Paper Helicopter Flight Time Optimization

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Introduction

In order to determine the ideal wing dimensions to maximize the flight time of paper helicopters, experiments were conducted using a 2^2 factorial design, with wing width and wing length as the factors. Four replicates were used unless otherwise noted, with two $\frac{3}{4}$" binder clips added for weight.

We found flight times at heights that could be achieved in the classroom were short enough that the natural variation in our response times while operating a stopwatch were large compared to the flight times themselves. In order to minimize the effect of such variability, we chose the stairwell in the Communications Facility building as the location for our testing. All helicopters were dropped from the first landing below the 4^{th} floor.

ANOVA Analysis of 2^2 Factorial Design

Preliminary testing suggested that very low levels for length or width resulted in much faster descents than levels near the middle-to-upper portion of the range of possible values within the given design constraints. For this reason, we chose to exclude dimensions less than 1.75". We chose 2.25"and 3.75" as the low and high levels for Factor A (wing width) and 2.25" and 4.20" as the low and high levels of B, which gave us $d_0 = (3.00, 3.23)$. Our design space is illustrated in Figures 1 and 2 below.

Table 4 gives the mean times for helicopters d_1 through d_4, with d_4 appearing to have the longest flight time.

Design	d_1	d_2	d_3	d_4
Mean Flight Time (seconds)	5.66	8.07	7.86	10.46

Table 1: Mean flight times for first round of testing.

The ANOVA table from our 2^2 factorial analysis is duplicated in Figure 3.

Length and width are both significant, with p-values ≈ 0.000. The analysis suggests that interaction may not be significant, with a p-value of 0.197. Figure 4 provides the interaction plot and is consistent with this conclusion. However, it is noted that interaction appeared be significant in some of the preliminary testing.

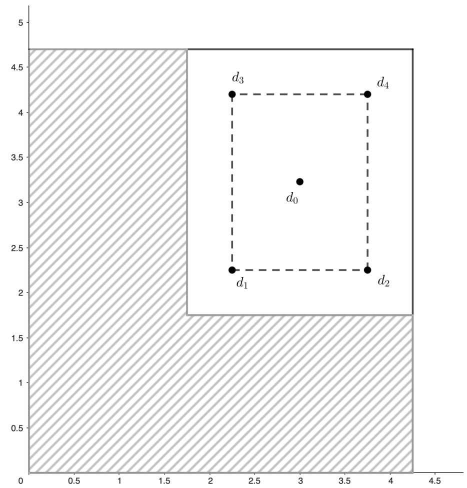

Figure 1: The maximal wing dimensions allowed within the given constraints is 4.25" by 4.70". Dimensions below 1.75", represented by the hatched area, were excluded based on preliminary testing.

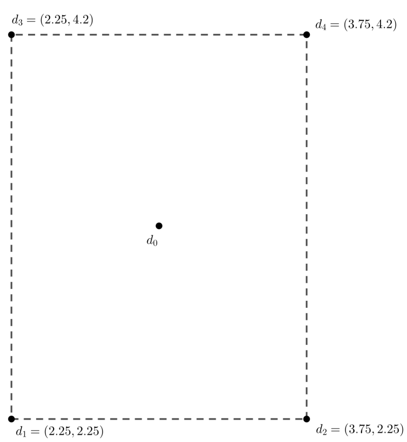

Figure 2: Initial 2^2 factorial design levels

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	3	46.2002	15.4001	797.41	0.000
Linear	2	46.1641	23.0820	1195.19	0.000
Width	1	25.0500	25.0500	1297.09	0.000
Length	1	21.1140	21.1140	1093.28	0.000
2-Way Interactions	1	0.0361	0.0361	1.87	0.197
Width*Length	1	0.0361	0.0361	1.87	0.197
Error	12	0.2318	0.0193		
Total	15	46.4319			

Figure 3: Minitab ANOVA Table for 2^2 Factorial Analysis

The normal probability plot for the residuals is given in Figure 5. While there are some slight irregularities, no major departures from normality can be observed.

