



Australia's National Science Agency

# Spatial error constraints reduce overfitting for potential field geophysical inversion

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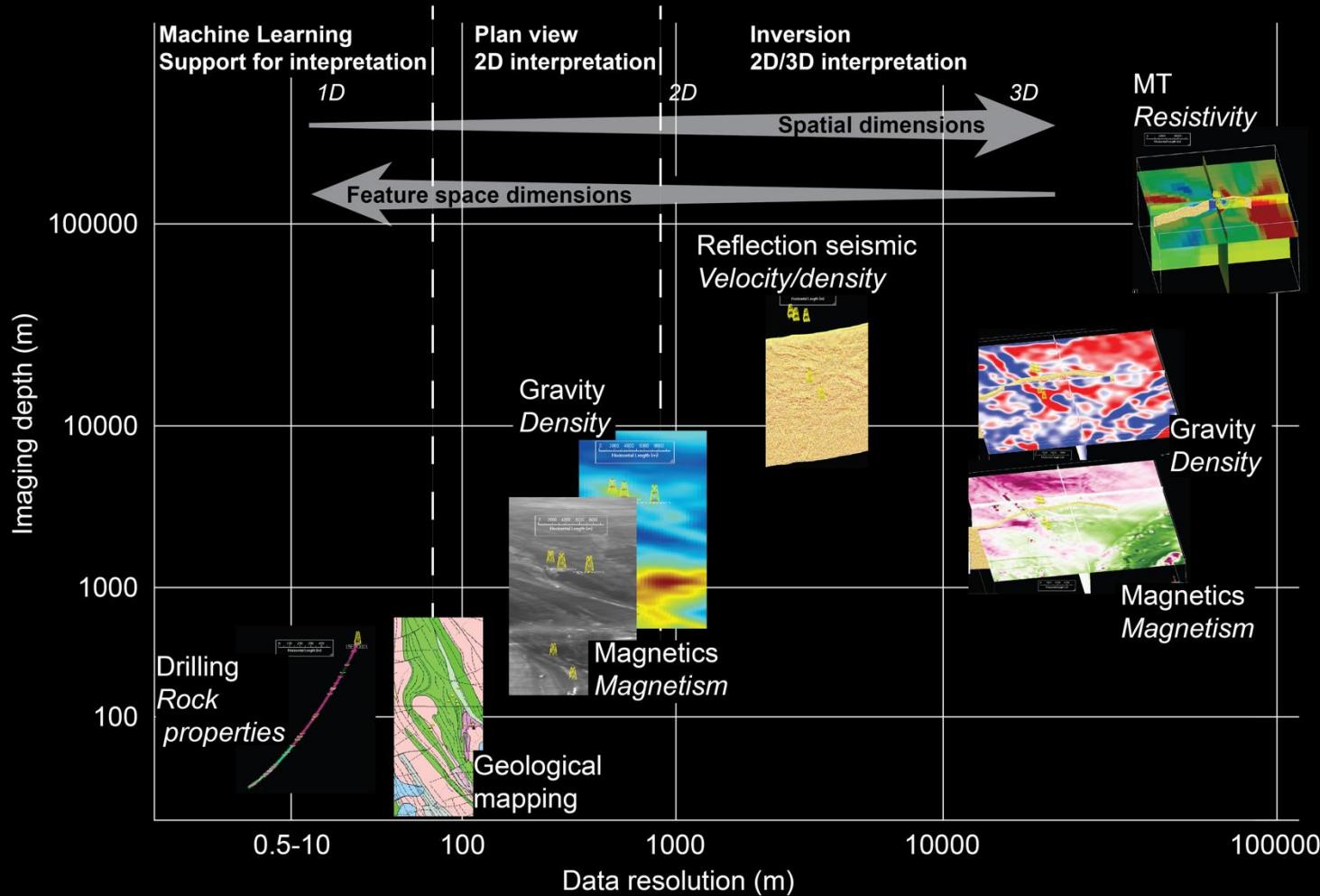
3D Geological Modelling Conference  
Fremantle, 8 & 9 April 2025



I would like to begin by acknowledging the lands of the Whadjuk people of the Nyoongar nation, and pay our respect to their Elders past and present.



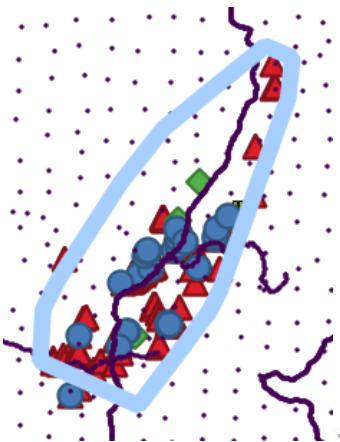
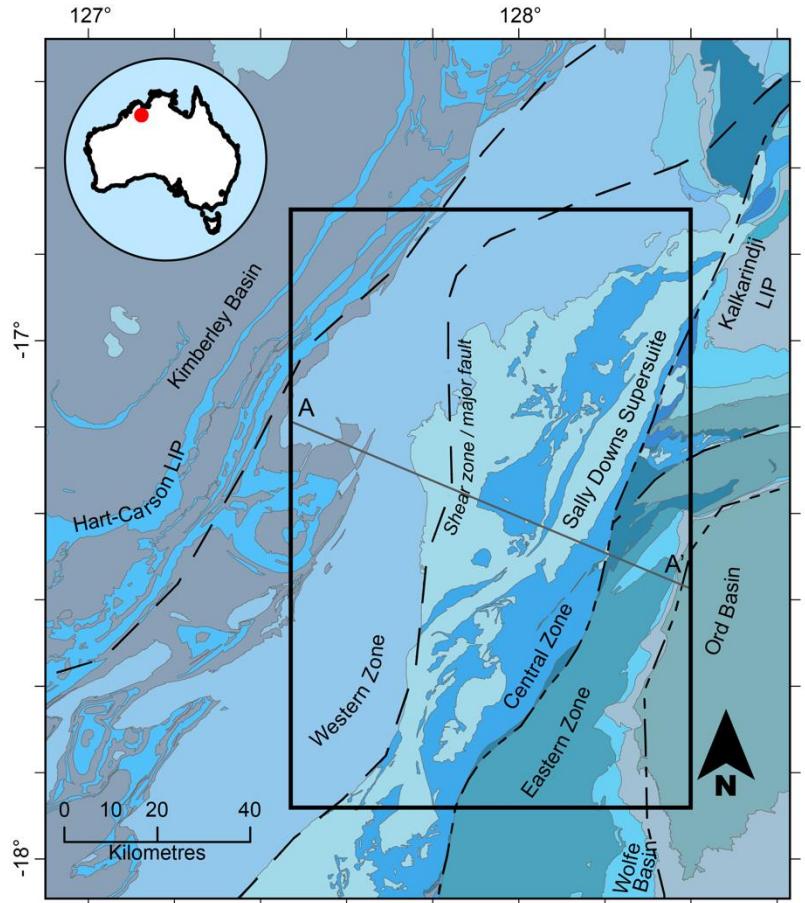
# Scale, Data, Dimensions



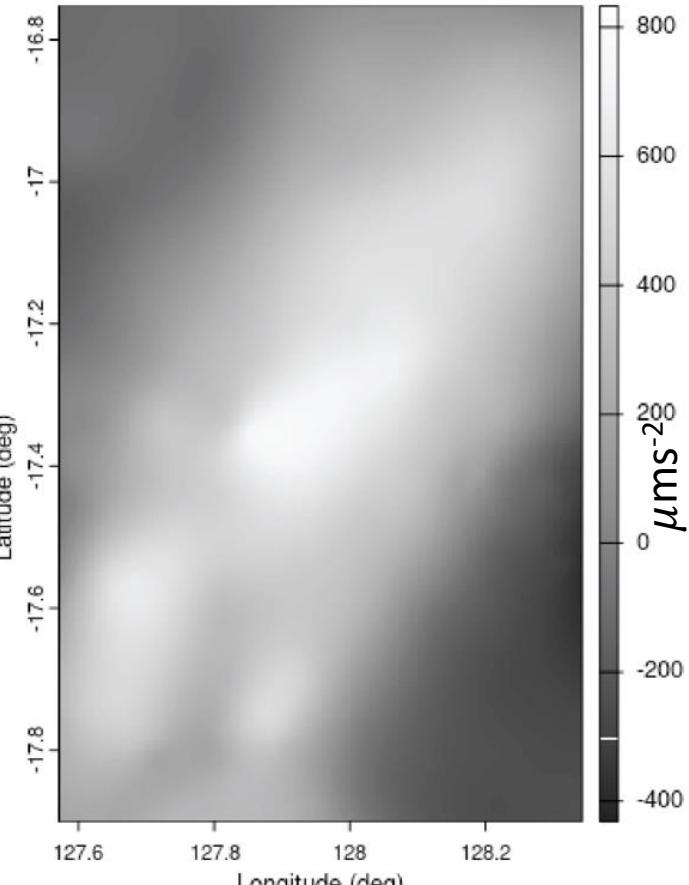
# Uncertainty in geological modelling

- Integration of different data
- Models are variable
- Inconsistencies
- Variability leads to uncertainty
- Exclusion of data
- Inclusion of interpretations

