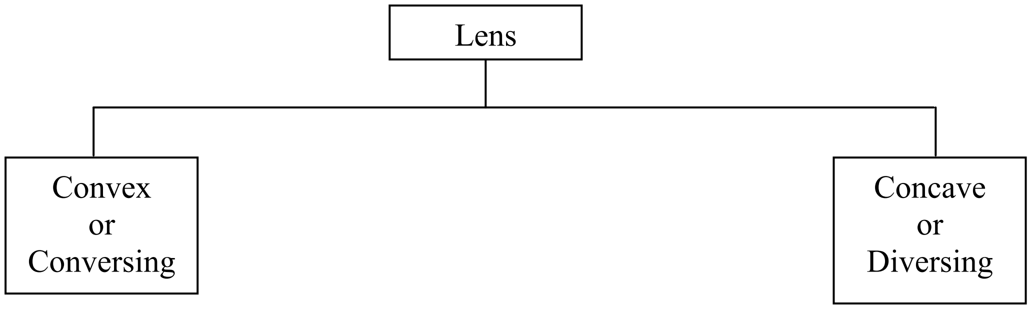
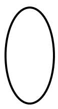
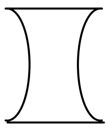
*** Refraction through Lenses:**

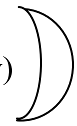
Lens: It is the piece of a transparent medium bounded by two or at least one spherical surface.



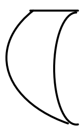
(i)  - double convex or biconvexo convex or convexo-concavex.

(ii)  - double concave or biconcave or concavo-concave.

(iii)  - palno-convex

(iv)  - concavo-convex

(v)  - plano-concave

(vi)  - convexo-concave.

*** Len's formula:**

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

Where, u = object distance
v = image distance
f = focal length of the lens

*** Sign Convention:**

(i) Real distance = (+)ve
and virtual distance = (-)ve

(ii) f = (+)ve for convex lens
and f = (-)ve for concave lens

*** Len's Maker's formula:**

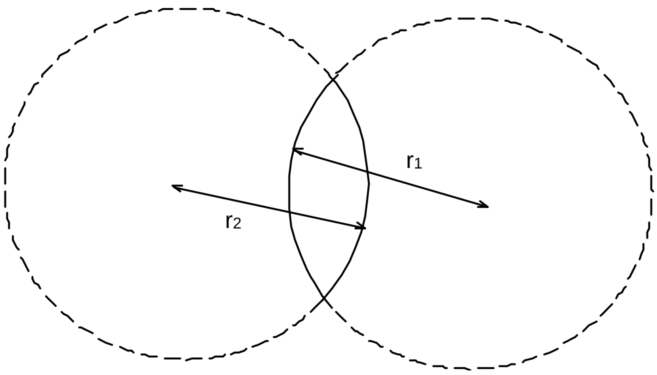
$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{r1} + \frac{1}{r2}\right]$$

Where, f = focal length of the lens

μ = Refractive index of the material of the lens.

$$\mu = \frac{Velocity\ of\ light\ in\ air\ (or\ vaccum\ medium)}{Velocity\ of\ light\ in\ that\ medium}$$

* Velocity of light(gas) > velocity of light in liquid > velocity of light in solid.



Spherical cases:-

(i) For equi-convex lens

$r_1 = r_2 = r$ (say)

$$\therefore \quad \frac{1}{f} = (\mu - 1)\frac{2}{r}$$

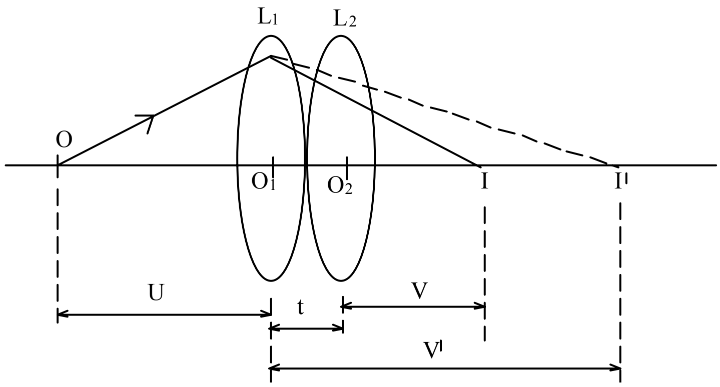
(ii) For plano-convex lens:-

$r_1 = \infty$ and $r_2 = r$

$$\therefore \quad \frac{1}{f} = (\mu - 1)\frac{1}{r} \qquad \left[\frac{1}{\infty} = 0\right]$$

(*) r_1 and r_2 = (+)ve for convex lens
and r_1 and r_2 = (-)ve for concave lens.

*** Combination of two thin lenses when places co-axially in contact**



Let,

L_1 and L_2 = Two thin convex lenses placed co-axially in contact.

f_1 and f_2 = Focal lengths of lenses of the L_1 and L_2 respectively.

t = The separation between the optical centers o_1 and o_2 respectively.

o = Point object situated on the axis.

$I^|$ = Real image of o formed by L_1 in the absences of lens L_2
= Virtual object of lens L_2

I = Real image of o formed by lens L_2.
= Real image of o formed by the combination of the lens.

u = oo_1 object distance for L_1.

v^1 = $o_1I^|$ image distance for L_1.

v = o_2I image distance for L_2.

For lens L_1 :

Using lens formula we can write.

$$\frac{1}{v^1}+\frac{1}{u}=\frac{1}{f} \quad \text{--------------} \quad \text{(I)}$$

For lens L_2 :

Using lens formula, we can write

$$\frac{1}{v}+\frac{1}{-o_2I^1}=\frac{1}{f_2} \qquad (\therefore \text{Object is virtual})$$

$$\therefore \quad \frac{1}{v}-\frac{1}{(v-t)}=\frac{1}{f_2} \quad \text{--------------} \quad \text{(II)}$$

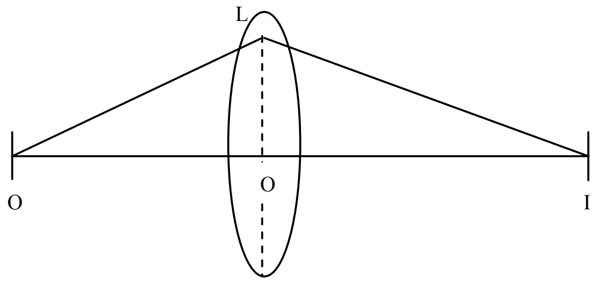
[$\therefore o_2I^| = o_1I^| - o_1o_2$]

Since, the lens are thin hence o_1 and o_2 lies very closed to each other. Thus, $v^1 - t \approx v^1$ as t is negligible with respect to v^1.

Thus, equation (II) becomes,

$$\frac{1}{u}-\frac{1}{v^1}=\frac{1}{f_2} \quad \text{--------------} \quad \text{(III)}$$

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f_1}+\frac{1}{f_2} \quad \text{--------------} \quad \text{(IV)}$$



* If the combination of lens replaced by a single lens so that it formed the real image of the same object of the same point and of the same magnification then it is said to be combined or equivalent focal length.

If f be combined focal length, then we can write.

$$\frac{1}{u}-\frac{1}{v^{|}}=\frac{1}{f_2} \text{ -------------- (V)}$$

Comparing equation (IV) and (V), we get:

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

Which is the required expression of combined focal length when the two thin lenses are placed co-axially in contact.

* Special caese:

(i) If L_1 is convex and L_2 is concave, then

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2} \qquad (\therefore\ f_1 = (+)\text{ve} \qquad f_2 = (-)\text{ve})$$

(ii) If the both lenses are concave from above equation, we can write,

$$\frac{1}{f}=-\frac{1}{f_1}+\frac{1}{f_2} \qquad (\therefore\ f_1 \text{ and } f_2 = (-)\text{ve})$$

(iii) Since required of focal ~~length of a lens~~ is called its power.

Hence, from the above equation, we can write.

$P = P_1 + P_2$ — P_1 and P_2 = power of lenses L_1 and L_2 respectively.

P = power of equivalent lens.

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u} \text{ -------------- (I)}$$

For convex lens:

When object is virtual u=(-)ve

$$\therefore \frac{1}{v}=\frac{1}{f}+\frac{1}{u} \qquad = +\text{ve always}$$

f = (+ve)

i.e. v=($\pm$)ve

Image = real (always)

For concave lens:

When object is virtual u = (-)ve

f = (-)ve

$$\therefore \frac{1}{v} = -\frac{1}{f} + \frac{1}{u}$$

a) If f>u then $\frac{1}{f} < \frac{1}{u}$

$$\therefore \frac{1}{v} = +ve \qquad \therefore v = (+)ve.$$

Thus, image = real

b) If f<u then $\frac{1}{f} > \frac{1}{u}$

$$\therefore \frac{1}{v} = -ve \qquad \therefore v = (-)ve.$$

Thus, image = virtual

#Unit of power of lens:

The S.I. unit is diopter (D)

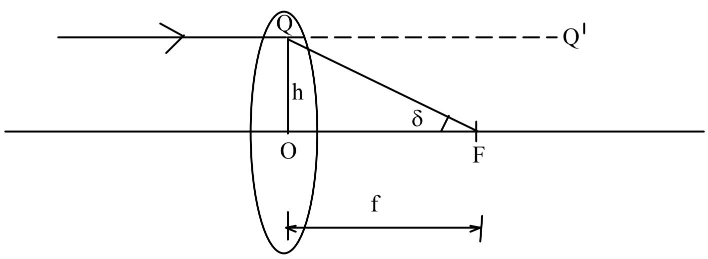
$$P = \frac{1}{f(meter)} = \frac{100}{f(cm)} D$$

If f=1m (or 100c) then p=1D.

Thus, the power of lens said to be 1 diopter if its focal length is 100 cm.

#

***Deviation of light produced by a thin lens:**



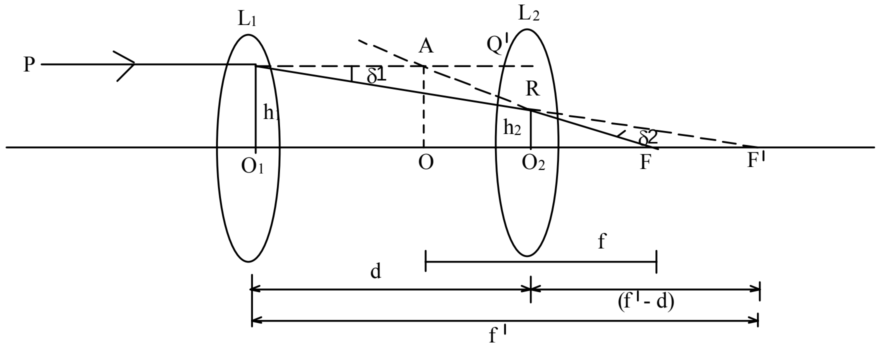
From $\triangle$ OQF

$\delta = \tan \delta \qquad (\therefore \delta = small\ for\ this\ lens)$

$\delta = \frac{OQ}{OF} \qquad \therefore \delta = \frac{h}{f}$

where h = height of incident optical centre.
f = focal length of the lens

 # Equivalent focal length of the combination of two thin lenses place co-axially some distance apart



Let, L_1 and L_2 are two thin convex lenses having their optical centre O_1 and O_2 respectively and are placed co-axially at a distance d i.e. O_1O_2=d, f_1f_2=focal lengths of the lenses L_1 and L_2 respectively.

A ray of light incident on L_1 parallel to the principle axis and refract a long QR which appears to converge at point 'F' the focus of length and finally refract along RF to meet at point F which the focus of equivalent lens.

δ_1 = deviation of light produced by L_1.

δ_2 = deviation of light produce by L_2.

h_1 = height of incident for lens L_1.

h_2 = height of incident for lens L_2.

Thus, we can write,

$$\delta_1 = \frac{h_1}{f_1} \quad \text{--------------} \quad \text{(I)}$$

$$\delta_2 = \frac{h_2}{f_2} \quad \text{--------------} \quad \text{(II)}$$

Here, total deviation produced by the combination of lenses is given by

$$\delta = \delta_1 + \delta_2$$

$$\therefore f = \frac{h_1}{f_1} + \frac{h_2}{f_2} \quad \text{--------------} \quad \text{(III)}$$

When PQ is equal to $Q^{|}$ and RF back to meet A. When a normal arrow is draw at a point O on the axis it (AO) represent the position of combine lenses. So that O and F are its optical centre and focus respectively. If f be the focal length of the equivalent lens (OF = f) then,

We can write,

$$\delta = \frac{h^{|}}{f} \quad \text{--------------} \quad \text{(IV)}$$

Thus, from equations (III) and (IV) we get,

$$\frac{h^{|}}{f}=\frac{h_1}{f_1}+\frac{h_2}{f_2} \quad \text{-------------} \quad (V)$$

From geometry, we can prove that

$\triangle QO_1F^{|}$ and $\triangle RO_2F^{|}$ are similar.

$$\therefore \frac{QO_1}{RO_2}=\frac{O_1f^{|}}{O_2f^{|}}$$

$$or, \frac{h_1}{h_2}=\frac{f^{|}}{(f_{|}-d)}$$

$$or, h_2=\frac{(f_1-d)h_1}{f^{|}} \quad \text{-------------} \quad (VI)$$

Thus, using equations (VI) in (V, we get:

$$\frac{h_1}{f}=\frac{h_1}{f_1}+\frac{(f^{|}-d)h_1}{f_2f_1}$$

$$or, \frac{1}{f}=\frac{1}{f_1}+\frac{f}{f_1f_2}-\frac{d}{f_1f_2}$$

$$or, \frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1f_2} \quad \text{-------------} \quad (VII)$$

Which is the required expression for combined focal lens when two thin lens are placed co-axially. Some distance apart.

Special cases:

(i) If the lens are placed co-axially in contact i.e. d=0.

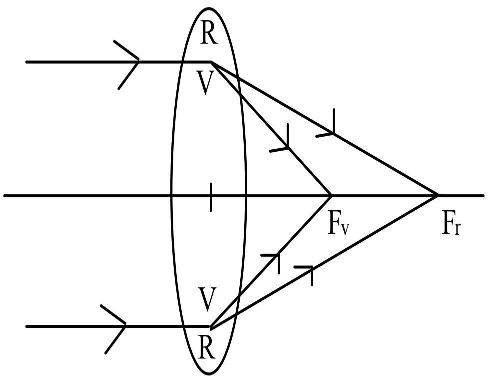
Hence, from above equation, we get:

$$\frac{1}{f_1}=\frac{1}{f_2}+\frac{1}{f_2}$$

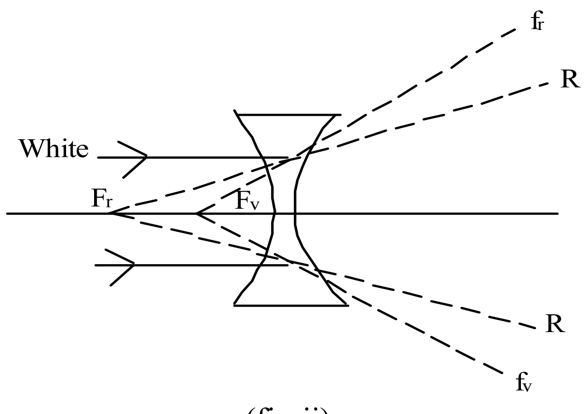
(ii) If P_1 and P_2 are powers of lenses L_1 and L_2 respectively the power of equivalent lens is given by,

$$P = P_1+P_2 - dP_1P_2$$

 # Chromatic aberration:



(fig.-i)



(fig-ii)

The inability of a lens to focus different colour of light to a single point when a parallel beam of light(white) is allow to pass through it parallel to principal axis is called chromatic aberration.

The chromatic aberration is produced by convex lens is taken to positive and (-ve) for concave lens. The difference between the focal length for red and violet rays of light is the measure of chromatic aberration.

*** Reason:**

When white light is allow to pass through lens it split up into constituent colour due to prismatic cution.

From lens maker formula, we know that,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

Where, f= focal length of the lens

μ =refractive index of material of lens.

r_1 and r_2 = radii of curvature of the lens.

The focal lengths of a given lens depend upon its refractive index that is higher the refractive index and lower will be the focal length and vice-versa.

For violet colour light:

Refractive index of material of lens is maximum for violet colour of light and is focal length is minimum. Similarly refractive index of lens is minimum for red colour of light and its focal length is maximum and so on.

 # Expression for chromatic aberration of lens:

Let,

f_v, f, f_r = the focal length of lens for violet, mean and red rays respectively.

μ_v, μ, μ_r = refractive indices of the lens for violet, mean and red rays respectively.

r_1 and r_2 = radii of curvature of the lens

w = dispersive power of the material of the lens.

Now from len's maker's formula, we can write:

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \text{ for mean ray.}$$

$$\therefore \frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{f(\mu - 1)} \text{ ------------- (I)}$$

Again, for violet ray, we ca write:

$$\frac{1}{f_v} = (\mu_v - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{f_v} = \frac{\mu_v - 1}{f(\mu - 1)} \text{------------- (II) \quad Using equation (I)}$$

Similarly, for red ray we can write,

$$\frac{1}{f_r} = \frac{\mu_r - 1}{f(\mu - 1)} \text{ -------------- (III)}$$

Subtracting equation (III) from (II), we get:

$$\frac{1}{f_v} - \frac{1}{f_r} = \frac{(\mu_v - 1) - (\mu_r - 1)}{f(\mu - 1)}$$

$$or, \frac{f_r - f_v}{f_v f_r} = \frac{\mu_v - 1 - \mu_r + 1}{f(\mu - 1)}$$

$$or, \frac{f_r - f_v}{f_v f_r} = \frac{\mu_v - \mu_r}{f(\mu - 1)}$$

$$\therefore f_r - f_v = \frac{w}{f} \times f_v f_r$$

Where, $w = \left(\dfrac{\mu_v - \mu_r}{\mu - 1} \right)$

If f be the geometric mean of f_v and f_r then, we can write:

$f^2 = f_v f_r$ -------------- (V)

Thus, equation (IV) becomes,

$$f_r - f_v = \frac{w}{f} \times f^2$$

$$f_r - f_v = w \times f \text{ -------------- (VI)}$$

i.e. chromatic aberration = dispersive power of the lens × focal length of lens.

Thus, equation (VI) is required equation for chromatic aberration produced by lens.

* Special cases:

(i) for convex lens:

f = (+ve)

$\therefore$ chromatic aberration = (+)ve (w = (+)ve always)

(ii) for concave lens:

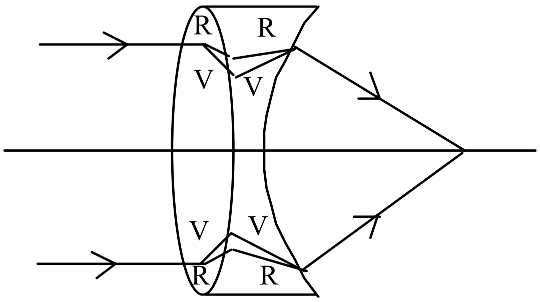
f = (-)ve

$\therefore$ chromatic aberration = (-)ve (unit = meter)

*** Removal of chromatic aberration or Achromatism:**

The process of removing chromatic aberration of the lens id called Achromatism and the combination of the lenses. Free from chromatic aberration is called Achromat or Achromatic combination of lens.

(i) when two lens are placed co-axially in contact to remove chromatic aberration a convex lens another concave lens of suitable material and suitable focal length are placed co-axially in contact.



Let, L_1 and L_2 = A convex lens and concave lens are placed co-axially contact to have no chromatic aberration.

f_v, f, f_r = focal length of lens $L_1$for violet mean and red rays respectively.

$f^{|}_v$, $f^{|}$, $f^{|}_r$ = focal length of lens L_2 for violet mean and red rays respectively.

μ_v, μ, μ_r = refractive indices of lens L_1 for violet, mean and red ray respectively.

w and $w^{|}$ = dispersive power of lens L_1 and L_2 respectively.

For lens L_1:

Using lens maker formula for mean rays

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$ where, r = radius of curvature of lens L_1.

$$\therefore \frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{f(\mu - 1)}$$

Again, for violet ray,

$$\frac{1}{f_v} = (\mu_v - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$or, \frac{1}{f_v} = \frac{\mu_v - 1}{f(\mu - 1)} \text{ ------------- (II)}$$ using equation (I)

Similarly, for lens L_2 we can write

$$\frac{1}{f_v} = \frac{\mu_{v|} - 1}{f^{|}(\mu^{|} - 1)} \text{ -------------------- (III)}$$ for violet colour

If f_v be the combined focal length for violet colour of light then, we can write

$$\frac{1}{F_v} = \frac{1}{f_v} - \frac{f}{f_v^{|}}$$

$$\therefore \frac{1}{F_v} = \frac{\mu_v - 1}{f(\mu - 1)} + \frac{\mu_v - 1}{f^{|}(\mu^{|} - 1)} \text{ ------------- (II)}$$ using equations (II) and (III)

Similarly, combined focal length for red colour of light i.e. F_r is given by,

$$\frac{1}{F_r} = \frac{\mu_r - 1}{f(\mu - 1)} + \frac{\mu_r^{|} - 1}{f^{|}(\mu^{|} - 1)} \text{ -------------------- (V)}$$

For achromatism

$F_v = F_r$

$$\frac{1}{F_v} = \frac{1}{F_r}$$

Putting the value of F_v and F_r

$$or, \frac{\mu_v - 1}{f(\mu - 1)} + \frac{\mu_v}{f(\mu^{|} - 1)} = \frac{\mu_r - 1}{f(\mu - 1)} + \frac{\mu_r^{|} - 1}{f^{|}(\mu^{|} - 1)} \qquad u\sin g\ equations\ (IV)\ and\ (V)$$

$$or, \frac{(\mu_v - 1) - (\mu_r - 1)}{f(\mu - 1)} = \frac{-(\mu_v^{|} + 1) + (\mu_r^{|} - 1)}{f^{|}(\mu^{|} - 1)}$$

$$or, \frac{\mu_v - 1 - \mu_r + 1}{f(\mu - 1)} = \frac{-(\mu_v^{|} - 1 - \mu_r^{|} - 1)}{f^{|}(\mu^{|} - 1)}$$

$$or, \frac{\mu_v - \mu_r}{f(\mu - 1)} = \frac{\mu_v^{|} - \mu_r^{|}}{f^{|}(\mu^{|} - 1)}$$

$$or, \frac{w}{f} = \frac{-w^{|}}{f^{|}} \text{ ------------- } (VI)$$

$$where,\quad w = \frac{\mu_v - \mu_r}{(\mu - 1)} \quad and \quad w^{|} = \frac{\mu_v^{|} - \mu_r^{|}}{(\mu^{|} - 1)}$$

Which is required condition for Achromatism.

For equation (VI) we have

$$\frac{w}{w^{|}} = -\frac{f}{f^{|}} \text{ ------------ } (VII) \qquad \therefore \frac{w}{w^{|}} = (+)ve$$

Hence, to have R.H.S. = (+)ve if one lens is convex another must be concave.

Again, from equation (VII), we have,

$$\frac{w}{f} + \frac{w^{|}}{f^{|}} = 0$$

If $w = w^{|}$ i.e. the lenses are of same material then

$$w\left(\frac{1}{f} + \frac{1}{f^{|}}\right) = 0$$

$$or, \frac{1}{f} + \frac{1}{f^{|}} = 0 \qquad (\because w \neq 0)$$

$$\frac{1}{F} = 0 \qquad F = Combined\ focal\ length\ of\ mean\ ray.$$

$$F = \infty$$

Thus, lenses should not be a same materials. They must be of different materials. Again, combined focal length of mean ray is given as:

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f^{|}}$$

For conversing nature of combination F must be (+)ve i.e.

$$\frac{1}{f}+\frac{1}{f^{|}}=(+)ve$$

$\therefore$ f = (+)ve and

$f^{|}$ = (-)ve

$\therefore$ $f<f^{|}$

Focal length of convex must be smaller than concave lens so that system of lens behave as convex lens.

Again, from equation (VII)

$$\frac{w}{f}+\frac{w^{|}}{f^{|}}=0$$

$\therefore$ f = (+)ve and

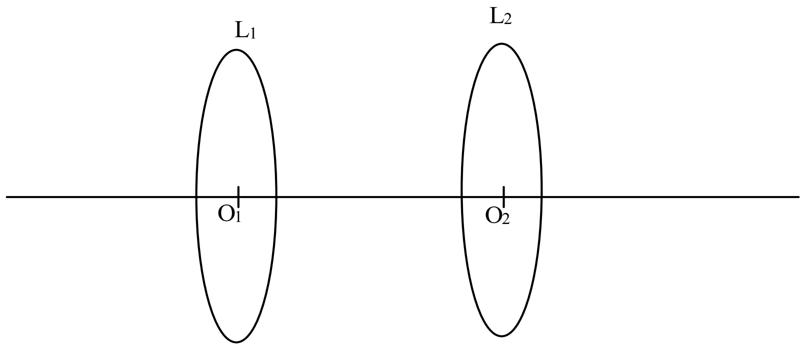
$f^{|}$ = (-)ve

$\therefore$ $f<f^{|}$ (for conversing nature of the system of the lens)

$\therefore$ $w < w^{|}$ i.e. Dispersive power of convex lens must be smaller than that of concave lens.

(II) Condition of Achromatic when lenses are placed co-axially at some distance apart:

Chromatic aberration can be removed when two thin lenses of same material are placed co-axially at some distance apart so that their separation equal to the average of the focal length of the two lenses.



$$d=\frac{f_1+f_2}{2}$$

Let, f_1 and f_2 = focal length of two lenses L_1 and L_2 (same material) respectively.

d = separation between two lens placed co-axially so that there is no chromatic aberration.

μ_v, μ, μ_r = Refractive indices of materials of lenses of violet, mean, red rays respectively.

f_v , f_r = focal length of lens L_1 for violet and red rays respectively.

= focal length of lens L_2 for violet and red rays respectively.

F_v , F_r = combined focal lengths for violet and red rays respectively.