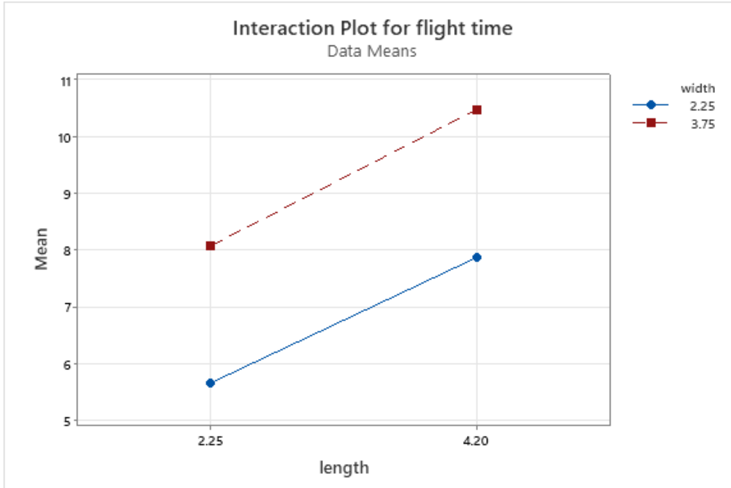

Figure 4: Interaction Plot for Width and Length. No prominent interaction is visible.

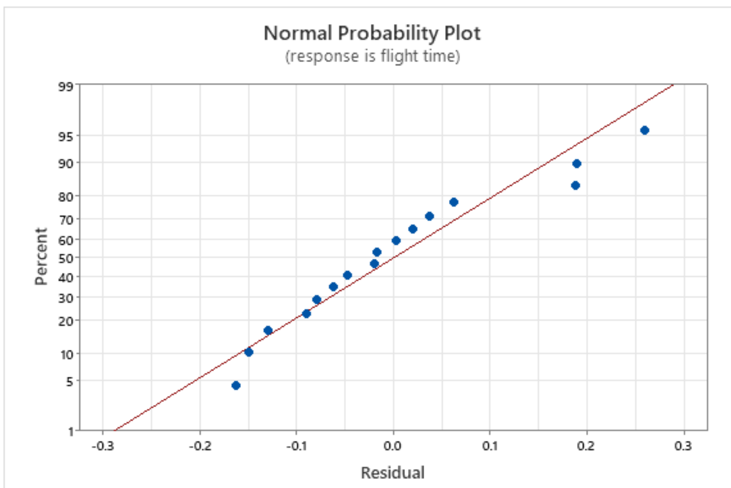

Figure 5: Normal Probability Plot for Residuals

For checking the assumption of constant variance, the plot of the absolute value of the residuals versus the fitted values is shown in Figure 6. The scatter appears to be randomly distributed and therefore our model assumptions are satisfied.

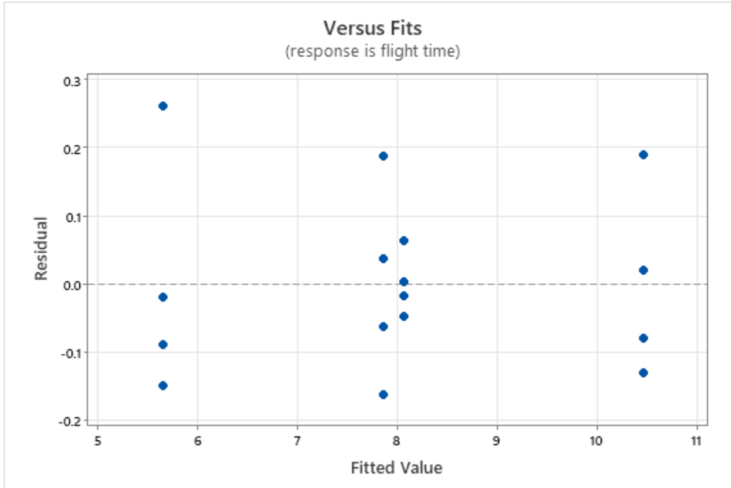

Figure 6: Residuals Versus Fitted Values

Fisher's LSD method shows that $d_1 < d_2 = d_3 < d_4$, confirming our initial impression that flight times for d_4 were longer than the others.

