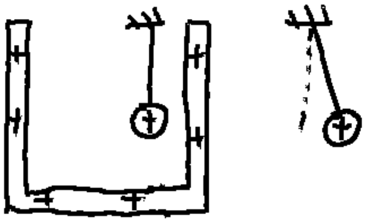
物理学引论（荣誉）Ⅱ

第10章 静电场

§10.1 库仑定律

库仑定律的建立过程

1755年 B.Franklin 的"金属桶实验"



桶内：球不偏 ，桶外：球偏

$F \propto \frac{1}{r^2}$ —— 猜想实验

1773年 H.Cavendish "同心金属球实验"

内表面严格不带电

T_1 $F_{回} = -F\theta$ $T = 2\pi\sqrt{\frac{ml}{F}}$

理论 $T \propto \sqrt{\frac{1}{F}}$

实验 $T \propto r$

$\Rightarrow F \propto \frac{1}{r^2}$

库仑定律 $\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\vec{e_r} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^3}\vec{r}$

真空介电常数 $\varepsilon_0 = \frac{1}{4\pi k} = 8.854\times10^{-12}\ (C^2\cdot N^{-1} m^{-1})$

T_2 $\frac{F_{电}}{F_{引}} = \frac{\frac{e^2}{4\pi\varepsilon_0}\cdot\frac{1}{r^2}}{\frac{Gm^2}{r^2}} = \frac{1}{4\pi\varepsilon_0 G m_e m_p} \approx 2\times10^{39}$

Q: 库仑定律在多小的尺度上还成立?

$\lambda_c = \frac{\hbar}{m_e c}$ vacuum polarization "真空极化"

库仑力满足叠加原理

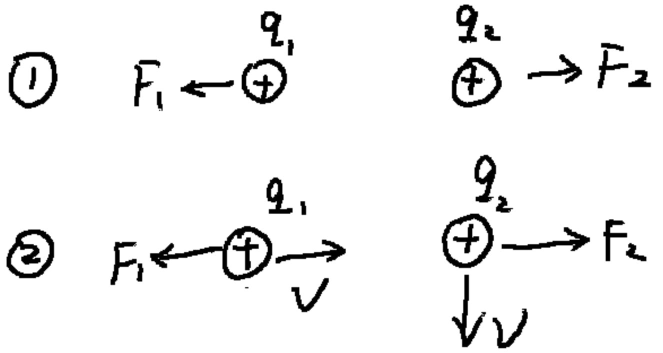
T_3 $\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 Q}{|\vec{R}-\vec{r_1}|^3}(\vec{R}-\vec{r_1}) + \frac{1}{4\pi\varepsilon_0}\frac{q_2 Q}{|\vec{R}-\vec{r_2}|^3}(\vec{R}-\vec{r_2})$

T4 $E_k = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_1} - \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_2} \approx 10^9 J$

电荷守恒

电荷运动不变性，即具有相对论不变性

T5 第二种 $F_1 \neq F_2$

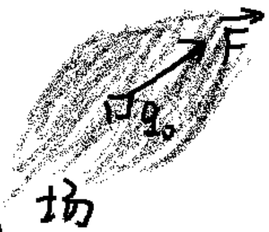


产生磁场

10.2 电场

电荷作用的中介物质

电场强度



$\vec{E} \overset{def}{=} \frac{\vec{F}}{q_0}$ (N/C, V/m)

电荷电量小 > 影响源电荷分布

线度小

T_6 $\quad \vec{E}_{r_1} = \dfrac{q(\vec{r_2}-\vec{r_1})}{4\pi\varepsilon_0 |\vec{r_2}-\vec{r_1}|^3}$

点电荷场强 $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q\vec{r}}{r^3}$

电场的计算

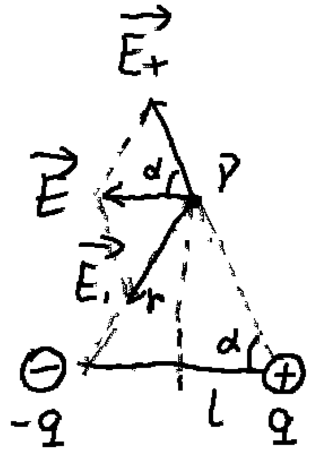
场强的矢量叠加原理

$\vec{E} = \sum_i \vec{E_i} \qquad \vec{E} = \int_Q \dfrac{dq\,\vec{r}}{4\pi\varepsilon_0 r^3}$

线电荷 $dq = \lambda dl$

面电荷 $dq = \sigma ds$

体电荷 $dq = \rho dV$



Q: $(-q, l, q)$, (中垂 r), $E=?$

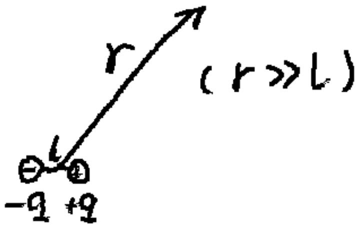
$E = 2E_+\cos\alpha$ (方向垂直中垂线向左)

$E_+ = \dfrac{q}{4\pi\varepsilon_0 R^2} \qquad \cos\alpha = \dfrac{l}{2R} \qquad R = \sqrt{(\frac{l}{2})^2 + r^2}$

得 $E = \dfrac{ql}{4\pi\varepsilon_0 (r^2 + \frac{l^2}{4})^{3/2}}$

当 $r \gg l$ 时, $\quad E = \dfrac{ql}{4\pi\varepsilon_0 r^2}$

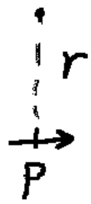
电偶极子 (Electric dipole)



电偶极矩 $p = ql$

方向由负指向正

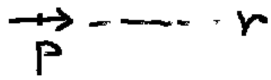
中远距离看水分子



已知 $\vec{p}$ 及 r

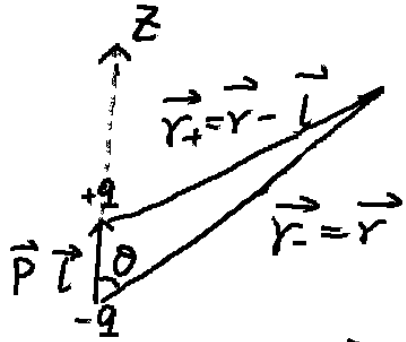
$$\vec{E} = -\frac{\vec{p}}{4\pi\varepsilon_0 r^3} \quad (\text{中垂线})$$

$$\vec{E} = \frac{2\vec{p}}{4\pi\varepsilon_0 r^3} \quad (\text{延长线})$$



补充内容:

一般情况



$$\vec{E}_- = \frac{-q\vec{r}_-}{4\pi\varepsilon_0 r_-^3}$$

$$\vec{E}_+ = \frac{q\vec{r}_+}{4\pi\varepsilon_0 r_+^3}$$

$\vec{r}_- = (x, y, z)$

$\vec{r}_+ = (x, y, z-l)$

令 $\vec{f}(\vec{r}) = \frac{q}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3}$

则 $\vec{E} = \vec{E}_- + \vec{E}_+ = \vec{f}(\vec{r}_+) - \vec{f}(\vec{r}_-)$

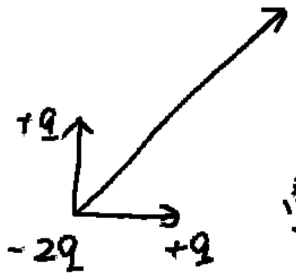
$= \vec{f}(x, y, z-l) - \vec{f}(x, y, z)$

令 $\Delta = -l$ 则 $\vec{E} = \Delta \frac{\vec{f}(x, y, z+\Delta) - \vec{f}(x, y, z)}{\Delta} = -l\frac{\partial \vec{f}}{\partial z}$.

T7: 计算 $\frac{\partial}{\partial z}\left(\frac{\vec r}{r^3}\right)$

$$\frac{\partial}{\partial z}\left(\frac{\vec r}{r^3}\right)=\frac{\partial}{\partial z}\left(\frac{x\vec i+y\vec j+z\vec k}{(x^2+y^2+z^2)^{3/2}}\right)=\frac{x^2+y^2-2z^2}{(x^2+y^2+z^2)^{5/2}}\vec k .$$

$$\vec E=\frac{1}{4\pi\varepsilon_0 r^3}\left(\frac{3pz\vec r}{r^3}-p\vec k\right)$$



叠加

一般 $\vec E=\frac{1}{4\pi\varepsilon_0 r^3}\left(\frac{3(\vec p\cdot\vec r)\vec r}{r^2}-\vec p\right)$

线电荷

T8 $$dE_x=\frac{\lambda\,dx\cos\theta}{4\pi\varepsilon_0 r^2}=\frac{\lambda\cos\theta}{4\pi\varepsilon_0 d}d\theta$$

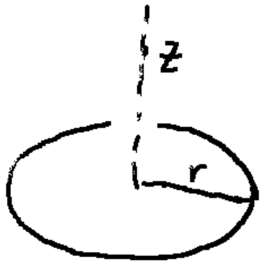
$$dE_y=\frac{\lambda\,dx\sin\theta}{4\pi\varepsilon_0 r^2}=\frac{\lambda\sin\theta}{4\pi\varepsilon_0 d}d\theta$$

无限长：柱状分布　　半无限长类似分析

$$\begin{cases}E_x=\frac{\lambda}{4\pi\varepsilon_0 d}(\sin\theta_2-\sin\theta_1)\to 0\\ E_y=\frac{\lambda}{4\pi\varepsilon_0 d}(\cos\theta_1-\cos\theta_2)\to\frac{\lambda}{2\pi\varepsilon_0 d}\end{cases}$$

（注意顺序）

圆环 $E_z = \oint dE_z = \frac{qz}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}$



面电荷 圆盘 $= \int$ 圆环

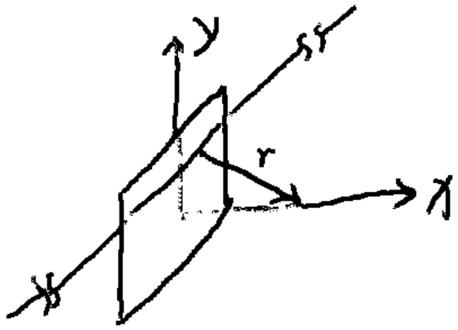
$$E = \frac{\sigma}{2\varepsilon_0}\left(1-\frac{z}{\sqrt{R^2+z^2}}\right)$$

$z \to +\infty$ $E = \frac{R^2\sigma}{4\varepsilon_0 z^2}$

$R \to +\infty$ $E = \frac{\sigma}{2\varepsilon_0}$.

$\vec{E} = \frac{\sigma}{2\varepsilon_0}\vec{n}$ 均强电场

Q:无限大均匀带电面σ，场点P，距离x，求$\vec{E}(x)$



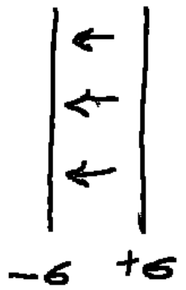
由对称性考虑x轴方向即可

T9: $dE = \frac{\sigma dy}{2\pi\varepsilon_0\sqrt{y^2+x^2}}$

$dE_x = dE\frac{x}{\sqrt{y^2+x^2}} = \frac{x\sigma dy}{2\pi\varepsilon_0(x^2+y^2)}$

$E = \int_{-\infty}^{+\infty}\frac{x\sigma}{2\pi\varepsilon_0(x^2+y^2)}dy = \frac{\sigma}{2\varepsilon_0}$

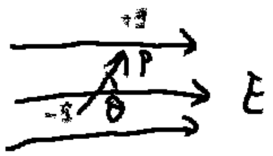
无限大面场强公式 $\vec{E} = \frac{\sigma}{2\varepsilon_0}\vec{n}$



$E = \frac{\sigma}{\varepsilon_0}$

电场力和力矩 $\vec{F} = q_0\vec{E}$

Q: 求电偶极子受的合力

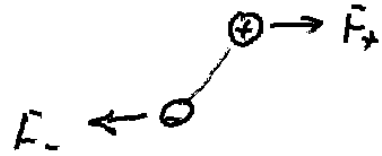


T10 $M = Eq\cdot\frac{l}{2}\sin\theta\cdot 2 = Eql\sin\theta$

方向竖直纸面向里 $\vec{M} = \vec{p}\times\vec{E}$

证明: $\vec{M} = \vec{r_+} \times \vec{F_+} + \vec{r_-} \times \vec{F_-}$

$= (\vec{r_+} - \vec{r_-}) \times (q\vec{E}) = \vec{P} \times \vec{E}$.



Q: Dipole in Non-Uniform Field

$\vec{p} = q\vec{l}$ $\quad \vec{E} = \vec{E}(x,y,z) = \vec{E}(\vec{r})$

$$\vec{F} = \vec{F_+} + \vec{F_-}$$

$$= q(\vec{E_+} - \vec{E_-})$$

$$= q(\vec{E}(\vec{r}+\vec{l}) - \vec{E}(\vec{r}))$$

$$= P\frac{\partial \vec{E}}{\partial z}$$

T11 $\quad \vec{F} = q(\vec{E}(x+P_x, y+P_y, z+P_z) - \vec{E}(x,y,z))$

$$= P_x\frac{\partial \vec{E_x}}{\partial x} + P_y\frac{\partial \vec{E_y}}{\partial y} + P_z\frac{\partial \vec{E_z}}{\partial z}.$$

$$= \vec{p} \cdot \nabla \vec{E}$$

∇: 梯度算子

§10.3 高斯定理

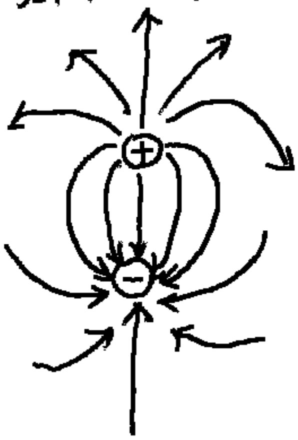
1、电场线

始于正电荷，止于负电荷

不相交　切线方向为切点电场方向

场强越强电场线越密.

