Discrete Math (Honor) 2021-Fall Homework-3

Instructor: Xiang YIN
Due: 2021.10.15 Friday in Class
(Please use A4 paper. Do not use exercise book!)

Problem 1. *(15 Points)*
Prove the following inferences using resolution method.

1. $(P \vee Q) \wedge (P \to R) \wedge (Q \to R) \Rightarrow R$
2. $(S \to \neg Q) \wedge (P \to Q) \wedge (R \vee S) \wedge (R \to \neg Q) \Rightarrow \neg P$
3. $\neg(P \wedge \neg Q) \wedge (\neg Q \vee R) \wedge \neg R \Rightarrow \neg P$

Answer:
$S = \{...\}$

1. (a) $P \vee Q, \neg P \vee R, \neg Q \vee R, \neg R$
   (b) $Q \vee R, \neg Q \vee R, \neg R$
   (c) $R, \neg R$
   (d) $\square$
2. (a) $(\neg S \vee \neg Q), (\neg P \vee Q), (R \vee S), (\neg R \vee \neg Q), \neg\neg P$
   (b) $(\neg S \vee \neg Q), (\neg P \vee Q), (R \vee S), (\neg R \vee \neg Q), P$
   (c) $(\neg S \vee \neg Q), Q, (R \vee S), (\neg R \vee \neg Q)$
   (d) $(\neg S \vee \neg Q), Q, (R \vee S), (\neg R \vee \neg Q)$
   (e) $\neg S, Q, (R \vee S), (\neg R \vee \neg Q)$
   (f) $Q, R, (\neg R \vee \neg Q)$
   (g) $Q, \neg Q$
   (h) $\square$
3. (a) $\neg P \vee \neg\neg Q, \neg Q \vee R, \neg R \Rightarrow, \neg\neg P$
   (b) $\neg P \vee \neg\neg Q, \neg Q \vee R, \neg R, P$
   (c) $\neg\neg Q, \neg Q \vee R, \neg R$
   (d) $\neg\neg Q, \neg Q$
   (e) $\square$

Problem 2. *(21 Points)*
Formalize each of the following sentences using predicate logic formula. (You can define your own predicate needed)

1. Everyone loves everyone except himself.
2. Every student except Alice is a friend of Bob.
3. Every student who walks talks.

4. Every boy who loves Alice hates every boy who Alice loves.
5. Every boy who loves Alice hates some boy who Cauchy loves.
6. There exists an unique boy who loves Alice but hates Bob.
7. There exists an unique boy who hates everyone who loves Bob.

Answer:
Note: There are infinite many correct answers...

1. Answer 1: $P(x, y)$: $x = y$, $Q(x, y)$: x doesn't love y, $(\forall x)(\forall y)(P(x, y) \leftrightarrow Q(x, y))$.
   Answer 2: $Love(x, y)$: x loves y, $(\forall x)(\forall y)Love(x, y) \wedge \neg Love(x, x)$.
2. Answer: $F(x, y)$: x is a friend of Bob, $E(x)$: $x \neq Alice$. $(\forall x)(F(x) \leftrightarrow E(x))$.
3. Define:
   P(x): x is a student
   T(x): x talks
   W(x): x walks
   Formalization:
   $\forall x(P(x) \rightarrow W(x) \rightarrow T(x))$
4. $P(x)$: x loves Alice, $F(y)$: Alice loves y, $Q(x, y)$: x hates y, D: all boys.
   $(\forall x)(\forall y)P(x) \wedge F(y) \rightarrow Q(x, y)$.
5. $P(x)$: x loves Alice, $F(y)$: Cauchy loves y, $Q(x, y)$: x hates y, D: all boys.
   $(\forall x)(\exists y)P(x) \wedge F(y) \rightarrow Q(x, y)$.
6. $P(x)$: x loves Alice, $Q(x)$: x hates Bob, $E(x, y)$: $x = y$
   $(\exists x)(\forall y)(P(x) \wedge Q(x) \rightarrow (P(y) \wedge Q(y) \rightarrow E(x, y)))$
7. $A(x, y)$: x hates y, $B(x)$: x loves Bob, $C(x, y)$: $x = y$
   $(\exists x)(\forall y)(B(y) \rightarrow A(x, y)) \wedge (\forall z)((B(y) \wedge A(z, y)) \rightarrow C(z, x))$

Problem 3. *(10 Points)*
For each of the following formulae, determine free variables and bound variables in it and determine the scope of each quantifier.

1. $(\forall x)(P(x) \rightarrow Q(x, y))$
2. $(\forall x)P(x, y) \rightarrow (\exists y)Q(x, y)$
3. $(\forall x)(\exists y)(P(x, y) \wedge Q(y, z) \vee (\exists x)R(x, y, z))$
4. $(\exists x)(P(x) \rightarrow Q(x)) \rightarrow (\exists y)R(y) \rightarrow S(z)$
5. $(\forall x)(P(x) \wedge (\exists y)Q(y)) \vee ((\forall x)P(x) \rightarrow Q(z))$

Answer

1. Free: y Bound: x
   Scope: $\forall x \mid P(x) \rightarrow Q(x, y)$
2. Free: y in $P(x, y)$ and x in $Q(x, y)$ Bound: x in $P(x, y)$ and y in $Q(x, y)$
   Scope:
   $\forall x \mid P(x, y)$
   $\exists y \mid Q(x, y)$
3. Free: z Bound: x y
   Scope:
   $\forall x \mid (\exists y)(P(x, y) \wedge Q(y, z) \vee (\exists x)R(x, y, z))$
   $\exists y \mid (P(x, y) \wedge Q(y, z) \vee (\exists x)R(x, y, z))$
   $\exists x \mid R(x, y, z)$

4. Free: z   Bound: x y
   Scope:
   $\exists x \mid P(x) \rightarrow Q(x)$
   $\exists y \mid R(y)$

5. Free: z   Bound: x y
   Scope:
   $\forall x \mid P(x) \wedge (\exists y) Q(y)$
   $\exists y \mid Q(y)$
   $\forall x \mid P(x)$