WAVELET COMPRESSION IN THE CONTEXT OF GRAVITY INVERSION

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Introduction

3D inversions require many computations. An increase in the amount and variety of data available to geophysicist has led to an increased interest in multiparameter inversions, an efficient compression scheme is required to decrease memory requirements. Decreasing memory requirements allows more models to be run on local desktops instead of supercomputers, which can be a significant hurdle in scientific research.

Tomofast-X uses wavelet compression to decrease the size of the sensitivity matrix in geophysical inversions. The goal of this research is to quantify the effect model size and compression rate have on an unconstrained inversion.

Wavelet Compression

Wavelet compression has a rich history in signal processing, computational mathematics, and geophysics starting in the 1980's (Daubechies 1992). There are two popular wavelets used in inversions, Daubichen4 and Haar, both included in Tomofast-X. This research uses the Haar wavelet.

Wavelet Compression Steps:

1. Transform sensitivity matrix, $\boldsymbol{S}$, row by row into the wavelet domain
2. Sort coefficients in decreasing order
3. Retain the first n coefficients determined by the compression rate, c_r, and discard the remaining coefficients
4. Calculate relative error
5. Transform sparse sensitivity matrix, $\boldsymbol{S}_r$, back into spatial domain and calculate final compression error c_e

$$c_r = \frac{n\ compressed}{n\ elements}$$

$$c_e = \frac{1}{n}\sum_{i=0}^{n}\frac{\|\boldsymbol{S}_i - \boldsymbol{S}_{r,i}\|}{\|\boldsymbol{S}_i\|}$$

After all rows of the model have been compressed the final compression error is the average relative error (Li and Oldenburg 2003, Martin et al. 2013). The compression rate is user defined and is difficult to determine without running multiple inversions over different rates.

Methodology

Generate 10 models → Run inversions varying compression rates → Determine rate required for error levels → Rerun at new Rates → Fit power law data

Model Generation

- 10 different models are generated with nCells ranging from 500,000 to 12 million
- Cell dimensions of 500m x 500m x500m
- nx=ny, nz=30 for all models
- Data is interpolated from the 2019 Australian National Gravity Grids (Lane et al. 2020) and gridded centrally onto of surface model cells
- Each model includes padding equal to the depth on either side

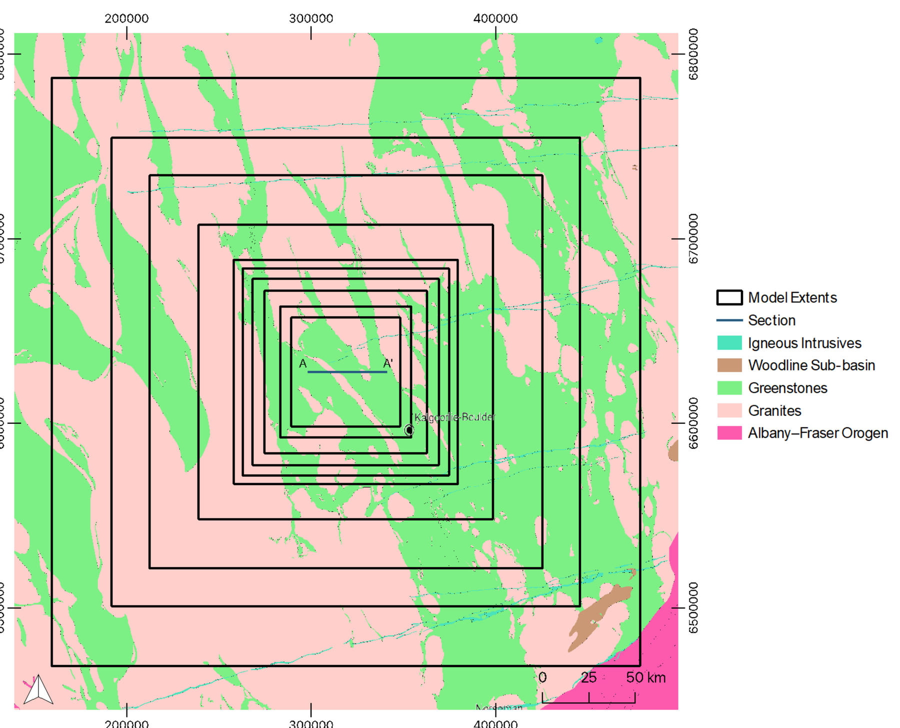


Figure 1: Model extents within the Yilgarn Craton overlaid by the 1:500K tectonic map of western Australia (GSWA,2022). Section A is the slice used in Figure 4.

