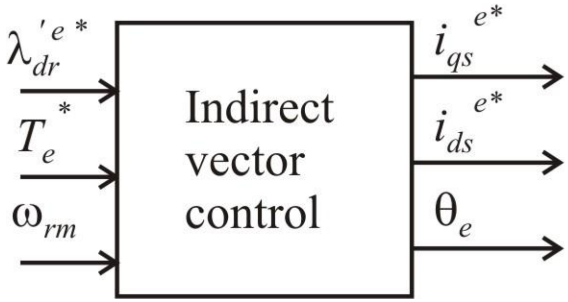
ECE 802, Electric Motor Control

Indirect Vector Control

Indirect Vector Control (Chapter 14)

- High-performance fast-response control
- Based on dynamic *q*-*d* model
- Rotor oriented (using rotor equations)

structure



Rotor Equations

induction motor model in the synchronous reference frame

$$v_{qr}'^{e} = r'_r i_{qr}'^{e} + (\omega_e - \omega_r)\lambda_{dr}'^{e} + p\lambda_{qr}'^{e} = 0 \qquad (1)$$

$$v_{dr}'^{e} = r'_r i_{dr}'^{e} - (\omega_e - \omega_r)\lambda_{qr}'^{e} + p\lambda_{dr}'^{e} = 0 \qquad (2)$$

$$\lambda_{qr}'^{e} = L'_{rr} i_{qr}'^{e} + L_M i_{qs}^{e} \qquad (3)$$

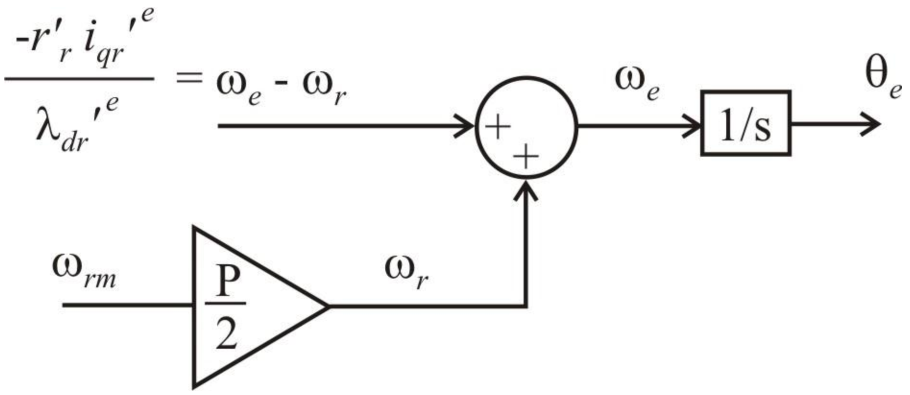
$$\lambda_{dr}'^{e} = L'_{rr} i_{dr}'^{e} + L_M i_{ds}^{e} \qquad (4)$$

$$T_e = \frac{3}{2}\frac{P}{2}(\lambda_{qr}'^{e} i_{dr}'^{e} - \lambda_{dr}'^{e} i_{qr}'^{e}) \qquad (5)$$

Note: Torque equation is not as straightforward as with the PMSM

If $\lambda_{qr}'^{e} = 0$, torque equation is straightforward
Design vector control so that

$$\omega_e - \omega_r = \frac{-r'_r i_{qr}'^{e}}{\lambda_{dr}'^{e}} \qquad (6)$$



Note, from (1) and (6)

$$p\lambda_{qr}'^{e} = -r'_r i_{qr}'^{e} - (\omega_e - \omega_r)\lambda_{dr}'^{e} = 0$$

so $\lambda_{qr}'^{e} = \lambda_{qr}'^{e}\Big|_{t=0} = 0$ (if motor is not energized at t = 0)

then $\lambda_{qr}'^{e} = 0$

From (2) with $\lambda_{qr}'^{e} = 0$

$$r'_r i_{dr}'^{e} + p\lambda_{dr}'^{e} = 0 \qquad (7)$$

Substitute (4) into (7)

$$r'_r i_{dr}'^{e} + L'_{rr}\, p i_{dr}'^{e} + L_M\, p i_{ds}^{e} = 0$$

From modulation (hysterisis or sine-triangle with compensation)

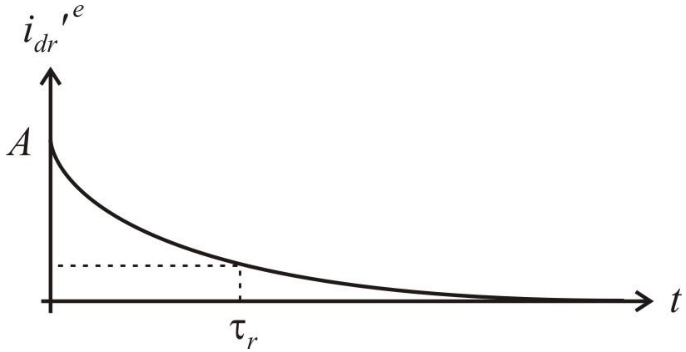
$$i_{ds}^{e} = i_{ds}^{e*} \qquad\qquad i_{qs}^{e} = i_{qs}^{e*}$$

Command constant *d*-axis current so $\quad p i_{ds}^{e*} = 0$

Then $\quad p i_{dr}'^{e} = \dfrac{-r'_r}{L'_{rr}} i_{dr}'^{e}$

$$\tau_r = \frac{L'_{rr}}{r'_r} \quad \text{(rotor time constant)}$$

Solution is: $i_{dr}'^{e} = Ae^{-t/\tau_r}$



after $t = 5\ \tau_r$, $i_{dr}'^{e} \approx 0$
(with constant i_{ds}^{e*})

with $i_{dr}^{'e} = 0 \qquad p\lambda_{qr}^{'e} = 0 \qquad \lambda_{qr}^{'e} = 0$

$$\omega_e - \omega_r = \frac{-r'_r i_{qr}'^e}{\lambda_{dr}'^e} \tag{8}$$

$$L'_{rr} i_{qr}'^e + L_M i_{qs}^e = 0 \tag{9}$$

$$\lambda_{dr}'^e = L_M i_{ds}^e \tag{10}$$

$$T_e = -\frac{3}{2}\frac{P}{2}\lambda_{dr}'^e i_{qr}'^e \tag{11}$$

Substitute (9) into (11)

$$T_e = \frac{3}{2}\frac{P}{2}\lambda_{dr}'^e \frac{L_M}{L'_{rr}} i_{qs}^e \tag{12}$$

With knowledge of $\lambda_{dr}'^{e}$ and L_M

Specifically command $i_{ds}^{e*} = \dfrac{\lambda_{dr}'^{e*}}{L_M}$ $\lambda_{dr}'^{e*}$ desired or rated $\lambda_{dr}'^{e}$

From (10) $\lambda_{dr}'^{e} = L_M i_{ds}^{e} = L_M i_{ds}^{e*} = \lambda_{dr}'^{e*}$

$$\lambda_{dr}'^{e} = \lambda_{dr}'^{e*}$$

For the *q*-axis, specifically command, $$i_{qs}^{e*} = \frac{2}{3}\frac{2}{P}\frac{L'_{rr}}{L_M}\frac{T_e^*}{\lambda_{dr}'^{e*}}$$

From (12) $$T_e = \frac{3}{2}\frac{P}{2}\lambda_{dr}'^{e}\frac{L_M}{L'_{rr}}\left(\frac{2}{3}\frac{2}{P}\frac{L'_{rr}}{L_M}\frac{T_e^*}{\lambda_{dr}'^{e*}}\right)$$

$$T_e = T_e^*$$

From (8) and (9)

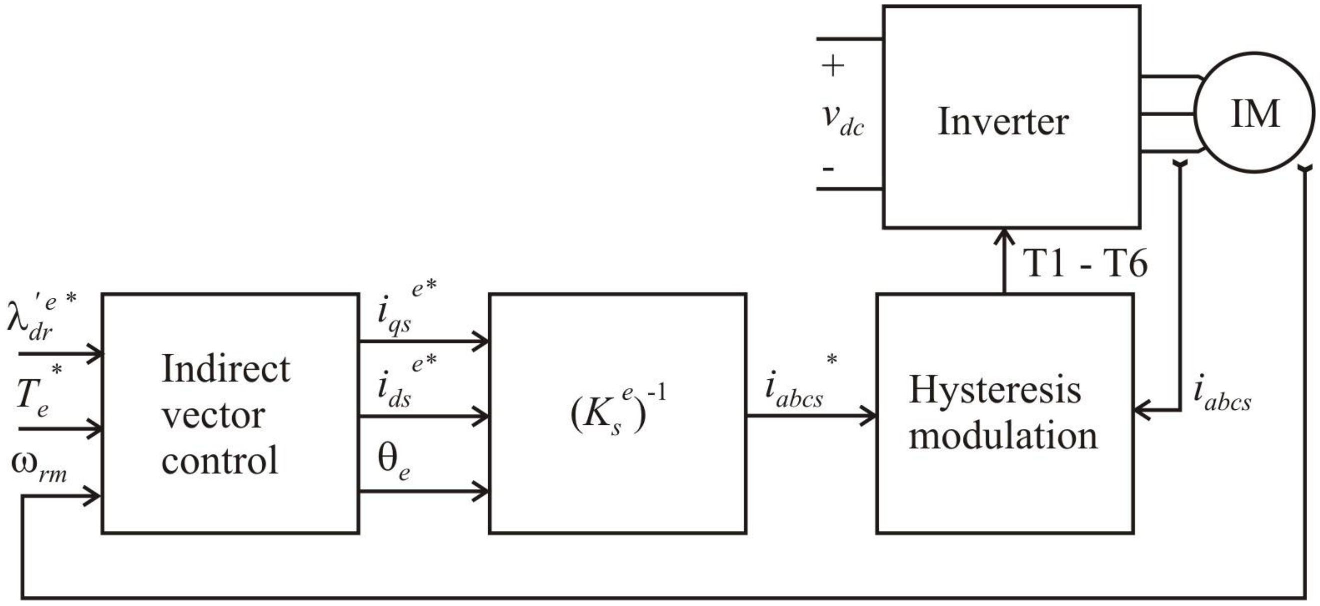
$$\omega_e - \omega_r = \frac{r'_r}{L'_{rr}} \frac{L_M}{\lambda_{dr}'^{\,e}} i_{qs}^e$$

