

交错网格与完全匹配层

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有限差分法是对介质模型，也就是对计算区域先进行离散网格化，将描述介质中传播的波动微分方程，利用微商和差商的近似关系，直接化为有限差分方程来求解，模拟波的传播。

地震勘探中的有限差分根据域的不同可分为时域有限差分 and 频域有限差分。根据网格不同可分为同位网格^[1]、交错网格^{[2][3]}、旋转网格^{[4][5]}等。对于边界反射波的处理，有早期的旁轴近似吸收边界条件^[6]、指数型吸收边界条件^[7]和现在比较流行的完全匹配层吸收边界^[8]。本文主要介绍了时域交错网格有限差分方法与完全匹配层吸收边界条件，并对简单的二维声波方程和弹性波方程给出了差分格式。

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一、什么是交错网格

交错网格是将不同的地震波场分量定义在整网格点和半网格点上，合理地安排地震波场分量在网格上的相对位置，可以方便地求取所需分量的差分。同时，它将波场分裂为 x 和 z 方向上的两个分量，将二阶位移微分方程分裂为若干个一阶速度—应力方程对波场进行求解。在交错网格中，假设 u^x 和 u^z 分别定义在 x 和 z 方向的半网格点上，则它们对 x 和 z 方向的中心差分格式为^[9]

$$\begin{cases} L_x(u_{i,j}^x) = \frac{1}{\Delta x} \sum_{n=1}^N C_n^{(N)} (u_{i+\frac{2n-1}{2},j}^x - u_{i-\frac{2n-1}{2},j}^x) \\ L_z(u_{i,j}^z) = \frac{1}{\Delta z} \sum_{n=1}^N C_n^{(N)} (u_{i,j+\frac{2n-1}{2}}^z - u_{i,j-\frac{2n-1}{2}}^z) \end{cases} \quad (1)$$

其中， u 为地震波场值， u^x 和 u^z 为它的两个方向分量， Δx 和 Δz 为 x 和 z 方向的空间间隔， $C_n^{(N)}$ 为差分系数， $2N$ 为差分的空间阶数。

二、什么是完全匹配层

吸收边界条件的思想就是在需要计算场值的区域之外加上一定厚度的吸收边界层，当波运行到计算边界时候，不会发生反射，而是直接穿透边界进入所加的吸收边界层，对吸收边界层设置一定的参数，从而起到吸收超出边界的波的作用。

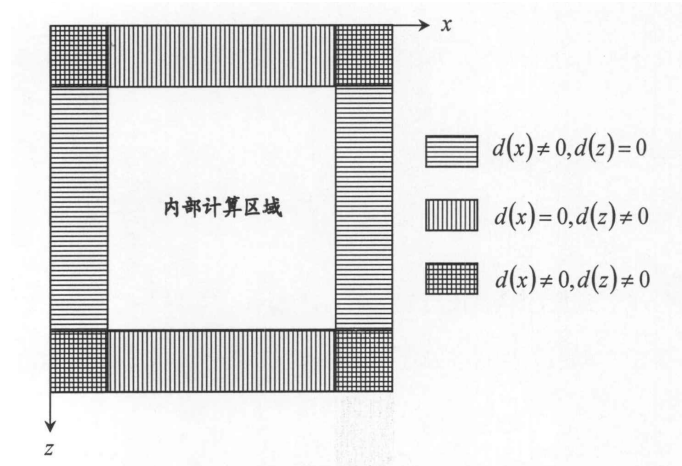


图 1: 完全匹配层吸收边示意图 (图中: $d(x) = d_x$, $d(z) = d_z$)

在时域有限差分方法波场模拟中，完全匹配层(PML)吸收边界条件将波场分量在吸收边界区域分裂，分别对各个分裂的波场分量赋以不同的耗损。在计算区域截断边界外，PML 层是一种非物理的特殊吸收介质，该层的波阻抗与相邻介质的波阻抗完全匹配，因而入射波将无反射地穿过界面进行 PML 层，同时，由于 PML 层为有耗介质，进入 PML 层的入射波将迅速衰减，最终实现消弱边界反射的效果。

PML 吸收边界具体做法如图 1 所示, 在内部计算区域, 采用一般的速度—应力方程, 而在 PML 层区域内, 在频率空间域对方程中的 x 和 z 方向偏导分别作如下替换:

$$\frac{\partial}{\partial x} \rightarrow \frac{i\omega}{i\omega + d_x} \frac{\partial}{\partial x}, \quad \frac{\partial}{\partial z} \rightarrow \frac{i\omega}{i\omega + d_z} \frac{\partial}{\partial z}$$

其中, ω 为角频率, d_x 和 d_z 分别为 x 和 z 方向的阻尼因子。

例如, 对于如下方程:

$$\frac{\partial u}{\partial t} = A \frac{\partial v}{\partial x}$$

在内部计算区域, 我们采用上式求解即可。而在 PML 层内, 我们应对方程作一些调整。上式对应的频率空间域方程为:

$$i\omega u = A \frac{\partial v}{\partial x}$$

在 PML 层内, 对 x 方向偏导进行替换, 得到如下方程:

$$i\omega u = A \frac{i\omega}{i\omega + d_x} \frac{\partial v}{\partial x}, \text{ 也即 } (i\omega + d_x)u = A \frac{\partial v}{\partial x}$$

其在时间空间域的表达形式为:

$$\left(\frac{\partial}{\partial t} + d_x\right)u = A \frac{\partial v}{\partial x} \quad (2)$$

因此, 我们在 PML 层内可采用上式求解, 即可实现 PML 吸收边界层内衰减。

在对上式左侧采用差分近似的实际过程中, 我们有两种近似方案。先假设上式右侧经空间差分近似后的结果为

$$A \frac{\partial v_k}{\partial x} \approx \spadesuit|_k$$

其中 ∂v_k 为 $k\Delta t$ 时刻 v 的偏导, Δt 为时间步长。同时假设 u 定义在半时间网格点上, 则第一种近似方案为:

$$\left(\frac{\partial}{\partial t} + d_x\right)u_k = \frac{\partial u_k}{\partial t} + d_x \cdot u_k = \frac{u_{k+1/2} - u_{k-1/2}}{\Delta t} + d_x \cdot \frac{u_{k+1/2} + u_{k-1/2}}{2}$$

将上式代入式 (2), 最终, 我们得到第一种近似下的时间递推关系式为:

$$u_{k+1/2} = \frac{2 - \Delta t \cdot d_x}{2 + \Delta t \cdot d_x} \cdot u_{k-1/2} + \frac{2\Delta t}{2 + \Delta t \cdot d_x} \cdot \spadesuit|_k \quad (3)$$

第二种近似方案为:

$$\left(\frac{\partial}{\partial t} + d_x\right)u_k = \frac{\partial u_k}{\partial t} + d_x \cdot u_k = \frac{u_{k+1/2} - u_{k-1/2}}{\Delta t} + d_x \cdot u_{k-1/2}$$

将其代入式 (2), 最终, 我们得到第二种近似下的时间递推关系式为:

$$u_{k+1/2} = (1 - \Delta t \cdot d_x)u_{k-1/2} + \Delta t \cdot \spadesuit|_k \quad (4)$$

其实, 我们可以将内部计算区域和 PML 层区域的方程统一起来, 当 $d_x = d_z = 0$ 时 PML 层区域的方程转化为内部计算区域的方程, 编程时我们可以考虑统一采用 PML 层区域的方程形式求解, 只需特别地在内部计算区域令 $d_x = d_z = 0$ 即可。

那么, 衰减因子 d_x 和 d_z 如何给定? 对于上边界或左边界, 文献^[10] 给出了形如下式的衰减因子:

$$d_*(i) = d_{0*} \left(\frac{i}{n_{pml*}} \right)^p$$

其中, $*$ 表示 x 或 z , i 为从内部有效计算区域边界起算的 PML 层数, n_{pml*} 为在 $*$ 方向上所加载的单边 PML 层网格点数, 典型地 p 的取值范围为 $1 \sim 4$ 。另外,

$$d_{0*} = \log \left(\frac{1}{R} \right) \frac{\tau V_s}{n_{pml*} \Delta*}$$

或

$$d_{0*} = \frac{\tau V_s}{\Delta*} (c_1 + c_2 n_{pml*} + c_3 n_{pml*}^2)$$

其中, R 为理论反射系数; τ 为微调参数, 取值范围为 $3 \sim 4$; V_s 为横波波速; $\Delta*$ 为在 $*$ 方向上的网格间距; c_i 为多项式系数。对于 R 或 c_i 的取值如下:

$$\begin{cases} R = 0.01, & \text{当 } n_{pml*} = 5 \\ R = 0.001, & \text{当 } n_{pml*} = 10 \\ R = 0.0001, & \text{当 } n_{pml*} = 20 \end{cases} \quad \text{或} \quad \begin{cases} c_1 = \frac{8}{15} \\ c_2 = \frac{-3}{100} \\ c_3 = \frac{1}{1500} \end{cases}$$

三、空间上任意偶数阶差分近似

在交错网格方法中, 波场分量的导数是在相应的分量网格节点之间的半程上计算的。因此, 我们可以用下式计算方程中的一阶空间导数:

$$\frac{\partial u}{\partial x} = \frac{1}{\Delta x} \sum_{n=1}^N \left\{ C_n^{(N)} \left[u \left(x + \frac{2n-1}{2} \Delta x \right) - u \left(x - \frac{2n-1}{2} \Delta x \right) \right] \right\} + O(\Delta x^{2N}) \quad (5)$$

上式中待定系数 $C_n^{(N)}$ 的准确求取是确保一阶空间导数的 $2N$ 阶差分精度的关键。将 $u \left(x + \frac{2n-1}{2} \Delta x \right)$ 和 $u \left(x - \frac{2n-1}{2} \Delta x \right)$ 在 x 处 Taylor 展开后可以发现, 通过求解下列方程组即可确定待定系数 $C_n^{(N)}$:

$$\begin{bmatrix} 1^1 & 3^1 & 5^1 & \cdots & (2N-1)^1 \\ 1^3 & 3^3 & 5^3 & \cdots & (2N-1)^3 \\ 1^5 & 3^5 & 5^5 & \cdots & (2N-1)^5 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1^{2N-1} & 3^{2N-1} & 5^{2N-1} & \cdots & (2N-1)^{2N-1} \end{bmatrix} \begin{bmatrix} C_1^{(N)} \\ C_2^{(N)} \\ C_3^{(N)} \\ \vdots \\ C_N^{(N)} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

其解为:

$$C_m^{(N)} = \frac{(-1)^{m+1} \prod_{i=1, i \neq m}^N (2i-1)^2}{(2m-1) \prod_{i=1, i \neq m}^N |(2m-1)^2 - (2i-1)^2|} \quad (6)$$

四、交错网格中的声波方程

如我们所常见的，在各向同性介质中，二维声波波动方程可表示为：

$$\frac{\partial^2 P}{\partial t^2} = v_P^2 \left(\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial z^2} \right) \quad (7)$$

