Transmission Expansion Planning by Quantum Annealing

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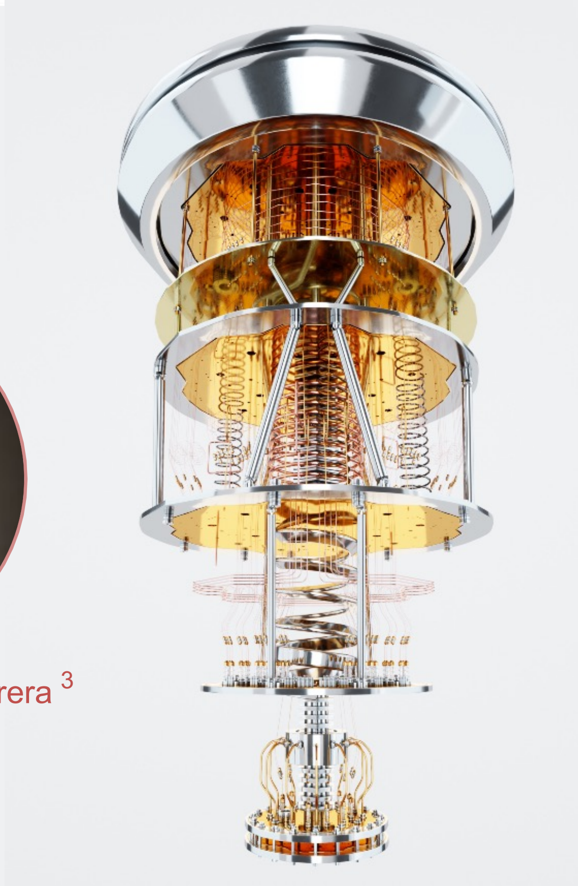

Source: Mckinsey Quantum Technology Monitor April 2023

A thesis submitted in fulfilment of the requirements for the degree of Master in Quantum Computing at Nebrija University.

3 DLR Deutsches Zentrum für Luft- und Raumfahrt
German Aerospace Center

2 BBVA

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- Conclusions and Future Work

The estimated value at stake for QC in the four industries most likely to see impact first has now reached nearly $1.3 trillion.

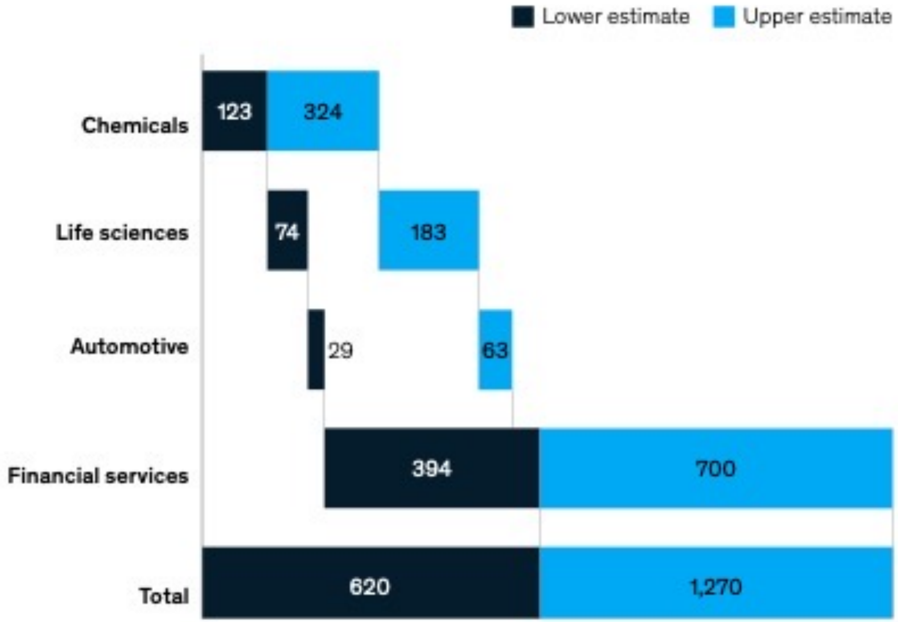
Source: Mckinsey Quantum Technology Monitor April 2023

Economic value + Incremental ++ Significant +++ Disruptive

Industry	Key segment for QC	2025–30	2030–35
Global energy and materials	Oil and gas	+	++
	Sustainable energy	+	+++
	Chemicals	++	+++
Life sciences	Pharmaceuticals	++	+++
Advanced industries	Automotive	++	++
	Aerospace and defense	+	++
	Advanced electronics	+	++
	Semiconductors	+	++
Finance	Financial services	++	+++
Telecom, media, and technology	Telecom	+	++
	Media	+	+
Travel, transport, and logistics	Logistics	+	++