Inversions

- For each model 8 different inversions are initially run using compression rates between 10^{-4} (very compressed) and 1.0 (no compression)
- Interpolating from the compression error of these 8 inversions the compression rate is calculated to obtain an error of 0.5,1, 3, 5, and 7%
- 5 additional inversions are run for each model size using the interpolated compression rates
- Each inversion is run on UWA HPC kaya using 36 CPUs

Results

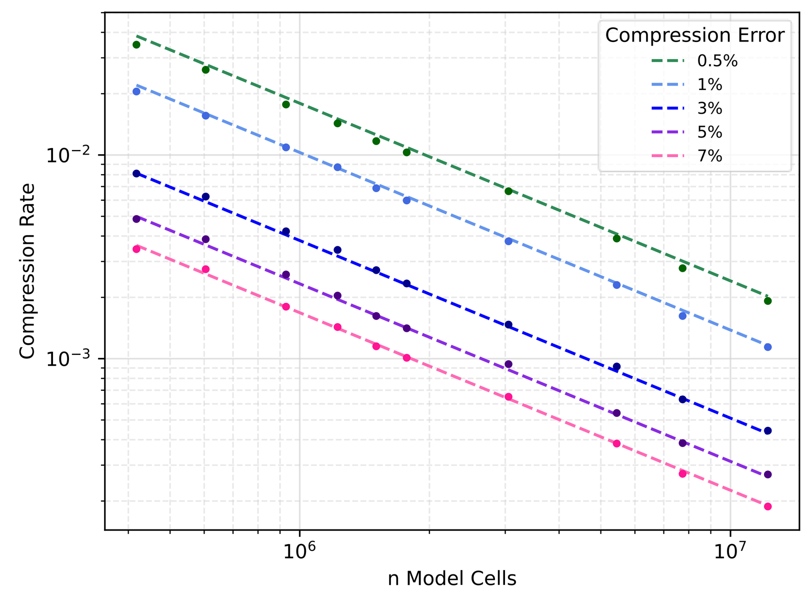
For most models, a maximum compression error of 1% is required to decrease visual artifacts in the final model.

At 7% compression error the distribution of densities remains accurate but appears overly distorted

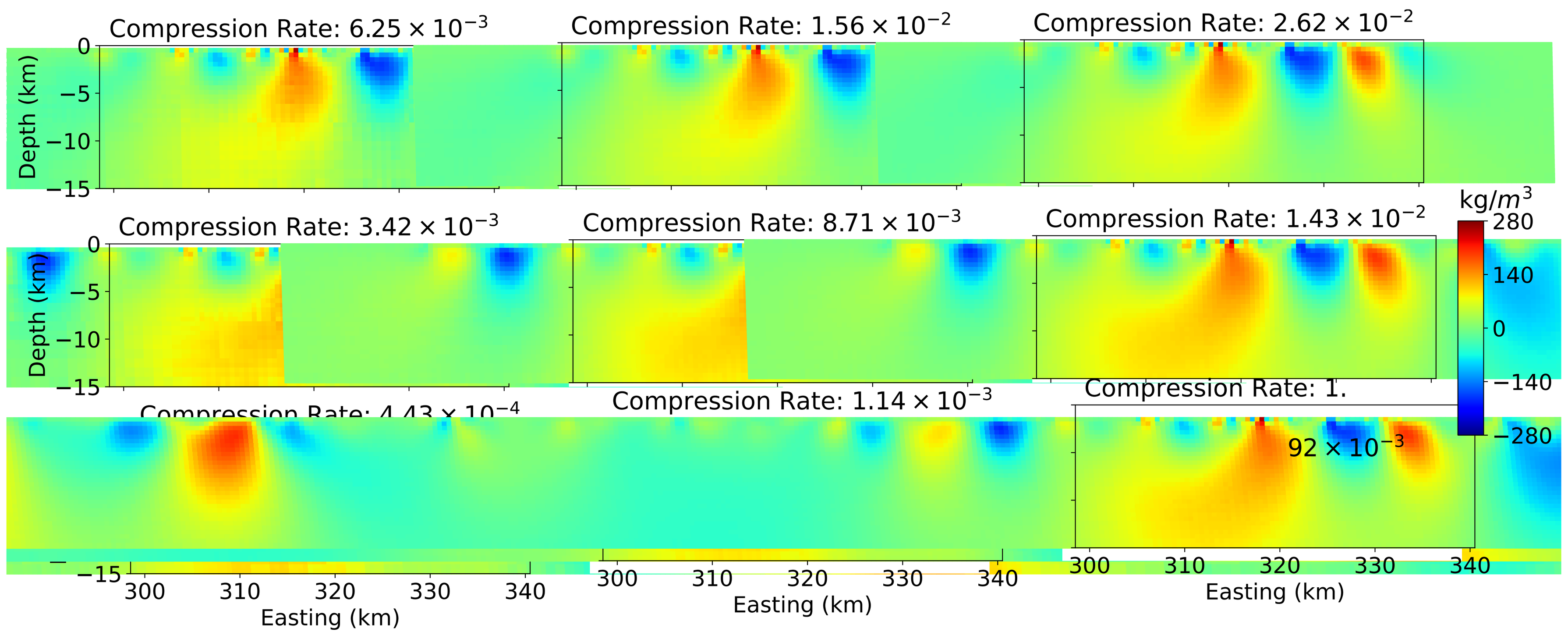
$$c_r(c_e, n) = \frac{1}{(a_1 + a_2 c_e)n^b}$$

c_r: compression rate	$a_1 = 4.86 \times 10^{-4}$
c_e: compression error	$a_2 = 8.36 \times 10^{-5}$
n: number of model cells	$b = 0.872$