其中， P 为压力波场或位移波场， v_P 为介质声波波速。

在交错网格中，我们将不同的波场分量定义在不同的网格点上，这就需要我们采用多波场分量的方程来进行波场模拟。在各向同性介质中，二维声波一阶速度—应力方程可表示为：

$$\begin{cases} \frac{\partial P}{\partial t} = -\rho v_P^2 \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} \right) \\ \frac{\partial v_x}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial x} \\ \frac{\partial v_z}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial z} \end{cases} \quad (8)$$

其中， v_x 和 v_z 分别为在 x 和 z 方向的质点运动速度波场分量， ρ 为介质密度。

在方程 (8) 中的第一个等式两边同时对 t 求偏导，交换等式右侧对时间求导和对空间求导的先后顺序，再结合方程 (8) 中的后两个等式，即可得到如式 (7) 所示的波动方程。

在 PML 吸收边界中，我们在 x 和 z 方向上采取不同的阻尼衰减因子，由于方程 (8) 中的第一个等式同时包含了对 x 和 z 方向的偏导，因此，还需要对该式作进一步拆分：

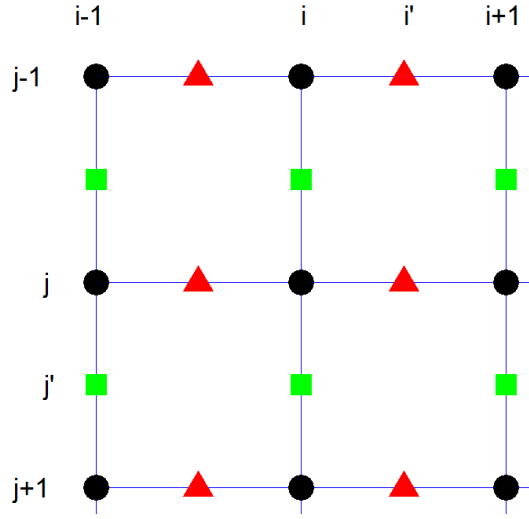
$$\begin{cases} P = P_x + P_z \\ \frac{\partial P_x}{\partial t} = -\rho v_P^2 \frac{\partial v_x}{\partial x} \\ \frac{\partial P_z}{\partial t} = -\rho v_P^2 \frac{\partial v_z}{\partial z} \end{cases} \quad (9)$$

其中， P_x 和 P_z 分别为应力波场 P 在 x 和 z 方向上的分量。

根据式 (8) 和 (9)，引入 PML 吸收边界条件，得到：

$$\begin{cases} \left(\frac{\partial}{\partial t} + d_x \right) v_x = -\frac{1}{\rho} \frac{\partial P}{\partial x} \\ \left(\frac{\partial}{\partial t} + d_z \right) v_z = -\frac{1}{\rho} \frac{\partial P}{\partial z} \\ \left(\frac{\partial}{\partial t} + d_x \right) P_x = -\rho v_P^2 \frac{\partial v_x}{\partial x} \\ \left(\frac{\partial}{\partial t} + d_z \right) P_z = -\rho v_P^2 \frac{\partial v_z}{\partial z} \end{cases} \quad (10)$$

按照如图 2 所示波场分量和参数排布方式，我们在时间上采用如式 (4) 所示的递推格式，在空间上采用如式 (5) 所示的任意偶数阶差分近似，可以得到在 PML 层内采用第二种近似下的时间二阶差分精度、空间 $2N$ 阶差分精度的交错网格有限差分声波方程时间递推格

图 2: 声波交错网格示意图 (•: $P, \rho v_p^2$; ▲: $v_x, 1/\rho$; ■: $v_z, 1/\rho$)

式如下:

$$\left\{ \begin{array}{l} v_x|_{i+1/2,j}^k = (1 - \Delta t \cdot d_x) v_x|_{i+1/2,j}^{k-1} - \frac{\Delta t}{\rho \Delta x} \sum_{n=1}^N \left\{ C_n^{(N)} [P|_{i+1/2+(2n-1)/2,j}^{k-1/2} - P|_{i+1/2-(2n-1)/2,j}^{k-1/2}] \right\} \\ v_z|_{i,j+1/2}^k = (1 - \Delta t \cdot d_z) v_z|_{i,j+1/2}^{k-1} - \frac{\Delta t}{\rho \Delta z} \sum_{n=1}^N \left\{ C_n^{(N)} [P|_{i,j+1/2+(2n-1)/2}^{k-1/2} - P|_{i,j+1/2-(2n-1)/2}^{k-1/2}] \right\} \\ P_x|_{i,j}^{k+1/2} = (1 - \Delta t \cdot d_x) P_x|_{i,j}^{k-1/2} - \frac{\rho v_p^2 \Delta t}{\Delta x} \sum_{n=1}^N \left\{ C_n^{(N)} [v_x|_{i+(2n-1)/2,j}^k - v_x|_{i-(2n-1)/2,j}^k] \right\} \\ P_z|_{i,j}^{k+1/2} = (1 - \Delta t \cdot d_z) P_z|_{i,j}^{k-1/2} - \frac{\rho v_p^2 \Delta t}{\Delta z} \sum_{n=1}^N \left\{ C_n^{(N)} [v_z|_{i,j+(2n-1)/2}^k - v_z|_{i,j-(2n-1)/2}^k] \right\} \end{array} \right. \quad (11)$$

其中, $P = P_x + P_z$, $v_x|_{i+1/2,j}^k$ 为空间网格点 $((i+1/2)\Delta x, j\Delta z)$ 处在 $k\Delta t$ 时刻 v_x 的值, Δx 和 Δz 分别为 x 和 z 方向上空间差分步长。

五、交错网格中的弹性波方程

对于弹性波方程, 如我们所常见的, 在各向同性介质中, 二维波动方程可表示为:

$$\left\{ \begin{array}{l} \rho \frac{\partial^2 u_x}{\partial t^2} = \frac{\partial}{\partial x} \left[\lambda \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \right) + 2\mu \frac{\partial u_x}{\partial x} \right] + \frac{\partial}{\partial z} \left[\mu \left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} \right) \right] \\ \rho \frac{\partial^2 u_z}{\partial t^2} = \frac{\partial}{\partial z} \left[\lambda \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \right) + 2\mu \frac{\partial u_z}{\partial z} \right] + \frac{\partial}{\partial x} \left[\mu \left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} \right) \right] \end{array} \right. \quad (12)$$

其中, u_x 和 u_z 分别为 x 和 z 方向上的位移, ρ 为介质密度, $\lambda = \rho(v_p^2 - 2v_s^2)$ 和 $\mu = \rho v_s^2$ 为介质拉梅常数, v_p 和 v_s 分别为介质的纵波速度和横波速度。

另外，我们有弹性动力学方程如下^[3]：

$$\begin{cases} \rho \frac{\partial^2 u_x}{\partial t^2} = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} \\ \rho \frac{\partial^2 u_z}{\partial t^2} = \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z} \\ \tau_{xx} = (\lambda + 2\mu) \frac{\partial u_x}{\partial x} + \lambda \frac{\partial u_z}{\partial z} \\ \tau_{zz} = (\lambda + 2\mu) \frac{\partial u_z}{\partial z} + \lambda \frac{\partial u_x}{\partial x} \\ \tau_{xz} = \mu \left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right) \end{cases} \quad (13)$$

其中， $(\tau_{xx}, \tau_{zz}, \tau_{xz})$ 为应力张量。不难发现，我们将方程 (13) 的后三个等式代入前两个等式中，即可得到如式 (12) 所示的波动方程。

然而，仅有上式，由于含有对时间的二阶偏导项，我们并不能将 PML 吸收边界条件直接引进来。我们将质点运动速度波场分量 $v_x = \partial u_x / \partial t$ 和 $v_z = \partial u_z / \partial t$ 引入上式，得到如下二维弹性波一阶速度—应力方程：

$$\begin{cases} \frac{\partial v_x}{\partial t} = \frac{1}{\rho} \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} \right) \\ \frac{\partial v_z}{\partial t} = \frac{1}{\rho} \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z} \right) \\ \frac{\partial \tau_{xx}}{\partial t} = (\lambda + 2\mu) \frac{\partial v_x}{\partial x} + \lambda \frac{\partial v_z}{\partial z} \\ \frac{\partial \tau_{zz}}{\partial t} = (\lambda + 2\mu) \frac{\partial v_z}{\partial z} + \lambda \frac{\partial v_x}{\partial x} \\ \frac{\partial \tau_{xz}}{\partial t} = \mu \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right) \end{cases} \quad (14)$$

但是，由于上式的每一个等式中都同时含有对 x 和 z 的偏导，依然不能直接引入 PML 边界条件。接下来，我们需要对上式中的每一个等式作如式 (9) 所示的拆分，进一步得到：

$$\begin{cases} v_x = v_x^x + v_x^z, & \frac{\partial v_x^x}{\partial t} = \frac{1}{\rho} \frac{\partial \tau_{xx}}{\partial x}, & \frac{\partial v_x^z}{\partial t} = \frac{1}{\rho} \frac{\partial \tau_{xz}}{\partial z} \\ v_z = v_z^x + v_z^z, & \frac{\partial v_z^x}{\partial t} = \frac{1}{\rho} \frac{\partial \tau_{xz}}{\partial x}, & \frac{\partial v_z^z}{\partial t} = \frac{1}{\rho} \frac{\partial \tau_{zz}}{\partial z} \\ \tau_{xx} = \tau_{xx}^x + \tau_{xx}^z, & \frac{\partial \tau_{xx}^x}{\partial t} = (\lambda + 2\mu) \frac{\partial v_x^x}{\partial x}, & \frac{\partial \tau_{xx}^z}{\partial t} = \lambda \frac{\partial v_z^z}{\partial z} \\ \tau_{zz} = \tau_{zz}^x + \tau_{zz}^z, & \frac{\partial \tau_{zz}^x}{\partial t} = \lambda \frac{\partial v_x^x}{\partial x}, & \frac{\partial \tau_{zz}^z}{\partial t} = (\lambda + 2\mu) \frac{\partial v_z^z}{\partial z} \\ \tau_{xz} = \tau_{xz}^x + \tau_{xz}^z, & \frac{\partial \tau_{xz}^x}{\partial t} = \mu \frac{\partial v_z^z}{\partial x}, & \frac{\partial \tau_{xz}^z}{\partial t} = \mu \frac{\partial v_x^x}{\partial z} \end{cases} \quad (15)$$

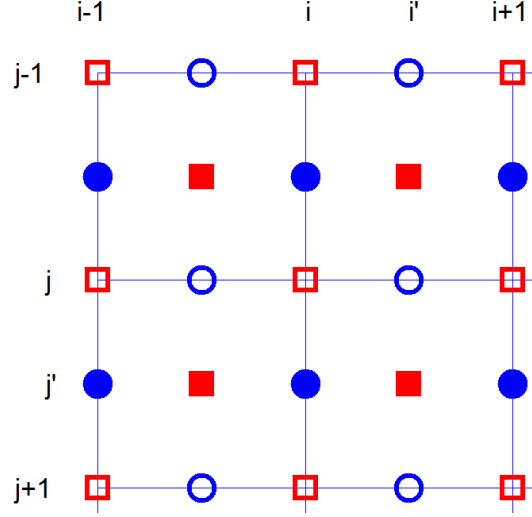


图 3: 弹性波交错网格示意图 (\square : $v_x, 1/\rho$; \blacksquare : $v_z, 1/\rho$; \circ : $\tau_{xx}, \tau_{zz}, (\lambda + 2\mu), \lambda$; \bullet : τ_{xz}, μ)