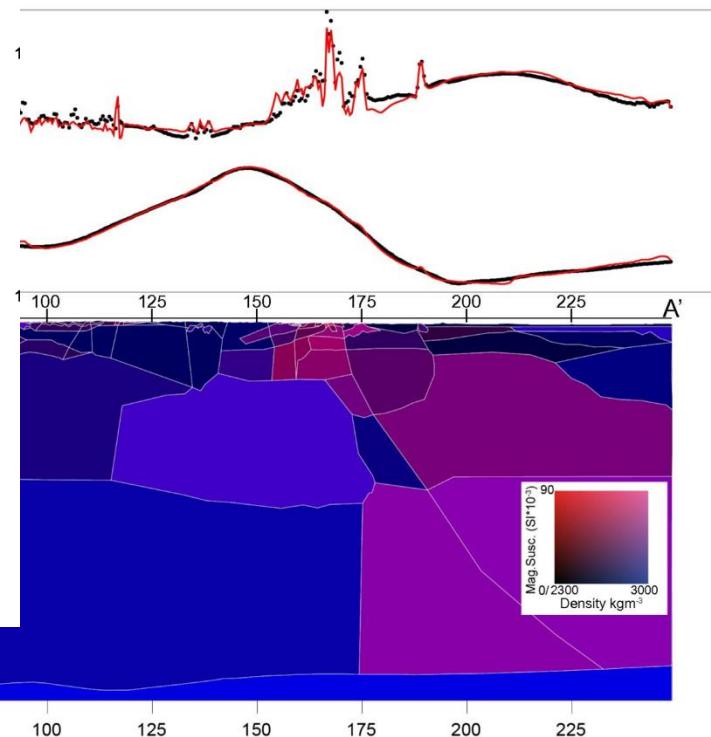
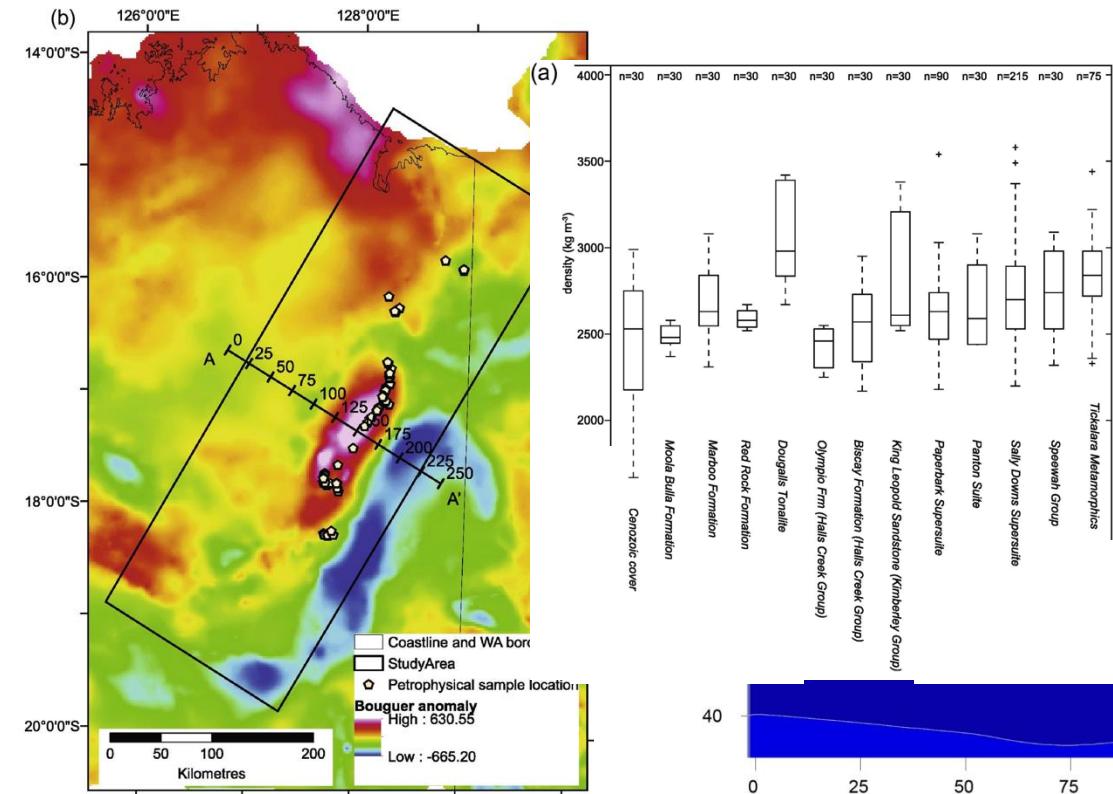
## Bouguer gravity anomaly



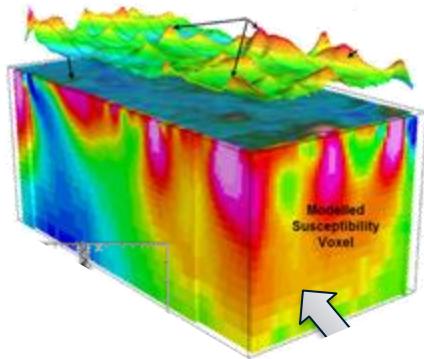
- Co
- ◆ V
- ▲ Ni
- + REE



# Forward modelling



# Geophysical inversion



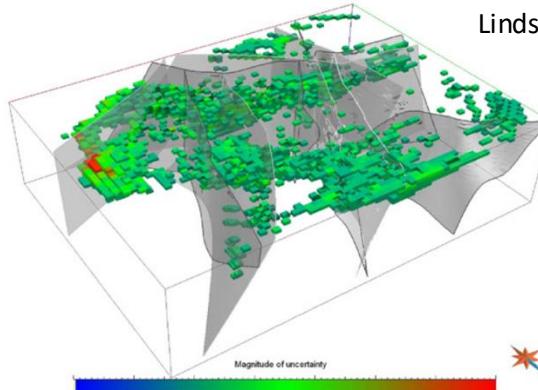
magnetic susceptibility ( $\text{SI} \cdot \text{m}^{-1}$ )

Thanks to Jeremie Giraud

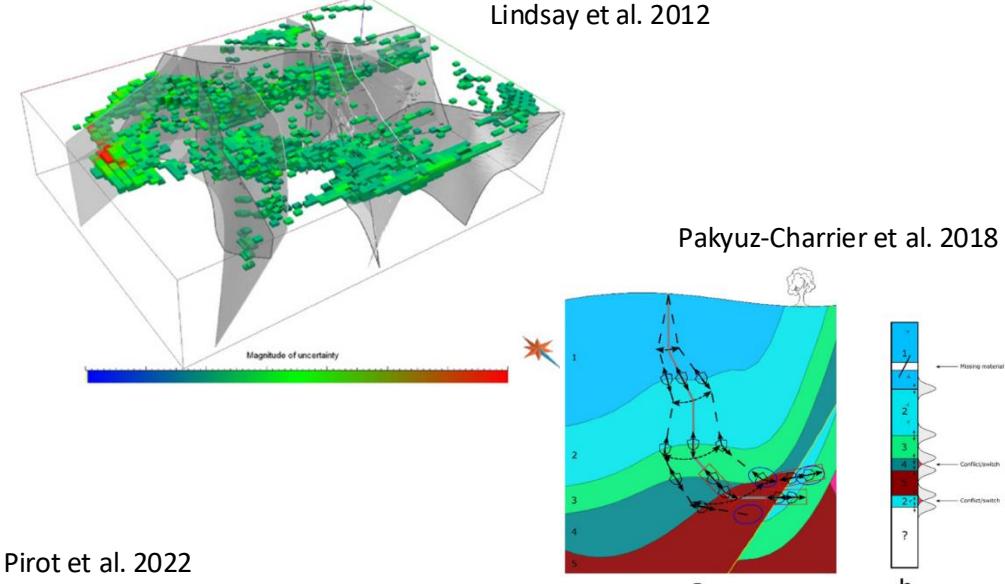


# Previous work

- Geological model uncertainty
  - Generating ensembles varying orientation measurements
- Improved statistical treatment
- Improved perturbation schemes
- Topological variations

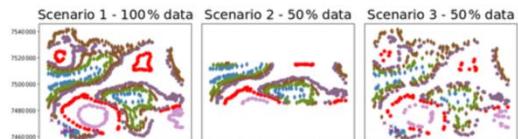


Lindsay et al. 2012

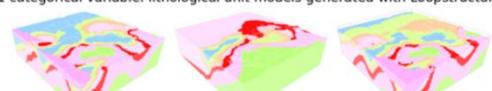


Pakyuz-Charrier et al. 2018

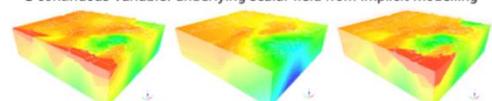
Pirot et al. 2022



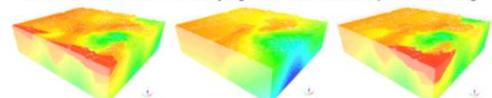
1 categorical variable: lithological unit models generated with Loopstructural