T12



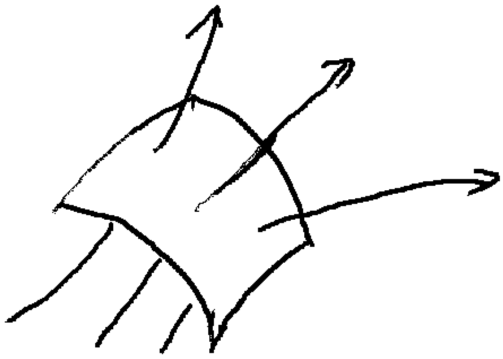
T13　$E \propto \frac{\Delta N}{\Delta S \cos\theta}$

$$\frac{\Delta N}{\Delta S_\perp} \propto E$$

$$dN = E\,dS_\perp = E\,dS\cos\theta$$

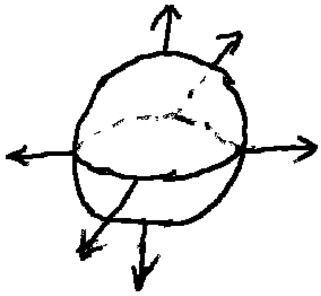
2. 电通量 (flux)

$$d\Phi \stackrel{def}{=} \vec{E} \cdot d\vec{S}$$



曲面 $\Phi = \iint_S \vec{E} \cdot d\vec{S}$

闭合面



约定：外法向为正方向

$$\Phi = \oint\!\!\oint_S \vec{E} \cdot d\vec{S}$$

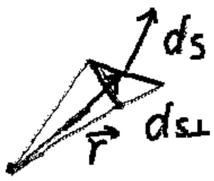
电场线穿出为正，穿入为负.

T14 $\Phi = \pi r^2 E \sin\theta$

T15 $\Phi = 4\pi r^2 \cdot \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} = \frac{q}{\varepsilon_0}$

$$\left(\Phi = \oint\!\!\oint \vec{E} d\vec{s} = \oint\!\!\oint E ds \right)$$

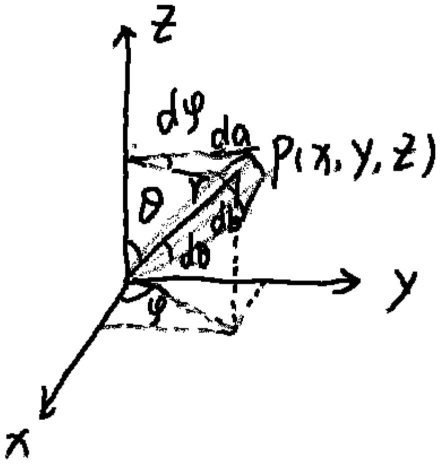
立体角



$$d\Omega = \frac{ds_\perp}{r^2}$$

$$\overset{def}{=} \frac{\vec{e_r}\cdot d\vec{s}}{r^2} = \sin\theta \, d\theta \, d\varphi$$ —球坐标系

球坐标系



$(x,y,z) \rightleftarrows (r,\theta,\varphi)$

$ds = da \cdot db$ $\quad$ ds—球面上的小面元

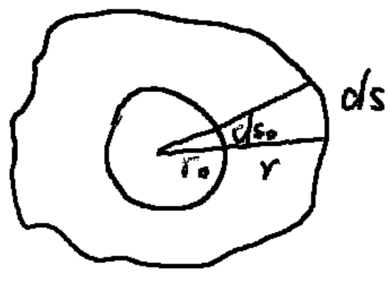
$$\begin{cases} db = r\,d\theta \\ da = r\sin\theta \, d\varphi \end{cases}$$

单位球面投影面积（根据定义可以小于0）.

Q: 计算任意闭合曲面对面内一点O所张的立体角.

$$\Omega = \oiint \frac{\vec{e_r}\cdot d\vec{s}}{r^2}$$

$$= \oiint \frac{ds_0}{r_0^2} = 4\pi$$



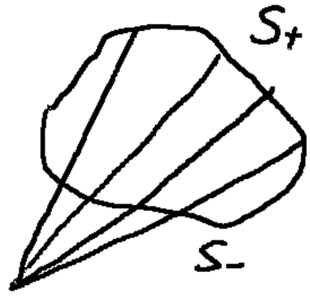
$\Phi = \frac{q}{\varepsilon_0}$

T16 $\Phi_1 = \frac{q}{\varepsilon_0} \cdot \frac{\Delta\Omega}{4\pi} = \frac{\Delta\Omega q}{4\pi\varepsilon_0}$.

Q: 任意闭合曲，曲面外一点 O，计算积分

$$\oiint \frac{\vec{e_r} \cdot d\vec{s}}{r^2}$$

$$= \iint \left(\frac{\vec{e_r} \cdot d\vec{s}}{r^2} + \frac{\vec{e_r} \cdot d\vec{s}'}{r'^2} \right) = 0$$



$\Phi_{E外} = 0$

$N_q = \frac{q}{\varepsilon_0}$.

$$\oiint \vec{E} \cdot d\vec{s} = \begin{cases} q/\varepsilon_0 \\ 0 \end{cases}$$

3. 高斯定理

$$\oiint \vec{E} \cdot d\vec{s} = \frac{1}{\varepsilon_0} \sum_i q_i \quad 离散$$

$$= \frac{1}{\varepsilon_0} \iiint_V \rho dv \quad 连续$$

例 引力场定义为 $\vec{E_g} = \frac{\vec{F}}{m_0}$

$\vec{F}_{引} = -\frac{GMm}{r^2}$ $\vec{F}_{库} = \frac{q}{4\pi\varepsilon_0 r^2}$

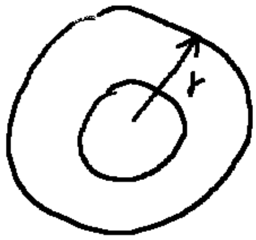
$\Phi = -4\pi GM$

4. 高斯定理的应用

求电场分布

球对称　　柱对称　　面对称

(1) 球外 $(r \geq R)$



① q分布对称性 $\rightarrow \vec{E}$对称性

② 选择S.

③ $\oiint_S \vec{E} \cdot d\vec{s} = \oiint_S E ds = E \oiint_S ds = 4\pi r^2 E$

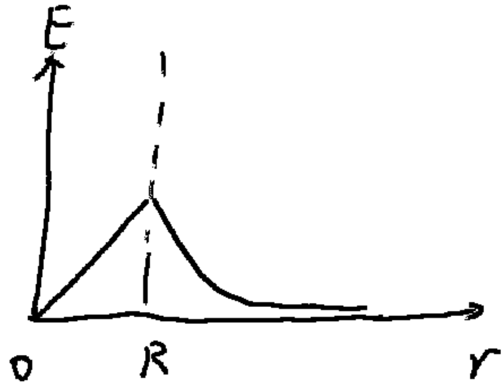
$= \frac{q}{\varepsilon_0}$

得 $\vec{E} = \frac{q \vec{e_r}}{4\pi\varepsilon_0 r^2}$

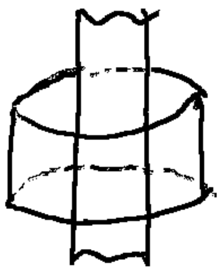
T18 球内 $\frac{1}{\varepsilon_0}\frac{4}{3}\pi r^3\rho = 4\pi r^2 E$

得 $\vec{E} = \frac{r\vec{e_r}}{3\varepsilon_0}$

$$E = \begin{cases} \frac{\rho}{3\varepsilon_0}\vec{r} = \frac{q}{4\pi\varepsilon_0 R^3}\vec{r} & (r<R) \\ \frac{q}{4\pi\varepsilon_0 r^3}\vec{r} & \end{cases}$$



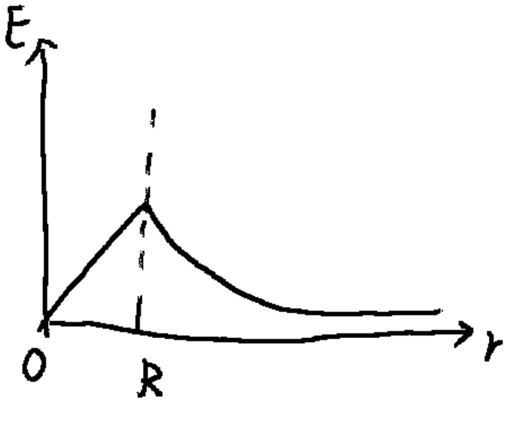
Q: 求无限长圆柱体 (R,ρ) 的电场分布



$$E\cdot 4\pi r l = \frac{l\pi R^2\rho}{\varepsilon_0}$$

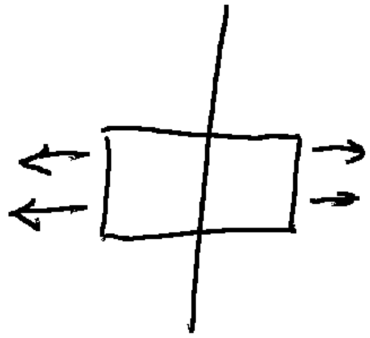
得 $\vec{E} = \begin{cases} \frac{\lambda}{2\pi\varepsilon_0 r}\vec{e}_r & (\lambda=\pi R^2\rho) \quad r>R \\ \frac{\lambda\vec{r}}{2\pi\varepsilon_0 R^2} & (r\le R) \end{cases}$

λ 单位：C/m



Q：无限大均匀带电平面σ的电场分布

$$\oint_S \vec{E}\cdot d\vec{S} = 2E\Delta S = \frac{\Delta S\sigma}{\varepsilon_0}$$



得 $E = \dfrac{\sigma}{2\varepsilon_0}$

T19：已知$E_1 = 5$　$E_2 = 3$　求σ　（单位 V/m）

$$(E_1 + E_2)\Delta S = \frac{\Delta S\sigma}{\varepsilon_0}$$

得 $\sigma = 8\varepsilon_0$

T20　$$\frac{q_0}{4\pi\varepsilon_0 r^2}e^{-r}\cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

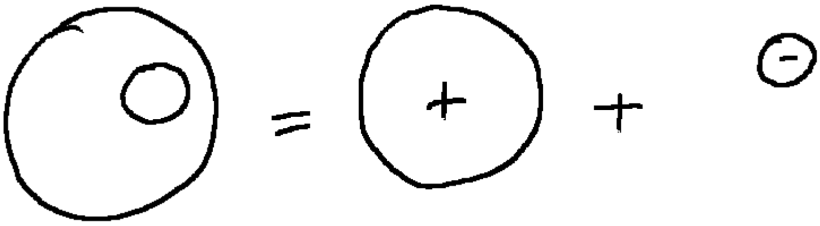
$$q = q_0 e^{-r}$$

$$\frac{dq}{dr} = -q_0 e^{-r}$$

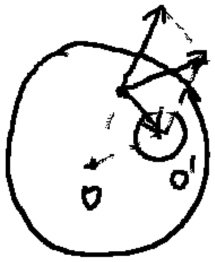
$$\rho(r) = \frac{dq}{4\pi r^2 dr} = -\frac{q_0}{4\pi r^2}e^{-r}$$

类似氢原子电荷分布

Q：均匀带电球（R,ρ），内有一半径为r的空腔，证明空腔内为匀强电场.



补偿法



$$\vec{E}=\vec{E_+}+\vec{E_-}=\frac{\rho}{3\varepsilon_0}(\vec{r_+}-\vec{r_-})=\frac{\rho}{3\varepsilon_0}\overrightarrow{OO'}$$

高斯定理微分形式

$$\oint\!\!\!\oint_S \vec{E}\cdot d\vec{S}=\frac{1}{\varepsilon_0}\iiint_V \rho dV=\frac{1}{\varepsilon_0}\rho\Delta V$$ ΔV很小，以致可以认为ρ不变

$$\oint\!\!\!\oint_S \vec{A}\,d\vec{S}=\iiint_V(\nabla\cdot\vec{A})dv$$ —— 数学中的高斯定理

散度：$\nabla\vec{A}=\frac{\partial A_x}{x}+\frac{\partial A_y}{y}+\frac{\partial A_z}{z}$.

$$\iiint_{\Delta V}(\nabla\cdot\vec{E})dV=\frac{1}{\varepsilon_0}\rho\Delta V$$

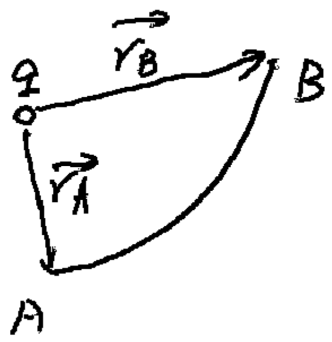
$$(\nabla\cdot\vec{E})\Delta V=\frac{1}{\varepsilon_0}\rho\Delta V$$

得 $\nabla\cdot\vec{E}=\frac{1}{\varepsilon_0}\rho$. 即 $\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{y}+\frac{\partial A_z}{z}=\frac{\rho}{\varepsilon_0}$.

divergence

§10.4 电势

Q: 计算功 (q, q_0, r_A, r_B, L)



$$A = \int_A^B \vec{F}\cdot d\vec{r}$$

$$= \int_A^B \frac{q_0 q}{4\pi\varepsilon_0 r^3}\vec{r}\,d\vec{r}$$

$$= \frac{q_0 q}{4\pi\varepsilon_0}\left(\frac{1}{r_A}-\frac{1}{r_B}\right)$$ 与L无关，只与初末态有关.

点电荷电场 $\oint_L \vec{E}\cdot d\vec{l} = 0$ 环路积分为0.