至此，我们可以在上式的时间微分中引入 PML 层吸收边界，得到：

$$\left\{ \begin{array}{ll} \left(\frac{\partial}{\partial t} + d_x \right) v_x^x = \frac{1}{\rho} \frac{\partial \tau_{xx}}{\partial x}, & \left(\frac{\partial}{\partial t} + d_z \right) v_x^z = \frac{1}{\rho} \frac{\partial \tau_{xz}}{\partial z} \\ \left(\frac{\partial}{\partial t} + d_x \right) v_z^x = \frac{1}{\rho} \frac{\partial \tau_{xz}}{\partial x}, & \left(\frac{\partial}{\partial t} + d_z \right) v_z^z = \frac{1}{\rho} \frac{\partial \tau_{zz}}{\partial z} \\ \left(\frac{\partial}{\partial t} + d_x \right) \tau_{xx}^x = (\lambda + 2\mu) \frac{\partial v_x^x}{\partial x}, & \left(\frac{\partial}{\partial t} + d_z \right) \tau_{xx}^z = \lambda \frac{\partial v_z^z}{\partial z} \\ \left(\frac{\partial}{\partial t} + d_x \right) \tau_{zz}^x = \lambda \frac{\partial v_x^x}{\partial x}, & \left(\frac{\partial}{\partial t} + d_z \right) \tau_{zz}^z = (\lambda + 2\mu) \frac{\partial v_z^z}{\partial z} \\ \left(\frac{\partial}{\partial t} + d_x \right) \tau_{xz}^x = \mu \frac{\partial v_z^z}{\partial x}, & \left(\frac{\partial}{\partial t} + d_z \right) \tau_{xz}^z = \mu \frac{\partial v_x^x}{\partial z} \end{array} \right. \quad (16)$$

其中，

$$\left\{ \begin{array}{l} v_x = v_x^x + v_x^z \\ v_z = v_z^x + v_z^z \\ \tau_{xx} = \tau_{xx}^x + \tau_{xx}^z \\ \tau_{zz} = \tau_{zz}^x + \tau_{zz}^z \\ \tau_{xz} = \tau_{xz}^x + \tau_{xz}^z \end{array} \right. \quad (17)$$

按照如图 3 所示波场分量和参数排布方式，我们在时间上采用如式 (4) 所示的递推格式，在空间上采用如式 (5) 所示的任意偶数阶差分近似，可以得到在 PML 层内采用第二种近似下的时间二阶差分精度、空间 $2N$ 阶差分精度的交错网格有限差分弹性波方程时间递推

格式如下：

$$\left\{ \begin{aligned}
 v_x^x|_{i,j}^{k+1/2} &= (1 - \Delta t \cdot d_x) v_x^x|_{i,j}^{k-1/2} + \frac{\Delta t}{\rho \Delta x} \sum_{n=1}^N \left\{ C_n^{(N)} [\tau_{xx}|_{i+(2n-1)/2,j}^k - \tau_{xx}|_{i-(2n-1)/2,j}^k] \right\} \\
 v_x^z|_{i,j}^{k+1/2} &= (1 - \Delta t \cdot d_z) v_x^z|_{i,j}^{k-1/2} + \frac{\Delta t}{\rho \Delta z} \sum_{n=1}^N \left\{ C_n^{(N)} [\tau_{xz}|_{i,j+(2n-1)/2}^k - \tau_{xz}|_{i,j-(2n-1)/2}^k] \right\} \\
 v_z^x|_{i+1/2,j+1/2}^{k+1/2} &= (1 - \Delta t \cdot d_x) v_z^x|_{i+1/2,j+1/2}^{k-1/2} + \frac{\Delta t}{\rho \Delta x} \sum_{n=1}^N \left\{ C_n^{(N)} [\tau_{xz}|_{i+1/2+(2n-1)/2,j+1/2}^k - \right. \\
 &\quad \left. \tau_{xz}|_{i+1/2-(2n-1)/2,j+1/2}^k] \right\} \\
 v_z^z|_{i+1/2,j+1/2}^{k+1/2} &= (1 - \Delta t \cdot d_z) v_z^z|_{i+1/2,j+1/2}^{k-1/2} + \frac{\Delta t}{\rho \Delta z} \sum_{n=1}^N \left\{ C_n^{(N)} [\tau_{zz}|_{i+1/2,j+1/2+(2n-1)/2}^k - \right. \\
 &\quad \left. \tau_{zz}|_{i+1/2,j+1/2-(2n-1)/2}^k] \right\} \\
 \tau_{xx}^x|_{i+1/2,j}^{k+1} &= (1 - \Delta t \cdot d_x) \tau_{xx}^x|_{i+1/2,j}^k + \frac{(\lambda + 2\mu)\Delta t}{\Delta x} \sum_{n=1}^N \left\{ C_n^{(N)} [v_x|_{i+1/2+(2n-1)/2,j}^{k+1/2} - \right. \\
 &\quad \left. v_x|_{i+1/2-(2n-1)/2,j}^{k+1/2}] \right\} \\
 \tau_{xx}^z|_{i+1/2,j}^{k+1} &= (1 - \Delta t \cdot d_z) \tau_{xx}^z|_{i+1/2,j}^k + \frac{\lambda \Delta t}{\Delta z} \sum_{n=1}^N \left\{ C_n^{(N)} [v_z|_{i+1/2,j+(2n-1)/2}^{k+1/2} - \right. \\
 &\quad \left. v_z|_{i+1/2,j-(2n-1)/2}^{k+1/2}] \right\} \\
 \tau_{zz}^x|_{i+1/2,j}^{k+1} &= (1 - \Delta t \cdot d_x) \tau_{zz}^x|_{i+1/2,j}^k + \frac{\lambda \Delta t}{\Delta x} \sum_{n=1}^N \left\{ C_n^{(N)} [v_x|_{i+1/2+(2n-1)/2,j}^{k+1/2} - \right. \\
 &\quad \left. v_x|_{i+1/2-(2n-1)/2,j}^{k+1/2}] \right\} \\
 \tau_{zz}^z|_{i+1/2,j}^{k+1} &= (1 - \Delta t \cdot d_z) \tau_{zz}^z|_{i+1/2,j}^k + \frac{(\lambda + 2\mu)\Delta t}{\Delta z} \sum_{n=1}^N \left\{ C_n^{(N)} [v_z|_{i+1/2,j+(2n-1)/2}^{k+1/2} - \right. \\
 &\quad \left. v_z|_{i+1/2,j-(2n-1)/2}^{k+1/2}] \right\} \\
 \tau_{xz}^x|_{i,j+1/2}^{k+1} &= (1 - \Delta t \cdot d_x) \tau_{xz}^x|_{i,j+1/2}^k + \frac{\mu \Delta t}{\Delta x} \sum_{n=1}^N \left\{ C_n^{(N)} [v_z|_{i+(2n-1)/2,j+1/2}^{k+1/2} - \right. \\
 &\quad \left. v_z|_{i-(2n-1)/2,j+1/2}^{k+1/2}] \right\} \\
 \tau_{xz}^z|_{i,j+1/2}^{k+1} &= (1 - \Delta t \cdot d_z) \tau_{xz}^z|_{i,j+1/2}^k + \frac{\mu \Delta t}{\Delta z} \sum_{n=1}^N \left\{ C_n^{(N)} [v_x|_{i,j+1/2+(2n-1)/2}^{k+1/2} - \right. \\
 &\quad \left. v_x|_{i,j+1/2-(2n-1)/2}^{k+1/2}] \right\}
 \end{aligned} \right. \quad (18)$$

其中，各波场分量之间还包含如式 (17) 所示关系， $v_x^x|_{i,j}^{k+1/2}$ 为空间网格点 $(i\Delta x, j\Delta z)$ 处在 $(k + 1/2)\Delta t$ 时刻 v_x^x 的值。

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附录 声波：TDFDAWFS2DSG

Matlab 程序

```

function TDFDAWFS2DSG

% TDFDAWFS2DSG
% This is a program of Time Domain Finite Difference Acoustic Wave Field Simulating with 2-Dimension
% Staggered Grid.
5 % Written by Tche.L. from USTC, 2016,6.

clc; clear; close all;
% format long;

10 %% Input parameters

nx = 101;           % the number of grid nodes in x-direction.
nz = 101;           % the number of grid nodes in z-direction.
npmlz = 20;         % the number of grid nodes in top and bottom side of PML absorbing boundary.
15 npmlx = 20;       % the number of grid nodes in left and right side of PML absorbing boudary.
sx = 50;            % the grid node number of source position in x-direction.
sz = 50;            % the grid node number of source position in z-direction.
dx = 5;             % the grid node interval in x-direction; Unit: m.
dz = 5;             % the grid node interval in z-direction; Unit: m.
20 nt = 500;         % the number of time nodes for wave calculating.
dt = 1e-3;          % the time node interval; Unit: s.
nppw = 12;          % the node point number per wavelength for dominant frequency of Ricker
                    % wavelet source.
ampl = 1.0e0;        % the amplitude of source wavelet.
xrcvr = 1:3:nx;      % the grid node number in x-direction of reciver position on ground.
25 nodr = 3;          % half of the order number for spatial difference.

%% Determine the difference coefficients

B = [1 zeros(1,nodr - 1)]';
30 A = NaN*ones(nodr,nodr);
for i = 1:1:nodr
    A(i,:) = (1:2:2*nodr - 1).^(2*i - 1);
end
C = A\B;
35

%% Model and source

Nz = nz + 2*npmlz;
Nx = nx + 2*npmlx;
40
vp = 2000*ones(Nz,Nx);           % the velocity of
    acoustic wave of model; Unit: m/s.
rho = 1000*ones(Nz,Nx);          % the density of model;
    Unit: kg/m^3.
rho(fix(Nz/3):end,fix(Nx/2):end) = 500;
vp(fix(Nz/3):end,fix(Nx/2):end) = 1000;
45
f0 = min(vp(:))/(min(dx,dz)*nppw); % the dominant frequency
    of source Ricker wavelet; Unit: Hz.
t0 = 1/f0;                       % the time shift of
    source Ricker wavelet; Unit: s; Suggest: 0.02 if fm = 50, or 0.05 if fm = 20.
t = dt*(1:1:nt);
src = (1 - 2*(pi*f0.*(t - t0)).^2).*exp(-(pi*f0*(t - t0)).^2); % the time series of

