det(\mathbf{A} - \lambda\mathbf{I}) = 0$

Example:

Table 8.6-1 Synchronous Machine Eigenvalues for Rated Conditions

Hydro Turbine Generator	Steam Turbine Generator
$-3.58 \pm j\cancel{77}$ 377	$-4.45 \pm j377$
$-1.33 \pm j8.68$	$-1.70 \pm j10.5$
-24.4	-32.2
-22.9	-11.1
-0.453	-0.349
	-0.855

60-Hz damped
$\tau = 1/3.58 = 0.27$s

rotor oscillations
$T = 2\pi/8.68 = 0.72$s
$\tau = 1/1.33 = 0.75$s

rotor circuit time constants
field: $\tau = 1/0.453 = 2.215$s

Fault Simulation

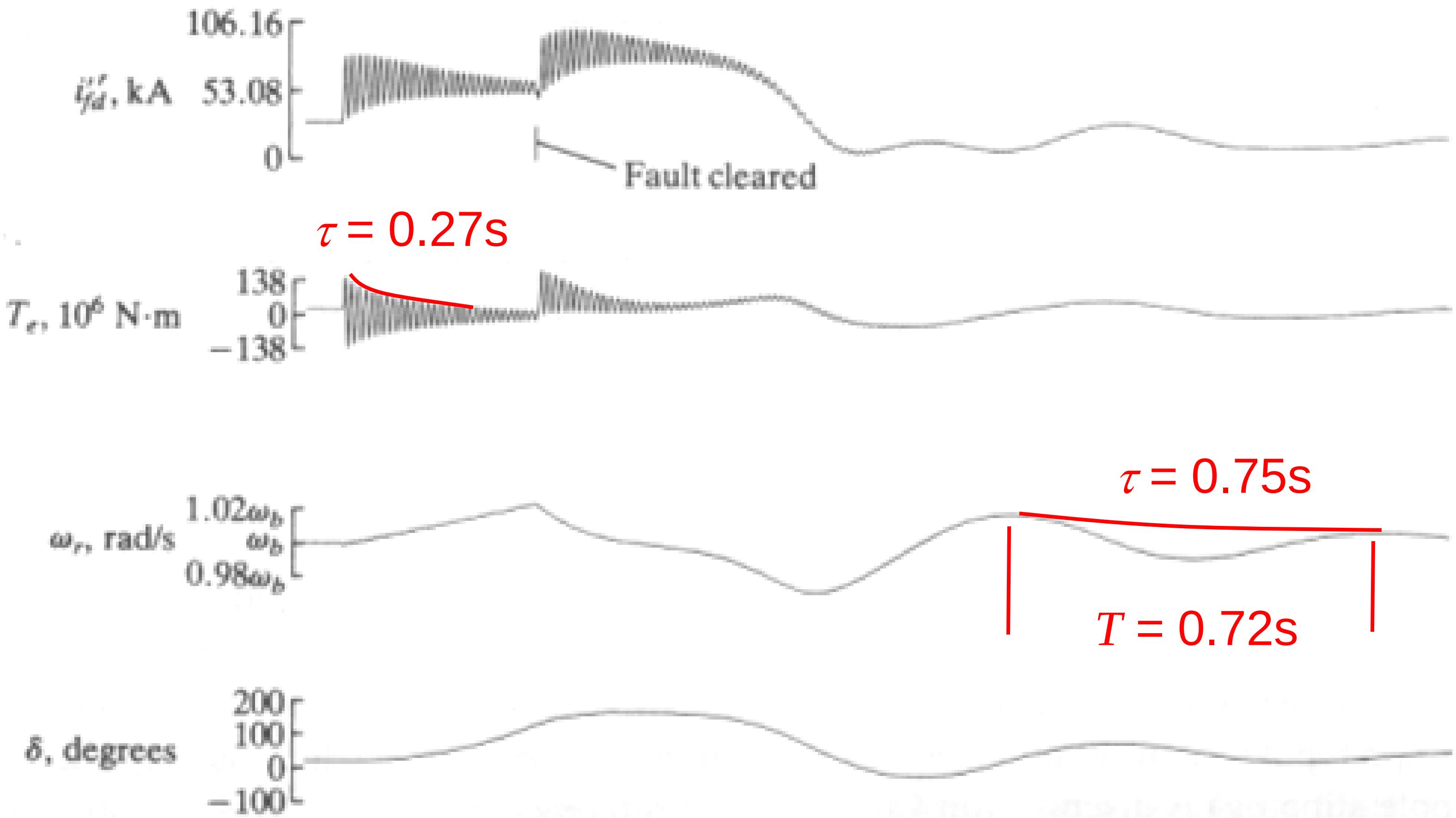


Figure 5.11-1 Dynamic performance of a hydro turbine generator during a 3-phase fault at the terminals.

