An integrated and interoperable platform enabling 3D stochastic geological modelling

Loop

Introduction to null space analysis, the example of gravity and magnetics

Presented by Jeremie Giraud

With contributions from: Mary Ford, Vitaliy Ogarko, Guillaume Caumon, Paul Cupillard, Lachlan Grose, Roland Martin

Main sponsor for this work

Jeremie Giraud has received funding from the European Union's Horizon 2020 research and innovation programme[1] under the Marie Skłodowska-Curie grant agreement No. 101032994.

[1] for more info about project:
https://cordis.europa.eu/project/id/101032994

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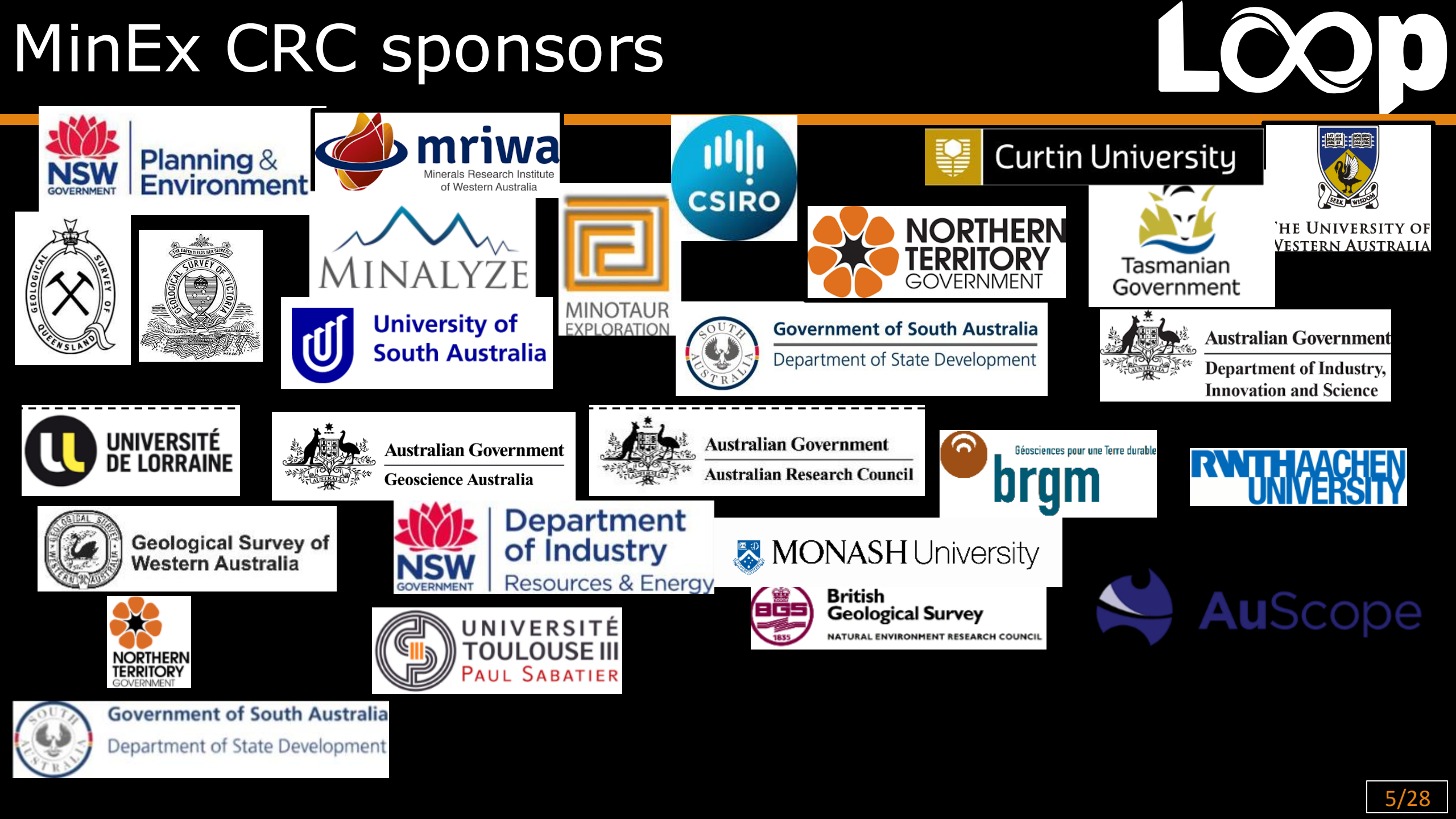
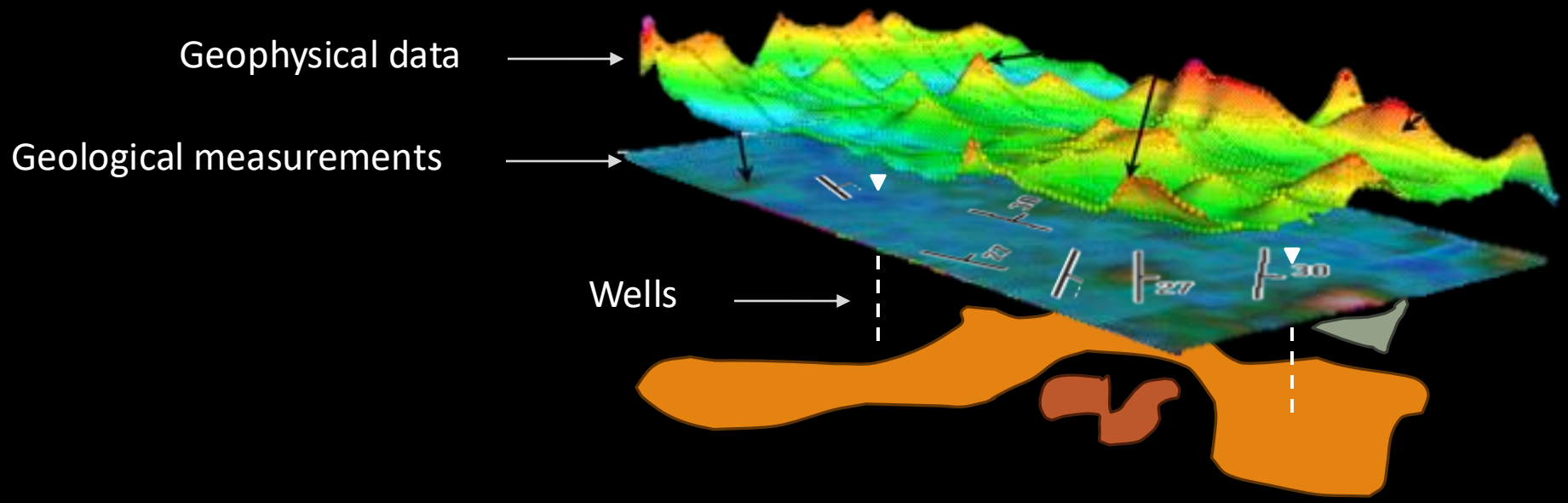


Motivation and Objectives



Exploration of alternative scenarios

Non-uniqueness: many models, same data!

Nullspace concept. examples

- Additive nullspace

- Discriminative nullspace

Same classification!

Geophysical models

e.g., geophysical data

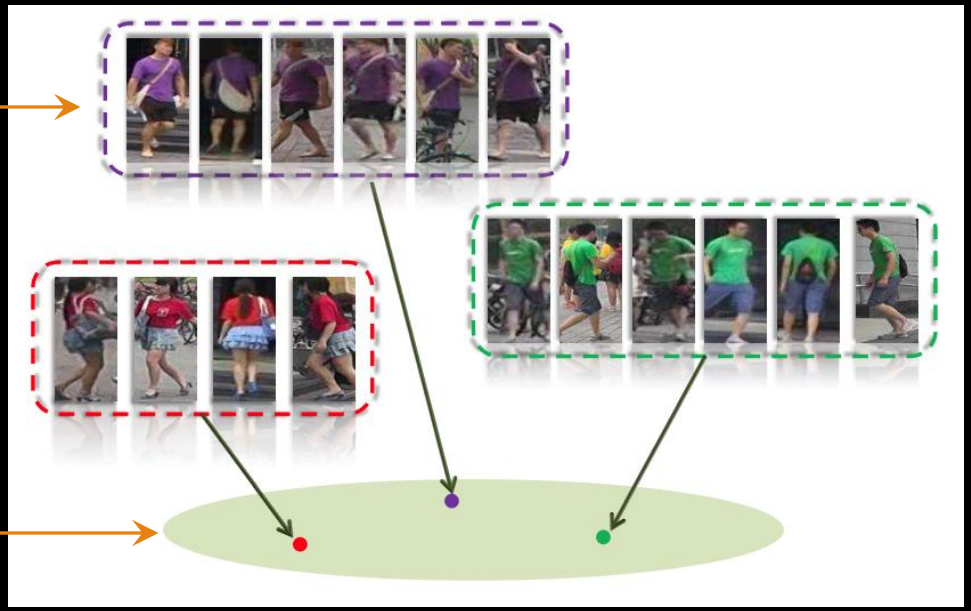


Image (top): Cook, Zare, Alina and Gader [2020] Outlier Detection through Null Space Analysis of Neural Networks, https://arxiv.org/pdf/2007.01263.pdf
Image (bottom): L. Zhang, T. Xiang and S. Gong [2016] Learning a Discriminative Null Space for Person Re-identification, doi: 10.1109/CVPR.2016.139