电势 $V_1 - V_2 \overset{def}{=\!=} \int_1^2 \vec{E}\cdot d\vec{l}$ $\quad -dV = \vec{E}\cdot d\vec{l}$

电势零点 $V(P_0)\equiv 0$ $\quad V(P) = \int_P^{P_0}\vec{E}\cdot d\vec{l}$

T21 $\quad V_R = \int_R^\infty \frac{q}{4\pi\varepsilon_0 r^2} = \frac{q}{4\pi\varepsilon_0 R}$

Q: 求均匀带电球体 (R, q) 电势分布 $\quad V(\infty)\equiv 0$.

1) $r > R$ $\quad V(r) = \frac{q}{4\pi\varepsilon_0 r}$

2) $r \leq R$ $\quad V(r) = \int_r^\infty \vec{E}\cdot d\vec{l} = \int_r^R \vec{E}_{内}\cdot d\vec{l} + \int_R^\infty \vec{E}_{外}\cdot d\vec{l}$

$$= \frac{q(3R^2 - r^2)}{8\pi\varepsilon_0 R}$$

Q：均匀带电球面 电势分布

Q：求无限长均匀带电直线（电荷密度λ）外任一点 P(r) 的电势，

① 若以无限远为势能零点.

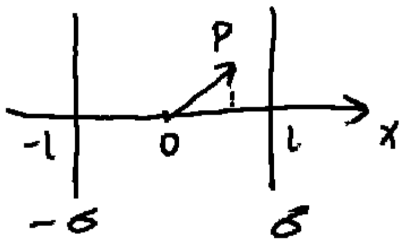
$$V(r)=\int_r^{\infty}\vec{E}\cdot d\vec{l}=\frac{\lambda}{2\pi\varepsilon_0}\ln r\Big|_r^{\infty}$$

此时无限远对分析不等价.

② $V(r_0)=0$.

$$V(r)=\frac{\lambda}{2\pi\varepsilon_0}\ln r\Big|_r^{r_0}=\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_0}{r}$$

T22



$\frac{\sigma}{\varepsilon_0}\cdot x \quad |x|<l$

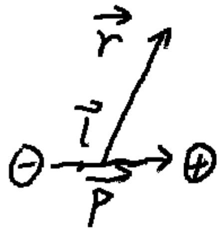
电势的叠加原理

(由场强叠加原理可知) $V_p = V_1 + V_2 + \cdots + V_n$.

$$V = \sum_i \frac{q_i}{4\pi\varepsilon_0 r_i} \qquad V = \iiint_Q \frac{dq}{4\pi\varepsilon_0 r}$$

Q: 计算电偶极子的电势分布 ($\vec{p}, \vec{r}, \theta$)

$$\vec{p} = q\vec{l}$$



$$V = V_+ + V_- = \frac{q}{4\pi\varepsilon_0 r_+} - \frac{q}{4\pi\varepsilon_0 r_-}$$

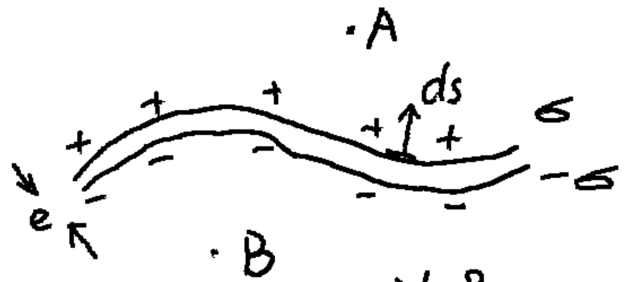
$$= \frac{q}{4\pi\varepsilon_0} \frac{(r_- - r_+)}{r_+ r_-} \approx \frac{ql\cos\theta}{4\pi\varepsilon_0 r^2}$$

$(\vec{p}, \vec{r})$ $\quad V(\vec{r}) = \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}$

T23 $$\int_0^L \frac{Q\frac{dx}{L}}{4\pi\varepsilon_0\sqrt{(x-a)^2+b^2}} = \frac{Q}{4\pi\varepsilon_0 L}\int_{-a}^{L-a}\frac{dx}{\sqrt{x^2+b^2}}$$

$$= \frac{Q}{4\pi\varepsilon_0 L} \ln \frac{L-a+\sqrt{L^2-2aL+2a^2}}{(\sqrt{2}+1)a}$$

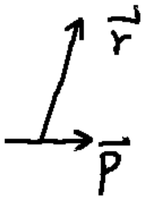
电偶极层&电偶极层强度



V_A ?

$$d\vec{p} = \sigma\, ds\, \vec{l}$$

$$\vec{\tau} = \frac{d\vec{p}}{ds} = \sigma \vec{l} \quad [C/m]$$



$$V = \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}$$

$$dV_A = \frac{\vec{r}\cdot d\vec{p}}{4\pi\varepsilon_0 r^3} = \frac{\vec{r}\cdot d\vec{s}}{r^3}\left(\frac{\sigma l}{4\pi\varepsilon_0}\right)$$

$$V_A = \left[\iint_S \frac{\vec{r}\cdot d\vec{s}}{r^3}\right]\frac{\sigma l}{4\pi\varepsilon_0} = \Omega_A \frac{\sigma l}{4\pi\varepsilon_0}$$

$$V_B = -\Omega_B \frac{\sigma l}{4\pi\varepsilon_0}$$

$$\Delta V = V_A - V_B = (\Omega_A + \Omega_B)\frac{\sigma l}{4\pi\varepsilon_0}.$$

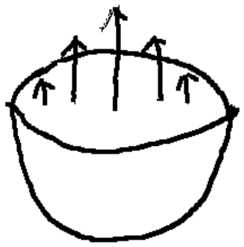
当A, B两点无限接近电偶极层时

$$\Omega_A + \Omega_B \to 4\pi$$

$$V_A - V_B \to \frac{\tau}{\varepsilon_0}.$$

T_{24} $V=\int_0^{\pi}\frac{R\lambda_0\sin\theta\,d\theta}{4\pi\varepsilon_0 r}$ $r=\sqrt{(x-R\cos\theta)^2+(R\sin\theta)^2}$

$=\frac{\lambda_0}{2\pi\varepsilon_0}$



等势面.

电势梯度

$\overrightarrow{grad}(V)=\vec{\nabla}V\overset{def}{=}\frac{dV}{dn}\vec{e_n}$

大小等于沿着等势面法线方向的空间变化率，指向电势增加的方向.

T_{25} $\overrightarrow{grad}(V)=\frac{dV}{dn}\vec{e_n}=-a\vec{i}$.

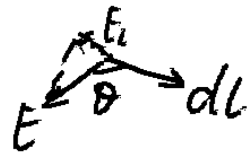
Q: 求点电荷q在$\vec{r}$点电势梯度

$V=\frac{q}{4\pi\varepsilon_0 r}$ $\overrightarrow{grad}(V)=-\frac{q}{4\pi\varepsilon_0 r^2}\vec{e_r}$

$$\overrightarrow{\text{grad}}(V) = -\vec{E}$$

Q：电势的方向导数与电场强度分量

$$dV = -\vec{E}\cdot d\vec{l} = -E\,dl\cos\theta$$



$$= -E_l\,dl$$

得 $E_l = -\dfrac{\partial V}{\partial l}$

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}.$$

故 $\vec{E} = -\left(\dfrac{\partial V}{\partial x}\vec{i} + \dfrac{\partial V}{\partial y}\vec{j} + \dfrac{\partial V}{\partial z}\vec{k}\right) = -\vec{\nabla} V$

T_{26} $\quad \vec{E}(x,y) = -\dfrac{\partial U(x,y)}{\partial x}\vec{i} - \dfrac{\partial U(x,y)}{\partial y}\vec{j}$

Q：由电偶极子分布：$V(\theta, r) = \dfrac{p\cos\theta}{4\pi\varepsilon_0 r^2}$ 计算 (E_r, E_θ)

$$E_r = -\frac{\partial V}{\partial r}\Big|_\theta = \frac{p\cos\theta}{2\pi\varepsilon_0 r^3}$$

$$E_\theta = -\frac{\partial V}{dn}\Big|_r = -\frac{1}{r}\frac{\partial V}{\partial \theta}. = \frac{p\sin\theta}{4\pi\varepsilon_0 r^3}$$

电势能 $W = qV$

$$W_1 - W_2 = \int_1^2 q\vec{E} \cdot dl$$

T26 $W = -l\cos\theta qE = -\vec{P} \cdot \vec{E}$

电偶极子 $W = -\vec{P} \cdot \vec{E}$

第11章 导体和电介质

导体：$\rho \sim 10^{-8} \sim 10^{-7}\,\Omega m$

半导体：$\rho \sim 10^{-3} \sim 10^{+5}\,\Omega m$

绝缘体：10^{+9} 10^{+17}

超导：$\rho \sim 10^{-26}\,\Omega m$

§11.1 导体

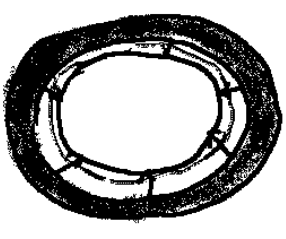
静电平衡 $I=0, I_s=0$ $\vec{E}_{内}=0$.

整个导体成为等势体.

$\rho_B=0$（由$\vec{E}_{内}=0$及高斯定理知）

$\vec{E}=\frac{\sigma}{\varepsilon_0}\vec{n}$（由等势及高斯定理知）

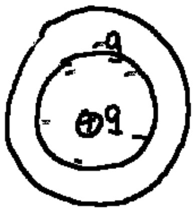
腔内无电荷，$U_{腔}=U_0$，$E_{腔内}=0$，$\sigma_{内}=0$.



反证法 可证 $U_{S_1}=U_{S_2}$

$U_{腔} = U_0 \rightarrow \vec{E}_{腔} = 0. \rightarrow \sigma_i = 0.$

Q：静电平衡导体空腔中有电荷



导体接地：只保证电势为零

T27

$E_{腔内} = 0$ ① ③

$E_{s外} = \frac{\sigma_{外}}{\varepsilon_0}$ ① ② ③ ④

$U_3 = U_{s内} = U_{s外}$ ① ③

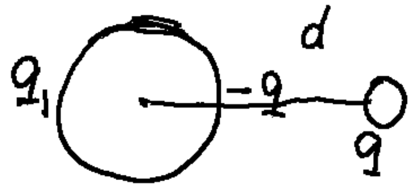
$E_{s内} = \frac{\sigma_{内}}{\varepsilon_0} = 0$ ① ③

T28 (b) (c)

$$\begin{cases} \dfrac{\sigma}{2\varepsilon_0} & \sigma\text{产生的分场} \\ \dfrac{\sigma}{\varepsilon_0} & \text{合场} \end{cases}$$

The Van de Graff Generator

T29 $$\frac{q}{4\pi\varepsilon_0 d} + \frac{q_{总}}{4\pi\varepsilon_0 R} = 0$$



$$q_{总} = -\frac{R}{d}q.$$

静电学边值问题的唯一性定理

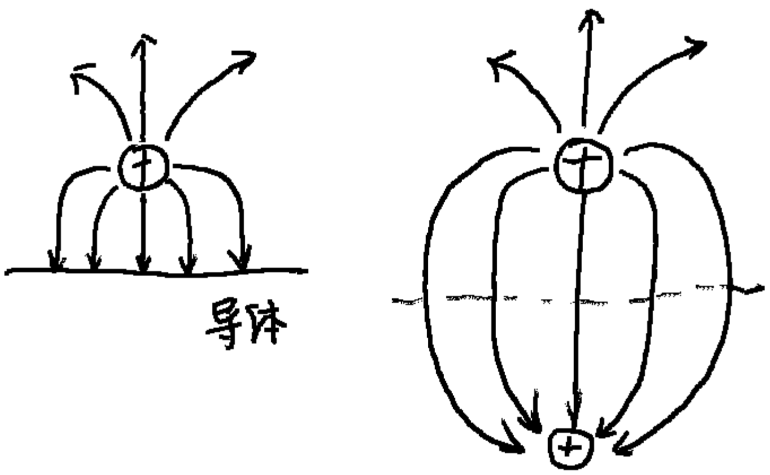
空间电荷分布确定，该空间的电场分布由各个导体的电势（或电量）及区域边界上的电势（或电场）唯一确定.

偏心对球壳外电场分布无影响

金属球壳内的电场分布不受外部影响.

电像法

Q：点电荷 q，无限大导体板，求E分布（以及导体板上的σ分布）



静电屏蔽

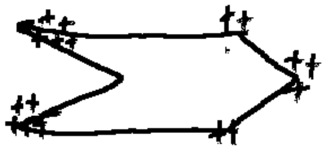
接地空腔导体可屏蔽内部电荷对外部的影响.

