Lab 6

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Load the Boston Housing data and create the vector y and the design matrix X.

```
data(Boston, package = "MASS")
y = Boston$medv
intecp = rep(1, nrow(Boston))
X = as.matrix(cbind(intecp, Boston[, 1 : 13]))
```

Find the OLS estimate and OLS predictions without using `lm`.

```
b = solve(t(X) %*% X) %*% t(X) %*% y
b
```

```
##                  [,1]
## intecp   3.645949e+01
## crim    -1.080114e-01
## zn       4.642046e-02
## indus    2.055863e-02
## chas     2.686734e+00
## nox     -1.776661e+01
## rm       3.809865e+00
## age      6.922246e-04
## dis     -1.475567e+00
## rad      3.060495e-01
## tax     -1.233459e-02
## ptratio -9.527472e-01
## black    9.311683e-03
## lstat   -5.247584e-01
```

```
yhat = X %*% b
yhat
```

```
##           [,1]
## 1   30.0038434
## 2   25.0255624
## 3   30.5675967
## 4   28.6070365
## 5   27.9435242
## 6   25.2562845
## 7   23.0018083
## 8   19.5359884
## 9   11.5236369
## 10  18.9202621
## 11  18.9994965
## 12  21.5867957
## 13  20.9065215
## 14  19.5529028
## 15  19.2834821
## 16  19.2974832
## 17  20.5275098
```

```
## 18  16.9114013
## 19  16.1780111
## 20  18.4061360
## 21  12.5238575
## 22  17.6710367
## 23  15.8328813
## 24  13.8062853
## 25  15.6783383
## 26  13.3866856
## 27  15.4639765
## 28  14.7084743
## 29  19.5473729
## 30  20.8764282
## 31  11.4551176
## 32  18.0592329
## 33   8.8110574
## 34  14.2827581
## 35  13.7067589
## 36  23.8146353
## 37  22.3419371
## 38  23.1089114
## 39  22.9150261
## 40  31.3576257
## 41  34.2151023
## 42  28.0205641
## 43  25.2038663
## 44  24.6097927
## 45  22.9414918
## 46  22.0966982
## 47  20.4232003
## 48  18.0365509
## 49   9.1065538
## 50  17.2060775
## 51  21.2815254
## 52  23.9722228
## 53  27.6558508
## 54  24.0490181
## 55  15.3618477
## 56  31.1526495
## 57  24.8568698
## 58  33.1091981
## 59  21.7753799
## 60  21.0849356
## 61  17.8725804
## 62  18.5111021
## 63  23.9874286
## 64  22.5540887
## 65  23.3730864
## 66  30.3614836
## 67  25.5305651
## 68  21.1133856
## 69  17.4215379
## 70  20.7848363
## 71  25.2014886
```

```
## 72  21.7426577
## 73  24.5574496
## 74  24.0429571
## 75  25.5049972
## 76  23.9669302
## 77  22.9454540
## 78  23.3569982
## 79  21.2619827
## 80  22.4281737
## 81  28.4057697
## 82  26.9948609
## 83  26.0357630
## 84  25.0587348
## 85  24.7845667
## 86  27.7904920
## 87  22.1685342
## 88  25.8927642
## 89  30.6746183
## 90  30.8311062
## 91  27.1190194
## 92  27.4126673
## 93  28.9412276
## 94  29.0810555
## 95  27.0397736
## 96  28.6245995
## 97  24.7274498
## 98  35.7815952
## 99  35.1145459
## 100 32.2510280
## 101 24.5802202
## 102 25.5941347
## 103 19.7901368
## 104 20.3116713
## 105 21.4348259
## 106 18.5399401
## 107 17.1875599
## 108 20.7504903
## 109 22.6482911
## 110 19.7720367
## 111 20.6496586
## 112 26.5258674
## 113 20.7732364
## 114 20.7154831
## 115 25.1720888
## 116 20.4302559
## 117 23.3772463
## 118 23.6904326
## 119 20.3357836
## 120 20.7918087
## 121 21.9163207
## 122 22.4710778
## 123 20.5573856
## 124 16.3666198
## 125 20.5609982
```

```
## 126 22.4817845
## 127 14.6170663
## 128 15.1787668
## 129 18.9386859
## 130 14.0557329
## 131 20.0352740
## 132 19.4101340
## 133 20.0619157
## 134 15.7580767
## 135 13.2564524
## 136 17.2627773
## 137 15.8784188
## 138 19.3616395
## 139 13.8148390
## 140 16.4488147
## 141 13.5714193
## 142  3.9888551
## 143 14.5949548
## 144 12.1488148
## 145  8.7282236
## 146 12.0358534
## 147 15.8208206
## 148  8.5149902
## 149  9.7184414
## 150 14.8045137
## 151 20.8385815
## 152 18.3010117
## 153 20.1228256
## 154 17.2860189
## 155 22.3660023
## 156 20.1037592
## 157 13.6212589
## 158 33.2598270
## 159 29.0301727
## 160 25.5675277
## 161 32.7082767
## 162 36.7746701
## 163 40.5576584
## 164 41.8472817
## 165 24.7886738
## 166 25.3788924
## 167 37.2034745
## 168 23.0874875
## 169 26.4027396
## 170 26.6538211
## 171 22.5551466
## 172 24.2908281
## 173 22.9765722
## 174 29.0719431
## 175 26.5219434
## 176 30.7220906
## 177 25.6166931
## 178 29.1374098
## 179 31.4357197
```

```
## 180 32.9223157
## 181 34.7244046
## 182 27.7655211
## 183 33.8878732
## 184 30.9923804
## 185 22.7182001
## 186 24.7664781
## 187 35.8849723
## 188 33.4247672
## 189 32.4119915
## 190 34.5150995
## 191 30.7610949
## 192 30.2893414
## 193 32.9191871
## 194 32.1126077
## 195 31.5587100
## 196 40.8455572
## 197 36.1277008
## 198 32.6692081
## 199 34.7046912
## 200 30.0934516
## 201 30.6439391
## 202 29.2871950
## 203 37.0714839
## 204 42.0319312
## 205 43.1894984
## 206 22.6903480
## 207 23.6828471
## 208 17.8544721
## 209 23.4942899
## 210 17.0058772
## 211 22.3925110
## 212 17.0604275
## 213 22.7389292
## 214 25.2194255
## 215 11.1191674
## 216 24.5104915
## 217 26.6033477
## 218 28.3551871
## 219 24.9152546
## 220 29.6865277
## 221 33.1841975
## 222 23.7745666
## 223 32.1405196
## 224 29.7458199
## 225 38.3710245
## 226 39.8146187
## 227 37.5860575
## 228 32.3995325
## 229 35.4566524
## 230 31.2341151
## 231 24.4844923
## 232 33.2883729
## 233 38.0481048
```

```
## 234 37.1632863
## 235 31.7138352
## 236 25.2670557
## 237 30.1001074
## 238 32.7198716
## 239 28.4271706
## 240 28.4294068
## 241 27.2937594
## 242 23.7426248
## 243 24.1200789
## 244 27.4020841
## 245 16.3285756
## 246 13.3989126
## 247 20.0163878
## 248 19.8618443
## 249 21.2883131
## 250 24.0798915
## 251 24.2063355
## 252 25.0421582
## 253 24.9196401
## 254 29.9456337
## 255 23.9722832
## 256 21.6958089
## 257 37.5110924
## 258 43.3023904
## 259 36.4836142
## 260 34.9898859
## 261 34.8121151
## 262 37.1663133
## 263 40.9892850
## 264 34.4463409
## 265 35.8339755
## 266 28.2457430
## 267 31.2267359
## 268 40.8395575
## 269 39.3179239
## 270 25.7081791
## 271 22.3029553
## 272 27.2034097
## 273 28.5116947
## 274 35.4767660
## 275 36.1063916
## 276 33.7966827
## 277 35.6108586
## 278 34.8399338
## 279 30.3519266
## 280 35.3098070
## 281 38.7975697
## 282 34.3312319
## 283 40.3396307
## 284 44.6730834
## 285 31.5968909
## 286 27.3565923
## 287 20.1017415
```