Four industries expected to see first impact

Value at stake with incremental impact of QC by 2035, $ billion



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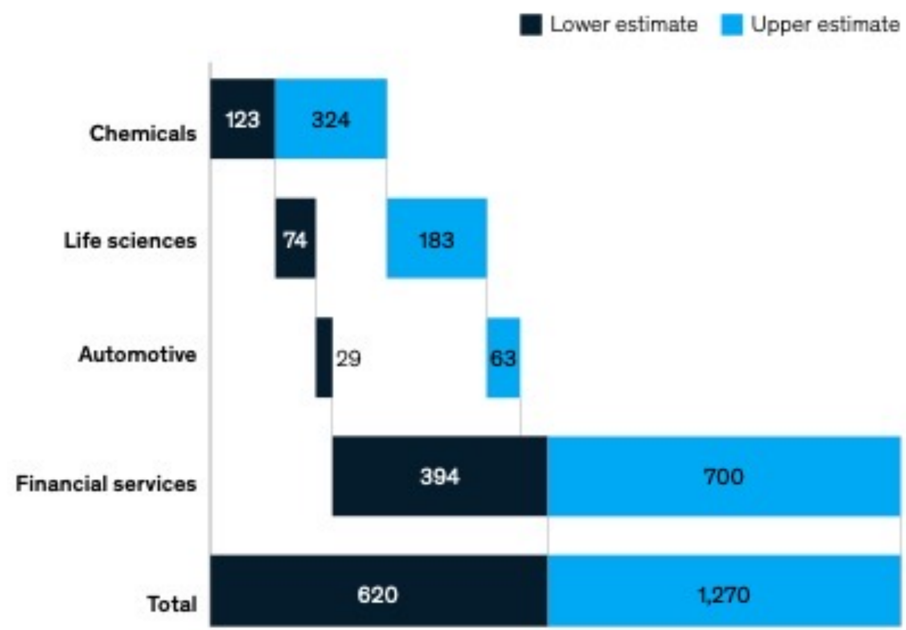

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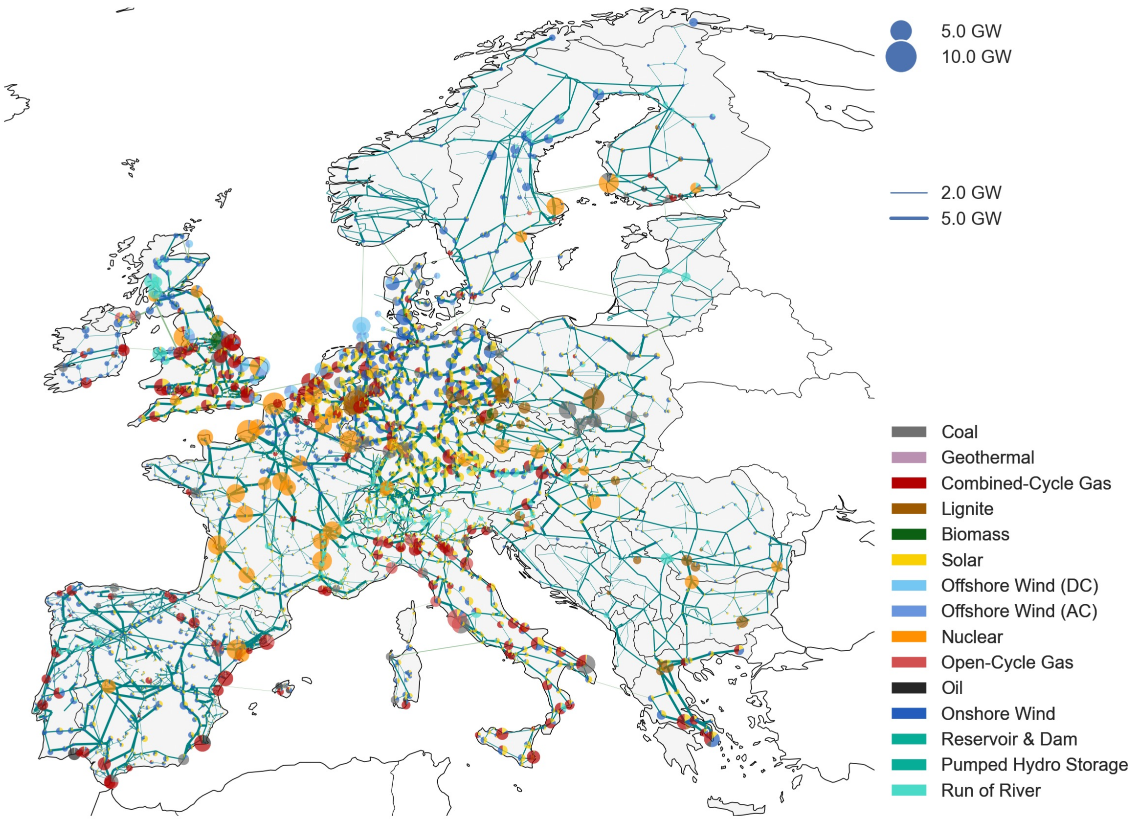
| Statement of the Problem>



Objective: distribute the energy across a network in the most effective way so that we minimize the energy loss.

Background: the smart grid provide data about the current status of the grid. There are predictions of **future scenarios** where the demand at each node is predicted and the network has to be able to fulfil the demand.

How: there are many ways of improving the efficiency of a network but we focus on the transmission expansion planning (TEP) problem. TEP aims at finding the optimal way of expanding the network by adding new lines between nodes.