Figure 2: Compression rate vs model size for each compression error..



A section of three different models increasing top to bottom (n=0.6 million, 1.2 million, and 12 million cells) are shown below, over the section A-A' in Figure 1. From left to right the compression error level decreases (3,1, and 0.5%)



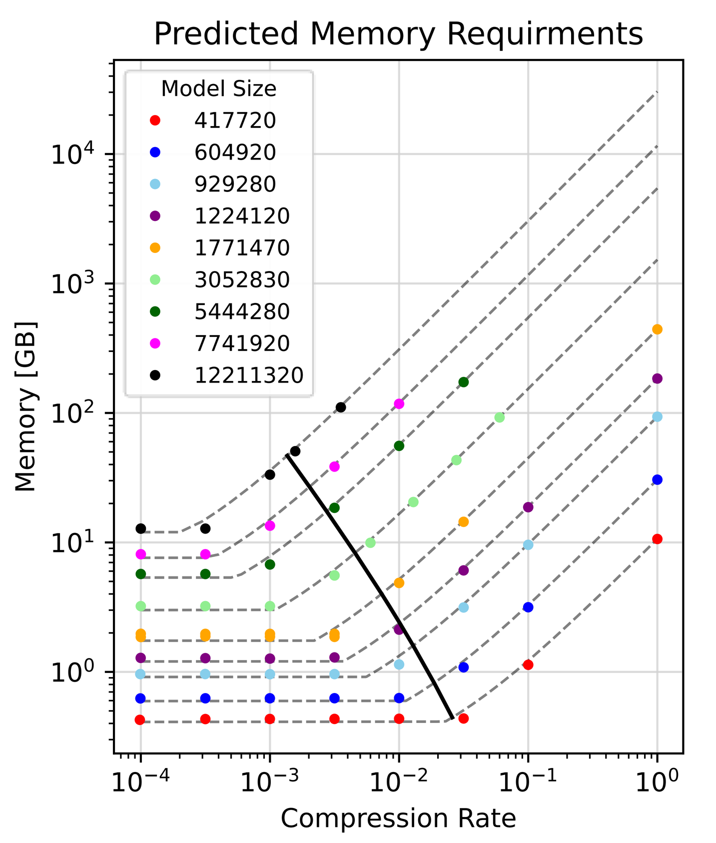


Figure 4: Memory requirements for different model sizes and compression rates. Black line indicates the 1% compression error.

Conclusion

The larger the size of the model the smaller compression rate is required to achieve the compression error.

When the memory required to calculate the sensitivity matrix eclipses the memory required to run the inversion increasing the compression rate no longer affects the memory (horizontal grey segments in predicted memory, Figure 4).

By creating an equation that will predicted the compression rate given a desired compression error and model size we have removed some of the guess work that accompanies model compression.

The benefit of using wavelet compression is that is allows models to be run that previously required too much memory.

Although this research focused on unconstrained gravity inversions, we have also investigated using unconstrained magnetic inversions.

In addition to Haar wavelets, we also explored Daubechies wavelets which performed better, but costs more in time.

References

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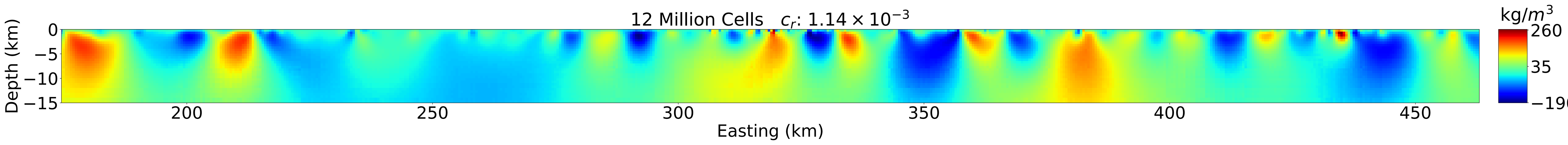

Figure 5: extent of center x slice from the12 million unconstrained gravity inversion. With compression error of 1%, compression rate 1.14×10^{-3}. This inversion was computed using 36 CPUs, used 37 GB of memory and had a run time of 22 hours.

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