```
## 288 27.0420667
## 289 27.2136458
## 290 26.9139584
## 291 33.4356331
## 292 34.4034963
## 293 31.8333982
## 294 25.8178324
## 295 24.4298235
## 296 28.4576434
## 297 27.3626700
## 298 19.5392876
## 299 29.1130984
## 300 31.9105461
## 301 30.7715945
## 302 28.9427587
## 303 28.8819102
## 304 32.7988723
## 305 33.2090546
## 306 30.7683179
## 307 35.5622686
## 308 32.7090512
## 309 28.6424424
## 310 23.5896583
## 311 18.5426690
## 312 26.8788984
## 313 23.2813398
## 314 25.5458025
## 315 25.4812006
## 316 20.5390990
## 317 17.6157257
## 318 18.3758169
## 319 24.2907028
## 320 21.3252904
## 321 24.8868224
## 322 24.8693728
## 323 22.8695245
## 324 19.4512379
## 325 25.1178340
## 326 24.6678691
## 327 23.6807618
## 328 19.3408962
## 329 21.1741811
## 330 24.2524907
## 331 21.5926089
## 332 19.9844661
## 333 23.3388800
## 334 22.1406069
## 335 21.5550993
## 336 20.6187291
## 337 20.1609718
## 338 19.2849039
## 339 22.1667232
## 340 21.2496577
## 341 21.4293931
```

```
## 342 30.3278880
## 343 22.0473498
## 344 27.7064791
## 345 28.5479412
## 346 16.5450112
## 347 14.7835964
## 348 25.2738008
## 349 27.5420512
## 350 22.1483756
## 351 20.4594409
## 352 20.5460542
## 353 16.8806383
## 354 25.4025351
## 355 14.3248663
## 356 16.5948846
## 357 19.6370469
## 358 22.7180661
## 359 22.2021889
## 360 19.2054806
## 361 22.6661611
## 362 18.9319262
## 363 18.2284680
## 364 20.2315081
## 365 37.4944739
## 366 14.2819073
## 367 15.5428625
## 368 10.8316232
## 369 23.8007290
## 370 32.6440736
## 371 34.6068404
## 372 24.9433133
## 373 25.9998091
## 374  6.1263250
## 375  0.7777981
## 376 25.3071306
## 377 17.7406106
## 378 20.2327441
## 379 15.8333130
## 380 16.8351259
## 381 14.3699483
## 382 18.4768283
## 383 13.4276828
## 384 13.0617751
## 385  3.2791812
## 386  8.0602217
## 387  6.1284220
## 388  5.6186481
## 389  6.4519857
## 390 14.2076474
## 391 17.2122518
## 392 17.2988727
## 393  9.8911664
## 394 20.2212419
## 395 17.9418118
```

```
## 396 20.3044578
## 397 19.2955908
## 398 16.3363278
## 399  6.5516232
## 400 10.8901678
## 401 11.8814587
## 402 17.8117451
## 403 18.2612659
## 404 12.9794878
## 405  7.3781636
## 406  8.2111586
## 407  8.0662619
## 408 19.9829479
## 409 13.7075637
## 410 19.8526845
## 411 15.2230830
## 412 16.9607198
## 413  1.7185181
## 414 11.8057839
## 415 -4.2813107
## 416  9.5837674
## 417 13.3666081
## 418  6.8956236
## 419  6.1477985
## 420 14.6066179
## 421 19.6000267
## 422 18.1242748
## 423 18.5217713
## 424 13.1752861
## 425 14.6261762
## 426  9.9237498
## 427 16.3459065
## 428 14.0751943
## 429 14.2575624
## 430 13.0423479
## 431 18.1595569
## 432 18.6955435
## 433 21.5272830
## 434 17.0314186
## 435 15.9609044
## 436 13.3614161
## 437 14.5207938
## 438  8.8197601
## 439  4.8675110
## 440 13.0659131
## 441 12.7060970
## 442 17.2955806
## 443 18.7404850
## 444 18.0590103
## 445 11.5147468
## 446 11.9740036
## 447 17.6834462
## 448 18.1269524
## 449 17.5183465
```

```
## 450 17.2274251
## 451 16.5227163
## 452 19.4129110
## 453 18.5821524
## 454 22.4894479
## 455 15.2800013
## 456 15.8208934
## 457 12.6872558
## 458 12.8763379
## 459 17.1866853
## 460 18.5124761
## 461 19.0486053
## 462 20.1720893
## 463 19.7740732
## 464 22.4294077
## 465 20.3191185
## 466 17.8861625
## 467 14.3747852
## 468 16.9477685
## 469 16.9840576
## 470 18.5883840
## 471 20.1671944
## 472 22.9771803
## 473 22.4558073
## 474 25.5782463
## 475 16.3914763
## 476 16.1114628
## 477 20.5348160
## 478 11.5427274
## 479 19.2049630
## 480 21.8627639
## 481 23.4687887
## 482 27.0988732
## 483 28.5699430
## 484 21.0839878
## 485 19.4551620
## 486 22.2222591
## 487 19.6559196
## 488 21.3253610
## 489 11.8558372
## 490  8.2238669
## 491  3.6639967
## 492 13.7590854
## 493 15.9311855
## 494 20.6266205
## 495 20.6124941
## 496 16.8854196
## 497 14.0132079
## 498 19.1085414
## 499 21.2980517
## 500 18.4549884
## 501 20.4687085
## 502 23.5333405
## 503 22.3757189
```

```
## 504 27.6274261
## 505 26.1279668
## 506 22.3442123
```

Write a function spec'd as follows:

```
#' Orthogonal Projection
#'
#' Projects vector a onto v.
#'
#' @param a   the vector to project
#' @param v   the vector projected onto
#'
#' @returns   a list of two vectors, the orthogonal projection parallel to v named a_parallel,
#'            and the orthogonal error orthogonal to v called a_perpendicular
orthogonal_projection = function(a, v){
  a_parallel = (v %*% t(v) %*% a) / ( sum(v^2) )
  a_perpendicular = a - a_parallel
  list("a_parallel" = a_parallel, "a_perpendicular" = a_perpendicular)
}
orthogonal_projection(c(1,2,3,4), c(1,2,3,4))
```

```
## $a_parallel
##      [,1]
## [1,]    1
## [2,]    2
## [3,]    3
## [4,]    4
##
## $a_perpendicular
##      [,1]
## [1,]    0
## [2,]    0
## [3,]    0
## [4,]    0
```

```
orthogonal_projection(c(1, 2, 3, 4), c(0, 2, 0, -1))
```

```
## $a_parallel
##      [,1]
## [1,]    0
## [2,]    0
## [3,]    0
## [4,]    0
##
## $a_perpendicular
##      [,1]
## [1,]    1
## [2,]    2
## [3,]    3
## [4,]    4
```

```
result = orthogonal_projection(c(2, 6, 7, 3), c(1, 3, 5, 7))
t(result$a_parallel) %*% result$a_perpendicular
```

```
##              [,1]
## [1,] 7.105427e-15
```

```
result$a_parallel + result$a_perpendicular
```

```
##      [,1]
## [1,]    2
## [2,]    6
## [3,]    7
## [4,]    3
```

```
result$a_parallel / c(1, 3, 5 ,7)
```

```
##           [,1]
## [1,] 0.9047619
## [2,] 0.9047619
## [3,] 0.9047619
## [4,] 0.9047619
```

Try to project onto the column space of X by projecting on each vector of X individually and adding up the projections. You can use the function `orthogonal_projection`.

```
sumOrthProj = rep(0, nrow(X))
for (j in 1 : ncol(X)){
  sumOrthProj = sumOrthProj + orthogonal_projection(y, X[, j])$a_parallel
  }
head(sumOrthProj)
```

```
##           [,1]
## [1,] 177.3425
## [2,] 185.6013
## [3,] 177.7175
## [4,] 171.7247
## [5,] 177.3255
## [6,] 175.5639
```

How much double counting occurred? Measure the magnitude relative to the true LS orthogonal projection.