Motivation: energy system models are getting larger and more complex due to the integration of decentralized weather-dependent renewable energy sources, intermittent loads, sector coupling and the increase of storage components.

| Statement of the Problem>

Symbol	Description	Type
N	Set of nodes of the network	Set
H	Set of snapshots	Set
C	Set of candidate transmission lines	Set
C_k	Set of candidate transmission lines from all nodes to node k	Set
E	Set of existing transmission lines	Set
E_k	Set of existing transmission lines from all nodes to node k	Set
x_{kl}	Transmission line from node k to l	Binary
f_{kl}^0	Power flow in existing line from node k to l	Integer
$\bar{f}_{kl}^0$	Maximum power flow in existing line from node k to l	Integer
f_{kl}^1	Power flow in candidate line from node k to l	Integer
$\bar{f}_{kl}^1$	Maximum power flow in candidate line from node k to l	Integer
r_k	Shedding load at node k	Integer
$d_k(h)$	Demand of node k at snapshot h	Integer
$g_k(h)$	Current generation at node k at snapshot h	Integer
$\bar{g}_k$	Maximum generation at node k	Integer
c_{kl}	Investment cost of transmission line from node k to l	Real
$c_k^{(oc)}$	Annualised operational cost per MWh of generator g_k	Real
c_k	Cost of shedding load at node k	Real

Table 4.1: Description of variables involved in TEP problems.

| Statement of the Problem>

$$\min_{\mathbf{x},\mathbf{g},\mathbf{r},\mathbf{f}^0,\mathbf{f}^1} \quad \underbrace{\sum_{kl\in C} c_{kl}x_{kl}}_{\text{Investment cost}} + \underbrace{\sum_{h\in H}\sum_{k} c_k^{(oc)} g_k(h)}_{\text{Operational cost}} + \underbrace{\sum_{h\in H}\sum_{k} r_k(h)c_k}_{\text{Load shedding cost}}$$

(Power Balance) $$d_k(h) - \left(\sum_{l\in E_k} f_{kl}^0(h) + \sum_{l\in C_k} f_{kl}^1(h) + g_k(h) + r_k(h)\right) = 0\,, \quad \forall k \in N, h \in H,$$

(Existing circuit flow limits) $$\left|f_{kl}^0(h)\right| - \bar{f}_{kl}^0(h) \leq 0\,, \quad \forall kl \in E, h \in H,$$

(Candidate circuit flow limits) $$\left|f_{kl}^1(h)\right| - \bar{f}_{kl}^1(h)x_{kl} \leq 0\,, \quad \forall kl \in C, h \in H,$$

(Node generation limits) $$g_k(h) - \bar{g}_k(h) \leq 0\,, \quad \forall k \in N, h \in H,$$

(Node loads limits) $$r_k(h) - d_k(h) \leq 0\,, \quad \forall k \in N, h \in H,$$

(Positive variables) $$\mathbf{d}, \mathbf{g}, \mathbf{f}^0, \mathbf{f}^1 \geq 0,$$

(Binary type) $$x_{kl} \in \{0,1\}\,, \quad \forall kl \in C,$$

Heuristic Methods: | Simulated Annealing >

Objective function:

$$\min_{\vec{x}} f(\vec{x}) = \min_{\vec{x}} \sum_{v} \sum_{u} \sum_{i=0}^{i<n} D_{u,v} x_{v,i} x_{u,i+1},$$

Visit each node just once in the whole path:

$$\sum_{i=0}^{i<n} x_{u,i} = 1 \quad \forall\, u \in R,$$

Each stop of the tour has one node:

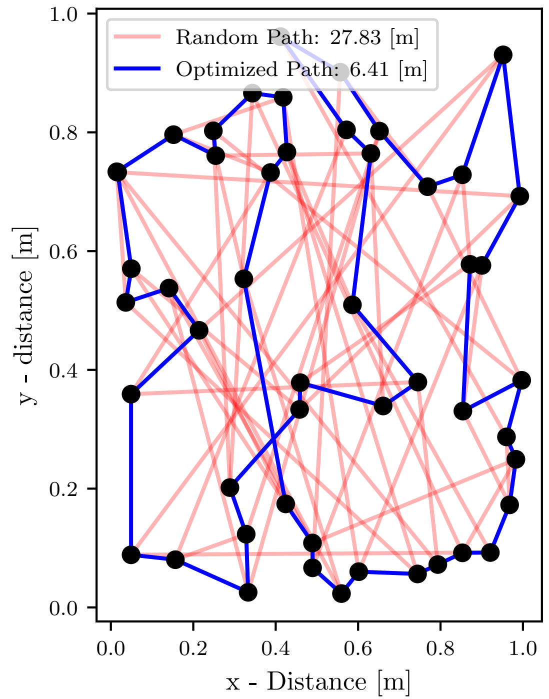
$$\sum_{u} x_{u,i} = 1 \quad \forall\, i \in P.$$