Synchronous Machine Reduced-Order Equations (Chapter 8)

Neglecting stator transients:

- Provides approximate transient prediction
- Works better for larger machines
- Reduces number of states
  - Helpful in power system simulations
- Model inputs are currents
  - Good for average-value models of current-regulated power electronic converters

Neglect Stator Transients

Neglect $p\lambda_{qs}^{e}$ $p\lambda_{ds}^{e}$

In the synchronous reference frame

$$v_{qs}^{e} = -r_s i_{qs}^{e} + \frac{\omega_e}{\omega_b}\Psi_{ds}^{e} + \underbrace{p\Psi_{qs}^{e}}_{0}$$

$$v_{ds}^{e} = -r_s i_{ds}^{e} - \frac{\omega_e}{\omega_b}\Psi_{qs}^{e} + \underbrace{p\Psi_{ds}^{e}}_{0}$$

Accurate in steady-state,
approximately true in transient-state

Transform to Rotor Reference Frame

Frame-to-frame transformation: $${}^{e}K^{r} = \begin{bmatrix} \cos(\theta_r - \theta_e) & -\sin(\theta_r - \theta_e) & 0 \\ \sin(\theta_r - \theta_e) & \cos(\theta_r - \theta_e) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^{e}K^{r} v_{qd0s}^{e} = -r_s\, {}^{e}K^{r} i_{qd0s}^{e} + \frac{\omega_e}{\omega_b}\, {}^{e}K^{r} \Psi_{dqs}^{e} \qquad \Psi_{dqs}^{e} = \begin{bmatrix} \Psi_{ds}^{e} \\ -\Psi_{qs}^{e} \\ 0 \end{bmatrix}$$

$$\downarrow$$

$$\frac{\omega_e}{\omega_b}\left[\Psi_{ds}^{e}\cos(\theta_r - \theta_e) + \Psi_{qs}^{e}\sin(\theta_r - \theta_e)\right] = \frac{\omega_e}{\omega_b}\Psi_{ds}^{r}$$

$$\frac{\omega_e}{\omega_b}\left[\Psi_{ds}^{e}\sin(\theta_r - \theta_e) - \Psi_{qs}^{e}\cos(\theta_r - \theta_e)\right] = -\frac{\omega_e}{\omega_b}\Psi_{qs}^{r}$$

$$v_{qd0s}^{r} = -r_s i_{qd0s}^{r} + \frac{\omega_e}{\omega_b}\Psi_{dqs}^{r} \qquad \Psi_{dqs}^{r} = \begin{bmatrix} \Psi_{ds}^{r} \\ -\Psi_{qs}^{r} \\ 0 \end{bmatrix}$$

Reduced-Order Model Equations

$$\boxed{\begin{aligned} v_{qs}^{r} &= -r_s i_{qs}^{r} + \frac{\omega_e}{\omega_b}\psi_{ds}^{r} \\ v_{ds}^{r} &= -r_s i_{ds}^{r} - \frac{\omega_e}{\omega_b}\psi_{qs}^{r} \end{aligned}}$$

$$v_{kq1}^{\prime r} = r_{kq1}^{\prime} i_{kq1}^{\prime r} + \frac{p}{\omega_b}\psi_{kq1}^{\prime r}$$

$$v_{kq2}^{\prime r} = r_{kq2}^{\prime} i_{kq2}^{\prime r} + \frac{p}{\omega_b}\psi_{kq2}^{\prime r}$$

$$v_{fd}^{\prime r} = r_{fd}^{\prime} i_{fd}^{\prime r} + \frac{p}{\omega_b}\psi_{fd}^{\prime r}$$

$$v_{kd}^{\prime r} = r_{kd}^{\prime} i_{kd}^{\prime r} + \frac{p}{\omega_b}\psi_{kd}^{\prime r}$$