```
sumOrthProj / yhat
```

```
##           [,1]
## 1     5.910661
## 2     7.416470
## 3     5.813919
## 4     6.002884
## 5     6.345853
## 6     6.951296
## 7     8.691341
## 8    11.148683
## 9    19.871200
## 10   11.345854
## 11   11.643105
## 12    9.655794
## 13    9.329048
## 14    9.730293
## 15   10.206143
## 16    9.721051
## 17    8.689919
## 18   11.835372
## 19   10.802933
```

```
## 20   10.332542
## 21   16.568954
## 22   11.335570
## 23   13.100863
## 24   15.209876
## 25   13.147691
## 26   14.768919
## 27   13.146267
## 28   13.683798
## 29   10.449348
## 30    9.615908
## 31   18.311396
## 32   11.227886
## 33   23.189793
## 34   14.210895
## 35   14.628384
## 36    7.566072
## 37    8.007963
## 38    7.554885
## 39    7.548862
## 40    6.386094
## 41    5.775690
## 42    5.882496
## 43    6.530023
## 44    6.795216
## 45    7.740312
## 46    7.819215
## 47    8.689039
## 48   11.084142
## 49   23.588503
## 50   11.124324
## 51    9.273747
## 52    8.252859
## 53    6.644966
## 54    7.713807
## 55   15.551774
## 56    7.187983
## 57    9.044364
## 58    6.877590
## 59    9.289698
## 60    9.722491
## 61   12.033009
## 62   11.966590
## 63    8.795372
## 64    9.414125
## 65    8.652110
## 66    6.866540
## 67    8.476144
## 68    8.774432
## 69   11.133699
## 70    9.102714
## 71    6.900316
## 72    8.199341
## 73    7.004984
```

```
## 74    7.226426
## 75    6.942073
## 76    7.940294
## 77    8.660062
## 78    8.122664
## 79    9.351979
## 80    8.362056
## 81    6.540045
## 82    7.304924
## 83    7.124111
## 84    7.554720
## 85    7.063346
## 86    6.214067
## 87    7.886034
## 88    6.566771
## 89    5.719955
## 90    5.505551
## 91    6.261892
## 92    6.246109
## 93    6.890952
## 94    6.532225
## 95    7.676570
## 96    5.729987
## 97    6.987225
## 98    4.855574
## 99    4.601449
## 100   5.270771
## 101   7.880296
## 102   7.426405
## 103   8.852713
## 104   9.735666
## 105   9.131698
## 106  10.764080
## 107  11.713379
## 108   9.375948
## 109   8.780735
## 110  10.139101
## 111   9.150301
## 112   7.423232
## 113   9.731909
## 114   9.923722
## 115   7.699714
## 116   9.689474
## 117   8.332015
## 118   8.235026
## 119   9.490487
## 120   9.283870
## 121   9.167556
## 122   9.082761
## 123  10.209300
## 124  13.309969
## 125  10.196204
## 126   9.131814
## 127  14.860871
```

```
## 128  14.538406
## 129  11.750455
## 130  15.865058
## 131  11.025317
## 132  11.388863
## 133  10.927960
## 134  14.066853
## 135  16.356883
## 136  13.046616
## 137  13.895850
## 138  11.447417
## 139  16.352287
## 140  13.627782
## 141  16.840221
## 142  59.668586
## 143  17.679838
## 144  18.954229
## 145  26.324018
## 146  18.239333
## 147  12.976190
## 148  26.958868
## 149  23.339506
## 150  14.914879
## 151  10.375446
## 152  11.542708
## 153  11.616815
## 154  12.173099
## 155  10.855253
## 156  11.249235
## 157  14.544679
## 158   5.983270
## 159   6.801784
## 160   8.247291
## 161   6.812143
## 162   5.365453
## 163   5.684315
## 164   5.577436
## 165   8.334924
## 166   7.723049
## 167   5.454290
## 168   8.377645
## 169   7.624672
## 170   7.672963
## 171   9.029523
## 172   8.463821
## 173   7.914424
## 174   6.107522
## 175   6.495013
## 176   5.288278
## 177   6.706342
## 178   5.988441
## 179   5.575209
## 180   4.894688
## 181   4.979287
```

```
## 182   5.868350
## 183   5.007036
## 184   5.493430
## 185   7.696028
## 186   6.925809
## 187   4.565691
## 188   5.740158
## 189   5.818237
## 190   5.656392
## 191   6.419455
## 192   6.583554
## 193   6.010027
## 194   5.890193
## 195   5.989549
## 196   4.967317
## 197   5.820175
## 198   6.512080
## 199   6.163124
## 200   7.415124
## 201   7.259434
## 202   7.220706
## 203   5.557519
## 204   5.008910
## 205   4.854505
## 206   7.748389
## 207   7.911605
## 208  11.108943
## 209   9.333790
## 210  13.894175
## 211  10.251368
## 212  13.677936
## 213   9.512994
## 214   7.052794
## 215  16.832092
## 216   7.361689
## 217   8.149528
## 218   6.912252
## 219   9.278101
## 220   7.616102
## 221   6.575950
## 222   9.691250
## 223   6.764018
## 224   6.273228
## 225   4.827985
## 226   4.738376
## 227   4.980056
## 228   5.708122
## 229   4.754171
## 230   5.303871
## 231   7.600438
## 232   5.595941
## 233   4.919282
## 234   5.012274
## 235   6.673621
```

```
## 236   7.269253
## 237   7.301067
## 238   5.726931
## 239   6.603715
## 240   6.888473
## 241   7.538683
## 242   8.719446
## 243   8.518055
## 244   6.826710
## 245  13.318163
## 246  16.684206
## 247  10.305690
## 248  11.003761
## 249   9.839479
## 250   8.307363
## 251   8.069536
## 252   7.601048
## 253   7.999475
## 254   6.857909
## 255   9.267645
## 256  10.189042
## 257   5.684631
## 258   4.329537
## 259   5.180554
## 260   5.352865
## 261   5.380558
## 262   5.050617
## 263   4.649308
## 264   5.603390
## 265   5.220233
## 266   6.258139
## 267   6.177355
## 268   4.488101
## 269   4.429672
## 270   8.485796
## 271   8.341916
## 272   6.392067
## 273   6.482513
## 274   6.097819
## 275   5.906462
## 276   5.565552
## 277   6.322659
## 278   6.210644
## 279   6.194772
## 280   4.779296
## 281   4.688133
## 282   5.110612
## 283   5.126844
## 284   5.135968
## 285   6.865861
## 286   7.308242
## 287  11.045675
## 288   7.689121
## 289   7.812485
```

```
## 290   7.733285
## 291   6.182611
## 292   6.045868
## 293   6.478338
## 294   6.984656
## 295   7.705271
## 296   6.512145
## 297   6.982427
## 298  10.319550
## 299   7.140008
## 300   6.508511
## 301   7.136281
## 302   6.927096
## 303   6.672039
## 304   5.806403
## 305   5.629823
## 306   6.119028
## 307   5.371305
## 308   5.811331
## 309   6.514872
## 310   7.952130
## 311   9.298012
## 312   6.527059
## 313   8.247498
## 314   7.378693
## 315   7.619630
## 316   9.376786
## 317  11.490406
## 318  10.705019
## 319   7.782840
## 320   8.950467
## 321   7.359624
## 322   7.364077
## 323   7.986783
## 324   9.871253
## 325   7.152633
## 326   7.090399
## 327   7.587434
## 328   9.831731
## 329   8.259594
## 330   7.047863
## 331   8.285094
## 332   9.840387
## 333   8.114268
## 334   8.218869
## 335   8.500449
## 336   8.767836
## 337   9.010689
## 338   9.739688
## 339   7.961549
## 340   8.460632
## 341   8.524500
## 342   6.386787
## 343   8.430817
```

```