In terms of commanded current,

$$\omega_e - \omega_r = \frac{r'_r}{L'_{rr}} \frac{i_{qs}^{e*}}{i_{ds}^{e*}}$$

Vector Control Implementation Steps

Example: 5hp (3.7 kW) induction motor



- command $\lambda_{dr}'^{e*} = \sqrt{2}\left|\tilde{\Lambda}'_{ar}\right| = 0.385\,\mathrm{V\cdot s}$
- wait $5\tau_r = 1.5\,\mathrm{sec}$
- command step in torque

constant speed

$$\omega_{rm} = 1750\,\mathrm{rpm} = 183.3\,\mathrm{rad/sec}$$

hysteresis $h = 0.5\,\mathrm{A}$

IM Steady-State Calculations

$$RPM := \frac{2\cdot\pi\cdot rad}{60\cdot s}$$

IM parameters

$r_s := 0.4\Omega$ $P := 4$ $r'_r := 0.2266\Omega$ $lagging := 1$

$L_{ls} := 5.73mH$ $L_M := 64.4mH$ $L'_{lr} := 4.64mH$

$L_{ss} := L_{ls} + L_M$ $L_{ss} = 70.1mH$

$L'_{rr} := L'_{lr} + L_M$ $L'_{rr} = 69mH$

operating conditions

$f_e := 60\cdot Hz$ $\omega_e := 2\cdot\pi\cdot f_e$ $\omega_e = 377\frac{rad}{s}$

$\omega_{rm} := 1750RPM$ $\omega_{rm} = 183.3\frac{rad}{s}$

$V_{LL} := 220\cdot V$ $V_s := \frac{V_{LL}}{\sqrt{3}}$ $V_s = 127V$

synchronous speed (no-load speed)

$\omega_{em} := \left(\frac{2}{P}\right)\cdot\omega_e$ $\omega_{em} = 188.5\frac{rad}{s}$

$\omega_{em} = 1800RPM$

slip

$\omega_r := \frac{P}{2}\cdot\omega_{rm}$ $\omega_r = 366.5\frac{rad}{s}$

$s := \frac{\omega_e - \omega_r}{\omega_e}$ $s = 0.0278$

IM Steady-State Calculations

impedances

$Z_s := r_s + j \cdot \omega_e \cdot L_{ls}$ $\quad |Z_s| = 2.2\Omega$ $\quad \arg(Z_s) = 79.5\text{deg}$

$Z_m := j \cdot \omega_e \cdot L_M$ $\quad |Z_m| = 24.3\Omega$ $\quad \arg(Z_m) = 90\text{deg}$

$Z'_r := \frac{r'_r}{s} + j \cdot \omega_e \cdot L'_{lr}$ $\quad |Z'_r| = 8.34\Omega$ $\quad \arg(Z'_r) = 12.1\text{deg}$

$$Z_f := \frac{1}{\frac{1}{Z_m} + \frac{1}{Z'_r}}$$ $\quad |Z_f| = 7.43\Omega$ $\quad \arg(Z_f) = 29.5\text{deg}$

$Z_{in} := Z_s + Z_f$ $\quad |Z_{in}| = 9\Omega$ $\quad \arg(Z_{in}) = 40.3\text{deg}$

currents

$V_{as} := V_s \cdot e^{j \cdot 0}$

$I_{as} := \frac{V_{as}}{Z_{in}}$ $\quad |I_{as}| = 14.1\text{A}$ $\quad \arg(I_{as}) = -40.3\text{deg}$

$I'_{ar} := -I_{as} \cdot \frac{Z_m}{Z_m + Z'_r}$ $\quad |I'_{ar}| = 12.6\text{A}$ $\quad \arg(I'_{ar}) = 157.1\text{deg}$

IM Steady-State Calculations

torque and power

$$T_e := 3\cdot\frac{P}{2}\cdot\left(\left|I'_{ar}\right|\right)^2\cdot\frac{r'_r}{s\cdot\omega_e} \qquad T_e = 20.5\,N\cdot m$$

$$T_e := \frac{3\cdot\left(\frac{P}{2}\right)\cdot\omega_e\cdot L_M{}^2\cdot r'_r\cdot s\cdot\left(\left|V_{as}\right|\right)^2}{\left[r_s\cdot r'_r + s\cdot\omega_e{}^2\cdot\left(L_M{}^2 - L_{ss}\cdot L'_{rr}\right)\right]^2 + \omega_e{}^2\cdot\left(r'_r\cdot L_{ss} + s\cdot r_s\cdot L'_{rr}\right)^2} \qquad T_e = 20.5\,N\cdot m$$

$$\theta := \arg\left(Z_{in}\right) \qquad \theta = 40.3\,deg$$

$$pf := \cos(\theta) \qquad pf = 0.763\ \text{lagging}$$

$$P_{in} := 3\cdot\left|V_{as}\right|\cdot\left|I_{as}\right|\cdot pf \qquad P_{in} = 4.104\,kW$$

$$P_{out} := T_e\cdot\omega_{rm} \qquad P_{out} = 3.757\,kW$$

$$P_{out} = 5\,hp$$

$$eff := \frac{P_{out}}{P_{in}} \qquad eff = 91.6\%$$

IM Steady-State Calculations

magnetizing flux linkage

$I_{am} := I_{as} + I'_{ar}$ $\quad |I_{am}| = 4.32A$ $\quad \arg(I_{am}) = -100.8deg$

$\Lambda_m := L_M \cdot I_{am}$ $\quad \boxed{|\Lambda_m| = 0.278V \cdot sec}$

$V_m := V_{as} - Z_s \cdot I_{as}$ $\quad |V_m| = 105V$ $\quad \arg(V_m) = -10.8deg$

$\Lambda_m := \frac{V_m}{\omega_e}$ $\quad |\Lambda_m| = 0.278V \cdot sec$

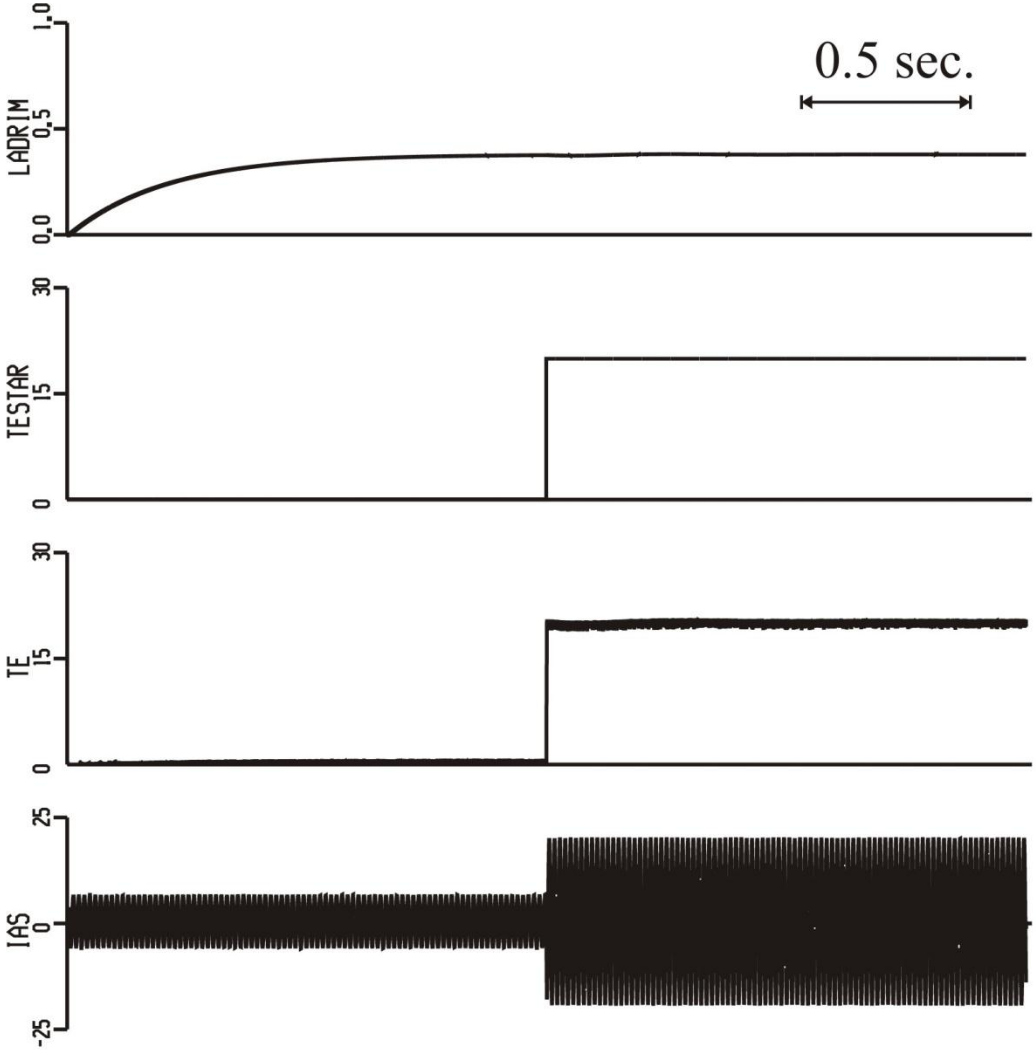
rotor flux linkage

$\Lambda'_r := \frac{-I'_{ar} \cdot \frac{r'_r}{s}}{j \cdot \omega_e}$ $\quad |\Lambda'_r| = 0.272V \cdot sec$ $\quad \boxed{\sqrt{2} \cdot |\Lambda'_r| = 0.385V \cdot sec}$