T_{30} $\quad \sigma_1 \cdot 4\pi R_1^2 + \sigma_2 \cdot 4\pi R_2^2 = Q_1 + Q_2$

$$\frac{\sigma_1 \cdot 4\pi}{4\pi \varepsilon_0 R_1} = \frac{\sigma_2 4\pi}{4\pi \varepsilon_0 R_2}$$

$$\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$$

T_{31}



§11.2 电容器及电容

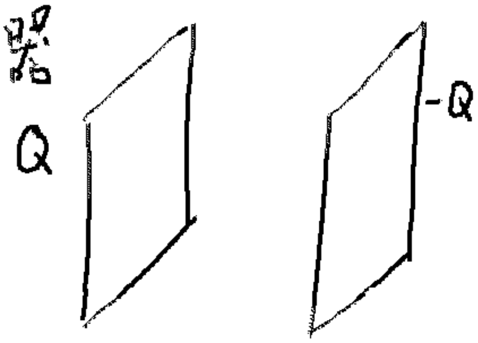
1. 孤立导体的电容

$C = \frac{Q}{U}$ 单位 $F = C \cdot V^{-1}$

$1F = 10^6 \mu F = 10^{12} pF.$

T_{32} $\quad C = \frac{Q}{U} = \frac{Q}{\frac{Q}{4\pi\varepsilon_0 R}} = 4\pi\varepsilon_0 R$

电容器



$C=\frac{Q}{\Delta U}$

Q：计算平行板电容器

$$E=\frac{\sigma}{\varepsilon_0}=\frac{Q}{S\varepsilon_0}$$

$$\Delta U=Ed=\frac{Qd}{S\varepsilon_0}$$

$$C=\frac{\varepsilon_0 S}{d}\propto S,\ \frac{1}{d}.$$

T_{33} $$E=\frac{Q}{4\pi\varepsilon_0 r^2}$$

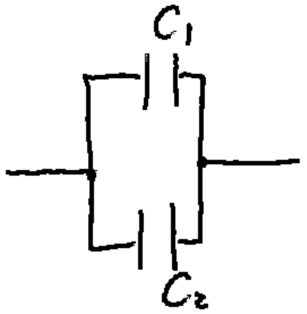
$$\Delta U=\int_{R_A}^{R_B}E\cdot(R_A-R_B)$$

$$C=\frac{Q}{\Delta U}=\frac{4\pi\varepsilon_0 R_A R_B}{R_B-R_A}$$

圆柱形电容器 $C=\frac{2\pi\varepsilon_0 l}{\ln R_B/R_A}$

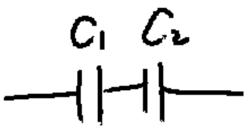
电容器的串、并联

并联 $C=\sum_{i=1} C_i$



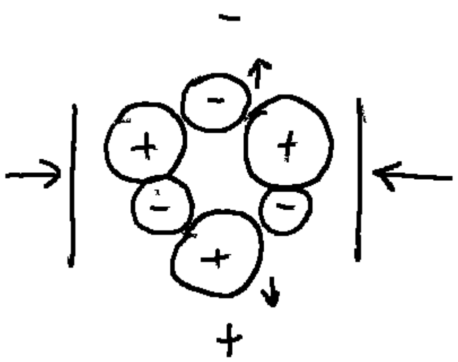
$C=C_1+C_2$

串联 $\frac{1}{C}=\sum_{i=1}\frac{1}{C_i}$.



$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}$

T34 $\frac{1}{C}=\frac{1}{C_1+C_2}+\frac{1}{C_3}$ 得 $C=1\mu F$.



电介质（绝缘体）

电介质的极化

1) 极性分子

$\vec{p} \neq \vec{0}$ 分子有固有电偶极矩

2) 无极性分子

$\vec{p} = \vec{0}$ 分子无固有电偶极矩

有极分子的转向极化 $\vec{M} = \vec{p} \times \vec{E}$ $\vec{p}$大致与$\vec{E}$同向.

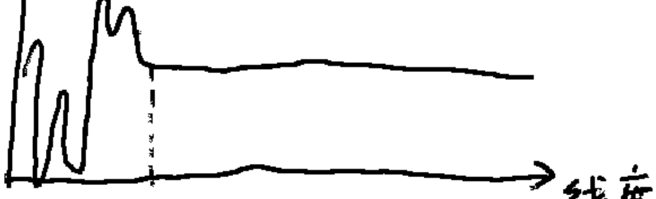
无极分子的位移极化 $\vec{p}$平格与$\vec{E}$同向

极化强度

宏观电磁学

线度：宏观小，
微观大.

“介质连续”



极化强度

$$\vec{P} \overset{def}{=\!=} \frac{\sum \vec{p_i}}{\Delta V} [C/m^2]$$ ΔV宏观小，微观大

例：已知 $n, \vec{p}$，求 $\vec{P}$

$$\vec{P} = \frac{\sum_V \vec{p_i}}{V} = \frac{N\vec{p}}{V} = n\vec{p}$$

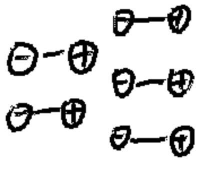
T35 $\vec{P} = n\vec{p} = \rho_+ \vec{l}$

与介质，外电场有关

表面极化电荷

在介质表面出现电荷（均匀），称为极化（束缚）电荷

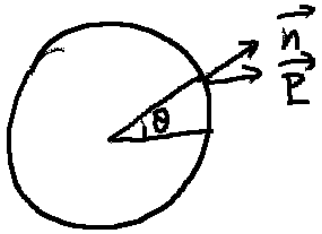
也可能在体内出现极化电荷（不均匀）



T36 $\quad \sigma' = \dfrac{dq'}{ds} = \dfrac{l\cos\theta\, ds\, n q}{ds} = nq\vec{l}\cdot\vec{e_n}$

$$\sigma' = \vec{P}\cdot\vec{n} \quad (= P_n = P\cos\theta)$$

T37



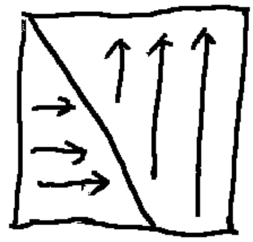
$$\sigma = P\cos\theta$$

例：求两种电介质的分界面上，极化电荷面密度（$\vec{P_1}$, $\vec{P_2}$, $\vec{e_{n_1}}$已知）

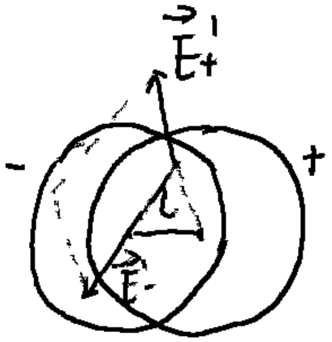
$$\sigma_1' = \vec{P_1}\cdot\vec{e_{n_1}}$$

$$\sigma_2' = \vec{P_2}\cdot\vec{e_{n_2}} = -\vec{P_2}\cdot\vec{e_{n_1}}$$

$$\sigma' = \sigma_1' + \sigma_2' = (\vec{P_1} - \vec{P_2})\cdot\vec{e_{n_1}}$$



T38



$$\vec{E}' = \vec{E_+}' + \vec{E_-}'$$

$$= \frac{\rho_e^+}{3\varepsilon_0}\vec{r_+} + \frac{\rho_e^-}{3\varepsilon_0}\vec{r_-} = -\frac{\vec{P}}{3\varepsilon_0}$$

退化场

极化率

$$\vec{P} = \chi_e \varepsilon_0 \vec{E} \quad \text{(线性介质)}$$

较弱（指向 $\vec{E}$）

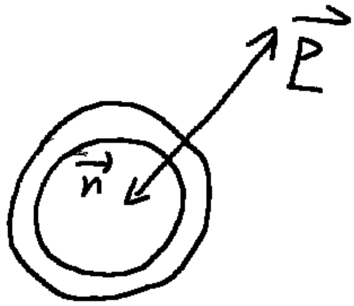
极化率，由介质本身性质决定.

$\varepsilon_r \stackrel{def}{=} \chi_e + 1$ 相对介电常数

真空中 $\chi_e = 0$. $\varepsilon_r = 1$

$\varepsilon = \varepsilon_0 \varepsilon_r = \varepsilon_0 (1+\chi_e)$ 介电常数

T39 $P = (\varepsilon_r - 1)\varepsilon_0 \dfrac{U}{d}$



T40 $E = \dfrac{Q+q'}{4\pi\varepsilon_0 R^2}$

$P = (\varepsilon_r - 1)\varepsilon_0 E$

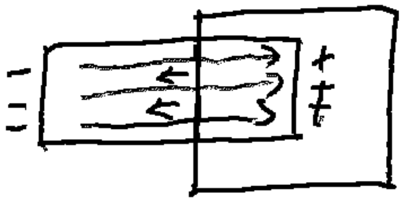
$P = -\dfrac{q'}{4\pi R^2}$

注意符号

得 $q' = -\dfrac{\varepsilon_r - 1}{\varepsilon_r} Q$

体极化电荷

Q：已知 P，求 S 内的 q'.



$$\begin{cases} \oint\!\!\oint_S \vec{P}\cdot d\vec{S} = -P\Delta S \\ P\Delta S = q' \end{cases}$$

$$q' = -\oint\!\!\oint \vec{P}\cdot d\vec{S}$$

T41 $q = -\oint\!\!\oint \vec{P}\cdot d\vec{S} = -kx^2\Delta S$

$$\rho' = \frac{-dkx^2\Delta S}{dx\Delta S} = -2kx$$

P线由负束缚电荷处发出，终止在正束缚电荷.

电位移~~矢~~量

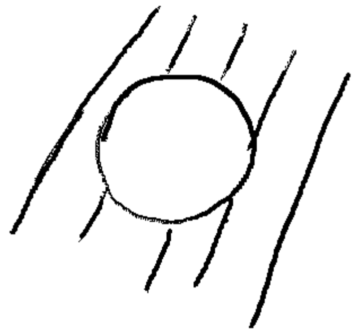
$$\vec{D} \overset{def}{=} \varepsilon_0\vec{E} + \vec{P} \quad [C/m^2]$$

1) 在真空中 $\vec{D} = \varepsilon_0\vec{E}$

2) 在线性介质 $\vec{P} = \chi_e\varepsilon_0\vec{E}$ $\vec{D} = (\chi_e+1)\varepsilon_0\vec{E} = \varepsilon_r\varepsilon_0\vec{E} = \varepsilon\vec{E}$.

$$\vec{D}=\varepsilon_r\varepsilon_0\vec{E}=\varepsilon\vec{E}$$（线性介质）

Q：均匀带电球（R，Q），包围均匀无限大各向同性电介质 ε_r，求 $\vec{D}$ 分布



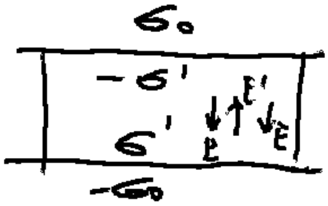
$$\vec{E}=\frac{Q+q'}{4\pi\varepsilon_0 r^2}\vec{e_r}$$

$$q'=-\frac{\varepsilon_r-1}{\varepsilon_r}Q$$

$$\vec{P}=(\varepsilon_r-1)\varepsilon_0\vec{E}$$

$$\vec{D}=\varepsilon_0\vec{E}+\vec{P}=\frac{Q}{4\pi r^2}\vec{e_r}$$

平行板电容器（S，$\pm\sigma_0$），电介质（ε_r，d） 求 D



T42
$$\begin{cases} E=E_0-E' \\ E_0=\dfrac{\sigma_0}{\varepsilon_0},\ E'=\dfrac{\sigma'}{\varepsilon_0} \\ \sigma'=P \\ P=(\varepsilon_r-1)\varepsilon_0 E \end{cases}$$

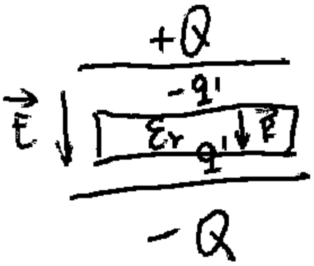
得 $E=\dfrac{\sigma_0}{\varepsilon_r\varepsilon_0}$，$P=\dfrac{\varepsilon_r-1}{\varepsilon_r}\sigma_0$

$$D=\varepsilon_0 E+P=\sigma_0$$

Q：薄板"永电体"，极化强度为 $\vec{P}$，求 $\vec{D}$，$\vec{E}$ 分布

	$\vec{P}$	$\vec{E}$	$\vec{D}$
板外	0	0	0
板内	P	$-\frac{\sigma'}{\varepsilon_0}=-\frac{P}{\varepsilon_0}$	0

T42-b



$$\begin{cases} \vec{E}=\dfrac{Q-q'}{S\varepsilon_0} \\ P=\dfrac{q'}{S} \\ \vec{D}=\varepsilon\vec{E}+\vec{P} \\ P=\varepsilon_0(\varepsilon_r-1)E \end{cases}$$

得 $\vec{D}=\sigma_0$

T+3

$$\oint_S \vec{E'}\cdot d\vec{s} = -\frac{\sigma'}{\varepsilon_0}\Delta S$$

$$\oint_S \vec{E_0}\cdot d\vec{s} = \frac{\sigma}{\varepsilon_0}\Delta S$$

$$\oint_S \vec{P}\, d\vec{s} = -\sigma'\Delta S$$

$$\oint_S \vec{E}\cdot d\vec{s} = \frac{\sigma'-\sigma}{\varepsilon_0}\Delta S$$

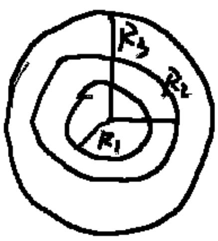
$$\oint_S (\varepsilon_0\vec{E}+\vec{P})\cdot d\vec{s} = -\frac{\sigma}{\varepsilon_0}\Delta S$$

高斯定理

$$\oint_S \vec{D}\cdot d\vec{s} = q_0$$ → 自由电荷

Q：球形电容器（$R_1, R_3, \pm q$），两层均匀电介质（$\varepsilon_{r1}, \varepsilon_{r2}, R_2$）.

求D, E和C.