## 344   7.737793
## 345   7.274381
## 346  11.736803
## 347  13.250632
## 348   9.105400
## 349   8.006187
## 350   9.303835
## 351  10.146250
## 352  11.073689
## 353  13.287098
## 354   9.436634
## 355  16.387820
## 356  13.977663
## 357  14.725115
## 358  12.455065
## 359  12.613567
## 360  13.041782
## 361  10.898330
## 362  13.286399
## 363  13.494326
## 364  13.681064
## 365   7.288281
## 366  15.990648
## 367  15.436951
## 368  20.995886
## 369   9.632030
## 370   8.092950
## 371   7.672383
## 372   9.702397
## 373  10.201896
## 374  44.062459
## 375 353.388173
## 376  10.274070
## 377  14.819090
## 378  12.928754
## 379  17.074551
## 380  15.744463
## 381  21.043938
## 382  14.335925
## 383  19.492258
## 384  20.045501
## 385  79.871474
## 386  33.757532
## 387  44.399396
## 388  48.759299
## 389  41.660873
## 390  18.154357
## 391  14.787073
## 392  14.624165
## 393  26.666119
## 394  12.519440
## 395  14.303409
## 396  12.678116
## 397  13.338453
```

```
## 398  15.729422
## 399  43.594644
## 400  23.908052
## 401  23.228046
## 402  14.802620
## 403  14.248572
## 404  20.428196
## 405  37.507191
## 406  35.823279
## 407  31.980928
## 408  12.201001
## 409  18.534607
## 410  12.469947
## 411  15.965974
## 412  14.131574
## 413 145.211267
## 414  21.391096
## 415 -64.364659
## 416  26.511600
## 417  18.300269
## 418  36.865340
## 419  44.642881
## 420  16.508682
## 421  12.940361
## 422  13.763437
## 423  13.064494
## 424  17.682921
## 425  15.073905
## 426  24.540455
## 427  13.455100
## 428  17.219690
## 429  16.655413
## 430  18.764439
## 431  12.810184
## 432  12.827557
## 433  10.407136
## 434  13.930590
## 435  15.134688
## 436  18.829396
## 437  16.599566
## 438  28.345929
## 439  52.768402
## 440  20.083432
## 441  21.182246
## 442  15.210066
## 443  13.754771
## 444  14.569707
## 445  22.383846
## 446  20.609287
## 447  14.393901
## 448  14.332165
## 449  14.925251
## 450  14.884619
## 451  14.258090
```

```
## 452  13.301809
## 453  13.798287
## 454  11.715381
## 455  15.718414
## 456  14.943886
## 457  18.439055
## 458  18.021972
## 459  14.530696
## 460  13.730143
## 461  13.093840
## 462  12.561008
## 463  12.838585
## 464  11.230805
## 465  12.185031
## 466  13.214758
## 467  16.023082
## 468  14.921109
## 469  14.911339
## 470  13.168825
## 471  12.459148
## 472  10.778469
## 473  10.972020
## 474   9.476923
## 475  15.272746
## 476  15.835979
## 477  12.451084
## 478  22.647788
## 479  13.337612
## 480  11.437749
## 481  10.275873
## 482   8.915264
## 483   8.505154
## 484  11.091519
## 485  12.153505
## 486  10.833144
## 487  12.799854
## 488  11.122687
## 489  20.055817
## 490  29.464586
## 491  67.203324
## 492  17.532022
## 493  14.655283
## 494   9.095363
## 495   9.083673
## 496  11.186260
## 497  14.539348
## 498  10.317491
## 499   9.057901
## 500  10.605788
## 501   9.709512
## 502   7.911682
## 503   8.275729
## 504   6.791971
## 505   7.208885
```

```
## 506   8.307545
```

Convert X into Q where Q has the same column space as X but has orthogonal columns. You can use the function `orthogonal_projection`. This is essentially gram-schmidt.

```
Q = matrix(NA, nrow = nrow(X), ncol = ncol(X))
Q[, 1] = X[, 1] #first column stays the same
#Q[, 2] = orthogonal_projection(X[, 2], X[, 1])$a_perpendicular
#Q[, 3] = orthogonal_projection(X[, 3], Q[, 2])$a_parallel +
#            orthogonal_projection(X[,3], Q[, 1])$a_parallel
#Q[, 4] = orthogonal_projection(X[, 4], Q[, 3])$a_parallel +
#            orthogonal_projection(X[, 4], Q[, 2])$a_parallel+
#            orthogonal_projection(X[, 4], Q[, 1])$a_parallel
for(j in 2 : ncol(X)){
   Q[ , j] = X[ , j]

   for(j0 in 1 : (j - 1)){
    Q[ , j] = Q[ , j] - (orthogonal_projection(X[ , j], Q[ , j0])$a_parallel)
  }
}
head(Q)
```

```
##      [,1]      [,2]       [,3]      [,4]        [,5]         [,6]
## [1,]    1 -3.607204   4.675641 -7.008889 -0.04710427  0.089429446
## [2,]    1 -3.586214 -13.312949 -4.749312 -0.06618241 -0.043347289
## [3,]    1 -3.586234 -13.312960 -4.749307 -0.06618247 -0.043347251
## [4,]    1 -3.581154 -13.310199 -9.640571 -0.04861445 -0.003114958
## [5,]    1 -3.544474 -13.290261 -9.649702 -0.04850894 -0.003184462
## [6,]    1 -3.583674 -13.311569 -9.639944 -0.04862170 -0.003110183
##             [,7]        [,8]       [,9]      [,10]      [,11]      [,12]
## [1,] -0.02387917   5.6213677 -0.4710629 -3.9409950  70.857436 -1.3904706
## [2,]  0.07722025  19.9133375  0.9390974 -2.8368530 -19.813048 -0.7644025
## [3,]  0.84122013   0.6655076  0.8905778 -3.7378212 -11.063616 -0.1438224
## [4,]  0.49714449 -10.3247147  1.3136625 -1.1185303 -10.232003  0.7785626
## [5,]  0.64635236  -2.2135279  1.5205996 -1.1462130  -9.841071  0.7867933
## [6,] -0.07086980   3.6521280  1.3436340 -0.4522033 -16.700021  0.3203511
##           [,13]      [,14]
## [1,]  8.9282409 -4.0730020
## [2,]  3.0628248 -2.1681642
## [3,]  2.6197087 -2.3622542
## [4,]  0.4844616 -2.7345457
## [5,]  2.8782413 -0.4190544
## [6,] -2.7082320 -4.1184588
```

```
#test the rank
pacman::p_load(Matrix)
rankMatrix(Q)
```

```
## [1] 14
## attr(,"method")
## [1] "tolNorm2"
## attr(,"useGrad")
## [1] FALSE
## attr(,"tol")
## [1] 1.123546e-13
```

```r
dim(Q)
```

```
## [1] 506  14
```

```r
ncol(X)
```

```
## [1] 14
```

```r
#we want columns to be orthogonal so lets do dot products
#0s everywhere except for diagnol
t(Q) %*% Q
```

```
##                [,1]          [,2]          [,3]          [,4]
##  [1,]  5.060000e+02 -1.544542e-12 -8.473222e-13 -1.064282e-11
##  [2,] -1.544542e-12  3.736322e+04  1.833200e-12  1.820544e-12
##  [3,] -8.473222e-13  1.833200e-12  2.636490e+05  4.443779e-12
##  [4,] -1.064282e-11  1.820544e-12  4.443779e-12  1.477223e+04
##  [5,]  4.116152e-14  3.180789e-14  1.194600e-13  7.313386e-13
##  [6,]  2.738278e-13  2.109771e-13  1.129652e-14  5.964510e-12
##  [7,] -4.435674e-12  2.954414e-12 -1.170175e-12  6.620642e-11
##  [8,] -2.233413e-11 -3.858247e-12  9.720225e-12 -1.070166e-10
##  [9,] -6.893375e-13  3.677059e-12 -1.001865e-12  1.132529e-10
## [10,]  2.939871e-12 -5.329071e-12 -3.808509e-12 -9.987211e-11
## [11,]  4.102674e-11  1.738272e-10 -2.785328e-12  3.081497e-09
## [12,] -1.135136e-11  8.789414e-12  7.247536e-13  2.656571e-10
## [13,]  4.072831e-10  1.519851e-10 -5.897505e-11  4.968760e-09
## [14,] -1.388312e-11  1.529799e-11 -5.783818e-12  3.403625e-10
##                [,5]          [,6]          [,7]          [,8]
##  [1,]  4.116152e-14  2.738278e-13 -4.435674e-12 -2.233413e-11
##  [2,]  3.180789e-14  2.109771e-13  2.954414e-12 -3.858247e-12
##  [3,]  1.194600e-13  1.129652e-14 -1.170175e-12  9.720225e-12
##  [4,]  7.313386e-13  5.964510e-12  6.620642e-11 -1.070166e-10
##  [5,]  3.218831e+01 -2.675475e-14 -1.918830e-13 -2.806644e-13
##  [6,] -2.675475e-14  2.591084e+00 -1.536766e-12 -2.640610e-11
##  [7,] -1.918830e-13 -1.536766e-12  2.029377e+02  3.697231e-10
##  [8,] -2.806644e-13 -2.640610e-11  3.697231e-10  1.617318e+05
##  [9,] -3.403527e-13 -1.304247e-12  2.052783e-11  2.128964e-12
## [10,]  5.884182e-14 -4.850051e-12  3.982170e-11  5.506209e-10
## [11,] -1.479150e-11 -1.340538e-10  6.804788e-10  1.165498e-08
## [12,] -9.342527e-13 -5.017084e-12  5.508982e-11  3.352234e-10
## [13,] -1.553480e-11 -9.259564e-11  1.604291e-09  6.060120e-09
## [14,] -1.191491e-12 -1.036152e-11  1.720823e-11  2.285184e-09
##                [,9]         [,10]         [,11]         [,12]
##  [1,] -6.893375e-13  2.939871e-12  4.102674e-11 -1.135136e-11
##  [2,]  3.677059e-12 -5.329071e-12  1.738272e-10  8.789414e-12
##  [3,] -1.001865e-12 -3.808509e-12 -2.785328e-12  7.247536e-13
##  [4,]  1.132529e-10 -9.987211e-11  3.081497e-09  2.656571e-10
##  [5,] -3.403527e-13  5.884182e-14 -1.479150e-11 -9.342527e-13
##  [6,] -1.304247e-12 -4.850051e-12 -1.340538e-10 -5.017084e-12
##  [7,]  2.052783e-11  3.982170e-11  6.804788e-10  5.508982e-11
##  [8,]  2.128964e-12  5.506209e-10  1.165498e-08  3.352234e-10
##  [9,]  5.742738e+02 -4.222489e-11 -4.938201e-10  1.419753e-11
## [10,] -4.222489e-11  1.664085e+04  2.342631e-09 -6.246736e-11
## [11,] -4.938201e-10  2.342631e-09  1.602478e+06 -1.758217e-09
## [12,]  1.419753e-11 -6.246736e-11 -1.758217e-09  1.319301e+03
```

```
## [13,]  3.850618e-10 -2.053042e-09 -1.707542e-08  4.196387e-09
## [14,]  6.702461e-11  2.036771e-10  1.914600e-09  3.358842e-10
##               [,13]         [,14]
##  [1,]  4.072831e-10 -1.388312e-11
##  [2,]  1.519851e-10  1.529799e-11
##  [3,] -5.897505e-11 -5.783818e-12
##  [4,]  4.968760e-09  3.403625e-10
##  [5,] -1.553480e-11 -1.191491e-12
##  [6,] -9.259564e-11 -1.036152e-11
##  [7,]  1.604291e-09  1.720823e-11
##  [8,]  6.060120e-09  2.285184e-09
##  [9,]  3.850618e-10  6.702461e-11
## [10,] -2.053042e-09  2.036771e-10
## [11,] -1.707542e-08  1.914600e-09
## [12,]  4.196387e-09  3.358842e-10
## [13,]  3.198118e+06 -8.166268e-11
## [14,] -8.166268e-11  8.754864e+03
```

Make Q's columns orthonormal.

```
for(j in 1 : ncol(Q)){
  Q[, j] = Q[, j] / sqrt(sum(Q[, j]^2))  #Qj/size of Qj akka the norm of Qj aka sum of Qji squared
}
```

Verify Q^T is the inverse of Q.

```
t(Q) %*% Q
```

```
##                [,1]          [,2]          [,3]          [,4]
##  [1,]  1.000000e+00 -1.170938e-16  7.329207e-17 -3.932090e-15
##  [2,] -1.170938e-16  1.000000e+00  1.566672e-17  6.763727e-17
##  [3,]  7.329207e-17  1.566672e-17  1.000000e+00 -5.826231e-17
##  [4,] -3.932090e-15  6.763727e-17 -5.826231e-17  1.000000e+00
##  [5,]  3.044440e-16  4.510281e-17  3.794708e-19  1.051744e-15
##  [6,]  7.548107e-15  6.550750e-16  5.526721e-17  3.046028e-14
##  [7,] -1.379756e-14  1.082847e-15 -2.208520e-16  3.826098e-14
##  [8,] -2.475017e-15 -7.361733e-17  5.084908e-17 -2.164291e-15
##  [9,] -1.269384e-15  7.773730e-16  2.385245e-18  3.891581e-14
## [10,]  1.098514e-15 -2.138047e-16 -9.540979e-18 -6.627464e-15
## [11,]  1.463239e-15  7.455516e-16  4.065758e-17  2.017742e-14
## [12,] -1.382228e-14  1.229485e-15  2.602085e-17  6.014552e-14
## [13,]  1.006416e-14  2.636644e-16 -5.095750e-17  2.289555e-14
## [14,] -6.628812e-15  8.515324e-16 -1.021318e-16  2.996148e-14
##                [,5]          [,6]          [,7]          [,8]
##  [1,]  3.044440e-16  7.548107e-15 -1.379756e-14 -2.475017e-15
##  [2,]  4.510281e-17  6.550750e-16  1.082847e-15 -7.361733e-17
##  [3,]  3.794708e-19  5.526721e-17 -2.208520e-16  5.084908e-17
##  [4,]  1.051744e-15  3.046028e-14  3.826098e-14 -2.164291e-15
##  [5,]  1.000000e+00 -2.882202e-15 -2.479679e-15 -1.329232e-16
##  [6,] -2.882202e-15  1.000000e+00 -6.696465e-14 -4.081119e-14
##  [7,] -2.479679e-15 -6.696465e-14  1.000000e+00  6.453291e-14
##  [8,] -1.329232e-16 -4.081119e-14  6.453291e-14  1.000000e+00
##  [9,] -2.511229e-15 -3.385638e-14  6.016531e-14  1.811702e-16
## [10,]  3.783866e-17 -2.339584e-14  2.159926e-14  1.060024e-14
## [11,] -2.035237e-15 -6.567132e-14  3.771779e-14  2.284709e-14
## [12,] -4.422678e-15 -8.574749e-14  1.065354e-13  2.301436e-14
```

```
## [13,] -1.515213e-15 -3.213684e-14  6.298919e-14  8.422896e-15
## [14,] -2.182933e-15 -6.870822e-14  1.289062e-14  6.070513e-14
##                [,9]         [,10]         [,11]         [,12]
##  [1,] -1.269384e-15  1.098514e-15  1.463239e-15 -1.382228e-14
##  [2,]  7.773730e-16 -2.138047e-16  7.455516e-16  1.229485e-15
##  [3,]  2.385245e-18 -9.540979e-18  4.065758e-17  2.602085e-17
##  [4,]  3.891581e-14 -6.627464e-15  2.017742e-14  6.014552e-14
##  [5,] -2.511229e-15  3.783866e-17 -2.035237e-15 -4.422678e-15
##  [6,] -3.385638e-14 -2.339584e-14 -6.567132e-14 -8.574749e-14
##  [7,]  6.016531e-14  2.159926e-14  3.771779e-14  1.065354e-13
##  [8,]  1.811702e-16  1.060024e-14  2.284709e-14  2.301436e-14
##  [9,]  1.000000e+00 -1.368133e-14 -1.628602e-14  1.636278e-14
## [10,] -1.368133e-14  1.000000e+00  1.449112e-14 -1.325676e-14
## [11,] -1.628602e-14  1.449112e-14  1.000000e+00 -3.825694e-14
## [12,]  1.636278e-14 -1.325676e-14 -3.825694e-14  1.000000e+00
## [13,]  8.986952e-15 -8.906396e-15 -7.539284e-15  6.461352e-14
## [14,]  2.987671e-14  1.688667e-14  1.612241e-14  9.881852e-14
##               [,13]         [,14]
##  [1,]  1.006416e-14 -6.628812e-15
##  [2,]  2.636644e-16  8.515324e-16
##  [3,] -5.095750e-17 -1.021318e-16
##  [4,]  2.289555e-14  2.996148e-14
##  [5,] -1.515213e-15 -2.182933e-15
##  [6,] -3.213684e-14 -6.870822e-14
##  [7,]  6.298919e-14  1.289062e-14
##  [8,]  8.422896e-15  6.070513e-14
##  [9,]  8.986952e-15  2.987671e-14
## [10,] -8.906396e-15  1.688667e-14
## [11,] -7.539284e-15  1.612241e-14
## [12,]  6.461352e-14  9.881852e-14
## [13,]  1.000000e+00 -4.839878e-16
## [14,] -4.839878e-16  1.000000e+00
```

Project Y onto Q and verify it is the same as the OLS fit.