Thus, we can write,

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1.f_2} \quad ------------(I)$$

$$\frac{1}{F_v}=\frac{1}{f_v}+\frac{1}{f_v^{|}}-\frac{d}{f_v.f_v^{|}} \quad ----------(II)$$

$$\frac{1}{F_r}=\frac{1}{f_r}+\frac{1}{f_r^{|}}-\frac{d}{f_r.f_r^{|}} \quad ----------(III)$$

For lens L_1 :

Using lens Maker's formula for violet rays we have

$$\frac{1}{f_v}=\frac{\mu_v-1}{f_1(\mu-1)} \quad ----------------(IV)$$

For red rays:

$$\frac{1}{f_r}=\frac{\mu_r-1}{f_1(\mu-1)} \quad -----------------(V)$$

For lens L_2 :

$$\frac{1}{f_v}=\frac{\mu_v-1}{f_2(\mu-1)} \quad ----------------(VI)$$

and, $\frac{1}{f_r}=\frac{\mu_r-1}{f_2(\mu-1)} \quad ----------------(VII)$

Using equations (IV) and (VI) in equation (II), we get:

$$\frac{1}{F_v}=\frac{\mu_v-1}{f_1(\mu-1)}+\frac{\mu_v-1}{f_2(\mu-1)}-\frac{d(\mu_v-1)^2}{(\mu-1)^2 f_1 f_2} \quad -----------(VIII)$$

Similarly, from equation (III), (V) and (VII)

$$\frac{1}{F_r}=\frac{\mu_r-1}{f_1(\mu-1)}+\frac{\mu_r-1}{f_2(\mu-1)}-\frac{d(\mu_r-1)^2}{(\mu-1)^2 f_1 f_2} \quad -----------(IX)$$

For Achromatism,

$$F_v=F_r$$

$$or,\ \frac{1}{F_v}=\frac{1}{F_r}$$

$$or,\ \frac{\mu_v-1}{f_1(\mu-1)}+\frac{\mu_v-1}{f_2(\mu-1)}-\frac{d(\mu_v-1)^2}{(\mu-1)^2 f_1 f_2}=\frac{\mu_r-1}{f_1(\mu-1)}+\frac{\mu_r-1}{f_2(\mu-1)}-\frac{d(\mu_r-1)^2}{(\mu-1)^2 f_1 f_2}$$

$$or, \frac{1}{f_1}+\frac{1}{f_2}\left(\frac{\mu_v-1-\mu_r-1}{\mu-1}\right)=\frac{d}{(\mu-1)^2 f_1 f_2}\left\{(\mu_v-1)^2-(\mu_r-1)^2\right\}$$

$$or,\left(\frac{1}{f_1}+\frac{1}{f_2}\right)\left(\frac{\mu_v-\mu_r}{\mu-1}\right)=\frac{d}{(\mu-1)^2 f_1 f_2}\left\{(\mu_v-1+\mu_r-1)\,(\mu_v-1-\mu_r+1)\right\}$$

$$or,\left(\frac{1}{f_1}+\frac{1}{f_2}\right)\left(\frac{\mu_v-\mu_r}{\mu-1}\right)=\frac{d}{(\mu-1)^2 f_1 f_2}\left\{(\mu_v+\mu_r-2)\,(\mu_v-\mu_r)\right\}$$

$$or, \left(\frac{1}{f_1}+\frac{1}{f_2}\right)\left(\frac{\mu_v-\mu_r}{\mu-1}\right)=\frac{d}{(\mu-1)^2 f_1 f_2}(2\mu-2)(\mu_v-\mu_r)$$

$$or, \left(\frac{1}{f_1}+\frac{1}{f_2}\right)\left(\frac{\mu_v-\mu_r}{\mu-1}\right)=2d(\mu-1)(\mu_v-\mu_r)$$

$$\mu=\frac{\mu_v+\mu_r}{2}$$

$$or, \left(\frac{1}{f_1}+\frac{1}{f_2}\right)\left(\frac{\mu_v-\mu_r}{\mu-1}\right)=\frac{2d}{f_1 f_2}\left(\frac{\mu_v-\mu_r}{\mu-1}\right)$$

$$or, \frac{1}{f_1}+\frac{1}{f_2}=\frac{2d}{f_1 f_2}$$

Multiplying by f_1f_2 both sides,

$$or, f_2+f_1=2d$$

$$or, d=\frac{f_1+f_2}{2}$$

Which is required condition for achromatism. #

Formula:

(I) $\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$

* For equi-convex lens

$$\frac{1}{f}=(\mu-1)\frac{2}{r} \qquad [r_1=r_2=r \quad say]$$

* For plano-convex

$$\frac{1}{f}=(\mu-1)\frac{1}{r} \qquad [r_1=\infty \qquad r_2=r \quad say]$$

(II) $\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$

When lenses are placed co-axially contact.

$\therefore P=P_1+P_2$ [P_1P_2 = power of lenses]

(III) $\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$

$$\therefore P=P_1+P_2-dP_1P_2$$

$$\therefore P=\frac{1}{f(in\ mm)}D$$

$$=\frac{100}{f(cm)}D$$

(IV) Chromatic aberration:

aberration = $f_r - f_v$

= $w \times f$ (w is the dispersive power)

$$w = \left(\frac{\mu_v - \mu_r}{\mu - 1} \right)$$

For achromatism:

$$\frac{w_1}{f_1} = -\frac{w_2^{|}}{f_2}$$

Numericals:

(iii) The dispersive power's for crown and flint glass are 0.015 and 0.030 respectively. Calculate the focal length of the lenses which form an achromatic doublet of focal length 60cm when placed in contact and are made up of crown and flint glasses.

So|^n:

(iv) The dispersive power's of crown and flint glasses are 0.015 and 0.03 respectively. The refractive indices for the mean rays are 1.52 and 1.65. If one of the surface of the flint glass lens is plane. Calculate the radii of curvature.

So|^n:

(3) An ambulance emitting a whine at 1600 Hz overtakes and passes a cyclist pedaling the bike at 18 Km/h. After being passed, the cyclist hears a frequency of 1590 Hz. How fast is ambulance moving?

So|^n: Here,

Frequency of source (ambulane), f = 1600 Hz.

Frequency of observer (cyclist), $f^{|}$ = 1590 Hz.

Velocity of observer (cyclist), v_o = 18Km/h = 5m/s

Velocity of ambulance, v_s = 332m/s

$\therefore$ we know that when ambulance and cyclist both are moving towards each other, the of sound heard by the cyclist is given by,

$$f^{|} = \left(\frac{v + v_o}{v + v_s} \right) f$$

$$or, 1590 = \left(\frac{332 + 5}{332 + v_s} \right) \times 1600$$

$$or, v_s = 7.12\ m/s$$

This is required velocity of ambulance crossing the road.

(1) (b) A small body of mass 0.1kg is undergoing a S.H.M. of amplitude 0.1m and period 2 sec. (i) what is the maximum force on body (ii) If the oscillations are produced in the spring, what should be the force constant?

So|^n: Given,

(i) Mass of a body (m) = 0.1Kg

Amplitude (X_m) = 0.1m

Period (T) = 2sec.

Maximum force, F_{max} = ?

F_{max} = m.a_{max}

= m(w^2×m)

$$= m\left(\frac{2\pi}{T}\right)^2 X_m \qquad \left(\therefore w = \frac{2\pi}{t}\right)$$

$$= \frac{4\pi^2 m X_m}{T^2}$$

$$= \frac{4\times(3.14)^2\times 0.1\times 0.1}{2\times 2}$$

$$= \frac{0.394384}{4}$$

= 0.098596 N

(ii) Here, maximum force, F_{max} = 0.098596 N

Amplitude (X_m) = 0.1m

Force constant or spring constant, K=?

$$\therefore K = \frac{F_{max}}{X_m}$$

$$= \frac{0.098596}{0.1}$$

= 0.98596

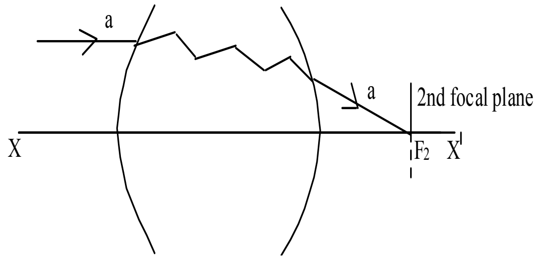
*** cardinal point: (or Gauss points):**

Gauss said that if in an optical system the positions of certain specific points called cardinal point of the system , be known, the system may be treated as a 'single unit'. The position and size of the image of an the object may then directly be obtain by same relations as used for thin lenses or single surfaces, however, complicated the system may be.

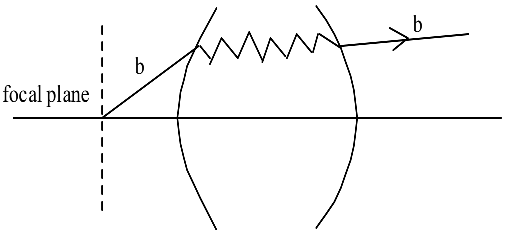
There are six cardinal points of an optical system:

Two focal points, two principal points and two nodal points.

(I) Focal points:



(fig-i)



(fig-ii)

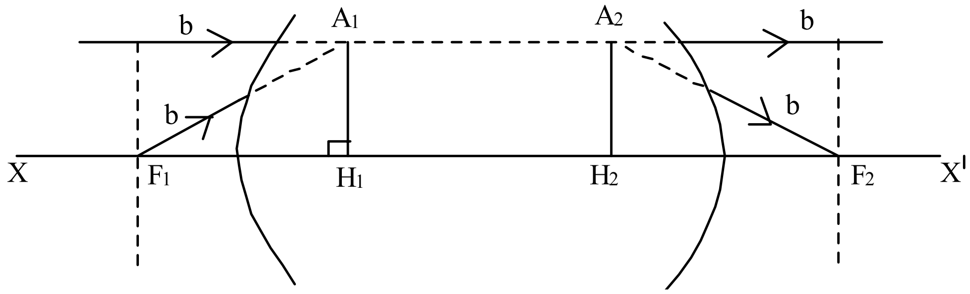
conjugate to points at infinity.

The image point on the axis which corresponds to the object at infinity is called second focal point of the system. It is denoted by I_2 (fig-I).

Similarly, the object point on the axis which corresponds to the image at infinity is called the 1st focal point of the system. It is denoted by F (fig-II).

Then, the planes through f_1 and f_2 and perpendicular to the axis are called the 1st focal plane and IInd focal plane respectively.

(II) Principal points (or unit points):

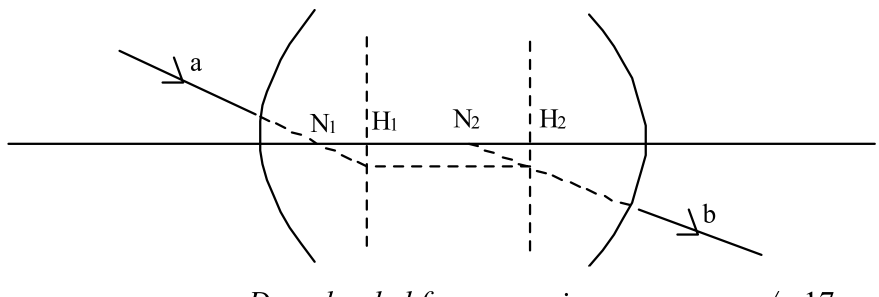


The principal points are a pair of conjugate points on the principal axis of the optical system having unit linear magnification.

If the incoming and outgoing rays are extended, each pair will intersect at points A_1 and A_2 on a plane surface. These planes are known as 1st and II principal planes and their points of intersection with the axis i.e. H_1 and H_2 being the first and second principal points respectively. The points A_1 and A_2 are conjugate point and $A_1H_1 = A_2H_2$.

The distance of 1st focal point F_1 from the principal point H_1 i.e. H_1F_1 is called the 1st focal length (f_2) of the system. Similarly, the distance H_2F_2 is called the 2nd foacl length (f_2) of the system. If the medium be same on the two sides of the system, then $f_1=f_2$.

(III) Nodal points:



The nodal points are a pair of conjugate points on the principal axis of the system, having unit angular magnification.

They are such that an incident ray directed towards the one nodal point emerges parallel to its original direction through the other nodal point. In fig. N_1 and N_2 are the first and second nodal point. The planes passing normally through are called the nodal planes. It is to be noted that $N_1H_1 = N_2H_2$, If the medium of the two sides of the system be same then, the principal point coincide with the nodal point and these are then called equivalent point.

Equivalent focal length:
(When two lenses are placed co-axially at some distance apart):

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Numerical problems:
(i) Two convex lenses having focal length 5cm 2cm are placed co-axially at c distance of 3cm. Find the equivalent focal length and the positions of the principal point.

So|^n: $f_1 = 5cm$
$f_2 = 2cm$
$d = 3cm$
Equivalent focal length (f) = ?
Positions of the principal points = ?

We know,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$= \frac{1}{5} + \frac{1}{2} - \frac{3}{5 \times 2}$$

$$= \frac{1}{5} + \frac{1}{2} - \frac{3}{10}$$

$$= \frac{4 + 10 - 6}{20}$$

$$= \frac{2}{5}$$

$$f = \frac{5}{2} = 2.5cm$$

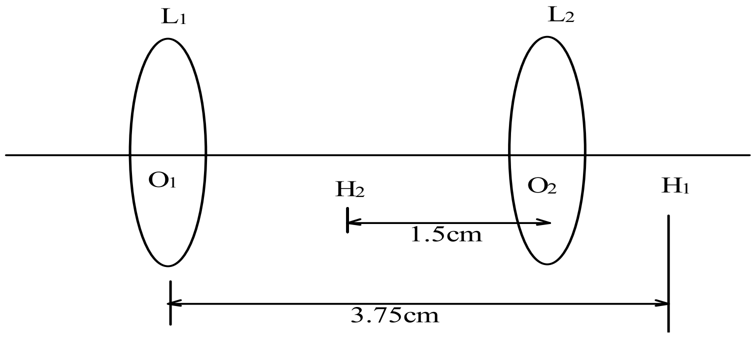
Again, the distance of first principal point from 1st lens is given by,

$$\alpha = \frac{d \times f}{f_2} = \frac{3 \times 2.5}{2} = \frac{7.5}{2} = 3.75cm$$

The distance of 2nd principal point from 2nd lens is given by,

$$\beta = \frac{d \times f}{f_1} = \frac{3 \times 2.5}{5} = .5cm$$ [Here real distance = (+)ve]

$\therefore f = 2.5cm, \quad \alpha = 3.7cm, \quad \beta = 1.5cm$



Also, determine the positions of the focal points.

The distance of first focal points from 1st lens $O_1F_1 = f(1-d/f_2)$

$$= 2.5\left(1-\frac{3}{2}\right)$$

$$= 2.5\times -0.5 = 1.25cm$$

Similarly, $O_2f_2 = f\left(1-\frac{d}{f_1}\right)$

$$= 2.5\left(1-\frac{3}{5}\right) = 1cm$$

(ii) Two thin convex lenses of focal length 20cm and 5cm are placed 10cm apart. Calculate the positions of the principal point of this combination.

So|n: Given,

$f_1 = 20cm$

$f_2 = 5cm$

$d = 10cm$

position of the principal point = ?

$\therefore$ we know that,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$= \frac{1}{20} + \frac{1}{5} - \frac{10}{20\times 5}$$

$$= \frac{1}{20} + \frac{1}{5} - \frac{10}{100}$$

$$= \frac{5+20-10}{100}$$

$$\frac{1}{f} = \frac{15}{100}$$

$$15f = 100$$

$$f = \frac{100}{15} = 6.66cm$$

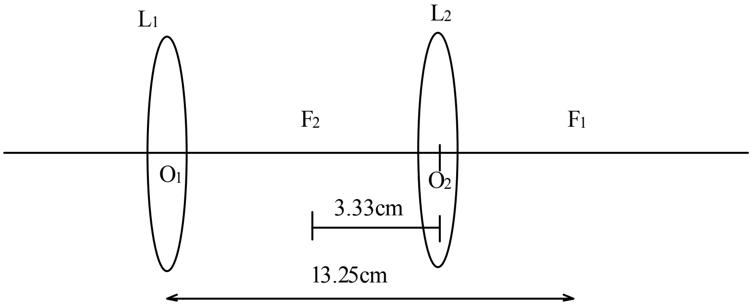
$\therefore f = 6.66cm$

Again, the distance of first principal point from 1st lens

$$= \alpha = \frac{d \times f}{f_2} = \frac{10 \times 6.66}{5} = 13.25$$

The distance of 2nd principal point from 2nd lens

$$= \beta = \frac{d \times f}{f_1} = \frac{10 \times 6.66}{20} = 3.33cm$$



Also, determine the positions of the focal points.

The distance of first focal points from 1st lens O_1F_1 =

$$f\left(1 - \frac{d}{f_2}\right) = 6.66\left(1 - \frac{10}{5}\right) = 6.66 \times -1 = -6.66cm$$

Similarly, O_2F_2 =

$$f\left(1 - \frac{d}{f_1}\right) = 6.66\left(1 - \frac{10}{20}\right) = 6.66(1 - 0.5) = 3.33cm$$

(iii) Two thin conversing lenses of powers 5D and 4D are placed co-axially 10cm apart. Find the focal length of the combination and the positions of the principal points.

So|n: Given,

$P_1 = 5D$

$P_2 = 4D$

$d = 10cm$

Here, $P_1 = \frac{1}{f_1} = \frac{100}{f_1}D$

$$or,\ f_1 = \frac{100}{P_1}D = \frac{100}{5D}D = 20cm$$

And, $P_2 = \frac{1}{f_2} = \frac{100}{f_2}D$

$$or,\ f_2 = \frac{100}{P_2}D = \frac{100}{4D}D = 25cm$$

We know,

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

$$or,\ \frac{1}{f}=\frac{1}{20}+\frac{1}{25}=\frac{5+4}{100}=\frac{9}{10}$$

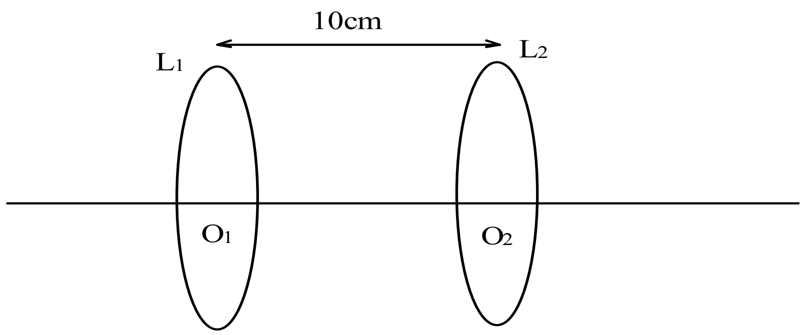
$$or,\ f=\frac{100}{9}=11.11cm$$

The distance of 1st principal points from 1st lens is given by

$$\alpha=\frac{d\times f}{f_2}=\frac{10\times 11.11}{25}=4.444cm$$

The distance of 2nd principal point from 2nd lens is given by

$$\beta=\frac{d\times f}{f_1}=\frac{11.11\times 10}{20}=5.555cm$$



Also, determine the positions of the focal points.

The distance of 1st focal points from 1st lens O_1F_1 =

$$f\left(1-\frac{d}{f_2}\right)=11.11\left(1-\frac{10}{25}\right)=11.11\times 0.6=6.666cm$$

Similarly, O_2F_2 =

$$f\left(1-\frac{d}{f_1}\right)=11.11\left(1-\frac{10}{20}\right)=11.11(1-0.5)=5.555cm$$

(iv) Two identical thin convex lenses of focal lens 8cm each are co-axial and 4cm apart. Find the equivalent focal length and the positions of the principal points. Also, find the position of the object for which the image is form at ∞.

So|n:

Monochromatic aberration:

The aberration arises for the single colour of light in a lens is called monochromatic aberration.

The monochromatic aberrations are:

I. Spherical aberration

II.Coma
III.Astigmatism
IV.Curvature of the field
V.Distortion

(I) Spherical aberration:

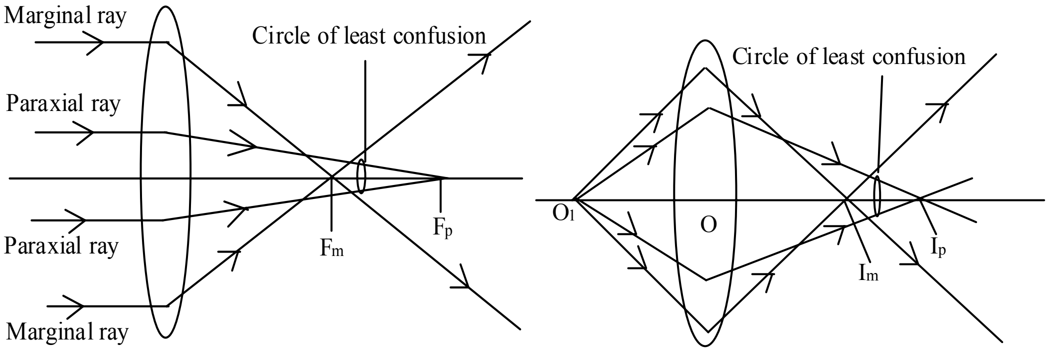


fig-i(axial point object at ∞) fig-ii(object point on the axis at finite distance)

When a parallel beam of light is allowed to pass through a lens, parallel to its principal axis(fig-i) the marginal rays converse to a point F_m closer to the lens after refraction and the paraxial rays after refraction converse to the point F_p away from the lens This in ability of a lens to focus the different rays of incident beam to a single point is called spherical aberration. The different between the focal length due to the paraxial rays and marginal rays is the measure of longitudinal spherical aberration.

The points where the marginal rays and paraxial rays cross each other after refraction, a small circle or a small circular path is obtain which is known as the circle of least confusion which is the closest to the point image of a point object. The radius of the circle of least confusion is the measure of lateral spherical aberration.

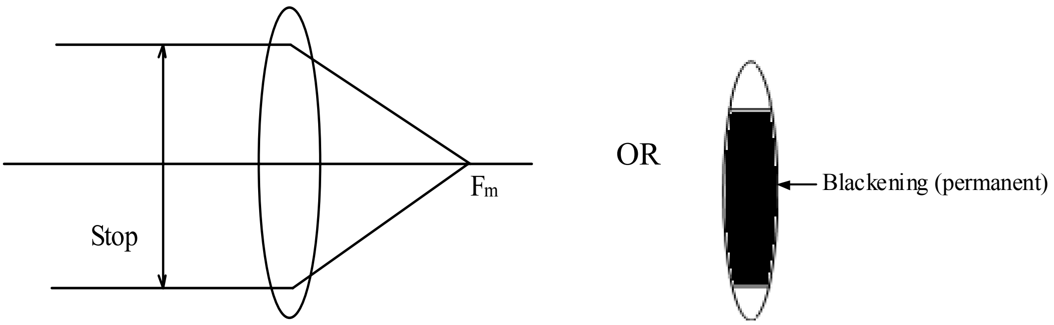
Similarly when a point object is placed at a finite distance from the lens on its principal axis, its image due to marginal rays i.e. I_m is form closer to the lens and as the image due to the paraxial rays I_p is formed away from the lens (fig-ii).

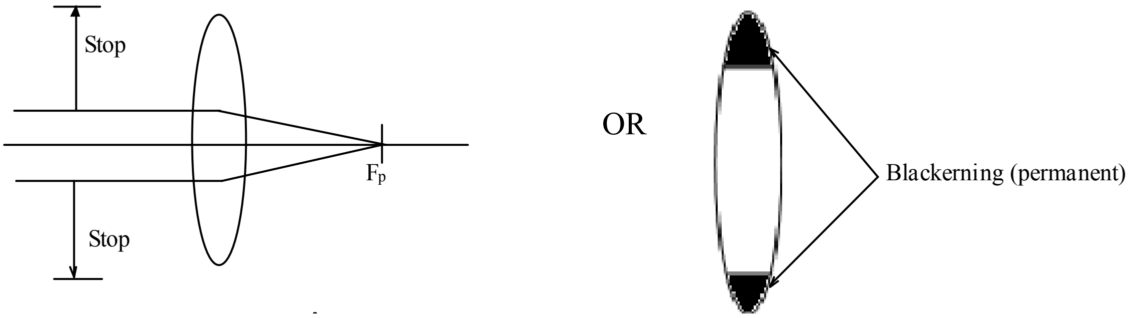
The spherical aberration arises due to the fact that the different zones of a lens have different radii of curvature and hence, difference focal length. The focal length of a lens for outer zones is minimum for this. Similarly, the focal length of a lens is maximum for central zones due to maximum radius of curvature.

The spherical aberration due to a convex lens is taken to be +ve and it is-ve for concave lens.

*** Minimisation of spherical aberration:**

(i) On using stops

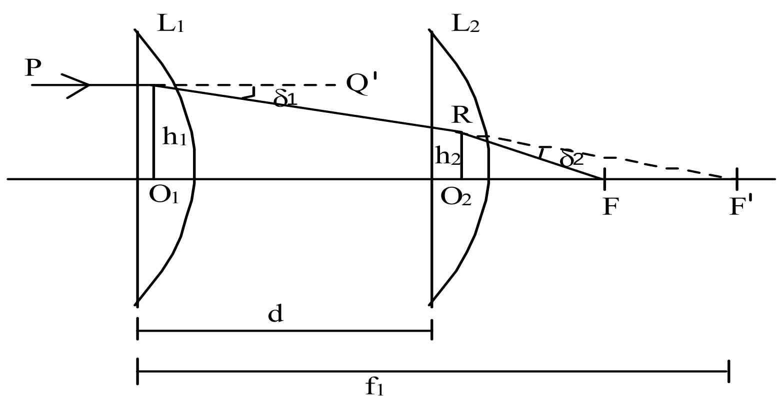




(ii) When two plano-convex lenses are placed co-axially at a suitable distance apart. The spherical aberration can be minimize. The distance between the two lenses must be equal to the different between the focal length of the lenses.

i.e. $d = f_1 - f_2$

where, f_1 and f_2 are the focal lengths of the lenses and d is the distance between them.



Let, L_1 and L_2 = two thin plano-convex lenses placed co-axially at a distance d so that the spherical aberration is minimum.

F_1 and F_2 = the focal length of the lenses L_1 and L_2 respectively.

h_1 and h_2 = height of incident rays for L_1 and L_2 respectively.

δ_1 and δ_2 = deviation of light produce by L_1 and L_2 respectively.

For minimum spherical aberration,

$$\delta_1 = \delta_2$$

$$\therefore \frac{h_1}{f_1} = \frac{h_2}{f_2}$$

$$\therefore \frac{h_1}{h_2} = \frac{f_1}{f_2} ----------(i)$$

From similar triangles QO_1F_1 and RO_2F_1

We can write,

$$\frac{h_1}{h_2} = \frac{O_1F_1}{O_2F_2}$$

$$or,\ \frac{h_1}{h_2} = \frac{f_1}{f_1 - d}$$

$$or,\ \frac{f_1}{f_2} = \frac{f_1}{f_1 - d} \qquad (using\ \ equation\,(i))$$

$$or,\ f_2 = f_1 - d$$

$$or,\ f_1 - f_2 = d\ \ --------(ii)$$

(II) coma:

The aberration, affects rays that came from object points situated slightly off the axis of the lens. In fact the image of such a point is from to have a egg-like or comet-like shape. The coma arises due to the following reasons.

(i) The different zones of the lens produced different lateral magnification.

$$\frac{height\ of\ the\ image}{height\ of\ object} = lateral\ magnification$$

For the non-axial point objects i.e. there is a decrease or increase of lateral magnification with the height of narrow circular zones of the lens.

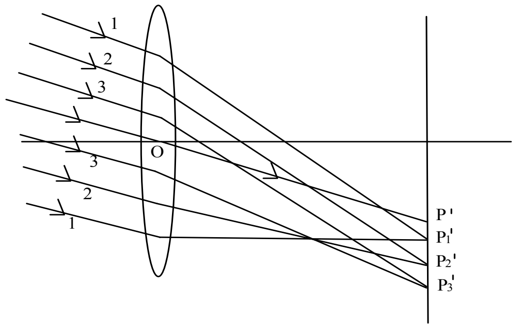
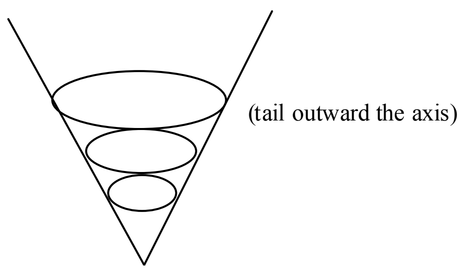


Fig-I (Non- axial point at ∞)



(+)ve coma

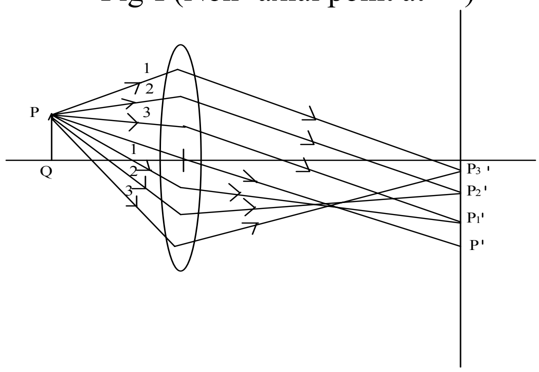
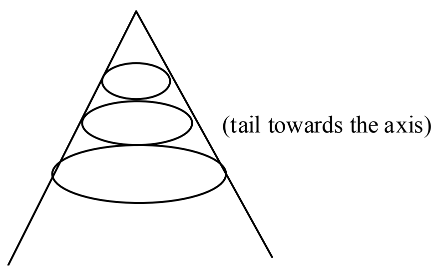


Fig-II (Non-axial object at a finite distance)



(-)ve coma

Hence, the rays from a point on passing through different zones (1,1), (2,2), (3,3), etc. of the lens are focus at different points $P_1^|, P_2^|, P_3^|$,etc. respectively.