rotor time constant

$\tau_r := \frac{L'_{rr}}{r'_r}$ $\quad \tau_r = 0.3s$

$5 \cdot \tau_r = 1.5s$

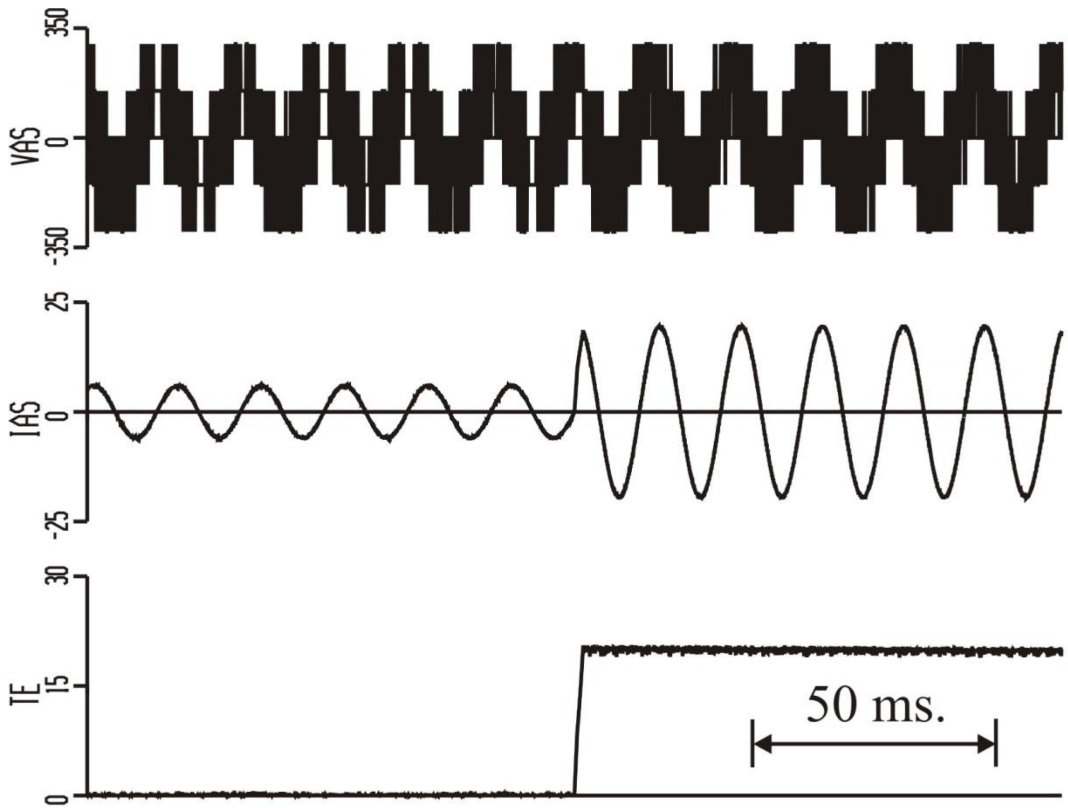


Vector controlled induction motor with hysteresis control

$P = 4$ $\quad$ $L_M = 64.43$ mH $\quad$ $\lambda_{dre}'^* = 0.385$ V-sec $\quad$ $h = 0.5$ A

$r_s = 0.3996\ \Omega$ $\quad$ $r_r' = 0.2266\ \Omega$ $\quad$ $T_e^* = 20$ N-m

$L_{ls} = 5.73$ mH $\quad$ $L_{lr}' = 4.64$ mH $\quad$ $\omega_{rm} = 1750$ RPM $= 183.3$ rad/sec



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$L_{ls} = 5.73$ mH	$L_{lr}' = 4.64$ mH	$\omega_{rm} = 1750$ RPM $= 183.3$ rad/sec	

Vector Control Terminology

Define vectors $\vec{\lambda}_{qdr}'^{e} = \lambda_{qr}'^{e} - j\lambda_{dr}'^{e}$ $\vec{i}_{qdr}'^{e} = i_{qr}'^{e} - ji_{dr}'^{e}$

Vector control ensures, $\lambda_{qr}'^{e} = 0$ $i_{dr}'^{e} = 0$

$$\vec{\lambda}_{qdr}'^{e} = -j\lambda_{dr}'^{e} \qquad \vec{i}_{qr}'^{e} = i_{qr}'^{e}$$

For motor operation T_e* > 0 so $i_{qs}^{e*} > 0$

$$i_{qr}'^{e} = \frac{-L_M}{L_{rr}'} i_{qs}^{e*} \quad \text{so} \quad i_{qr}'^{e} < 0$$

Synchronous reference frame vectors

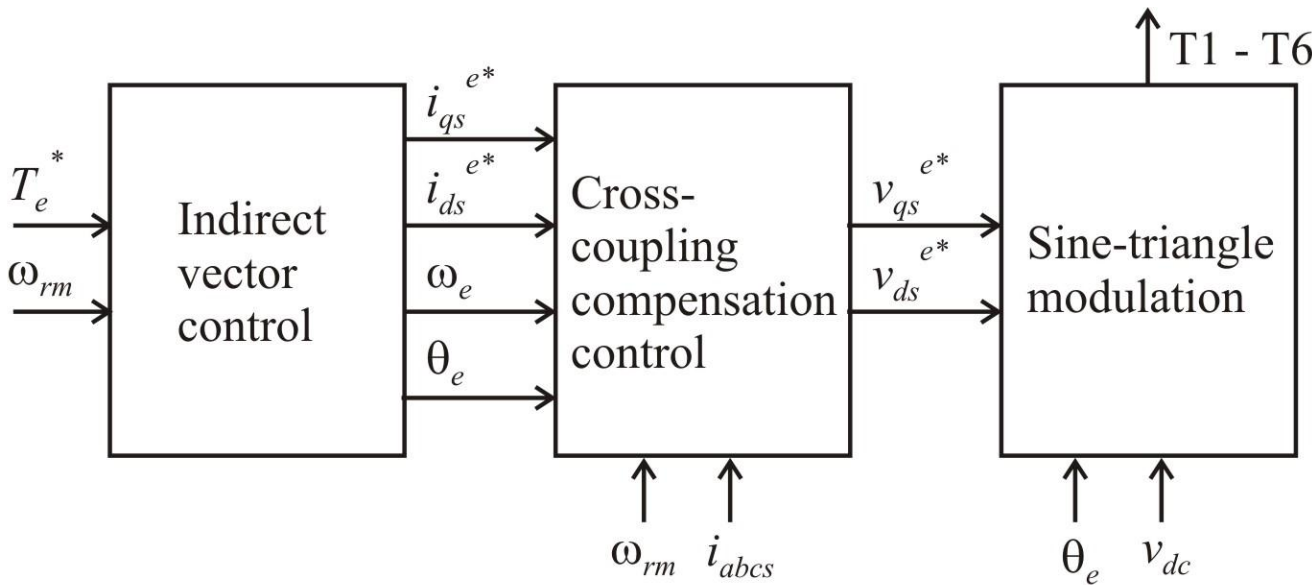
Note $\left|\vec{\lambda}_{qdr}'^{e} \times \vec{i}_{qdr}'^{e}\right| = \left|\vec{\lambda}_{qdr}'^{e}\right| \left|\vec{i}_{qdr}'^{e}\right| \sin\left(\angle \vec{i}_{qdr}'^{e} - \angle \vec{\lambda}_{qdr}'^{e}\right) = \lambda_{dr}'^{e} i_{qr}'^{e}$

from before, $T_e = \frac{3}{2}\frac{P}{2}(\lambda_{qr}'^{e} i_{dr}'^{e} - \lambda_{dr}'^{e} i_{qr}'^{e})$

with $\lambda_{qr}'^{e} = 0$ $i_{dr}'^{e} = 0$ $T_e = -\frac{3}{2}\frac{P}{2}\lambda_{dr}'^{e} i_{qr}'^{e}$