```

```

    source wavelet.
50 % The source wavelet formula refers to the equations (18) of Collino and Tsogka, 2001.

    %% Perfectly matched layer absorbing factor

    % R = 1e-6; % Recommend: $R = 1e-2,
    if $nabsr = 5; $R = 1e-3, if $nabsr = 10; $R = 1e-4, if $absr = 20.
55 % dpml0z = log(1/R)*3*max(vp(:))/(2*npmlz);
    dpml0z = 3*max(vp(:))/dz*(8/15 - 3/100*npmlz + 1/1500*npmlz^2);
    dpmlz = zeros(Nz,Nx);
    dpmlz(1:npmlz,:) = (dpml0z*((npmlz: - 1:1)./npmlz).^2)*ones(1,Nx);
    dpmlz(npmlz + nz + 1:Nz,:) = dpmlz(npmlz: - 1:1,:);
60 dpml0x = 3*max(vp(:))/dx*(8/15 - 3/100*npmlx + 1/1500*npmlx^2);
    dpmlx = zeros(Nz,Nx);
    dpmlx(:,1:npmlx) = ones(Nz,1)*(dpml0x*((npmlx: - 1:1)./npmlx).^2);
    dpmlx(:,npmlx + nx + 1:Nx) = dpmlx(:,npmlx: - 1:1);
    % The PML formula refers to the equations (2) and (3) of Marcinkovich and Olsen, 2003.
65
    %% Wavefield calculating

    rho1 = rho; % or = [(rho(:,1:end - 1) + rho(:,2:end))./2 (2*rho(:,end) - rho(:,end - 1))
    ];
    rho2 = rho; % or = [(rho(1:end - 1,:) + rho(2:end,:))./2; (2*rho(end,:) - rho(end - 1,:))
    ];
70
    Coeffi1 = (2 - dt.*dpmlx)./(2 + dt.*dpmlx);
    Coeffi2 = (2 - dt.*dpmlz)./(2 + dt.*dpmlz);
    Coeffi3 = 1./rho1./dx.*(2*dt./(2 + dt.*dpmlx));
    Coeffi4 = 1./rho2./dz.*(2*dt./(2 + dt.*dpmlz));
75 Coeffi5 = rho.*(vp.^2)./dx.*(2*dt./(2 + dt.*dpmlx));
    Coeffi6 = rho.*(vp.^2)./dz.*(2*dt./(2 + dt.*dpmlz));

    % ++++++ approximate coefficient ++++++
    % Coeffi1 = 1 - dt.*dpmlx;
80 % Coeffi2 = 1 - dt.*dpmlz;
    % Coeffi3 = 1./rho./dx.*dt;
    % Coeffi4 = 1./rho./dz.*dt;
    % Coeffi5 = rho.*(vp.^2)./dx.*dt;
    % Coeffi6 = rho.*(vp.^2)./dz.*dt;
85 % -----

    NZ = Nz + 2*nodr; % All values of the
    outermost some columns are set to zero to be a boundary condition: all of wavefield values
    beyond the left and right boundary are null.
    NX = Nx + 2*nodr; % All values of the
    outermost some rows are set to zero to be a boundary condition: all of wavefield values beyond
    the top and bottom boundary are null.

90 Znodes = nodr + 1:NZ - nodr;
    Xnodes = nodr + 1:NX - nodr;
    znodes = nodr + npmlz + 1:nodr + npmlz + nz;
    xnodes = nodr + npmlx + 1:nodr + npmlx + nx;
    nsrcz = nodr + npmlz + sz;
95 nsrcx = nodr + npmlx + sx;

    Ut = NaN*ones(NZ,NX,2); % the wavefield value
    preallocation.
    Uz = zeros(NZ,NX,2); % The initial condition:
    all of wavefield values are null before source excitation.
    Ux = zeros(NZ,NX,2); % The initial condition:
    all of wavefield values are null before source excitation.

```

```

100 Vz = zeros(NZ,NX,2); % The initial condition:
    all of wavefield values are null before source excitation.
Vx = zeros(NZ,NX,2); % The initial condition:
    all of wavefield values are null before source excitation.
Psum = NaN*ones(Nz,Nx);
U = NaN*ones(nz,nx,nt);
105
tic;
for it = 1:1:nt
    fprintf('The calculating time node is: it = %d\n',it);
    Ux(nsrcz,nsrcx,1) = Ux(nsrcz,nsrcx,1) + ampl*src(it)./2;
110    Uz(nsrcz,nsrcx,1) = Uz(nsrcz,nsrcx,1) + ampl*src(it)./2;
    Ut(:, :, 1) = Ux(:, :, 1) + Uz(:, :, 1);
    U(:, :, it) = Ut(znodes, xnodes, 1);
    Psum(:, :) = 0;
    for i = 1:1:nodr
115        Psum = Psum + C(i).*(Ut(Znodes,Xnodes + i,1) - Ut(Znodes,Xnodes + 1 - i,1));
    end
    Vx(Znodes,Xnodes,2) = Coeffi1.*Vx(Znodes,Xnodes,1) - Coeffi3.*Psum;
    Psum(:, :) = 0;
    for i = 1:1:nodr
120        Psum = Psum + C(i).*(Ut(Znodes + i,Xnodes,1) - Ut(Znodes + 1 - i,Xnodes,1));
    end
    Vz(Znodes,Xnodes,2) = Coeffi2.*Vz(Znodes,Xnodes,1) - Coeffi4.*Psum;
    Psum(:, :) = 0;
    for i = 1:1:nodr
125        Psum = Psum + C(i).*(Vx(Znodes,Xnodes - 1 + i,2) - Vx(Znodes,Xnodes - i,2));
    end
    Ux(Znodes,Xnodes,2) = Coeffi1.*Ux(Znodes,Xnodes,1) - Coeffi5.*Psum;
    Psum(:, :) = 0;
    for i = 1:1:nodr
130        Psum = Psum + C(i).*(Vz(Znodes - 1 + i,Xnodes,2) - Vz(Znodes - i,Xnodes,2));
    end
    Uz(Znodes,Xnodes,2) = Coeffi2.*Uz(Znodes,Xnodes,1) - Coeffi6.*Psum;
    Ut(:, :, 1) = Ut(:, :, 2);
    Uz(:, :, 1) = Uz(:, :, 2);
135    Ux(:, :, 1) = Ux(:, :, 2);
    Vz(:, :, 1) = Vz(:, :, 2);
    Vx(:, :, 1) = Vx(:, :, 2);
end
toc;
140
%% Plotting

% Wavefield Snapshot
figure;% colormap gray;
145 clims = [min(U(:)) max(U(:))]./5;
for it = 1:5:nt
    imagesc((0:nx - 1).*dx,(0:nz - 1).*dz,U(:, :, it));%clims);
    set(gca,'xaxislocation','top'); axis equal; axis([0 (nx - 1)*dx 0 (nz - 1)*dz]);
    colorbar; xlabel('x distance (m)'); ylabel('z depth (m)');
150    title(sprintf('the snapshot of %.1f ms',it*dt*1e3),'position',[(nx - 1)*dx/2,(nz - 1)*dz*(1 +
        0.07)]);
    pause(0.01);
end

% Synthetic Seismogram
155 synggram(:, :) = U(1,xrcvr,:);
synmax = max(abs(synggram(:)));
rcvrintv = (xrcvr(2) - xrcvr(1))*dx;

```

```

synggram = synggram./syngmax.*(rcvrintv/2);
[nsyn,~] = size(synggram);
160 figure; hold on;
    for i = 1:1:nsyn
        plot(synggram(i,:) + (xrcvr(i) - 1)*dx,t.*1e3);
    end
xlabel('x distance (m)'); ylabel('travel time (ms)');
165 title('Synthetic Seismogram','position',[(xrcvr(1) + xrcvr(nsyn))*dx/2,t(end)*1e3*(1 + 0.07)]);
set(gca,'axislocation','top');
set(gca,'YDir','reverse');
hold off;

170 end

%% References

% Collino and Tsogka, 2001. Geophysics, Application of the perfectly matched absorbing layer model to
    the linear elastodynamic problem in anisotropic heterogeneous media.
175 % Marcinkovich and Olsen, 2003. Journal of Geophysical Research, On the implementation of perfectly
    mathced layers in a three-dimensional fourth-order velocity-stress finite difference scheme.

```

Fortran 程序

```

MODULE InputPara

    IMPLICIT NONE

    PUBLIC
5    INTEGER, PARAMETER :: nx = 101, nz = 101                ! nx: the total number of grid nodes
        in x-direction; nz: the total number of grid nodes in z-direction.
    INTEGER, PARAMETER :: npmlx = 20, npmlz = 20              ! npmlx: the total number of grid
        nodes in top and bottom side of PML absorbing boundary; npmlz: the total number of grid nodes in
        left and right side of PML absorbing boundary.
    INTEGER, PARAMETER :: sx = 50, sz = 50                    ! sx: the grid node number of source
        position in x-direction; sz: the grid node number of source position in z-direction.
    INTEGER, PARAMETER :: dx = 5, dz = 5                      ! dx: the grid node interval in x-
        direction; dz: the grid node interval in z-direction; Unit: m.
10    INTEGER, PARAMETER :: nt = 500                          ! the total number of time nodes for
        wave calculating.
    REAL, PARAMETER :: dt = 1.0E-3                            ! the time node interval, Unit: s.
    INTEGER, PARAMETER :: nppw = 12                           ! the total node point number per
        wavelength for dominant frequency of Ricker wavelet source.
    REAL, PARAMETER :: amp = 1.0E0                             ! the amplitude of source wavelet.
    INTEGER, PARAMETER :: nodr = 3                             ! half of the order number for
        spatial difference.
15    INTEGER, PARAMETER :: irstr = 1                          ! the node ID of starting reciver
        point.
    INTEGER, PARAMETER :: nrintv = 3                           ! the total node number between each
        two adjacent receivers.
    INTEGER, PARAMETER :: itstr = 1                            ! the time node ID of the first
        snapshot.
    INTEGER, PARAMETER :: ntintv = 5                           ! the total time node number between
        each two followed snapshot.

20    REAL :: src(nt)                                           ! the time series of source wavelet.
    REAL :: vp(nz, nx), rho(nz, nx)                           ! vp: the velocity of acoustic wave
        of model, Unit: m/s; rho: the density of model, Unit: kg/m^3.
    INTEGER, PARAMETER :: nrcvr = CEILING(REAL(nx)/nrintv)     ! the total number of all receivers.