Search for Optimal Design

In order to find the optimal wing design, we performed a regression analysis as well. Since some of our preliminary testing suggested interaction was significant, we opted to include the interaction term in our regression model, although we calculated both for comparison. With interaction, we obtained the model

$$y = -0.164 + 1.459x_1 + 0.983x_2 + 0.0650x_1x_2$$

where x_1 represents width (as defined in the project handout) and x_2 represents length. The gradient of y is

$$\nabla y = (1.459 + 0.0650x_2, 0.983 + 0.0650x_1)$$

with

$$\begin{aligned}\nabla y(d_0) &= (1.459 + 0.0650(3.23), 0.983 + 0.0650(3.00)) \\ &= (1.67, 1.17).\end{aligned}$$

Calculation of the gradient for the regression model without the interaction term also yielded $(1.67, 1.17)$.

Using the gradient, we located three additional wing dimensions for testing. Since we left only $\frac{1}{2}$ inch of space above and to the right of our high levels in our 2^2 design, we opted to include one point d_5 with dimensions less than the high levels of A and B, with the expectation that the data points obtained from d_5 would be useful in analyzing our data later. We chose $d_5 = (3.50, 3.57)$, $d_6 = (4.00, 3.92)$, and $d_7 = (4.25, 4.10)$. These design points are illustrated in the design space in Figure 7, along with the gradient vector.

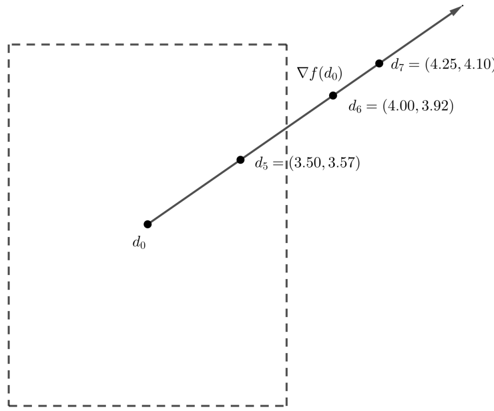

Figure 7: Design space with updated design points based on gradient analysis

Additional helicopters were built based on these new design points and tested in the same location. Unfortunately, these helicopters did not perform as expected, with all new designs having lower mean flight times than d_4 (see Table 2).

Design	d_5	d_6	d_7
Mean Flight Time (seconds)	9.05	8.80	9.29

Table 2: Mean flight times for second round of testing.

Our analysis suggests that interaction is not significant, with the p-value of 0.629 for the interaction term. This is dropped, giving us the second order regression model

$$y = -5.41 + 12.61x_1 - 6.15x_2 - 1.825x_1^2 + 1.136x_2^2.$$

We find some model inadequacy problems here. The variance is not constant, with greater variance for d_5 through d_7 than d_1 through d_4, as seen in Figure 8. Furthermore, the p-value for the Anderson-Darling test is less than 0.005, suggesting that the standardized residuals are not normally distributed.

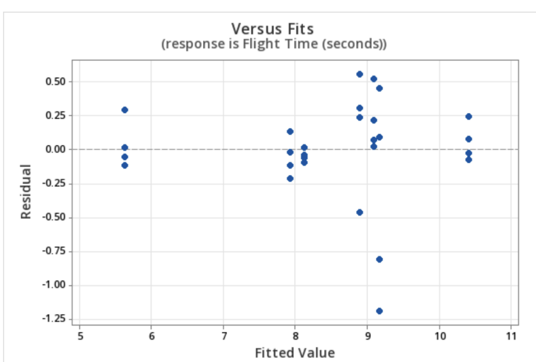

Figure 8: The residual plot for our 2nd order regression model suggests nonconstant variance.