$\vec{D}$分布对称性由(q_0, q')的对称性确定

$\vec{D}$具有球对称性→ S为球面

$$\oint_S \vec{D}\cdot d\vec{s} = 4\pi r^2 D = q \qquad \vec{D}=\frac{q}{4\pi r^2}\vec{e_r}\ (R_1<r<R_3)$$

$$\vec{E_1}=\frac{\vec{D}}{\varepsilon_{r_1}\varepsilon_0}=\frac{q\vec{e_r}}{4\pi\varepsilon_{r_1}\varepsilon_0 r^2}\qquad \vec{E_2}=\frac{q\vec{e_r}}{4\pi\varepsilon_{r_2}\varepsilon_0 r^2}$$

T44 $D\cdot\Delta S=\Delta S\,\sigma_0$ 得 $D=\sigma_0$

$$\vec{E_1}=\frac{\vec{D}}{\varepsilon_{r_1}\varepsilon_0}=\frac{\sigma_0}{\varepsilon_0\varepsilon_{r_1}}$$

$$\vec{E_2}=\frac{\vec{D}}{\varepsilon_{r_2}\varepsilon_0}=\frac{\sigma_0}{\varepsilon_0\varepsilon_{r_2}}$$

$$\Delta U=\frac{\sigma_0}{\varepsilon_{r_1}}d_1+\frac{\sigma_0}{\varepsilon_{r_2}}d_2$$

$$C=\frac{Q}{\Delta U}=\frac{\varepsilon_0\sigma_0 S}{\frac{\sigma_0}{\varepsilon_{r_1}}d_1+\frac{\sigma_0}{\varepsilon_{r_2}}d_2}=\frac{\varepsilon_0(\varepsilon_{r_1}+\varepsilon_{r_2})S}{\varepsilon_{r_2}d_1+\varepsilon_{r_1}d_2}.$$

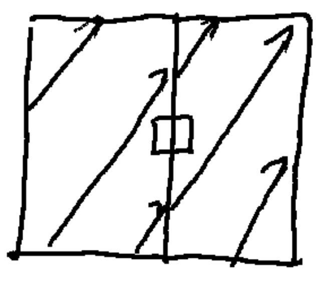
目前

$\vec{D}$ 无意义,是辅助量

静电荷边值问题

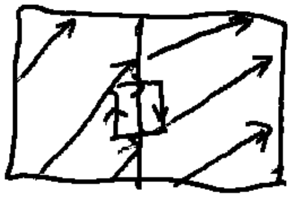
Q:两种介质界面上无自由电荷分布,两侧的E·D有什么关系?

$$\begin{cases}E_{t1}=E_{t2}\\ D_{n1}=D_{n2}\end{cases}$$



$$\oint_S \vec{D}\, d\vec{S} = 0 \qquad \vec{D_1}\cdot\Delta\vec{S_1} + \vec{D_2}\cdot\Delta\vec{S_2} = 0$$

$$D_{n1} = D_{n2}$$



$$\oint_l \vec{E}\cdot d\vec{l} = 0$$

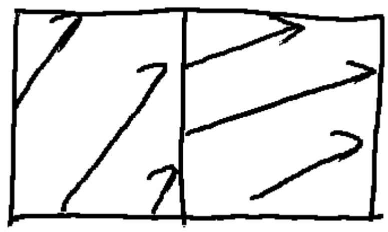
$$\vec{E_1}\cdot\Delta\vec{l_1} + \vec{E_2}\cdot\Delta\vec{l_2} = 0$$

$$E_{t1} = E_{t2}.$$

D法在分界面处连续不断,但E线不连续.

Q: 证明在界面上E(or D)线的折射满足

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2}$$



$$D_1\cos\theta_1 = D_2\cos\theta_2$$

$$E_1\sin\theta_1 = E_2\sin\theta_2$$

$$\frac{E_1}{D_1}\tan\theta_2 = \frac{E_2}{D_2}\tan\theta_2$$

$$\frac{E_1}{D_1} = \frac{1}{\varepsilon_1} \qquad \frac{E_2}{D_2} = \frac{1}{\varepsilon_2}$$

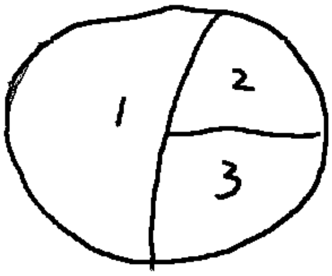
T45 $\dfrac{\tan\theta_1}{\tan\theta_2} = \dfrac{1}{\varepsilon_r}$ 得 $\theta_2 = \arctan\dfrac{13\sqrt{3}}{6}$

$$E_2\sin\theta_2 = E_1\sin\theta_1 \qquad E_2 = 1.03\times10^4\ V/m.$$

$$\sigma' = -(\varepsilon_r - 1)\varepsilon_0 E_2\cos\theta_2 = -1.27\times10^{-7}\ C/m^2$$

极化电荷对 $\vec{D}$ 分布是有影响的

有介质时的唯一性定理

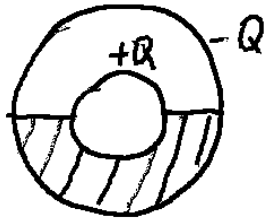


$\begin{cases}\text{电荷分布}\\ \text{边界}\end{cases}$ 且 $\begin{cases}D_{in} = D_{jn}\\ E_{it} = E_{jt}\end{cases}$

空间电荷分布确定、ε_i 分布确定，在各个子区域的边界上满足 $E_{it} = E_{jt}$ $D_{in} = D_{jn}$

该空间的电场分布由各个导体的电势（或电量）及整个区域边界上的电势（或电场）唯一确定.

Q: 球形电容器（Q, -Q, ε），求两半球中 E、D 分布、



猜解
$$\begin{cases} \vec{E_1} = \dfrac{A}{r^2}\vec{e_r} \\ \vec{E_2} = \dfrac{A}{r^2}\vec{e_r} \end{cases}$$

$$\Rightarrow \begin{cases} \vec{D_1} = \dfrac{\varepsilon_0 A}{r^2}\vec{e_r} \\ \vec{D_2} = \dfrac{\varepsilon_0 A}{r^2}\vec{e_r} \end{cases}$$

（$\vec{D_1}$，$\vec{D_2}$）半球对称性

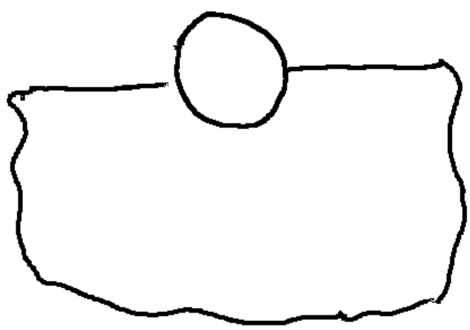
$$\oint\!\!\!\oint_S \vec{D}\, d\vec{S} = 2\pi r^2 D_1 + 2\pi r^2 D_2 = 2\pi\varepsilon_0 A + 2\pi\varepsilon A$$

$$A = \frac{Q}{2\pi(\varepsilon_0+\varepsilon)}$$

$E_{1t} = E_{2t}$　　$D_{1n} = D_{2n}$ $\Rightarrow$ 试探解是唯一的

Q: 带电金属球（R, Q），半个球处在电介质 ε_r 中，则球正下方 $r>R$ 处的 E、D

解同上

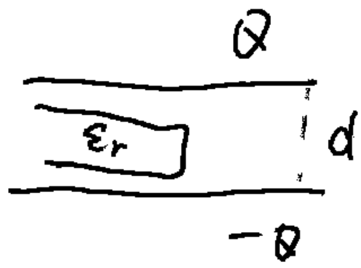


$$\oiint \vec{D}\cdot d\vec{S} = 2\pi r^2 D_1 = 2\pi r^2 \sigma_0 上$$

$$\oiint \vec{D}\cdot d\vec{S} = 2\pi r^2 D_2 = 2\pi r^2 \sigma_0 下$$

$$\sigma_0^{下} > \sigma_0^{上}$$

T46 $\vec{E_1} = \vec{E_2}$



$$\vec{D_1} = (\varepsilon_r+1)\varepsilon_0 \vec{E} \quad \vec{D_2} = \varepsilon_0 \vec{E}$$

$$\frac{S}{2}D_1 + \frac{S}{2}D_2 = Q$$

$$E = \frac{2Q}{\varepsilon_0(\varepsilon_r+1)S}$$

§11.4 静电场的能量

T47 $\dfrac{q_1 q_2}{4\pi\varepsilon_0 r_1} + \dfrac{q_2 q_3}{4\pi\varepsilon_0 r_2} + \dfrac{q_3 q_1}{4\pi\varepsilon_0 r_3}$

$$W = \sum_{i=1}^{N} \left(\frac{1}{2} q_i U_i\right)$$

U_i : q_i 处的总电势.

一个带电体的总电势能？

$$U = U_{Q-dq} = U_Q - U_{dq}$$

$$W = \frac{1}{2}\int_Q U_{Q-dq}\, dq$$

$$U_{dq} = \frac{dq}{4\pi\varepsilon_0 \delta} = \frac{\frac{4}{3}\pi\delta^3\rho}{4\pi\varepsilon_0\delta} \sim \delta^2 \to 0$$

故 $W = \frac{1}{2}\int_Q U\, dq = \frac{Q^2}{8\pi\varepsilon_0 R}$.

T48 $W = \frac{1}{2}\sum Q_i U_i$

$$= \frac{Q^2}{2C_0}$$

$$= \frac{1}{2}Q\Delta U = \frac{1}{2}C_0\Delta U^2$$

Q：充满电介质 ε 电容器 C，带电 Q 时的静电能

$$W = \frac{Q^2}{2C} = \frac{1}{2}CU^2 = \frac{1}{2}QU.$$

2. 电场能量密度

$W = \frac{1}{2} Q U$

$$\begin{cases} Q = S\sigma_0 = SD \\ U = Ed \end{cases}$$

$W = \frac{1}{2}(Sd)ED = \frac{1}{2}VED = \frac{1}{2}V(\vec{E}\cdot\vec{D})$

$w = \frac{W}{V} = \frac{1}{2}\vec{E}\cdot\vec{D} \quad (J/m^3)$

$W = \frac{1}{2}\iiint \vec{E}\cdot\vec{D}\,dV$

T49 $w = \frac{1}{2} \vec{E} \cdot \vec{D}$

$$E = \frac{q}{4\pi r^2 \varepsilon}$$

$$D = \frac{q}{4\pi r^2}$$

$$w = \frac{q^2}{32\pi^2 r^4 \varepsilon}$$

$$W = \int_{R_1}^{R_2} \frac{q^2}{32\pi^2 r^4 \varepsilon} \cdot 4\pi r^2 dr$$

$$= \frac{q^2}{8\pi\varepsilon} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$W_e = \frac{1}{2} \frac{q^2}{C}$ —— 储能公式

Q：同轴长直圆柱筒 (l、a、b)，中间介质 ε。求电容

$D = \frac{\lambda}{2\pi r}$ $E = \frac{\lambda}{2\pi\varepsilon r}$

$$W = \int_a^b \frac{1}{2} D E \cdot 2\pi r l dr$$

$\Rightarrow$ $W = \frac{Q^2}{2C}$.

T5.

$r > r_e \qquad \vec{E} = \dfrac{-e}{4\pi\varepsilon_0 r^2}\vec{e_r} \qquad w_2 = \frac{1}{2}DE$

$r < r_e \qquad \vec{E} = \dfrac{\rho}{3\varepsilon_0}\vec{r} = \dfrac{-e}{\frac{4}{3}\pi r_e^3} \qquad w_1 = \frac{1}{2}DE \qquad D = \varepsilon_0 E$

$$W = \int_0^{r_e} w_2\, 4\pi r^2 dr + \int_{r_e}^{\infty} w_1\, 4\pi r^2 dr = \frac{3}{5}\frac{e^2}{4\pi\varepsilon_0 r_e}$$

Q：设电子半径为 r_0 的导体球，静电能与其静能同量级试估算电子的经

$$W = m_e c^2$$

$$W \sim \frac{e^2}{4\pi\varepsilon_0 r_e}\cdot\frac{3}{5}$$

$$r_e \sim 2.8\times10^{-15}\,\mathrm{m})$$

静电能（电场能）

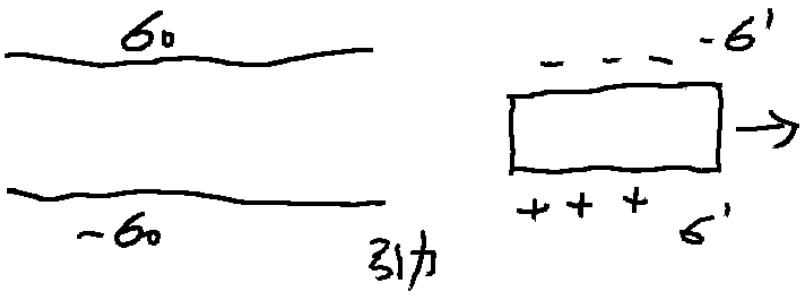
$W=\frac{1}{2}\sum_i q_i U_i$ $W=\frac{1}{2}\int_Q dq\, U$

$W_e=\frac{1}{2}U_c Q=\frac{1}{2}CU_c^2=\frac{1}{2}\frac{Q^2}{C}$

$w_e=\frac{1}{2}DE=\frac{1}{2}\vec{D}\cdot\vec{E}$

自能：单个带电体的电场能

互能：带电体间的势能

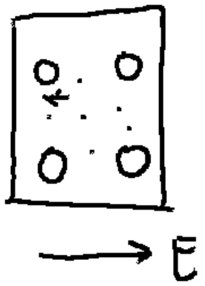


"极化能" "热能"

第12章 稳恒磁场

§12.1 电流和电源

漂移运动

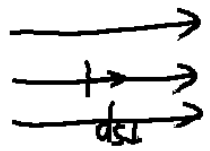


一、电流密度

载流子：电子、空穴、离子…　　$q>0$

　　　　⊖　　⊕

$$\vec{j} \overset{def}{=} \frac{dI}{dS}\vec{n}$$



"⊥单位截面"电流强度

则 $\vec{j} = \dfrac{nq\vec{v}_d\,dS}{dS} = nq\vec{v}_d$

电流强度

$$dI = \vec{j} \cdot d\vec{S}$$

$$I = \iint_S \vec{j} \cdot d\vec{S}$$

电荷守恒定律

$$\oiint_S \vec{j} \cdot d\vec{S} = -\frac{dq}{dt}$$

若空间电荷分布不随时间变化（稳恒电流条件）

$$\oiint_S \vec{j} \cdot d\vec{S} = 0.$$ steady current

稳恒电场满足：高斯定理；保守场，可引入电势

恒定电场	静电场
激发磁场	电荷静止，不激发电场
$\vec{E}_S \neq 0$	内部 $\vec{E} = 0$
有电势差	等势
能量	不需要能量

欧姆定律

$$dI = \frac{-dU}{\rho \cdot dl/ds} \qquad dU = -E\,dl$$

$$dI = \frac{1}{\rho} E\,ds$$

$$\Rightarrow \quad \vec{j} = \frac{1}{\rho}\vec{E} = \sigma \vec{E} \quad (\sigma = \frac{1}{\rho} \text{ 电导率})$$ 欧姆定律微分形式