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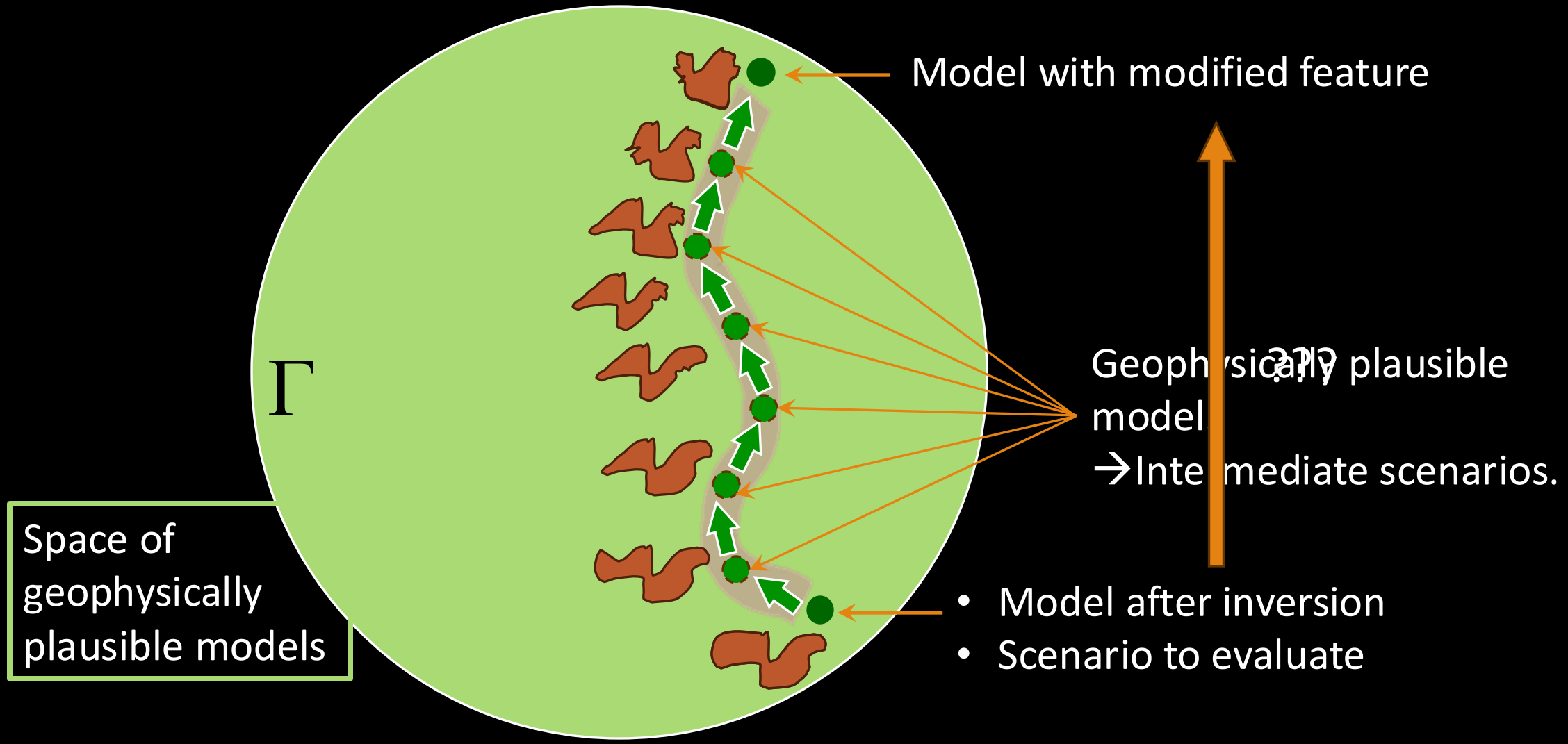
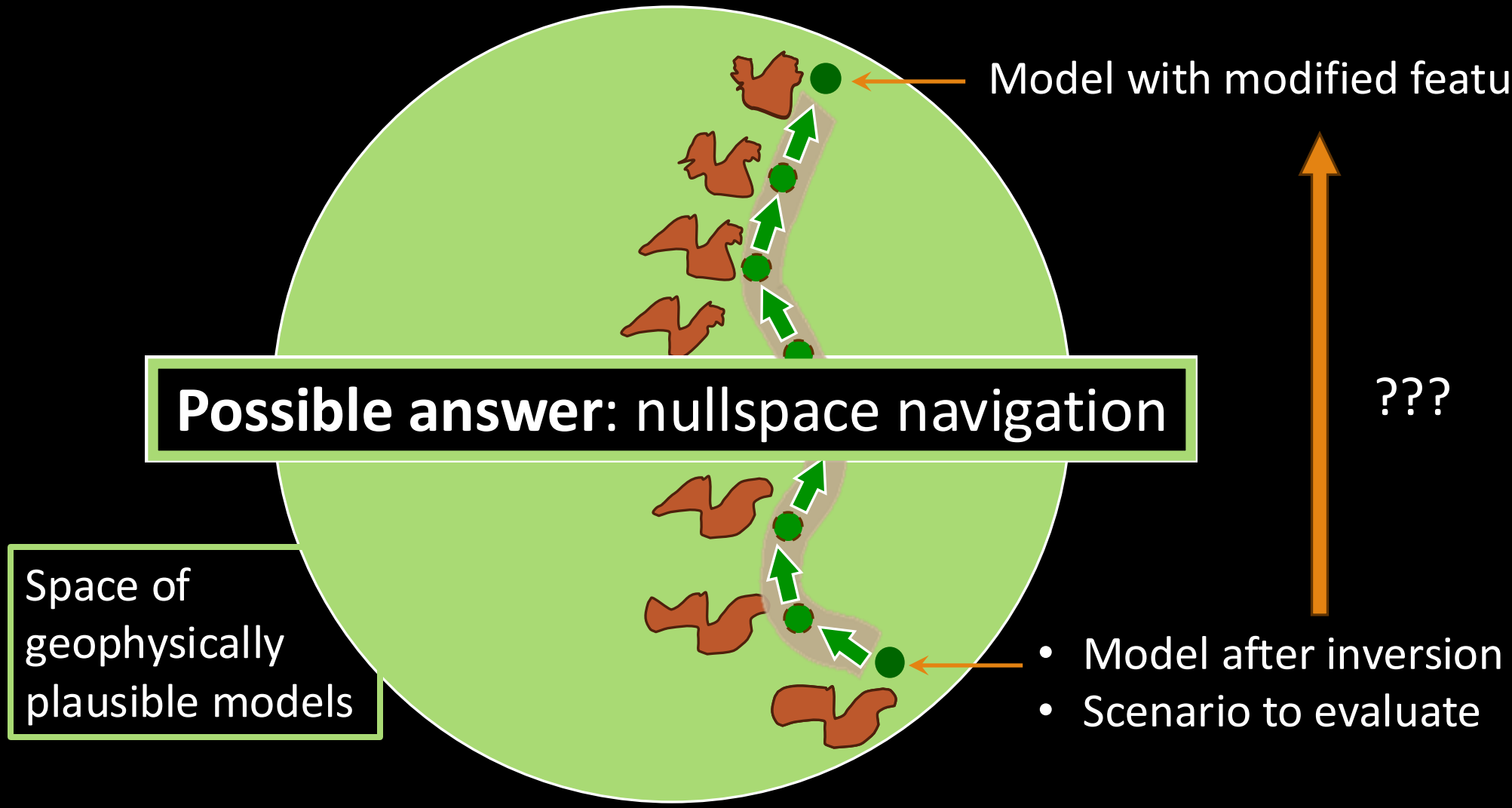


Question: what if a given feature is added or removed?

