Discrete Math (Honor) 2021-Fall Homework-10

Instructor: Xiang YIN

Due: Submit Online Before Final Exam

(Please use A4 paper. Do not use exercise book!)

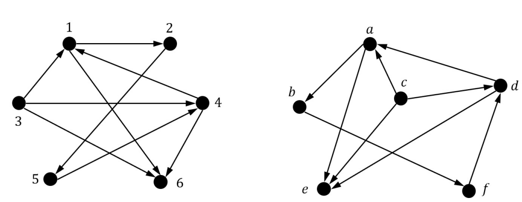
Problem 1. *(10 Points)*
Determine whether each of the following statements is true or false.

1. For any simple graph $G = \langle V, E\rangle$, if $\forall v, v' \in V : d(v) + d(v') \geq |V|$, then G has a Hamilton path. (True / False)
2. Any complete graph contains a Hamilton circuit. (True / False)
3. For any simple graph $G = \langle V, E\rangle$, if $\forall v \in V : d(v) \geq \frac{n}{2}$, then G has a Hamilton circuit. (True / False)
4. For any undirected graph $G = \langle V, E\rangle$, we have $\sum_{v\in V} d(v) = 2|E|$. (True / False)
5. Connected graph G is a tree if and only if each edge of G is a bridge. (True / False)

Problem 2. *(10 Points)*

1. If G has 7 vertices, where 6 vertices have degree 3 and one vertex has degree 6, then G has 12 edges.
2. If G has n vertices and m edges, where the degree of each vertex is either k or $k + 1$, then G contains $n(k+1)-2m$ vertices whose degrees are k.
3. If undirected graph G has 12 edges, 6 vertices with degree 3, and the degrees of the rest of the vertices are smaller than 3, then G contains at least 3 vertices.

Problem 3. *(10 Points)*
Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.

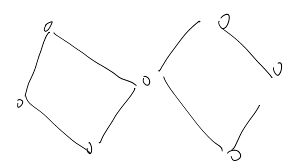


$f(1) = a$
$f(2) = b$
$f(3) = c$
$f(4) = d$
$f(5) = f$
$f(6) = e$

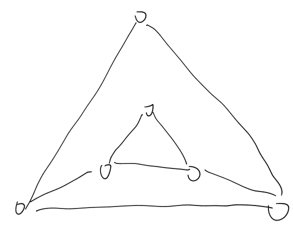
Problem 4. *(10 Points)*

1. Provide a simple graph with at least 4 vertices such that it has an Euler circuit but has no Hamilton circuit.
2. Provide a simple graph with at least 5 vertices such that it has a Hamilton circuit but has no Euler circuit.

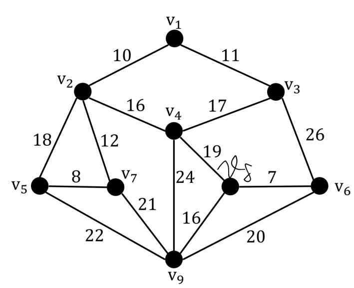
1.



2.



Problem 5. *(10 Points)*
Construct the minimum spanning tree of the following graph using both the Kruskal's Algorithm and the Prim's Algorithm (specific which edge is added at each step).



K's:

pick $e_1=(v_6,v_8)$
$E \leftarrow E \setminus \{e_1\}$
$E_T \leftarrow \{e_1\}$

pick $e_2=(v_5,v_7)$
$E \leftarrow E \setminus \{e_2\}$
$E_T \leftarrow E_T \cup \{e_2\}$

pick $e_3=(v_1,v_2)$
$E \leftarrow E \setminus \{e_3\}$
$E_T \leftarrow E_T \cup \{e_3\}$

pick $e_4=(v_1,v_3)$
...

pick $e_5=(v_2,v_7)$
...

pick $e_6=(v_2,v_4)$
...

pick $e_7=(v_9,v_8)$
...

pick $e_8=(v_3,v_4)$
$E \leftarrow E \setminus \{e_8\}$

pick $e_9=(v_4,v_8)$
$E \leftarrow E \setminus \{e_9\}$
$E_T \leftarrow E_T \cup \{e_9\}$

$E_T=\{e_1, e_2 \cdots e_7, e_9\}$

$T=\langle V, E_T \rangle$

P's: pick v_1, $V_T \leftarrow \{v_1\}$, $E_T \leftarrow \emptyset$

$e_1=(v_1,v_2)$
$V_T \leftarrow V_T \cup \{v_2\}$
$E_T \leftarrow \{e_1\}$

$e_2=(v_1,v_3)$
$V_T \leftarrow V_T \cup \{v_3\}$
$E_T \leftarrow E_T \cup \{e_2\}$

$e_3=(v_2,v_7)$
...

$e_4=(v_5,v_7)$
...

$e_5=(v_2,v_4)$
...

$e_6=(v_4,v_8)$
...

$e_7=(v_6,v_8)$
...

$e_8=(v_8,v_9)$
...

$T=\langle V_T, E_T \rangle$