```

```

INTEGER :: xrcvr(nrcvr)                                ! the grid node number in x-direction
               of reciver position on ground.

25 INTEGER, PRIVATE :: i

PRIVATE nppw, amp
PRIVATE ModelVpRho, SrcWavelet

30 CONTAINS
  SUBROUTINE IntlzInputPara()
    xrcvr = [ (istr + (i - 1)*nrntv, i = 1, nrcvr) ]
    CALL ModelVpRho()
    CALL SrcWavelet()
35  END SUBROUTINE IntlzInputPara
  SUBROUTINE ModelVpRho()
    ! here you can reset $vp$ and $rho$ for the model.
    vp = 2000
    vp(nz/3:nz, nx/2:nx) = 1000
40    rho = 1000
    rho(nz/3:nz, nx/2:nx) = 500
  END SUBROUTINE ModelVpRho
  SUBROUTINE SrcWavelet()
    ! here you can reset $src$ for the source wavelet.
45    REAL :: f0, t0, pi = 3.1415926
    REAL :: t(nt)
    f0 = MINVAL(vp)/(MIN(dx, dz)*nppw)
    t0 = 1/f0
    t = [ (i*dt, i = 1, nt) ]
50    src = amp*(1 - 2*(pi*f0*(t - t0))**2)*EXP( - (pi*f0*(t - t0))**2)
  END SUBROUTINE SrcWavelet

END MODULE InputPara

55 MODULE WaveExtrp

  USE InputPara
  IMPLICIT NONE

60  PRIVATE
  REAL :: C(nodr)                                ! the difference coefficients of
               spatial the $2*nodr$-th order difference approximating.
  INTEGER, PARAMETER :: Nzz = nz + 2*npmlz, Nxx = nx + 2*npmlx ! Nzz: the total number of grid nodes
               in z-direction of compute-updating zone including PML layer; Nxx: the total number of grid
               nodes in x-direction of compute-updating zone including PML layer.
  REAL :: vpp(Nzz, Nxx), rho0(Nzz, Nxx)           ! vpp: the velocity of the expanded
               model including PML layer, Unit: m/s; rho0: the density of the expanded model including PML
               layer, Unit: kg/m^3.
  REAL :: dpmlz(Nzz, Nxx), dpmlx(Nzz, Nxx)         ! dpmlz: the PML damping factor in z-
               direction; dpmlx: the PML damping factor in x-direction.
65  REAL :: Coef1(Nzz, Nxx), Coef2(Nzz, Nxx), &
               & Coef3(Nzz, Nxx), Coef4(Nzz, Nxx), &
               & Coef5(Nzz, Nxx), Coef6(Nzz, Nxx) ! Coef1 ~ Coef6: the coefficients of
               wavefield time-extrapolating formula.

  INTEGER :: i, j

70  REAL, PUBLIC :: P(nz, nx, nt)                ! the calculating wavefield component
               varying with time.

  PUBLIC WaveExec

```

```

75 CONTAINS
SUBROUTINE WaveExec()
  CALL CalC()
  CALL ModelExpand()
  CALL CalCoefs()
80 CALL CalWave()
END SUBROUTINE WaveExec
SUBROUTINE CalC()
  REAL :: rtemp1, rtemp2
  DO i = 1,nodr,1
    rtemp1 = 1.0
    rtemp2 = 1.0
    DO j = 1,nodr,1
      IF(j == i) CYCLE
      rtemp1 = rtemp1*((2*j - 1)**2)
90 rtemp2 = rtemp2*ABS((2*i - 1)**2 - (2*j - 1)**2)
    END DO
    C(i) = ( - 1)**(i + 1)*rtemp1/((2*i - 1)*rtemp2)
  END DO
END SUBROUTINE CalC
95 SUBROUTINE ModelExpand()
  vpp = 0.0
  rhoo = 0.0
  vpp(npmlz + 1:npmlz + nz, npmlx + 1:npmlx + nx) = vp
  rhoo(npmlz + 1:npmlz + nz, npmlx + 1:npmlx + nx) = rho
100 DO i = 1,npmlx,1
  vpp(:, i) = vpp(:, npmlx + 1)
  vpp(:, npmlx + nx + i) = vpp(:, npmlx + nx)
  rhoo(:, i) = rhoo(:, npmlx + 1)
  rhoo(:, npmlx + nx + i) = rhoo(:, npmlx + nx)
105 END DO
DO i = 1,npmlz,1
  vpp(i, :) = vpp(npmlz + 1, :)
  vpp(npmlz + nz + i, :) = vpp(npmlz + nz, :)
  rhoo(i, :) = rhoo(npmlz + 1, :)
  rhoo(npmlz + nz + i, :) = rhoo(npmlz + nz, :)
110 END DO
END SUBROUTINE ModelExpand
SUBROUTINE CalDpml()
  REAL :: dpml0z, dpml0x
115 dpml0z = 3*MAXVAL(vp)/dz*(8.0/15 - 3.0/100*npmlz + 1.0/1500*(npmlz**2))
  DO i = 1,npmlz,1
    dpmlz(i, :) = dpml0z*((REAL(npmlz - i + 1)/npmlz)**2)
  END DO
  dpmlz(npmlz + nz + 1:Nzz, :) = dpmlz(npmlz:1:-1, :)
120 dpml0x = 3*MAXVAL(vp)/dx*(8.0/15 - 3.0/100*npmlx + 1.0/1500*(npmlx**2))
  DO i = 1,npmlx,1
    dpmlx(:, i) = dpml0x*((REAL(npmlx - i + 1)/npmlx)**2)
  END DO
  dpmlx(:, npmlx + nx + 1:Nxx) = dpmlx(:, npmlx:1:-1)
125 END SUBROUTINE CalDpml
SUBROUTINE CalCoefs()
  CALL CalDpml()
  Coef1 = (2 - dt*dpmlx)/(2 + dt*dpmlx)
  Coef2 = (2 - dt*dpmlz)/(2 + dt*dpmlz)
130 Coef3 = (2*dt/(2 + dt*dpmlx))/(rhoo*dx)
  Coef4 = (2*dt/(2 + dt*dpmlz))/(rhoo*dz)
  Coef5 = (2*dt/(2 + dt*dpmlx))*(rhoo*(vpp**2)/dx)
  Coef6 = (2*dt/(2 + dt*dpmlz))*(rhoo*(vpp**2)/dz)
END SUBROUTINE CalCoefs
135 SUBROUTINE CalWave()

```



```

140 INTEGER :: it
INTEGER, PARAMETER :: Nzzz = Nzz + 2*nodr, Nxxx = Nxx + 2*nodr
INTEGER :: znds(nz) = [ (nodr + npmlz + i, i = 1,nz,1) ], &
& xnds(nx) = [ (nodr + npmlx + i, i = 1,nx,1) ]
145 INTEGER :: Zznds(Nzz) = [ (nodr + i, i = 1,Nzz,1) ], &
& Xxnds(Nxx) = [ (nodr + i, i = 1,Nxx,1) ]
INTEGER :: nsrctz = nodr + npmlz + sz, nsrctx = nodr + npmlx + sx
REAL :: Pt(Nzzz, Nxxx, 2) = 0, &
& Pz(Nzzz, Nxxx, 2) = 0, Px(Nzzz, Nxxx, 2) = 0, &
145 & vz(Nzzz, Nxxx, 2) = 0, vx(Nzzz, Nxxx, 2) = 0
REAL :: SpcSum(Nzz, Nxx)
DO it = 1,nt,1
WRITE(*,"(A,G0)") 'The calculating time node is: it = ',it
Px(nsrctz, nsrctx, 1) = Px(nsrctz, nsrctx, 1) + src(it)/2
150 Pz(nsrctz, nsrctx, 1) = Pz(nsrctz, nsrctx, 1) + src(it)/2
Pt(:, :, 1) = Px(:, :, 1) + Pz(:, :, 1)
P(:, :, it) = Pt(znds, xnds, 1)
SpcSum = 0
DO i = 1,nodr,1
155 SpcSum = SpcSum + C(i)*(Pt(Zznds, Xxnds + i, 1) - Pt(Zznds, Xxnds + 1 - i, 1))
END DO
vx(Zznds, Xxnds, 2) = Coef1*vz(Zznds, Xxnds, 1) - Coef3*SpcSum
SpcSum = 0
DO i = 1,nodr,1
160 SpcSum = SpcSum + C(i)*(Pt(Zznds + i, Xxnds, 1) - Pt(Zznds + 1 - i, Xxnds, 1))
END DO
vz(Zznds, Xxnds, 2) = Coef2*vz(Zznds, Xxnds, 1) - Coef4*SpcSum
SpcSum = 0
DO i = 1,nodr,1
165 SpcSum = SpcSum + C(i)*(vx(Zznds, Xxnds - 1 + i, 2) - vx(Zznds, Xxnds - i, 2))
END DO
Px(Zznds, Xxnds, 2) = Coef1*Px(Zznds, Xxnds, 1) - Coef5*SpcSum
SpcSum = 0
DO i = 1,nodr,1
170 SpcSum = SpcSum + C(i)*(vz(Zznds - 1 + i, Xxnds, 2) - vz(Zznds - i, Xxnds, 2))
END DO
Pz(Zznds, Xxnds, 2) = Coef2*Pz(Zznds, Xxnds, 1) - Coef6*SpcSum
Pt(:, :, 1) = Pt(:, :, 2)
Pz(:, :, 1) = Pz(:, :, 2)
175 Px(:, :, 1) = Px(:, :, 2)
vz(:, :, 1) = vz(:, :, 2)
vx(:, :, 1) = vx(:, :, 2)
END DO
END SUBROUTINE CalWave
180
END MODULE WaveExtrp

!***** TDFDAWFS2DSG *****
! Time Domain Finite Difference Acoustic Wave Field Simulating with 2-Dimension Staggered Grid
185 ! Written by Tche. L. from USTC, 2016,7
! References:
! Collino and Tsogka, 2001. Geophysics, Application of the perfectly matched absorbing layer model
to the linear elastodynamic problem in anisotropic heterogeneous media.
! Marcinkovich and Olsen, 2003. Journal of Geophysical Research, On the implementation of perfectly
mathced layers in a three-dimensional fourth-order velocity-stress finite difference scheme.
!*****
190 PROGRAM TDFDAWFS2DSG

USE InputPara
USE WaveExtrp
IMPLICIT NONE

```

```

195 CHARACTER(LEN = 128) :: SnapFile = './Snapshot/Snapshot_****.dat'      ! the snapshot file name
    template.
    CHARACTER(LEN = 128) :: SyntFile = 'SyntRcrd.dat'                      ! the synthetic record file
    name.
    REAL    :: SyntR(nrcvr, nt)
    INTEGER :: i

200 CALL IntlzInputPara()
    CALL WaveExec()
    DO i = itstr, nt, ntintv
        WRITE(SnapFile(21:24), "(I4.4)") i
205     CALL Output(TRIM(SnapFile), nz, nx, P(:, :, i))
    END DO
    DO i = 1, nt, 1
        SyntR(:, i) = P(1, xrcvr, i)
    END DO
210 CALL Output(TRIM(SyntFile), nrcvr, nt, SyntR)

END PROGRAM TDFDAWFS2DSG

SUBROUTINE Output(Outfile, M, N, OutA)
215 IMPLICIT NONE
    CHARACTER(LEN = *) , INTENT(IN) :: Outfile
    INTEGER, INTENT(IN) :: M, N
    REAL, INTENT(IN)    :: OutA(M, N)
    CHARACTER(LEN = 40) :: FmtStr
220 INTEGER :: i, j
    INTEGER :: funit
    WRITE(FmtStr, "(',G0,'E15.6')") N
    OPEN(NEWUNIT = funit, FILE = Outfile, STATUS = 'UNKNOWN')
    DO i = 1, M, 1
225     WRITE(funit, FmtStr) (OutA(i, j), j = 1, N, 1)
    END DO
    CLOSE(funit)
END SUBROUTINE Output

```

附录 弹性波：TDFDEWFS2DSG

Matlab 程序

```

function TDFDEWFS2DSG

% TDFDEWFS2DSG
% This is a program of Time Domain Finite Difference Elastic Wave Field Simulating with 2-Dimension
% Staggered Grid.
5 % Written by Tche.L. from USTC, 2016.7.

clc; clear; close all;
% format long;

10 %% Input parameters

nx = 159;           % the number of grid nodes in x-direction.
nz = 159;           % the number of grid nodes in z-direction.
npmlz = 20;         % the number of grid nodes in top and bottom side of PML absorbing boundary.
15 npmlx = 20;       % the number of grid nodes in left and right side of PML absorbing boudary.
sx = 80;            % the grid node number of source position in x-direction.
sz = 80;            % the grid node number of source position in z-direction.
dx = 5;             % the grid node interval in x-direction; Unit: m.
dz = 5;             % the grid node interval in z-direction; Unit: m.
20 nt = 500;        % the number of time nodes for wave calculating.
dt = 1e-3;          % the time node interval; Unit: s.
nppw = 12;          % the node point number per wavelength for dominant frequency of Ricker
                    % wavelet source.
ampl = 1.0e0;       % the amplitude of source wavelet.
xrcvr = 1:3:nx;     % the grid node number in x-direction of reciver position on ground.
25 nodr = 1;         % half of the order number for spatial difference.

%% Determine the difference coefficients

B = [1 zeros(1,nodr - 1)]';
30 A = NaN*ones(nodr,nodr);
for i = 1:1:nodr
    A(i,:) = (1:2:2*nodr - 1).^(2*i - 1);
end
C = A\B;

35 %% Model and source

Nz = nz + 2*npmlz;
Nx = nx + 2*npmlx;

40 vp = 2000*ones(Nz,Nx);           % the velocity of P-wave
    of model; Unit: m/s.
vs = 1000*ones(Nz,Nx);           % the velocity of S-wave
    of model; Unit: m/s.
rho = 1000*ones(Nz,Nx);          % the density of model;
    Unit: kg/m^3.
% vp(fix(Nz/3):end,fix(Nx/2):end) = 1500;

45 lmd = rho.*(vp.^2 - 2.*vs.^2);   % the lame parameter
    lambda of elastic wave of model.
mu = rho.*vs.^2;                 % the lame parameter mu
    of elastic wave of model.