(ii) Each zones forms the images of a point in the form of a circle known as "comatic circle" of which radius increases as the radius of the zone increases. Thus, the image of the non-axial point consists of an expanding series of overlapping comatic circle.

If the lateral magnification form the outer zone is large than that for the central zone (fig-i), the tail of the comatic circle extends outwards from the principal axis at the coma is said to be (+)ve.

If the lateral magnification form the inner zone is smaller than that for the central zone (fig-ii) the tail of the comatic circle extends inwards from the principal axis at the coma is said to be (-)ve.

*** Reduction of coma (minimization of coma):**

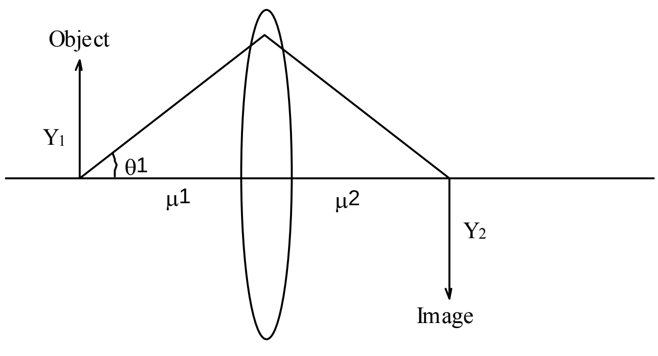
(i) On using the stops at suitable distance.

(ii) By Abbe's sine condition:

For a lens or system of lenses free from spherical aberrations coma is completely eliminated. If Abbe's sine condition,

$$\mu_1 y_1 Sin\theta_1 = \mu_2 y_2 Sin\theta_2$$

Is satisfied for all the zones, where μ_1, μ_2 are the refractive indices of the object and image spaces. Y_1 and y_2 are the heights of the object and image respectively for a zone and θ_1 *and* θ_2 are the angles which the conjugate rays make the axis.



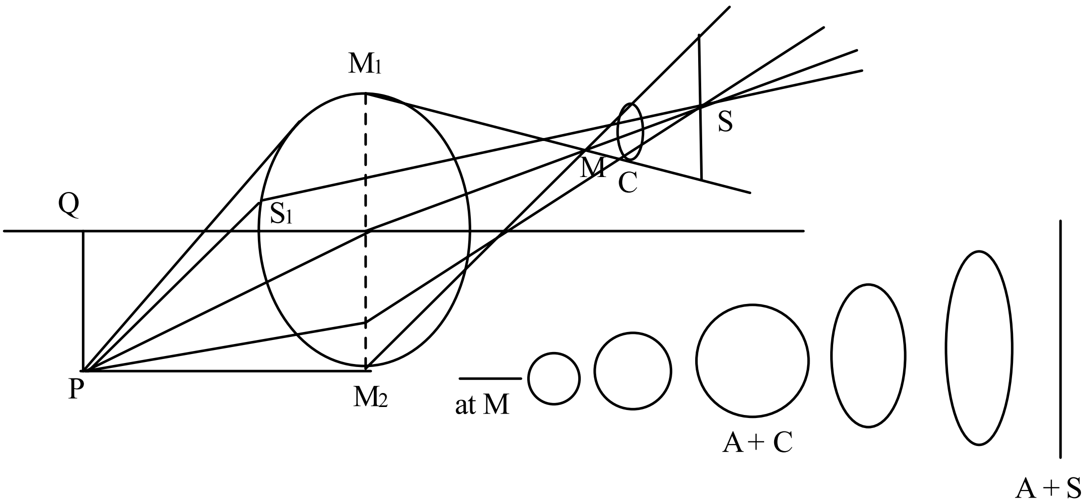
Thus, the absence of coma requires that,

$$\frac{Sin\theta_1}{Sin\theta_2} = \frac{\mu_1 y_1}{\mu_2 y_2} = cons\tan t \;\; for \;\; all \;\; values \;\; of \;\; \theta$$

The lens satisfy the above condition is called alplanatic lens.

(III) Astigmatism (or not a point):

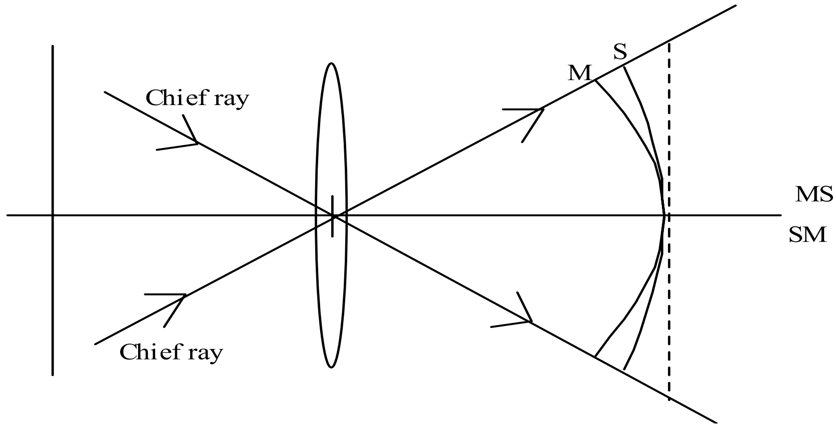
Even if a lens free from spherical aberration and coma, the image of point object situated far off the axis is not a point but consists of two mutually perpendicular lines some distance apart. This aberration is called astigmatism (means not at a point).



M_1M_2 = Meridional section,

S_1S_2 = sagital sections.

The rays from object point P situated far of the axis, passing through the meridional plane M_1 and M_2 (i.e. plane containing the point P and the axis of the lns) of the lens meet in a horizontal line at M while those passing through sagittal plane S_1S_2 (i.e. plane containing point P and perpendicular to meridional plane) meet in a vertical line at S. M and S are called the primary and secondary images respectively. When a screen is placed between M and S, an irregular patch of light is obtained and at point where the rays passing through the two sections cross each other i.e. at C, the path becomes circle which is known as the circle of least confusion and is the closest approach to the point image.



When the object is plane ad extended then from each point of the object there is a primary image and a secondary image. The locus of the primary and secondary images are curved surfaces M and S respectively which coincide with each other at a point on the axis of the lens. The distance between M and S measure along the chief rays is called astigmatic difference.

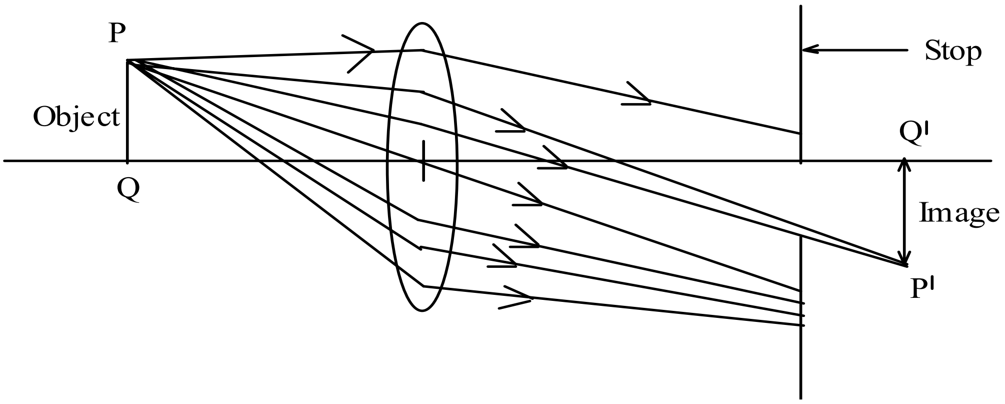
The astigmatism produced by a convex lens is taken to be the +ve and –ve for a concave lens.

*** Reason of astigmatism:**

For an off axial point the focal length of any narrow co-axial circular zone of the lens is not the same at every point of it, being list in maridional sections and maximum in the sagittal section but in each case less than the focal length.

* Removal of astigmatism:

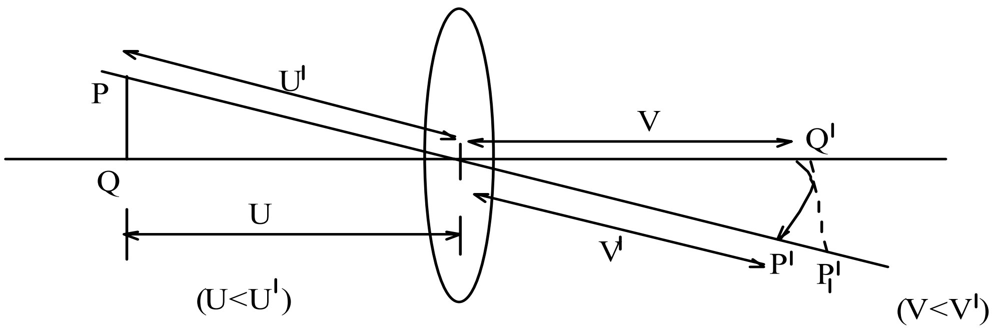
(i) Astigmatism difference may be reduced on using a top at a suitable position so that only less oblique rays are permitted to form the image.



(ii) For a lens system the astigmatism may be eliminated by adjusting their relative position. The lens system free from astigmatism is called an astigmate.

(IV) Curvature of the field:

If a lens or lens system is free from spherical aberration, an astigmatism even then the image of the plane and extended object PQ is in general a curved surface so that if a screen be placed at $Q^{|}$ perpendicular to the axis the complete image $P^{|}Q^{|}$ will not be focused. This defect is called curvature of the field.



The curvature of the field arises due to the fact that the points away from the axis (such as P) are at a greater distance from the centre O of the lens then the axial point Q. Hence, image $P^{|}$ is formed at a smaller distance then $Q^{|}$ (image of Q).

* Removal of the curvature of the field:

(i) for a single lens the curvature of the field is minimized by placing a stop in a suitable position in front of the lens.

(ii) For the system of thin lenses the curvature of the field is given by

$$\frac{1}{R}\sum_{i=1}^{n}\frac{1}{\mu_i f_i}$$

Where, R = radius of the curvature of the final image.

μ = refractive index of the lens.

f = focal length of the lens.

Thus, for no curvature of the field the required condition is

$$\sum_{i=1}^{n}\frac{1}{\mu_i f_i} = 0 \qquad [R = \infty \ \ for \ \ plane \ \ surface]$$

For only two lenses the above condition provides

$$\frac{1}{\mu_1 f_1} + \frac{1}{\mu_2 f_2} = 0$$

$$or, \ \frac{\mu_2 f_2 + \mu_1 f_1}{(\mu_1 f_1)(\mu_2 f_2)} = 0$$

$$\therefore \mu_2 f_2 + \mu_1 f_1 = 0$$

Which is known as petzralls condition. Since, μ_1 and μ_2 are always (+)ve. Hence, from above equation it is clear that, f_1 and f_2 should be of opposite sign. Thus, a combining a convex lens and a concave lens of suitable material and suitable focal lengths the curvature of the field is removed.

(V) Distortion:

When a stop is used with a lens to minimized the various aberrations, the image of a plane square shaped object placed perpendicular to the axis is not of the same shape as the object. This defect is called distortion.

It arises due to the variation of magnification with the lateral distance of an object point from the lens axis.

There are two types of distortions.

(i) pin cushion distortion

(ii) barrel shaped distortion

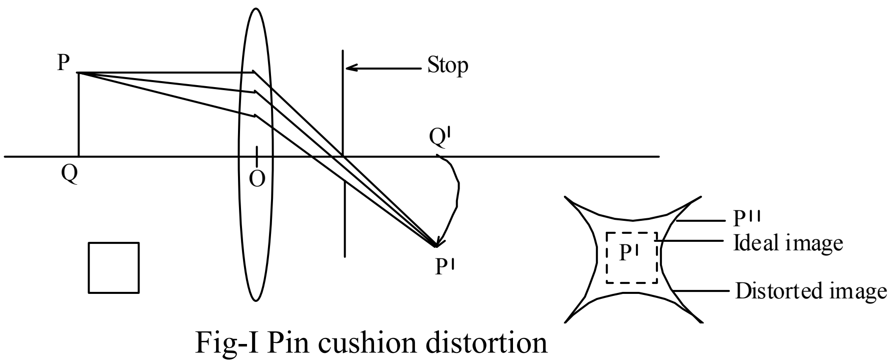


Fig-I Pin cushion distortion

If the stop is placed on the image side then the to form the image of the point P, those rays are used for which, the object distance is smaller than the ray passing through the centre O of the lens. Thus, the magnification of the outermost part of the plane object is greater than that of the central part producing pin cushion distortion.

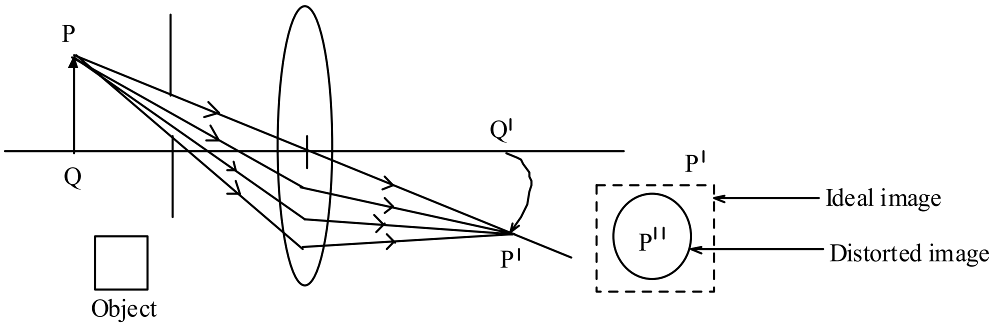
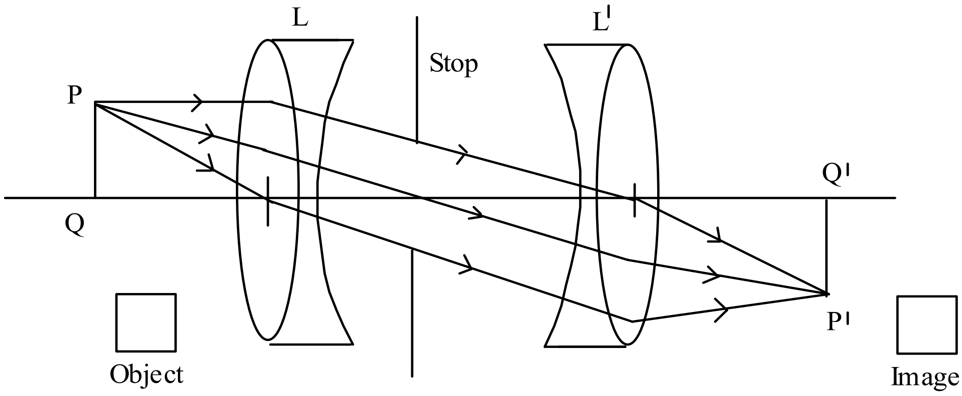


Fig-II Barrel shaped distortion

If the stop is placed on the object side then the central part of the image are magnified more than the outr parts and distortion produced is called the barrel shaped distortion i.e., if the magnification decreases with increase in the lateral distance of the object point then barrel shaped distortion is produced.

 # Removal of distortion:

A single thin lens without stop is free from distortion practically for all object distances. However, it cannot be free from all other aberrations. The distortion can be minimized.



The distortion can be minimized on employing two lens system well corrected from other aberrations and placed symmetrically on opposite sides of a stop or aperture so that the pin cushion produce by the first lens system is compensated by the barrel shape distortion produced by the second lens system.

Physical Optics:

Equation of a traveling wave (or progressive wave)

$y = aSin(\omega t + \phi)$

Where, y = displacement at time t

a = amplitude

ω = angular frequency

= $2\pi\eta$

η = frequency

ϕ = phase difference

 # Principle of superposition of wave:

It states that when two or more waves moving simultaneously through a medium in the same line (or very closed to each other) with same velocity superpose on each other the displacement of a particle due to resultant beam is the vector sum of the individual displacement of the same particle in same time due to the various waves.

If $\vec{y}_1, \vec{y}_2, \vec{y}_3, ...$ be individual displacement of a particle in time t due to various wave then from the principle of superposition of the wave the displacement of the same particle in same time due to resultant wave is given by,

$$\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + ...$$

It is to be noted that the displacement of a particle due to a wave does not depend upon the presence of another waves.

 # Interference of waves: **group "A" or "B"**

If two or more waves of same frequency (or wave length), same amplitude (or nearly same amplitude) and having in same phase direction with same velocity superpose on each other then the intensity of resultant wave alternately becomes maximum and minimum in a regular manner. This phenomenon of redistribution of the energy of the waves of same frequency is called interference of the waves.

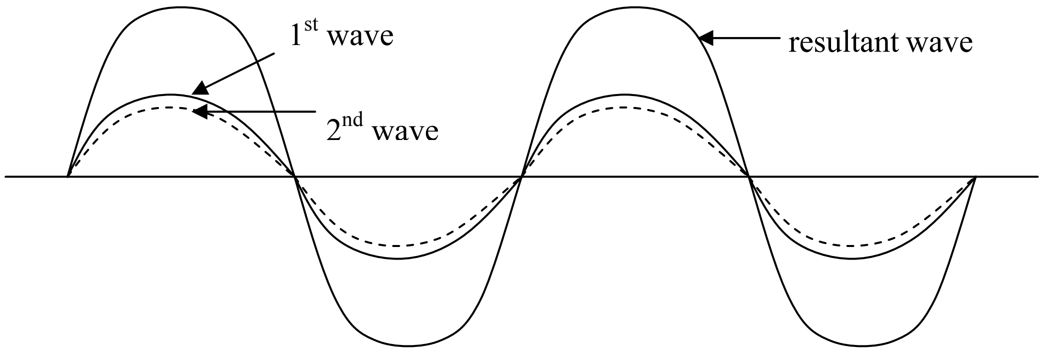
There are two types of the interference of the waves:

(i) constructive interference

(ii) destructive interference

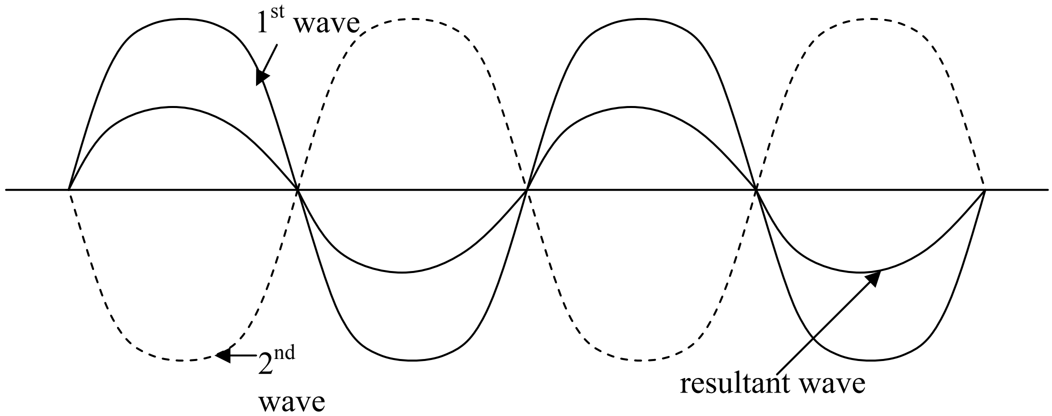
(i) Constructive Interference:

The interference is said to be constructive if the superposition of the waves gives rise to the maximum intensity of the resultant wave.



(ii) Destructive Interference:

The interference is said to be destructive if the superposition of the waves gives fall to the minimum intensity of the resultant wave.



 # Theory of Interference of waves:

Let $y_1 = a_1 Sin\omega t -----(i)$

and $y_2 = a_2 Sin(\omega t + \phi) -----(ii)$

be two waves of same frequency $(\therefore \omega = 2\pi f,\ f = frequency)$ moving simultaneously in the same direction superpose on each other to produce the interference of the wave.

Where, a_1 *and* a_2 = nearly equal amplitude ($a_1 > a_2$ say) respectively.

ϕ = phase difference between the waves.

Thus, from the principle of superposition of the waves the resultant wave is given by,

$$y = y_1 + y_2$$

$$or,\ y = a_1 Sin\omega t + a_2 Sin(\omega t + \phi) \qquad Using\ equations\ (i)\ and\ (ii)$$

$$or,\ y = a_1 Sin\omega t + a_2 Sin\omega t . Cos\phi + a_2 Cos\omega t . Sin\phi$$

$$or,\ y = Sin\omega t(a_1 + a_2 Cos\phi) + Cos\omega t(a_2 Sin\phi)$$

Putting

$$A\,Cos\delta = a_1 + a_2 Cos\phi \;-----(iii)$$

and, $$A\,Sin\delta = a_2 Sin\phi \;-----(iv)$$

we get,

$$y = A\left[Sin\,\omega t\,Cos\delta + Cos\,\omega t\,Sin\delta\right]$$

$$y = A\,Sin(\omega t + \delta) \;-----(v)$$

Which represents the resultant wave having amplitude A and Phase difference δ with respect to the wave given by equation (i).

Squaring and adding equation (iii) and (iv),

We get,

$$A^2\left(Cos^2\delta + Sin^2\delta\right) = a_1{}^2 + 2a_1a_2Cos\phi + a_2{}^2Cos^2\phi + a_2{}^2Sin^2\phi$$

$$or,\; A^2 = a_1{}^2 + 2a_1a_2Cos\phi + a_2{}^2(Cos^2\phi + Sin^2\phi)$$

$$\therefore A = \sqrt{a_1{}^2 + 2a_1a_2Cos\phi + a_2{}^2} \;-----(vi)$$

Which determines the amplitude of the resultant wave.

Now, dividing equation (iv) by (iii), we get,

$$\tan\delta = \frac{a_2 Sin\phi}{a_1 + a_2 Cos\phi} \;-----(vii)$$

Which determines the phase difference δ.

For constructive Interference:

A = maximum

[I α amplitude^2, I=intensity=energy per sec per unit are]

From equation (vi)

$$A = a_1 + a_2 \;(\max imum)$$

When $Cos\phi = +1$

$$\therefore \phi = 0, 2\pi, 4\pi, 6\pi, ...$$

i.e. $\phi = 2\eta\pi$

where η = 0, 1, 2, 3, 4, …………….

Hence, for constructive interference the phase difference between the waves must be even number multiple of π (or integral multiple of 2π).

Again,

Phase difference $= \frac{2\pi}{\lambda} \times path\ difference$

$$\therefore 2n\pi = \frac{2\pi}{\lambda} \times path\ difference$$

Path difference $= n\lambda \;\left(or,\; 2n \times \frac{\lambda}{2}\right)$

Where, n = 0, 1, 2, 3, ……………….

λ = wave length

Thus, for constructive interference the path difference between the waves must be integral multiple of λ (even no. multiple of λ).

For destructive interference:

A = minimum

From equation (vi), we have

$A = a_1 - a_2 \ (\min imum)$

When $Cos\phi = -1$

$\therefore \phi = \pi, 3\pi, 5\pi, 7\pi, ...$

$\phi = (2n+1)\pi$

Where, n =0, 1, 2, 3, ……………………

Thus, for the destructive interference, the phase difference between the waves must be odd no. multiple of π .

Again, Phase difference $= \frac{2\pi}{\lambda} \times Path\ difference$

$\therefore Path\ difference = (2n+1)\frac{\lambda}{2}$

Where, n =0, 1, 2, 3, ……………………

Thus, for destructive interference the path difference between the waves must be odd no. multiple of λ/2.

Intensity Distribution:

$\therefore Intensity\, \alpha\, amplitude^2$

Thus, $I_{\max.} = A_{\max.}{}^2 = (a_1 + a_2)^2$

And, $I_{\min.} = A_{\min.}{}^2 = (a_1 - a_2)^2$

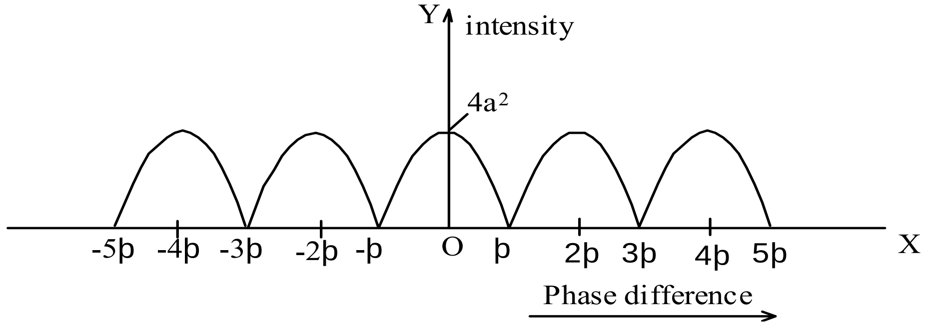
If $a_1 = a_2 = a$ (say)

Then, $I_{\max.} = Aa^2$

$I_{\min.} = 0$

But, net expected intensity of wave = I_1+I_2 = $a^2 + a^2 = 2a^2$ which is the average of $I_{max.}$ and $I_{min.}$.

Hence, the interference of waves is in accordance with the law of conservation of energy. It is to be noted that the total energy from a point of minima is transfer to the point of maxima and hence, $I_{max.}$ and $I_{min.}$ become respectively $4a^2$ and 0.



Necessary condition for interference of wave:

(i) The waves must have same frequency or wave length.

(ii) The waves must have same amplitude or nearly same amplitude.

(iii) The waves have in same phase or they may have constant phase difference.

(iv) The waves must travel in the same direction in the same line or very close to each other with same velocity.

(v) The sources must be coherent.

(vi) If the waves are plan polarized then, their plane of polarization must be same.

Coherent Sources:

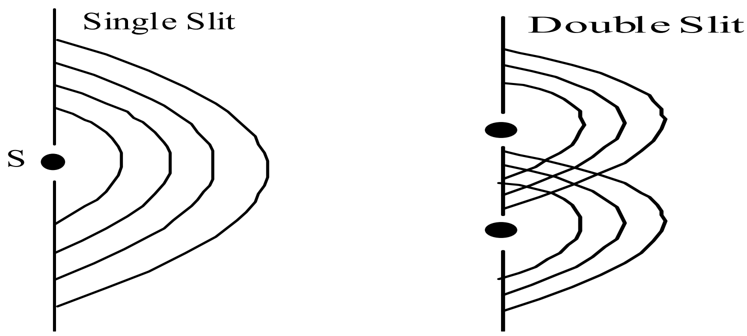
Two sources are said to be coherent if they emit waves of same frequency or wave length, same amplitude (or nearly same amplitude) and having in same phase (or have a constant phase difference).

Two separate identical sources are never coherent hence, the coherence sources are obtain from a single parent source. To produce coherence sources their must be either division of wave front (for example: double slit, fresnel's bimirror and biprism, etc.) or division of amplitude (for example: thin film of transparent medium)

Production of Coherent Sources:

On using Double slit:

A double slit is used to produce the coherent sources in which division of wavefront takes place.



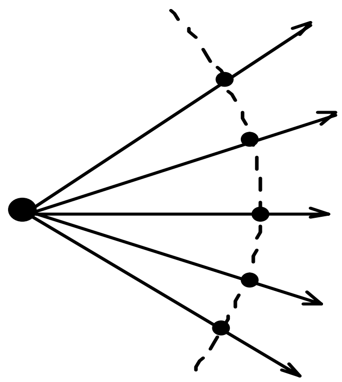
Here, S = a source of monochromatic light placed closer to a single slit.

S_1S_2 = a double slit placed symmetrically with respect to x and parallel to the single slit so that SS_1=SS_2.

