P135. 11.3

1. (1) $\frac{1}{2}$ (6) 0 (7) $\ln\frac{a}{b}$ (8) $\frac{1}{6}$

(12) $-\frac{1}{2}$ (14) 1 (18) $e^{-\frac{1}{2}}$ (19) $e^{-\frac{1}{2}}$

2. (1) 洛必达 $A=g'(0)$

(2) $f(x)$在$x=0$处可导且导数连续

3. $$\lim_{h\to 0}\frac{f(x_0+2h)-2f(x_0+h)+f(x_0)}{h^2}$$

$$=\lim_{h\to 0}\frac{2f'(x_0+2h)-2f'(x_0+h)}{2h}$$

$$=\lim_{h\to 0}\frac{f'(x_0+2h)-f'(x_0+h)}{h}$$

$$=\lim_{h\to 0}\frac{f'(x_0+2h)-f'(x_0)+f'(x_0)-f'(x_0+h)}{h}$$

$$=2\lim_{h\to 0}\frac{f'(x_0+2h)-f'(x_0)}{2h}-\lim_{h\to 0}\frac{f'(x_0+h)-f'(x_0)}{h}$$

$$=2f''(x_0)-f''(x_0)$$

$$=f''(x_0)$$

9. $\cancel{\lim_{n\to\infty}\frac{n-(n+1)}{}}$ proof: $x_n\downarrow\to 0$

$$\lim_{n\to\infty}\frac{(n+1)-n}{\frac{1}{x_{n+1}^2}-\frac{1}{x_n^2}}=\lim_{n\to\infty}\frac{x_{n+1}^2x_n^2}{x_n^2-x_{n+1}^2}=\lim_{n\to\infty}\frac{\sin^2x_n\cdot x_n^2}{x_n^2-\sin^2x_n}$$

由带Peano余项的Maclaurin公式

$$\sin^2x=x^2-\frac{1}{3}x^4+o(x^4)$$

$$\therefore\lim_{n\to\infty}\frac{(n+1)-n}{\frac{1}{x_{n+1}^2}-\frac{1}{x_n^2}}=\lim_{n\to\infty}\frac{x_n^2\cdot x_n^2}{x_n^2-(x_n^2-\frac{1}{3}x_n^4+o(x_n^4))}=3$$

由Stolz定理 $\lim_{n\to\infty}nx_n^2=3$

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11.5

1. (1) $1+2x+x^2-\frac{2}{3}x^3-\frac{5}{6}x^4-\frac{1}{15}x^5+o(x^5)$

(3) $x-\frac{1}{3}x^3+\frac{1}{10}x^5+o(x^5)$

2. (3) $\frac{1}{2}$ (5) $\frac{1}{2}$ (6) $\frac{1}{3}$

3. ~~$f(x)=f(\frac{a+b}{2})+f'(\frac{a+b}{2})(x-\frac{a+b}{2})+\frac{f''(\frac{a+b}{2})}{2}(x-\frac{a+b}{2})^2+o(x-\frac{a+b}{2})^2$~~

由Taylor公式

$$f(x)=f(\frac{a+b}{2})+f'(\frac{a+b}{2})(x-\frac{a+b}{2})+\frac{f''(\xi)}{2}(x-\frac{a+b}{2})^2$$

$$f(a)=f(\frac{a+b}{2})+f'(\frac{a+b}{2})\cdot\frac{a-b}{2}+\frac{f''(\xi_1)}{2}\cdot(\frac{a-b}{2})^2$$

$$f(b)=f(\frac{a+b}{2})+f'(\frac{a+b}{2})\frac{b-a}{2}+\frac{f''(\xi_2)}{2}(\frac{a-b}{2})^2$$

$\therefore f(a)-2f(\frac{a+b}{2})+f(b)=\frac{(b-a)^2}{4}\cdot f''(\xi)$

由达布…… $\exists\xi, f''(\xi)=\frac{f''(\xi_1)+f''(\xi_2)}{2}$

7. $a=\frac{4}{3}$, $b=-\frac{1}{3}$

8. 由Taylor公式

$$f(x)=f(a)+\frac{f''(\xi_1)}{2}(x-a)^2,\ \xi_1\in(a,b)$$

$$f(x)=f(b)+\frac{f''(\xi_2)}{2}(x-b)^2,\ \xi_2\in(a,b)$$

$\therefore f(\frac{a+b}{2})=f(a)+\frac{f''(\xi_1)}{2}(\frac{b-a}{2})^2=f(b)+\frac{f''(\xi_2)}{2}(\frac{b-a}{2})^2$

$\therefore \frac{4}{(b-a)^2}|f(b)-f(a)|=|\frac{f''(\xi_1)-f''(\xi_2)}{2}|$

$\leq\frac{1}{2}(|f''(\xi_1)|+|f''(\xi_2)|)$

$\leq f''(\xi)$，其中 $\xi=\xi_1$ 或 ξ_2.