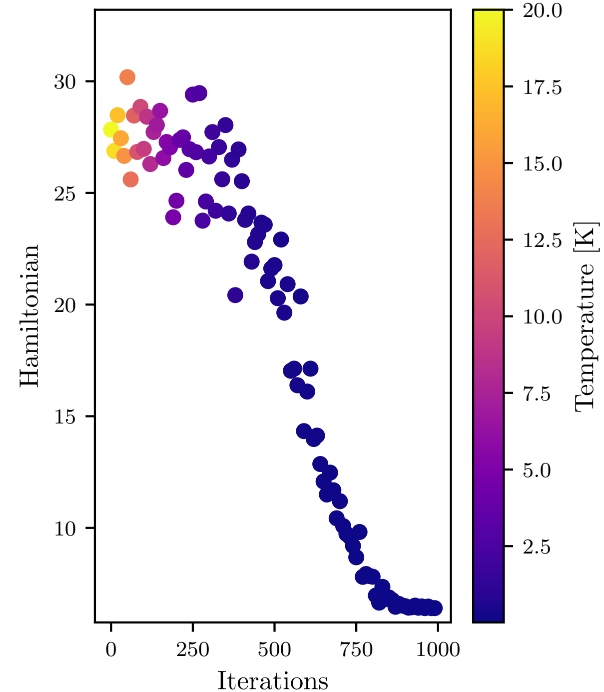
Constrains in cost function by Lagrange multipliers:

$$\min_{\vec{x}} f(\vec{x}) = \min_{\vec{x}} \sum_{v} \sum_{u} \sum_{i=0}^{i<n} D_{u,v} x_{v,i} x_{u,i+1}$$

$$-\sum_{i=0}^{i<n} \lambda_i \left(\sum_{u} x_{u,i} - 1 \right)^2 - \sum_{u} \lambda_u \left(\sum_{i=0}^{i<n} x_{u,i} - 1 \right)^2 .$$