1 continuous variable: underlying scalar-field from implicit modelling

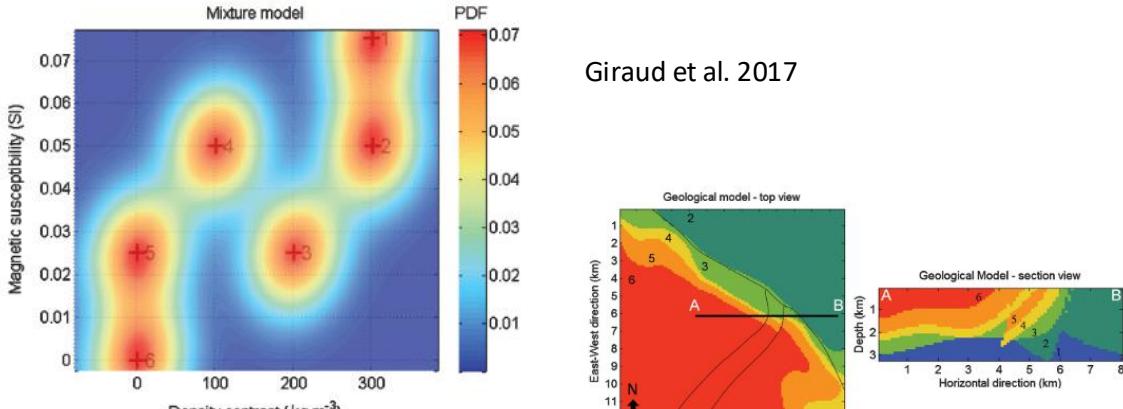


Thiele et al. 2016



# Previous work

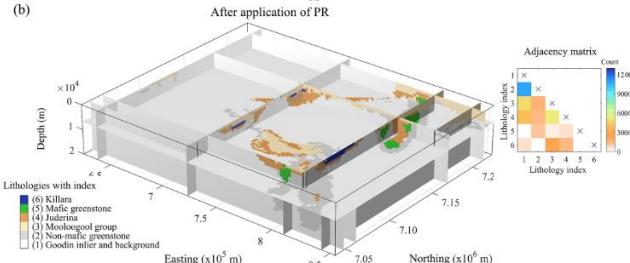
- Geophysical inversion



- Petrophysical uncertainty

- Geological model uncertainty

- Geological plausibility



Giraud et al. 2020

Goodin inlier and background							
		0	0	0	0	0	0
Felsic greenstone	0	0	0	0	0	0	0
	0	0	0	0	0	0	0
Mooloogool group	0	0	0	0	0	0	0
	1	0	0	0	0	0	0
Juderina formation	0	0	0	0	0	0	0
	0	1	0	0	0	0	0
Mafic greenstone	0	0	0	0	0	0	0
	0	0	1	0	0	0	0
Killara	0	0	0	0	0	0	0
	0	0	0	0	0	1	0

Goodin inlier and background

Felsic greenstone

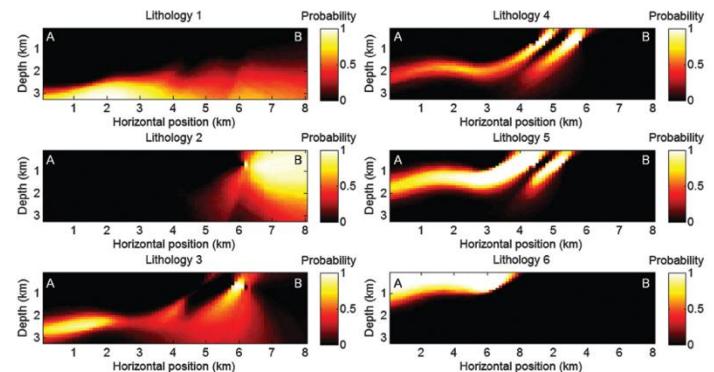
Mooloogool group

Juderina formation

Mafic greenstone

Killara

This table provides the adjacency matrix for the geological model, showing connections between different lithologies (1-6) across the domain. The matrix is symmetric, indicating bidirectional connections.



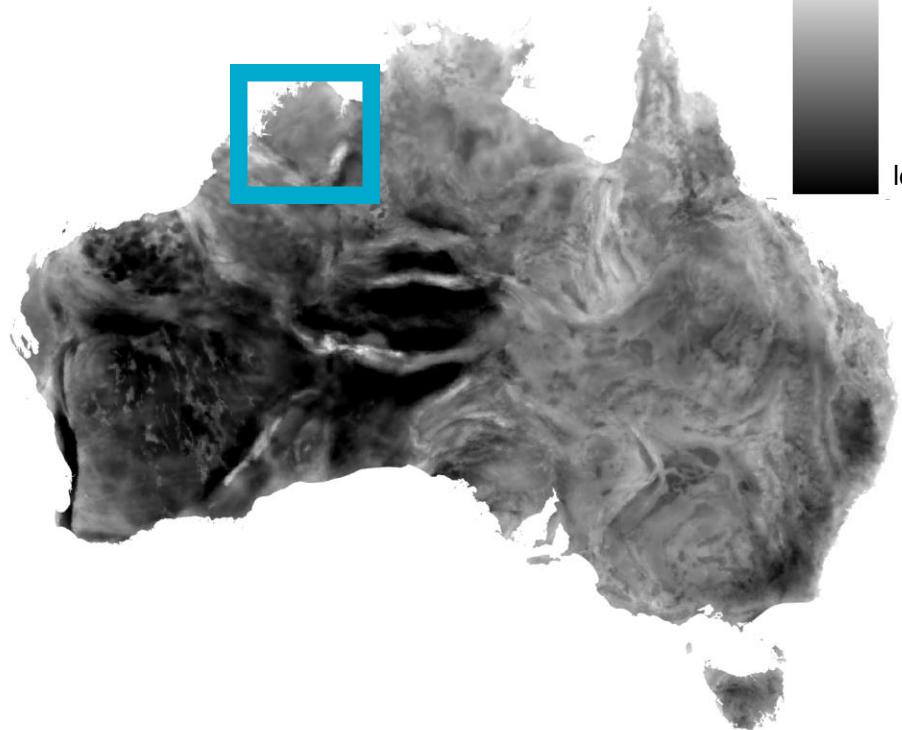
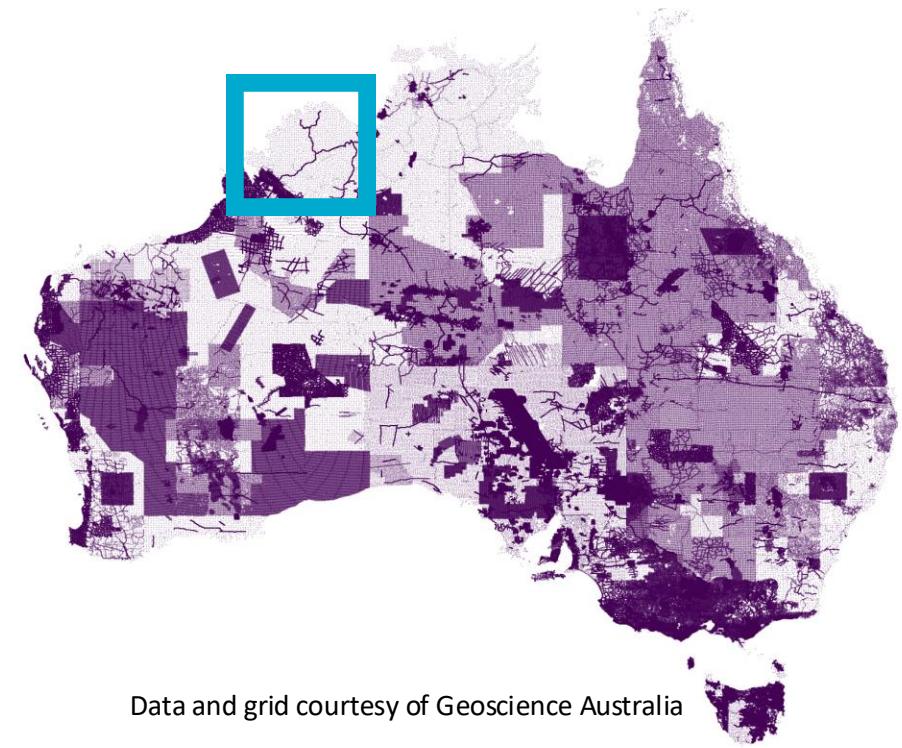
# An Australian gravity model and its data

~1.6 M data (continental scale compilation)

Gravitational acceleration ('gravity')