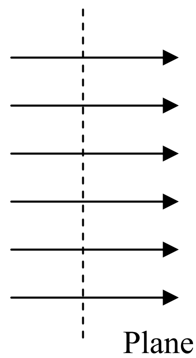
The spherical wave from S gets divided into two parts on passing through the double slit. Thus, S_1 and S_2 behave as coherent sources which are virtual sources.

Wavefront:

The locus of the points in the S place vibrating in same phase.



For a point source, the wave front is spherical at finite and wave front is plane wave front at ∞ from the source.



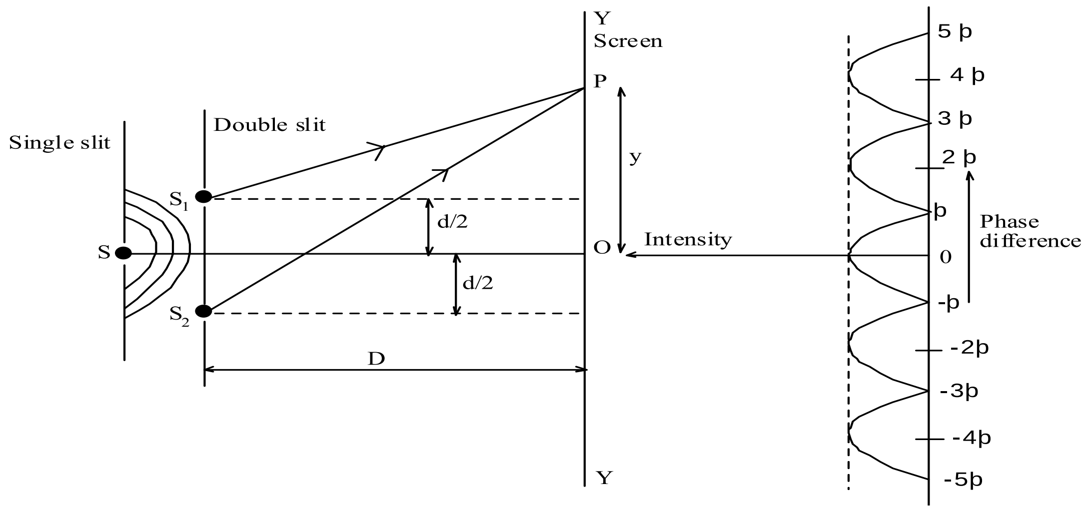
Plane wavefront

For line source the wave front is cylindrical.

 # Theory of interference fringes:- Young's double slit experiment:

Young experimentally explained the interference of light on the basis of Huygen's wave theory, on using a double slit. He found a central bright fringe surrounded by alternate dark and bright fringes of equal width on either side of it.

Experimental arrangement:



Here, S = A source of monochromatic light placed closer to a single slit.

S_1S_2 = A double slit placed symmetrically with respect to x and parallel to the single slit so that $SS_1=SS_2$

XY = A screen placed parallel to the single slit, on which the interference pattern is obtained.

d = separation between the slit S_1 and S_2 (i.e. slit width)

D = Separation between the double slit and the screen.

O = The point on the screen equidistance from S_1 and S_2 where central bright fringe is obtained.

P = An arbitary point on the screen, where wave from S_1 and S_2 superpose on each other to produce the interference pattern.

y = OP, the separation of the point P, from the centre O.

λ = wavelength of light used.

The path difference between the waves from S_1 and S_2 which superpose on each other at point P is given by

$$x = S_2P - S_1P \text{ -----}(i)$$

Now,

$$S_2P^2 - S_1P^2 = \left\{D^2 + \left(y + \frac{d}{2}\right)^2\right\} - \left\{D^2 + \left(y - \frac{d}{2}\right)^2\right\}$$

$$or,\ (S_2P + S_1P)(S_2P - S_1P) = \left(y + \frac{d}{2}\right)^2 - \left(y - \frac{d}{2}\right)^2$$

$$or,\ (S_2P + S_1P)x = 4.y.\frac{d}{2}$$

$$\therefore\ x = \frac{2yd}{(S_2P + S_1P)} \text{ -----}(ii)$$

Since, point P lies very close to O. Hence, $SP_2 \approx S_1P$.

Thus, equation (ii) becomes,

$$\therefore\ x = \frac{2yd}{2D} = \frac{yd}{D} \text{ -----}(iii)$$

For bright fringe:

Path difference = $n\lambda$ where n =0, 1, 2, 3, ……….

$$\therefore \frac{yd}{D} = n\lambda \qquad using\ equation\ (iii)$$

Thus, $y_n = \frac{n\lambda D}{d}$ -----(iv) where n =0, 1, 2, 3, …………

Which determines the separation of n^{th} bright fringe from centre.

Putting n =0, 1, 2, 3, ………. in above equation

we get,

the separation of the various bright fringes from the centre.

i.e. $y_0 = 0$, (i.e. centre is bright)

$y_1 = \frac{\lambda D}{d}$ (1st bright fringe)

$y_2 = \frac{2\lambda D}{d}$ (2nd bright fringe)

$y_3 = \frac{3\lambda D}{d}$ (3rd bright fringe)

$y_n = \frac{n\lambda D}{d}$ (nth bright fringe)

Thus, the separation between two consequtive bright fringe is

$$\beta = y_1 - y_0 = y_2 - y_1 = y_3 - y_2 = ... = \frac{\lambda D}{d} \quad -----(v)$$

i.e. the bright fringes are equispaces.

For dark fringe:

$\therefore Path\ Difference = (2n+1)\frac{\lambda}{2}$ where, n =0, 1, 2, 3, ……….

$\therefore \frac{y_{nd}}{D} = (2n+1)\frac{\lambda}{2}$ (Using equation (iii))

$\therefore y_n = \frac{(2n+1)\lambda D}{2d} \quad -----(vi)$ where, n =0, 1, 2, 3, ……….

Which determines the separation of (n+nth) dark fringe from centre.

Putting n =0, 1, 2, 3, ………. In above equation.

We get, the separation of various dark fringes from the centre.

i.e. $y_0 = \frac{\lambda D}{2d}$ (1st dark fringe) i.e. centre is not dark.

$y_1 = \frac{3\lambda D}{2d}$ (2nd dark fringe)

$y_2 = \frac{5\lambda D}{2d}$ (3rd dark fringe)

$y_3 = \frac{7\lambda D}{2d}$ (4th dark fringe)

$y_n = \frac{(2n+1)\lambda D}{2d}$ [(n+1)th dark fringe]

Hence, the separation between two consecutive dark fringe is

$$\beta^{|} = y_1 - y_0 = y_2 - y_1 = y_3 - y_2 = ... = \frac{\lambda D}{d} \quad -----(vii)$$

Thus, the dark fringes are equi-spaced.

Hence, from equations (v) and (vii) it is clear that the dark and bright fringes are equi-spaced and the separation between two consecutive bright or dark fringes is

$$\left[\beta = \frac{\lambda D}{d}\right] -----(viii)$$

Where β = fringe width

Thus,

$\beta \ \alpha \ \lambda$

$\beta \ \alpha \ D$

$\beta \ \alpha \ \frac{1}{d}$

From equations (i), we can write,

$$\lambda = \frac{\beta d}{D} -----(ix)$$

Thus, on knowing β (fringe width), d and D the wavelength of monochromatic light can be determine.

Note: If the entire experimental experiment is emerge in a liquid of refractive index (μ).

In this case the fringe width is given by,

$\beta^{|} = \frac{\lambda^{|} D}{d}$ where, $\lambda^{|}$ = wave length of light in liquid = $\frac{\lambda}{\mu}$ (in air)

$$\therefore \beta^{|} = \frac{\lambda D}{\mu d} = \frac{\beta \ (in \ air)}{\mu}$$

Thus, the fringe width decreases.

Formulae:

(i) $\beta = \frac{\lambda D}{d}$

Where, β = fringe width.

= separation between two consecutive bright (or dark) fringes or lines or bands.

D = separation between the slit and double slit.

d = slit width.

= separation between the two slit.

= separation between the two coherent sources.

= separation between the two virtual sources.

(ii) For bright fringe:

$$y_n = \frac{n\lambda D}{d}$$

Where, y_n = separation of n_{th} bright fringe from centre.

(iii) For dark fringe:

$$y_n = \frac{(2n+1)\lambda D}{2d}$$

Where, y_n = separation of n_{th} dark fringe from centre.

(iv) $c = f\lambda$

Where, f = frequency.

(*) For 10^{th} bright fringe, n = 10
For 10^{th} dark fringe, n = 9

Numericals:

1) Two slits are 0.3mm apart and are placed 50cm from a screen. What be the distant between the second and 3^{rd} dark lines of the interference pattern, when the slits are eliminated by a light of 6000 A° wavelength?

So|^n: Given,

d = 0.3mm = 0.3×10^{-3}m
D = 50cm = 0.5m
λ = 6000 A° = 6.0×10^{-7}m
β =?

$$\beta = \frac{\lambda D}{d} = \frac{6.0\times10^{-7}\times0.5}{0.3\times10^{-3}} = 10\times10^{-4} = 10^{-3} = \frac{1}{1000} = 0.001m$$

2) In Young 's slit experiment the distance between the centre of the interference pattern and the 10^{th} bright fringe on either side of it is 3.44cm and the distance between the slits & the screen is 2.0m. If the wavelength of light is used 5.39×10^{-7}m, then determine the slit separation.

So|^n: Given,

n = 10 (for 10^{th} bright fringe)
y_n = 3.4cm = 3.44×10^{-2}m
λ = 5.39×10^{-7}m
D = 2.0m
d =?

$$y_n = \frac{n\lambda D}{d}$$

$$\therefore d = \frac{n\lambda D}{y_n} = \frac{10\times5.39\times10^{-7}\times2}{3.44\times10^{-2}} = 3.15\times10^{-4}m$$

3) Two slits in Young's experiment are 0.02cm apart. The interference fringes for light of wavelength 600 nanometer are formed on a screen 80cm away. Calculate the distance of (i) fifth bright fringe and (ii) the 7^{th} dark fringe from the centre of the screen.

So|^n: Given,

$d = .02cm = 0.02\times10^{-2}m$

$D = 80cm = 0.80m$

$\lambda = 600$ nanometer $= 6.00\times10^{-7}m$ $[1_{nm} = 10^{-9}m]$

Here, For 5^{th} bright fringe,

$n = 5$

y_n =?

$$y_n = \frac{n\lambda D}{d}$$

$$y_5 = \frac{5\times6\times10^{-7}\times0.8}{0.02\times10^{-2}} = 1.2\times10^{-2}\,m$$

And, for 7^{th} dark fringe,

$n = 6$

y_n =?

$$y_n = \frac{(2n+1)\lambda D}{2d}$$

$$y_6 = \frac{(2\times6+1)\times6\times10^{-7}\times0.8}{2\times0.02\times10^{-2}} = 1.56\times10^{-2}\,m$$

4) In young's double slit experiment, light has a frequency of $6\times10^{14}H_Z$ (or sec^{-1}). The distance between the centers of adjacent bright fringes is 0.75mm. If the screen is 1.5m away, what is the distance between the slits.

So|^n: Given,

$f = 6\times10^{14}H_Z$

$\beta = 0.75mm = 0.75\times10^{-3}m$

$D = 1.5m$

d =?

$$c = f\lambda$$

$$\therefore \lambda = \frac{c}{f} = \frac{3\times10^{8}}{6\times10^{14}} = 5\times10^{-7}\,m$$

We have,

$$\beta = \frac{\lambda D}{d}$$

$$\therefore d = \frac{\lambda D}{\beta} = \frac{5\times10^{-7}\times1.5}{0.75\times10^{-3}} = 1.7\times10^{-3}\,m$$

5) In Young's double slit experiment, the wavelength of light used is 400nm. The screen is at a distance of 0.80m from the slits, which are 0.20mm apart. Calculate the fringe width, determine the change in fringe width, if the entire experimental arrangement is emerge in water (μ=1.33).

So|^n: Given,

$\lambda = 400\text{nm} = 4.0 \times 10^{-7}\text{m}$

$D = 0.80\text{m}$

$d = 0.20\text{mm} = 0.20 \times 10^{-3}\text{m}$

$\beta = ?$

Again for water, $\mu = 1.33$

$\beta - \beta^{|} = ?$ Where, $\beta^{|}$ = fringe width in water

Multiple Reflection: Division of Amplitude

The coherent sources are also produced by the multiple reflection of light on the two faces of a thin of transparent medium in which division of amplitude takes place.

It is to be noted that when light is incident on a thin film of a transparent media a part of it get reflected and remaining refracted or transmitted.

According to 'Fresnel' the fraction of the amplitude of light reflected is

$r = \dfrac{\mu - 1}{\mu + 1}$ (for normal incidence)

Where, μ = refractive index of the transparent medium.

For glass ($\mu = 1.5$)

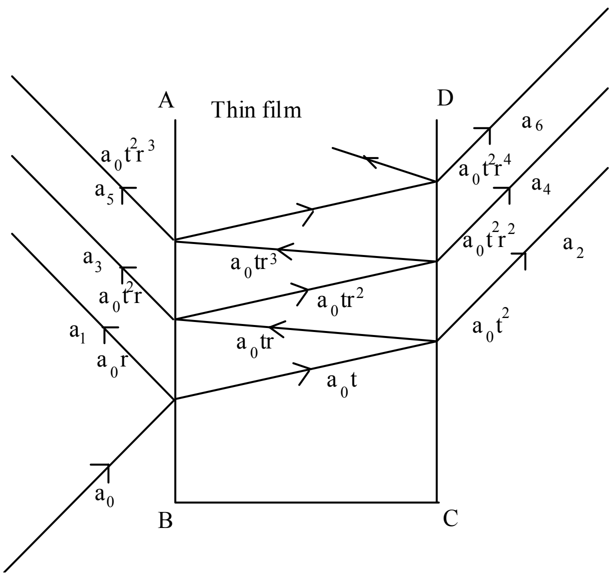
$\therefore r = 0.2$

If t be the fraction of the amplitude of light transmitted through the medium then

$t^2 + r^2 = 1$

$t = \sqrt{1 - r^2}$

$\therefore t = 0.98$ (Putting $r = 0.2$)



Let, ABCD = A thin film of transparent medium.

a_o = amplitude of the incident beam.

r = fraction of amplitude of light reflected

t = fraction of amplitude of light transmitted.

When a beam of light is incident on the phase AB of the thin film a part of which is reflected another part is transmitted to incident on CD which suffers a multiple reflection between the two faces of the film and hence the beams $a_1, a_3, a_5, \ldots\ldots\ldots$ are obtained in the reflection side and beams $a_2, a_4, a_6, \ldots\ldots\ldots$ on transmission side.

Now the amplitude of the beam in reflection sides are

$a_1 = a_o r$

$a_3 = a_o t^2 r$

$a_5 = a_o t^2 r^3$

$a_7 = a_o t^2 r^5$

Thus, the amplitudes of the beam in reflection sides are

$1 : t^2 : t^2r^2 : t^2r^4$

Hence, the first two beams are of nearly equal amplitude ($\therefore 7 \approx 1$) and the amplitudes of the rest beams are negligibly small.

Again the amplitudes of the beam in transmission side

$a_2 = a_o t^2$

$a_4 = a_o t^2 r^2$

$a_6 = a_o t^2 r^4$

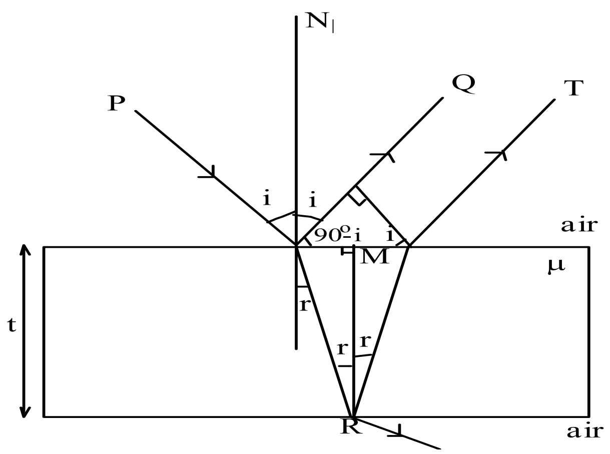
$a_8 = a_o t^2 r^6$

Hence, the beams are in the ration of

$1 : r^2 : r^4 : r^6 : \ldots\ldots\ldots$

Thus, the beams are of rapidly falling amplitudes in the transmission side.

 # Interference of light due to a thin film (for reflected light):



Let t = the thickness of a uniform thin film of a transparent medium of refractive index μ.

PO = A ray of light incident at a point O of the 1st face of the film and gets partly reflected along OQ and partly refracted along OR incident at point R of the 2nd face of the film and partly reflected along RS to incident at S of the first point of the face and finally a part of it emerges along ST.

$N_1N_1^|$ = A normal drawn at point O of first face of the film.

RM = Another normal drawn from point R to the point M of the first face of the film.

SN = Third normal drawn from point S to OQ.

$\angle PON_1 = \angle N_1OQ = i$, = angle of incidence = $\angle OSN$ (from geometry)

$\angle N_1^{|}OR = r$ = angle of refraction = $\angle ORM = \angle MRS$ (from geometry, law of ref.)

The coherent beams OQ and ST superpose on each other to produce interference pattern. The beams OQ and ST have common path up to O from P and beyond normal SN they cover equal path hence path difference between them is given by

$x = (OR + RS)_{film} - ON_{air}$

or, $x = \mu (OR + RS)_{air} - ON_{air}$

$\therefore x = \mu (OR + RS) - ON$ - - - - - (i)

From right angled triangle ORM, we have

$$Cos\, r = \frac{RM}{OR} = \frac{t}{OR}$$

$$\therefore OR = \frac{t}{Cos\, r} -----(ii)$$

From geometry, we can prove that,

ΔORM and ΔMRS are congruent

$$\therefore OR = RS = \frac{t}{Cos\, r} -----(iii) \qquad \text{[Using equations (ii)]}$$

Again, OM = MS

Thus, OS = 2OM - - - - - (iv)

$$\therefore OS = 2t\, Tan\, r$$

Now, from right angle triangle OSN, we have

$$Sin\, i = \frac{ON}{OS}$$

$$or,\ ON = OS\, Sin\, i$$

$$\therefore ON = 2t\, Tan\, r\, Sin\, i -----(v) \qquad \text{[Using equation (iv)]}$$

Thus, using equations (iii) and (v) in equation (i), we get,

$$x = \frac{2\mu t}{Cos\, r} - 2t\, Tan\, r\, Sin\, i$$

$$\therefore x = \frac{2\mu t}{Cos\, r} - 2t\frac{Sin\, r}{Cos\, r} \times \mu\, Sin\, r \qquad \left[\therefore \mu = \frac{Sin\, i}{Sin\, r}\right]$$

$$or,\ x = \frac{2\mu t}{Cos\, r}\left(1 - Sin^2 r\right)$$

$$\therefore x = 2\mu t\, Cos\, r -----(vi)$$

Which is the apparent path difference.

Since, the ray OQ is reflected from the denser medium (i.e. film) and the ray ST originated from point R is backed by rarer medium (i.e. air) hence there is a phase difference π or an equivalent path difference of $\lambda \backslash 2$ between hem. Hence, the correct path difference is given by,

$$x^{|} = 2\mu t\, Cos\, r \pm \lambda/2 \; -----(vii)$$

For constructive interference:

$$\therefore\ path\, difference = n\lambda \qquad \text{where, } n = 0, 1, 2, 3, \ldots\ldots\ldots$$

$$\therefore 2\mu t\, Cos\, r - \frac{\lambda}{2} = n\lambda$$

$$or,\ 2\mu t Cos\, r = n\lambda + \frac{\lambda}{2}$$

$$\therefore 2\mu t\, Cos\, r = (2n+1)\frac{\lambda}{2} \; -----(viii) \qquad \text{where, } n = 0, 1, 2, 3, \ldots\ldots\ldots$$

Which is the condition of constructive interference for which film appears bright.

For destructive interference:

$$path\, difference = (2n+1)\,\lambda/2 \qquad \text{where, } n = 0, 1, 2, 3, \ldots\ldots\ldots$$

$$or,\ 2\mu t\, Cos\, r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$or,\ 2\mu t\, Cos\, r = (2n+1)\frac{\lambda}{2} + \frac{\lambda}{2}$$

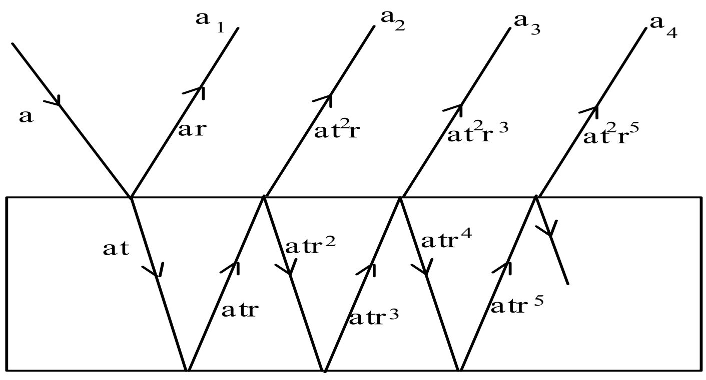
$$or,\ 2\mu t\, Cos\, r = (n+1)\lambda$$

Since, n is an integer and $(n+1)$ is also an integer. Hence, above equation may be written as,

$$2\mu t\, Cos\, r = n\lambda \qquad \text{where, } n = 0, 1, 2, 3, \ldots\ldots\ldots$$

Which is the condition for the destructive interference due to which the film appears the film appears to be dark.

Since, the amplitude of the beam OQ and ST are nearly equal. Hence, the interference pattern is not perfect and the minimum intensity is never zero. However, when we consider the multiple reflection of light between the two faces of the thin film the minimum intensity of light reduces to zero.



If r and t be the coefficient of the amplitude of reflected and transmitted light respectively due to the film, then the amplitude of the beam in the reflected side are

$a_1 = a_r$
$a_2 = at^2r$
$a_3 = at^2r^3$
$a_4 = at^2r^5$

where, a = the amplitude of the incident beam.

Now, the resultant amplitude of the beams

$a_2, a_3, a_4, a_5, \ldots\ldots\ldots$ (i.e. 2nd, 3rd, 4th, ……………. beams)

Now, the resultant beams of

i.e. $A = a_2 + a_3 + a_4 + \ldots\ldots\ldots\ldots$

or, $A = at^2r + at^2r^3 + at^2r^5 + \ldots\ldots\ldots$

or, $A = at^2r\,(1 + r^2 + r^4 + \ldots\ldots\ldots)$

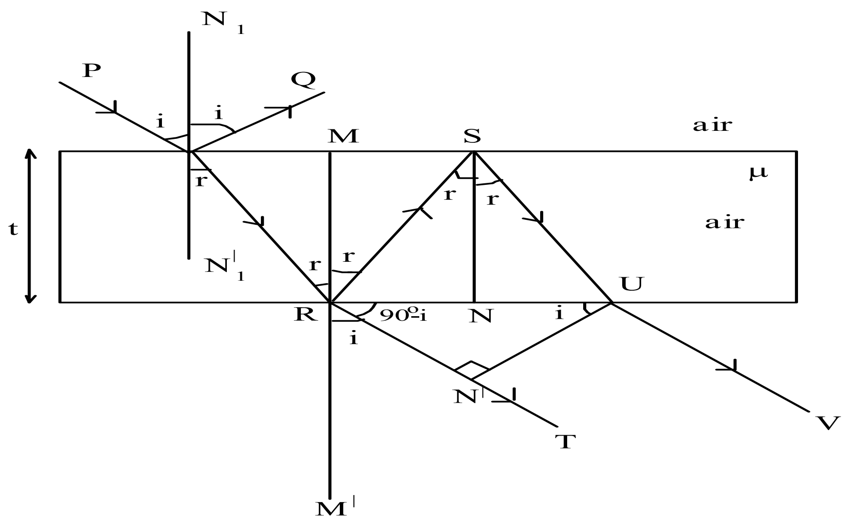
$$= at^2r \times \left(\frac{1}{1-r^2}\right)$$

or, $A = \dfrac{at^2r}{t^2}$ $\qquad [\therefore t^2 + r^2 = 1]$

$\therefore$ A = ar

Which is the amplitude of the 1st beam. Thus, the interference pattern becomes perfect dark due to multiple reflection.

*** Interference of light due to a thin film (for transmitted light):**



Numericals:

1. A parallel beam of light (λ = 5890 A°) is incident on a thin glass plate (μ=1.5) such that the angle of refraction into the plate is 60°. Calculate the smallest thickness of the glass plate which will appear dark by reflection.

So|n: Given,

λ=5890 A° = 5890×10^{-8}cm

$\mu = 1.5$

r = 60°

n = 1 [for the smallest thickness of film]

For refracted light,

$\therefore 2\mu t \, Cos\, r = n\lambda$

$$or,\ t = \frac{n\lambda}{2\mu Cos\, r} = \frac{1\times 5890\times 10^{-8}}{2\times 1.5\times Cos\, 60^\circ} = 1.059\times 10^{-8}\, cm$$

For reflected light,

$\therefore 2\mu t \, Cos\, r = n\lambda$

$$or,\ t = \frac{n\lambda}{2\mu Cos\, r} = \frac{1\times 5890\times 10^{-8}}{2\times 1.5\times Cos\, 60^\circ} = 1.059\times 10^{-8}\, cm$$

2. A soap film of refractive index 4/3 and of thickness 1.5×10^{-4}cm is illuminated by white incident at angle of 60°. The light reflected by it is examine by a spectroscope in which is found a dark band corresponding to a wavelength of 5×10^{-5}cm. Calculate the order of interference of the dark band.

So|^n: Given,

3. A soap film 5×10^{-5}cm thick is viewed at a n angle of 35° to the normal, find the wavelength of light in the visible spectrum which will be absent from the reflected light (μ=4/3)

So|^n: Given,

t = 5×10^{-5}cm = 5×10^{-7}m

i = 35°

μ = 4/3

The wavelength of light in the visible spectrum which are absent in the reflected light = ?

From Snell's law,

$$\mu = \frac{Sin\, i}{Sin\, r}$$

$$or,\ Sin\, r = \frac{Sin\, i}{\mu}$$

$$\therefore r = Sin^{-1}\frac{Sin\, i}{\mu} = Sin^{-1}\frac{Sin\, 35^\circ}{4/3} = Sin^{-1}\frac{0.573}{1.333} = Sin^{-1}0.4298 = 25.45^\circ$$

For reflected light, we have

$2\mu t\, Cos\, r = n\lambda \ -----(i)$ [for dark fringe]

Putting n =1,

$$\therefore \lambda_1 = 2\mu t \, Cos\, r = 2 \times \frac{4}{3} \times Cos\, 25.45^\circ = 2 \times 1.333 \times 0.902 = 2.405cm$$

Again, putting n = 2,

$$\therefore \lambda_2 = \frac{2.405}{2} = 1.202cm$$

4. A beam of parallel rays is incident at an angle of 30^o with the normal on a plane parallel film of thickness 4×0^{-5}cm and refractive index 1.50 so that, the reflected light whose wavelength is 7.539×10^{-5}cm will be strengthen (bright) by reinforcement.