Motivation and Objectives



Generalities

→ Transition from one model that fits the data to another

« Null space shuttles » (Deal and Nollet 1996, de Vit et al 2012)

→ Modifying the modify the model WHILE maintaining data fit

« Hamiltonian null-space shuttles » (Fichtner and Zunino, 2019)

(inspired from Brownian mouvement)

→ Maintain the Hamiltonian $\boldsymbol{H}$ constant

$$\boldsymbol{H = misfit\ term + perturbation\ term}$$

Should not change

Should change

Nullspace shuttles

- Hamiltonian null-space shuttles (Fichtner and Zunino, 2019) (inspired from Brownian mouvement)
  - Misfit term // potential energy

$$\psi^d = \psi^d(\boldsymbol{d}_{obs.}^{geophy}, \boldsymbol{m}) = \left\| \boldsymbol{d}_{obs.}^{geophy} - \underbrace{\boldsymbol{S}\boldsymbol{m}}_{\text{Calculated data (pot. fields: linear)}} \right\|_2^2$$

  - Perturbation term // kinetic energy

$$K(\boldsymbol{p}) = \frac{1}{2}\boldsymbol{p}^T \boldsymbol{M}^{-1}\boldsymbol{p}$$

  - Hamiltonian

$$H(\boldsymbol{m}, \boldsymbol{p}) = \text{potential energy} + \text{kinetic energy}$$
$$H(\boldsymbol{m}, \boldsymbol{p}) = \textbf{constant}$$
$$H(\boldsymbol{m}, \boldsymbol{p}) = \psi^d(\boldsymbol{d}_{obs.}^{geophy}, \boldsymbol{m}) + K(\boldsymbol{p})$$

Nullspace shuttles

- Hamiltonian null-space shuttles (Fichtner and Zunino, 2019) (inspired from Brownian mouvement)
  - Misfit term // potential energy

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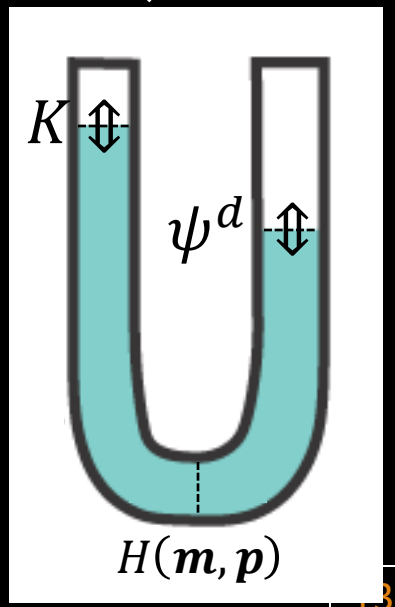


Image of unmodified tubes with water from: http://deuns.chez.com/ps/pression/pression4.html

Nullspace shuttles

- Solving the equations
  - Useful quantities
    - Derivative of potential energy

$$\frac{\partial}{\partial \boldsymbol{m}}\left(\psi^{d}\left(\boldsymbol{d}_{obs.}^{geophy}, \boldsymbol{m}\right)\right) = -\boldsymbol{S}^{T}\left(\boldsymbol{d}_{obs.}^{geophy} - \boldsymbol{S}\boldsymbol{m}\right)$$

    - Derivative of perturbation term

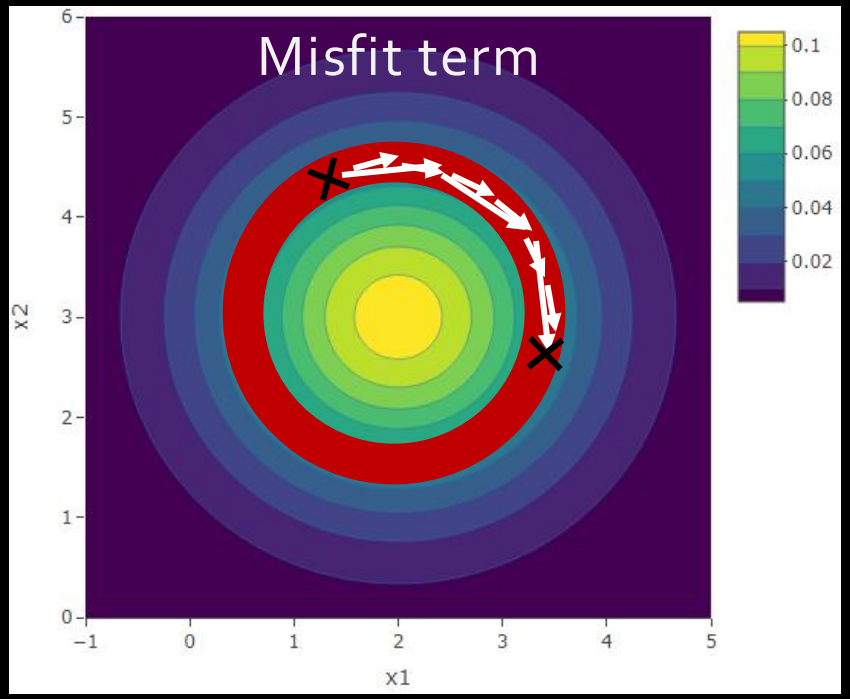
$$\frac{\partial}{\partial \boldsymbol{m}}\left(K(\boldsymbol{p})\right) = \boldsymbol{\delta pert} \cdot \boldsymbol{M}^{-1}$$

  - Constraints
    - Depth weighting – same as for inversion
    - Inequality / Positivity constraints, e.g. no mag susc. < 0.
    - Prior model constraints

Iterative numerical scheme: leapfrog method.

Parameters

- Step length (Δt)
- Data misfit tolerance ε
  - Explore far from starting point, quickly
  - Or small step by step modifications