```

```

f0 = min(vs(:))/(max(dx,dz)*nppw); % the dominant frequency
    of source Ricker wavelet; Unit: Hz.
50 t0 = 1/f0; % the time shift of
    source Ricker wavelet; Unit: s; Suggest: 0.02 if fm = 50, or 0.05 if fm = 20.
t = dt*(1:1:nt);
src = (1 - 2*(pi*f0*(t - t0)).^2).*exp(-(pi*f0*(t - t0)).^2); % the time series of
    source wavelet.
% The source wavelet formula refers to the equations (18) of Collino and Tsogka, 2001.

55 %% Perfectly matched layer absorbing factor

% R = 1e-6; % Recommend: $R = 1e-2$,
    if $nabsr = 5; $R = 1e-3, if $nabsr = 10; $R = 1e-4, if $absr = 20.
% dpml0z = log(1/R)*3*max(vs(:))/(2*npmlz);
dpml0z = 3*max(vs(:))/dz*(8/15 - 3/100*npmlz + 1/1500*npmlz^2);
60 dpmlz = zeros(Nz,Nx);
dpmlz(1:npmlz,:) = (dpml0z*((npmlz - 1:1)./npmlz).^2)*ones(1,Nx);
dpmlz(npmlz + nz + 1:Nz,:) = dpmlz(npmlz - 1:1,:);
dpml0x = 3*max(vs(:))/dx*(8/15 - 3/100*npmlx + 1/1500*npmlx^2);
dpmlx = zeros(Nz,Nx);
65 dpmlx(:,1:npmlx) = ones(Nz,1)*(dpml0x*((npmlx - 1:1)./npmlx).^2);
dpmlx(:,npmlx + nx + 1:Nx) = dpmlx(:,npmlx - 1:1);
% The PML formula refers to the equations (2) and (3) of Marcinkovich and Olsen, 2003.

%% Wavefield calculating
70
Coeffi1 = (2 - dt.*dpmlx)./(2 + dt.*dpmlx);
Coeffi2 = (2 - dt.*dpmlz)./(2 + dt.*dpmlz);
Coeffi3 = 2*dt./(2 + dt.*dpmlx)./rho./dx;
Coeffi4 = 2*dt./(2 + dt.*dpmlz)./rho./dz;
75 Coeffi5 = 2*dt./(2 + dt.*dpmlx).*(lmd + 2.*mu)./dx;
Coeffi6 = 2*dt./(2 + dt.*dpmlz).*lmd./dz;
Coeffi7 = 2*dt./(2 + dt.*dpmlx).*lmd./dx;
Coeffi8 = 2*dt./(2 + dt.*dpmlz).*(lmd + 2.*mu)./dz;
Coeffi9 = 2*dt./(2 + dt.*dpmlx).*mu./dx;
80 Coeffi10 = 2*dt./(2 + dt.*dpmlz).*mu./dz;

% ++++++ approximate coefficient ++++++
% Coeffi1 = 1 - dt.*dpmlx;
% Coeffi2 = 1 - dt.*dpmlz;
85 % Coeffi3 = dt./rho./dx;
% Coeffi4 = dt./rho./dz;
% Coeffi5 = (lmd + 2.*mu).*dt./dx;
% Coeffi6 = lmd.*dt./dz;
% Coeffi7 = lmd.*dt./dx;
90 % Coeffi8 = (lmd + 2.*mu).*dt./dz;
% Coeffi9 = mu.*dt./dx;
% Coeffi10 = mu.*dt./dz;
%

95 NZ = Nz + 2*nodr;
NX = Nx + 2*nodr;

Znodes = nodr + 1:NZ - nodr;
Xnodes = nodr + 1:NX - nodr;
100 znodes = nodr + npmlz + 1:nodr + npmlz + nz;
xnodes = nodr + npmlx + 1:nodr + npmlx + nx;
nsrcz = nodr + npmlz + sz;
nsrcx = nodr + npmlx + sx;

105 vxt = zeros(NZ,NX,2);

```

```

vxx = zeros(NZ,NX,2);
vxz = zeros(NZ,NX,2);
vzt = zeros(NZ,NX,2);
vzx = zeros(NZ,NX,2);
110 vzz = zeros(NZ,NX,2);
txxt = zeros(NZ,NX,2);
txxx = zeros(NZ,NX,2);
txxz = zeros(NZ,NX,2);
tzzt = zeros(NZ,NX,2);
115 tzzx = zeros(NZ,NX,2);
tzzz = zeros(NZ,NX,2);
txzt = zeros(NZ,NX,2);
txzx = zeros(NZ,NX,2);
txzz = zeros(NZ,NX,2);
120 Psum = NaN*ones(Nz,Nx);

P = NaN*ones(nz,nx,nt);

tic;
125 for it = 1:1:nt
    fprintf('The calculating time node is: it = %d\n',it);
    %% load source
    txxx(nsrcz,nsrcx,1) = txxx(nsrcz,nsrcx,1) + ampl*src(it)./4;
    txxz(nsrcz,nsrcx,1) = txxz(nsrcz,nsrcx,1) + ampl*src(it)./4;
130 tzzx(nsrcz,nsrcx,1) = tzzx(nsrcz,nsrcx,1) + ampl*src(it)./4;
    tzzz(nsrcz,nsrcx,1) = tzzz(nsrcz,nsrcx,1) + ampl*src(it)./4;
    txxt(:, :, 1) = txxx(:, :, 1) + txxz(:, :, 1);
    tzzt(:, :, 1) = tzzx(:, :, 1) + tzzz(:, :, 1);
    P(:, :, it) = txxt(znodes,xnodes,1);
135 % P(:, :, it) = tzzt(znodes,xnodes,1);
    % P(:, :, it) = txzt(znodes,xnodes,1);
    %% calculate v_x
    Psum(:, :) = 0;
    for i = 1:1:nodr
140 Psum = Psum + C(i).*(txxt(Znodes,Xnodes + i - 1,1) - txxt(Znodes,Xnodes - i,1));
    end
    vxx(Znodes,Xnodes,2) = Coeffi1.*vxx(Znodes,Xnodes,1) + Coeffi3.*Psum;
    Psum(:, :) = 0;
    for i = 1:1:nodr
145 Psum = Psum + C(i).*(txzt(Znodes + i - 1,Xnodes,1) - txzt(Znodes - i,Xnodes,1));
    end
    vxz(Znodes,Xnodes,2) = Coeffi2.*vxz(Znodes,Xnodes,1) + Coeffi4.*Psum;
    vxt(:, :, 2) = vxx(:, :, 2) + vxz(:, :, 2);
    % P(:, :, it) = vxt(znodes,xnodes,2);
150 %% calculate v_z
    Psum(:, :) = 0;
    for i = 1:1:nodr
        Psum = Psum + C(i).*(txzt(Znodes,Xnodes + i,1) - txzt(Znodes,Xnodes - i + 1,1));
    end
155 vzx(Znodes,Xnodes,2) = Coeffi1.*vzx(Znodes,Xnodes,1) + Coeffi3.*Psum;
    Psum(:, :) = 0;
    for i = 1:1:nodr
        Psum = Psum + C(i).*(tzzt(Znodes + i,Xnodes,1) - tzzt(Znodes - i + 1,Xnodes,1));
    end
160 vzz(Znodes,Xnodes,2) = Coeffi2.*vzz(Znodes,Xnodes,1) + Coeffi4.*Psum;
    vzt(:, :, 2) = vxz(:, :, 2) + vzz(:, :, 2);
    % P(:, :, it) = vzt(znodes,xnodes,2);
    %% calculate tau_{xx} and tau_{zz}
    Psum(:, :) = 0;
165 for i = 1:1:nodr
        Psum = Psum + C(i).*(vxt(Znodes,Xnodes + i,2) - vxt(Znodes,Xnodes - i + 1,2));
    end

```

```

end
txxx(Znodes,Xnodes,2) = Coeffi1.*txxx(Znodes,Xnodes,1) + Coeffi5.*Psum;
tzzx(Znodes,Xnodes,2) = Coeffi1.*tzzx(Znodes,Xnodes,1) + Coeffi7.*Psum;
170 Psum(:, :) = 0;
for i = 1:1:nodr
    Psum = Psum + C(i).*(vzt(Znodes + i - 1,Xnodes,2) - vzt(Znodes - i,Xnodes,2));
end
txxz(Znodes,Xnodes,2) = Coeffi2.*txxz(Znodes,Xnodes,1) + Coeffi6.*Psum;
175 tzzz(Znodes,Xnodes,2) = Coeffi2.*tzzz(Znodes,Xnodes,1) + Coeffi8.*Psum;
txxt(:, :, 2) = txxx(:, :, 2) + txxz(:, :, 2);
tzzt(:, :, 2) = tzzx(:, :, 2) + tzzz(:, :, 2);
%% calculate tau_{xz}
Psum(:, :) = 0;
180 for i = 1:1:nodr
    Psum = Psum + C(i).*(vzt(Znodes,Xnodes + i - 1,2) - vzt(Znodes,Xnodes - i,2));
end
txzx(Znodes,Xnodes,2) = Coeffi1.*txzx(Znodes,Xnodes,1) + Coeffi9.*Psum;
Psum(:, :) = 0;
185 for i = 1:1:nodr
    Psum = Psum + C(i).*(vxt(Znodes + i,Xnodes,2) - vxt(Znodes - i + 1,Xnodes,2));
end
txzz(Znodes,Xnodes,2) = Coeffi2.*txzz(Znodes,Xnodes,1) + Coeffi10.*Psum;
txzt(:, :, 2) = txzx(:, :, 2) + txzz(:, :, 2);
190 %% exchange for next cycle
vxx(:, :, 1) = vxx(:, :, 2);
vxz(:, :, 1) = vxz(:, :, 2);
vxt(:, :, 1) = vxt(:, :, 2);
vzx(:, :, 1) = vzx(:, :, 2);
195 vzz(:, :, 1) = vzz(:, :, 2);
vzt(:, :, 1) = vzt(:, :, 2);
txxx(:, :, 1) = txxx(:, :, 2);
txxz(:, :, 1) = txxz(:, :, 2);
txxt(:, :, 1) = txxt(:, :, 2);
200 tzzx(:, :, 1) = tzzx(:, :, 2);
tzzz(:, :, 1) = tzzz(:, :, 2);
tzzt(:, :, 1) = tzzt(:, :, 2);
txzx(:, :, 1) = txzx(:, :, 2);
txzz(:, :, 1) = txzz(:, :, 2);
205 txzt(:, :, 1) = txzt(:, :, 2);
end
toc;

%% Plotting
210 % Wavefield Snapshot
figure;% colormap gray;
clims = [min(P(:)) max(P(:))]/5;
for it = 1:5:nt
    imagesc((0:nx - 1).dx,(0:nz - 1).dz,P(:, :, it));%clims);
    set(gca,'xaxislocation','top'); axis equal; axis([0 (nx - 1)*dx 0 (nz - 1)*dz]);
    colorbar; xlabel('x distance (m)'); ylabel('z depth (m)');
    title(sprintf('the snapshot of %.1f ms',it*dt*1e3),'position',[(nx - 1)*dx/2,(nz - 1)*dz*(1 +
        0.07)]);
    pause(0.01);
220 end

% Synthetic Seismogram
syngam(:, :) = P(1,xrcvr,:);
synmax = max(abs(syngam(:)));
225 rcvrintv = (xrcvr(2) - xrcvr(1))*dx;
syngam = syngam./synmax.*(rcvrintv/2);