So|^n: Given,

$t = 4\times0^{-5}$cm

$\mu = 1.50$

$i = 30^o$

To prove the film = bright for reflected light

For constructive interference, we have

$$2\mu t \, Cos\, r = N \times \lambda/2 \qquad \text{(for reflected light)}$$

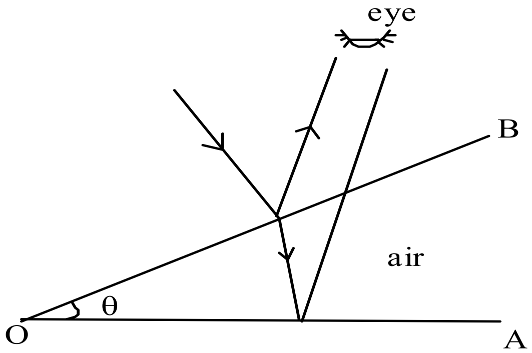
Where, N = an odd number

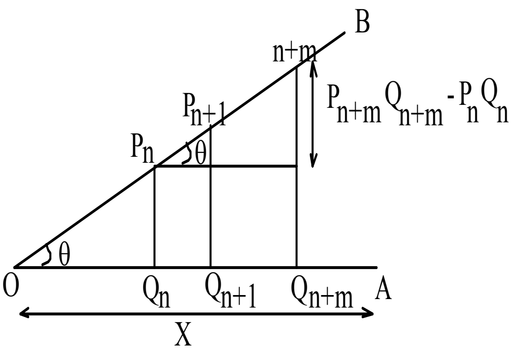
$$\mu = \frac{Sin\, i}{Sin\, r}$$

$$\therefore r = Sin^{-1}\left(\frac{Sin\, i}{\mu}\right)$$

*** Interference due to wedge shaped film:**

Two plane surfaces OA and OB inclined a an angle θ (i.e. $\angle$AOB = θ) enclose a wedge shaped film, of which thickness increases from O to A. For reflected monochromatic light, a system of equidistant interference fringes are seen which are parallel to the line of intersection of the two surfaces when air film is viewed. The effect is best observed formed small angle of incidence.





Let the points P_n, P_{n+1}, ………, P_{n+m} be the positions occupied by n_{th}, $(n+1)_{th+}$, ………, $(n+m)_{th}$ bright fringes respectively so that the corresponding thickness of the air film are P_nQ_n, $P_{n+1}Q_{n+1}$, ………, $P_{n+m}Q_{n+m}$ respectively.

Now, for bright fringe, we have

$$2\mu t\, Cos\, r = (2n+1)\frac{\lambda}{2} \qquad \text{[for reflected light]}$$

For small angle of incidence,

Cos r = 1

For air, $\mu = 1$

Thus, from above equation, we can write,

$$2t = (2n+1)\frac{\lambda}{2} -----(i)$$

For film of thickness P_nQ_n, the equation (i) becomes:

$$2P_nQ_n = (2n+1)\frac{\lambda}{2} -----(ii)$$

Similarly, for next bright fringe (i.e. $(n+1)_{th}$ fringe) at point P_{n+1}, we can write

$$2P_{n+1}Q_{n+1} = \{2(n+1)+1\}\frac{\lambda}{2} -----(iii)$$

Subtracting equation (ii) from (iii), we get

$$2(P_{n+1}Q_{n+1} - P_nQ_n) = \lambda$$

$$\therefore P_{n+1}Q_{n+1} - P_nQ_n = \frac{\lambda}{2} -----(iv)$$

Which is increase in the thickness of the air film due to two consequtive bright fringe. Thus, for m bright fringes between P_n and P_{n+m}, we can write

$$P_{n+m}Q_{n+m} - P_nQ_n = m\frac{\lambda}{2} -----(v)$$

Now, from fig,

$$\theta = \frac{P_{n+m}Q_{n+m} - P_nQ_n}{Q_nQ_{n+m}}$$

$$\therefore \theta = \frac{m\lambda}{2x} -----(vi) \qquad \text{where, } x = Q_nQ_{n+m}$$

Thus, θ can be determined. Now, the fringe width

$$\beta = \frac{x}{m} = \frac{\lambda}{2\theta} -----(vii) \qquad [\theta \text{ must be in radian}]$$

Numerical Problems:

1. Two glass plates enclose a wedge shape air film, touching at one edge and are separated by a wire of 0.05mm diameter at a distance of 5cm from the edge. Calculate the fringe width, when a monochromatic light of wavelength 6000A^o from a source falls normally on the film.

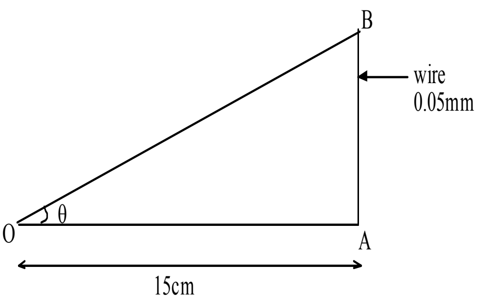
So|^n: Given,

Diameter of the wire (AB) = 0.05mm = 0.005cm

Distance OA = 15cm

$\lambda = 6000A^o = 6.0\times10^{-5}$cm

Fringe width, β = ?



From, ΔOAB

$$\theta = \frac{AB}{OA} \qquad [\therefore \theta = \text{small}, \therefore \text{Tan}\,\theta = \theta]$$

$$\therefore \theta = 3.33\times10^{-4}\ radian$$

Now, fringe width

$$\beta = \frac{\lambda}{2\theta}$$

$$\therefore \beta = \frac{6.05\times10^{-5}}{2\times3.33\times10^{-4}} = 0.09075cm$$

2. Light of wavelength 600_{nm} falls normally on a thin wedge shape film of refractive index 1.4, forming fringes that are 2mm apart. Find the angle of the wedge.

So|^n: Given,

$\lambda = 600_{nm} = 600\times10^{-9}$m = 6.0×10^{-7}m

μ = 1.4

β = 2mm = 2×10^{-3}m

θ = ?

Now, $\beta = \dfrac{\lambda}{2\theta\mu}$

$$\therefore \theta = \frac{\lambda}{2\mu\beta} = 1.07\times10^{-4}\,rad.$$

3. A glass wedge of an angle 0.01rad. is illuminated by monochromatic light of 6000 A^o falling normally on it. At what distance from the edge of the wedge, will be 10th fringe be observed by reflected light.

So|^n: Given,

4. The interference fringes are produce with monochromatic light falling normally on a wedge shaped film of cellophane whose refractive index is 1.40. The angle of the wedge is 10 seu of an arc and the distance between the successive fringes is 0.5cm. Calculate the wavelength of light sed.

So|^n: Given,

5. A square piece of cellophane film with index of refraction 1.5 has a wedge shaped section so that its thickness at two opposite sides are t_1 and t_2. If with a light of wavelength 6000A^o, the numbers of fringes appearing in the film is 10 then calculate the difference: $T_2 - T_1$.

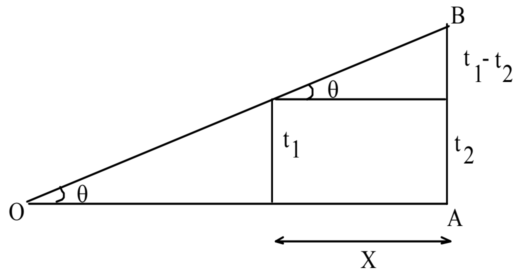
So|^n: Given,

$\mu = 1.5$

$\lambda = 6.0 \times 10^{-5}$cm

m = 10

$t_2 - t_1 = ?$



Now, $\theta = \dfrac{t_2 - t_1}{x}$ $-----(i)$

Again, x = 10β, where β = fringe width

$$\therefore \beta = \frac{x}{10} \quad -----(ii)$$

Also, $\beta = \dfrac{\lambda}{2\theta\mu}$

$$\therefore \frac{x}{10} = \frac{\lambda \times x}{2\mu \times (t_2 - t_1)}$$ [Using equation (i) and (ii)]

$$\therefore t_2 - t_1 = \frac{10\lambda}{2\mu} = \frac{10 \times 6.0 \times 10^{-5}}{2 \times 1.5} = 2 \times 10^{-4}\, cm$$

Newton's Rings:

When a plano-convex lens L of large focal length is placed on a plane glass plate P, it encloses a thin film of air between the lower surface of the land and upper surface of the glass plate of which thickness is minimum at point of contact and gradually increases from centre outwards with a monochromatic source of light alternate bright and dark rings are obtained so that the thickness of each ring is uniform but with the increase in the order of the fringe the thickness of the fringe decreases. These circular rings are called Newton's rings having centre at the point of contact of the lens and glass plate.

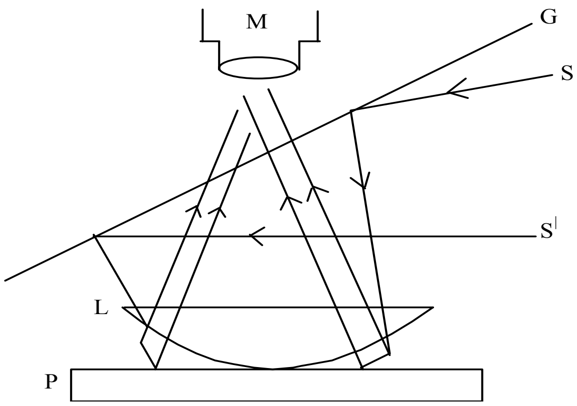


Fig-I

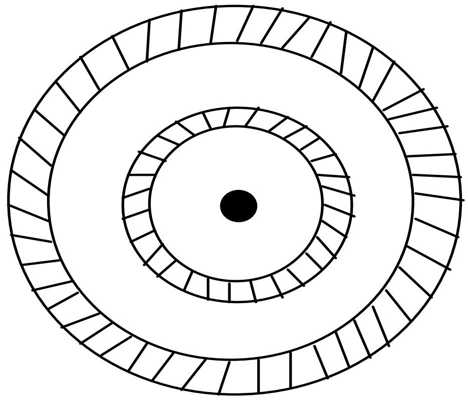


Fig-II Newton's rings for reflected rays

Experimental arrangement:

$SS^{|}$ = an extended source of monochromatic light

L = A plano-convex lens of large focal length placed on the plane glass plate P

G = Another glass plate placed at an angle of 45^o with horizontal

M = A microscope used to see the interference pattern

The light coming from $SS^{|}$ gets partly reflected on G towards the thin film enclosed between the lens L and glass plate P. The reflection of light at the lower surface of the lens and the upper surface of the plate P forms a coherent beam of light which superpose on each other to produce the interference pattern which consists of alternate bright and dark circular rings with the centre dark.

Theory:

Let, O = point of contact between lens and glass plate.

t = thickness of air film at a distant r from O.

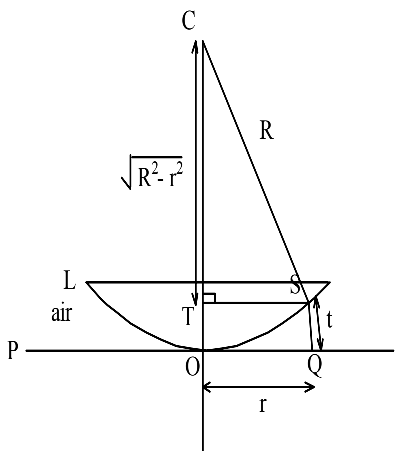
i.e. OQ = r.

R = The radius of curvature of the curve surface of the lens.

R = CO = CS

Where, C is the centre of curvature.

λ = wavelength of monochromatic light.



Now,

$$t = SQ = OT$$

$$or,\ t = CO - TC$$

$$or,\ t = R - \left(R^2 - r^2\right)^{\frac{1}{2}}$$

$$or,\ t = R - R\left(1 - \frac{r^2}{2R^2}\right)\text{[Using binomial theorem]}$$

$$or,\ t = R\left\{1 - 1 + \frac{r^2}{2R^2}\right\}$$

$$or,\ t = \frac{r^2}{2R^2}\ \ -----(i)$$

For thin film,

Path difference is given by,

$$x = 2\mu t\, Cos\, r$$

For air, $\mu = 1$

For small angle of incidence, $Cos\, r = 1$

Thus, above equation becomes

$$x = 2t$$

$$or,\ x = 2 \times \frac{r^2}{2R} \qquad \text{[Using equation (i)]}$$

$$\therefore x = \frac{r^2}{R} \quad -----(ii)$$

For dark rings:

Path difference, $x = n\lambda$ where, n =0, 1, 2, 3, ………..

$$or, \frac{r_n^{\ 2}}{R} = n\lambda$$

$$\therefore r_n = \sqrt{n\lambda R} \quad -----(iii)$$ where, n =0, 1, 2, 3, ………..

which determines the radius of n_{th} dark rings.

If D_n be the diameter of nt_h dark fringe then we can write,

$$D_n = 2r_n = 2\sqrt{n\lambda r} \quad -----(iv)$$ where, n =0, 1, 2, 3, ………..

putting n=0, we get D_n=0, i.e. centre is dark.

Similarly, for n=1, 2, 3, ... the diameter of 1st, 2nd, 3rd, ………. dark rings can be determined from above equation.

For bright rings:

$$\therefore x = (2n-1)\frac{\lambda}{2}$$ where, n =0, 1, 2, 3, ………..

$$or, \frac{r_n^{\ 2}}{R} = (2n-1)\frac{\lambda}{2}$$ [Using equation (ii]

$$\therefore r_n = \sqrt{\frac{(2n-1)\lambda R}{2}} \quad -----(v)$$

which determines the radius of n_{th} bright rings.

If D_n be the diameter of nth bright ring then, we can write,

$$D_n = 2r_n = 2\sqrt{\frac{(2n-1)\lambda R}{2}} \quad -----(vi)$$ where, n =0, 1, 2, 3, ………..

putting n=1, 4, 9 and 16 in equation (iv), we get

the diameter of 1st, 4th, 9th, 16th dark ring

i.e. $D_1 = 2\sqrt{\lambda R} \quad -----(vii)$

$$D_4 = 4\sqrt{\lambda R} \quad -----(viii)$$

$$D_9 = 6\sqrt{\lambda R}$$

$$and, D_{16} = 16\sqrt{\lambda R}$$

Thus, it is found that $D_4 - D_{16} = D_{16} - D_9 = 2\sqrt{\lambda R}$.

Thus, when the order of the fringe increases its width (thickness) decreases. That is the fringes come closer and closer to each other.

*** Determination of wavelength of monochromatic light:**

The experimental arrangement is setup to obtain the Newton's rings for reflected light on using a source of monochromatic light.

The diameter's of n_{th} and $(n+m)_{th}$ dark rings are determined by traveling microscope.

If D_n and (D_{n+m}) be the diameters of n_{th} and $(n+m)_{th}$ dark rings respectively. Then, from equation (iv), we can write,

$$D_n^{\ 2} = 4n\lambda R \ \ -----(viii)$$

and, $$(D_{n+m})^2 = 4(n+m)\lambda R \ \ -----(ix)$$

Subtracting equation (viii) from (ix), we get,

$$(D_{n+m})^2 - D_n^{\ 2} = 4m\lambda R$$

$$\therefore \lambda = \frac{(D_{n+m})^2 - D_n^{\ 2}}{4mR}$$

which determines the wavelength of monochromatic light.

(Q) What are Newtons's rings? Describe the theory of Newton's rings.

(Q) What are Newton's rings? Describe an experiment to determine the wavelength of a monochromatic light.

Newton's rings (for transmitted light):

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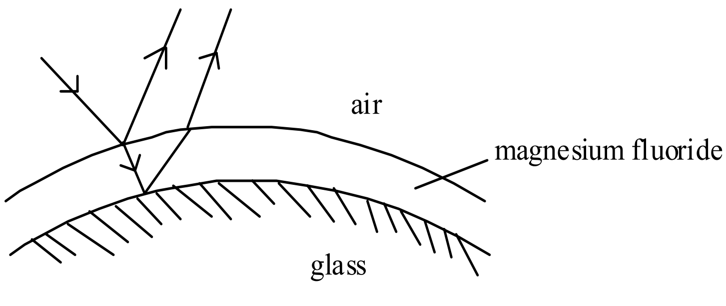
Blooming:

Whenever light is incidence on a lens or a lens system a small percentage of it is reflected from the egg surface of the lens and produced a background of unfocused light due to which the intensity and hence clarity of the final image is reduced. There is also reduction in intensity of image due to less transmission of light through the lens or lens system.



The amount of reflected light can be reduced by evaporating a thin coating of fluoride salt. For example, magnesium fluoride on the surfaces of the lens.

When light is incident a part of wavelength λ is reflected from air fluoride surfaces and the remainder penetrates the coating and partly reflected

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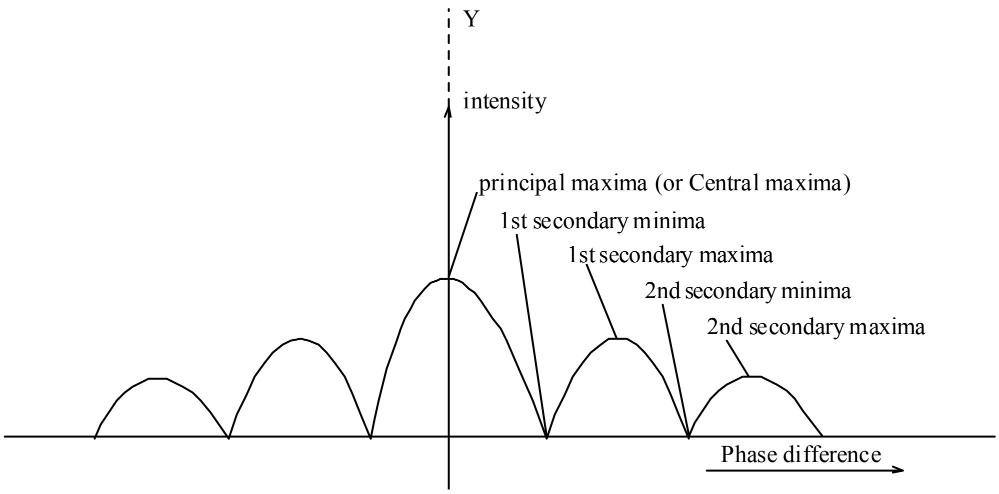
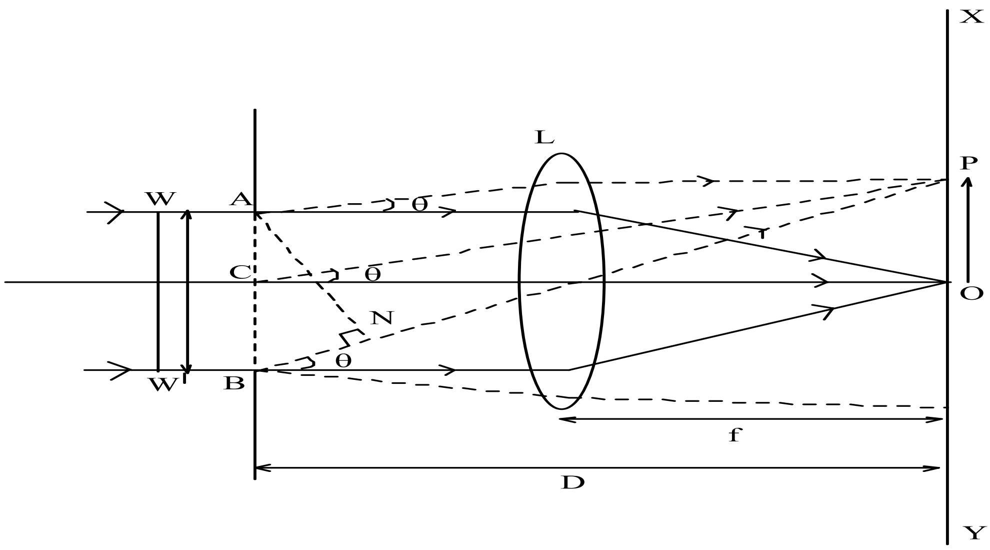
*** Diffraction of light at single slit:**



For secondary minimum:



Path difference, $BN = ABSin\theta$

Path difference = $n\lambda$ where, n =0, 1, 2, 3, ………..

$\therefore dSin\theta_n = n\lambda$

$or, Sin\theta_n = \frac{n\lambda}{d}$

$or, \theta_n = \frac{n\lambda}{d}$ [Since θ_n=Small, $\therefore$ Sin $\theta_n = \theta_n$]

Again, distance of n_{th} secondary minimum from centre,

$\theta_n = \frac{y_n}{D}$ $-----(ii)$ [where, y_n= , D=]

Thus, $\frac{n\lambda}{d} = \frac{y_n}{D}$

$\therefore y_n = \frac{n\lambda D}{d}$ [where d=slit width, n=1, 2, 3, …..]

The distance of 1st secondary minimum, from centre

$y_1 = \frac{\lambda D}{d}$, [putting n=1]

The width of principal max (or central maximum):

$= 2y_1 = \frac{2\lambda D}{d}$

Since, lens is placed very closed to the slit. Hence,

$D \approx f$

p is replaced by f in above equation.

For secondary maximum:

$dSin\,\theta_n = (2n+1)\frac{\lambda}{2}$ where, n =0, 1, 2, 3, ………..

$\theta_n = \frac{(2n+1)\lambda}{2d}$

Again, $\frac{y_n}{D} = \frac{(2n+1)\lambda}{2d}$

$y_n = \frac{(2n+1)\lambda D}{2d}$

Numericals:

1. A screen is placed 2m away from a single slit. Calculate the slit width, if the 1st diffraction minima light 5mm on either side of maxima. The incident plane wave have a wavelength of 5000 A°.

So|n: Given,

D = 2m = 200cm

n = 1 (1st minima)

y_1 = 5mm = 0.5cm

λ = 5000A° = 5.0×10^{-5}cm

d = ?

$\therefore y_n = \frac{n\lambda D}{d}$ [for minima]

$\therefore y_1 = \frac{\lambda D}{d}$

Thus, $d = \frac{\lambda D}{y_1} = \frac{5.0\times10^{-5}\times 200}{0.5} = 0.02cm$

2. A slit of width 0.15cm is illuminated by light of wavelength 5×10^{-5}cm and a diffraction pattern is obtain on a screen 2.1m away. Calculate the width of central maximum.

So|n: Given,

d = 0.15cm

$\lambda = 5\times10^{-5}$cm

D = 2.1m = 210cm

Width of central maximum = ?

Now, the distance of 1st secondary minimum from the centre is given by

$$y_1 = \frac{\lambda D}{d} \quad -----(i)$$

Thus, the width of central maximum

$$= 2y_1 = \frac{2\lambda D}{d} = \frac{2\times5\times10^{-5}\times210}{0.15} = 0.14cm$$

3. Light of wavelength 5600A° passed through a slit 0.1cm wide and forms a diffraction pattern on a screen 1.8m away. What is the width of central maximum. Also, find the width of the central maximum. When the apparatus is emerge in water (μ = 4/3).

So|n: Given,

d = 0.1cm

$\lambda = 5.6\times10^{-5}$cm

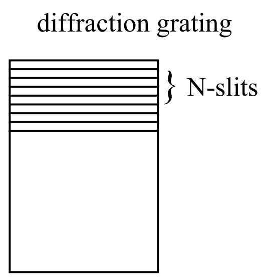
D = 1.8m = 180cm

Width of central maximum = ?

Again, width of central maximum, when the apparatus is immerge in water = ?

For water (μ =4/3)

 # Diffraction due to a grating (or diffraction grating):



If N be the number of lines per unit length then,

$$a+b = \frac{1}{N}$$

where, $a+b$ =grating element

a = width of space

b = width of a line

$$(a+b)\,Sin\,\theta_n = n\lambda$$

where, n =0, 1, 2, 3, ………..

[* Maximum value of $Sin\,\theta_n$ = 1]

Numericals:

1. Light is incident normally on a grating 0.5cm and 2500 lines. Find the angle of diffraction for the principal maximum of two sodium lines in the first order spectrum.

[$\lambda_1 = 5890A^o$ and $\lambda_2 = 5896A^o$]

So|^n: Given,

Width of the grating = 0.5cm

Number of lines = 2500

$n = 1$

$\lambda_1 = 5890A^o$

$\lambda_2 = 5896A^o$

θ_1 and $\theta_2 = ?$

Now, number of lines per unit length of he grating

$$N = \frac{2500}{0.5} cm^{-1} = 5000 cm^{-1}$$

Now, the grating element is given by,

$$a + b = \frac{1}{N} = \frac{1}{5000} cm = 2 \times 10^{-4} cm$$

Now, For wavelength λ_1:

$\therefore (a+b) Sin\theta_1 = 1 \times \lambda_1$ [$\therefore$ n = 1]

$\theta_1 =$

For λ_2:

2. A parallel beam of mono-chromatic light is allowed to incident normally on a plane grating having 1250 lines cm^{-1} and a second order spectral line is observed to be deviated through 30^o. Calculate the wavelength of the spectral line.

So|^n: Given,

N = 1250 lines cm^{-1}

n = 2

$\theta_2 = 30^o$

$\lambda = ?$

Now, $a + b = \frac{1}{N} = \frac{1}{1250} cm$ [$\lambda = 2 \times 10^{-4}$cm]

$\therefore (a+b)\, Sin\theta_n = n\lambda$

$\therefore (a+b)\, Sin\theta_2 = 2\lambda$

3. Find the highest order spectrum, which may be seen with monochromatic light os wavelength 6000A^o by means of a diffraction grating with 5000 lines per cm.

So|^n: Given,

$Sin\theta_n = 1$ [for highest order]

$\lambda = 6.0\times10^{-5}$cm

N = 5000 lines cm^{-1}

Maximum value of n = ?

4. A plane grating has $5000 lines per inch. Find the angle of spectrum of the 5048A° and 5016A° lines of helium in the 2nd order spectrum.

So|^{n}: Given,

N = 15000 lines per inch

$= \dfrac{1500}{2.54}$ lines per cm

5. A plane transmission grating have 6000 lines cm^{-1} and is used to obtain a spectrum of light from a sodium lamp in the 2nd order. Calculate the angular separation between the two sodium lines of wavelength 5890A° and 5896A°.

So|^{n}: Given,

6. How many orders will be visible if the wavelength of the incidence radiation is 5000A° and the number of lines on the grating is 2620 in one inch.

So|^{n}: Given,

A = 5000A°

= 5.0×10^{-5}cm

N = 2620 lines/inch

$= \dfrac{2620}{2.54}\ lines/inch$

$Sin\,\theta_n = 1$ (max.)

maximum value of n = ?

$$a+b = \frac{1}{N} = \frac{1}{2.54} = 0.393$$

we know,

$$(a+b)\,Sin\,\theta_n = n\lambda$$

$$or,\ n = \frac{\lambda}{a+b} = \frac{5.0\times10^{-5}}{0.393} = 12.72\times10^{-5}$$

7. A diffraction grating used at normal incidence gives a line, λ_1 = 6000A° in a certain order superpose (meet together) on another line λ_2 =4500A° of the next higher order. If the angle of refraction is 30°, howmany lines are there in a centimeter in the grating.

So|^{n}: Given,

$\lambda_1 = 6000A^o = 6.0\times10^{-5}$cm

$\lambda_2 = 4500A^o = 4.5\times10^{-5}$cm

$\theta_1 = \theta_2 = 30^o$

$n_2 = n_1 + 1$

N = ?