T51b $\quad j_1 = \gamma_1 E_1$ 得 $E_1 = \frac{j_1}{\gamma_1}$

$$j_2 = \gamma_2 E_2 \qquad j_2 = j_1 \ , \ E_2 = \frac{j_1}{\gamma_2}$$

$$\sigma_0 = (E_2 - E_1)\varepsilon_0 = j_1 \varepsilon_0 \left(\frac{1}{\gamma_2} - \frac{1}{\gamma_1}\right)$$

T52 $\quad j = \frac{I}{2\pi r^2}$

$$E = \frac{j}{\sigma} = \frac{I}{2\pi\sigma r^2}$$

$$\Delta U = -\int_{r_1}^{r_2} E(r)\,dr = \frac{I}{2\pi\sigma}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

电导率 Paul Drude 模型

The (free) elections : a classical ideal gas

Elections are scattered randomly by nuclei

The average time between collisions (The scattering time) τ

The interaction between elections is neglected

Q: $(e, \vec{E}, \tau)$，求电子的漂移速度 $\vec{V_d} = \langle \vec{V}(t) \rangle$

$$\vec{V}(t) = \vec{V_0} + \vec{a}t$$

$$\vec{a} = -\frac{e\vec{E}}{m_e}$$

$$\vec{V}(t) = \vec{V_0} - \frac{e\vec{E}}{m_e}t$$

T53 $\vec{V} = -\frac{e\vec{E}}{m_e}\tau$

$$\vec{j} = n(-e)\vec{V_d} = \frac{ne^2\tau}{m}\vec{E}$$

$$\sigma = \frac{ne^2\tau}{m}$$

方法2：
$$\begin{cases} \langle \vec{v}(t) \rangle = \vec{v}_d \\ -e\vec{E} + \vec{f} = 0 \\ \vec{f} = -\dfrac{m\vec{v}_d}{\tau} \quad \text{(假设)} \end{cases}$$

……

$$\tau = \frac{l}{\bar{v}}$$

$$\bar{v} \sim \sqrt{T}$$

$$\sigma \sim \frac{1}{\sqrt{T}}$$

$\sigma \sim \frac{1}{T}$（实际，需要用量子统计，考虑杂质影响）

2. 焦耳定律的微分形式

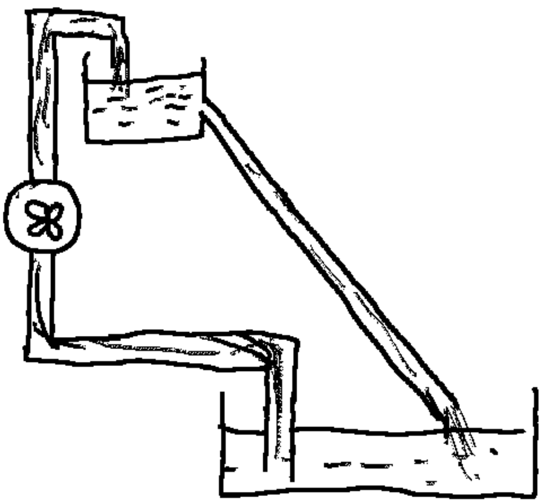
$$\begin{cases} dP = (dI)^2 dR \\ dI = j\,dS = (\sigma E)\,dS \\ dR = \dfrac{1}{\sigma}\dfrac{dl}{dS} \end{cases} \Rightarrow \quad w = \frac{dP}{dV} = \sigma E^2 \ (W/m^3)$$
$$= \rho j^2$$

TJ4 $I_0 = \gamma E(r)\cdot 4\pi r^2$ $E(r) = \sqrt{\frac{I_0}{4\pi r^2 \gamma k}}$

$$W = \int_a^b \gamma E^2(r)\cdot 4\pi r^2 dr = \int_a^b k\left(\frac{I_0}{4\pi r^2 \gamma k}\right)^{\frac{3}{2}} 4\pi r^2 dr$$

Q：如何来维持导体中恒定的电流？

仅依靠电力能维持回路中的恒定电流？



1、非静电力

在电源内存在使正电荷从电源的负极聚积到正极的作用力

$$\vec{E_k} = \frac{\vec{F_k}}{q} \quad (V/m)$$

洛伦兹力、旋电场，温差电源扩散作用、化学电池中溶解和沉积过程

2. 电源电动势

$\varepsilon = \frac{W}{q}$ 把单位正电荷由负极移向正极非静电力所作的功.

$$W = \int_{-}^{+} \vec{F_k} \cdot d\vec{l} = q \int_{-}^{+} \vec{E_k} \cdot d\vec{l}$$

$$\varepsilon = \frac{W}{q} = \int_{-}^{+} \vec{E_k} \cdot d\vec{l}$$

$\varepsilon = \oint_l \vec{E_k} \cdot d\vec{l}$ 多个电源

T35 $$\oint_l \vec{E} \cdot d\vec{l} = 0$$

$$\oint_l \vec{E_k} \cdot d\vec{l} = \varepsilon$$

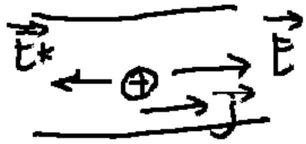
$$\oint_l (\vec{E_k} + \vec{E}) \cdot d\vec{l} = \varepsilon$$

$$\int_a^b \vec{E} \cdot d\vec{l} = -IR$$

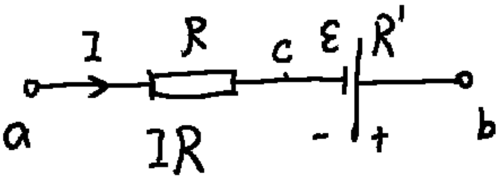
3. 电源内部的欧姆定律

$$\vec{V}_d \propto \vec{E} + \vec{E}_k$$

$$\vec{j} = \sigma(\vec{E} + \vec{E}_k)$$



可以证明含源电路欧姆定律



$$\vec{j} = \gamma'(\vec{E} + \vec{E}_k)$$

$$V_{cb} = V_c - V_b$$
$$= \int_c^b \vec{E} \cdot d\vec{l}$$
$$= \int_c^b \left(\frac{\vec{j}}{\gamma'} - \vec{E}_k\right) d\vec{l}$$

$$\int_c^b \frac{\vec{j} \cdot d\vec{l}}{\gamma'} = \int_c^b \frac{j\,dl\,s}{\gamma' S} = I\int_c^b \frac{dl}{\gamma' S'} = IR'$$

故 $V_{cb} = IR' - \varepsilon$

§12.2 磁场

磁铁产生磁场 — 分子电流产生磁场

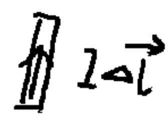
(大量分子电流定向排列的结果 —— 安培假说

定义 1: $\vec{F} \underline{\underline{def}}\ q\vec{V}\times\vec{B}$

磁感应强度：磁场的大小，可根据运动电荷中受力来量度.

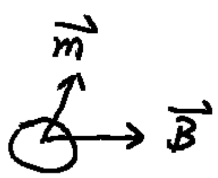
T_{56} $\vec{F} = q\vec{V}\times\vec{B} = -ev_0(B_1\vec{k} - B_2\vec{j})$

定义 2: $\Delta\vec{F} = I\Delta\vec{l}\times\vec{B}$



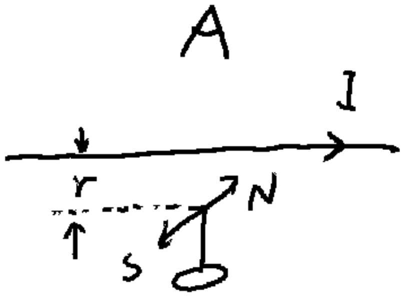
电流元

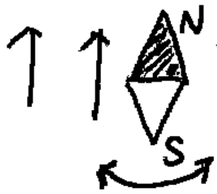
定义 3: $\vec{M} = \vec{m}\times\vec{B}$



§12.3 毕奥—萨伐尔定律

Biot-Savart 磁针振荡测量法





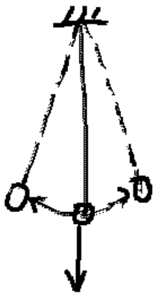
力场

$T^2 \propto r$ (实验)

$F \propto \frac{1}{r^2}$ (理论)

$\Rightarrow F \propto \frac{1}{r}$

单摆



$$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{lm}{mg}} = 2\pi\sqrt{\frac{lm}{G}} \rightarrow G \propto \frac{1}{T^2}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I d\vec{l} \times \vec{e_r}}{r^2} = \frac{\mu_0}{4\pi} \cdot \frac{I d\vec{l} \times \vec{r}}{r^3}$$

$\mu_0 = 4\pi \times 10^{-7}$ (T·m/A) 真空的磁导率

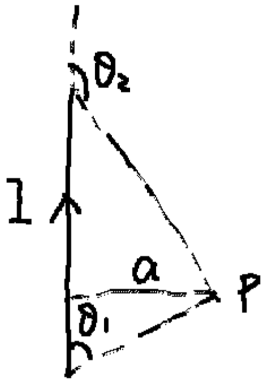
$$\vec{B} = \int_L d\vec{B}$$

T57 $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \vec{e_r}}{r^2}$

$d\vec{B_1} = \vec{0}$ $\quad d\vec{B_2}$ ⊙ $\quad d\vec{B_3}$ ⊗

T58 b

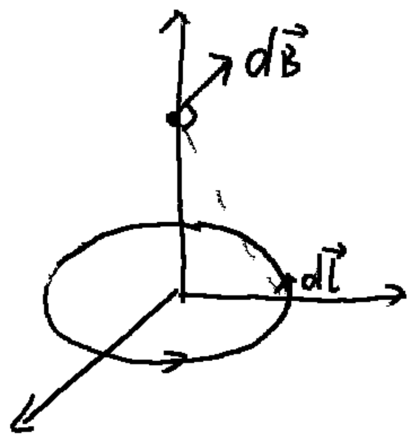
例：求直电流的磁场 $(I, \theta_1, \theta_2, a)$



$$dB = \frac{\mu_0}{4\pi} \frac{I dz \sin\theta}{r^2}$$

$$= \frac{\mu_0 I}{4\pi a} \sin\theta d\theta$$

$$B = \frac{\mu_0 I}{4\pi a} (\cos\theta_1 - \cos\theta_2)$$



T59 $dB = \frac{\mu_0}{4\pi} \frac{I dl}{R^2 + z^2}$

$$dB_z = \frac{\mu_0}{4\pi} \frac{I dl}{r^2}$$

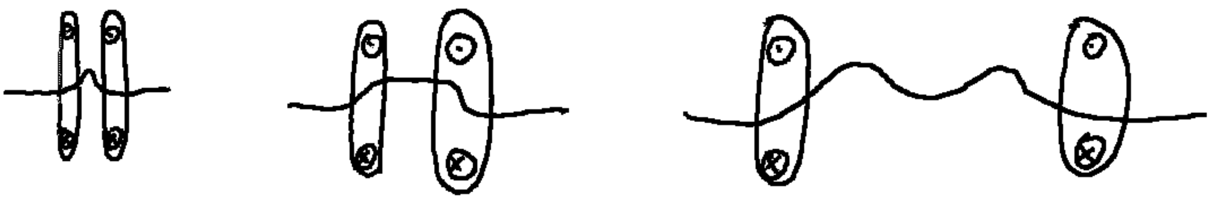
$$B = \frac{\mu_0 I R^2}{2(z^2+R^2)^{3/2}}$$

磁矩：$\vec{m} = I\vec{S}$ （$A\,m^2$）.　　$\vec{S} = \pi R^2 \vec{k}$　　$\vec{m} = NI\vec{S}$

$$\vec{B} = \frac{\mu_0 \vec{m}}{2\pi (R^2+z^2)^{3/2}}.$$

$$z=0 \quad \vec{B_0} = \frac{\mu_0 \vec{m}}{2\pi R^3}$$

$$z \gg R \quad \vec{B} = \frac{\mu_0 \vec{m}}{2\pi z^3}$$



T60　$dB = \frac{\mu_0}{4\pi} \frac{nqvsdl\,\vec{e_r}}{r^2}$

载流子数

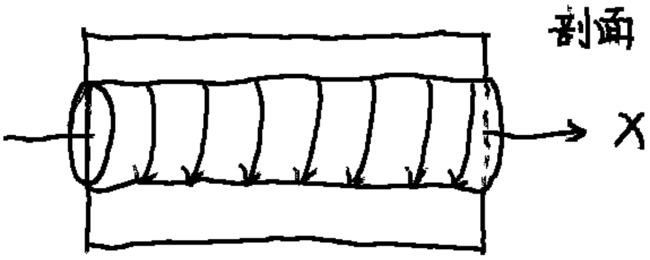
$$dB = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \vec{r}}{r^3} (nsdl)$$

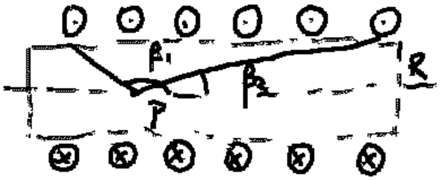
$$\vec{B} = \frac{d\vec{B}}{dN} = \frac{\mu_0}{4\pi} \frac{q\vec{v}\times\vec{r}}{r^3} \quad (v \ll c)$$

例：计算螺线管轴线上的磁场（I, n, β_1, β_2 给定）

n：数密度 $= \frac{N}{L}$（匝/m）

"密绕"



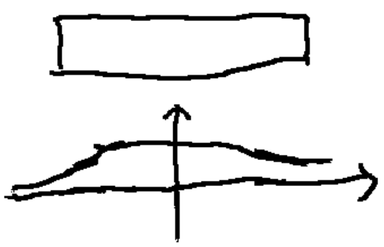


T61 $$dB = \frac{\mu_0 I n R^2 dx}{2(R^2+x^2)^{3/2}} \qquad x = R\cot\beta$$

$$B = \int_{\beta_1}^{\beta_2} dB(\beta) = \frac{\mu_0 n I}{2}(\cos\beta_2 - \cos\beta_1)$$

无限长 $B = \mu_0 n I$

半无限长端面中点 $B = \frac{\mu_0 n I}{2}$.