high

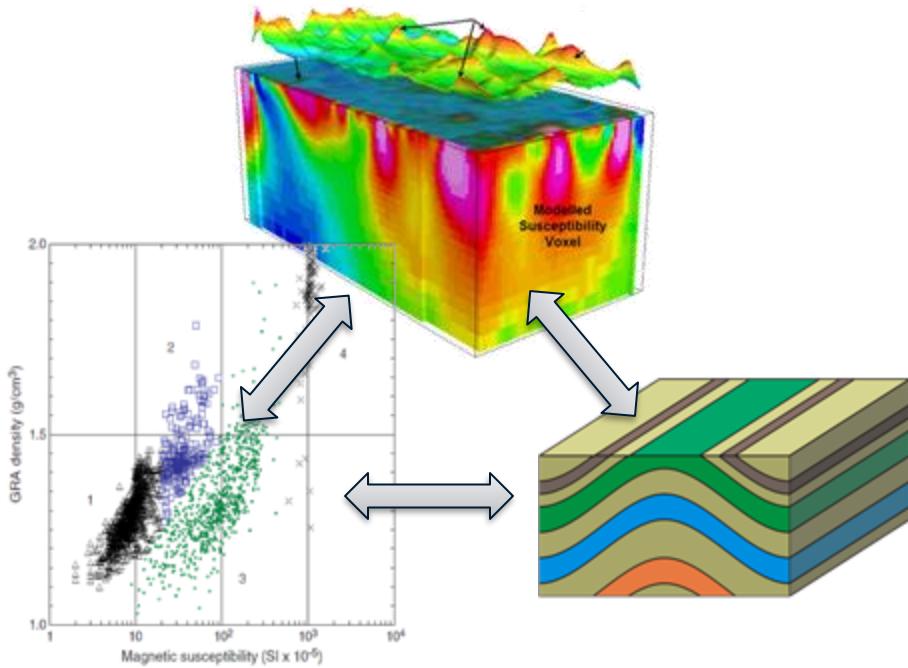
low



Data and grid courtesy of Geoscience Australia

# Why is this important?

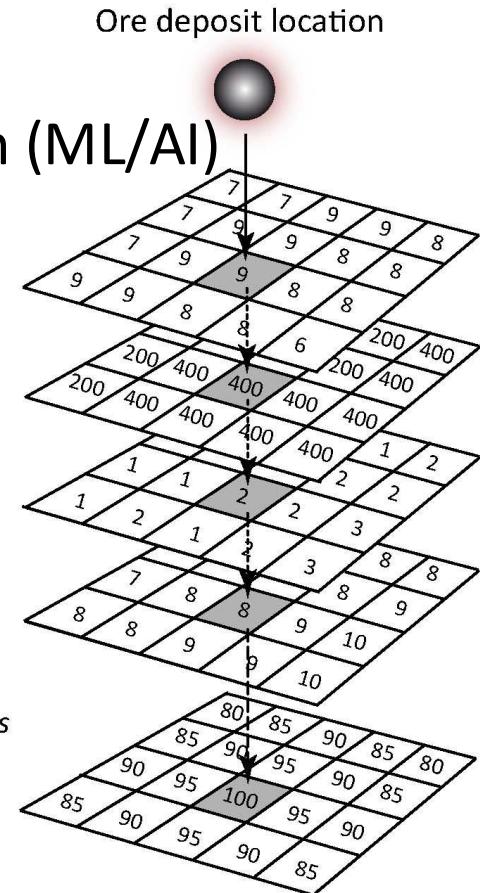
- Inversion



- Data fusion (ML/AI)

Lithology  
+  
Distance  
to faults  
+  
Magnetic  
response  
+  
Gravity  
response  
↓  
*Calculate Weights*  
=

Posterior  
probability



Thanks to Jeremie Giraud

Lindsay et al 2014

# Modelling

- INLA
- Integrated Nested Laplacian Approximation
- Rue et al 2009, r-inla.org
- Performs approximate Bayesian inference in latent Gaussian models
- Generalised linear mixed models, **spatial**, spatio-temporal models
- SPDE (Stochastic Partial Differential Equation)

From Cressie et al 2021 using SST (Sea Surface Temp data)

Table 1 Comparison of spatial predictions of SST residuals in the Brazil-Malvinas confluence zone using the five R packages, FRK, INLA, LatticeKrig, mgcv, and gstat, and using a C++ implementation of MRA provided by Huang et al. (2019)

Method	RMSPE	COV90	IS90	CRPS	Run time (minutes)
FRK	0.46	0.88	2.21	0.23	8.10
INLA	0.46	0.93	2.18	0.24	0.86
LatticeKrig	0.45	0.91	2.09	0.23	9.63
mgcv	0.45	0.90	2.12	0.23	5.73
MRA	0.44	0.92	2.06	0.23	NA <sup>a</sup>
gstat	0.44	0.91	2.07	0.22	7.56

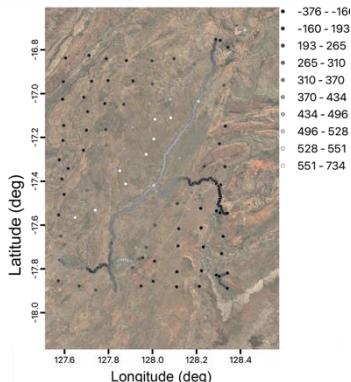
## FRK v2

(not used in this presentation)

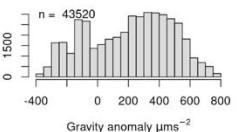
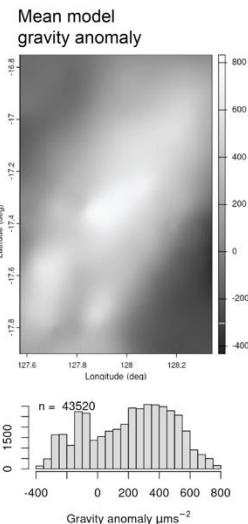
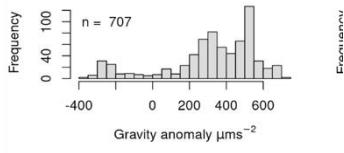
- Zammit-Mangion et al 2021
- Sainsbury-Dale et al 2024

- Pre-processing
- High pass filter (remove regional)

Gravimetry  
measurements

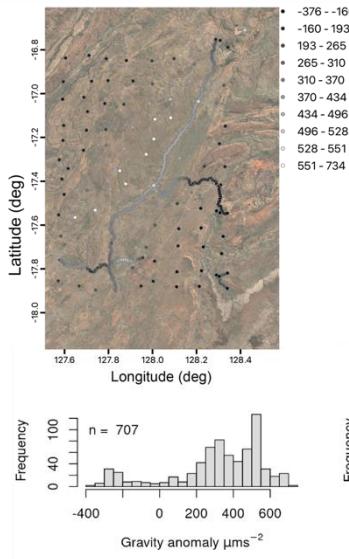


Geostatistical  
interpolation

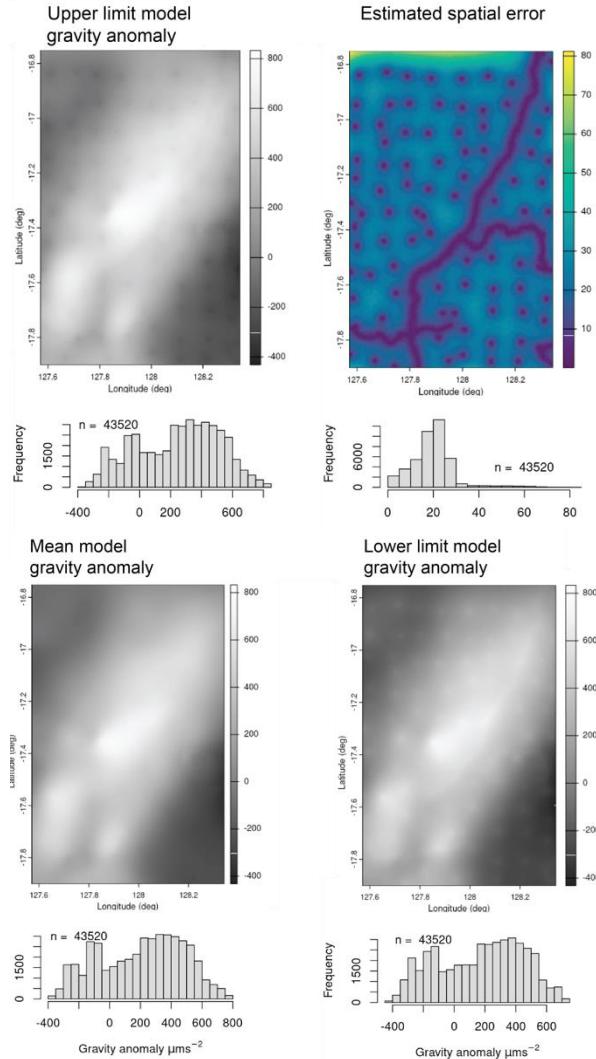


- Pre-processing
  - High pass filter (remove regional)

Gravimetry  
measurements



## Geostatistical interpolation



Lindsay et al 2025  
In review

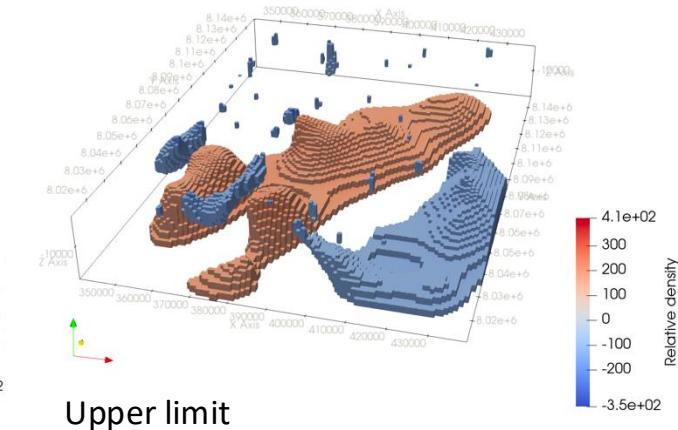
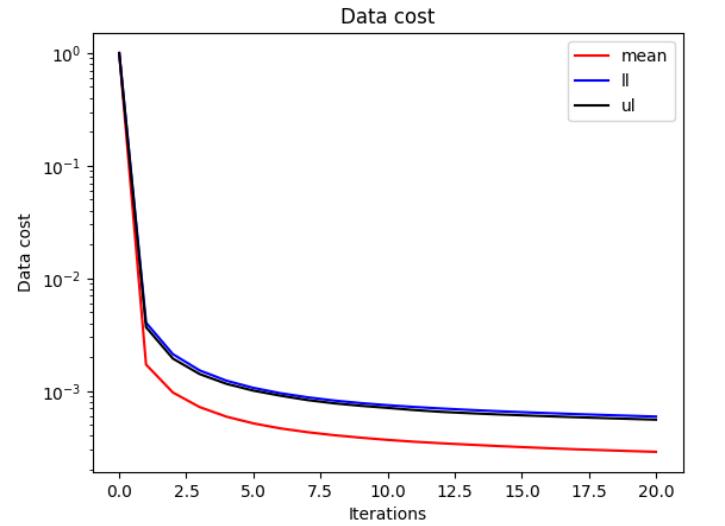
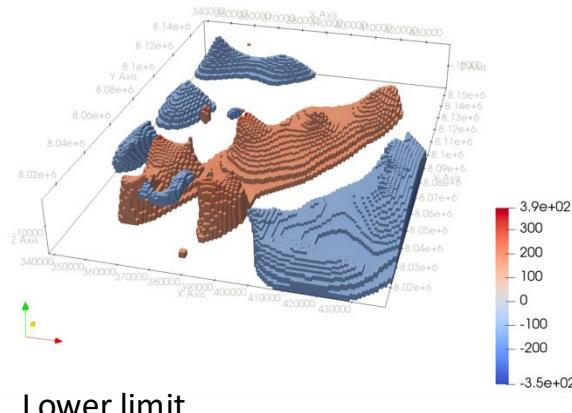
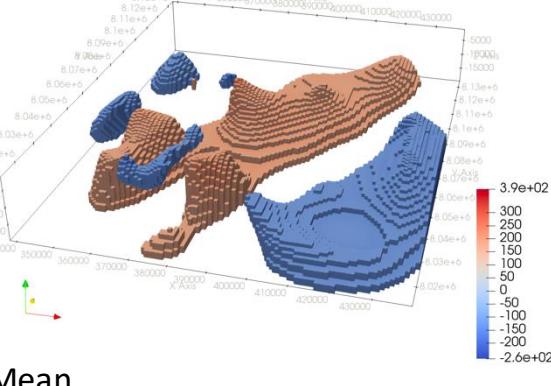
lower limit = 10<sup>th</sup> %ile  
upper limit = 90<sup>th</sup> %ile