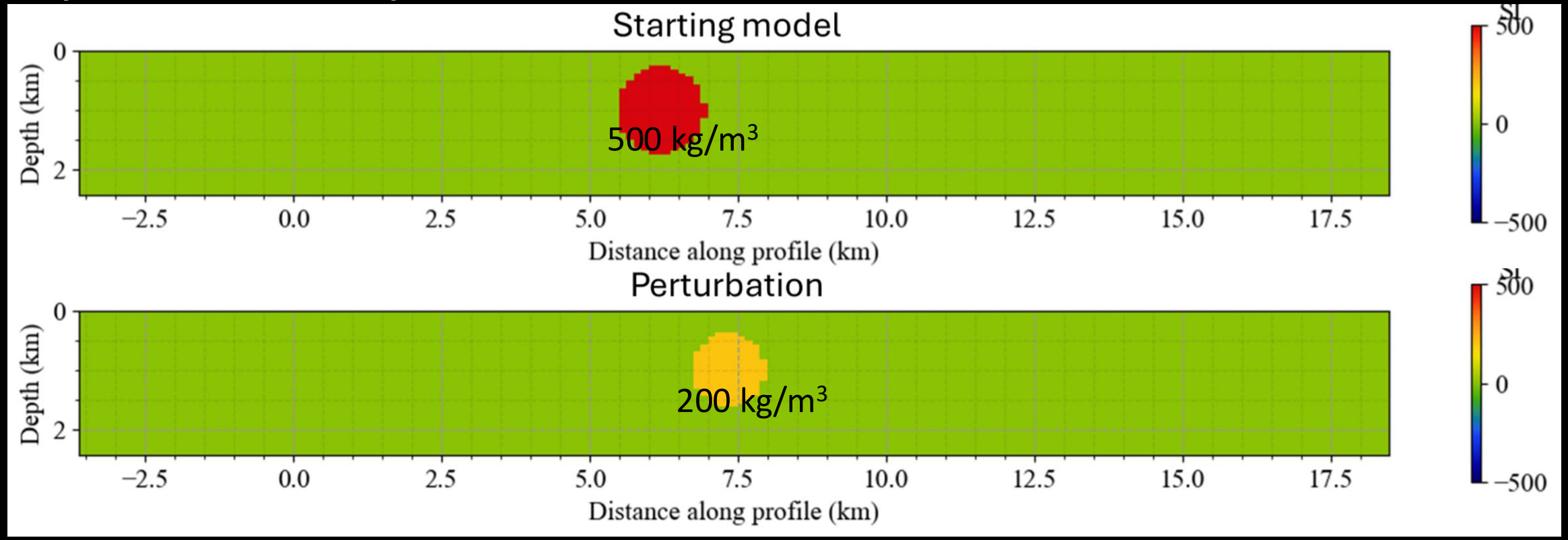


One model per step: explore space of equivalent models

Examples that can be reproduced using the notebook

Visual examples - gravi

- Synthetic, conceptual model



Visual examples - gravi