T62 C

$$\oint\!\!\oint_S \vec{B}\cdot d\vec{S}=0$$ 无单独的N极或S极

对称理论

安培环路定理

$$\oint_l \vec{B}\cdot d\vec{l}=\oint_L \frac{\mu_0 I}{2\pi r}\vec{e}_t(dl\cdot\vec{e}_t)=\mu_0 I$$

T63 $$\oint_L \vec{B}\cdot d\vec{l}=\oint \frac{\mu_0 I}{2\pi r}\vec{e}_t(dl\cdot\vec{e}_t)=0.$$

结论

$$\oint_L \vec{B}\cdot d\vec{l}=\begin{cases}\pm\mu_0 I & \text{电流穿过回路}\\ 0 & \text{电流不穿过回路}\end{cases}$$

多根载流导线的磁场：代数求和

$$\oint_L \vec{B}\cdot d\vec{l}=\mu_0\sum I$$

T64 $\oint_L \vec{B} \cdot d\vec{l} = \mu_0 (I_2 - I_1)$

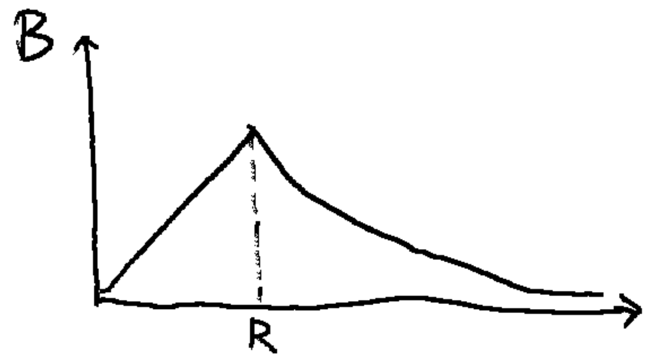
T65 $\oint_L \vec{B} \cdot d\vec{l} = 2\mu_0 I$

Q：无限长柱形均匀电流 (I, R)，分析导体内外 B 分布对称性

$$\oint_L \vec{B} \cdot d\vec{l} = \mu_0 I \qquad \vec{B} = \frac{\mu_0 I}{2\pi r} \vec{e_t}$$

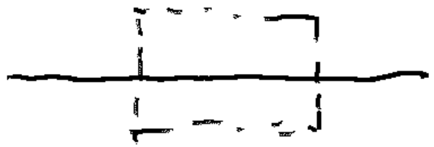
T66 $\oint_L \vec{B} \cdot d\vec{l} = \mu_0 I \cdot \frac{r^2}{R^2}$

$$\vec{B} = \frac{\mu_0 I r}{2\pi R^2} \vec{e_t}$$



有限长：为满足稳恒电流，磁场分布不满足对称性

Q：无限大薄导体板均匀分布电流（δ/$A\cdot m^{-1}$），求 B 分布



$B=\frac{\mu_0}{2}\delta$

T67　板内　$2Bl=\mu_0 2xlj$　　$B=\mu_0 jx$

板外　$B=\frac{1}{2}\mu_0\delta=\frac{1}{2}\mu_0 jd$

Q：证明无限长密绕螺线管（I, $n=N/L$）内为匀强磁场

$B_{内}=\mu_0 nI$　　$B_{外}=0$

实际＝理想＋直线

Q：计算细螺线环内的磁感应强度分布（$N, R \gg r, I$）

$r \ll (R_1, R_2)$

$$\oint_L \vec{B}\cdot d\vec{l} = 2\pi R B = \mu_0 N I$$

$$B = \frac{\mu_0 I N}{2\pi R} = \mu_0 I n$$

磁场的旋涡

旋度 $\frac{1}{\Delta S}\oint_l \vec{B}\cdot d\vec{l}$

数学公式

$$\oint_l \vec{A}\cdot d\vec{l} = \iint_S (\nabla\times\vec{A})\cdot d\vec{S}$$

$$\oint_l \vec{B}\cdot d\vec{l} = \iint_{\Delta S} (\nabla\times\vec{B})\cdot d\vec{S} = \mu_0 \Delta I$$

$$(\nabla\times\vec{B})\cdot\Delta\vec{S} = \mu_0\vec{j}\cdot\Delta\vec{S}$$

$$\nabla\times\vec{B} = \mu_0\vec{j}$$ 微分形式

1. 安培定律

Q: 电流元 $I d\vec{l}$ 在磁场 $\vec{B}$ 中所受的洛伦兹力?

一个 q: $\vec{f} = q\vec{v} \times \vec{B}$

$I d\vec{l}$: $d\vec{F} = \vec{f} \, dN = nqSV(d\vec{l} \times \vec{B})$

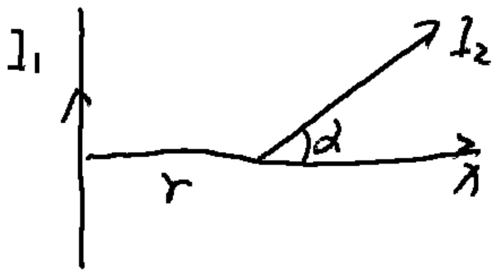
$$d\vec{F} = I \, d\vec{l} \times \vec{B}$$

Q: 无限长直 I_1, 电流 I_2 同一平面 (r, L, α) 求 L 所受的磁场力

$dF = I_2 dl B$

$B = \frac{\mu_0 I_1}{2\pi x}$

$F = \int_r^{r+L\cos\alpha} I_2 dl B$



T68 $F = \int_0^{\pi} BI \, dl \sin\theta$

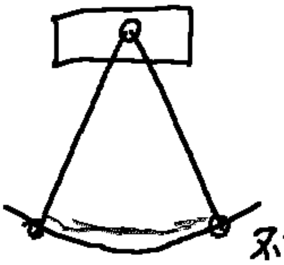
$= \int_0^{\pi} BIR \sin\theta \, d\theta$

$= 2BIR$

安培的四个示零实验

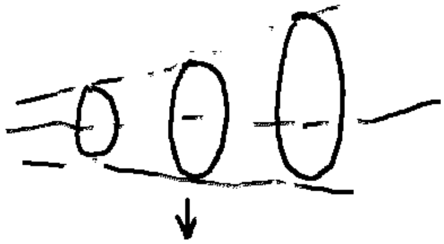
无定向秤  电流反向，作用力也反向 $d\vec{F} \propto I d\vec{l}$

 电流元是矢量 $I_1 d\vec{l} + I_2 d\vec{l} = I d\vec{l}$



不动

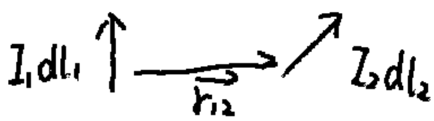
作用在电流元上的力与其垂直



力与距离平方成反比

所受合力为0

$$d\vec{F}_{12} = \frac{\mu_0}{4\pi} \frac{I_2 d\vec{l}_2 \times I_1 d\vec{l}_1 \times \vec{r}_{12}}{r_{12}^3}$$



$$d\vec{F} = I d\vec{l} \times \vec{B}$$

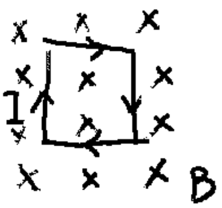
$$d\vec{F} = I_2 d\vec{l} \times d\vec{B}_1 = \frac{\mu_0}{4\pi} \frac{I_2 d\vec{l}_2 \times (I_1 d\vec{l}_1 \times \vec{e}_{21})}{r_{21}^2}$$

T71 $\quad d\vec{f}_{21} = 0$

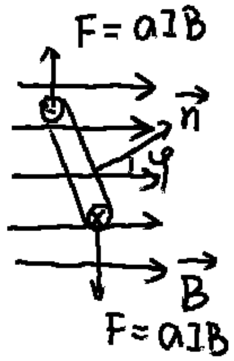
$$d\vec{F}_{12} = \frac{\mu_0}{4\pi} \frac{I_1 I_2 dl_1 dl_2}{r^2}$$

2. 磁力矩

Q: 求线框受到的磁力矩



已知 a, b, I, B $\qquad F=0, \ M=0$



已知 a, b

I, B, φ.

磁矩 $\vec{m} = abI\vec{n}$

力偶: $F = aIB$

力矩: $M = Fb\sin\varphi = abIB\sin\varphi$

$= mB\sin\varphi$

磁力矩 $\qquad \vec{M} = \vec{m} \times \vec{B}$.

推广: 任意形状

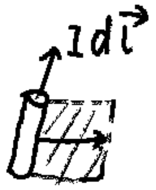
$\vec{M} = \vec{m} \times \vec{B} \qquad \vec{m} = NI\vec{S}$

T69 $\quad \vec{m} = \frac{1}{2}\pi R^2 I \vec{n} \qquad \vec{M} = \vec{m} \times \vec{B} = \frac{1}{2}\pi R^2 BI \vec{e}_k$

其中 $\vec{e}_k$ 方向竖直向下

3. 安培力的功

Q：在外磁场 $\vec{B}$ 中电流元 $I d\vec{l}$ 移动 $d\vec{r}$ 位移安培力的功为何？



$$dA = d\vec{F}\cdot d\vec{r}$$
$$= (I d\vec{l}\times\vec{B})\cdot d\vec{r}$$
$$= I\,(d\vec{r}\times d\vec{l})\cdot\vec{B}$$
$$= I\,(d\vec{s}\cdot\vec{B})$$

$$dA = I\,d\Phi \Rightarrow A = I\Delta\Phi$$

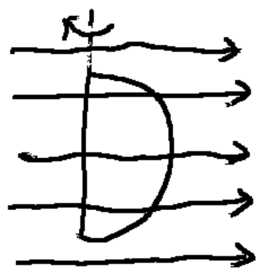
Q：闭合电流，安培力的功

$$A = I(\Phi_1 - \Phi_2) = I\Delta\Phi$$

$\Delta\Phi$：运动导线扫过的磁通量，闭合线圈处末态磁通之增量

Q：半圆环 (R, I)，B，求线圈转过 90°，磁力矩做了多少功？

$$A = I(\phi_f - \phi_i) = IB\frac{\pi R^2}{2}$$



其中 $\phi_f = \frac{\pi R^2}{2} B$ （$\vec{m}$ 与 $\vec{B}$ 同向）

I与B成 RHR，$\Phi>0$；否则$\Phi\le 0$.

T70　$A = I_2\Delta\Phi = -I_2\int_b^{b+a}\frac{\mu_0 I_1}{2\pi r}a\cdot dr = \frac{\mu_0}{2\pi}I_1 I_2\ln\frac{b}{b+a}$

Q：安培力作功与洛仑兹力不做功矛盾吗？

$\vec{u} = \vec{v}_d + \vec{v}$

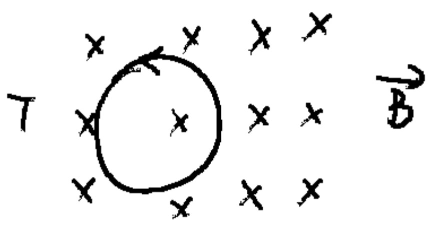
$\vec{F} = (q\vec{u}\times\vec{B})\cdot\vec{u} = 0$

$dA = d\vec{F}\cdot\vec{v}\,dt$

导线运动时，安培力只是洛仑兹力一个分量

§12.6 带电粒子的运动

1、洛仑兹力公式



$F = q(\vec{v}\times\vec{B}+\vec{E})$

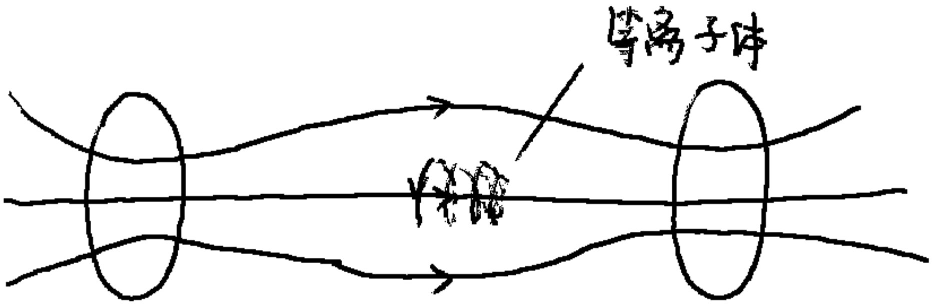
$R = \frac{mv_0}{qB}$　$w = \frac{qB}{m}$　$T = \frac{2\pi m}{qB}$

$\frac{q}{m}$：荷质比　　与v无关.