$$\begin{aligned}
\psi_{qs}^{r} &= -X_{ls} i_{qs}^{r} + X_{mq}(-i_{qs}^{r} + i_{kq1}^{\prime r} + i_{kq2}^{\prime r}) \\
\psi_{ds}^{r} &= -X_{ls} i_{ds}^{r} + X_{md}(-i_{ds}^{r} + i_{fd}^{\prime r} + i_{kd}^{\prime r}) \\
\psi_{kq1}^{\prime r} &= X_{lkq1}^{\prime} i_{kq1}^{\prime r} + X_{mq}(-i_{qs}^{r} + i_{kq1}^{\prime r} + i_{kq2}^{\prime r}) \\
\psi_{kq2}^{\prime r} &= X_{lkq2}^{\prime} i_{kq2}^{\prime r} + X_{mq}(-i_{qs}^{\prime} + i_{kq1}^{\prime r} + i_{kq2}^{\prime r}) \\
\psi_{fd}^{\prime r} &= X_{lfd}^{\prime} i_{fd}^{\prime r} + X_{md}(-i_{ds}^{r} + i_{fd}^{\prime r} + i_{kd}^{\prime r}) \\
\psi_{kd}^{\prime r} &= X_{lkd}^{\prime} i_{kd}^{\prime r} + X_{md}(-i_{ds}^{r} + i_{fd}^{\prime r} + i_{kd}^{\prime r})
\end{aligned}$$

Eigenvalue Examples

Full-order model Eigenvalues

Table 8.6-1 Synchronous Machine Eigenvalues for Rated Conditions

Hydro Turbine Generator	Steam Turbine Generator
$-3.58 \pm j\cancel{77}$ 377	$-4.45 \pm j377$
$-1.33 \pm j8.68$	$-1.70 \pm j10.5$
-24.4	-32.2
-22.9	-11.1
-0.453	-0.349
	-0.855

Reduced-order model Eigenvalues

Table 9.6-2 Synchronous Machine Eigenvalues for Rated Conditions Calculated with Stator Electric Transients Neglected

Hydro Turbine Generator	Steam Turbine Generator
$-1.33 \pm j8.68$	$-1.70 \pm j10.5$
-24.4	-32.2
-22.9	-11.1
-0.453	-0.855
	-0.350

stator transients gone
(no $\pm j377$ term)