Cross-Coupling Compensation Control

Interface vector control to voltage-source modulation



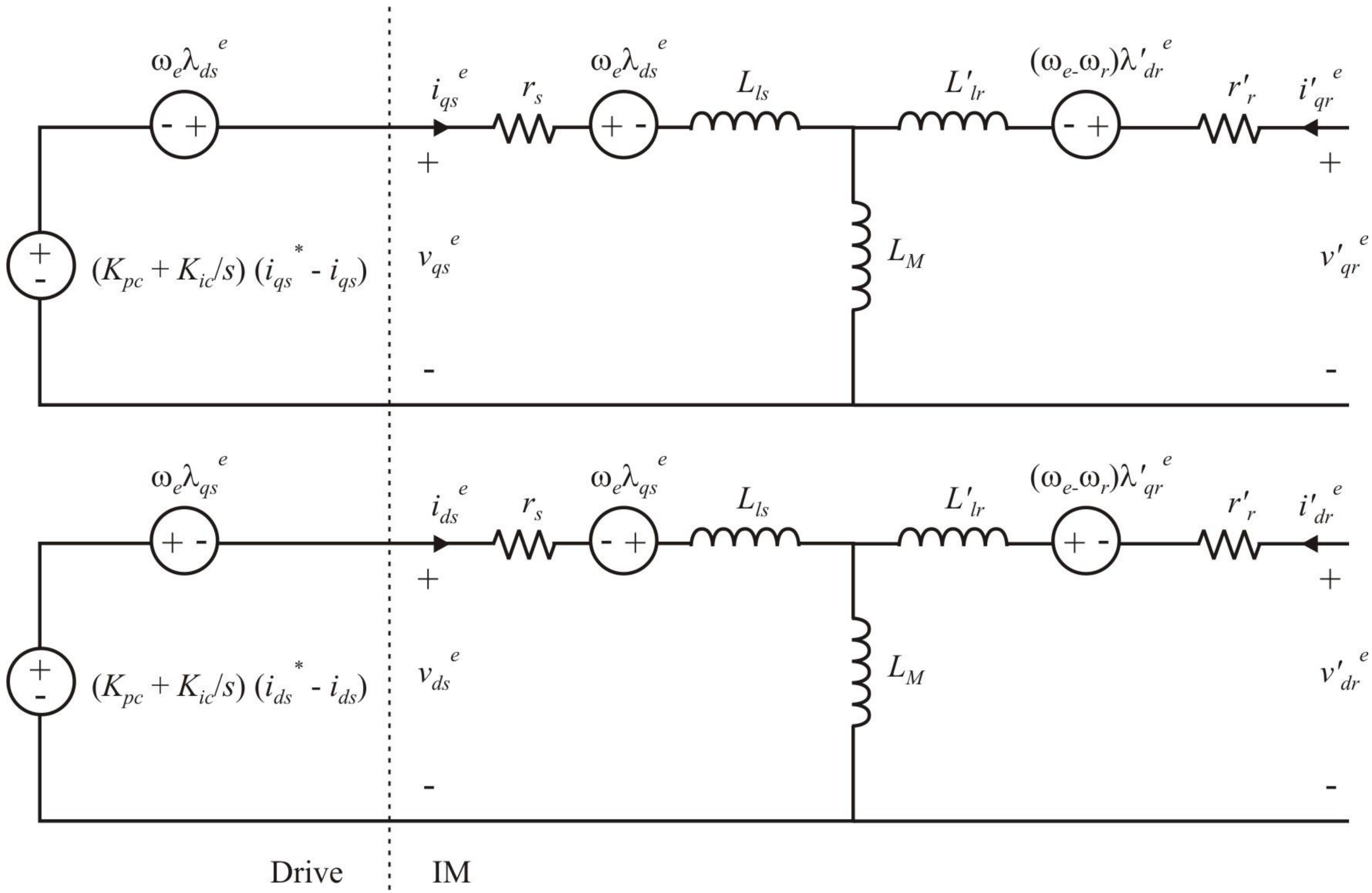
Compensation Control

add $\omega_e\lambda_{qs}^{e}$ and $\omega_e\lambda_{ds}^{e}$ to v_{qs}^{e*} and v_{ds}^{e*}

command

$$v_{qs}^{e*} = \left(K_{pc} + \frac{K_{ic}}{s}\right)\left(i_{qs}^{e*} - i_{qs}^{e}\right) + \omega_e\lambda_{ds}^{e}$$

$$v_{ds}^{e*} = \left(K_{pc} + \frac{K_{ic}}{s}\right)\left(i_{ds}^{e*} - i_{ds}^{e}\right) - \omega_e\lambda_{qs}^{e}$$



Next step : Compute λ_{qs}^{e} and λ_{ds}^{e} from i_{qs}^{e} , i_{ds}^{e}

Stator Flux Linkage from Commanded Currents

Note: Vector control ensures $\lambda_{qr}'^{e} = 0 \quad i_{dr}'^{e} = 0$

From machine equations:

$$\lambda_{ds}^{e} = L_{ls} i_{ds}^{e} + L_M \left(i_{ds}^{e} + i_{dr}'^{e} \right) = L_{ss} i_{ds}^{e} + \cancelto{0}{L_M i_{dr}'^{e}}$$

$$\lambda_{ds}^{e} = L_{ss} i_{ds}^{e*}$$

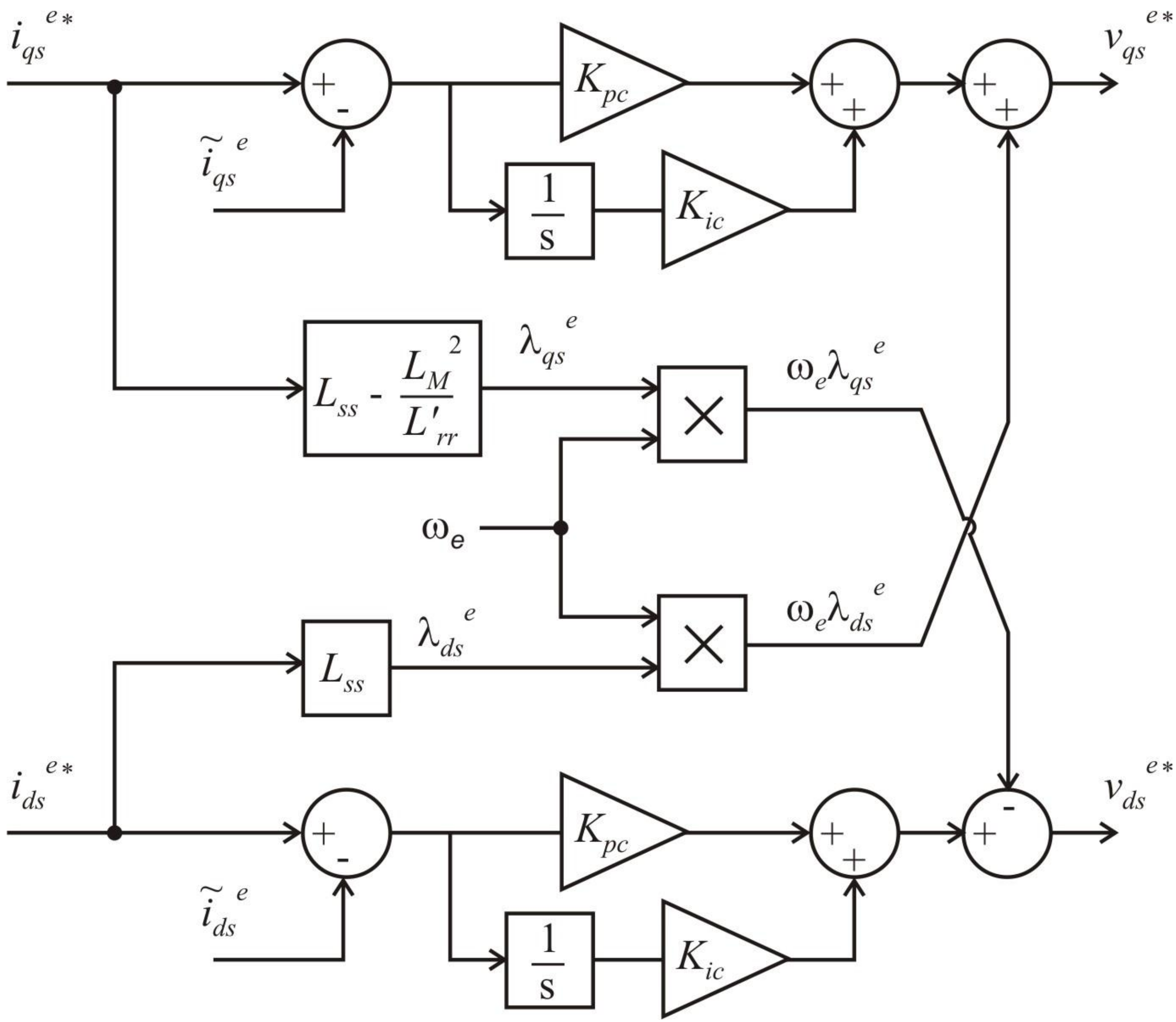
$$\lambda_{qr}'^{e} = L'_{lr} i_{qr}'^{e} + L_M \left(i_{qs}^{e} + i_{qr}'^{e} \right) = L'_{rr} i_{qr}'^{e} + L_M i_{qs}^{e} = 0$$

$$\longrightarrow \quad i_{qr}'^{e} = \frac{-L_M}{L'_{rr}} i_{qs}^{e}$$

$$\lambda_{qs}^{e} = L_{ls} i_{qs}^{e} + L_M \left(i_{qs}^{e} + i_{qr}'^{e} \right) = L_{ss} i_{qs}^{e} + L_M i_{qr}'^{e} = L_{ss} i_{qs}^{e} - \frac{L_M^{\,2}}{L'_{rr}} i_{qs}^{e}$$

$$\lambda_{qs}^{e} = \left(L_{ss} - \frac{L_M^{\,2}}{L'_{rr}} \right) i_{qs}^{e*}$$