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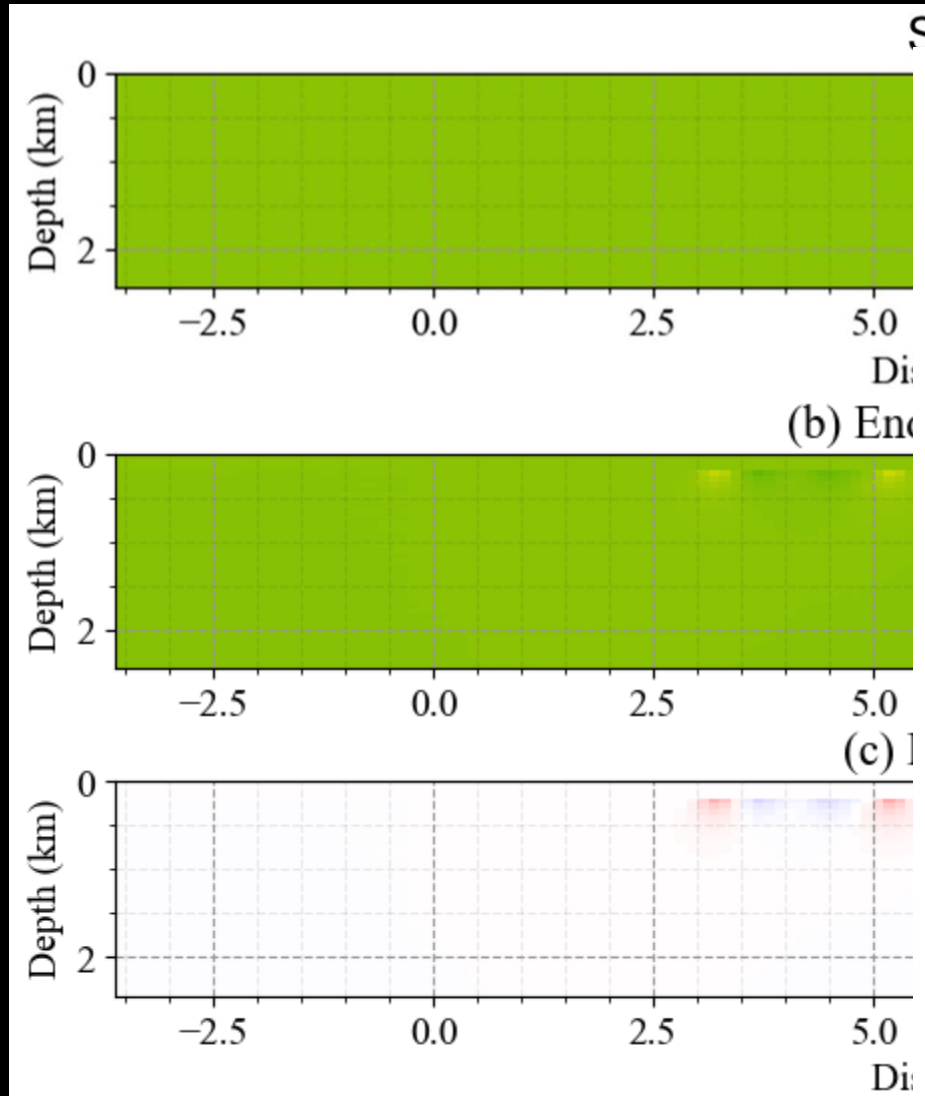
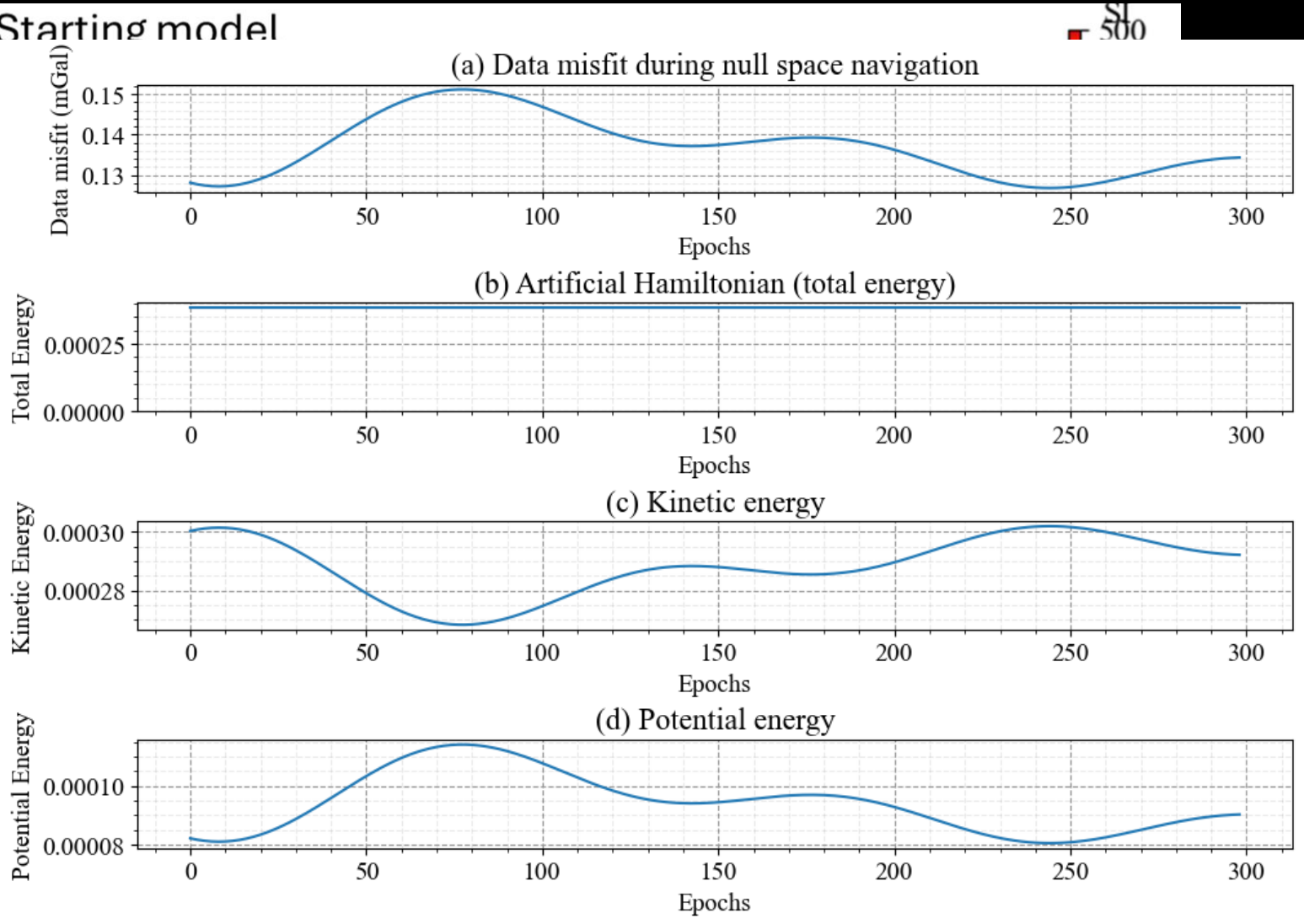
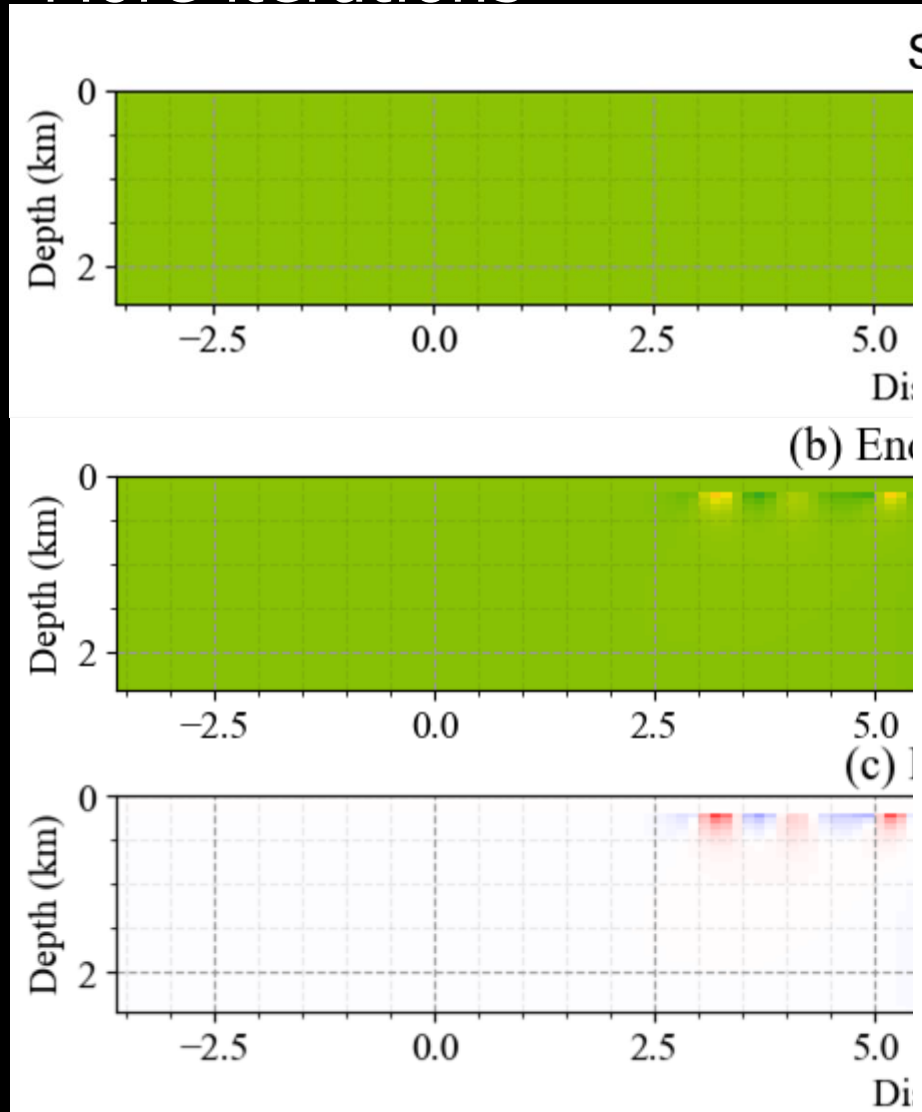
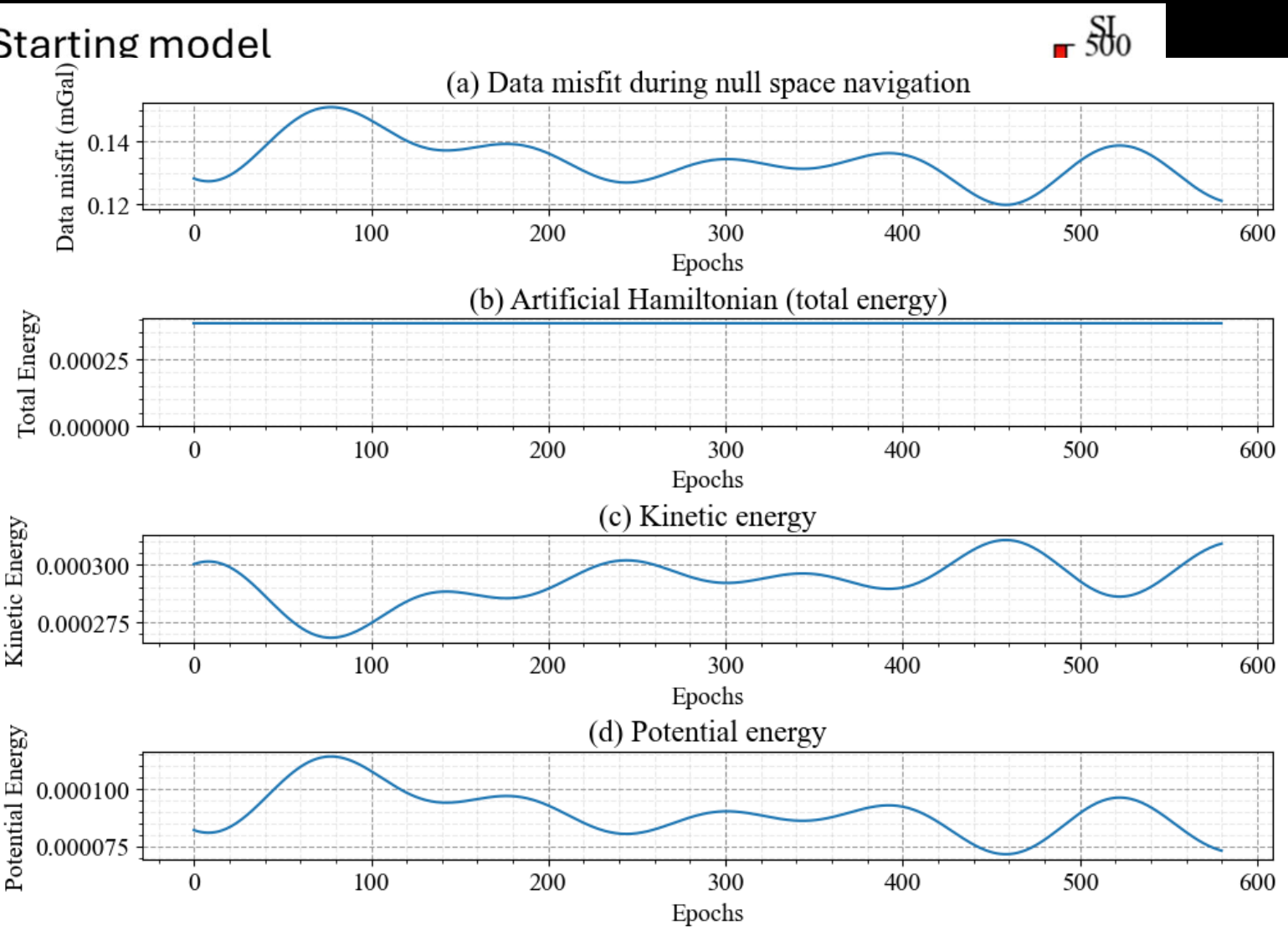