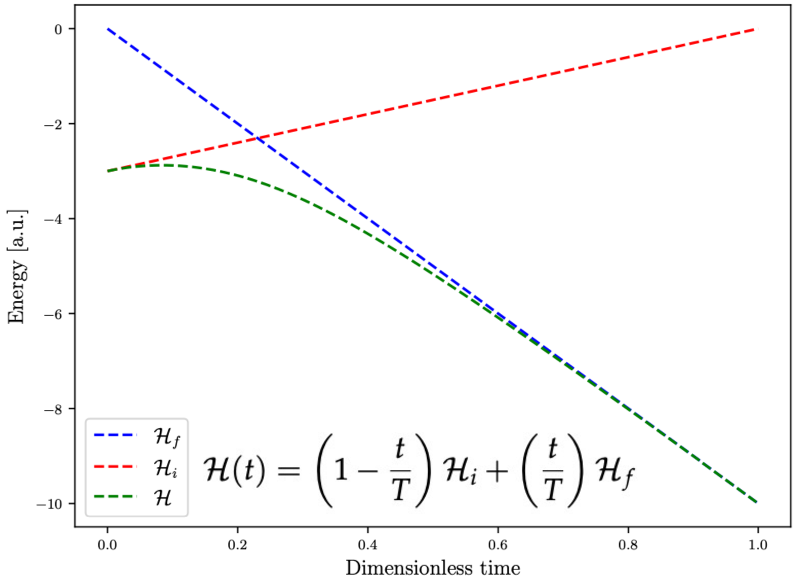
Travelling Salesman Problem



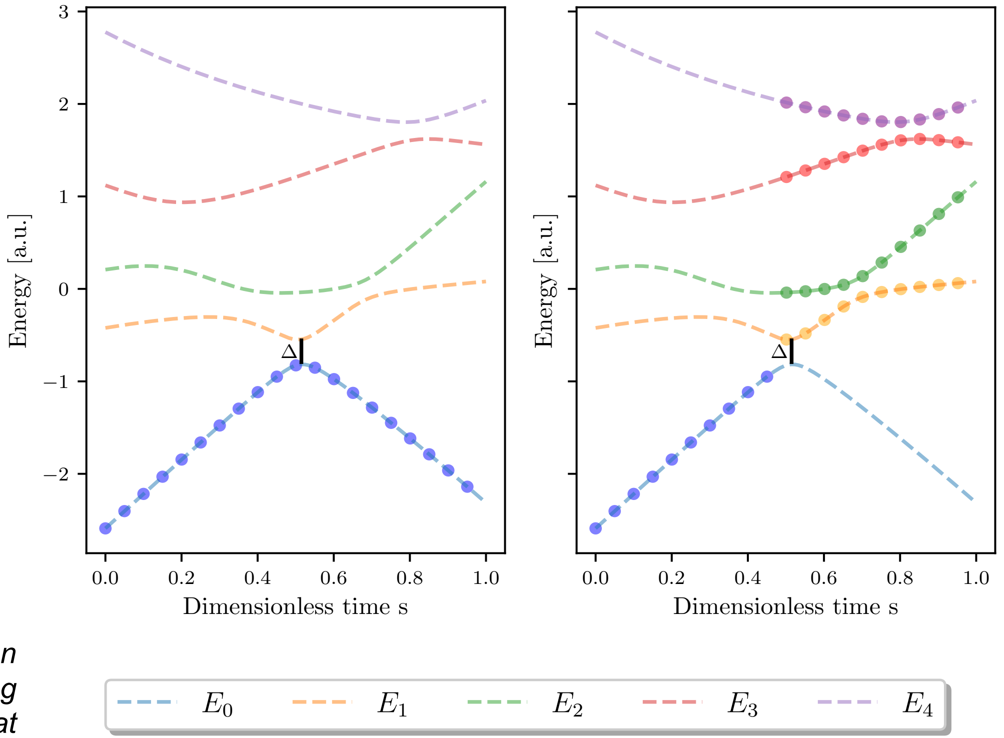


Heuristic Methods: | Quantum Annealing>

Interpolation of initial and final Hamiltonians:



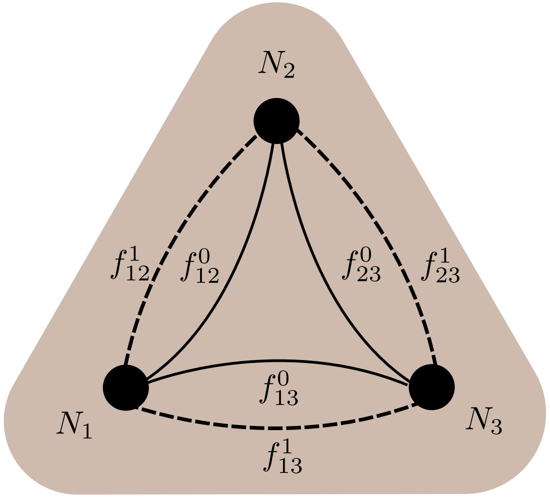
Eigenenergies as function of dimensionless time:



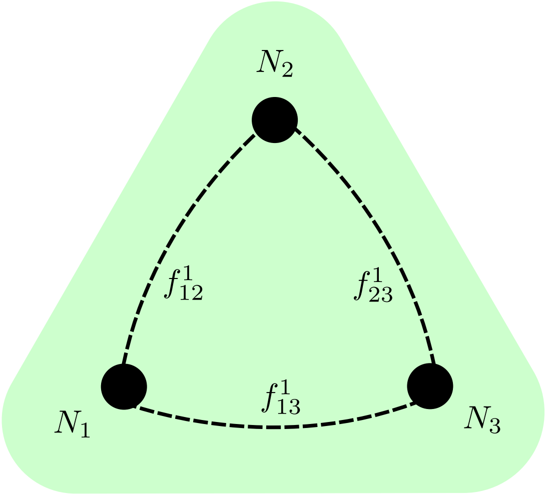
Quoting Sarandy and Lidar:

The theorem posits, roughly, that if a state is an instantaneous eigenstate of a sufficiently slowly varying Hamiltonian at one time, then it will remain an eigenstate at later times, while its eigenenergy evolves continuously.

TEP: | Small Network>



Brownfield model: Network considering current transmission lines (solid lines) and candidate lines (dashed lines).

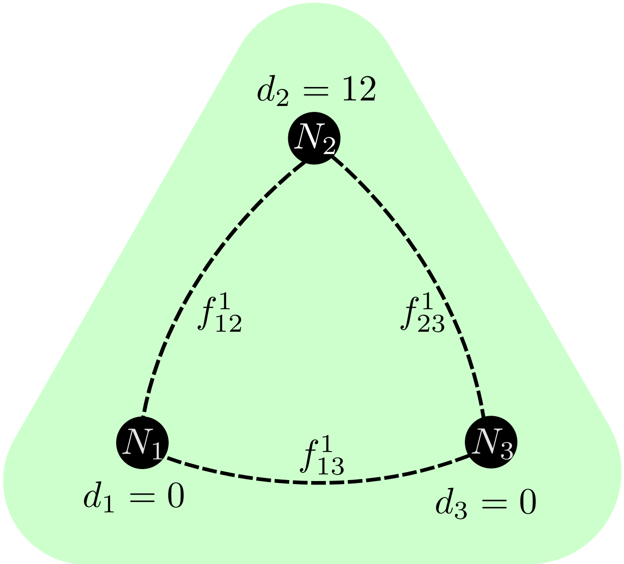


Greenfield model: Network considering just candidate lines (dashed lines).

TEP: | Small Network>

- Single demand at node 2.
- One time snapshot.
- Maximum capacity per generator 10 units of energy.

The demand cannot be fulfilled by its own generator and it requires to build at least one transmission line.



Symbol	Description	Value
$\mathbf{c} = [c_{12}, c_{13}, c_{23}]$	Investment cost	$[10, 20, 30]$
$\mathbf{c}^{\mathrm{oc}} = [c_1^{\mathrm{oc}}, c_2^{\mathrm{oc}}, c_3^{\mathrm{oc}}]$	Operational cost	$[10, 5, 2]$
$\mathbf{d} = [d_1, d_2, d_3]$	Demands	$[0, 12, 0]$
$\bar{\mathbf{g}} = [\bar{g}_1, \bar{g}_2, \bar{g}_3]$	Maximum capacity	$[10, 10, 10]$
$\bar{\mathbf{f}}^1 = [f^1_{12}, f^1_{13}, f^1_{23}]$	Maximum Power Flow in candidate line	$[10, 10, 10]$

Table 4.2: Investment cost, operational cost, demand, maximum capacity per generator and maximum power flow per candidate line for a three node network.