Using our new regression model, we plotted a response surface (see Figure 9) and located its highest point. Visually, this appeared to be around 11.25 seconds. However, looking at our mesh data, the point $x_1 = 3.42$ and $x_2 = 4.70$ corresponds to 12.56 seconds. (This can be verified by plugging these values into the 2^{nd} order regression model above.)

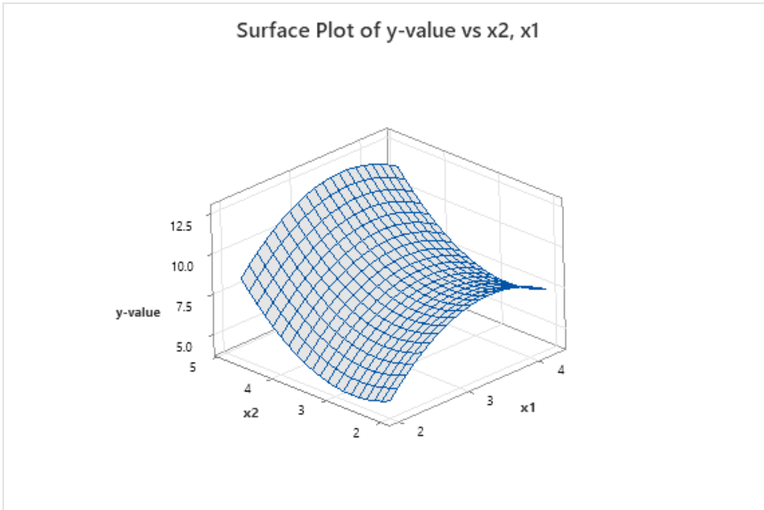

Figure 9: The peak value in our surface plot is 12.56 at $(3.42, 4.70)$.

Using these values, we constructed our final helicopter $d_{max} = (3.42, 4.70)$.

We obtained three "good" flights with d_{max} but were unable to get the wings to open on subsequent replicates. Using only the three good replicates, we obtained a mean of 9.01. Unfortunately, we were unable to run a Fisher's LSD comparison. We repeatedly obtained an error from our general linear model noting that Length and Width*Length could not be estimated and were excluded from the model. However, it is clear our model was unable to maximize flight time, as 9.01 is significantly less than the predicted mean of 12.56 from our regression model as well as the mean of d_4 from our first round of testing.

As an alternative, we generated a new regression model that included data from all helicopters and calculated 95% confidence intervals on the mean flight time for each design. This method suggests the mean flight time for d_{max} is equal to those for d_4, d_6, and d_7.

Conclusion

Clearly, our experimental model was unable to maximize flight time. It's unclear exactly what went wrong. Our initial regression model after the first round of testing appeared robust, passing the model adequacy tests.

However, as noted in the discussion above, our second order model after our second round of testing clearly had inadequacies, failing the tests for constant variance and normally distributed residuals. Because of this, it was unsurprising that d_{max} did not meet its predicted flight time. During testing of d_{max}, we noticed that the helicopter wobbled a bit as it descended. This had happened previously in some of our preliminary testing and seemed to be at least somewhat related to the weight of the helicopter. This is a factor we played with informally in our preliminary testing and we found two binder clips seemed optimal, with one clip tending to result in models that listed as they fell and three clips causing them to drop like rocks.

There was probably some listing happening with models d_6 and d_7 that may have been the cause of our inconstant variance. This was something we struggled with because our initial models did not fly well with additional weight, but our "optimized" models seemed to need more.

If we were to repeat this experiment, we would consider a 2^3 design with weight as a third factor. We would also use lighter clips in order to have more flexibility for design modification.

Another factor we discussed was the construction material. We might consider a thicker, more rigid paper for our helicopters which we suspect might have helped d_{max} to fly better.