# Discrete Math-2021 Fall-Quiz-1

Name:

**Problem 1.** *(10 Points)* Determine if each of the following propositional formulas is tautology, contradiction or satisfiable.

1. $(\neg(P \leftrightarrow Q) \rightarrow ((P \wedge \neg Q) \vee (\neg P \wedge Q)))$ ( tautology / contradiction / satisfiable)
2. $(P \wedge \neg(Q \rightarrow P)) \wedge (R \wedge Q) \vee R$ ( tautology / contradiction / satisfiable)
3. $P \rightarrow (Q \rightarrow P)$ ( tautology / contradiction / satisfiable)
4. $(P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$ ( tautology / contradiction / satisfiable)
5. $\neg(Q \rightarrow R) \wedge R$ ( tautology / contradiction / satisfiable)

**Answer:** 1. tautology, 2. satisfiable, 3. tautology, 4. tautology , 5. contradiction.

**Problem 2.** *(10 Points)* Write down formula $\alpha$ in both CNF and DNF based on the following truth table.

| $P$ | $Q$ | $R$ | $\alpha$ |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

**Answer:**

**CNF**

$(P \vee Q \vee \neg R) \wedge (P \vee \neg Q \vee R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee R)$

**CNF**

$(P \vee Q \vee \neg R) \wedge (\neg Q \vee R) \wedge (\neg P \vee Q)$

**CNF**

$(Q \vee \neg P) \wedge (Q \vee \neg R) \wedge (R \vee \neg P) \wedge (R \vee \neg Q)$

**DNF**

$(\neg P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \vee (P \wedge Q \wedge R)$

**DNF**

$(\neg P \wedge \neg Q \wedge \neg R) \vee (Q \wedge R)$

**Problem 3.** *(10 Points)* Write the following formula in CNF

$$P \rightarrow ((Q \wedge R) \wedge (P \vee (\neg Q \wedge \neg R)))$$

**Answer:** $(\neg P \vee Q) \wedge (\neg P \vee R)$ or $(\neg P \vee \neg Q \vee R) \wedge (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee Q \vee R)$

**Problem 4.** *(10 Points)* Prove the following inference

$$(P \wedge W) \rightarrow (R \vee S)\ ,\ Q \rightarrow (U \wedge W)\ ,\ U \rightarrow P\ ,\ \neg S \quad \vdash \quad Q \rightarrow R$$

**Answer:**

1. $Q$
2. $Q \to (U \wedge W)$
3. $U \wedge W$
4. $U$
5. $U \to P$
6. $P$
7. $W$
8. $P \wedge W$
9. $(P \wedge W) \to (R \vee S)$
10. $R \vee S$
11. $\neg S$
12. $R$

**Problem 5.** *(10 Points)*
Let $P(x)$ be "$x$ *is a student*", $Q(x)$ be "$x$ *is a course*", $L(x, y)$ be "$x$ *likes* $y$" and $E(x, y)$ be "$x = y$". Formalize each of the following sentences by predicate formula.

1. 有些学生喜欢所有课程.
   right: $(\exists x)(P(x) \wedge (\forall y)(Q(y) \to L(x, y)))$
   wrong: $(\exists x)(\forall y)((P(x) \wedge Q(y)) \to L(x, y))$
2. 每个学生都有不喜欢的课程.
   right: $(\forall x)(P(x) \to (\exists y)(Q(y) \wedge \neg L(x, y)))$
   wrong: $(\forall x)(\exists y)((P(x) \wedge Q(y)) \to \neg L(x, y))$
3. 每个学生只喜欢一门课程.
   right: $(\forall x)(P(x) \to (\exists y)(Q(y) \wedge L(x, y) \wedge (\forall z)((Q(z) \wedge L(x, z)) \to E(y, z))))$
   wrong: $(\forall x)(\exists y)((P(x) \wedge Q(y) \to L(x, y)) \wedge (\forall z)(Q(z) \wedge L(x, z) \to E(z, y)))$
4. 有些课程只有一个学生喜欢.
   right: $(\exists x)(Q(x) \wedge (\exists y)(P(y) \wedge L(y, x) \wedge (\forall z)((P(z) \wedge L(z, x)) \to E(y, z))))$
   wrong: $(\exists x)(\exists y)((P(x) \wedge Q(y)) \to L(y, x)) \wedge (\forall z)((P(z) \wedge L(z, x)) \to E(y, z))))$

**Problem 6.** 附加题 *(5 Points)*
Put the correct relation in "(?)"

1. $(\exists x)(\forall y)(\forall z)P(x, y, z)$ (?) $(\forall y)(\exists x)(\exists z)P(x, y, z)$ ($\Rightarrow$ / $\Leftarrow$ / $\Leftrightarrow$ / None)
2. $(\exists x)(\forall y)(\forall z)P(x, y, z)$ (?) $(\exists y)(\forall x)(\exists z)P(x, y, z)$ ($\Rightarrow$ / $\Leftarrow$ / $\Leftrightarrow$ / None)
3. $(\exists x)(\forall y)(\forall z)P(x, y, z)$ (?) $(\forall z)(\exists y)(\forall x)P(x, y, z)$ ($\Rightarrow$ / $\Leftarrow$ / $\Leftrightarrow$ / None)
4. $(\forall x)(\exists y)(\forall z)P(x, y, z)$ (?) $(\forall z)(\exists y)(\forall x)P(x, y, z)$ ($\Rightarrow$ / $\Leftarrow$ / $\Leftrightarrow$ / None)
5. $(\exists y)(\forall x)(\exists z)P(x, y, z)$ (?) $(\exists z)(\exists y)(\forall x)P(x, y, z)$ ($\Rightarrow$ / $\Leftarrow$ / $\Leftrightarrow$ / None)

**Answer:**1. $\Rightarrow$, 2. None, 3. None, 4. None, 5. $\Leftarrow$.