# Inversion run

- 20 iterations
- Model grid:
  - $102 \times 147 \times 45$
- Gravity data
  - 10880
- Depth weighted = Yes
- Spatial error = No

Tomofast-x  
Open-source inversion  
code

Reference density = 2670  
 $\text{kg/m}^3$   
Density values threshold  
 $> 150 \text{ kg/m}^3$   
 $< 105 \text{ kg/m}^3$



# An input representing coverage

- Can be input to Tomofast
- Locations of low and high data density
- Used to weight regions during inversion according to data availability

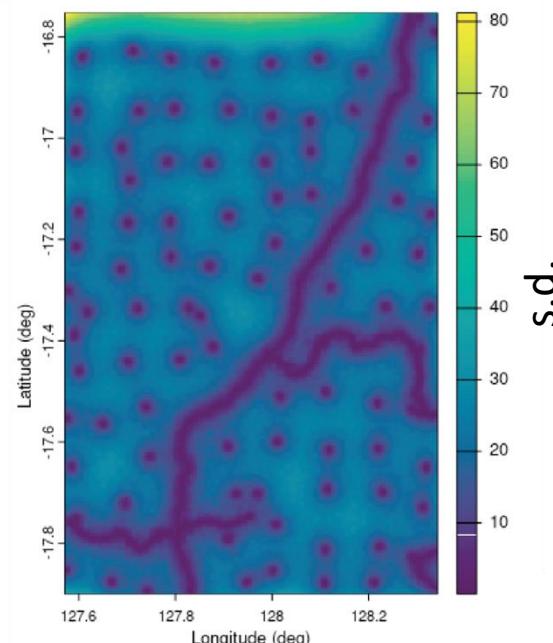
$$\Phi_d = \|\mathbf{W}_d(\mathbf{d}^{calc} - \mathbf{d}^{obs})\|_2^2,$$

Diagonal weighting matrix

$$W_d = 1/\sigma^2$$

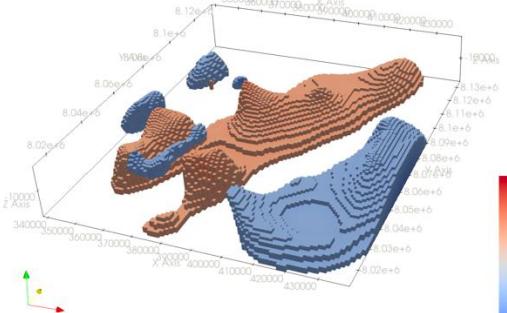
Data distance related uncertainty

Estimated spatial error



# Inversion run

- 20 iterations
- Model grid:
  - $102 \times 147 \times 45$
- Gravity data
  - 10880
- Depth weighted = Yes
- Grid covariance = Yes



Mean

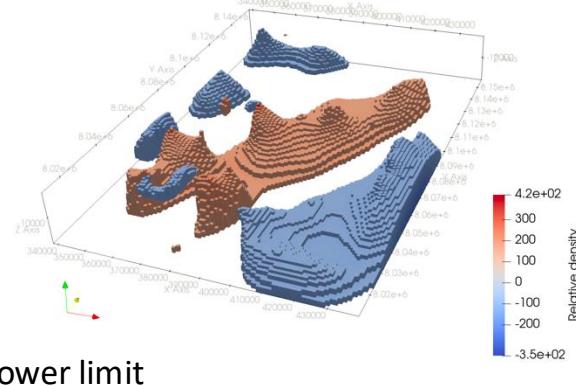
Tomofast-x  
Open-source inversion  
code

Reference density =  $2670 \text{ kg/m}^3$

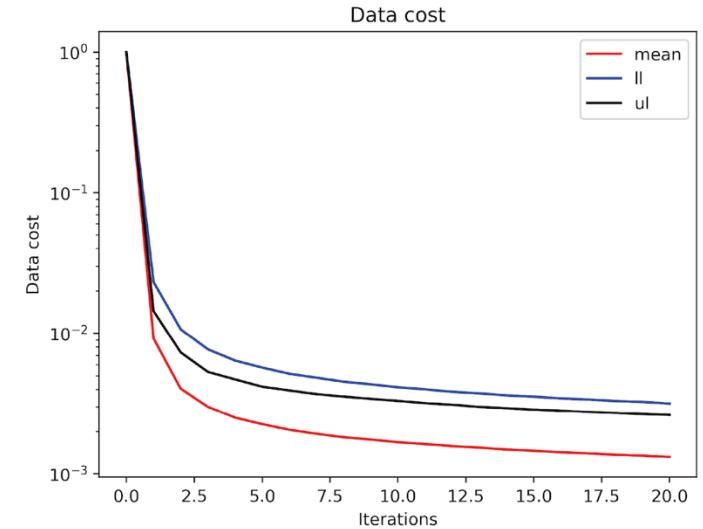
Density values threshold

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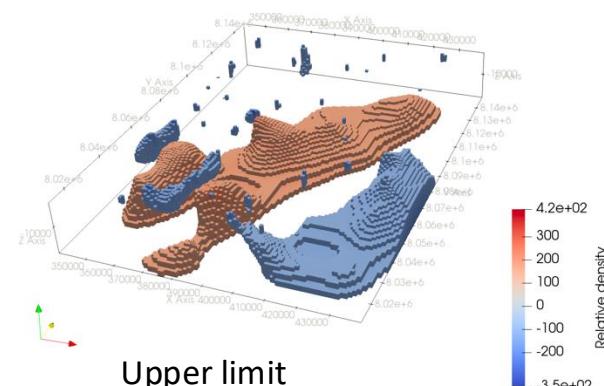
lower limit = 10<sup>th</sup> %ile  
upper limit = 90<sup>th</sup> %ile



Lower limit



Upper limit



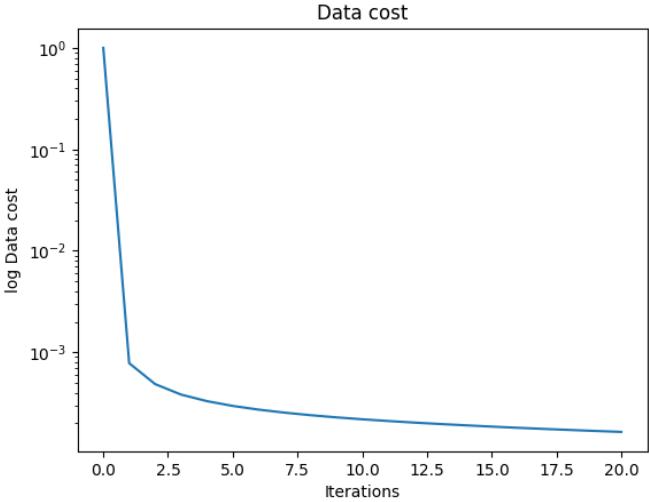
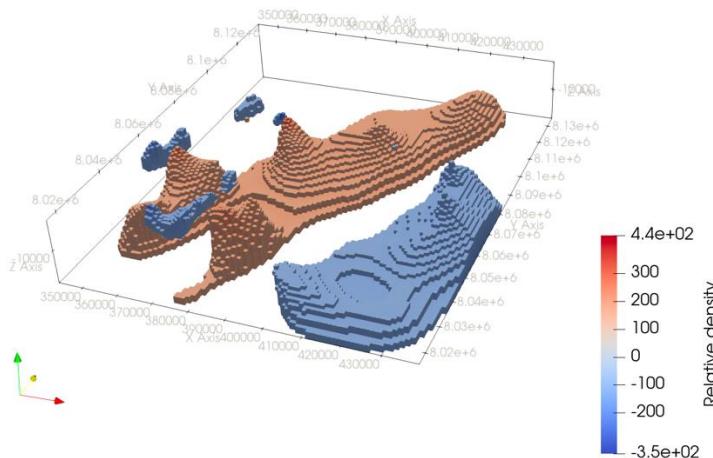
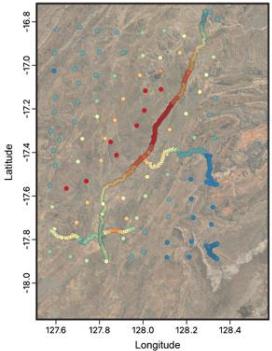
Relative density