且 $|f(\xi)|=\max(|f(\xi_1)|,|f(\xi_2)|)$

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P155. 11.8

1. (3) ~~由~~错误

$$f(x)=\begin{cases}\frac{x}{2}+x^2\sin\frac{1}{x}, & x\neq 0\\ 0, & x=0\end{cases}$$

(先证满足题目条件)

当 $x\neq 0$, $f'(x)=\frac{1}{2}+2x\sin\frac{1}{x}+x^2\cdot\cos\frac{1}{x}\cdot\left(-\frac{1}{x^2}\right)$

$=\frac{1}{2}+2x\sin\frac{1}{x}-\cos\frac{1}{x}$

取 $x=\frac{1}{2n\pi}$, $n=1,2,3,\cdots$

$f'(x)=\frac{1}{2}-1=-\frac{1}{2}<0$

$\therefore \forall\delta>0$, $\exists N=\left[\frac{1}{2\pi\delta}\right]+1$, $\forall n>N$, $x\in U(0,\delta)$

$f'(x)<0$，再由~~保~~保号性. □

(4) 错误

$$f(x)=\begin{cases}2-x^2\left(2+\sin\frac{1}{x}\right), & x\neq 0,\\ 2, & x=0.\end{cases}$$

同样, 取 $x=\frac{1}{2n\pi}$.

2. 反证: ~~假设 x_0 不为 $f(x)$~~ 不妨 x_0 为极大值点, 若 x_0 不为最大值

则 $\exists x_1\neq x_0$, $f(x_1)>f(x_0)$, 则取 $[a,b]=[\min\{x_0,x_1\},\max\{x_0,x_1\}]$

则 $f(x)\in C[a,b]$, 由 $\therefore f(x)$ 在 $[a,b]$ 上有最小值, 记为 $f(x_2)$, $x_2\in[a,b]$

$\because f(x_0)$ 为极大值 $\therefore\exists\delta>0$ $\forall x\in \mathring{U}(x_0,\delta)$, $f(x)<f(x_0)<f(x_1)$

$\therefore x_2$ 不为端点, $\therefore x_2$ 为 $[a,b]$ 上的极小值点, 矛盾!

□

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3. (3) $(0,n)\uparrow$, $(n,+\infty)\downarrow$

(4) $(0,+\infty)\uparrow$, $(-1,0)\downarrow$

(6) $(0,+\infty)\downarrow$

4. ~~(3) $x=1$处极小值0；$x=$~~

(3) 在$x=e^{-2}$处取极小值$-2e^{-1}$

(5) 在$x=0$处取极小值0，

在$x=\frac{2}{3}$处取极大值$(\frac{2}{3})^{\frac{2}{3}}e^{-\frac{2}{3}}$

6. $4e^{-2}$

8. (2) 令$f(x)=(x+1)\ln x-2(x-1)$，$x\in(1,+\infty)$

$f'(x)=\ln x+\frac{x+1}{x}-2$

$f''(x)=\frac{1}{x}-\frac{1}{x^2}=\frac{x-1}{x^2}>0$

$\therefore f'(x)>f'(1)=0$

$\therefore f(x)>f(1)=0$

$\therefore \frac{2(x-1)}{x+1}<\ln x$

(4) 要证 $\frac{1}{x}e^{-\frac{1}{x}}<xe^{-x}$

即证 $-\ln x-\frac{1}{x}<\ln x-x$

令$f(x)=2\ln x+\frac{1}{x}-x$，$x\in(0,1)$

$f'(x)=\frac{2}{x}-\frac{1}{x^2}-1$

$f''(x)=\frac{2}{x^3}-\frac{2}{x^2}=\frac{2(1-x)}{x^3}>0$

$\therefore f'(x)<f'(1)=0$

$\therefore f(x)>f(1)=0$ □.

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