For 1st line,

$(a+b)Sin\theta_1 = n_1\lambda_1 \quad -----(i)$

For 2nd line

$(a+b)Sin\theta_2 = (n_1+1)\lambda_{21} \quad -----(ii)$

Dividing equation (i) by (ii), we get,

$$\frac{(a+b)Sin\theta_1}{(a+b)Sin\theta_2} = \frac{n_1\lambda_1}{(n_1+1)\lambda_2}$$

$$or,\ 1 = \frac{n_1\lambda_1}{(n_1+1)\lambda_2}$$

$$or,\ (n_1+1)\lambda_2 = n_1\lambda_1$$

$$or,\ \frac{\lambda_2}{\lambda_1} = \frac{n_1}{(n_1+1)}$$

$$or,\ \frac{6.0\times10^{-5}}{4.5\times10^{-5}} = \frac{n_1}{(n_1+1)}$$

$$or,\ 1.333 = \frac{n_1}{(n_1+1)}$$

$$or,\ (n_1+1) = \frac{n_1}{1.333}$$

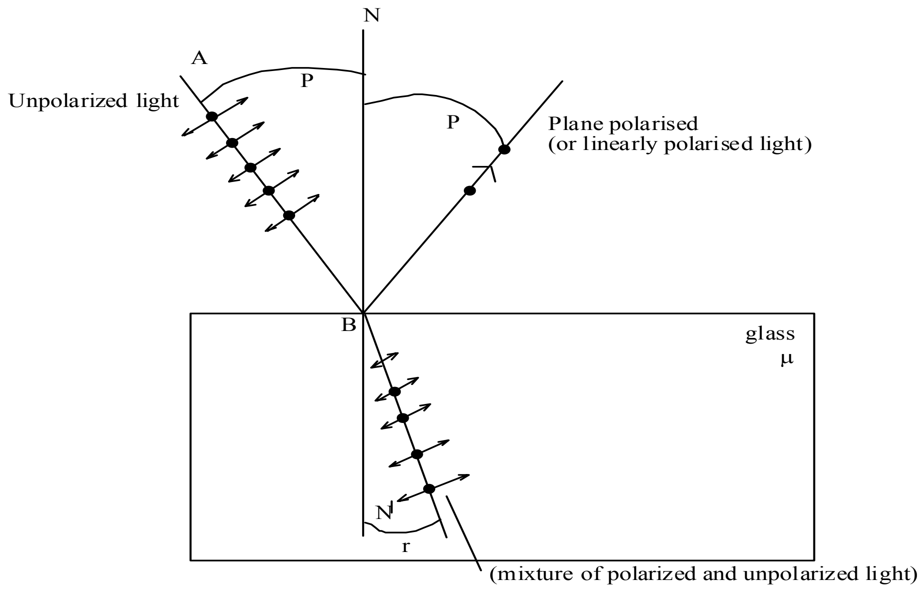
Putting the value of n_1 in equation (i),

$a + b = ?$ (cm)

Now, number of lines per cm,

$$N = \frac{1}{a+b} = ...$$

**** Polarization of light:**



P = angle of incidence for which reflected light is completely plane polarized
= angle of polarization or polarization angle
r = abgle of refraction

Brewster's law

$\mu = Tan\,P$

From Snell's law

$\mu = \dfrac{Sin\,P}{Sin\,r}$

$\therefore Tan\,P = \dfrac{Sin\,P}{Sin\,r}$

$or,\ \dfrac{Sin\,P}{Cos\,P} = \dfrac{Sin\,P}{Sin\,r}$

$or,\ Cos\,P = Sin\,r$

$or,\ Sin(90^o\text{-}P) = r$

$P + r = 90^o$

Thus, he reflected and refracted rays are at right angles to each other, when polarization takes place.

$\mu = \dfrac{1}{Sin\,c}$ [c = Critical angle]

Numericals:

1. Refractive index of water is 1.33. calculate the angle of polarization for light reflected from the surface of a lake.

So|^n: Given,

$\mu = 1.33$

P = ?

$\therefore \mu = Tan\,P$

$or,\ 1.33 = Tan\,P$

$\therefore P = Tan^{-1}(1.33) = 53.06^o$

2. A ray of light strikes a glass plate at an angle of incidence 60º. If the reflected and refracted rays are perpendicular to each other find the refractive index of the glass..

So|ⁿ: Given,

P = 60º

μ = ?

$\therefore r$ = 90º-P = 90º-60º = 30º

From Snell's law,

$$\mu = \frac{Sin\,P}{Sin\,r} = \frac{Sin\,60^o}{\text{Sin}\,30^o} = \frac{0.866}{0.5} = 1.732$$

3. A ray of light is incident on the surface of a glass plate of refractive index 1.557 at the polarizing angle. Calculate the angle of refraction.

So|ⁿ: Given,

μ = 1.557

r = ?

$\therefore \mu = Tan\,P$

$\therefore P = Tan^{-}(\mu) = Tan^{-1}(1.557) = 57.28^o$

Again, $P + r = 90^o$

$or,\ r$ = 90º-P = 90º-57.28º

$\therefore r = 32.72^o$

4. The polarizing angle for a medium is 60º. What will be the critical angle?

So|ⁿ: Given,

P = 60º

μ = Tan60º = 1.732

critical angle (c) = ?

$\therefore$ for critical angle,

$$\mu = \frac{1}{Sin\,c}$$

$$or,\ 1.732 = \frac{1}{Sin\,c}$$

$or,\ Sin\,c \times 1.732 = 1$

$$Sin\, c = \frac{1}{1.732} = 0.57736$$

$$c = Sin^{-1}(0.55736)$$

$\therefore$ Critical angle (c) = 35.2655º

"Group-C"

Electro-static Induction:

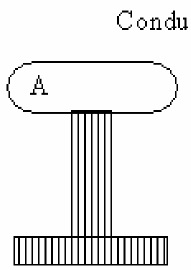


Fig. -I

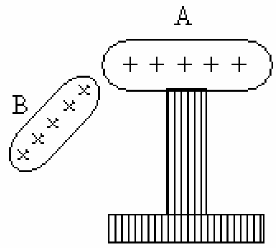


Fig. -II

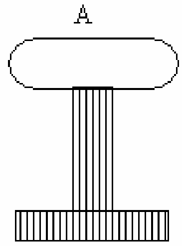


Fig. -III

The phenomenon in which a conductor gets temporarily charged in the presence of charged body is called electrostatic induction.

The charges produced on the conductor are called induced charges. There are two types of induced charges:

i. Induced bound charges
ii. Induced free charges

i. Induced bound charges: The charges at the end of the conductor closer to the charge body are called induced bound charges.
ii. Induced free charges: The charges at the end of the conductor away from the charge body are called induced free charges.

Inducing Charges:

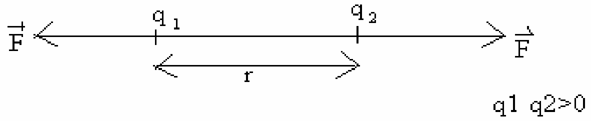
The charges responsible for electrostatic induction are called inducing charges.

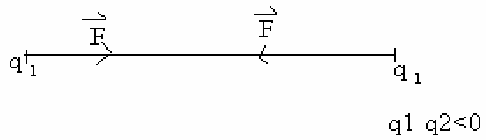
Experimentally it is found that,

i. Induced bound charge = Induced free charges
ii. Induced free (or bound) charges = Inducing charges

Coulomb's law:-

It states that the force of attraction or repulsion between two charges is directly proportional to the product of two charges and inversely proportional to the square of distance between them.





i.e. $F \alpha \frac{q_1 q_2}{r^2}$

$\alpha \frac{1}{d^2}$

where, F = magnitude of the force acting between two charge q_1 and q_2.

r = Separation between the charges = d

Thus,

$$F \alpha \frac{q_1 q_2}{r^2}$$

$$\therefore F = K\frac{q_1 q_2}{r^2} \quad -----(i)$$

where, K = Constant of proportionality

which depends upon the system of unit in which force, charge and distance are measure, and also on the nature of the medium in which charges are placed.

In S.I. unit for air,

$$K = \frac{1}{4\pi\varepsilon_o}\cdot\frac{q_1 q_2}{r^2} \quad -----(ii)$$

In C.G.S. system,

$K = 1$ for air

$$\therefore F = \frac{q_1 q_2}{r^2} \quad -----(iii)$$

Unit of charge

(i) S.I. unit of charge is C (columb)

$$\therefore F = \frac{1}{4\pi\varepsilon_o}\cdot\frac{q_1 q_2}{r^2}$$

If , $q_1 = q_2 = q$ (say)

$F = 9\times10^9$N

r = 1m

then,

$9\times10^9=9\times10^9\times q^2/r^2$

$\therefore q = \pm 1C$

Thus, 1 Coulomb is the charge which would repel and equal and similar charge by a force of 9×10^9N when placed 1m apart in air or vacuum.

(ii) The C.G.S. unit of charge is stat coulomb or electrostatic unit (esu) of charge.

$$\therefore F = \frac{q_1 q_2}{r^2}$$

if $q_1 = q_2 = q(say)$

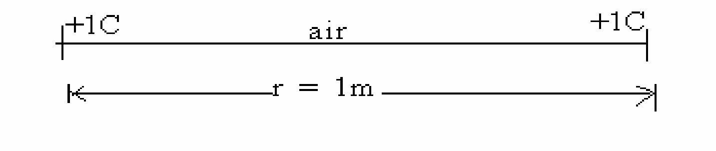
F = 1 dyne

r = 1cm

then, $q = \pm 1 stat - c.$

Relation between coulomb and stat-coulomb:

If two charges each of +1C be separated by a distance of 1m in air then force acting between them is 9×10^9N.



Let, 1C = n stat-c.

1N = 10^5 dyne

and, $I_m = 10^2$cm.

$\therefore F = \frac{q_1 q_2}{r^2}$ in C.G.S. system

$$\therefore 9\times10^9\times10^5 = \frac{n\times n}{\left(10^2\right)^2}$$

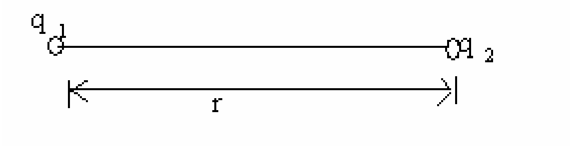
$$or,\ n^2 = 9\times10^{14}\times10^4$$

$$\therefore n = 3\times10^5$$

$\therefore 1C = 3\times10^9$ stat-C.

Relative permittivity (or Dielectric constant (k)):

The relative permittivity (or dielectric constant) of a medium may be defined as the ratio of the permittivity of that medium to the permittivity of air or vacuum.



i.e. $E_r\left(or\,K\right) = \frac{E}{E_o}$ $-----(i)$

where, E = permittivity of the medium

Let us consider that the charge q_1, q_2 are placed at a distance of r in air vacuum. Thsu, from coulomb's law the magnitude of force acting between them is given by

$$F = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2} \quad -----(ii)$$

If the same charges are placed at same distance in a medium of permittivity epsilon(ε). Then, the force acting between them is given by,

$$F^{|} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}$$

$\therefore F^{|} = \frac{1}{4\pi\varepsilon_o\varepsilon_r}\frac{q_1 q_2}{r^2}$ $-----(iii)$ [Using equation (i) alos putting $\varepsilon = \varepsilon_o$K]

Dividing (ii) by (iii)

$$\frac{F}{F^{|}} = E_r \quad -----(iv)$$

Thus, the relative permittivity or dielectric constant of a medium may also be defined as the ratio of the force acting between two given charge separated by given distance in air or vacuum to the force acting between same charges separated by same distance in that medium.

* $E_{air} = 1.005\ E_o$

$\therefore E_{air} \approx E_o$

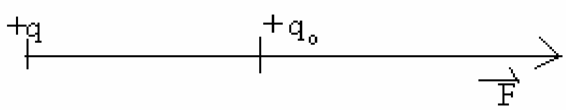
Electric Field

The space around a charge in which it can exert force in another charge is called electric field. It is a vector quantity.

Electric Intensity:

Intensity of electric field or strength of the electric field.

The electric intensity at a point in an electric field may be definite as the force experienced per unit +ve test charge at that point. It is denoted by E and is a vector quantity.



If F be the magnitude of the force experienced by a +ve test charge $+q_o$ placed at a point P in the electric field.

Due to charge +q at O, then the magnitude of electric intensity at point P is given by

$$E = \frac{E}{q_o}$$

In vector notation,

$$\vec{E} = \frac{\vec{F}}{q_o}$$

* **Units of Electric Intensity:**

(i) The Si unit is NC^{-1}. (or volt/meter)

(ii) The C.G.S. unit is dyne/stat-C.

Expression for Electric Intensity at a point due to a point charge

Let, +q = a + ve point charge placed at O in air.

P = a point in the electric field due to +q charge where electric intensity is to be determined.

r = OP

$+q_o$ = a +ve test charge placed at point P.

If F be the magnitude of the force acting on +ve test charge $+q_o$ at point P then, we can write,

$$F = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2} \quad -----(i) \quad \text{[Using Coulomb's law]}$$

where, ε_o = permittivity of air or vacuum

If E be the magnitude of the electric intensity at point P, then we can write,

$$E = \frac{F}{q_o} \quad -----(ii) \quad \text{[from definition]}$$

Thus, from equation (i) and (ii), we get,

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} \quad -----(iii) \quad [\text{along } \overrightarrow{OP}]$$

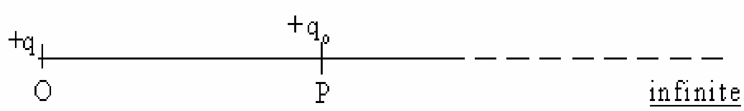
which is the required expression form the electric intensity at a point due to a point charge.

It is clear from above equation that for a given point charge

$$E \alpha \frac{1}{r^2}$$

*** Electric potential:**

The electric potential at a point in an electric field may be defined as the amount work done in bringing (without acceleration) a unit +ve test charge from infinity to that point. It is denoted by V and is a scalar quantity.



If W be the amount of work done in moving a +ve test charge $+q_o$ from infinity to a point P in the electric field due to charge +q at O then, the electric potential at point P is given by,

$$V = \frac{W}{q_o}$$

Units of Electric potential:

(i) The S.I. unit of electric potential = volt

$\therefore$ Electric potential = W/q_o

$\therefore$ Volt (V) = J/C

Thus, $1V = \dfrac{1J}{1C}$

Volt: The electric potential at a point is said to be one volt if 1 Joule of work is done in moving (without acceleration) +1 Coulomb of charge from infinity to that point in an electric field.

(ii) C.G.S. unit of electric potential is stat-V (or electro static unit (esu) of potential)

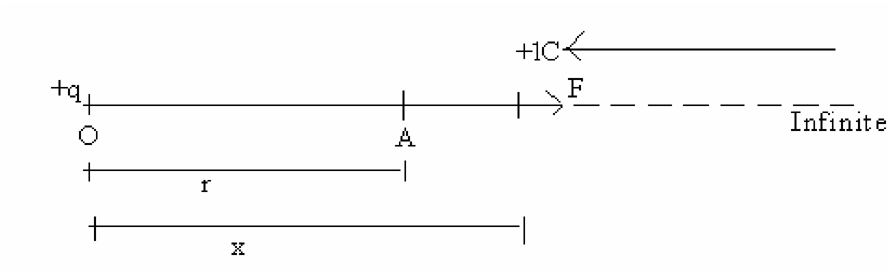
$$1 Stat - V = \frac{1 Erg}{1 stat - C}$$

Electric potential at a point due to a point charge:

Let, +q = a +ve point charge placed at O in air

A = a point in electric field due to +q charge, where electric potential is to be determined

r = OA



When a unit +ve test charge i.e. +1C is moved (without acceleration) from infinity to point A, at any time it reaches the point P so that OP = x. Thus, from coloumbs law, the magnitude of the force acting on +1C of charge is given by,

$$F = \frac{1}{4\pi\varepsilon_o}\frac{q\times 1}{r^2} \quad -----(i) \quad [\text{along } \overrightarrow{OP}]$$

where, ε_o = permittivity of air or vacuum

If dx be the infinitesimally small displacement of +1C of charge from P to Q then work done is given by,

$$dw = -Fdx \quad -----(ii)$$

the +ve sign shows that the work is done against the force.

Thus, the total work done in moving +1C of charge from infinity to A is given by,

$$W = \int_\infty^r dw = \int_\infty^r - Fdx \qquad \text{[From equation (ii)]}$$

$$or, \quad W = \frac{-q}{4\pi\varepsilon_o}\int_\infty^r \frac{1}{x^2} \qquad \text{[Using equation (i)]}$$

$$or, \quad W = -\frac{q}{4\pi\varepsilon_o}\int_\infty^r \frac{1}{x^2}dx$$

$$or, \quad W = -\frac{q}{4\pi\varepsilon_o}\left[-\frac{1}{x}\right]_\infty^r$$

$$or, \quad W = +\frac{q}{4\pi\varepsilon_o}\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r} \quad -----(iii) \qquad \left(\therefore \frac{1}{\infty} = 0\right)$$

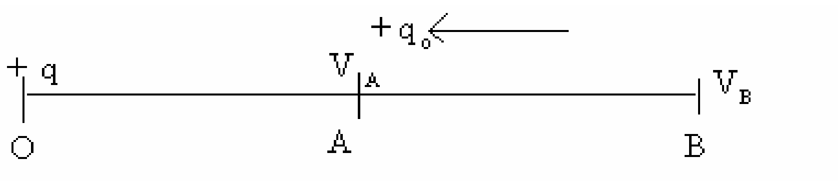
which is the required expression for electric potential at a point due to a point charge.

It is clear from above equation that for a given point charge,

$$V \alpha \frac{1}{r}$$

* **Electric potential difference:**

The electric potential difference between two points in an electric field may be defined as amount of work done in moving a unit +ve test charge from one point to another point.

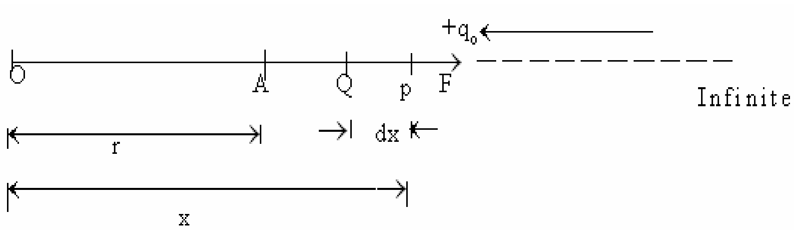


If W be the amount of work done in moving (without acceleration) a +ve test charge $+q_o$ from point B to point A in an electric field due to charge +q at O then electric potential difference between them is given by,

$$V_A - V_B = \frac{W}{q_o}$$

where, V_A and V_B = electric potential at point A and b respectively.

Expression for electric potential difference due to a point charge:



Let, +q = A + ve point charge placed at point O in air

A and B = two points in the electric field due to point charge +q, between which

electric potential difference is to be determined.

r = OA

R = OB (distance of B from O)

V_A and V_B = electric potential at point A and B respectively.

When a unit +ve test charge i.e. +1C is brought (without acceleration) from B to point A, at any time it reaches the point P so that OP = x. Thus, from Coulomb's law the magnitude of the force acting on +1C of charge is given by,

$$F = \frac{1}{4\pi\varepsilon_o}\frac{q}{x^2} \quad -----(i) \quad [\text{along } \overrightarrow{OP}]$$

where, ε_o = permittivity of air or vacuum.

If dx be the infinite small small displacement of 1C of charge from P to Q the work done is given by,

$$dw = -Fdx \quad -----(ii)$$

The –ve sign shows that the work is done against the force. Thus, total work done in moving 1C of charge from B to A is given by,

$$W = \int_R^r dw = \int_R^r -Fdx \quad \text{[From equation (ii)]}$$

$$or, \quad W = -\frac{q}{4\pi\varepsilon_o}\int_R^r \frac{1}{x^2}dx \qquad \text{[Using equation (i)]}$$

$$or, \quad W = -\frac{q}{4\pi\varepsilon_o}\left[-\frac{1}{x}\right]_R^r$$

$$\therefore \quad W = +\frac{q}{4\pi\varepsilon_o}\left[\frac{1}{r}-\frac{1}{R}\right] \quad -----(iii)$$

From definition,

$$W = V_A - V_B \quad -----(iv)$$

Thus, from equation (iii) and (iv), we get,

$$V_A - V_B = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r}-\frac{1}{R}\right) \quad -----(v)$$

which is the required expression for the electric potential difference due to a point charge.

Special Cases:

If point B is at infinity, then R = ∞ and $V_B = 0$.

Thus, equation (v) reduces to

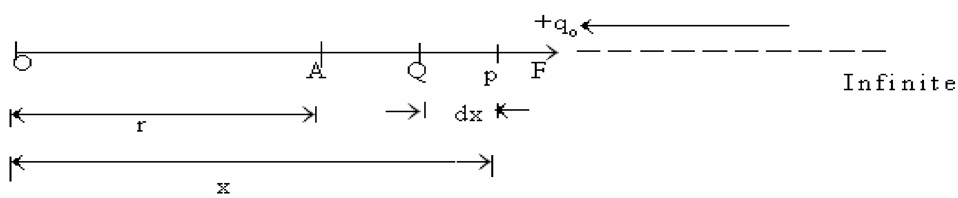
$$V_A = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$$

which is the electric potential at point A due to point charge +q.

*** Electric potential energy:**

The electric potential energy of a charge at a point in an electric field may be defined as the amount of work done in moving (without acceleration) that charge from infinity to that point.

*** Expression for electric potential energy due to a point charge:**



Let +q = a +ve point charge placed at a point O in air

A = a point in the electric field due to charge +q, where electric potential energy of a +ve test charge $+q_o$ is to be determined.

r = OA (distance of A from O)

When +ve test charge $+q_o$ is removed (without acceleration) from infinity to point A, at any time it reaches the point P so that OP = x.

Thus, from Coulomb's law, the magnitude of the force experienced by $+q_o$ charge at P is given by,

$$F = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{x^2} \quad -----(i) \qquad [\text{along } \overrightarrow{OP}]$$

If dx be the infinitesimally small displacement of $+q_o$ charge from P to Q then work done is given by,

$$dw = -Fdx \quad -----(ii)$$

Thus, the total work done in moving $+q_o$ charge from infinity to A is given by,

$$W = +\frac{qq_o}{4\pi\varepsilon_o}\left[\frac{1}{r}-\frac{1}{\infty}\right]$$

$$\therefore U = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{r} \quad -----(iii) \qquad \left[\therefore \frac{1}{\infty}=0\right]$$

which is the required expression from the electric potential energy due to a point charge.

If V be the electric potential at point A, then we can write,

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r} \quad -----(iv)$$

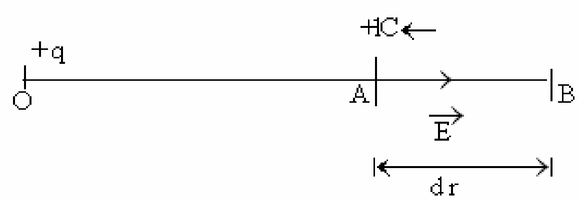
Thus, equation (iii) becomes $U = V \times q_o \quad -----(v)$

Unit of potential difference = Volt (S.I. Unit)

1 Vlot:

The electric potential difference between two points in an electric field is said to be 1V if 1J of work done in moving one coulomb charge from one point to another point.

*** Relation between electric intensity and electric potential:**



Let, +q = a positive point charge placed at point O in air.

A and B = two points very close to each other in the electric field due to charge +q

dr = separation between points A and B

dv = potential difference between the points A and B

Since, the points A and B are very close to each other, hence, the electric intensity (or electric field) between them is suppose to be uniform. From the definition of electric intensity we can write,

E = F ----- (i) when q_o = 1C [$\therefore$ E = F/q_o]

Where, E = magnitude of the electric intensity between A and B in the direction $\overrightarrow{OA}$.

Now, the amount of work done in moving unit +ve test charge i.e. +1C from B to A is given by

$$dW = -Fdr$$
$$or, \quad dd = -Edr$$
$$\therefore \quad E = -\frac{dv}{dr} \quad -----(ii)$$

which is required relation.

The quantity dv/dr is known as potential gradient i.e. the rate of change of electric potential with respect to distance is called potential gradient.

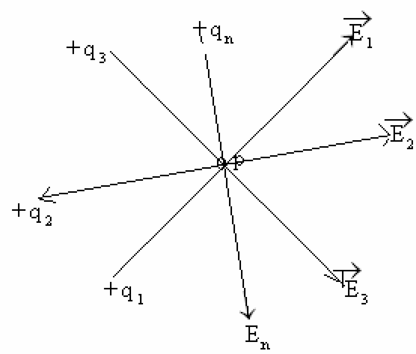
From above equation it is clear that, the electric intensity at a point in an electric field is the negative rate of charge of potential with respect to distance. The –ve sign shows that the direction of electric field (or electric intensity0 is the direction in which electric potential decreases with distance.

Note: E = dv/dr [neglecting –ve sign]

$\therefore$ E = volt/meter

$\therefore$ SI unit of electric intensity = Vm^{-1}

*** Electric Intensity at a point various charges (or principle of superposition of electric fields):**



The electric intensity at a point due to various charges is the vector sum of the electric fields at that point due to them.

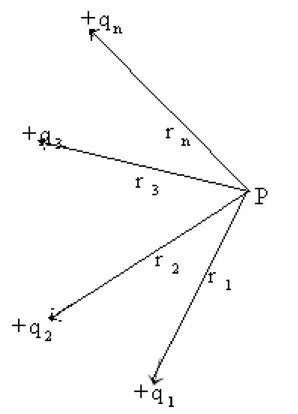
i.e. $\vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3} + ... + \vec{E_n}$

where, $\vec{E_1}, \vec{E_2}, \vec{E_3}, ...$ are the electric field at a point due to charges q_1, q_2, q_3, …… respectively.

This is known as the principle of superposition of electric fields.

It is to be noted that the electric field or electric intensity at a point due to a charge does not depend upon the presence of other charges.

*** Electric potential at a point due to various charges:**



The electric potential at a point due to various charges is the algebraic sum of the electric potential at that point due to them.

If V_1, V_2, V_3, …… be the electric potential at a point P due to charges q_1, q_2, q_3, …….. respectively then electric potential at point P due to various charges is give by

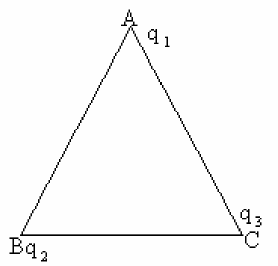
$$V = V_1 + V_2 + V_3 + ...$$

$$or, \quad V = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + ... + \frac{q_n}{r_n}\right)$$

where, r_1, r_2, r_3, ………, r_n = distances of q_1, q_2, q_3, ……… From point P respectively.

$$\therefore V = \frac{1}{4\pi\varepsilon_o}\sum_{i=1}^{n}\frac{q_i}{r_i}$$

The electric potential energy of the system of three charges placed at the corners of a triangle



The total electric potential energy of the system of charges is given by,

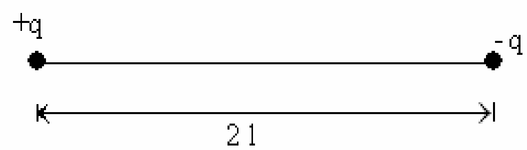
$$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1q_2}{AB} + \frac{q_2q_3}{BC} + \frac{q_1q_3}{AC}\right)$$

Remember:

(i) $\frac{1}{4\pi\varepsilon_o} = 9\times10^9\, Nm^2C^{-2}$

(ii) put the –ve sign of the charge (if given)

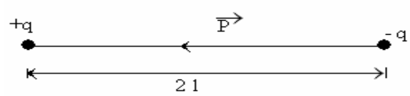
*** Electric dipole:**



When two equal and opposite charges are placed close to each other, they constitute an electric dipole. The separation between the two charges is known as the length of dipole and is denoted by 2ℓ.

*** Electric dipole moment:**

It is defined as the product as the product of the either charge and the separation between the charges.



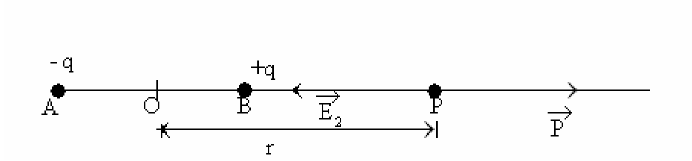
i.e. P = q ×2 ℓ

where, P = magnitude of electric dipole moment

q = charges

It is a vector quantity and its direction is from –ve charge to +ve charge i.e. opposite to the direction of electric field.

*** Electric field at a point on the axial line of an electric dipole:**



Let, AB = an electric dipole consisting of charges of magnitude q and has its length 2 ℓ

P = a point on the axis of the dipole where electric field is to be determined.

r = OP, distance of point P from the centre O of the dipole.

