P186　1(13,14,15,16)　2(5,6,7,8,9)　3(1,3)

补充练习　求不定积分　(1) $\int x\ln x\,dx$　(2) $\int \cos(\ln x)\,dx$

(3) $I=\int \frac{\cos x}{\sin x+\cos x}dx$　(4) $J=\int \frac{\sin x}{\sin x+\cos x}dx$

P193　1(1,10)　2(3,5,7,8)　3(4,5)　4(2,3,4,17,18)　5(2,4)

P199　3(2,3)　4(2,3)

P186　1(13) $\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}dx \xlongequal[1+\sqrt{1-x^2}]{\text{上下同乘}} \int \frac{2-x^2}{x^2}dx+\int\frac{2\sqrt{1-x^2}}{x^2}dx = -\frac{2}{x}-x+\int\frac{2\sqrt{1-x^2}}{x^2}dx$

$\xlongequal{令x=\sin t} -\frac{2}{x}-x+2\int\frac{\cos^2 t}{\sin^2 t}dt = -\frac{2}{x}-x-2\cot(t)-2t+C$

$=-\frac{2}{x}-x-2\arcsin x-\frac{2\sqrt{1-x^2}}{x}+C$

(14) 令 $\sqrt{1+e^{2x}}=t$ 则 $\int\frac{1}{\sqrt{1+e^{2x}}}dx=\int\frac{1}{t}\cdot\frac{t}{t^2-1}\cdot dt=\int\frac{1}{t^2-1}dt$

$=\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|+C = \frac{1}{2}\ln\left(\frac{\sqrt{1+e^{2x}}-1}{\sqrt{1+e^{2x}}+1}\right)+C$

(15) $\int\frac{\sin x+\cos x}{\sqrt[3]{\sin x-\cos x}}dx=\int\frac{1}{\sqrt[3]{\sin x-\cos x}}d(\sin x-\cos x)=\int\frac{1}{\sqrt[3]{u}}du=\frac{3}{2}(\sin x-\cos x)^{\frac{2}{3}}+C$

(16) $\int\frac{1}{\sqrt{1+x+x^2}}dx=\int\frac{1}{\sqrt{(x+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}}d(x+\frac{1}{2}) = \ln\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|+C$

P186.　2.(5) 分部积分　$x^n=\left(\frac{x^{n+1}}{n+1}\right)'$　$\frac{x^{n+1}}{(n+1)^2}\left[(n+1)\cdot\ln x-1\right]+C$

(6) 分部积分　$1=(x)'$　$x\cdot\arctan x-\frac{1}{2}\ln(1+x^2)+C$

(7) 分部积分　$x=(\frac{1}{2}x^2)'$　$\frac{1}{2}(1+x^2)\arctan x-\frac{1}{2}x+C$

(8) 分部积分　$1=(x)'$　$\int\frac{\sqrt{x}}{1+x}dx \longrightarrow \sqrt{x}=t$

$x\arcsin\sqrt{\frac{x}{1+x}}-\sqrt{x}+\arctan\sqrt{x}+C$

(9) 分部积分　$\frac{e^{\arctan x}}{\sqrt{1+x^2}} = \left(e^{\arctan x}\right)'$

$\frac{1+x}{2\sqrt{1+x^2}}e^{\arctan x}+C$

3.(1)　$n=0$　$I_0=x+C$

$n=1$　$I_1=-\cos x+C$

$n\geq 2$.　分部积分　$I_n=-\sin^{n-1}x\cos x+(n-1)I_{n-2}-(n-1)I_n$

即 $I_n=\frac{n-1}{n}I_{n-2}-\sin^{n-1}x\cos x\,\frac{1}{n}$

(3) $n=0 \quad I_0 = \arcsin x + C$

$n=1 \quad I_1 = -\sqrt{1-x^2} + C$

$n \geqslant 2$ 分部积分 $\frac{x}{\sqrt{1-x^2}} = (-\sqrt{1-x^2})' \quad I_n = -x^{n-1}\sqrt{1-x^2} + (n-1)(I_{n-2} - I_n)$