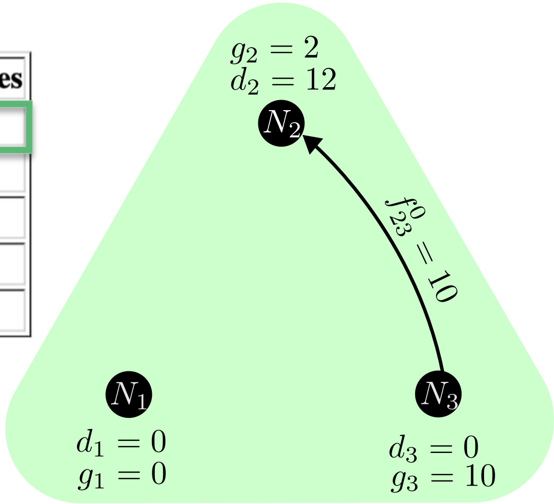
TEP: | Small Network>

Results:

	X_1_2	X_1_3	X_2_3	g_1	g_2	g_3	energy	is_feasible	is_satisfied	num_occurrences
9860	0	0	1	0	2	10	60.0	True	True	1
8980	0	0	1	0	3	9	63.0	True	True	1
8100	0	0	1	0	4	8	66.0	True	True	1
7220	0	0	1	0	5	7	69.0	True	True	1
9868	0	0	1	1	2	10	70.0	True	True	1

Indicates that the constraints are fulfilled.

The solution that minimizes the objective cost – which is the sum of investment cost and operational cost – decides to build the transmission line connecting node 3 and 2 (the most expensive one) and use the generator 3 to produce most of the required energy (the cheapest operational cost)



Symbol	Description	Value
$\mathbf{c} = [c_{12}, c_{13}, c_{23}]$	Investment cost	$[10, 20, 30]$
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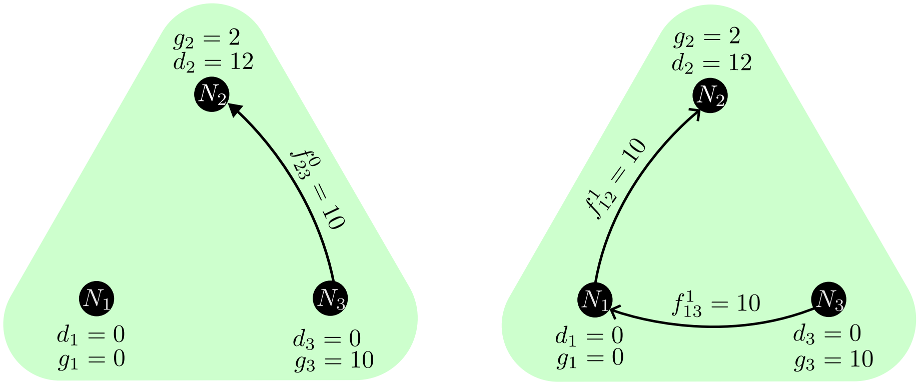
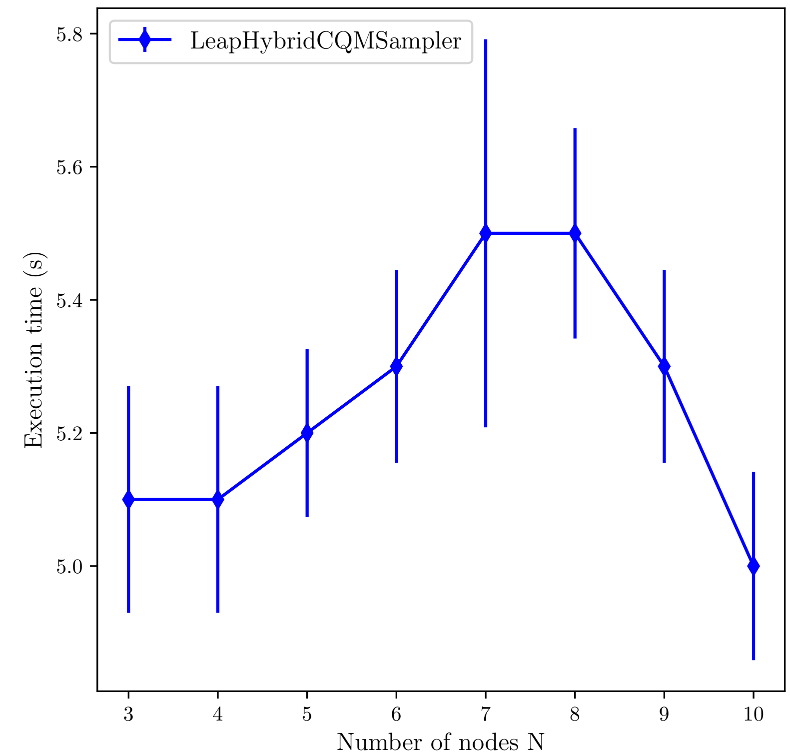
TEP: | Hybrid Solver>