Fault Simulation

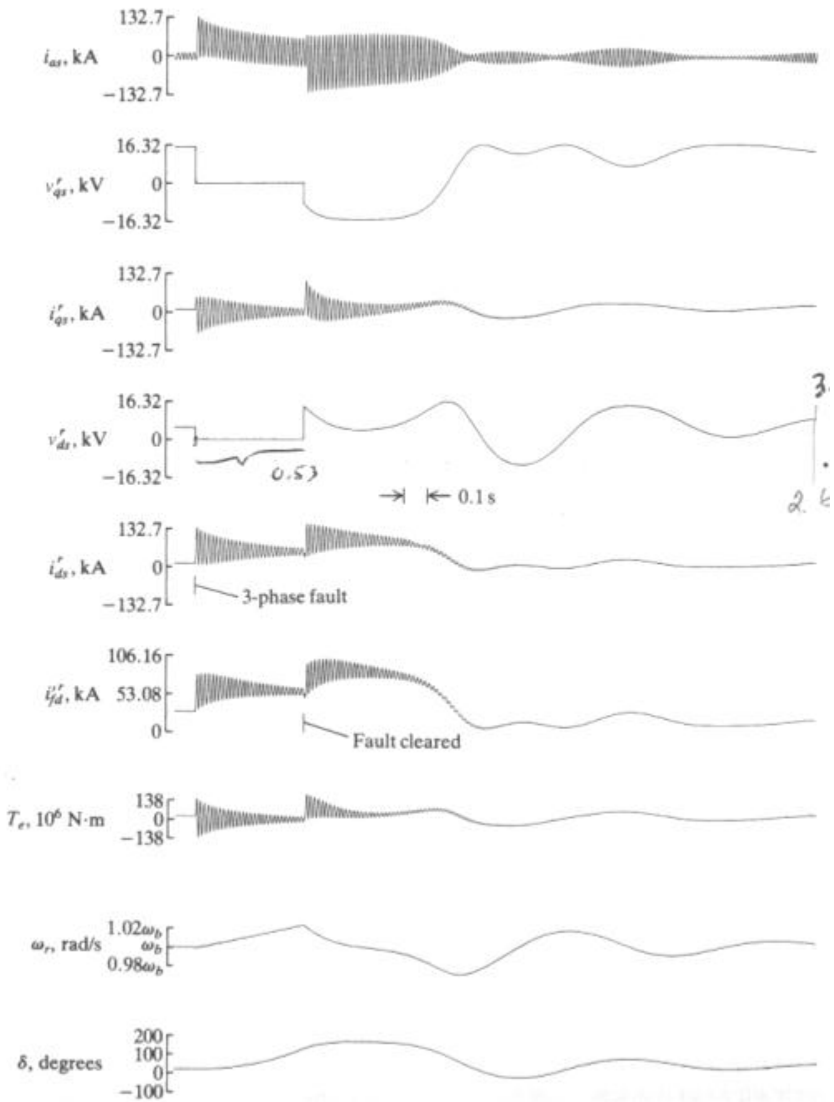


Figure 5.11-1 Dynamic performance of a hydro turbine generator during a 3-phase fault at the terminals.

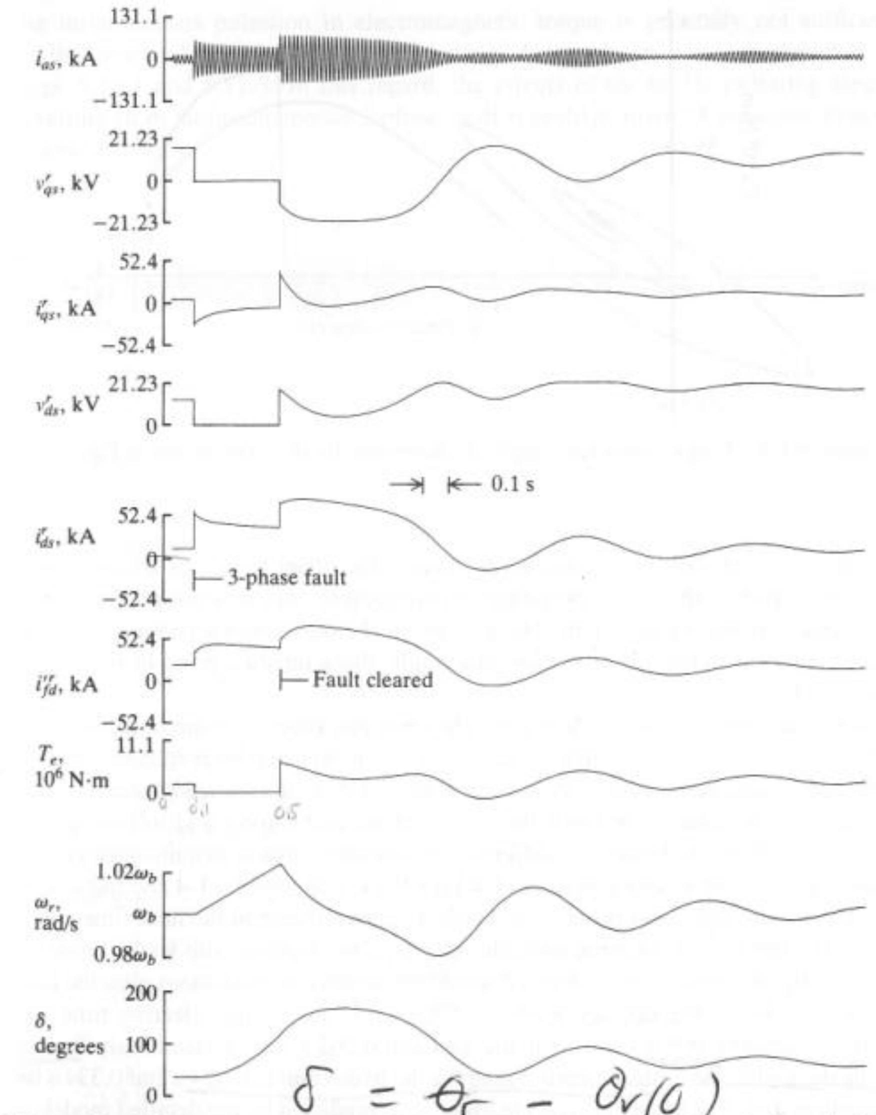


Figure 9.4-1 Dynamic performance of a hydro turbine generator during a 3-phase fault at the terminals predicted with stator electric transients neglected.

Full-order model

Reduced-order model