Visual examples - gravi

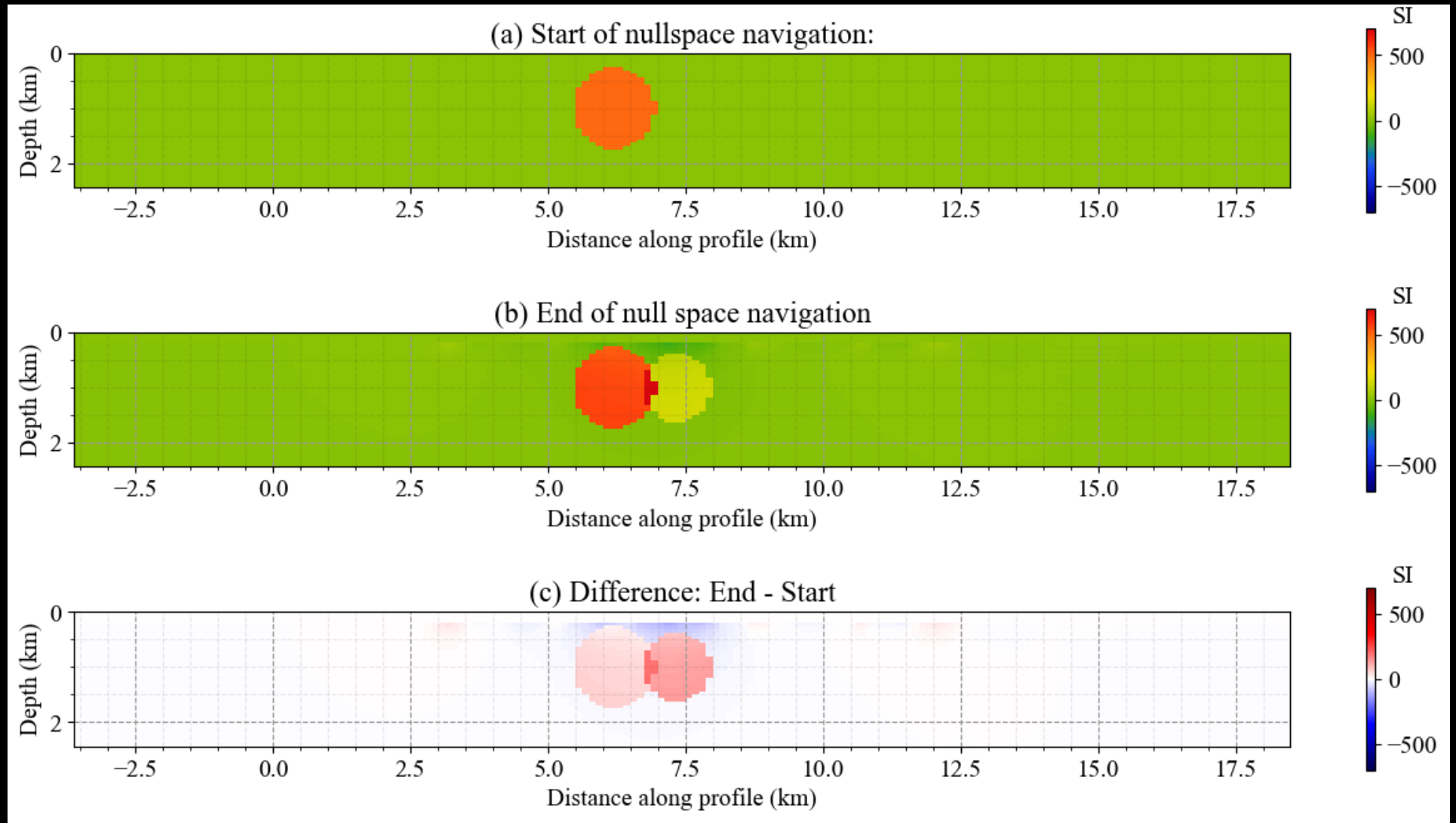
- More iterations





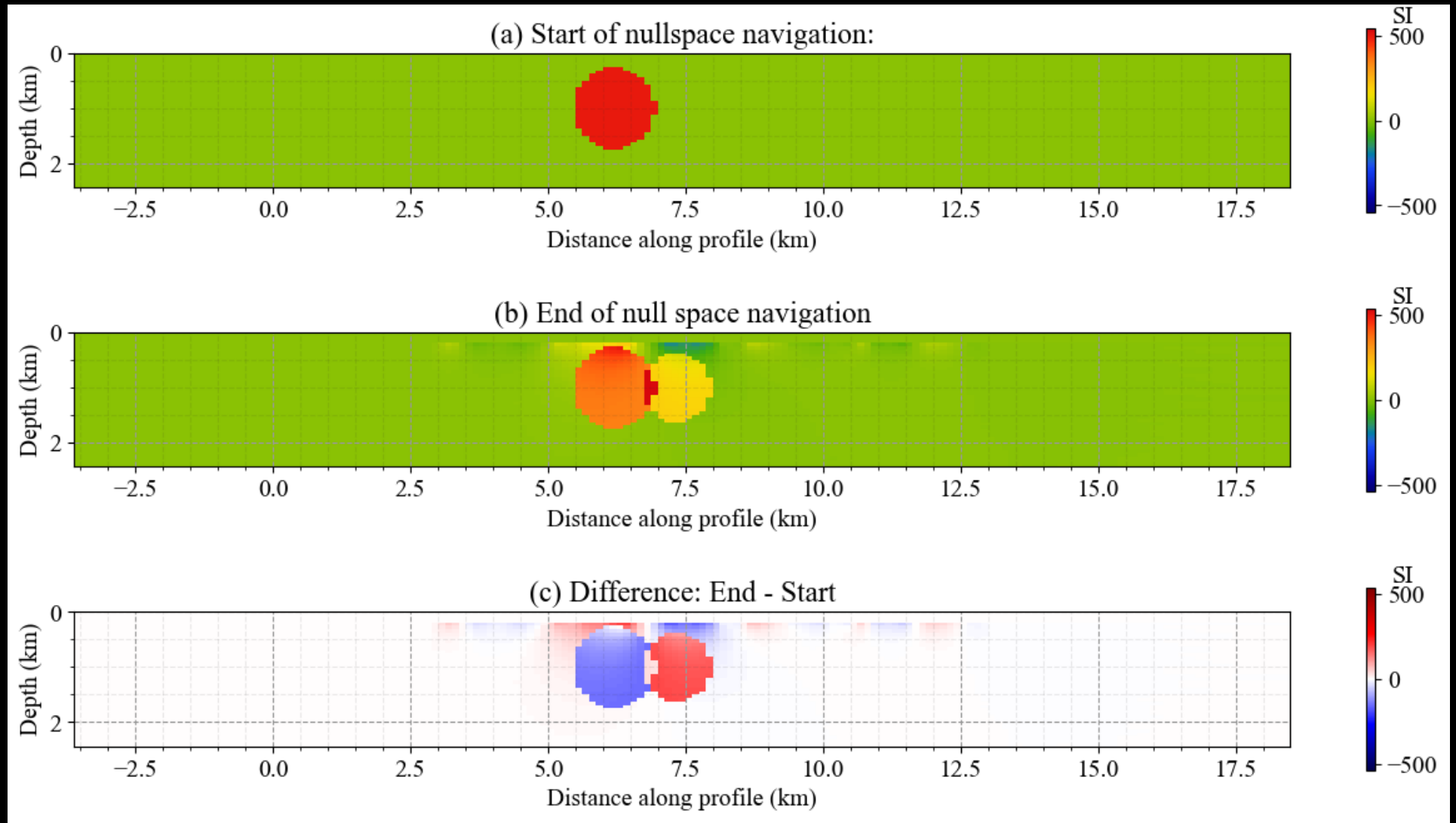
Visual examples - gravi

- Smallness constraints – <u>increase</u> magnitude of values in the model



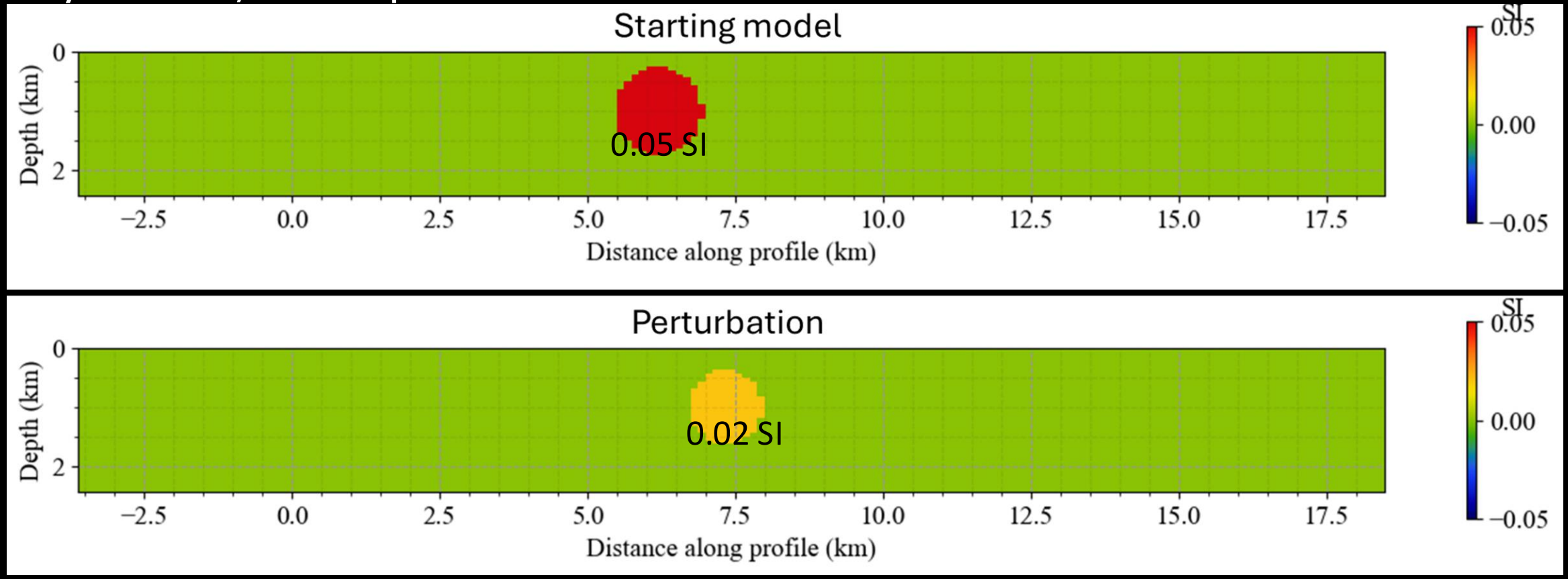
Visual examples - gravi

- Smallness constraints – reduce magnitude of values in the model

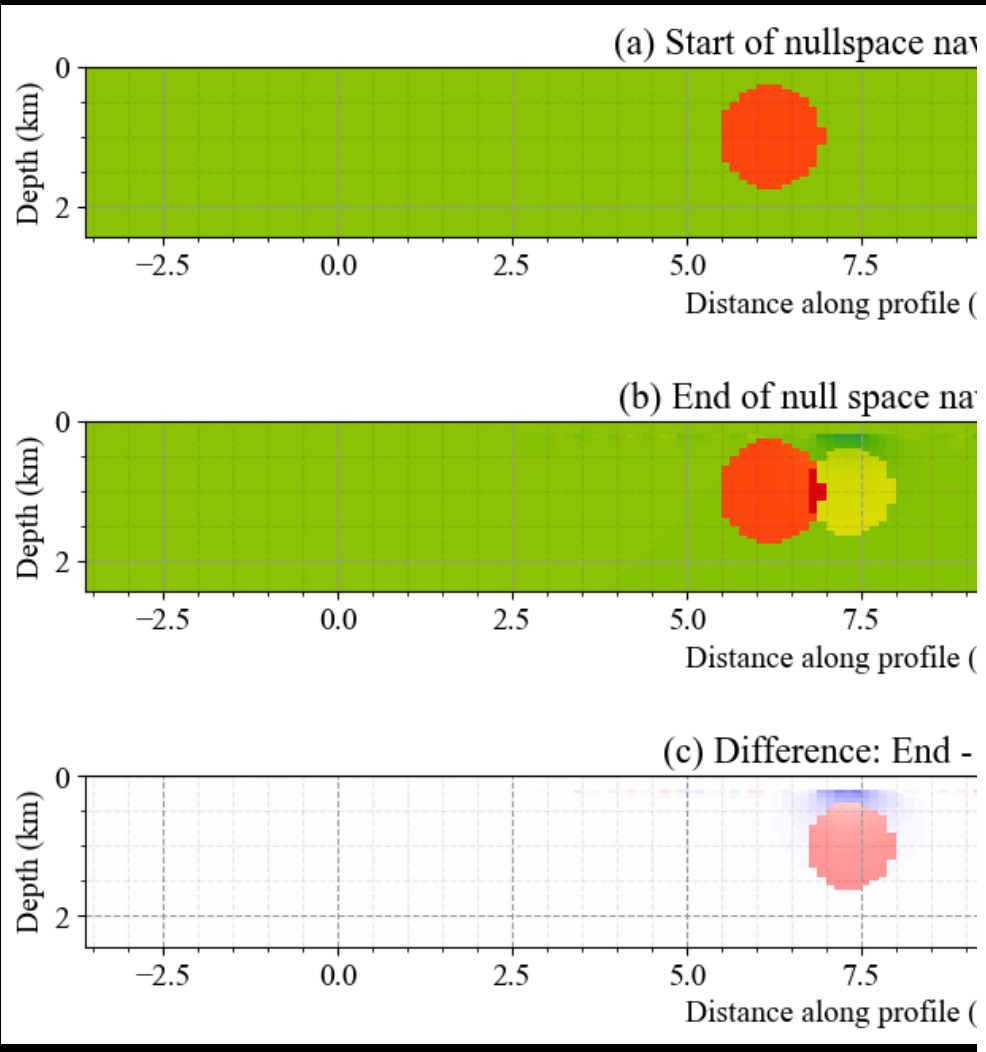