Now, the magnitude of electric field at point due to charge at B is given by,

$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{q}{BP^2} = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r-\ell)^2} \quad -----(i) \qquad [\text{along } \overrightarrow{OP}]$$

[$\therefore$ BP = OP – OB = r - ℓ]

similarly, the magnitude of electric field at point P due to charge at A is given by,

$$E_2 = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r+\ell)^2} \quad -----(ii) \qquad [\text{along } \overrightarrow{PO}]$$

Thus, the resultant electric field at point P due to the electric dipole at

$$E = E_1 - E_2 \qquad \left[\therefore \left|\overrightarrow{E_1}\right| > \left|\overrightarrow{E_2}\right|\right]$$

$$or, \quad E = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{(r-\ell)^2} - \frac{1}{(r+\ell)^2}\right] \quad \text{[Using equation (i) and (ii)]}$$

$$or, \quad E = \frac{q}{4\pi\varepsilon_o}\left[\frac{(r+\ell)^2 - (r-\ell)^2}{(r-\ell)^2(r+\ell)^2}\right]$$

$$or, \quad E = \frac{q}{4\pi\varepsilon_o}\left[\frac{4r\ell}{(r^2-\ell^2)}\right]$$

$$or, \quad E = \frac{1}{4\pi\varepsilon_o}\frac{2\times q\times 2\ell\times r}{(r^2-\ell^2)^2}$$

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_o}\frac{2\Pr}{(r^2-\ell^2)^2} \quad -----(iii) \qquad [\text{along } \overrightarrow{OP}]$$

$\therefore P = q \times 2\ell$, The electric dipole moment

which is the required expression.

For short dipole:

$\ell << r$

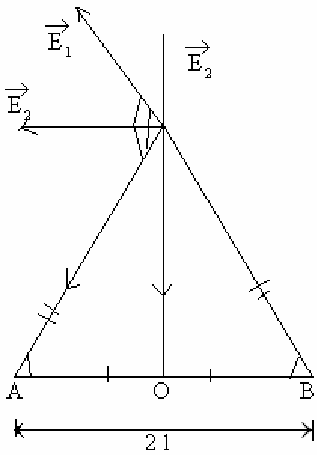
$\therefore r^2 - \ell^2 \approx r^2$

Thus, equation (iii) becomes,

$$\therefore E = \frac{1}{4\pi\varepsilon_o}\frac{2P}{r^3}$$

$$\therefore E \alpha \frac{1}{r^3}$$

*** Electric field at a point on the equatorial line of an electric dipole:**



Let, AB = an electric dipole consisting of charges +q and –q and has its length 2 ℓ

P = a point on the equatorial line (i.e. perp. drawn at the mid point of the dipole O)

Where, electric field is to be determined.

r = OP, distance of point P from the centre of the dipole

Now, the magnitude of the electric field at point P due to charge at B is given by,

$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{q}{BP^2} = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r^2+\ell^2)} \quad -----(i)$$

[along $\overrightarrow{BP}$]

[$\therefore BP^2 = OP^2 + OB^2$]

Similarly, magnitude of electric field at point P due to charge at point A is given by

$$E_2 = \frac{1}{4\pi\varepsilon_o}\frac{q}{AP^2} = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r^2+\ell^2)} \quad -----(ii) \quad [\text{along } \overrightarrow{PA}]$$

[$\therefore AP^2 = OP^2 + OA^2$]

Thus, it is clear from the above equation is that

$$\left|\overrightarrow{E_1}\right| = \left|\overrightarrow{E_2}\right|$$

Thus, the resultant electric field at point is parallel to BA, the axis of dipole.

The vertical components of $\overrightarrow{E_1}$ and $\overrightarrow{E_2}$ cancel each other and the resultant electric field at point P is due to the horizontal components of E_1 and E_2. Thus, the magnitude of the resultant electric field at point P is given by,

$$E = E_1 Cos\,\theta + E_2 Cos\,\theta$$

$$or, \quad E = 2E_1 Cos\,\theta \quad (\therefore E_1 = E_2)$$

$$or, \quad E = 2\times\frac{1}{4\pi\varepsilon_o}\times\frac{q}{\left(r^2+\ell^2\right)}\times\frac{\ell}{\sqrt{r^2+\ell^2}} \qquad \left[from\ \ \Delta OPB \quad PPB\theta = \frac{OB}{BP}\right]$$

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_o}\frac{P}{\left(r^2+\ell^2\right)^{3/2}} \quad -----(iii)$$

where, P = q × 2 ℓ , electric dipole moment.

Which is the required expression.

For short dipole:

$r >> \ell$

$\therefore r^2 + \ell^2 \approx r^2$

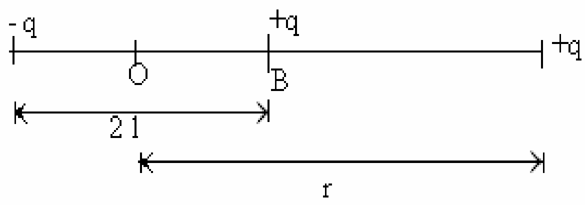
Thus, equation (iii) becomes,

$$E = \frac{1}{4\pi\varepsilon_o}\frac{P}{r^3} \quad -----(iv) \qquad \left(\therefore E\alpha\frac{1}{r^3}\right)$$

Note:

$$\frac{E_{axial}}{E_{equ.}} = 2 \qquad \text{[where, r = constant and dipole is same]}$$

***Electric potential at a point on the axial line of an electric dipole:**



Let, AB =

P = a point on the axial line of the dipole where electric potential is to be determined.

r = OP, distance of point P from the centre O of the dipole.

Now, the electric potential at a point P due to charge at B is given by,

$$V_1 = \frac{1}{4\pi\varepsilon_o}\frac{q}{BP} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r-\ell} \quad -----(i)$$

[$\therefore$ BP = OP – OB = r - ℓ]

Similarly, electric potential at point P due to charge at A is given by,

$$V_2 = \frac{1}{4\pi\varepsilon_o}\frac{q}{AP} = \frac{1}{4\pi\varepsilon_o}\frac{(-q)}{(r+\ell)} \quad -----(ii)$$

[$\therefore$ AP = OP + OA = r + ℓ]

Thus, net electric potential at point P due to the dipole is

$V = V_1 + V_2$

$$or, \quad V = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{r-\ell} - \frac{1}{r+\ell}\right] \qquad \text{[Using equation (i) or (ii)]}$$

$$or, \quad V = \frac{q}{4\pi\varepsilon_o}\left[\frac{r+\ell-r+\ell}{\left(r^2-\ell^2\right)}\right]$$

$$or, \quad V = \frac{q}{4\pi\varepsilon_o} \times \frac{2\ell}{\left(r^2-\ell^2\right)}$$

$$\therefore V = \frac{1}{4\pi\varepsilon_o}\frac{P}{\left(r^2-\ell^2\right)} \quad -----(iii)$$

where, P = q × 2 ℓ = Electric dipole moment

which is the required equation

For short dipole:

$r >> \ell$

$r^2 - \ell^2 \approx r^2$

Thus, equation (iii) becomes,

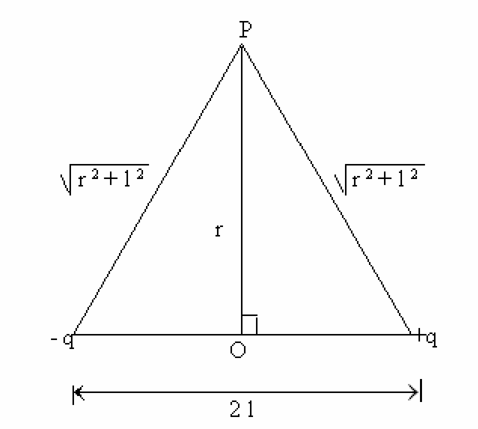
$$V = \frac{1}{4\pi\varepsilon_o}\frac{P}{r^2} \quad -----(iv) \qquad \left[\therefore V \alpha \frac{1}{r^2}\right]$$

*** Electric potential at a point on equatorial line of an electric dipole:**

$\therefore V = 0$

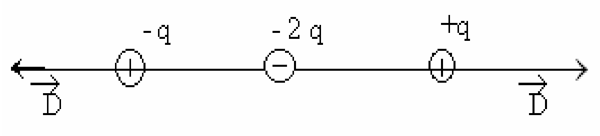
$$V_1 = \frac{1}{4\pi\varepsilon_o}\frac{q}{\sqrt{r^2+\ell^2}}$$

$$V_2 = \frac{1}{4\pi\varepsilon_o}\frac{(-q)}{\sqrt{r^2+\ell^2}}$$

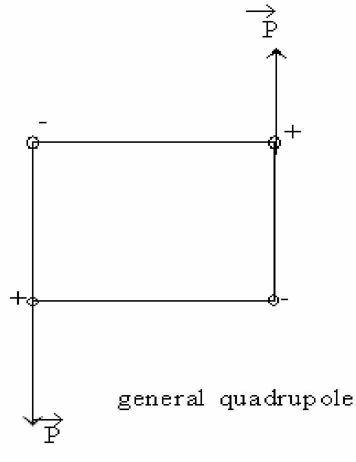


Quadrupole:

A quadrupole is a pair of two identical dipoles having their dipoles in opposite direction and separated by an infinitesimally small distance. Thus, a quadrupole is a system of point charges.



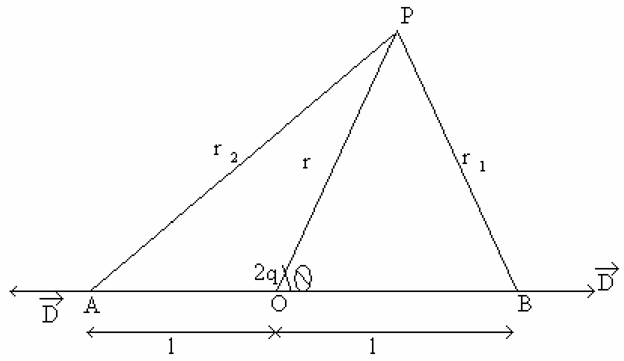
Linear quadrupole



general quadrupole

Electric potential and field due to a quardpole:

Let us consider a linear quardrupole consisting of two dipoles OA and OB having their –ve charges coincident at O and +ve charges separated from O by ℓ.



Let us consider a point P, distance r in a direction making an angle θ with the axis of quardrupole r is very very large in comparison to ℓ. (i.e. r >> ℓ).

Now, electric potential at point P is the sum of potential due to the charges thus, we can write,

$$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{q}{BP} + \frac{q}{AP} - \frac{2q}{OP}\right)$$

$$\therefore \quad V = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_1} + \frac{1}{r_2} - \frac{2}{r}\right) \quad -----(i)$$

Using the properties of triangle, we can write,

$$\therefore \quad r_1^2 = OP^2 + OB^2 - 2\cdot OP\cdot OB\cdot Cos\,\theta \qquad [\text{For } \Delta\text{OPB}]$$

$$or, \quad r_1^2 = r^2 + \ell^2 - 2\cdot r\ell Cos\,\theta$$

$$or, \quad r_1 = \left(r^2 + \ell^2 - 2\cdot r\ell Cos\,\theta\right)^{1/2}$$

$$or, \quad \frac{1}{r_1} = \left(r^2 + \ell^2 - 2\cdot r\ell Cos\,\theta\right)^{-1/2}$$

$$or, \quad \frac{1}{r_1} = \left\{r^2\left(1 + \frac{\ell^2}{r^2} - \frac{2\ell}{r}Cos\,\theta\right)\right\}^{-1/2}$$

$$or, \quad \frac{1}{r_1} = r^{-1}\left\{1 - \left(\frac{2\ell}{r}Cos\,\theta - \frac{\ell^2}{r^2}\right)\right\}^{-1/2}$$

Using the formula,

$$\left(1-x\right)^{-n} = 1 + nx + \frac{n(n+1)}{L^2}x^2 + \frac{n(n+1)(n+2)}{L^3}x^3 + ... \quad -----(ii)$$

(On neglecting the terms having more than 3rd power of ℓ/r)

Similarly, we can obtain the expression for r_2 on replacing θ by (π – θ) on equation (ii)

i.e. $\left(Cos(\pi-\theta) = -Cos\theta\right)$

$$\frac{1}{r_1} = \frac{1}{r}\left\{1 - \frac{\ell Cos\,\theta}{r} - \frac{\ell^2\left(1-3Cos^2\theta\right)}{2r^2} - \frac{\ell^3\left(5Cos^3\theta - 3Cos\,\theta\right)}{2r^3}\right\} \quad -----(iii)$$

Thus, using equation (ii) and (iii) in equation (i), we get,

$$V = \frac{1}{4\pi\varepsilon_o} \cdot \frac{1}{r}\left[2 - \frac{\ell^2}{r^2}(1 - 3Cos\,2\theta) - 2\right]$$

$$or, \quad V = \frac{1}{4\pi\varepsilon_o} \cdot \frac{1}{r}\left[2 - \frac{\ell^2}{r^2}(1 - 3Cos2\theta) - 2\right]$$

$$or, \quad V = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q\ell^2}{r^3}(3Cos2\theta - 1) \quad -----(iv)$$

which is required expression

Special cases:

(i) If θ = 0 i.e. point P lies on the axis, then

$$V = \frac{1}{4\pi\varepsilon_o} \cdot \frac{2q\ell^2}{r^3} \qquad \text{Thus, } V\alpha\frac{1}{r^3}$$

(ii) If θ = 90º, i.e. the point P lies on the equatorial line,

$$V = -\frac{1}{4\pi\varepsilon_o} \cdot \frac{q\ell^2}{r^3}$$

The –ve sign due to the fact that the –ve charges (i.e. –q) is closer with respect to te +ve charges.

Electric field:

The radial component of electric field (i.e. along the direction $\vec{r}$) is given by,

$$\therefore E_r = -\frac{dv}{dr}$$

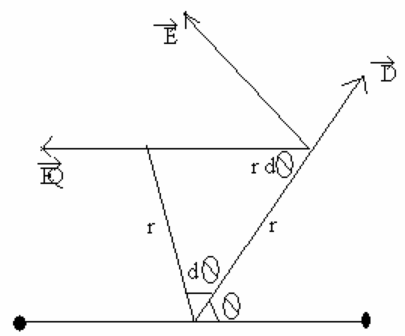
$$or, \quad E_r = -\frac{d}{dr}\left\{\frac{1}{4\pi\varepsilon_o} \cdot \frac{q\ell^2}{r^3}(3Cos^2\theta - 1)\right\} \qquad \text{[using equation (iv)]}$$

$$or, \quad E_r = -\frac{1}{4\pi\varepsilon_o} \cdot q\ell^2(3Cos^2\theta - 1) \cdot \frac{d}{dr}\left(\frac{1}{r^3}\right)$$

$$or, \quad E_1 = +\frac{1}{4\pi\varepsilon_o} \cdot \frac{3q^2}{r^4}(3Cos^2\theta - 1) \quad -----(v) \qquad \left[\frac{d}{dr}r^{-3} = -3r^{-4}\right]$$

Again, the component of electric field P in direction perpendicular to the direction of r (i.e. the angular component of the electric field) is given by,

$$E_Q = -\frac{dv}{rd\theta} = -\frac{1}{r} \cdot \frac{d}{d\theta}\left\{\frac{1}{4\pi\varepsilon_o} \cdot \frac{q\ell^2}{r^3}(3Cos^2\theta - 1)\right\}$$



$$or, \quad E_Q = -\frac{1}{4\pi\varepsilon_o} \cdot \frac{q\ell^2}{r^4} \cdot \frac{d}{d\theta}\left(3Cos^2\theta - 1\right)$$

$$or, \quad E_Q = +\frac{1}{4\pi\varepsilon_o} \cdot \frac{q\ell^2}{r^4} \times 6Cos\theta\left(Sin\theta\right) \qquad \left[\frac{d}{d\theta}\left(Cos^2\theta\right) = 2Cos\,\theta\left(-Sin\theta\right)\right]$$

$$\therefore \quad E_Q = \frac{1}{4\pi\varepsilon_o} \cdot \frac{6q\ell^2 Cos\theta.Sin\theta}{r^4} \quad -----(vi)$$

Thus, the resultant electric field at point P is given by,

$$\therefore \quad E = \sqrt{E_r^2 + E_Q^2}$$

$$or, \quad E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{3q\ell^2}{r^4} \sqrt{\left(3Cos^2\theta - 1\right)^2 + \left(2Cos\,\theta \times Sin\,\theta\right)^2}$$

$$or, \quad E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{3q\ell^2}{r^4} \sqrt{9Cos^2\theta - 6Cos^2\theta + 1 + 4Cos^2\theta\left(1 - Cos^2\theta\right)}$$

$$or, \quad E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{3q\ell^2}{r^4} \sqrt{9Cos^2\theta - 6Cos^2\theta + 1 + 4Cos^2\theta - 4Cos^2\theta}$$

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{3q\ell^2}{r^4} \sqrt{1 - 2Cos^2\theta + 5Cos^4\theta} \quad -----(vii)$$

which is the required expression.

Special Cases:

(i) If θ = 0º (i.e. point P lies on the axis of the quardrupole)

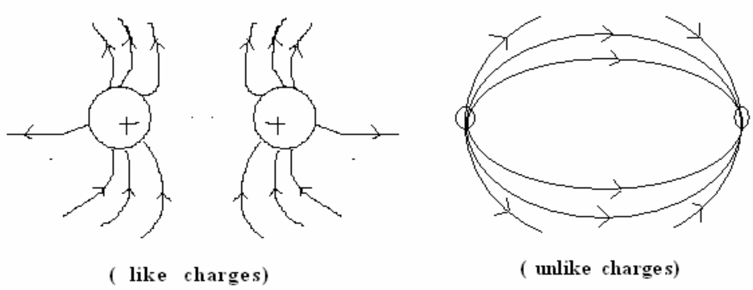
$$E = \frac{1}{4\pi\varepsilon_0} \frac{6q\ell^2}{r^4} \qquad \left[\therefore E\alpha \frac{1}{r^4}\right]$$

(ii) If θ = 90º (i.e. point D lies on the equatorial line)

$$E = \frac{1}{4\pi\varepsilon_0} \frac{3q\ell^2}{r^4} \qquad \left[\therefore E\alpha \frac{1}{r^4}\right]$$

Electric lines of force:

The electric lines of forces is the path along which a unit positive charge would move if it is free to do so.



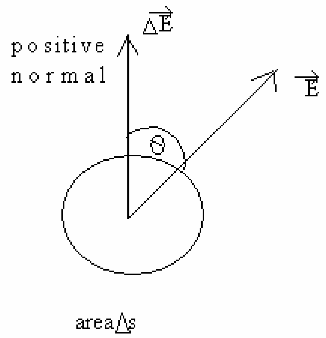
Again the electric lines of forces are the curves the tangent of drawn at any points of which determines the direction of electric field at that point.

Properties of electric lines of force:

(i) They start from +ve charge and last at the same magnitude of –ve charge.
(ii) They do not cross each other otherwise at the point of intersection there will be two directions of electric field which is impossible.
(iii) They have tendency to contact along the length. This shows that the unlike charges attract each other.
(iv) They exert lateral pressure on each other. This shows that the like charges repel each other.
(v) They are imaginary line and perpendicular to the surface of the charge conductor.
(vi) Number of lines of force due to unit +ve charge is $4\pi K$

Where, $K = \frac{1}{4\pi\varepsilon_0}$ (for charge +1C) and hence it is $\frac{1}{\varepsilon_o}$ (ε_o permittivity of air)

*** Electric Flux:**



Let, ΔS = a hypothetical area

E = uniform electric field which makes an angle θ with the +ve normal to the surface of area ΔS.

The quantity $\Delta\varphi = E\Delta S \cos\varphi = \vec{E}\cdot\vec{\Delta S}$ is called the flux of the electric field through chosen area.

Where, ΔS is area vector along the direction of the +ve normal.

The electric flux is a scalar quantity.

Using the techniques of integration the electric flux be given by

$$\phi = \int \vec{E}\cdot\vec{ds}$$

where, the integration has to be perform over the entire surface through which the flux is required.

For a close surface, we can write,

$$\phi = \oint \vec{E}\cdot\vec{ds}$$

$$\therefore \phi = \int \vec{E}\cdot\vec{ds} = \int EdsCos\theta = EACos\theta = \vec{E}\cdot\vec{A} \qquad \left[A = \int ds\right]$$

Special case:

(i) If θ = 0°, i.e. $\vec{E}$ is perpendicular to the area.

Then, $\phi = EA$ (maximum)

(ii) If θ = 90°, i.e. $\vec{E}$ is parallel to the area.

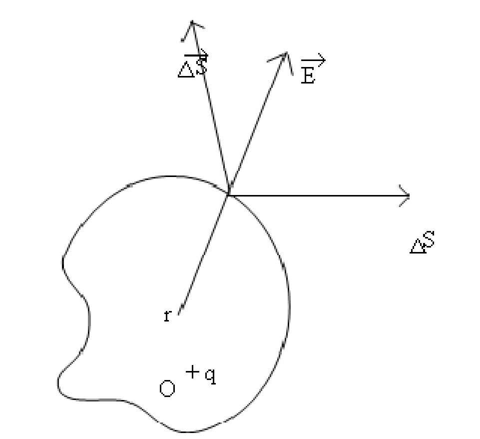
Then, $\phi = 0$ (minimum)

Gauss's Theorem:

Statement: The electric flux through a close surface whatever its shape may be, $1/\varepsilon_o$ times the charge enclosed by the surface where, ε_o is the permittivity of air or vacuum.

i.e. $\phi = \oint \vec{E} \cdot \vec{ds} = \dfrac{1}{\varepsilon_o} \times q$

where, q = charge enclosed by the surface.



Proof:

Let, q = A positive placed at point O inside a close surface.

P = A point on the surface

ΔS = small area on the surface around point P

r = OP, the separation of point P from O.

$\vec{E}$ = The electric field at point P due to charge +q (along $\overrightarrow{OP}$)

θ = angle made by the direction of $\vec{E}$ with the outward normal to area ΔS

Now, the magnitude of electric field at point P is given by

$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r^2} \quad -----(i)$$

If $\Delta\phi$ be the electric flux through area ΔS, then we can write,

$$\Delta\phi = \vec{E} \cdot \vec{ds} = E\Delta S\, Cos\,\theta \quad -----(ii)$$

Using equation (i) and (ii), we get

$$\Delta\phi = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r^2} \times \Delta S\, Cos\,\theta$$

$$\therefore \quad \Delta\phi = \frac{1}{4\pi\varepsilon_o} \cdot q\Delta\Omega \quad -----(iii)$$

where, $\Delta\Omega = \dfrac{\Delta S\, Cos\,\theta}{r^2}$, the solid angle subtended by area ΔS at point O.

Thus, the net flux through the entire surface is given by

$$\phi = \sum \Delta\phi = \frac{q}{4\pi\varepsilon_o} \cdot \sum \Delta\Omega$$

$$or, \quad \phi = \frac{q}{4\pi\varepsilon_o} \times 4\pi$$

$$\therefore \phi = \frac{1}{\varepsilon_o} \times q$$

which proves the Gauss' theorem.

Special case:

(i) If the charge q is the outside surface, in this case $\sum \Delta\phi = 0$ (i.e. no area is enclose).

$$\therefore \phi = 0$$

Charge distribution:

*** Application of Gauss' theorem:**

i. Linear distribution: Charge distributed in wire i.e. along the length of the conductor.

ii. Linear Charge density: denoted by (λ)

$$= \frac{\text{Charge given to the wire}}{\text{length of the wire}} = \frac{q}{l}$$

iii. Surface charge distribution: charge distributed over the entire surface of the conductor.

$$\text{Surface charge density } (\sigma) = \frac{q}{S}$$

where, S = surface area of conductor
q = charge given

iv. Volume charge distribution: Charge is distributed in the entire volume of the conductor.

$$\text{Volume charge density } (\rho) = \frac{\text{q}}{\text{V}}$$

where, q = charge given
V = volume of the conductor

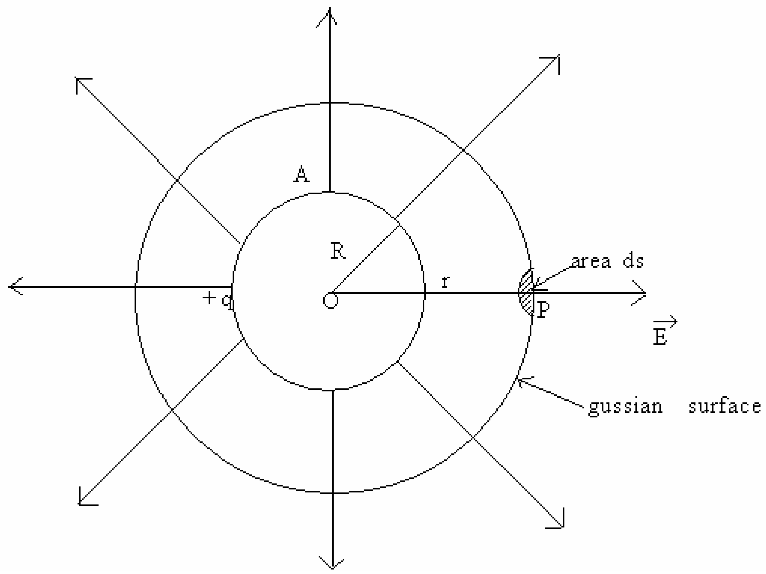
Application of Gauss' theorem:

(I) Electric field at a point due to a uniformly charge spherical shell (i.e. hollow sphere):

Let, A = a spherical shell of radius R which is given +q charge so that the charge is distributed uniformly over the entire surface.

P = A point where the electric field is to be determined

r = OP, the distance



of the centre of spherical O.

ds = small area around the point P

Let us imagine a Gaussian surface i.e. a Sphere of radius r through point P.

(a) when the point P is out side the sphere:

Now, the electric flux through the Gaussian surface is given by

$$\phi = \oint \vec{E} \cdot \vec{ds}$$

$$= \oint EdsCos\,0^o \qquad \left[\therefore \overline{E} \text{ is perpendicular to the surface}\right]$$

$$= E\oint ds$$

$$\therefore \phi = E \times 4\pi r^2 \quad -----(i) \qquad \left[\therefore \vec{E} \text{ area of sphere} = 4\pi r^2\right]$$

But from gauss' theorem,

$$\phi = \frac{1}{\varepsilon_o} \times q \quad -----(ii)$$

where, ε_o = permittivity of air or vacuum

Thus from equation (i) and (ii), we get,

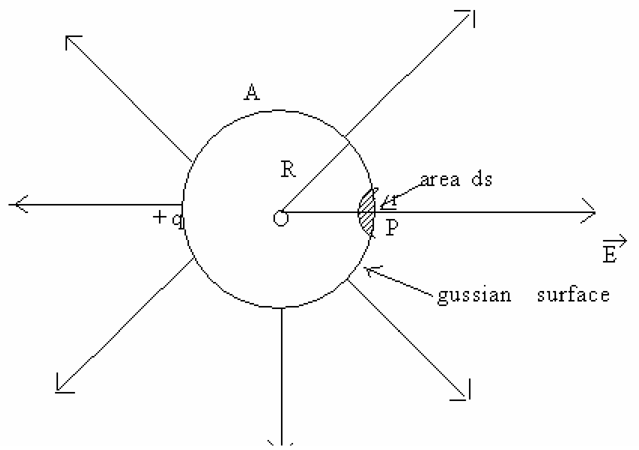
$$E \times 4\pi r^2 = \frac{1}{\varepsilon_o} \times q$$

$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r^2} \quad -----(iii)$$

which is the required expression.

Equation (iii) is identical to the experiment for the electric intensity at a point due a point charge +q separated by a distance r from it. Thus, the charge sphere behaves as if the total charge where concentrated at the centre, when point P lies outside the sphere.