LeapHybridCQMSampler

Main features:

- 20 min of free trial.
- Solve MILP problems.
- Cannot know how much quantum solver was involved in the resolution.





x_{12}	x_{13}	x_{23}	f_{12}	f_{13}	f_{21}	f_{23}	f_{31}	f_{32}	g_1	g_2	g_3	r_1	r_2	r_3	Prices
0	0	1	0	0	0	10	0	0	0	2	10	0	0	0	60
1	1	0	0	10	10	0	0	0	0	2	10	0	0	0	60

Quantum-Classical Iterative Method | Benders' decomposition algorithm >

Step 1: Initialization of Benders' decomposition

$t = 1$	Benders' iteration	(4.9)
$\underline{z}^0 = -\infty$	Lower bound	(4.10)
$\bar{z}^0 = \infty$	Upper bound	(4.11)
$\alpha^0 = 0$	Load shedding and operational cost	(4.12)
$\Pi^0_{f^1_{kl}} = 0 \quad \forall kl \in C$	Lagrange multipliers	(4.13)

Quantum-Classical Iterative Method | Benders' decomposition algorithm >

Step 2: Master problem solved by a quantum annealer

$$\min_{\mathbf{x}^t} \quad \underline{z}^t \tag{4.14a}$$

$$\text{s.t.} \quad \underline{z} \geq \sum_{kl \in C} c_{kl} x_{kl}^t + \alpha^{t-1} - \sum_{kl \in C} \Pi_{f_{kl}^1}^{t-1} \left(x_{kl}^t - x_{kl}^{t-1} \right), \tag{4.14b}$$

$$x_{kl} \in \{0,1\}, \qquad \forall kl \in C. \tag{4.14c}$$

Quantum-Classical Iterative Method
| Benders' decomposition algorithm >

Step 3: Slave problem solved by a classical solver

$$\min_{\mathbf{g},\mathbf{r},\mathbf{f}^0,\mathbf{f}^1} \quad \underbrace{\sum_k c_k^{(\mathrm{oc})} g_k}_{\text{Operational cost}} + \underbrace{\sum_k r_k c_k}_{\text{Load shedding cost}} \tag{4.15a}$$

$$\text{s.t.} \quad d_k - \Big(\sum_{l\in E_k} f_{kl}^0 + \sum_{l \in C_k} f_{kl}^1 + g_k + r_k\Big) = 0, \forall k \in N, \tag{4.15b}$$

$$\left|f_{kl}^0\right| - \bar{f}_{kl}^0 \leq 0, \qquad \forall kl \in E, \tag{4.15c}$$

$$\left|f_{kl}^1\right| - \bar{f}_{kl}^1 x_{kl}^t \leq 0, \qquad \forall kl \in C, \tag{4.15d}$$

$$g_k - \bar{g}_k \leq 0, \qquad \forall k \in N, \tag{4.15e}$$

$$r_k - d_k \leq 0, \qquad \forall k \in N. \tag{4.15f}$$

$$\bar{z}^t = \min\{\bar{z}^{t-1}, \sum_{kl \in C} c_{kl} x_{kl}^t + \alpha^t\} \qquad \text{Update upper bound} \tag{4.16}$$

Quantum-Classical Iterative Method
| Benders' decomposition algorithm >

Step 4: Stopping criterion

If $\left(\bar{z}^t - \underline{z}^t\right) / \bar{z}^t \leq \epsilon$, then we found the (sub)-optimal solution, else $t = t + 1$ and we have to repeat the algorithm from step 2 until the stopping criterion is satisfied.

Quantum-Classical Iterative Method | Benders' decomposition algorithm >

Benders' decomposition algorithm

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If $(\bar{z}^t - \underline{z}^t) / \bar{z}^t \leq \epsilon$, then we found the (sub)-optimal solution, else $t = t + 1$ and we have to repeat the algorithm from step 2 until the stopping criterion is satisfied.