Compensation Control Block Diagram



Practical Current Measurement Technique

τ = low-pass filter time constant (sec)

f_c = low-pass filter cut-off frequency (Hz)

set f_c lower than the switching frequency
but higher than the fundamental frequency

Determine T1 - T6 from v_{qs}^{e*}, v_{ds}^{e*}

modulation index

$$V_s^* = \frac{1}{\sqrt{2}}\sqrt{\left(v_{qs}^{e*}\right)^2 + \left(v_{ds}^{e*}\right)^2}$$

$$d = 2\sqrt{2}\,\frac{V_s^*}{v_{dc}}$$

phase shift

$$\phi_v^* = \tan^{-1}\left(\frac{-v_{ds}^{e*}}{v_{qs}^{e*}}\right)$$

$$\theta_c = \theta_e + \phi_v^*$$

duty cycles

$$d_a = d\cos\left(\theta_c\right) - \frac{d}{6}\cos\left(3\theta_c\right)$$

$$d_b = d\cos\left(\theta_c - \frac{2\pi}{3}\right) - \frac{d}{6}\cos\left(3\theta_c\right)$$

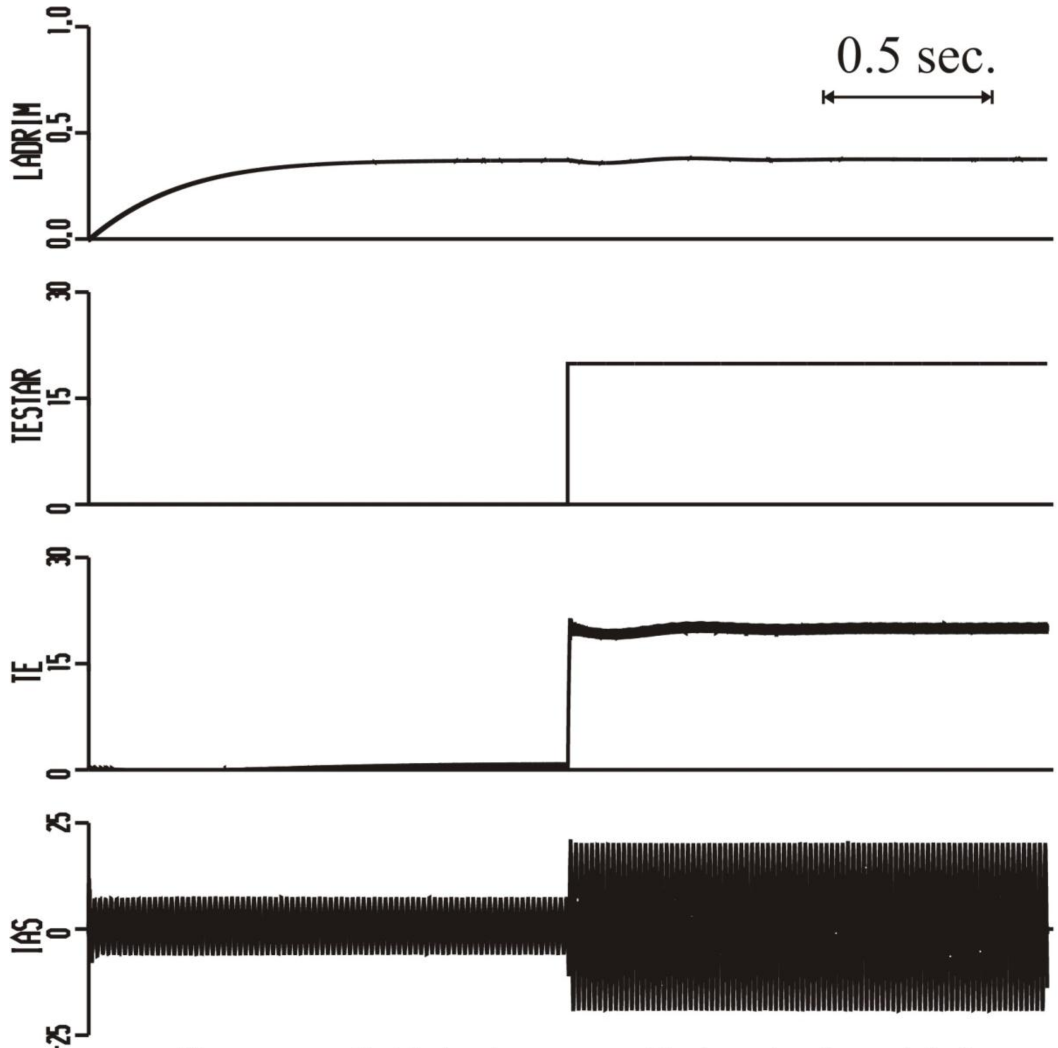
$$d_c = d\cos\left(\theta_c + \frac{2\pi}{3}\right) - \frac{d}{6}\cos\left(3\theta_c\right)$$

compare to a triangle waveform to obtain T1 - T6

Example from before: 5hp (3.7 kW) motor

constant speed with cross-coupling compensation control

- command $\lambda_{dr}'^{e*} = \sqrt{2}\left|\tilde{\Lambda}'_{ar}\right| = 0.385\,\mathrm{V}\cdot\mathrm{s}$
- wait $5\tau_r = 1.5\,\mathrm{sec}$
- command step in torque



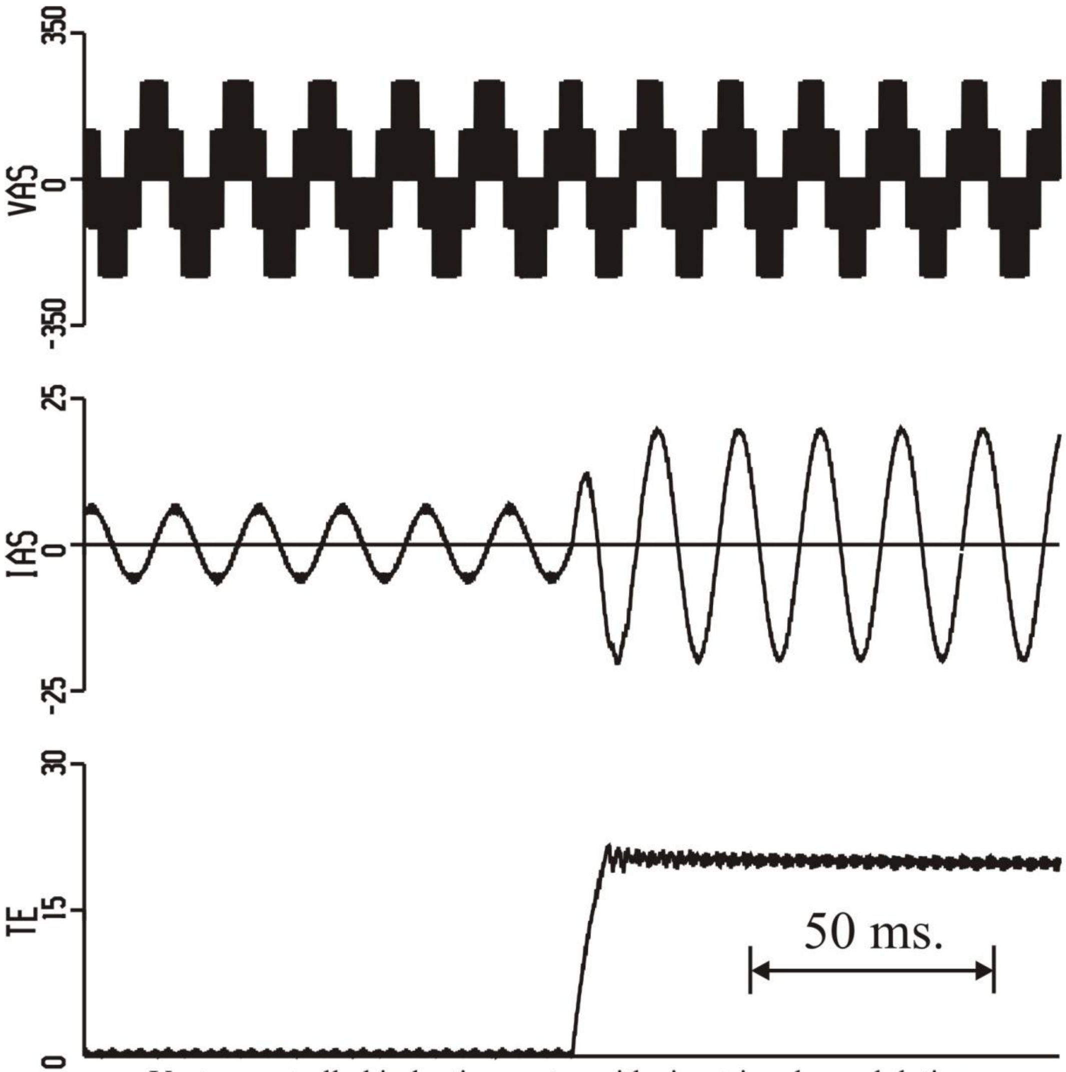
Vector controlled induction motor with sine-triangle modulation