```

```

[nsyn,~] = size(syngram);
figure; hold on;
for i = 1:1:nsyn
230   plot(syngram(i,:) + (xrcvr(i) - 1)*dx,t.*1e3);
end
xlabel('x distance (m)'); ylabel('travel time (ms)');
title('Synthetic Seismogram','position',[(xrcvr(1) + xrcvr(nsyn))*dx/2,t(end)*1e3*(1 + 0.07)]);
set(gca,'xaxislocation','top');
235 set(gca,'YDir','reverse');
hold off;

end

240 %% References

% Collino and Tsogka, 2001. Geophysics, Application of the perfectly matched absorbing layer model to
    the linear elastodynamic problem in anisotropic heterogeneous media.
% Marcinkovich and Olsen, 2003. Journal of Geophysical Research, On the implementation of perfectly
    mathced layers in a three-dimensional fourth-order velocity-stress finite difference scheme.

```

Fortran 程序

```

MODULE InputPara

    IMPLICIT NONE

    PUBLIC

5    INTEGER, PARAMETER :: nx = 160, nz = 160                ! nx: the total number of grid nodes
        in x-direction; nz: the total number of grid nodes in z-direction.
    INTEGER, PARAMETER :: npmlx = 20, npmlz = 20              ! npmlx: the total number of grid
        nodes in top and bottom side of PML absorbing boundary; npmlz: the total number of grid nodes in
        left and right side of PML absorbing boundary.
    INTEGER, PARAMETER :: sx = 80, sz = 80                    ! sx: the grid node number of source
        position in x-direction; sz: the grid node number of source position in z-direction.
    INTEGER, PARAMETER :: dx = 5, dz = 5                      ! dx: the grid node interval in x-
        direction; dz: the grid node interval in z-direction; Unit: m.
10   INTEGER, PARAMETER :: nt = 500                          ! the total number of time nodes for
        wave calculating.
    REAL, PARAMETER :: dt = 1.0E-3                            ! the time node interval, Unit: s.
    INTEGER, PARAMETER :: nppw = 12                           ! the total node point number per
        wavelength for dominant frequency of Ricker wavelet source.
    REAL, PARAMETER :: amp = 1.0E0                            ! the amplitude of source wavelet.
    INTEGER, PARAMETER :: nodr = 3                             ! half of the order number for
        spatial difference.
15   INTEGER, PARAMETER :: irstr = 1                          ! the node ID of starting reciver
        point.
    INTEGER, PARAMETER :: nrntv = 3                           ! the total node number between each
        two adjacent receivers.
    INTEGER, PARAMETER :: itstr = 1                            ! the time node ID of the first
        snapshot.
    INTEGER, PARAMETER :: ntintv = 5                          ! the total time node number between
        each two followed snapshot.

20   REAL :: src(nt)                                           ! the time series of source wavelet.
    REAL :: vp(nz, nx), vs(nz, nx), rho(nz, nx)              ! vp: the velocity of P-wave of model
        , Unit: m/s; vs: the velocity of S-wave of model, Unit: m/s; rho: the density of model, Unit: kg
        /m^3.
    INTEGER, PARAMETER :: nrcvr = CEILING(REAL(nx)/nrntv)      ! the total number of all receivers.

```

```

INTEGER :: xrcvr(nrcvr)                                ! the grid node number in x-direction
               of reciver position on ground.

25  INTEGER, PRIVATE :: i

PRIVATE nppw, amp
PRIVATE ModelVpRho, SrcWavelet

30  CONTAINS
    SUBROUTINE IntlzInputPara()
        xrcvr = [ (istr + (i - 1)*nrintv, i = 1, nrcvr) ]
        CALL ModelVpRho()
        CALL SrcWavelet()
35  END SUBROUTINE IntlzInputPara
    SUBROUTINE ModelVpRho()
        ! here you can reset $vp$ and $rho$ for the model.
        vp = 2000
        vs = 1000
40        rho = 1000
    END SUBROUTINE ModelVpRho
    SUBROUTINE SrcWavelet()
        ! here you can reset $src$ for the source wavelet.
        REAL :: f0, t0, pi = 3.1415926
        REAL :: t(nt)
45        f0 = MINVAL(vs)/(MIN(dx, dz)*nppw)
        t0 = 1/f0
        t = [ (i*dt, i = 1, nt) ]
        src = amp*(1 - 2*(pi*f0*(t - t0))**2)*EXP( - (pi*f0*(t - t0))**2)
50  END SUBROUTINE SrcWavelet

END MODULE InputPara

MODULE WaveExtrp

55  USE InputPara
    IMPLICIT NONE

PRIVATE

60  REAL :: C(nodr)                                ! the difference coefficients of
               spatial the $2*nodr$-th order difference approximating.
    INTEGER, PARAMETER :: Nzz = nz + 2*npmlz, Nxx = nx + 2*npmlx ! Nzz: the total number of grid nodes
               in z-direction of compute-updating zone including PML layer; Nxx: the total number of grid
               nodes in x-direction of compute-updating zone including PML layer.
    REAL :: vpp(Nzz, Nxx), vss(Nzz, Nxx), rho0(Nzz, Nxx) ! vpp: the velocity of P-wave of the
               expanded model including PML layer, Unit: m/s; vss: the velocity of S-wave of the expanded model
               including PML layer, Unit: m/s; rho0: the density of the expanded model including PML layer,
               Unit: kg/m^3.
    REAL :: lmdd(Nzz, Nxx), muu(Nzz, Nxx) ! lmdd: the lame parameter lambda of
               elastic wave of the expanded model including PML layer; muu: the lame parameter mu of elastic
               wave of the expanded model including PML layer.
    REAL :: dpmlz(Nzz, Nxx), dpmlx(Nzz, Nxx) ! dpmlz: the PML damping factor in z-
               direction; dpmlx: the PML damping factor in x-direction.
65  REAL :: Coef1(Nzz, Nxx), Coef2(Nzz, Nxx), &
               & Coef3(Nzz, Nxx), Coef4(Nzz, Nxx), &
               & Coef5(Nzz, Nxx), Coef6(Nzz, Nxx), &
               & Coef7(Nzz, Nxx), Coef8(Nzz, Nxx), &
               & Coef9(Nzz, Nxx), Coef0(Nzz, Nxx) ! Coef1 ~ Coef0: the coefficients of
               wavefield time-extrapolating formula.
70  INTEGER :: i, j

```



```

REAL, PUBLIC :: P(nz, nx, nt)                                ! the calculating wavefield component
    varying with time.

75 PUBLIC WaveExec

CONTAINS
SUBROUTINE WaveExec()
    CALL CalC()
80    CALL ModelExpand()
    CALL CalCoefs()
    CALL CalWave()
END SUBROUTINE WaveExec
SUBROUTINE CalC()
85    REAL :: rtemp1, rtemp2
    DO i = 1,nodr,1
        rtemp1 = 1.0
        rtemp2 = 1.0
        DO j = 1,nodr,1
90            IF(j == i) CYCLE
            rtemp1 = rtemp1*((2*j - 1)**2)
            rtemp2 = rtemp2*ABS((2*i - 1)**2 - (2*j - 1)**2)
        END DO
        C(i) = (- 1)**(i + 1)*rtemp1/((2*i - 1)*rtemp2)
95    END DO
END SUBROUTINE CalC
SUBROUTINE ModelExpand()
    vpp = 0.0
    vss = 0.0
100    rhoo = 0.0
    vpp(npmlz + 1:npmlz + nz, npmlx + 1:npmlx + nx) = vp
    vss(npmlz + 1:npmlz + nz, npmlx + 1:npmlx + nx) = vs
    rhoo(npmlz + 1:npmlz + nz, npmlx + 1:npmlx + nx) = rho
    DO i = 1,npmlx,1
105        vpp(:, i) = vpp(:, npmlx + 1)
        vpp(:, npmlx + nx + i) = vpp(:, npmlx + nx)
        vss(:, i) = vss(:, npmlx + 1)
        vss(:, npmlx + nx + i) = vss(:, npmlx + nx)
        rhoo(:, i) = rhoo(:, npmlx + 1)
110        rhoo(:, npmlx + nx + i) = rhoo(:, npmlx + nx)
    END DO
    DO i = 1,npmlz,1
        vpp(i, :) = vpp(npmlz + 1, :)
        vpp(npmlz + nz + i, :) = vpp(npmlz + nz, :)
115        vss(i, :) = vss(npmlz + 1, :)
        vss(npmlz + nz + i, :) = vss(npmlz + nz, :)
        rhoo(i, :) = rhoo(npmlz + 1, :)
        rhoo(npmlz + nz + i, :) = rhoo(npmlz + nz, :)
    END DO
    lmdd = rhoo*(vpp**2 - 2*(vss**2))
    muu = rhoo*(vss**2)
END SUBROUTINE ModelExpand
SUBROUTINE CalDpml()
    REAL :: dpml0z, dpml0x
125    dpml0z = 3*MAXVAL(vs)/dz*(8.0/15 - 3.0/100*npmlz + 1.0/1500*(npmlz**2))
    DO i = 1,npmlz,1
        dpmlz(i, :) = dpml0z*((REAL(npmlz - i + 1)/npmlz)**2)
    END DO
    dpmlz(npmlz + nz + 1:Nzz, :) = dpmlz(npmlz:1:-1, :)
130    dpml0x = 3*MAXVAL(vs)/dx*(8.0/15 - 3.0/100*npmlx + 1.0/1500*(npmlx**2))
    DO i = 1,npmlx,1
        dpmlx(:, i) = dpml0x*((REAL(npmlx - i + 1)/npmlx)**2)
    END DO