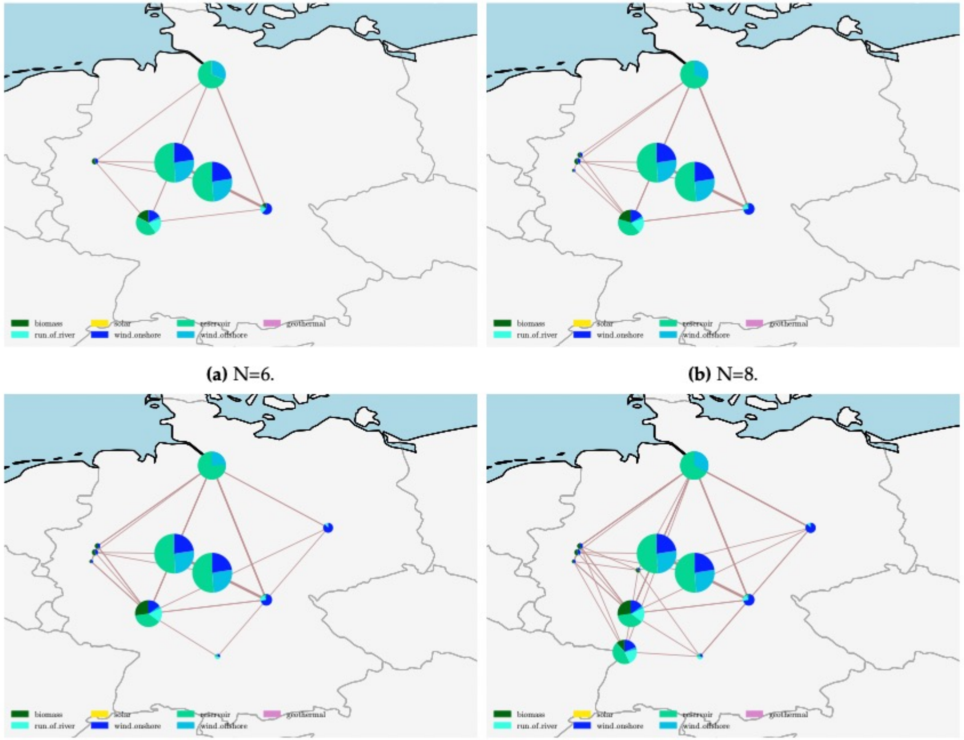
| Conclusions and Future Work>

PyPSA: this python package will read the raw data and produce the linear model files .lp

QUARK: this python package will reformulate the problem so that it can be sent to different quantum machines (hardware agnostic).

Benchmarking: once we have the interface to load the data and formulate the problem we can send it to different quantum machines and compare our hybrid quantum-classical method with those provide by companies such as D-Wave.

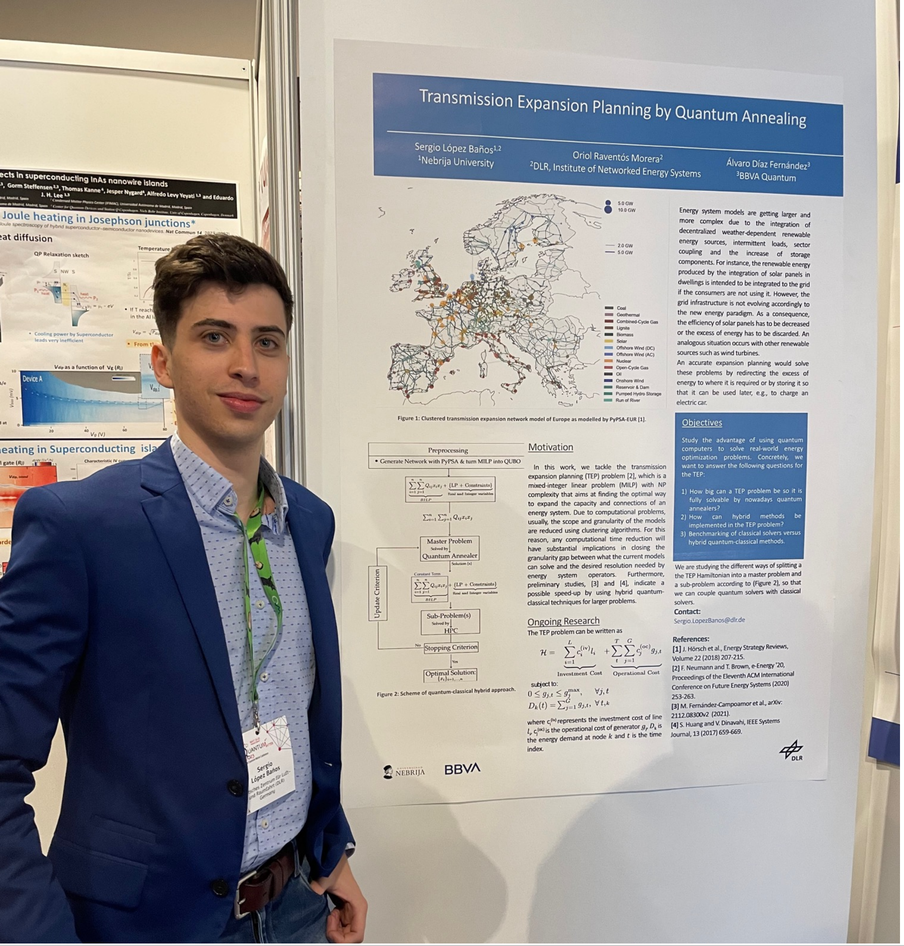
We expect that an specific hybrid quantum-classical model should work better than a general hybrid method. Furthermore, we will know exactly the amount of quantum solver employed.



(a) N=6. (b) N=8.

(c) N=10. (d) N=12.

Figure 6.1: Germany network generated with PyPSA for N cluster from eGo100 data.



Thank you for your time

QUANTUMatter2023 Conference: Poster presentation titled “Transmission Expansion Planning by Quantum Annealing”.

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