The Russian cards problem

Project report

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The Russian Cards problem is a logic problem. In the original version of the problem, first presented at the 2000 Moscow Mathematics Olympiad, seven cards are marked '0' to '6'. Agent A and B are both dealt three cards at random, while agent C is dealt a single card. Agent A must communicate to agent B what cards she has, and vice versa, without agent C gaining knowledge of whether some specific card is held by A or B. Some solutions to the problem are mathematical in nature, but solutions exclusively based on announcements and higher-order logic exist as well.

The five-, six-, and seven-hand solutions are means of solving the original problem that all share the same broad structure. These solutions rely on agent A providing a number of alternative hands, one of which is her own, in a way that guarantees at least two of those hands comply with the information available to C, but at most one hand complies with the information available to B. After this announcement B will have gained knowledge of A's hand, while C still considers two separate models possible.

Our research question

We would like to explore variants of the Russian cards problem where the agents hold different numbers of cards than in the original (3,3,1) version. We are interested in variants where C holds more than one card (3,3,2), where all agents hold a larger or smaller number of cards (4,4,1), (2,2,1), or where the three agents all receive different amounts of cards (3,2,1). Our central research question is:

> How many solutions exist to the Russian cards problem for varying distributions of cards over the three players?

We will construct Kripke models showcasing these variants, and determine what strategies from the original game still apply, with a focus on the n-hand solution strategies.

Looking at the literature on this topic, we find a generalization of the Russian cards problem to n players and $3(n-1)+1$ cards, implying the default distribution of three cards for each player except the observer, who receives one. We would instead explore permutations where the number of players is fixed, but the amount of cards each receives is different, which we deem different enough.

Implementation

The program will take as input the number of players, the number of cards and the number of cards that agent C has. It will then compute all possible *runs*, i.e. consecutive public announcements, by the other agents.

Relevance to multi-agent systems

The Russian card problem contains complicated epistemic situations where the knowledge that agents have of other agents' knowledge plays a key role in determining what models they consider accessible. A more general version of the Russian card problem could be the basis for cryptographic protocols in multi-agent systems.

This project could in general be relevant for situations in multi-agent systems where all agents need to communicate and combine their pieces of information, without some intruder learning this information.

Literature

- Hans van Ditmarsch - The Russian Card problem (original publication)
- A Zero-Error Source Coding Solution to the Russian Cards Problem
- Regular Model Checking Approach to Knowledge Reasoning over Parameterized Systems