```
cbind(Q %*% t(Q) %*% y, yhat)
```

```
##           [,1]       [,2]
## 1   30.0038434 30.0038434
## 2   25.0255624 25.0255624
## 3   30.5675967 30.5675967
## 4   28.6070365 28.6070365
## 5   27.9435242 27.9435242
## 6   25.2562845 25.2562845
## 7   23.0018083 23.0018083
## 8   19.5359884 19.5359884
## 9   11.5236369 11.5236369
## 10  18.9202621 18.9202621
## 11  18.9994965 18.9994965
## 12  21.5867957 21.5867957
## 13  20.9065215 20.9065215
## 14  19.5529028 19.5529028
## 15  19.2834821 19.2834821
## 16  19.2974832 19.2974832
## 17  20.5275098 20.5275098
```

```
## 18  16.9114013 16.9114013
## 19  16.1780111 16.1780111
## 20  18.4061360 18.4061360
## 21  12.5238575 12.5238575
## 22  17.6710367 17.6710367
## 23  15.8328813 15.8328813
## 24  13.8062853 13.8062853
## 25  15.6783383 15.6783383
## 26  13.3866856 13.3866856
## 27  15.4639765 15.4639765
## 28  14.7084743 14.7084743
## 29  19.5473729 19.5473729
## 30  20.8764282 20.8764282
## 31  11.4551176 11.4551176
## 32  18.0592329 18.0592329
## 33   8.8110574  8.8110574
## 34  14.2827581 14.2827581
## 35  13.7067589 13.7067589
## 36  23.8146353 23.8146353
## 37  22.3419371 22.3419371
## 38  23.1089114 23.1089114
## 39  22.9150261 22.9150261
## 40  31.3576257 31.3576257
## 41  34.2151023 34.2151023
## 42  28.0205641 28.0205641
## 43  25.2038663 25.2038663
## 44  24.6097927 24.6097927
## 45  22.9414918 22.9414918
## 46  22.0966982 22.0966982
## 47  20.4232003 20.4232003
## 48  18.0365509 18.0365509
## 49   9.1065538  9.1065538
## 50  17.2060775 17.2060775
## 51  21.2815254 21.2815254
## 52  23.9722228 23.9722228
## 53  27.6558508 27.6558508
## 54  24.0490181 24.0490181
## 55  15.3618477 15.3618477
## 56  31.1526495 31.1526495
## 57  24.8568698 24.8568698
## 58  33.1091981 33.1091981
## 59  21.7753799 21.7753799
## 60  21.0849356 21.0849356
## 61  17.8725804 17.8725804
## 62  18.5111021 18.5111021
## 63  23.9874286 23.9874286
## 64  22.5540887 22.5540887
## 65  23.3730864 23.3730864
## 66  30.3614836 30.3614836
## 67  25.5305651 25.5305651
## 68  21.1133856 21.1133856
## 69  17.4215379 17.4215379
## 70  20.7848363 20.7848363
## 71  25.2014886 25.2014886
```

```
## 72  21.7426577 21.7426577
## 73  24.5574496 24.5574496
## 74  24.0429571 24.0429571
## 75  25.5049972 25.5049972
## 76  23.9669302 23.9669302
## 77  22.9454540 22.9454540
## 78  23.3569982 23.3569982
## 79  21.2619827 21.2619827
## 80  22.4281737 22.4281737
## 81  28.4057697 28.4057697
## 82  26.9948609 26.9948609
## 83  26.0357630 26.0357630
## 84  25.0587348 25.0587348
## 85  24.7845667 24.7845667
## 86  27.7904920 27.7904920
## 87  22.1685342 22.1685342
## 88  25.8927642 25.8927642
## 89  30.6746183 30.6746183
## 90  30.8311062 30.8311062
## 91  27.1190194 27.1190194
## 92  27.4126673 27.4126673
## 93  28.9412276 28.9412276
## 94  29.0810555 29.0810555
## 95  27.0397736 27.0397736
## 96  28.6245995 28.6245995
## 97  24.7274498 24.7274498
## 98  35.7815952 35.7815952
## 99  35.1145459 35.1145459
## 100 32.2510280 32.2510280
## 101 24.5802202 24.5802202
## 102 25.5941347 25.5941347
## 103 19.7901368 19.7901368
## 104 20.3116713 20.3116713
## 105 21.4348259 21.4348259
## 106 18.5399401 18.5399401
## 107 17.1875599 17.1875599
## 108 20.7504903 20.7504903
## 109 22.6482911 22.6482911
## 110 19.7720367 19.7720367
## 111 20.6496586 20.6496586
## 112 26.5258674 26.5258674
## 113 20.7732364 20.7732364
## 114 20.7154831 20.7154831
## 115 25.1720888 25.1720888
## 116 20.4302559 20.4302559
## 117 23.3772463 23.3772463
## 118 23.6904326 23.6904326
## 119 20.3357836 20.3357836
## 120 20.7918087 20.7918087
## 121 21.9163207 21.9163207
## 122 22.4710778 22.4710778
## 123 20.5573856 20.5573856
## 124 16.3666198 16.3666198
## 125 20.5609982 20.5609982
```

```
## 126 22.4817845 22.4817845
## 127 14.6170663 14.6170663
## 128 15.1787668 15.1787668
## 129 18.9386859 18.9386859
## 130 14.0557329 14.0557329
## 131 20.0352740 20.0352740
## 132 19.4101340 19.4101340
## 133 20.0619157 20.0619157
## 134 15.7580767 15.7580767
## 135 13.2564524 13.2564524
## 136 17.2627773 17.2627773
## 137 15.8784188 15.8784188
## 138 19.3616395 19.3616395
## 139 13.8148390 13.8148390
## 140 16.4488147 16.4488147
## 141 13.5714193 13.5714193
## 142  3.9888551  3.9888551
## 143 14.5949548 14.5949548
## 144 12.1488148 12.1488148
## 145  8.7282236  8.7282236
## 146 12.0358534 12.0358534
## 147 15.8208206 15.8208206
## 148  8.5149902  8.5149902
## 149  9.7184414  9.7184414
## 150 14.8045137 14.8045137
## 151 20.8385815 20.8385815
## 152 18.3010117 18.3010117
## 153 20.1228256 20.1228256
## 154 17.2860189 17.2860189
## 155 22.3660023 22.3660023
## 156 20.1037592 20.1037592
## 157 13.6212589 13.6212589
## 158 33.2598270 33.2598270
## 159 29.0301727 29.0301727
## 160 25.5675277 25.5675277
## 161 32.7082767 32.7082767
## 162 36.7746701 36.7746701
## 163 40.5576584 40.5576584
## 164 41.8472817 41.8472817
## 165 24.7886738 24.7886738
## 166 25.3788924 25.3788924
## 167 37.2034745 37.2034745
## 168 23.0874875 23.0874875
## 169 26.4027396 26.4027396
## 170 26.6538211 26.6538211
## 171 22.5551466 22.5551466
## 172 24.2908281 24.2908281
## 173 22.9765722 22.9765722
## 174 29.0719431 29.0719431
## 175 26.5219434 26.5219434
## 176 30.7220906 30.7220906
## 177 25.6166931 25.6166931
## 178 29.1374098 29.1374098
## 179 31.4357197 31.4357197
```

```