# Station points only

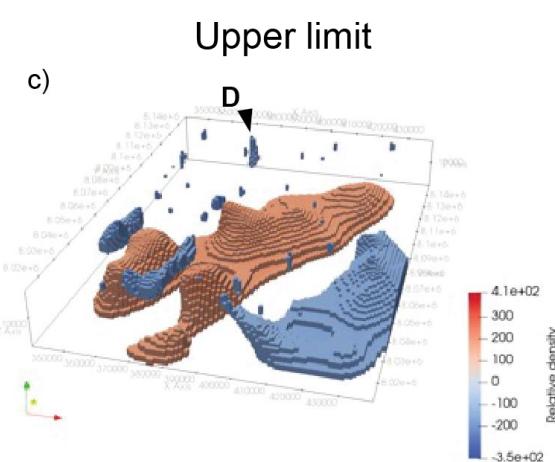
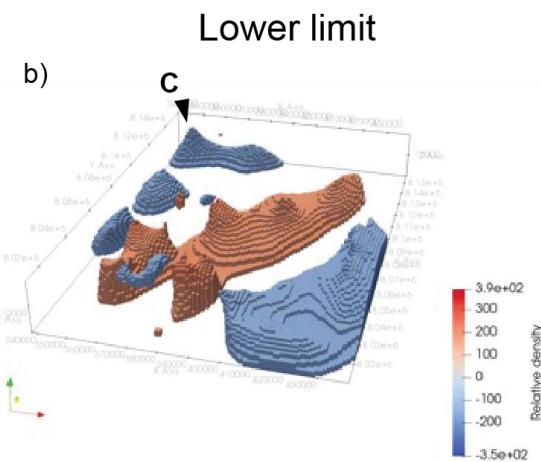
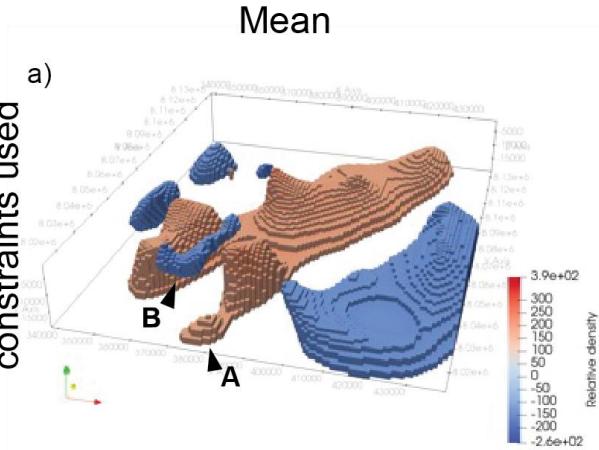
- 20 iterations
- Model grid:
  - $101 \times 147 \times 45$
- Gravity data
  - 707
- Depth weighted = Yes
- Grid covariance matrix = No

Reference density =  $2670 \text{ kg/m}^3$   
Density values threshold

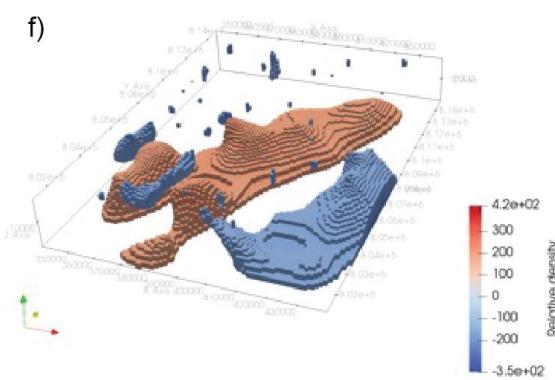
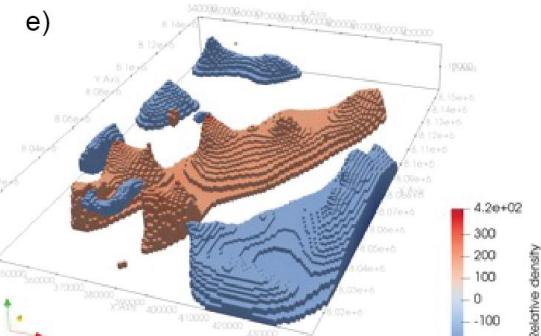
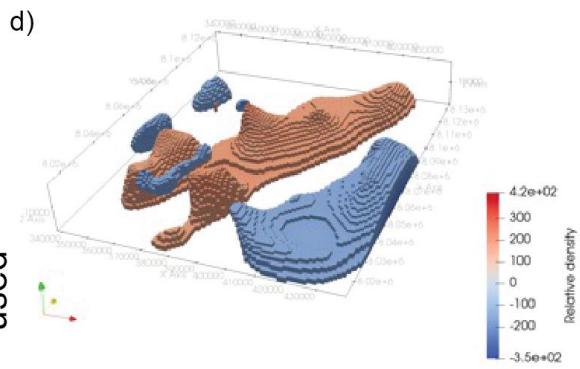
- $> 150 \text{ kg/m}^3$
- $< 105 \text{ kg/m}^3$