$P = 4$ $L_M = 64.43$ mH $\lambda_{dre}'^* = 0.385$ V-sec

$r_s = 0.3996\ \Omega$ $r_r' = 0.2266\ \Omega$ $T_e^* = 20$ N-m

$L_{ls} = 5.73$ mH $L_{lr}' = 4.64$ mH $\omega_{rm} = 1750$ RPM $= 183.3$ rad/sec

$K_{pc} = 50$ V/A $K_{ic} = 50$ V/A-sec



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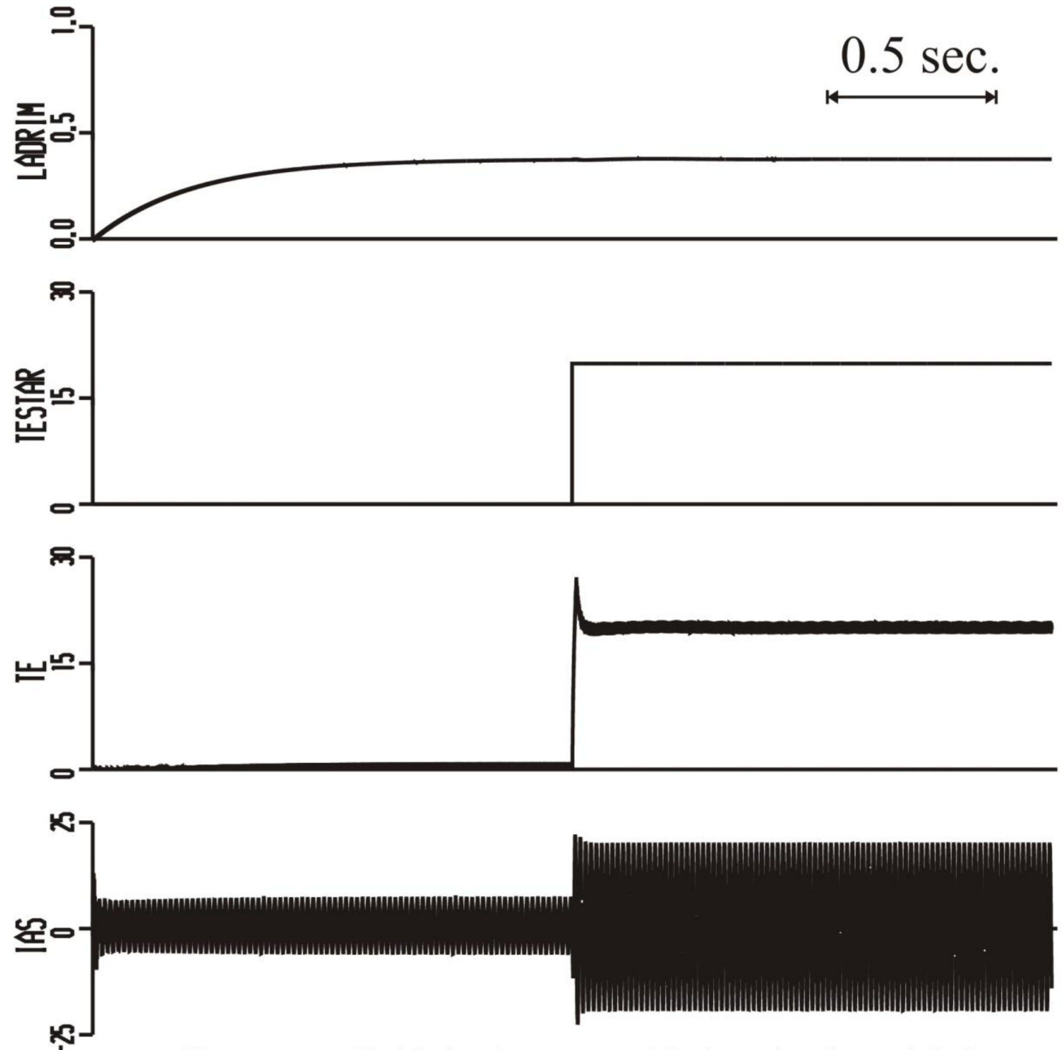

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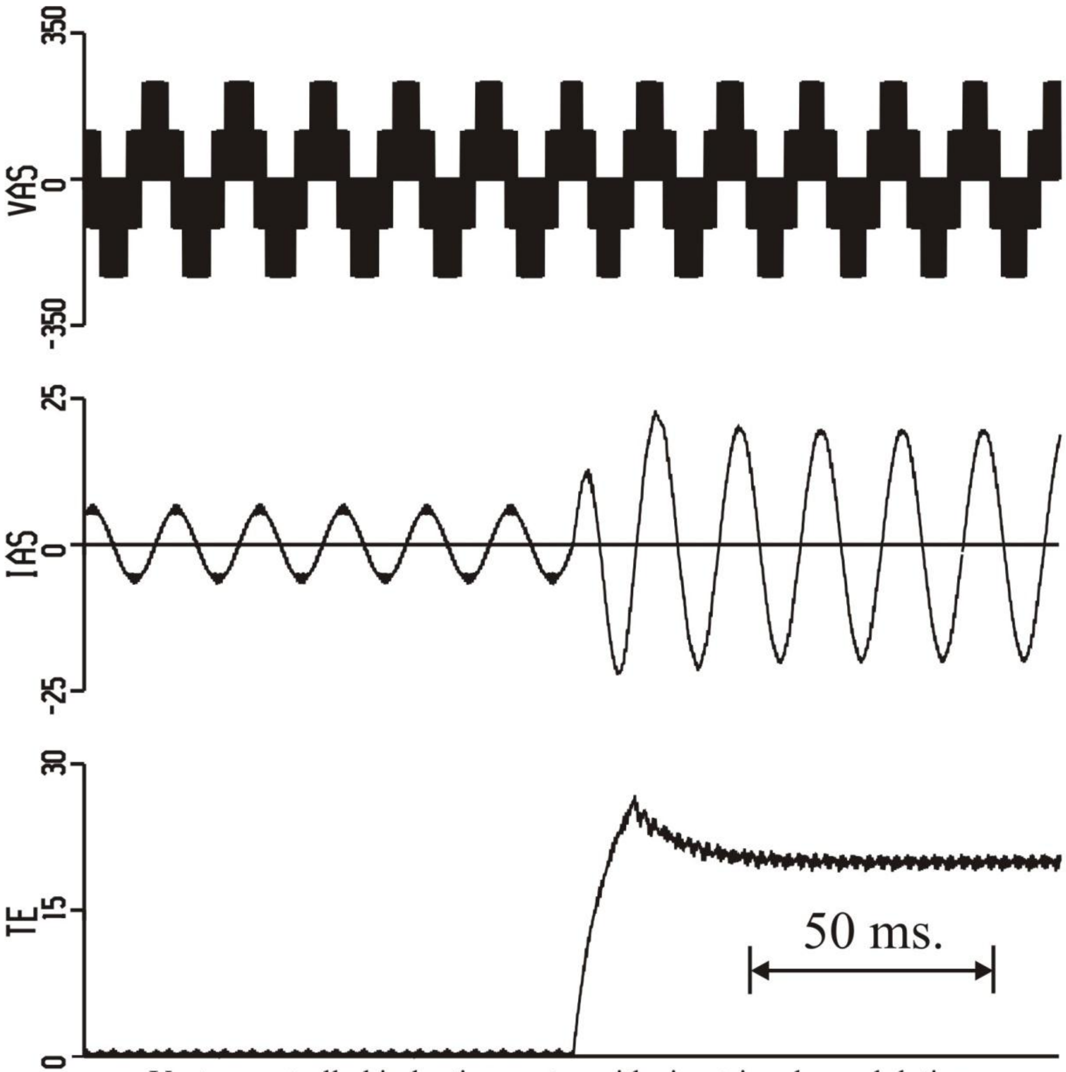
$K_{pc} = 50$ V/A	$K_{ic} = 5000$ V/A-sec	



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Indirect Vector Control

Based on rotor equations formulated in the synchronous reference frame

Based on controlling the flux and current to be at 90 degrees. The *d*-axis current sets the flux much like the field of a dc machine. The *q*-axis current sets the torque much like the armature of a dc machine.

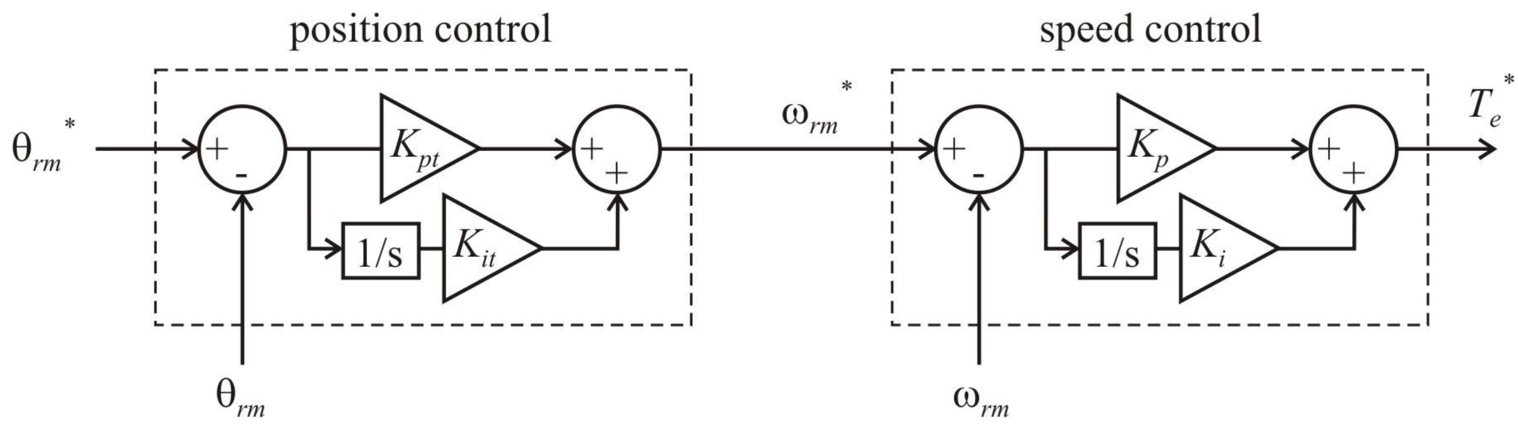
Implementation is straightforward if the machine parameters are known. The vector control can be followed by a current regulated control or a voltage-source control with cross-coupling compensation.