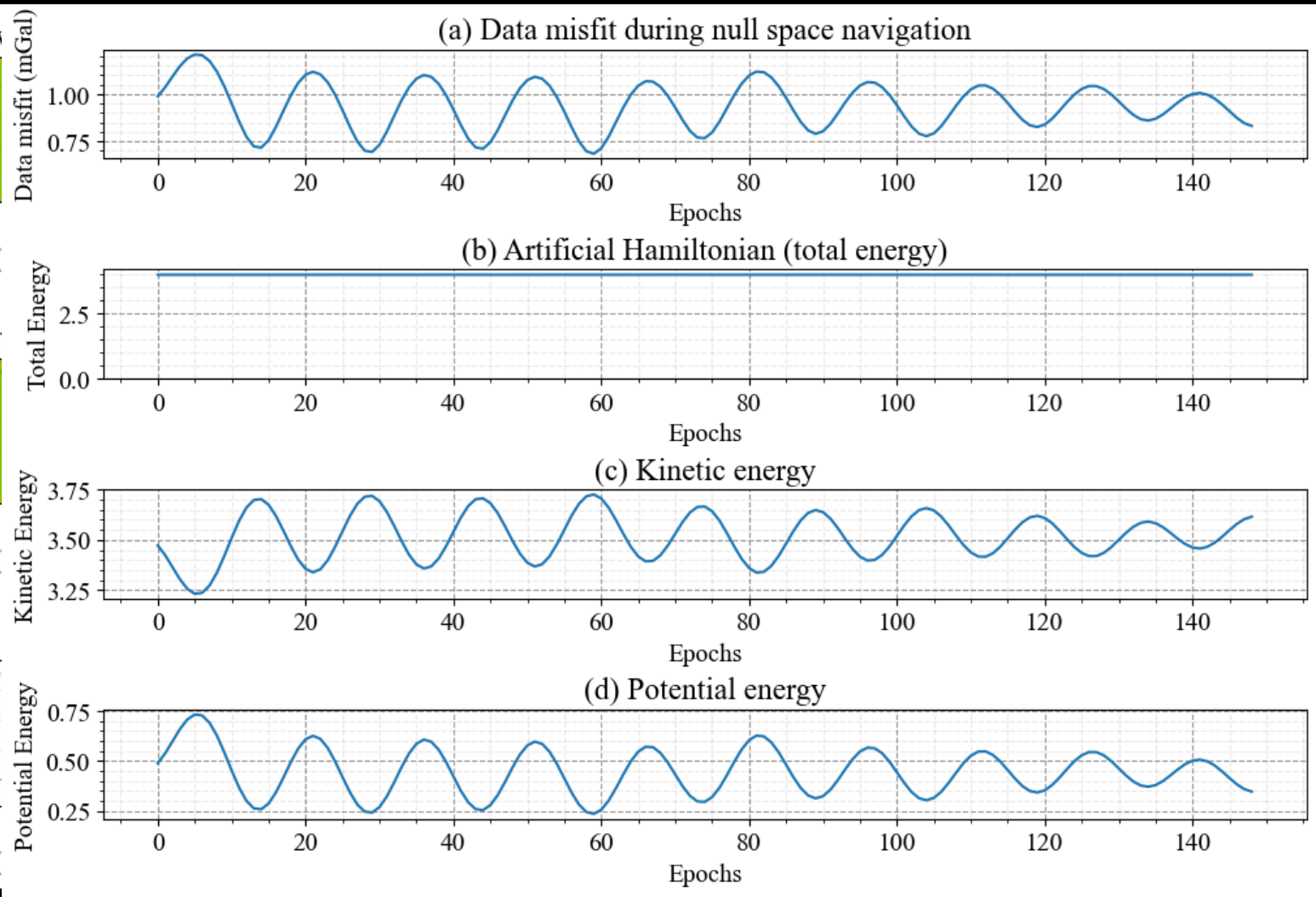
Visual examples - mag

- Synthetic, conceptual model

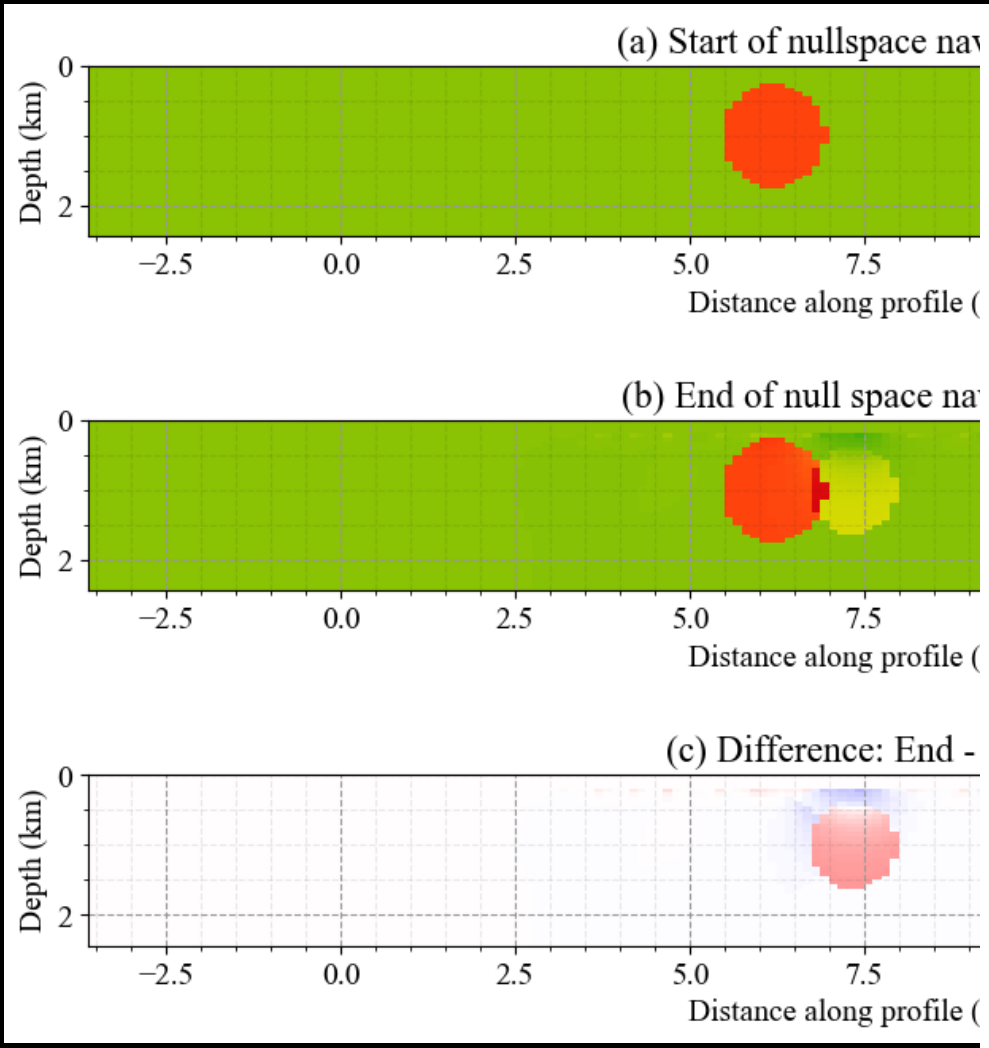


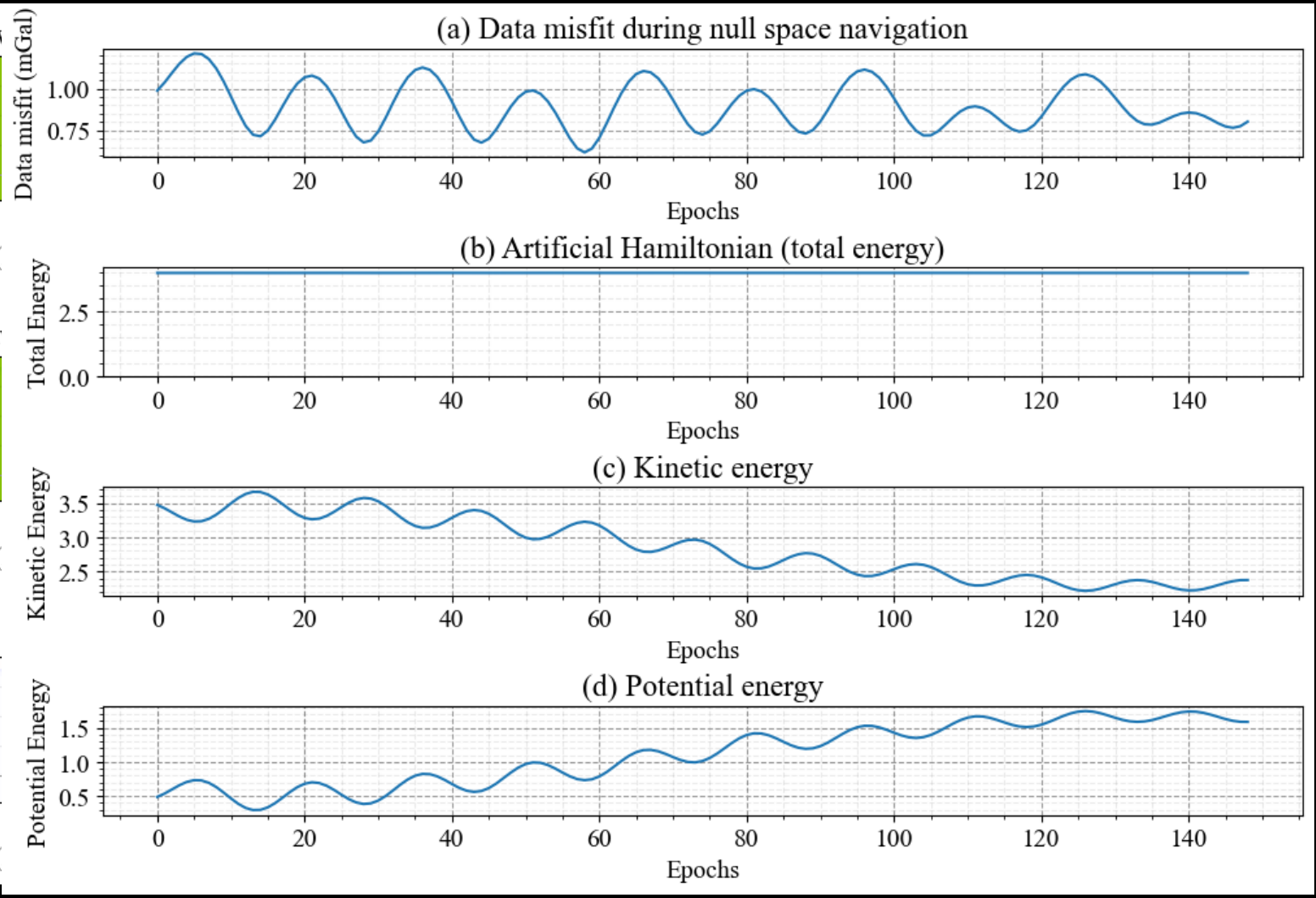
- Base case – no constraints



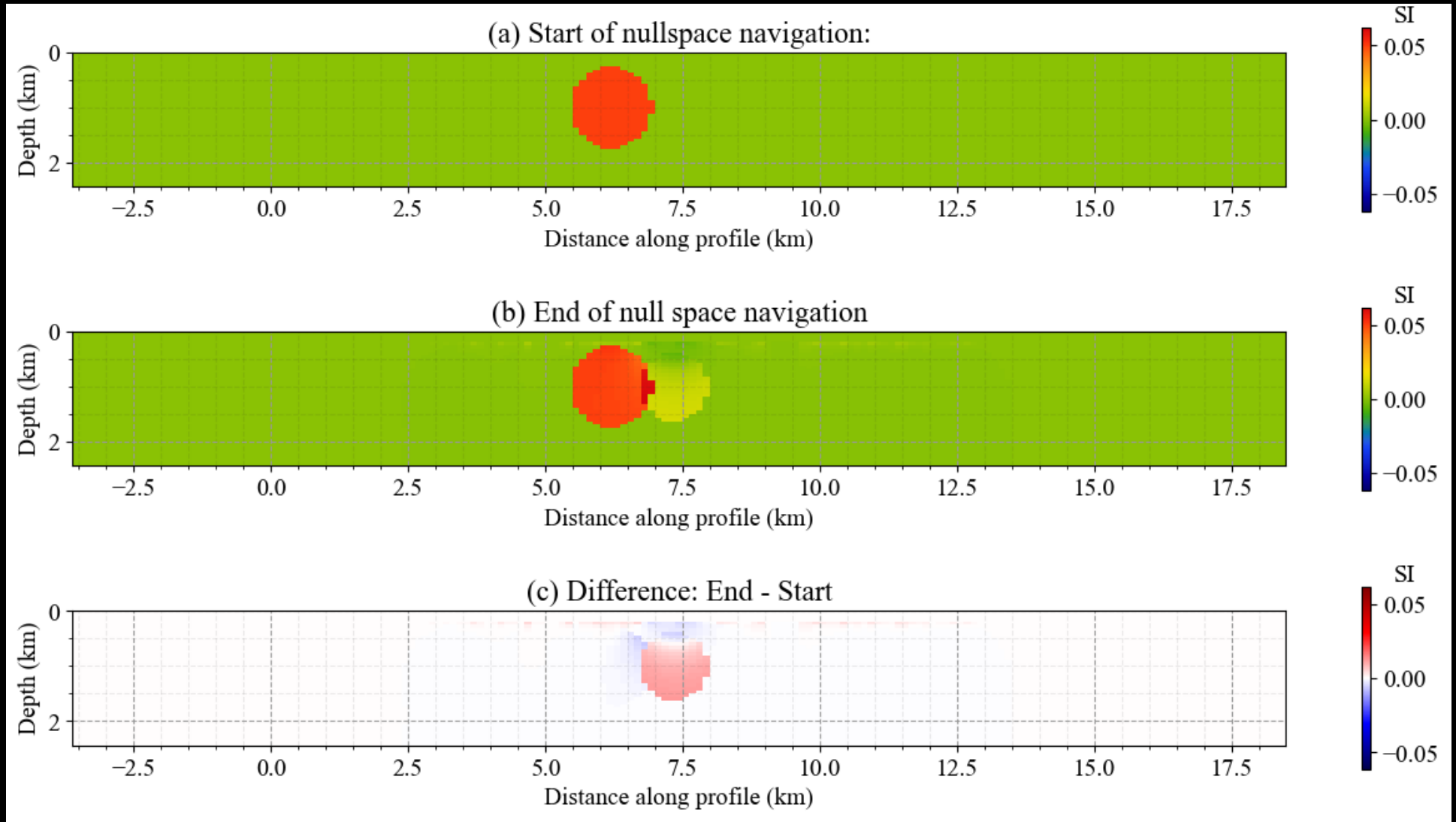


- Positivity constraints – still some flexibility in intermediate models





- Positivity constraints – almost no flexibility



References

- De Wit , R.W.L. , Trampert, J. & Van Der Hilst, R.D., 2012. Toward quanti- fying uncertainty in travel time tomography using the null-space shuttle, J. geophys. Res., 117, 1–20.
- Deal , M.M. & Nolet, G., 1996. Nullspace shuttles, Geophys. J. Int., 124, 372–380.
- Fichtner , A. & Zunino, A. 2019. Hamiltonian nullspace shuttles. Geophys. Res. Lett., 46, 644–651.

Questions?