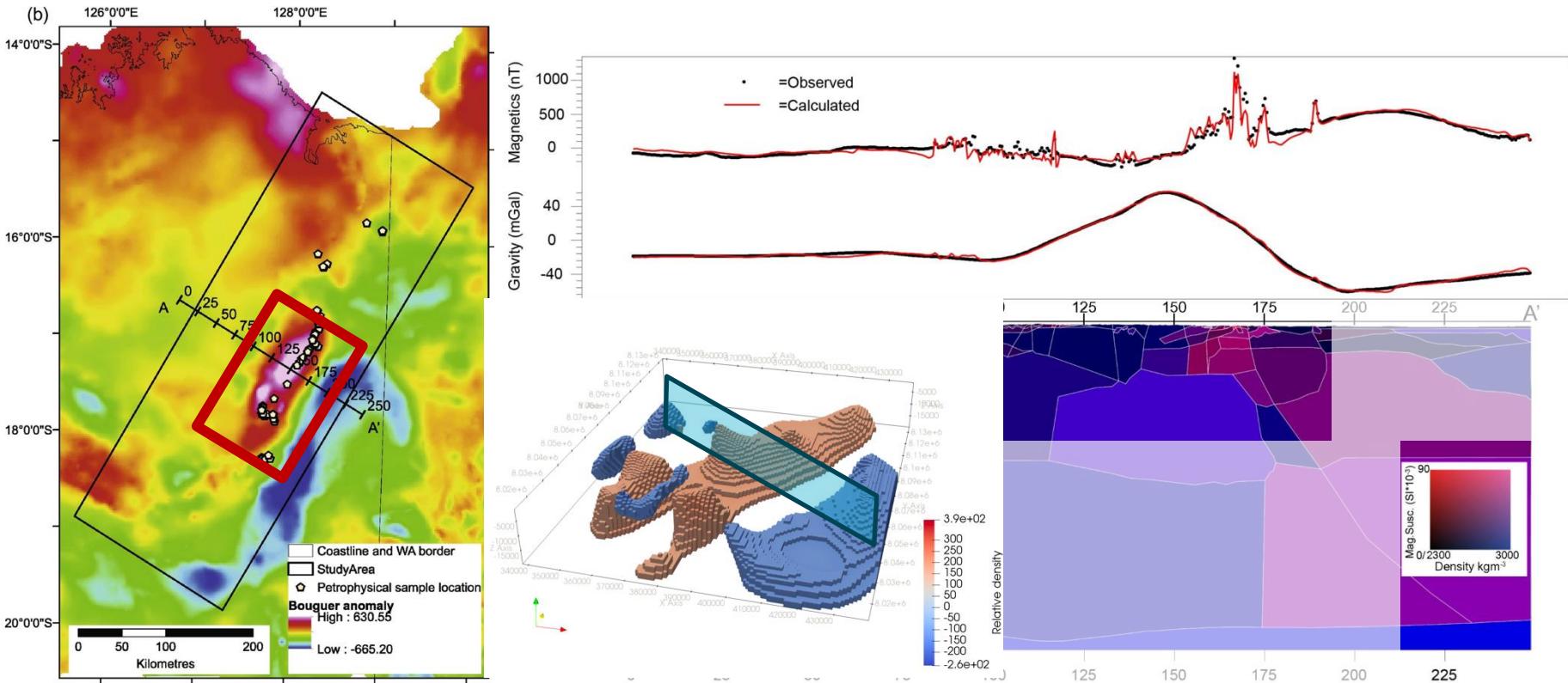
No spatial error  
constraints used



Spatial constraints  
used

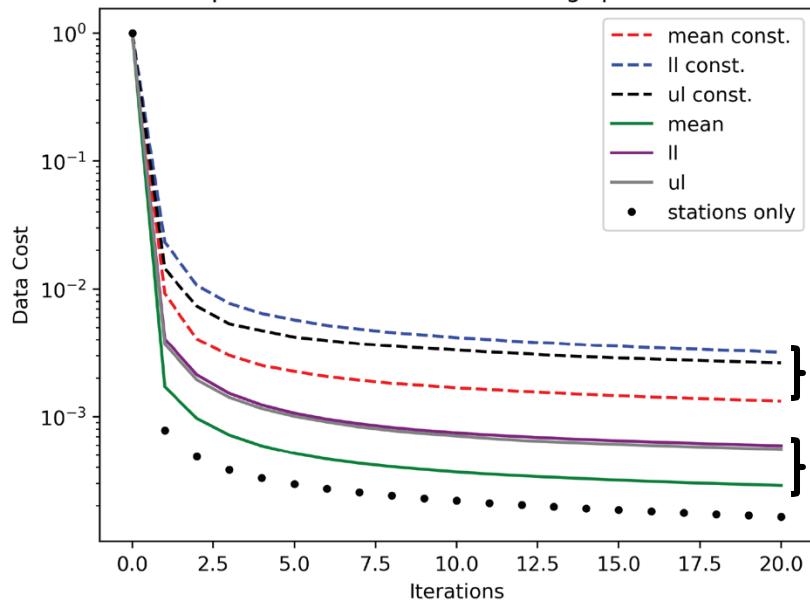


# Forward modelling



# Comparison data cost with/without covar

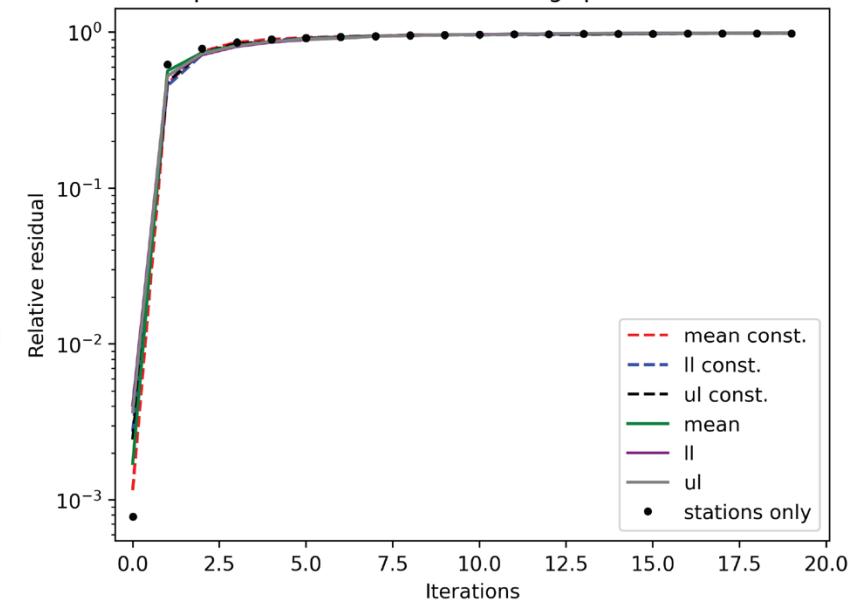
Data cost comparison with and without using spatial error constraints



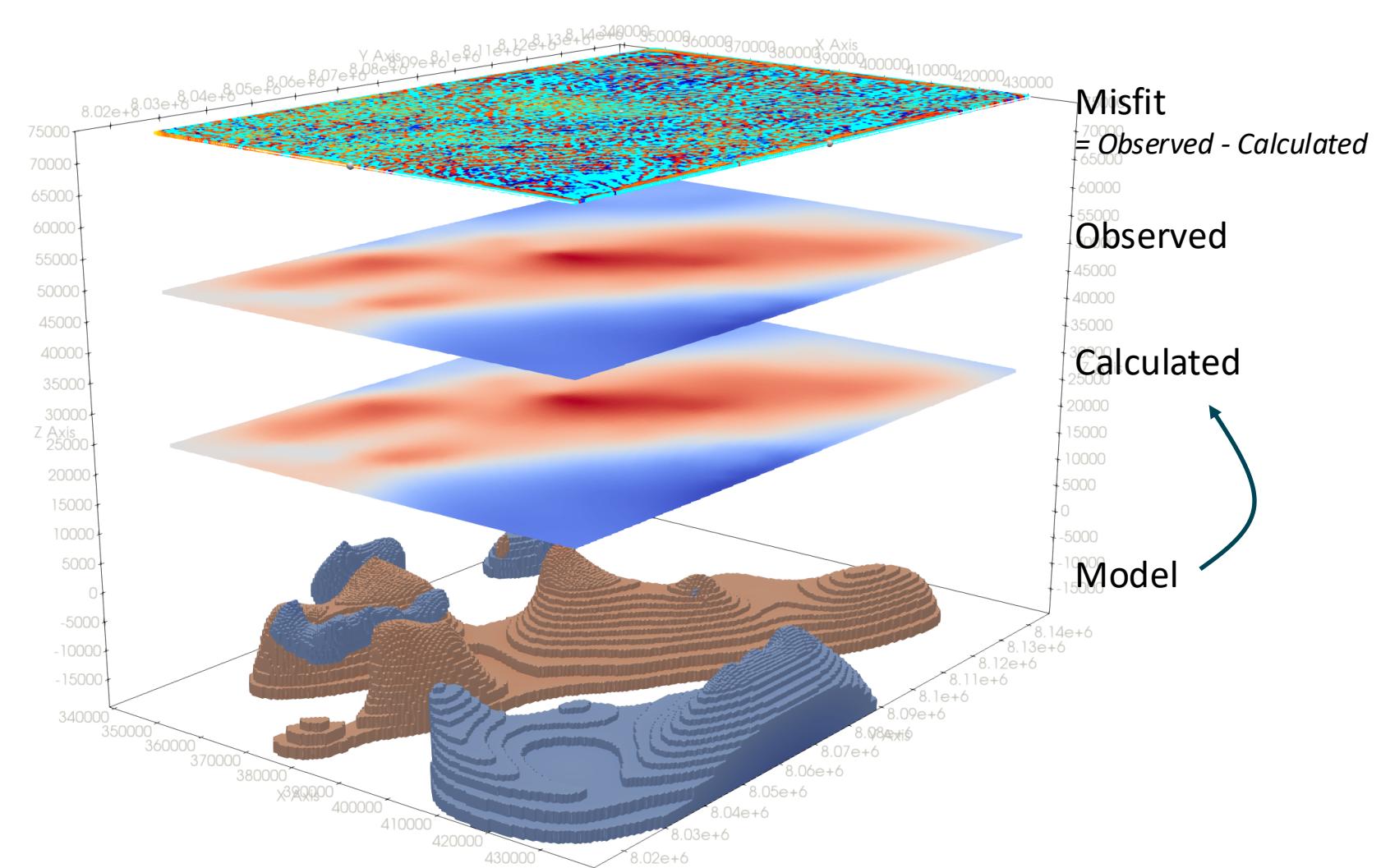
Using spatial  
error  
  
Not using  
spatial error

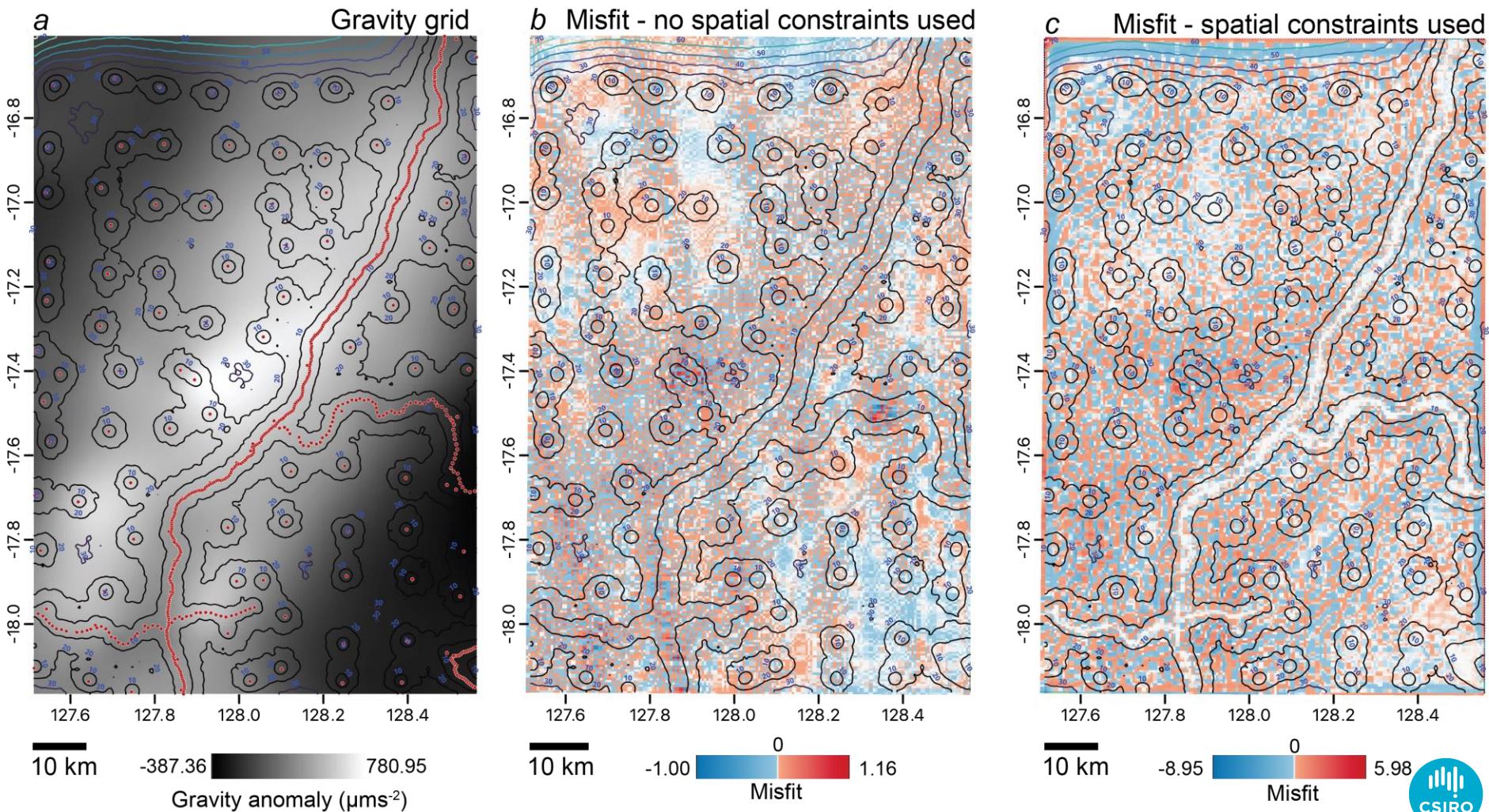
Data cost: normalised version of RMSE  
Low values are desired