```

```

END DO
dpmlx(:, npmlx + nx + 1:Nxx) = dpmlx(:, npmlx:1:-1)
135 END SUBROUTINE CalDpml
SUBROUTINE CalCoefs()
CALL CalDpml()
Coef1 = (2 - dt*dpmlx)/(2 + dt*dpmlx)
Coef2 = (2 - dt*dpmlz)/(2 + dt*dpmlz)
140 Coef3 = (2*dt/(2 + dt*dpmlx))/rho0/dx
Coef4 = (2*dt/(2 + dt*dpmlz))/rho0/dz
Coef5 = (2*dt/(2 + dt*dpmlx))*(l added + 2*muu)/dx
Coef6 = (2*dt/(2 + dt*dpmlz))*l added/dz
Coef7 = (2*dt/(2 + dt*dpmlx))*l added/dx
145 Coef8 = (2*dt/(2 + dt*dpmlz))*(l added + 2*muu)/dz
Coef9 = (2*dt/(2 + dt*dpmlx))*muu/dx
Coef10 = (2*dt/(2 + dt*dpmlz))*muu/dz
END SUBROUTINE CalCoefs
SUBROUTINE CalWave()
150 INTEGER :: it
INTEGER, PARAMETER :: Nzzz = Nzz + 2*nodr, Nxxx = Nxx + 2*nodr
INTEGER :: znds(nz) = [ (nodr + npmlz + i, i = 1,nz,1) ], &
& xnds(nx) = [ (nodr + npmlx + i, i = 1,nx,1) ]
INTEGER :: Zznds(Nzz) = [ (nodr + i, i = 1,Nzz,1) ], &
& Xxnds(Nxx) = [ (nodr + i, i = 1,Nxx,1) ]
155 INTEGER :: nsrcz = nodr + npmlz + sz, nsrcx = nodr + npmlx + sx
REAL :: vxt(Nzzz, Nxxx, 2) = 0, vxx(Nzzz, Nxxx, 2) = 0, &
& vxz(Nzzz, Nxxx, 2) = 0, vzt(Nzzz, Nxxx, 2) = 0, &
& vzx(Nzzz, Nxxx, 2) = 0, vzz(Nzzz, Nxxx, 2) = 0, &
160 & txxt(Nzzz, Nxxx, 2) = 0, txxx(Nzzz, Nxxx, 2) = 0, &
& txxz(Nzzz, Nxxx, 2) = 0, tzxt(Nzzz, Nxxx, 2) = 0, &
& tzzx(Nzzz, Nxxx, 2) = 0, tzzz(Nzzz, Nxxx, 2) = 0, &
& txzt(Nzzz, Nxxx, 2) = 0, txzx(Nzzz, Nxxx, 2) = 0, &
& txzz(Nzzz, Nxxx, 2) = 0, SpcSum(Nzz, Nxx) = 0
165 DO it = 1,nt,1
WRITE(*,"(A,G0)") 'The calculating time node is: it = ',it
!! load source
txxx(nsrcz, nsrxc, 1) = txxx(nsrcz, nsrxc, 1) + src(it)/4
txxz(nsrcz, nsrxc, 1) = txxz(nsrcz, nsrxc, 1) + src(it)/4
170 tzzx(nsrcz, nsrxc, 1) = tzzx(nsrcz, nsrxc, 1) + src(it)/4
tzzz(nsrcz, nsrxc, 1) = tzzz(nsrcz, nsrxc, 1) + src(it)/4
txxt(:, :, 1) = txxx(:, :, 1) + txxz(:, :, 1)
tzxt(:, :, 1) = tzzx(:, :, 1) + tzzz(:, :, 1)
P(:, :, it) = txxt(znds, xnds, 1);
175 ! P(:, :, it) = tzxt(znds, xnds, 1);
! P(:, :, it) = txzt(znds, xnds, 1);
!! calculate v_x
SpcSum = 0
DO i = 1,nodr,1
180 SpcSum = SpcSum + C(i)*(txxt(Zznds, Xxnds + i - 1, 1) - txxt(Zznds, Xxnds - i, 1))
END DO
vxx(Zznds, Xxnds, 2) = Coef1*vxx(Zznds, Xxnds, 1) + Coef3*SpcSum
SpcSum = 0
DO i = 1,nodr,1
185 SpcSum = SpcSum + C(i)*(txzt(Zznds + i - 1, Xxnds, 1) - txzt(Zznds - i, Xxnds, 1))
END DO
vxz(Zznds, Xxnds, 2) = Coef2*vxz(Zznds, Xxnds, 1) + Coef4*SpcSum
vxt(:, :, 2) = vxx(:, :, 2) + vxz(:, :, 2)
! P(:, :, it) = vxt(znds, xnds, 2)
190 !! calculate v_z
SpcSum = 0
DO i = 1,nodr,1
SpcSum = SpcSum + C(i)*(txzt(Zznds, Xxnds + i, 1) - txzt(Zznds, Xxnds - i + 1, 1))

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```

195      END DO
      vzx(Zznds, Xxnds, 2) = Coef1*vzx(Zznds, Xxnds, 1) + Coef3*SpcSum
      SpcSum = 0
      DO i = 1,nodr,1
          SpcSum = SpcSum + C(i)*(tzzt(Zznds + i, Xxnds, 1) - tzzt(Zznds - i + 1, Xxnds, 1))
      END DO
200      vzz(Zznds, Xxnds, 2) = Coef2*vzz(Zznds, Xxnds, 1) + Coef4*SpcSum
      vzt(:, :, 2) = vzx(:, :, 2) + vzz(:, :, 2)
      !
      P(:, :, it) = vzt(znds, xnds, 2)
      !! calculate tau_{xx} and tau_{zz}
      SpcSum = 0
205      DO i = 1,nodr,1
          SpcSum = SpcSum + C(i)*(vxt(Zznds, Xxnds + i, 2) - vxt(Zznds, Xxnds - i + 1, 2))
      END DO
      txxx(Zznds, Xxnds, 2) = Coef1*txxx(Zznds, Xxnds, 1) + Coef5*SpcSum
      tzzx(Zznds, Xxnds, 2) = Coef1*tzzx(Zznds, Xxnds, 1) + Coef7*SpcSum
210      SpcSum = 0
      DO i = 1,nodr,1
          SpcSum = SpcSum + C(i)*(vzt(Zznds + i - 1, Xxnds, 2) - vzt(Zznds - i, Xxnds, 2))
      END DO
      txxz(Zznds, Xxnds, 2) = Coef2*txxz(Zznds, Xxnds, 1) + Coef6*SpcSum
215      tzzz(Zznds, Xxnds, 2) = Coef1*tzzz(Zznds, Xxnds, 1) + Coef8*SpcSum
      txxt(:, :, 2) = txxx(:, :, 2) + txxz(:, :, 2)
      tzzt(:, :, 2) = tzzx(:, :, 2) + tzzz(:, :, 2)
      !! calculate tau_{xz}
      SpcSum = 0
220      DO i = 1,nodr,1
          SpcSum = SpcSum + C(i)*(vzt(Zznds, Xxnds + i - 1, 2) - vzt(Zznds, Xxnds - i, 2))
      END DO
      txxz(Zznds, Xxnds, 2) = Coef1*txxz(Zznds, Xxnds, 1) + Coef9*SpcSum
      SpcSum = 0
225      DO i = 1,nodr,1
          SpcSum = SpcSum + C(i)*(vxt(Zznds + i, Xxnds, 2) - vxt(Zznds - i + 1, Xxnds, 2))
      END DO
      txzz(Zznds, Xxnds, 2) = Coef2*txzz(Zznds, Xxnds, 1) + Coef0*SpcSum
      txzt(:, :, 2) = txzx(:, :, 2) + txzz(:, :, 2)
230      !! exchange for next cycle
      vxx(:, :, 1) = vxx(:, :, 2)
      vxz(:, :, 1) = vxz(:, :, 2)
      vxt(:, :, 1) = vxt(:, :, 2)
      vzx(:, :, 1) = vzx(:, :, 2)
235      vzz(:, :, 1) = vzz(:, :, 2)
      vzt(:, :, 1) = vzt(:, :, 2)
      txxx(:, :, 1) = txxx(:, :, 2)
      txxz(:, :, 1) = txxz(:, :, 2)
      txxt(:, :, 1) = txxt(:, :, 2)
240      tzzx(:, :, 1) = tzzx(:, :, 2)
      tzzz(:, :, 1) = tzzz(:, :, 2)
      tzzt(:, :, 1) = tzzt(:, :, 2)
      txzx(:, :, 1) = txzx(:, :, 2)
      txzz(:, :, 1) = txzz(:, :, 2)
245      txzt(:, :, 1) = txzt(:, :, 2)
      END DO
      END SUBROUTINE CalWave

END MODULE WaveExtrp
250
!***** TDFDEWFS2DSG *****
! Time Domain Finite Difference Elastic Wave Field Simulating with 2-Dimension Staggered Grid
! Written by Tche. L. from USTC, 2016,7
! References:

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255 ! Collino and Tsogka, 2001. Geophysics, Application of the perfectly matched absorbing layer model
      ! to the linear elastodynamic problem in anisotropic heterogeneous media.
      ! Marcinkovich and Olsen, 2003. Journal of Geophysical Research, On the implementation of perfectly
      ! matched layers in a three-dimensional fourth-order velocity-stress finite difference scheme.
      ! *****
PROGRAM TDFDEWFS2DSG

260  USE InputPara
      USE WaveExtrp
      IMPLICIT NONE

      CHARACTER(LEN = 128) :: SnapFile = './Snapshot/Snapshot_***.dat' ! the snapshot file name
      template.
265  CHARACTER(LEN = 128) :: SyntFile = 'SyntRcrd.dat' ! the synthetic record file
      name.
      REAL :: SyntR(nrcvr, nt)
      INTEGER :: i

      CALL IntlzInputPara()
270  CALL WaveExec()
      DO i = itstr, nt, ntintv
          WRITE(SnapFile(21:24), "(I4.4)") i
          CALL Output(TRIM(SnapFile), nz, nx, P(:, :, i))
      END DO
275  DO i = 1, nt, 1
          SyntR(:, i) = P(1, xrcvr, i)
      END DO
      CALL Output(TRIM(SyntFile), nrcvr, nt, SyntR)

280  END PROGRAM TDFDEWFS2DSG

SUBROUTINE Output(Outfile, M, N, OutA)
      IMPLICIT NONE
      CHARACTER(LEN = *) , INTENT(IN) :: Outfile
285  INTEGER, INTENT(IN) :: M, N
      REAL, INTENT(IN) :: OutA(M, N)
      CHARACTER(LEN = 40) :: FmtStr
      INTEGER :: i, j
      INTEGER :: funit
290  WRITE(FmtStr, "((' ', G0, 'E15.6')'") N
      OPEN(NEWUNIT = funit, FILE = Outfile, STATUS = 'UNKNOWN')
      DO i = 1, M, 1
          WRITE(funit, FmtStr) (OutA(i, j), j = 1, N, 1)
      END DO
295  CLOSE(funit)
END SUBROUTINE Output

```