## 180 32.9223157 32.9223157
## 181 34.7244046 34.7244046
## 182 27.7655211 27.7655211
## 183 33.8878732 33.8878732
## 184 30.9923804 30.9923804
## 185 22.7182001 22.7182001
## 186 24.7664781 24.7664781
## 187 35.8849723 35.8849723
## 188 33.4247672 33.4247672
## 189 32.4119915 32.4119915
## 190 34.5150995 34.5150995
## 191 30.7610949 30.7610949
## 192 30.2893414 30.2893414
## 193 32.9191871 32.9191871
## 194 32.1126077 32.1126077
## 195 31.5587100 31.5587100
## 196 40.8455572 40.8455572
## 197 36.1277008 36.1277008
## 198 32.6692081 32.6692081
## 199 34.7046912 34.7046912
## 200 30.0934516 30.0934516
## 201 30.6439391 30.6439391
## 202 29.2871950 29.2871950
## 203 37.0714839 37.0714839
## 204 42.0319312 42.0319312
## 205 43.1894984 43.1894984
## 206 22.6903480 22.6903480
## 207 23.6828471 23.6828471
## 208 17.8544721 17.8544721
## 209 23.4942899 23.4942899
## 210 17.0058772 17.0058772
## 211 22.3925110 22.3925110
## 212 17.0604275 17.0604275
## 213 22.7389292 22.7389292
## 214 25.2194255 25.2194255
## 215 11.1191674 11.1191674
## 216 24.5104915 24.5104915
## 217 26.6033477 26.6033477
## 218 28.3551871 28.3551871
## 219 24.9152546 24.9152546
## 220 29.6865277 29.6865277
## 221 33.1841975 33.1841975
## 222 23.7745666 23.7745666
## 223 32.1405196 32.1405196
## 224 29.7458199 29.7458199
## 225 38.3710245 38.3710245
## 226 39.8146187 39.8146187
## 227 37.5860575 37.5860575
## 228 32.3995325 32.3995325
## 229 35.4566524 35.4566524
## 230 31.2341151 31.2341151
## 231 24.4844923 24.4844923
## 232 33.2883729 33.2883729
## 233 38.0481048 38.0481048
```

```
## 234 37.1632863 37.1632863
## 235 31.7138352 31.7138352
## 236 25.2670557 25.2670557
## 237 30.1001074 30.1001074
## 238 32.7198716 32.7198716
## 239 28.4271706 28.4271706
## 240 28.4294068 28.4294068
## 241 27.2937594 27.2937594
## 242 23.7426248 23.7426248
## 243 24.1200789 24.1200789
## 244 27.4020841 27.4020841
## 245 16.3285756 16.3285756
## 246 13.3989126 13.3989126
## 247 20.0163878 20.0163878
## 248 19.8618443 19.8618443
## 249 21.2883131 21.2883131
## 250 24.0798915 24.0798915
## 251 24.2063355 24.2063355
## 252 25.0421582 25.0421582
## 253 24.9196401 24.9196401
## 254 29.9456337 29.9456337
## 255 23.9722832 23.9722832
## 256 21.6958089 21.6958089
## 257 37.5110924 37.5110924
## 258 43.3023904 43.3023904
## 259 36.4836142 36.4836142
## 260 34.9898859 34.9898859
## 261 34.8121151 34.8121151
## 262 37.1663133 37.1663133
## 263 40.9892850 40.9892850
## 264 34.4463409 34.4463409
## 265 35.8339755 35.8339755
## 266 28.2457430 28.2457430
## 267 31.2267359 31.2267359
## 268 40.8395575 40.8395575
## 269 39.3179239 39.3179239
## 270 25.7081791 25.7081791
## 271 22.3029553 22.3029553
## 272 27.2034097 27.2034097
## 273 28.5116947 28.5116947
## 274 35.4767660 35.4767660
## 275 36.1063916 36.1063916
## 276 33.7966827 33.7966827
## 277 35.6108586 35.6108586
## 278 34.8399338 34.8399338
## 279 30.3519266 30.3519266
## 280 35.3098070 35.3098070
## 281 38.7975697 38.7975697
## 282 34.3312319 34.3312319
## 283 40.3396307 40.3396307
## 284 44.6730834 44.6730834
## 285 31.5968909 31.5968909
## 286 27.3565923 27.3565923
## 287 20.1017415 20.1017415
```

```
## 288 27.0420667 27.0420667
## 289 27.2136458 27.2136458
## 290 26.9139584 26.9139584
## 291 33.4356331 33.4356331
## 292 34.4034963 34.4034963
## 293 31.8333982 31.8333982
## 294 25.8178324 25.8178324
## 295 24.4298235 24.4298235
## 296 28.4576434 28.4576434
## 297 27.3626700 27.3626700
## 298 19.5392876 19.5392876
## 299 29.1130984 29.1130984
## 300 31.9105461 31.9105461
## 301 30.7715945 30.7715945
## 302 28.9427587 28.9427587
## 303 28.8819102 28.8819102
## 304 32.7988723 32.7988723
## 305 33.2090546 33.2090546
## 306 30.7683179 30.7683179
## 307 35.5622686 35.5622686
## 308 32.7090512 32.7090512
## 309 28.6424424 28.6424424
## 310 23.5896583 23.5896583
## 311 18.5426690 18.5426690
## 312 26.8788984 26.8788984
## 313 23.2813398 23.2813398
## 314 25.5458025 25.5458025
## 315 25.4812006 25.4812006
## 316 20.5390990 20.5390990
## 317 17.6157257 17.6157257
## 318 18.3758169 18.3758169
## 319 24.2907028 24.2907028
## 320 21.3252904 21.3252904
## 321 24.8868224 24.8868224
## 322 24.8693728 24.8693728
## 323 22.8695245 22.8695245
## 324 19.4512379 19.4512379
## 325 25.1178340 25.1178340
## 326 24.6678691 24.6678691
## 327 23.6807618 23.6807618
## 328 19.3408962 19.3408962
## 329 21.1741811 21.1741811
## 330 24.2524907 24.2524907
## 331 21.5926089 21.5926089
## 332 19.9844661 19.9844661
## 333 23.3388800 23.3388800
## 334 22.1406069 22.1406069
## 335 21.5550993 21.5550993
## 336 20.6187291 20.6187291
## 337 20.1609718 20.1609718
## 338 19.2849039 19.2849039
## 339 22.1667232 22.1667232
## 340 21.2496577 21.2496577
## 341 21.4293931 21.4293931
```

```
## 342 30.3278880 30.3278880
## 343 22.0473498 22.0473498
## 344 27.7064791 27.7064791
## 345 28.5479412 28.5479412
## 346 16.5450112 16.5450112
## 347 14.7835964 14.7835964
## 348 25.2738008 25.2738008
## 349 27.5420512 27.5420512
## 350 22.1483756 22.1483756
## 351 20.4594409 20.4594409
## 352 20.5460542 20.5460542
## 353 16.8806383 16.8806383
## 354 25.4025351 25.4025351
## 355 14.3248663 14.3248663
## 356 16.5948846 16.5948846
## 357 19.6370469 19.6370469
## 358 22.7180661 22.7180661
## 359 22.2021889 22.2021889
## 360 19.2054806 19.2054806
## 361 22.6661611 22.6661611
## 362 18.9319262 18.9319262
## 363 18.2284680 18.2284680
## 364 20.2315081 20.2315081
## 365 37.4944739 37.4944739
## 366 14.2819073 14.2819073
## 367 15.5428625 15.5428625
## 368 10.8316232 10.8316232
## 369 23.8007290 23.8007290
## 370 32.6440736 32.6440736
## 371 34.6068404 34.6068404
## 372 24.9433133 24.9433133
## 373 25.9998091 25.9998091
## 374  6.1263250  6.1263250
## 375  0.7777981  0.7777981
## 376 25.3071306 25.3071306
## 377 17.7406106 17.7406106
## 378 20.2327441 20.2327441
## 379 15.8333130 15.8333130
## 380 16.8351259 16.8351259
## 381 14.3699483 14.3699483
## 382 18.4768283 18.4768283
## 383 13.4276828 13.4276828
## 384 13.0617751 13.0617751
## 385  3.2791812  3.2791812
## 386  8.0602217  8.0602217
## 387  6.1284220  6.1284220
## 388  5.6186481  5.6186481
## 389  6.4519857  6.4519857
## 390 14.2076474 14.2076474
## 391 17.2122518 17.2122518
## 392 17.2988727 17.2988727
## 393  9.8911664  9.8911664
## 394 20.2212419 20.2212419
## 395 17.9418118 17.9418118
```

