Discrete Math (Honor) 2021-Fall Homework-3

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Solution 1.

1. First suppose R is false. Let's turn it into a CNF:

$$(P \vee Q) \wedge (\neg P \vee R) \wedge (\neg Q \vee R) \wedge \neg R$$

Build resolve set S:

$$S = \{P \vee Q, \neg P \vee R, \neg Q \vee R, \neg R\}$$

Resolution:

① $P \vee Q$

② $\neg P \vee Q$

③ $\neg Q \vee R$

④ $\neg R$

⑤ $Q \vee R$

⑥ R

⑦ Contradiction.

2. CNF:

$$(\neg S \vee \neg Q) \wedge (\neg P \vee Q) \wedge (R \vee S) \wedge (\neg R \vee \neg Q) \wedge P$$
$$S = \{\neg S \vee \neg Q, \neg P \vee Q, R \vee S, \neg R \vee \neg Q, P, \neg S \vee \neg P, S \vee \neg Q\}$$

Resolution:

① P

② $\neg S \vee \neg P$

③ $\neg S$

④ $S \vee \neg Q$

⑤ $\neg Q$

⑥ $\neg P \vee Q$

⑦ $\neg P$

⑧ Contradiction.

3. CNF:

$$(\neg P \vee Q) \wedge (\neg Q \vee R) \wedge \neg R \wedge P$$
$$S = \{\neg P \vee Q, \neg Q \vee R, \neg R, P\}$$

Resolution:

① P

② $\neg P \vee Q$

③ Q

④ $\neg Q \vee R$

⑤ R

⑥ $\neg R$

⑦ Contradiction.

Solution 2.

$D: all\ people. Eq(x,y): x = y. Fri(x,y): x, y\ are\ firends. Lov(x,y): x\ loves\ y. Hat(x,y): x\ hates\ y.$

1. $(\forall x)(\forall y)(\neg Eq(x,y) \to Lov(x,y))$
2. $(\forall x)(Student(x) \wedge \neg Eq(x, Alice) \to Fri(x, Bob))$
3. $(\forall x)(student(x) \wedge Talkable(x) \to Walkable(x))$
4. $(\forall x)(Boy(x) \wedge Lov(x, Alice) \wedge (\forall y)Lov(Alice, y) \to Hat(x,y))$
5. $(\forall x)(Boy(x) \wedge Lov(x, Alice) \wedge (\forall y)Lov(Cauchy, y) \wedge Eq(x,y) \to Hat(x,y))$
6. $(\exists x)(Love(x, Alice) \wedge Hate(x, Bob)))$
7. $(\exists x)(Love(x, Alice) \wedge (\forall y)(Love(y, Bob) \wedge Hate(x,y)))$

Problem 1. *(10 Points)*
For each of the following formulae, determine free variables and bound variables in it and determine the scope of each quantifier.

1. $(\forall x)(P(x) \to Q(x,y))$
2. $(\forall x)P(x,y) \to (\exists y)Q(x,y)$
3. $(\forall x)(\exists y)(P(x,y) \wedge Q(y,z) \vee (\exists x)R(x,y,z))$
4. $(\exists x)(P(x) \to Q(x)) \to (\exists y)R(y) \to S(z)$
5. $(\forall x)(P(x) \wedge (\exists y)Q(y)) \vee ((\forall x)P(x) \to Q(z))$

Solution 3.

1. free variable:y

   bound variable:x

   the scope of $\forall x$ is $P(x) \to Q(x,y)$

2. free variable: the first y and the second x

   bound variable: the first x and the second y

   the scope of $\forall x$ is $P(x,y)$,the scope of $\exists y$ is $Q(x,y)$

3. free variable: z

   bound variable: x and y

   the scope of $\forall x$ is $\exists y)(P(x,y) \wedge Q(y,z) \vee (\exists x)R(x,y,z)$

   the scope of $\exists y$ is $(P(x,y) \wedge Q(y,z) \vee (\exists x)R(x,y,z)$

   the scope of $\exists x$ is $R(x,y,z)$

4. free variable: z

   bound variable: x and y

   the scope of $\exists x$ is $P(x) \to Q(x)$

   the scope of $\exists y$ is $R(y)$

5. free variable: z

   bound variable: x and y

   the scope of the first $\forall x$ is $P(x) \wedge (\exists y)Q(y)$

   the scope of $\exists y$ is $Q(y)$

   the scope of the second $\forall x$ is $P(x)$