Birla Institute of Technology and Science, Pilani-K K Birla Goa Campus

Numerical Methods for Partial Differential Equations (MATH F422)

Parabolic Partial Differential Equations

Lab Sheet-1

Write a MATLAB code using (i) method of lines (ii) forward in time and central in space (FT-CS) (iii) backward in time and central in space (BT-CS) and (vi) Crank-Nicolson scheme for the following parabolic partial differential equations.

Also discuss the von Neumann stability for FT-CS, BT-CS and Crank-Nicolson schemes.

1. Consider the following partial differential equation

$$u_t - u_{xx} = 0, \;\; 0 < x < 2, \; t > 0$$

with initial condition

$$u(x, 0) = g(x) = \sin(\pi/2)x, \; 0 \leq x \leq 2,$$

and the boundary conditions

$$u(0, t) = 0, \; u(2, t) = 0, \; t > 0.$$

The exact solution is given by

$$u(x, t) = e^{-\frac{\pi^2}{4}t} \sin\left(\pi/2\right)x.$$

Find the approximate solution and compare with the exact solution.

2. Consider the following partial differential equation

$$u_t - \frac{1}{16}u_{xx} = 0, \;\; 0 < x < 1, \; t > 0$$

with initial condition

$$u(x, 0) = g(x) = 2\sin(2\pi)x, \; 0 \leq x \leq 1,$$

and the boundary conditions

$$u(0, t) = 0, \; u(1, t) = 0, \; t > 0.$$

The exact solution is given by

$$u(x, t) = 2e^{-\frac{\pi^2}{4}t} \sin\left(2\pi\right)x.$$

Find the approximate solution and compare with the exact solution.

3. Consider the following partial differential equation

$$u_t - \frac{4}{\pi^2} u_{xx} = 0, \quad 0 < x < 4,\ t > 0$$

with initial condition

$$u(x,0) = g(x) = \sin(\pi/4)x\,(1 + 2\cos(\pi/4)x)\,, \quad 0 \le x \le 4,$$

and the boundary conditions

$$u(0,t) = 0,\ u(4,t) = 0,\ t > 0.$$

The exact solution is given by

$$u(x,t) = e^{-t}\sin(\pi/2)x + e^{-\frac{t}{4}}\sin(\pi/4)x.$$

Find the approximate solution and compare with the exact solution.

4. Consider the following partial differential equation

$$u_t - \frac{1}{\pi^2} u_{xx} = 0, \quad 0 < x < 1,\ t > 0$$

with initial condition

$$u(x,0) = g(x) = \cos\pi\left(x - \frac{1}{2}\right), \quad 0 \le x \le 1,$$

and the boundary conditions

$$u(0,t) = 0,\ u(1,t) = 0,\ t > 0.$$

The exact solution is given by

$$u(x,t) = e^{-t}\cos\pi\left(x - \frac{1}{2}\right).$$

Find the approximate solution and compare with the exact solution.