RRL comparison with and without using spatial error constraints



Relative residual: measure from the solver representing model variance. High values are good. Convergence is achieved when RR  $\rightarrow 1.0$

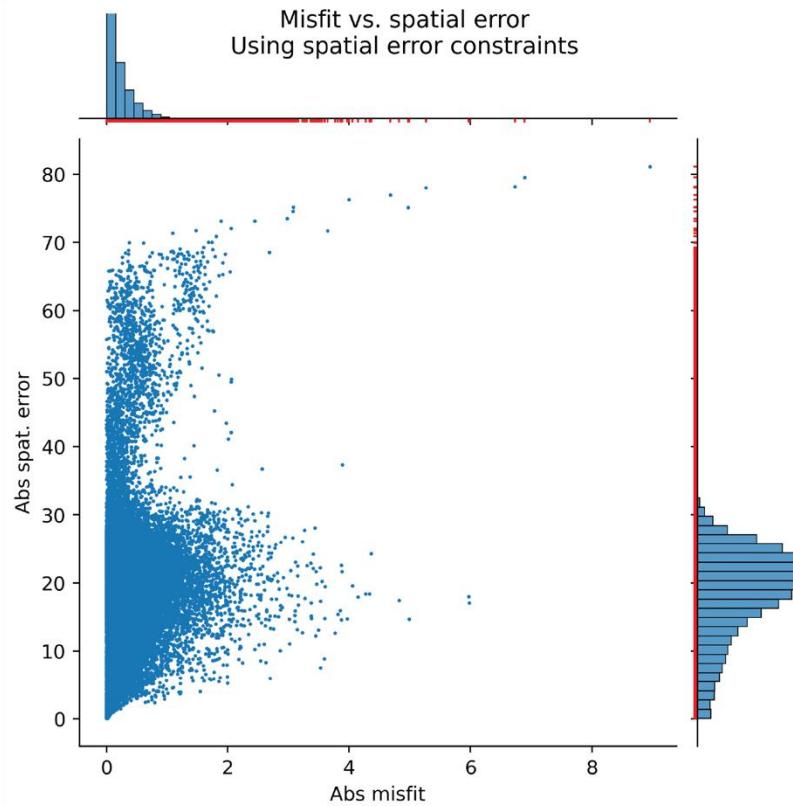
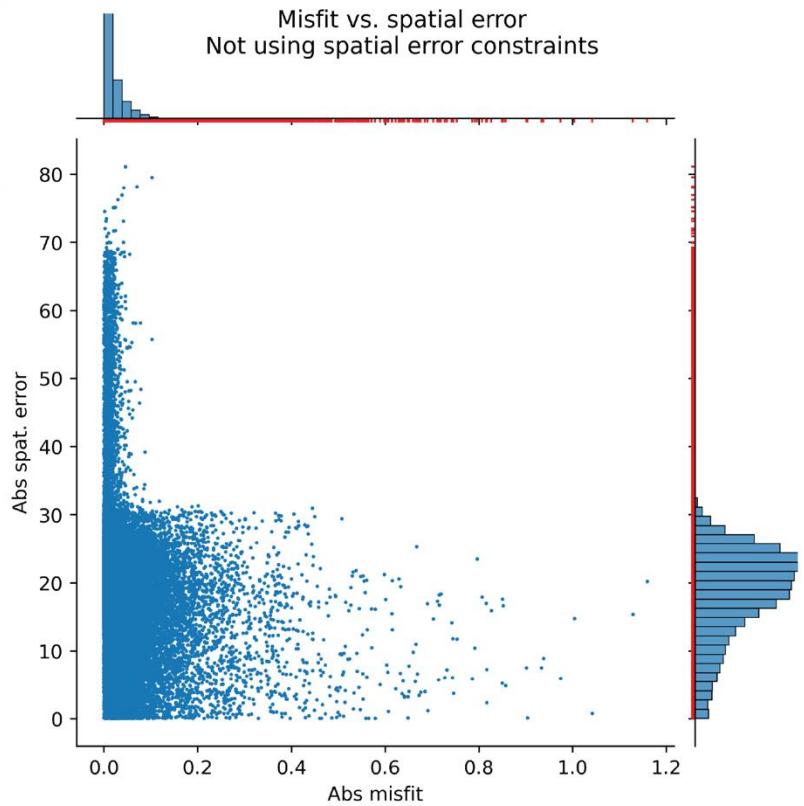




Contours represent spatial error with values as labelled.



# Misfit vs Spatial Error

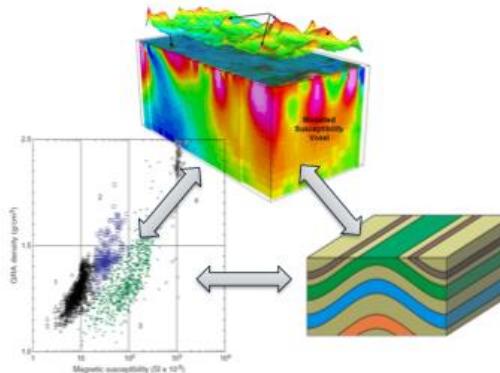


# Thoughts

- Magnetic data
  - Tends to be closer spaced
  - Effects of spatial error constraints lesser (?)
  - Near-surface noise fitting more potentially more likely
- L1/L2 regularisation (use this already)
- Cross-validation (withholding n% of the data)
- Bayesian methods
  - Using priors
  - Using Bayesian Information Criterion to assess
- Measures of overfitting (e.g. Durbin-Watson statistic)
  - Autocorrelation
- Applications to prospectivity analysis and machine learning

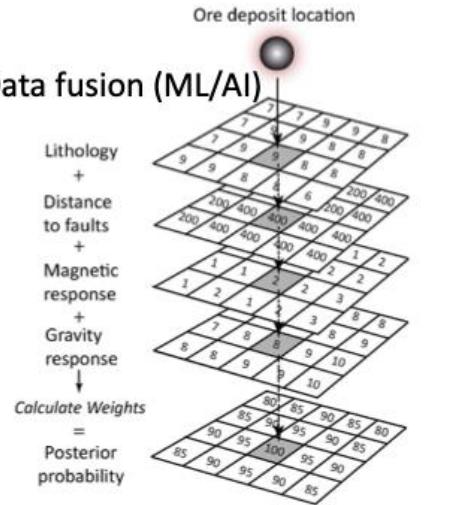
## Why is this important?

- Inversion



Thanks to Jeremie Giraud

- Data fusion (ML/AI)

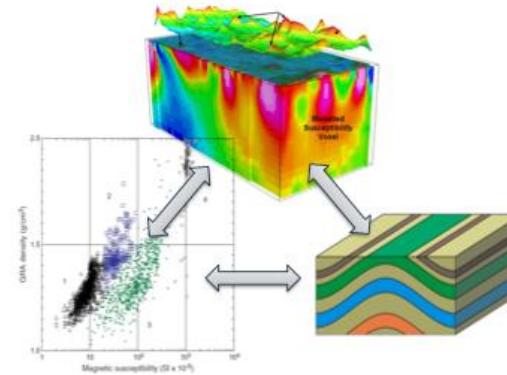


# Conclusions

- Near surface variations
- Spatial error reduces overfitting
- Benefits near-surface noise fitting
- Using different percentiles has no real impact

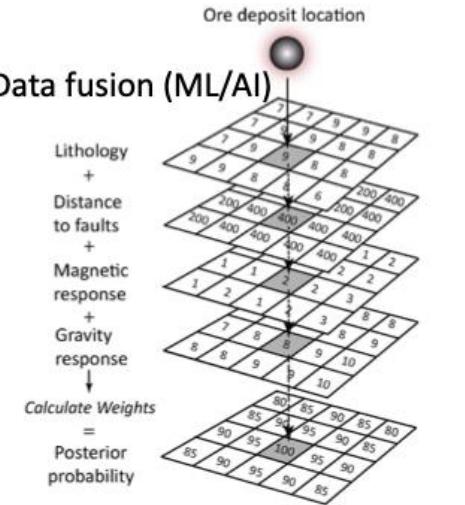
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Lindsay et al 2014



# Thank you for your attention

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Australia's National Science Agency



The image shows the front cover of a book. At the top right, the text "Under consideration" is displayed in white. The background features a silhouette of several people standing on a mountain peak against a sunset sky. A glowing blue wavy line, representing data or a model, runs across the landscape. A yellow rectangular box contains the text "Geological Society Special Publication". Below this, the title "Geological Modelling and Mapping in the Age of AI" is written in large, bold, white capital letters. At the bottom, the names of the editors are listed: Charlie Kirkwood, Kristine Asch, Mark Lindsay, Florian Wellmann, Michael Hillier, and Guillaume Caumon.

