Birla Institute of Technology and Science, Pilani-K K Birla Goa Campus

Numerical Methods for Partial Differential Equations (MATH F422)

Parabolic Partial Differential Equations

Lab Sheet-2

Write a MATLAB code using (i) forward in time and central in space (FT-CS) (ii) backward in time and central in space (BT-CS) and (iii) Crank-Nicolson scheme for the following parabolic partial differential equations.

Also discuss the von Neumann stability for FT-CS, BT-CS and Crank-Nicolson schemes.

1. Consider the following parabolic partial differential equation

$$u_t = u_{xx} + e^{-t}\cos(\pi x)\left(\pi^2 - 1\right), \quad 0 < x < 1,\ t > 0$$

with initial condition

$$u(x,0) = u_0(x) = \cos(\pi x),\ 0 \le x \le 1,$$

and the boundary conditions

$$\frac{\partial u}{\partial x}(0,t) = 0, \quad \frac{\partial u}{\partial x}(1,t) = 0,\ t > 0.$$

The exact solution is given by

$$u(x,t) = e^{-t}\cos(\pi x).$$

Find the approximate solution and compare with the exact solution.

2. Consider the following parabolic partial differential equation

$$u_t = u_{xx} + e^{-t}(x^2 - x + 2), \quad 0 < x < 1,\ t > 0$$

with initial condition

$$u(x,0) = u_0(x) = x(1-x),\ 0 \le x \le 1,$$

and the boundary conditions

$$u(0,t) + \frac{\partial u}{\partial x}(0,t) = e^{-t}, \quad \frac{\partial u}{\partial x}(1,t) = -e^{-t},\ t > 0.$$

The exact solution is given by

$$u(x,t) = e^{-t}\,x(1-x).$$

Find the approximate solution and compare with the exact solution.