```
## 396 20.3044578 20.3044578
## 397 19.2955908 19.2955908
## 398 16.3363278 16.3363278
## 399  6.5516232  6.5516232
## 400 10.8901678 10.8901678
## 401 11.8814587 11.8814587
## 402 17.8117451 17.8117451
## 403 18.2612659 18.2612659
## 404 12.9794878 12.9794878
## 405  7.3781636  7.3781636
## 406  8.2111586  8.2111586
## 407  8.0662619  8.0662619
## 408 19.9829479 19.9829479
## 409 13.7075637 13.7075637
## 410 19.8526845 19.8526845
## 411 15.2230830 15.2230830
## 412 16.9607198 16.9607198
## 413  1.7185181  1.7185181
## 414 11.8057839 11.8057839
## 415 -4.2813107 -4.2813107
## 416  9.5837674  9.5837674
## 417 13.3666081 13.3666081
## 418  6.8956236  6.8956236
## 419  6.1477985  6.1477985
## 420 14.6066179 14.6066179
## 421 19.6000267 19.6000267
## 422 18.1242748 18.1242748
## 423 18.5217713 18.5217713
## 424 13.1752861 13.1752861
## 425 14.6261762 14.6261762
## 426  9.9237498  9.9237498
## 427 16.3459065 16.3459065
## 428 14.0751943 14.0751943
## 429 14.2575624 14.2575624
## 430 13.0423479 13.0423479
## 431 18.1595569 18.1595569
## 432 18.6955435 18.6955435
## 433 21.5272830 21.5272830
## 434 17.0314186 17.0314186
## 435 15.9609044 15.9609044
## 436 13.3614161 13.3614161
## 437 14.5207938 14.5207938
## 438  8.8197601  8.8197601
## 439  4.8675110  4.8675110
## 440 13.0659131 13.0659131
## 441 12.7060970 12.7060970
## 442 17.2955806 17.2955806
## 443 18.7404850 18.7404850
## 444 18.0590103 18.0590103
## 445 11.5147468 11.5147468
## 446 11.9740036 11.9740036
## 447 17.6834462 17.6834462
## 448 18.1269524 18.1269524
## 449 17.5183465 17.5183465
```

```
## 450 17.2274251 17.2274251
## 451 16.5227163 16.5227163
## 452 19.4129110 19.4129110
## 453 18.5821524 18.5821524
## 454 22.4894479 22.4894479
## 455 15.2800013 15.2800013
## 456 15.8208934 15.8208934
## 457 12.6872558 12.6872558
## 458 12.8763379 12.8763379
## 459 17.1866853 17.1866853
## 460 18.5124761 18.5124761
## 461 19.0486053 19.0486053
## 462 20.1720893 20.1720893
## 463 19.7740732 19.7740732
## 464 22.4294077 22.4294077
## 465 20.3191185 20.3191185
## 466 17.8861625 17.8861625
## 467 14.3747852 14.3747852
## 468 16.9477685 16.9477685
## 469 16.9840576 16.9840576
## 470 18.5883840 18.5883840
## 471 20.1671944 20.1671944
## 472 22.9771803 22.9771803
## 473 22.4558073 22.4558073
## 474 25.5782463 25.5782463
## 475 16.3914763 16.3914763
## 476 16.1114628 16.1114628
## 477 20.5348160 20.5348160
## 478 11.5427274 11.5427274
## 479 19.2049630 19.2049630
## 480 21.8627639 21.8627639
## 481 23.4687887 23.4687887
## 482 27.0988732 27.0988732
## 483 28.5699430 28.5699430
## 484 21.0839878 21.0839878
## 485 19.4551620 19.4551620
## 486 22.2222591 22.2222591
## 487 19.6559196 19.6559196
## 488 21.3253610 21.3253610
## 489 11.8558372 11.8558372
## 490  8.2238669  8.2238669
## 491  3.6639967  3.6639967
## 492 13.7590854 13.7590854
## 493 15.9311855 15.9311855
## 494 20.6266205 20.6266205
## 495 20.6124941 20.6124941
## 496 16.8854196 16.8854196
## 497 14.0132079 14.0132079
## 498 19.1085414 19.1085414
## 499 21.2980517 21.2980517
## 500 18.4549884 18.4549884
## 501 20.4687085 20.4687085
## 502 23.5333405 23.5333405
## 503 22.3757189 22.3757189
```

```
## 504 27.6274261 27.6274261
## 505 26.1279668 26.1279668
## 506 22.3442123 22.3442123
```

Project Y onto the columns of Q one by one and verify it sums to be the projection onto the whole space.

```
y %*% Q[, 1, drop = FALSE]
```

```
##          [,1]
## [1,] 506.8629
```

```
y %*% Q[, 2, drop = FALSE]
```

```
##           [,1]
## [1,] -80.25449
```

```
y %*% Q[, 4, drop = FALSE]
```

```
##           [,1]
## [1,] -50.50977
```

```
y %*% Q[, 14, drop = FALSE]
```

```
##           [,1]
## [1,] -49.10029
```

```
project_y_Q <- 0
for (j in 1 : ncol(Q)){
  project_y_Q = (y %*% Q[, j, drop = FALSE])
  print(project_y_Q)
}
```

```
##          [,1]
## [1,] 506.8629
##           [,1]
## [1,] -80.25449
##          [,1]
## [1,] 59.61825
##           [,1]
## [1,] -50.50977
##          [,1]
## [1,] 39.11327
##          [,1]
## [1,] -8.73199
##          [,1]
## [1,] 104.5855
##           [,1]
## [1,] -9.500943
##           [,1]
## [1,] -42.18413
##           [,1]
## [1,] -5.842459
##           [,1]
## [1,] -18.15362
##           [,1]
## [1,] -36.18438
##          [,1]
## [1,] 24.35852
```

```
##           [,1]
## [1,] -49.10029
```

Verify the OLS fit squared lengh is the sum of squared lengths of each of the orthogonal projections.

Rewrite the "The monotonicity of SSR" demo from the lec06 notes. Comment every line in detail. Write about what the plots means.

```
#we have an array on n = 100 observations.
#first column is the intercept
#we create a matrix with n rows and the first column of 1's
#add a new random continuous predictor to each iteration
#that plot shows that for each additional predictor Rsquared increases.
#This is an example of overfitting because the additional predictors arent truly explaining y but rathe
```

Rewrite the "Overfitting" demo from the lec06 notes. Comment every line in detail. Write about what the plots means.

```
#set a training set and test set.. when you calculate your insample rmse you dont get a margin error be
```