则 $I_n = -\frac{x^{n-1}}{n}\sqrt{1-x^2} + \frac{n-1}{n} I_{n-2}$

P193 1.(1) $\int \frac{1}{(x+1)^2(x-1)}dx = \frac{1}{4}\int \frac{1}{x-1}dx - \frac{1}{4}\int \frac{x+3}{(x+1)^2}dx = \frac{1}{4}\ln\left|\frac{x-1}{x+1}\right| + \frac{1}{2(x+1)} + C$

(10) $\int \frac{x^2}{(x+1)^{100}}dx = \int \frac{(x-1)(x+1)+1}{(x+1)^{100}}dx = -\frac{1}{99(x+1)^{99}} + \frac{1}{49(x+1)^{98}} - \frac{1}{97(x+1)^{97}} + C$

补充练习: (1) 分部积分 $x = (\frac{1}{2}x^2)' \quad \frac{1}{2}x^2 \ln x - \frac{x^2}{4} + C$

(2) $\ln x = t$ 分部积分 $e^t = (e^t)' \quad \frac{[\sin(\ln x) + \cos(\ln x)]x}{2} + C$

(3)(4) $I = \int \frac{\cos x}{\sin x + \cos x}dx \quad J = \int \frac{\sin x}{\sin x + \cos x}dx$

则 $I + J = x + C$

$I - J = \int \frac{\cos x - \sin x}{\sin x + \cos x}dx = \int \frac{d(\sin x + \cos x)}{\sin x + \cos x} = \ln|\sin x + \cos x| + C$

从而 $I = \frac{x + \ln|\sin x + \cos x|}{2} + C$

$J = \frac{x - \ln|\sin x + \cos x|}{2} + C$

P193 2.(3) $t = \tan\frac{x}{2} \quad \frac{1}{2}\ln\left|\tan\frac{x}{2}\right| - \frac{1}{4}\tan^2\frac{x}{2} + C$

(5) 原式 $= \int \frac{2}{5+\cos 2x}dx \quad t = \tan x \quad \frac{1}{\sqrt{6}}\arctan\frac{\sqrt{2}\tan x}{\sqrt{3}} + C$

(7) 原式 $= \int \frac{d(\sin x)}{(2+\sin x)(1-\sin^2 x)}$ 令 $t = \sin x \quad -\frac{1}{3}\ln|2+\sin x| + \frac{1}{2}\ln|1+\sin x| - \frac{1}{6}\ln|1-\sin x| + C$

或者 $t = \tan\frac{x}{2}$ 原式 $= \int \frac{1}{\left(2 + \frac{2t}{1+t^2}\right)\frac{1-t^2}{1+t^2}} \frac{2dt}{1+t^2}$

$= \int \frac{1+t^2}{(1+t^2+t)(1-t^2)}dt = \int \frac{1+t^2}{(1-t^3)(1+t)}dt$

$= \int \frac{1}{1+t}dt + \int \frac{t^2}{1-t^3}dt = \ln\left|1+\tan\frac{x}{2}\right| - \frac{1}{3}\ln\left|1-\tan^3\frac{x}{2}\right| + C$

(8) $(\tan x)' = \sec^2 x \quad \sqrt{2}\cdot\arctan\frac{\tan x}{\sqrt{2}} - x + C$

3.(4) $t = \frac{1}{x}$ 原式 $= -\int \frac{dt}{\sqrt{2t^2+2t+1}} = -\frac{\sqrt{2}}{2}\int \frac{d(t+\frac{1}{2})}{\sqrt{(t+\frac{1}{2})^2 + (\frac{1}{2})^2}} = -\frac{\sqrt{2}}{2}\cdot\ln\left|t+\frac{1}{2}+\sqrt{(t+\frac{1}{2})^2+\frac{1}{4}}\right| + C$