T72 $R=\frac{mv_0\sin\theta}{qB}$ $\omega=\frac{qB}{m}$ $h=\frac{2\pi m}{qB}v_0\cos\theta$

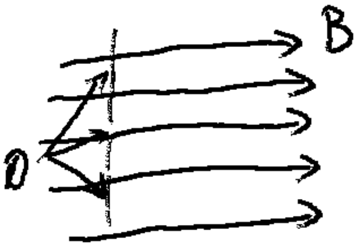
$\vec{m} \parallel -\vec{B}$

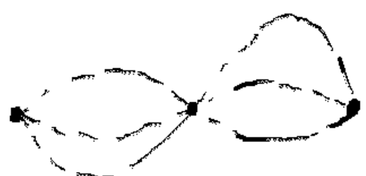
如果把载流运动等效成一个磁铁，则磁铁的极性与产生外磁场磁铁的极性相对（N-N，S-S）



磁约束

磁聚焦





周期相同，螺距相同

3.霍尔效应

T73 $\frac{U_H}{b}q = Bvq$ 得 $U_H = Bvb$

$$U_H = \frac{1}{nq}\frac{IB}{d} = \underbrace{R_H}_{\text{霍尔系数}}\frac{IB}{d}$$

半导体霍尔效应较明显

T74 (1) 上下面

(2) $\begin{cases} \sigma E_{ext} = \rho v \\ q\frac{V_H}{h} = qvB \end{cases}$

得 $B = \frac{V_H}{E_{ext}}\frac{\rho l}{\sigma h}$

第13章 磁介质

§13.1 顺磁性和抗磁性

电子轨道磁矩

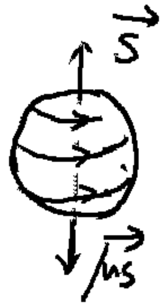
$$\mu_L = IS = \frac{Ve}{2\pi r}\pi r^2 = \frac{1}{2}eVr$$

T_{75} $L = rmV$

得 $\vec{\mu_L} = -\frac{e}{2m}\vec{L}$

电子的自旋磁矩

$$\vec{\mu_s} = -\frac{e}{m}\vec{S}$$



分子或原子的磁矩：为所有电子轨道和自旋磁矩的和

顺磁质 (paramagnet)

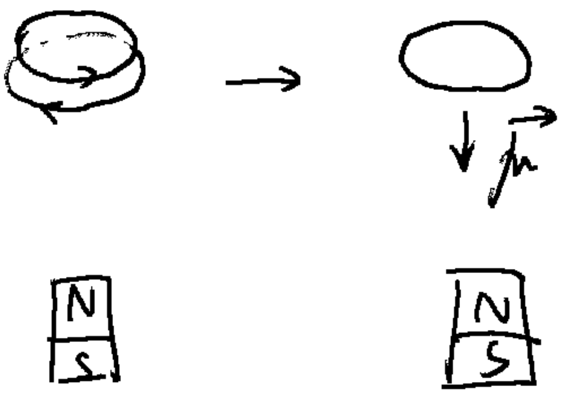
$\vec{\mu}_m \neq 0$ 铝、钠、铬、氧

抗磁质 (diamagnet)

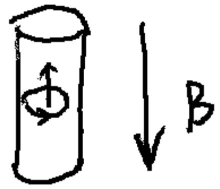
$\vec{\mu}_m = 0$ 铜、银

§13.2 磁化强度和磁化电流

外场中的顺磁质 转向磁化



抗磁介质的磁化

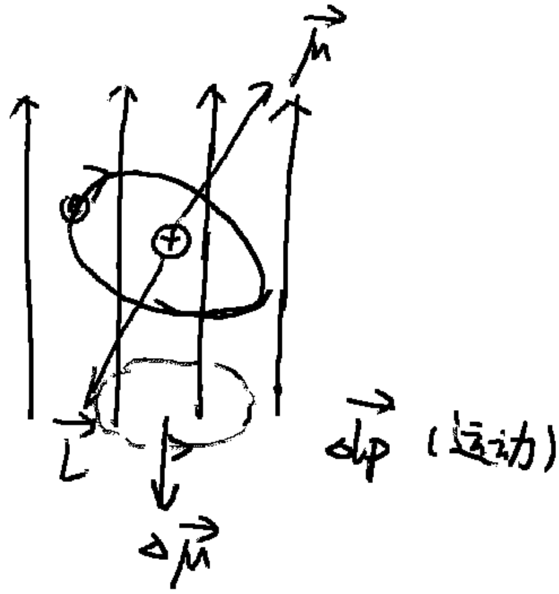


Q: 单电子原子处在外磁场 B中

$\vec{M} = \vec{\mu} \times \vec{B}$

$d\vec{L} = \vec{M}\, dt$

$\vec{L}(t+dt) = \vec{L}(t) + d\vec{L}$



$\Delta\vec{\mu} = -\frac{e}{2m}\Delta\vec{L} \Rightarrow \Delta\vec{\mu} \parallel (-\vec{B})$

感应的附加磁矩总是与外磁场方向相反 — 抗磁性 $\Delta\mu \parallel -B$

(正电荷旋转情况不变)

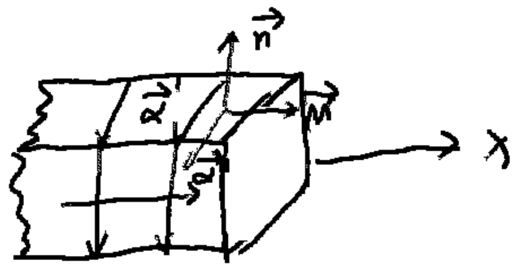
磁化强度等于介质单位体积中分子磁矩

$\vec{M} \overset{def}{=} \lim_{\Delta V \to 0} \frac{\sum_{i \in \Delta V} \vec{\mu_i}}{\Delta V}$ [A/m] ΔV 宏观小，微观大

面电流密度

T76 $\vec{M} = \lim\limits_{\Delta V \to 0} \frac{n \Delta V \vec{m}}{\Delta V} = n\vec{m}$

Q：细长磁介质棒，表面磁化电流面密度为 $\vec{\alpha'}$，沿切线，求棒内 $\vec{M}$



$$\begin{cases} M_t = I'S = \alpha' L s \\ M_t = M(Ls) \end{cases} \Rightarrow \alpha' = M$$

$$\vec{\alpha'} = \vec{M} \times \vec{n}$$

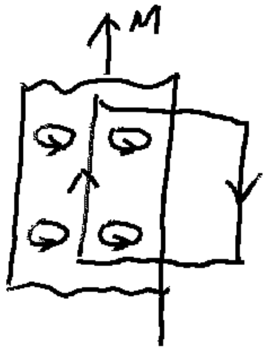
T77 $\vec{\alpha'} = \vec{M} \times \vec{n}$ $\quad I' = \alpha' d = Md$

$M_t = \pi r^2 I' = (\pi r^2 d) M$

$B' = \frac{\mu_0 I'}{2r} = \frac{\mu_0 M d}{2r}$

Q：磁介质被均匀磁化（$\vec{M}$），求$\vec{M}$对回路ABCD的线积分

$$\oint_L \vec{M}\cdot d\vec{l} = Ml = \alpha' l = I'$$



宏观：表面电流

微观：类似"面包圈"．

$$I' = \pi r^2 i n l = Ml$$

穿过任意回路L的磁化电流等于磁化强度沿回路的线积分．

§13.5 介质中的安培环路定理

Q：有磁介质

$$\oint_L \vec{B}\cdot d\vec{l} = \mu_0(I_0 + I')$$

$$I' = \oint_L \vec{M}\cdot d\vec{l}$$

$$\oint_L \left(\frac{\vec{B}}{\mu_0} - \vec{M}\right)\cdot d\vec{l} = I_0 \quad \text{（传导电流）}$$

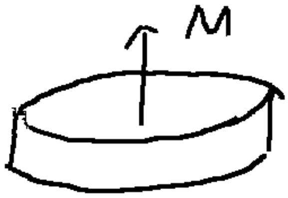
$\vec{H} - \vec{E}$

$\vec{B} - \vec{D}$

$$\vec{H} \stackrel{def}{=} \left(\frac{\vec{B}}{\mu_0} - \vec{M}\right) \quad \text{磁场强度}$$

Q：薄圆盘磁铁 $\vec{M}$，求中心附近的 $\vec{B}$、$\vec{H}$

$$\vec{B}_{i,e}' = \frac{\mu_0 M d}{2r}\vec{k}$$

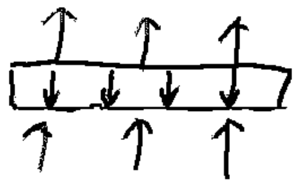


$$\vec{M}_i = M\vec{k}$$

$$\vec{M}_e = 0$$

$$\vec{H}_i = \frac{\vec{B}_i}{\mu_0} - \vec{M}_i = -M\left(1-\frac{d}{2r}\right)\vec{k} \downarrow$$

$$\vec{H}_e = \frac{\vec{B}_e}{\mu_0} - \vec{M}_e = \frac{d}{2r}M\vec{k} \uparrow$$



$\vec{H}$分布

T78

$$\vec{M} = M\vec{i}$$

$$\vec{B}_1 = \mu_0\alpha' = \mu_0\vec{M}$$

$$\vec{H}_1 = 0$$

$$\vec{B}_e = \frac{1}{2}\vec{B}_1 = \frac{1}{2}\mu_0\vec{M}$$

$$\vec{H}_{e1} = -\frac{1}{2}\vec{M}$$

$$\vec{H}_{e2} = \frac{1}{2}\vec{M}$$

对于各向同性的介质

$\vec{M} = \chi_m \vec{H}$　　χ_m 磁化率

$\mu_r = \chi_m + 1$　　μ_r 相对磁导率

$\mu = \mu_r \mu_0$　　μ 磁导率

真空：$\chi_m = 0$，$\mu_r = 1$，$\mu = \mu_0$

$$\vec{B} = \mu_0 (1 + \chi_m) \vec{H} = \mu_r \mu_0 \vec{H} = \mu \vec{H}.$$

顺磁质：$\chi_m > 0$，$\mu_r > 1$　　抗磁质：$\chi_m < 0$，$\mu_r < 1$.

磁介质中的环路定理

$$\begin{cases} \oint_L \vec{B} \cdot d\vec{l} = \mu_0 (I_0 + I') \\ \vec{H} = \dfrac{\vec{B}}{\mu_0} - \vec{M} \\ \oint_L \vec{M} \cdot d\vec{l} = I' \end{cases}$$

H对任意环路积分等于穿越回路面的传导电流的代数和.

$$\Rightarrow \oint_L \vec{H} \cdot d\vec{l} = I_0$$

T79 $H \cdot 2\pi r = I \qquad H = \dfrac{I}{2\pi r}$

$$M = (\mu_r - 1)H = \frac{(\mu_r - 1)I}{2\pi r}$$

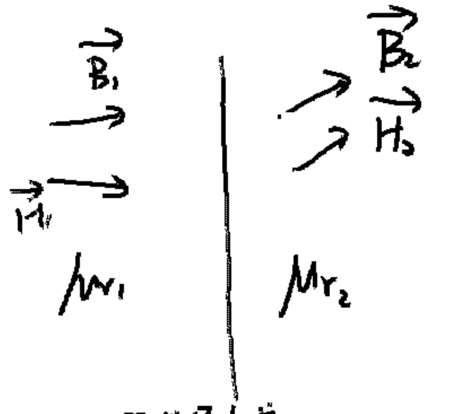
$$B = \mu H = \frac{\mu_r \mu_0 I}{2\pi r}$$

$$\alpha_1' = M = \frac{(\mu_r - 1)I}{2\pi R_1} \qquad \alpha_2' = \frac{(\mu_r - 1)I}{2\pi R_2}$$

T80 $2H \cdot l = nlI \qquad H = nI$

$$B = \mu_r \mu_0 H = \mu_r \mu_0 nI$$

$$\alpha' = (\mu_r - 1)H = (\mu_r - 1)nI$$

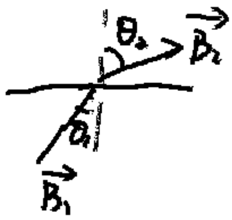


无传导电流

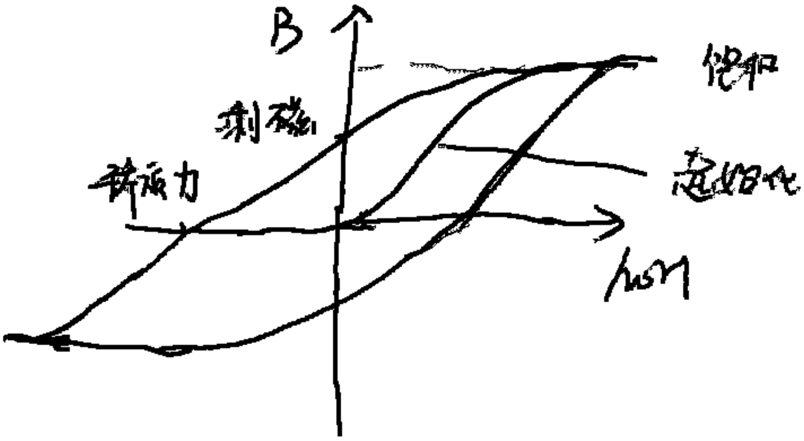
$$B_{1n} = B_{2n}$$

$$H_{1t} = H_{2t}$$

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}$$



§13.4 铁磁性



软磁：μ大，矫顽力Hc小，磁滞回线窄，磁滞损耗小

制作铁芯：电磁铁，变压器

硬磁：Br大，Hc大 充磁后不易退磁

制作永磁体

矩磁：剩磁大，矫顽磁力小

记忆元件.

第14章 电磁感应

电磁感性定律 $\varepsilon = -\dfrac{d\Phi}{dt}$

“坐标系”：$[\,\varepsilon\ (/\vec{E}_k),\ \Phi\ (/\vec{B})\,]$: RHR.

T81 Φ - $\dfrac{d\Phi}{dt}$ - ε+ 逆时针

跳环与自感现象

电磁阻尼

T82 $\varepsilon = \int_0^l B(x)\,\omega x\,dx$

$$B(x) = \frac{\mu_0 I}{2\pi(a + x\cos\theta)}$$

得 $\varepsilon = \frac{\mu_0 I \omega}{2\pi\cos\theta}\left(l - \frac{a}{\cos\theta}\ln\frac{a + l\cos\theta}{a}\right)$

T83 $\varepsilon = (\vec{v} \times \vec{B}) \cdot \overrightarrow{oa}$