Speed and Position Control

mechanical systems

- Regulator applications - keep output constant despite disturbances (inertia helps)
- Servo applications - rapidly change system output (inertia hinders)

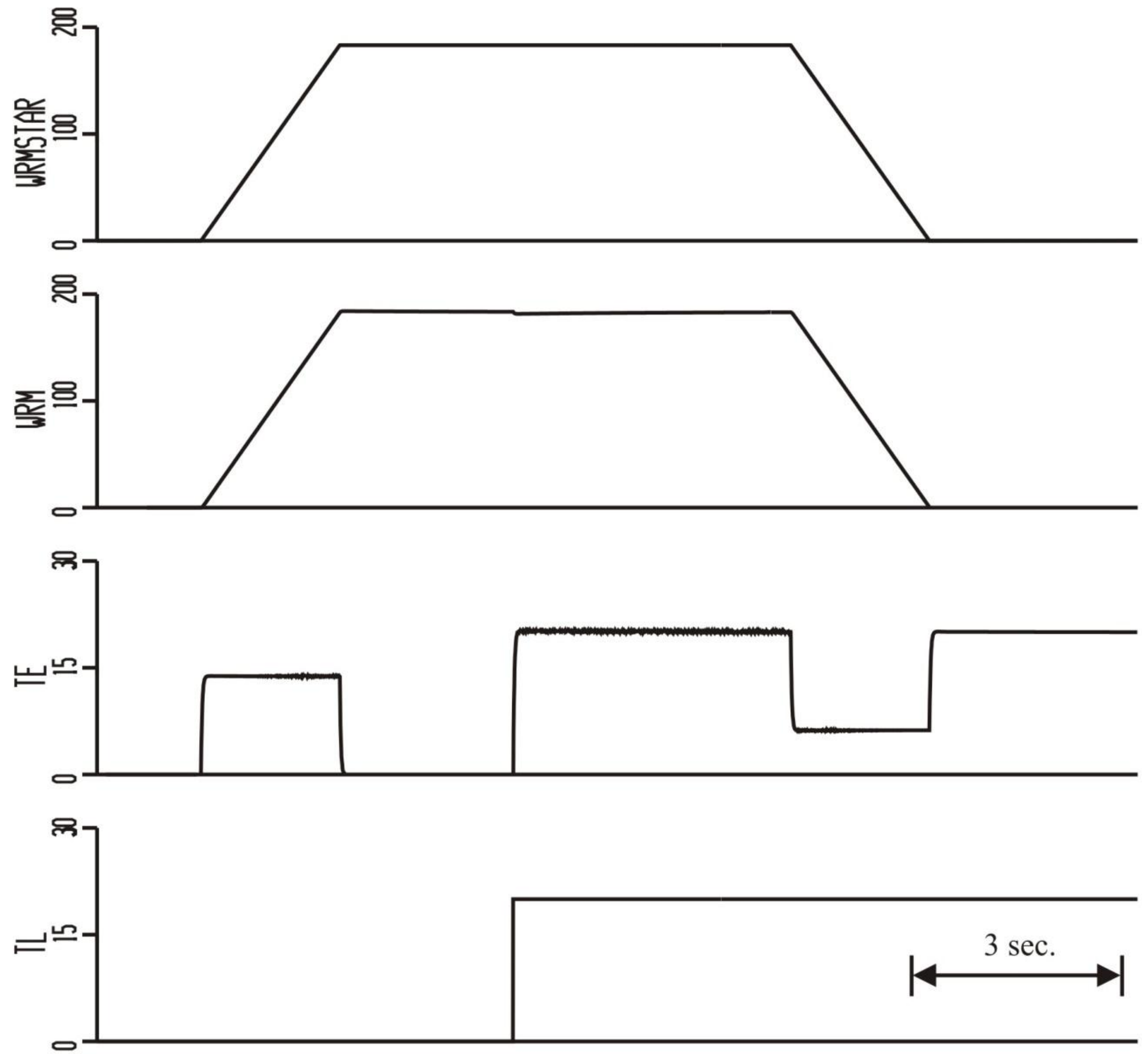
system control



Speed Control Example

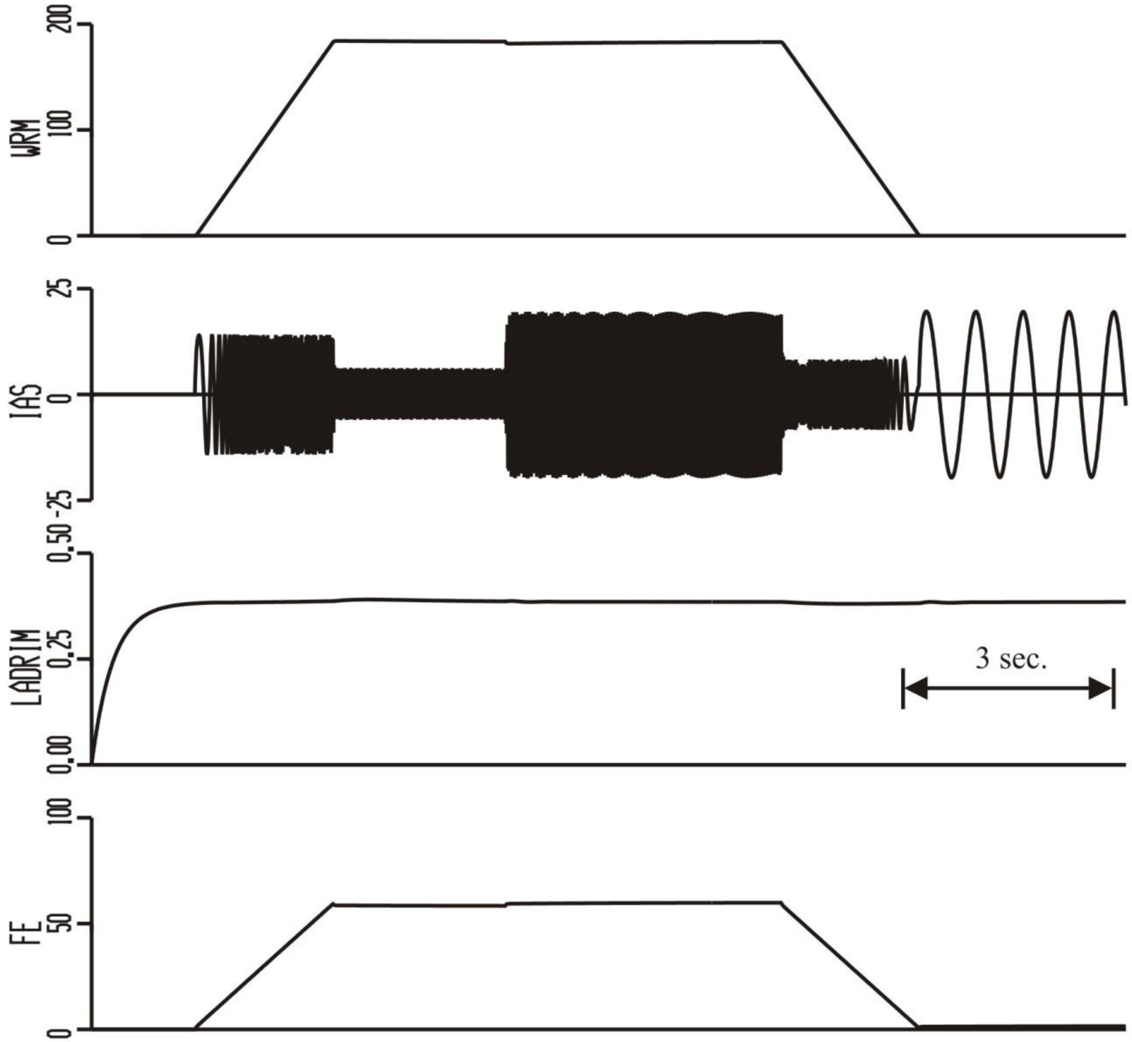
Motor from before 5hp (3.7 kW)
$J = 0.15\ \mathrm{kg \cdot m^2}$

- ramp speed command from zero to rated
- step load torque from zero to rated
- ramp speed from rated to zero while rated load torque is applied



Vector controlled induction motor with sine-triangle modulation and speed control

$P = 4$	$L_M = 64.43$ mH	$\lambda_{dre}'^* = 0.385$ V-sec
$r_s = 0.3996\ \Omega$	$r_r' = 0.2266\ \Omega$	$\omega_{rm} = 0$ to 1750 RPM (183.3 rad/sec)
$L_{ls} = 5.73$ mH	$L_{lr}' = 4.64$ mH	$T_L = 0$ to 20 N-m
$K_{pc} = 50$ V/A	$K_{ic} = 50$ V/A-sec	$K_p = 10$ N-m-sec $\quad K_i = 5$ N-m