$= -\frac{\sqrt{2}}{2}\cdot\ln\left|\frac{1}{x} + \frac{1}{2} + \sqrt{\frac{1}{x^2} + \frac{1}{x} + \frac{1}{2}}\right| + C$

(5) $x - \frac{1}{2} = \frac{3}{2}\sin t \quad \frac{9}{8}\arcsin\frac{2x-1}{3} + \frac{2x-1}{4}\sqrt{-x^2+x+2} + C$

4(2) $\int \frac{\tan x}{a^2\sin^2 x + b^2\cos^2 x}dx = \int \frac{\tan x}{a^2\tan^2 x + b^2}\cdot d\tan x \overset{t=\tan x}{=\!=} \int \frac{t}{a^2t^2+b^2}dt$

$= \frac{1}{2}\int \frac{dt^2}{a^2t^2+b^2} = \frac{1}{2a^2}\cdot \ln|a^2\tan^2 x + b^2| + C \quad (a \neq 0)$

若 $a=0$，则 原式 $= \int \frac{\tan x}{b^2}d\tan x = \frac{1}{2b^2}\tan^2 x + C$

(3) $e^x = t$ 原式 $= \int \frac{\ln t}{(t-1)^2}dt = -\frac{\ln t}{t-1} + \int \frac{dt}{t(t-1)} = -\frac{\ln t}{t-1} + \ln\left|\frac{t-1}{t}\right|$

$= -\frac{x}{e^x-1} + \ln\left|\frac{e^x-1}{e^x}\right| + C$

(4) 分部积分. $\int \frac{xe^x}{(1+x)^2}dx = -\frac{xe^x}{1+x} + \int \frac{(x+1)e^x}{1+x}dx = -\frac{xe^x}{1+x} + e^x + C$

$= \frac{e^x}{1+x} + C$

(17) $\arcsin x = t$ $\quad \frac{x}{2}\sqrt{1-x^2}\arcsin x - \frac{1}{4}[x^2 - (\arcsin x)^2] + C$

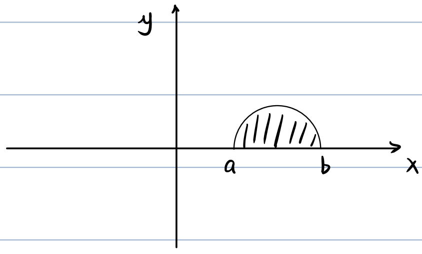
(18) $\arcsin x = t$ $\quad \frac{x}{2}\sqrt{1-x^2}\arcsin x - \frac{1}{4}[x^2 + (\arcsin x)^2] + C$

5.(2) $\cos^2 x = t$ $\quad f(x) = -\frac{1}{2}x^2 + x + C \quad x \in [0,1]$

(4) $e^x = t$ $\quad x\left(\frac{3}{2}\sin(\ln x) + \frac{1}{2}\cos(\ln x)\right) + C$

P199 3(2) 3

(3) $\frac{a-1}{\ln a}$ 讨论 $a=1$ 取 $\frac{i}{n}$ 然后求和求极限



4(2) $f(x) = \sqrt{(x-a)(b-x)}$

则 $(f(x))^2 + (x - \frac{a+b}{2})^2$

$= -x^2 - ab + (a+b)x + x^2 + (\frac{a+b}{2})^2 - (a+b)x = (\frac{b-a}{2})^2$

则 $f(x)$ 为 圆心为 $\frac{a+b}{2}$ 半径为 $\frac{b-a}{2}$ 的半圆.

从而 $S = \int_a^b \sqrt{(x-a)(b-x)}\,dx = \frac{1}{2}\cdot\pi\cdot(\frac{b-a}{2})^2 = \frac{(b-a)^2\pi}{8}$

(3) $f(x) = |x - \frac{a+b}{2}|$ 如右图所示.

则 $S = \int_a^b |x - \frac{a+b}{2}|dx = (\frac{b-a}{2})^2$