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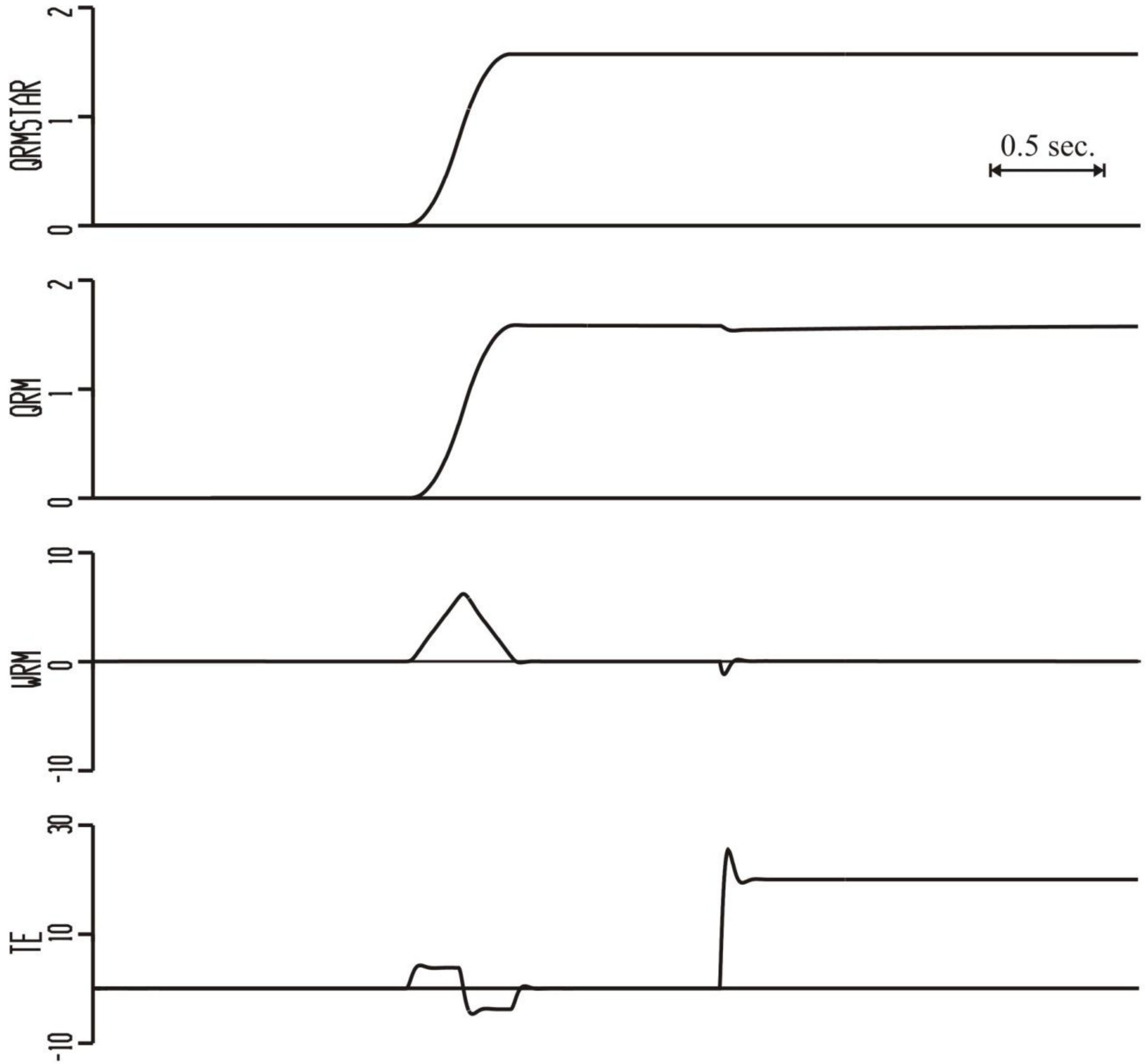
Position Control Example

Motor from before 5hp (3.7 kW)

- change position from zero to 90 degrees
- step torque from zero to rated



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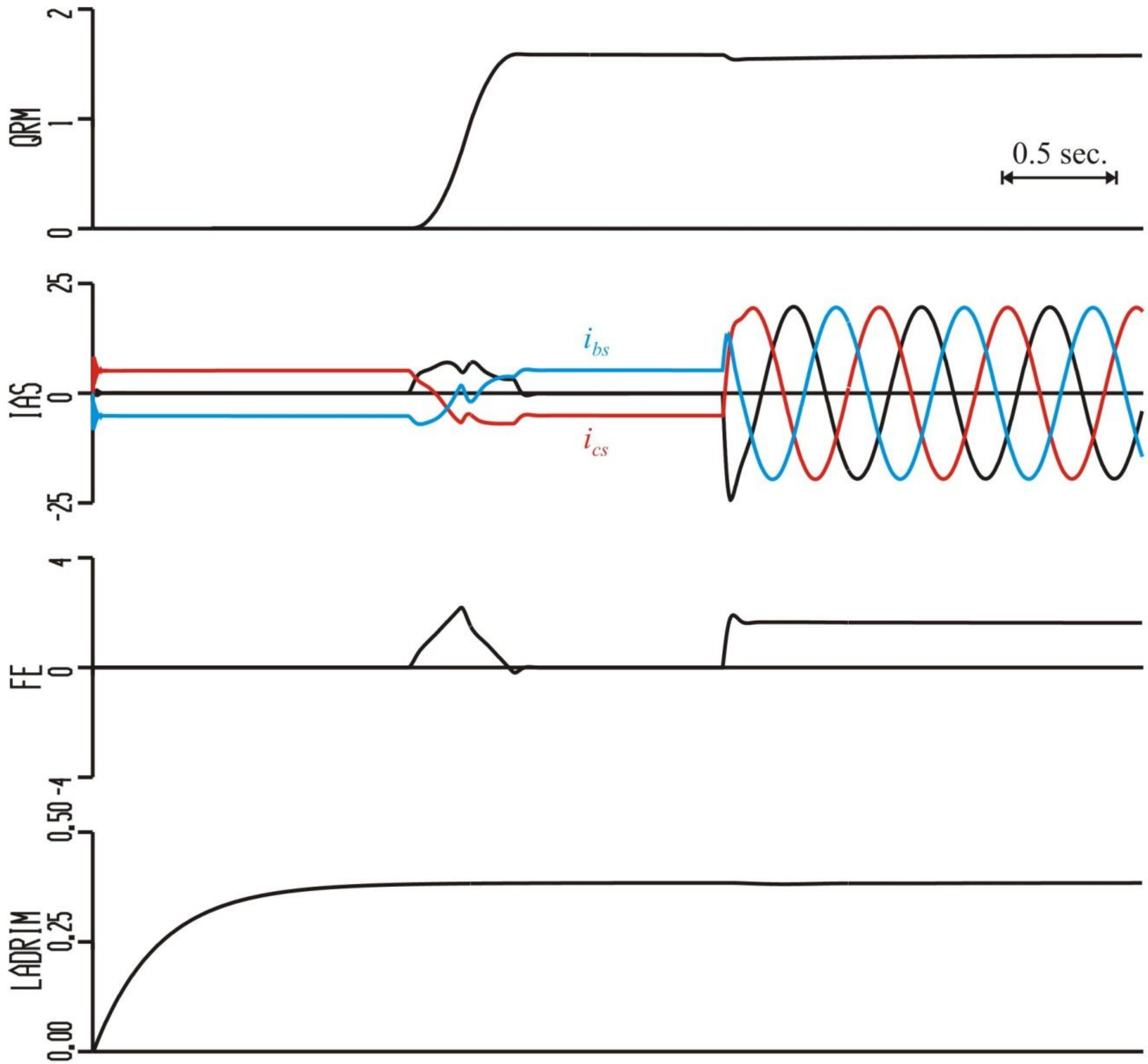
$L_{ls} = 5.73$ mH	$L_{lr}' = 4.64$ mH	$T_L = 0$ to 20 N-m	
$K_{pc} = 50$ V/A	$K_{ic} = 50$ V/A-sec	$K_p = 10$ N-m-sec	$K_i = 5$ N-m
		$K_{pt} = 50$ sec	$K_{it} = 20$



Vector controlled induction motor with sine-triangle modulation and position control

$P = 4$	$L_M = 64.43$ mH	$\lambda_{dre}'^{*} = 0.385$ V-sec	
$r_s = 0.3996\ \Omega$	$r_r' = 0.2266\ \Omega$	$\omega_{rm} = 0$ to 1750 RPM (183.3 rad/sec)	
$L_{ls} = 5.73$ mH	$L_{lr}' = 4.64$ mH	$T_L = 0$ to 20 N-m	
$K_{pc} = 50$ V/A	$K_{ic} = 50$ V/A-sec	$K_p = 10$ N-m-sec	$K_i = 5$ N-m
		$K_{pt} = 50$ sec	$K_{it} = 20$

Speed and Position Control Examples

Traditional PI controllers to regulated speed and position

The drive was implemented with a vector controlled induction machine using voltage-source modulation

Speed and position are commanded to move along a ramp and parabolic curve respectively to avoid commanding a step change in speed which realistically cannot be achieved