(b) **When point P lies on the surface of the sphere:**



In this case,

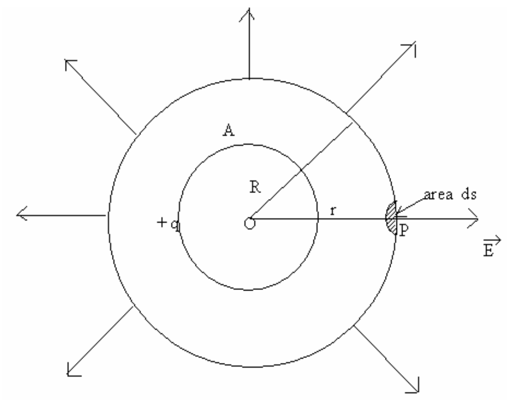
r = R

Thus, equation (iii) becomes,

$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{R^2} \quad -----(iv)$$

which is the required expression.

(c) When point P lies inside the sphere:



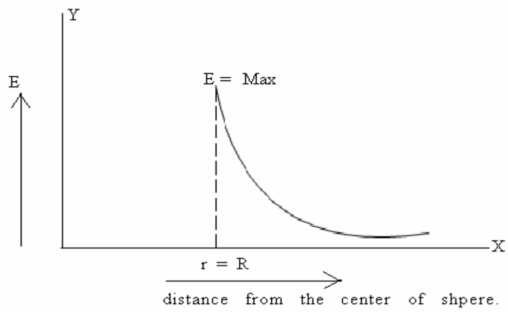
In this case, no charge is enclosed by the Gaussian's surface.

i.e. q = 0

Thus, from equation (iii), we get

$$E = 0 \quad -----(v)$$

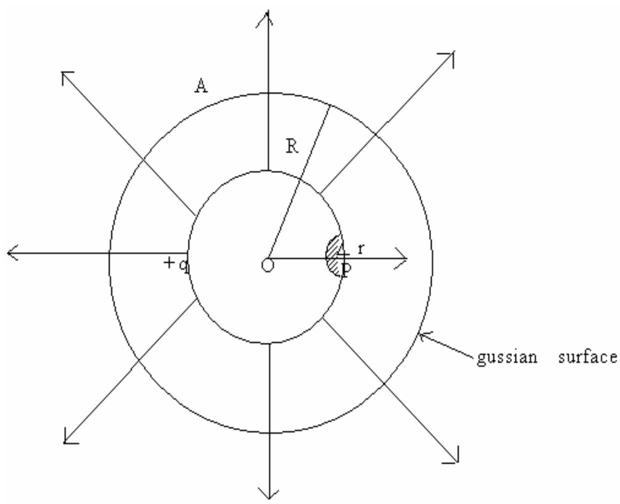
Thus, then is no electric field inside a charge hollow sphere.



(II) Electric field at a point due to a uniformly charges solid sphere:

(Not done)

(c) When $q^{|}$ be the charge enclose by the



Gaussian surface, then equation (iii), we can write,

$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q^{|}}{r^2} \quad -----(v)$$

Now, charge per unit volume of the given sphere is

$$\rho = \frac{q}{\frac{4}{3}\pi r^3} \quad -----(vi)$$

$$\left[\therefore \text{Volume of sphere} = \frac{4}{3} R^3 \right]$$

Since, the volume of the Gaussian sphere is 4/3 πr^3 hence the charge emclosed by the Gaussian surface is given by

$$q^{|} = \rho \times \frac{4}{3}\pi r^3$$

$$or, \quad q^{|} = \frac{q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3$$

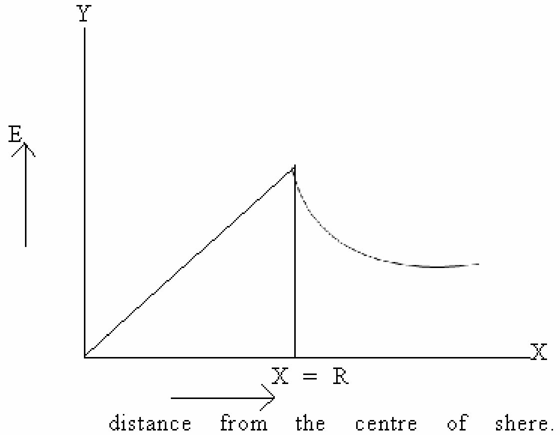
$$\therefore \quad q^{|} = \frac{qr^3}{R^3} \quad -----(vii)$$

Thus, from equation (v) and (vii), we get,

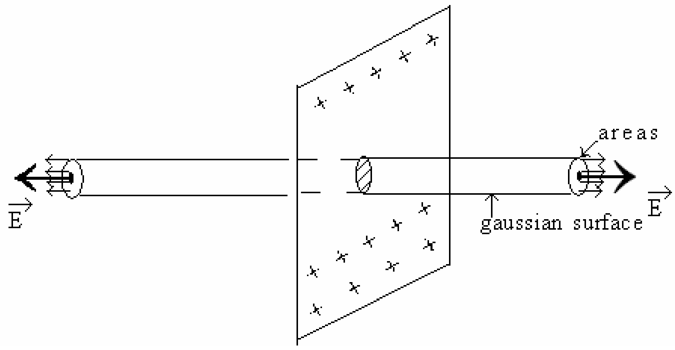
$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{1}{r^3} \times \frac{qr^3}{R^3}$$

$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{qr}{R^3} \quad -----(viii)$$

It is clear from above equation that E is directly proportion to r.



Electric field at a point due to an infinite plane sheet of charge:



Let, A = An infinite plane sheet of charge

σ = Surface charge density of the sheet

S = cross-sectional area of the Gaussian surface which is a cylinder

P = A point where electric field is to be determined

$\vec{E}$ = Electric field at point P due to the plane sheet of charge

Since, the lines of force are perpendicular to the surface of the sheet hence they only pass through the cross-section area of the Gaussian surface. Thus, surface integral of $\vec{E}.\vec{ds}$ over the entire surface of the cylinder is reduced to the surface integral due to two sides. Thus, from gausses theorem, surface integral due to two sides. Thus, from gausses theorem, we can write

$$\oint_S \vec{E} \cdot \vec{ds} = \frac{1}{\varepsilon_o} \times q$$

where, q = charges enclosed by the Gaussian surface

$$or, \quad \oint_{S1} \vec{E} \cdot \vec{ds} + \oint_{S2} \vec{E} \cdot \vec{ds} = \frac{1}{\varepsilon_o} \times q$$

$$or, \quad EE + ES = \frac{1}{\varepsilon_o} \times q \qquad \left[\therefore \vec{E} \cdot \vec{ds} = Eds\,Cos\,0^o = Eds\right]$$

$$\therefore 2ES = \frac{1}{\varepsilon_o} \times \sigma S$$

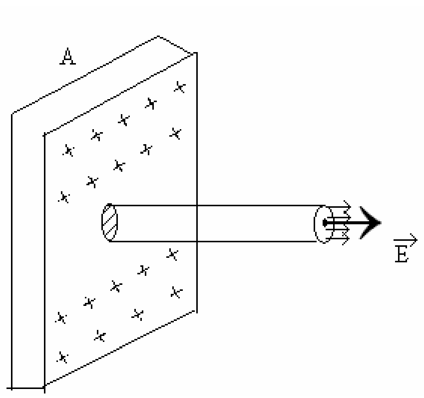
$$\therefore E = \frac{\sigma}{2\varepsilon_o}$$

which is the required expression.

It is clear from above equation that the magnitude of the electric field is independent of the distance from the sheet.

Practically an infinite sheet of charge does not exist. There result are correct for real charge sheet is the point under consideration are not very close to the edges and the distances from the sheet are small compare to the dimensions of the sheet.

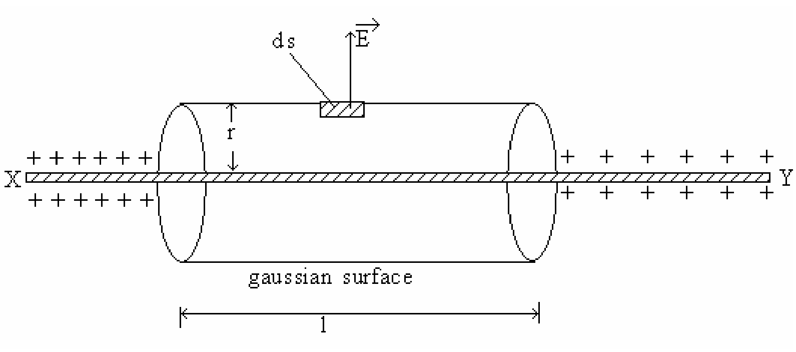
(IV) **Electric field at a point due to infinite charged conducting plate:**



Let, A = an infinite charge plane conductor
σ = Surface charge density of the plate
S = cross-sectional area of the Gaussian surface which is a cylinder.
P =
$\vec{E}$ = Electric field at point P due to the

Thus, the electric field does not depend upon the distance from the plate.

(V) **Electric intensity at a point due to a uniformly charged conductor (i.e. wire) of infinite length.**



Let, XY = A uniformly charged conducting wire
P = A point where electric intensity is to be determined
λ = Linear charge density of the conducting wire
ℓ = length of the Gaussian surface through point P, which is a cylinder of radius r (i.e. distance of point P from the conducting wire is r)

Since, the lines of force are perpendicular to the surface of the wire hence they only cross-through the curve area of the Gaussian surface and not through the cross-sectional area. Hence, total electric flux through the Gaussian surface is,

$$\phi = \oint_S \vec{E} \cdot \vec{ds} = \oint_S EdsCos0^o$$

$$or, \quad \phi = E\oint ds \qquad \left[\therefore \oint ds = \text{area of curved surface of the cylinder}\right]$$

$$\therefore \phi = E \times 2\pi rl \quad -----(i)$$

Now, from gauss' theorem, we have

$$\phi = \frac{1}{\varepsilon_o} \times q \quad -----(ii)$$

where, q = Charge enclose by the Gaussian surface

Thus, from equations (i) and (ii), we get,

$$E \times 2\pi rl = \frac{1}{\varepsilon_o} \times q\ ]$$

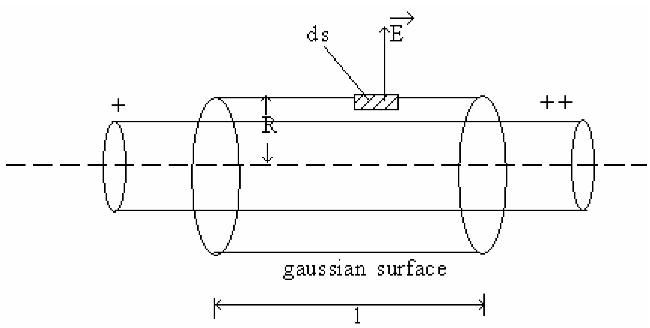
$$or, \quad E = \frac{1}{\varepsilon_o} \times \frac{q}{2\pi rl} \qquad \left[\therefore \lambda = \frac{q}{l}\right]$$

$$\therefore E = \frac{\lambda}{2\pi\varepsilon_o r} \quad -----(iii)$$

which is the required expression.

$$\therefore E \alpha \frac{1}{r}$$

(VI) **Electric field at a point due to a uniformly charge hollow cylinder of infinite length:**



Let, R = the radius of a uniformly charged hollow cylinder of infinite length

P = A point where electric field is to be determined

r = distance of point P from the axis of the cylinder

Let us consider a Gaussian surface through point P which is a cylinder of radius r and length l.

Since, the lines of force are perpendicular to the surface of the cylindrical hence they cross only through the curve surface of the Gaussian surface. Thus, the total electric flux due to curve, surface only.

(a) when point P is outside the cylinder:

The total electric flux through the Gaussian surface is given by,

$$\phi = \oint_S \vec{E} \cdot \vec{ds}$$

$$= \oint_S EdsCos\,0^o$$

$$or, \quad \phi = E\oint_S ds$$

$$\therefore \quad \phi = E \times 2\pi rl \quad -----(i)$$

But, $\phi = \frac{1}{\varepsilon_o} \times q$ [from gauss' theorem]

Where, q = charge enclosed by the Gaussian surface.

If σ be the surface charge density of the hollow cylinder then, we can write

$$\sigma = \frac{q}{2\pi R \times l} \quad -----(ii)$$

$$\therefore \quad q = \sigma \times 2\pi Rl \quad -----(iii)$$

Thus, equation (ii) becomes,

$$\phi = \frac{1}{\varepsilon_o} \times 2\pi Rl\sigma \quad -----(iv)$$

Hence, from equation (i) and (iv), we get,

$$E \times 2\pi rl = \frac{1}{\varepsilon_o} \times 2\pi Rlr$$

$$\therefore E = \frac{R\sigma}{\varepsilon_o r} \quad -----(v)$$

which is the required expression

(b) When point P is on the surface of the cylinder,

In this case r = R

$$\therefore \quad E = \frac{\sigma}{\varepsilon_o}$$

(c) When point C is inside the cylinder

$$E = 0$$

i.e. no charge is enclosed by the Gaussian surface

(VII) Electric field at a point due to a uniformly charged solid cylinder of infinite length (left Uncompleted)

If σ be the volume charge density of the cylinder, then,

$$\sigma = \frac{q}{\pi R^2 l}$$

$$\therefore q = \pi R^2 l\sigma \quad -----(iii)$$

Thus, equation (ii) becomes,

$$\phi = \frac{1}{\varepsilon_o} \times \pi R^2 l \sigma \quad -----(iv)$$

Hence, equation (i) and (iv), we get,

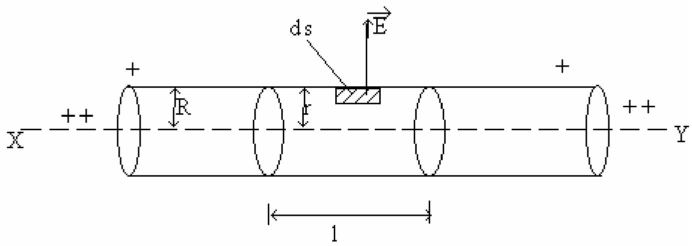
$$E \times 2\pi r l = \frac{1}{\varepsilon_o} \pi R^2 \sigma$$

$$E = \frac{R^2 \sigma}{2\varepsilon_o r} \quad -----(v)$$

which is the required expression

$$\left[\therefore E \alpha \frac{1}{r} \right]$$

(b) **When point P lies on the surface of the cylinder:**



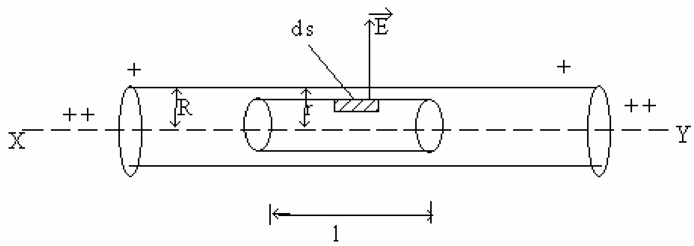
In this case,

r = R

$$E = \frac{R^2 \sigma}{2\varepsilon_o \times R}$$

$$E = \frac{R\sigma}{2\varepsilon_o} \quad -----(v)$$

(C) **When point P lies inside the cylinder:**



In this case, charge enclosed by the Gaussian surface is given by,

$$q = \sigma \times \pi r^2 l$$

Thus, $\phi = \frac{1}{\varepsilon_o} \times \sigma \times \pi r^2 l$

[Using equation (ii)]

$$or, \quad E \times 2\pi r l = \frac{1}{\varepsilon_o} \times \sigma \pi r^2 l \quad \text{[Using equation (i)]}$$

$$\therefore \quad E = \frac{\sigma \times r}{2\varepsilon_o}$$

$$\therefore E \alpha r$$

Capacitance of

Capacitor: When a conductor or capacitor is given charge its potential rises. It is found that,

$q \alpha V$

where, q = charge given

V = rise in potential

Thus, q = CV

Where, C = constant of proportionality known as capacitance of capacitor/conductor)

If V = 1

Then, q = C

Thus, the capacitance of a conductor or capacitor is numerically equal to the charge required to raise its potential through unity.

Units of capacitance:

(i) The S.I. unit of capacitance is farad (F):

$$\therefore q = CV$$

$$\therefore C = \frac{q}{V}$$

$$\therefore F = \frac{Coulombs}{Volt}$$

Thus, $1F = \dfrac{1C}{1V}$

Thus, capacitance of a conductor (capacitor) is said to be one farad if it required one columb of charge to raise its potential through 1 volt.

Since Farad is large quantity hence in practice the capacitance is measured in mF, μF, PF, etc.

1 mF = 10^{-3} F

1 μF = 10^{-6}F

(ii) The C.G.S. unit of capacitor is stat-F or esu of capacitance,

$$1\ \ Stat - F = \frac{1\ \ State - coulomb}{1\ \ Stat - V}$$

*** Relation between F and Stat-F:**

$$F = \frac{1C}{1V}$$

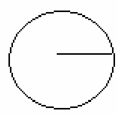
1C = 3×10^{-9} stat-C

$$1F = \frac{3\times10^{9}\, stat - C}{\frac{1}{300}\, Stat - V}$$

$$1V = \frac{1}{300}\ \ Stat - V$$

1F = 9×10^{11} stat-F

*** Capacitance of a spherical conductor:**



Let, r = radius of a spherical conductor

C = capacitance of the spherical conductor,

+q = charge given to the conductor

V = rise in the potential of the conductor

Thus,

$$C = \frac{q}{V} \quad -----(i)$$

Now,

$$V = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r} \quad -----(ii) \qquad \text{[In SI unit]}$$

Thus, from equation (i) and (ii), we get,

$$C = \frac{q}{\left(\frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r}\right)}$$

$$\therefore C = 4\pi\varepsilon_o r \quad -----(iii)$$

which is the required expression

In CGS system,

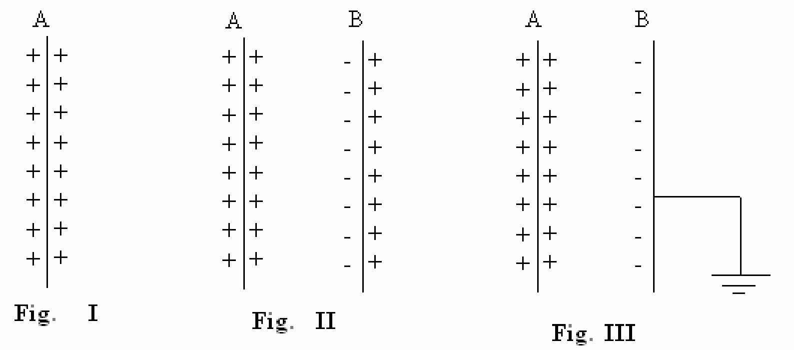
V = q/r

C = r [If r = 5cm, then C = 5stat-F]

CAPACITOR (or CONDENSER):

It is device used to store charge. A capacitor is an arrangement due to which the capacitance of a conductor is artificially increased.

Principal of capacitor:



A is a conductor which is given positive charge until its potential becomes constant. Hence it cannot accommodate further charges (fig-I)

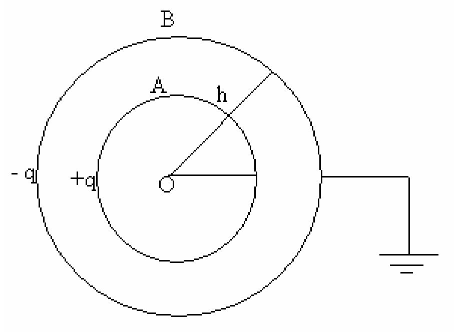
Now, uncharged conductor B is placed closer to A (fig-II). The electro-static induction takes place and the induced bound charges (i.e –ve charges) arc on the surface of 'B' closer to A and the induced free charges (i.e. +ve Charges) remain on the surface of the B away from A. Thus, there is a decrease in the potential A due to which it can acquire some charge.

Now, the conductor B is earthed while induced –ve charges (i.e. bound charges) remain on B. Thus, there is an appreciable decrease In the potential A due to which it can aquire more and more charges.

This is the principle of capacitor.

Spherical capacitor:

Let us consider a spherical capacitor which consists of two concentric spheres A and B of radii a and b respectively in which A is given +q charge and B is earthed.



Now, the capacitance of the conductor A is given by,

$$C^{|} = 4\pi\varepsilon_o a \quad -----(i)$$

where, ε_o = permittivity of air (or vacuum)

If C be the capacitance of the capacitor, then we can write

$$C = \frac{q}{V} \quad -----(ii)$$

where, V = net potential of A

Now, $V = V_A + V_B$

Where, V_A = Electric potential at any point of surface of A

V_B = Electric potential at any point of surface of B or inside it.

$$or, \quad V = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{a} + \frac{1}{4\pi\varepsilon_o} \cdot \frac{(-q)}{b}$$

$$or, \quad V = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$\therefore \quad V = \frac{q}{4\pi\varepsilon_o}\left(\frac{b-a}{ab}\right) \quad -----(iii)$$

which is the required expression for the capacitance of a spherical capacitor.

Since, $\frac{ab}{b-a} > a$

Thus, $C > C^{|}$ [from equation (i) and (iv)]

Thus, the capacitance of the conductor A is artificially increased.

Problem:

For the earth,

r = 6400km = 6400×1000m

$\varepsilon_o = 8.85\times10^{-12}\text{Fm}^{-1}$

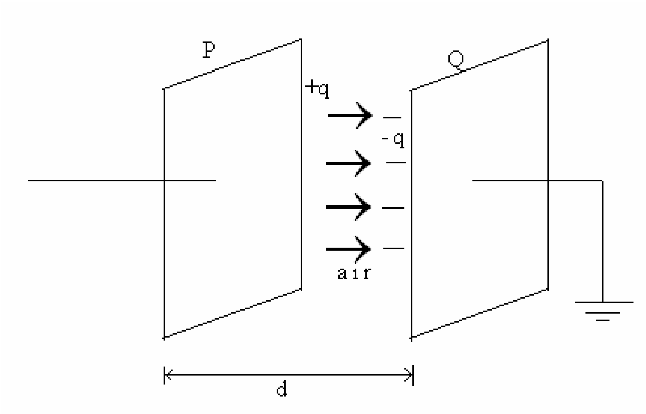
C = ?

We know that,

$$C = 4\pi\varepsilon_o r$$

Parallel plate capacitor:

It consists of two identical metallic plate placed parallel to each other in which one is given charge and another is earthed.



Let, P and Q = two metallic plates placed parallel to each other in which P is given +q charge and plate Q is earthed.

A = surface area of each plate

d = separation between the plate

$\vec{E}$ = uniform electric field between the plate.

V = potential difference between the plate.

C = capacitance of parallel plate capacitor.

Thus, $C = \frac{q}{V} \quad -----(i)$

Now, the magnitude of electric field between the plate is given by,

$E = \frac{\sigma}{\varepsilon_o}$ [where, σ = surface charge density]

From Gaussian theorem,

$\sigma = \frac{q}{A}$

ε_o = permittivity of air (or vacuum)

$E = \frac{q}{\varepsilon_o A}$

But, $E = \frac{V}{d}$ $[\therefore V = ED]$

Thus, $V = \frac{q}{\varepsilon_o A} d \quad -----(iii)$ [Using equation (ii)]

Thus, equation from (i) and (iii), we get,

$C = \frac{\varepsilon_o A}{d} \quad -----(iv)$

which is the required expression for a capacitance of a parallel plate capacitor with which in air between two plate.

If a medium of permittivity ε(epsilon) is between the plate then, using (iv) we can write,

$C = \frac{EA}{d} \quad -----(v)$

But, $E = \varepsilon_o K \quad -----(vi)$ [where, K = dielectric constant of the medium]

$\therefore K = \frac{E}{\varepsilon_o}$

$$C = \frac{\varepsilon_o KA}{d} \quad -----(vii)$$

which is the expression for the capacitance of parallel plate capacitor with a dielectric medium.

From the above equation it is clear that C is directly proportional to A

C α A

C α 1/d

C α K

Factors affecting C:

i. area

ii. separation

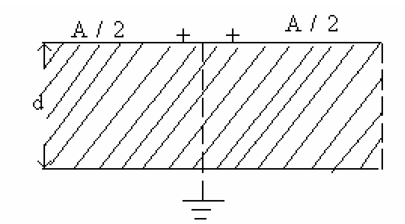
iii. K, K>1

C is increase

K → ε_r

Problem:

(I)



The capacitance of the parallel plate capacitor with air between the plate.

$$C = \frac{\varepsilon_o A}{d} \quad -----(i)$$

When two dielectric are introduced between the plate as shown in the figure, the system represents the parallel combination of two parallel plate capacitor.

Now, the capacitance of the capacitor with dielectric medium of dielectric constant K, is given by,

$$C_1 = \frac{\varepsilon_o K_1 \frac{A}{2}}{d}$$

$$C_1 = \frac{\varepsilon_o K_1 A}{2d} \quad -----(ii)$$

Similarly, the capacitance of another capacitor with K_2 is given by,

$$C_2 = \frac{\varepsilon_o K_2 A}{2d} \quad -----(iii)$$

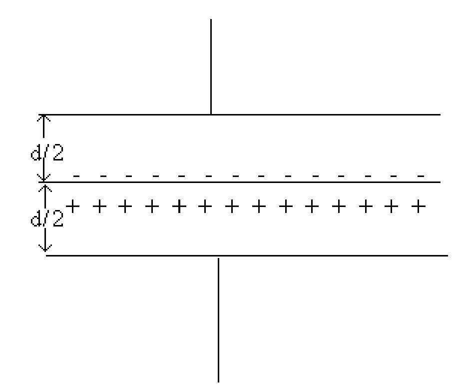
If $C^{\prime}$ be the capacitance of the system then, we can write,

$$d = C_1 + C_2$$

$$C^{|} = \frac{\varepsilon_{oA}}{2d}\left(K_1 + K_2\right)$$

$$\therefore C^{|} = C \times \left(\frac{K_1 + K_2}{2}\right) \quad -----(iv) \qquad \text{[New capacitor]}$$

(II)



When two dielectric are introduced between the plate as shown in the figure, the system represented the series combination of two parallel plate capacitor. Now, the capacitance of the capacitor with the dielectric medium of dielectric constant K, is given by,

$$C_1 = \frac{\varepsilon_o K_1 A}{d/2} = \frac{2\varepsilon_o K_1 A}{d} \quad -----(ii)$$

Similarly, for K_1

$$C_2 = \frac{2\varepsilon_o K_2 A}{d} \quad -----(iii)$$

If 'C' be the separation of the system, then we can write,

$$\frac{1}{C^{|}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C^{|} = \frac{C_1 C_2}{C_1 + C_2}$$

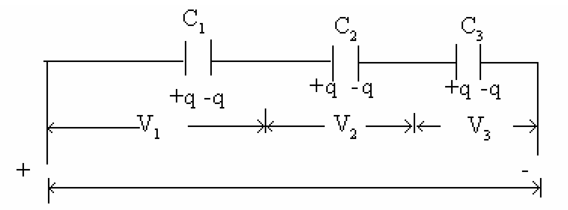
$$C^{|} = 4\left(\frac{\varepsilon_o A}{d}\right)^2 \frac{K_1 K_2}{\left(K_1 + K_2\right)} = \frac{2\varepsilon_o A}{d} \times \frac{K_1 K_2}{K_1 + K_2}$$

$$\therefore C^{|} = C \times \frac{2K_1 K_2}{K_1 + K_2} \quad -----(iv)$$

which is the required expression.

Grouping of the capacitor:

The capacitor are said to group in series if charge capacitor remains same when a potential difference is applied across the combination.



Let, C_1, C_2 and C_3 = the capacitance of capacitor group in series.

V = applied potential difference is applied across the combination.

q = charge on each capacitor

Since, the capacitor of the capacitor are different hence in present the potential difference between the plate and different capacitor is different.

If V_1, V_2, V_3 be the p.d. of the C_1, C_2, and C_3 respectively, then we can write,

$V = V_1 + V_2 + V_3$ ---------- (i)

But, $V_1 = \frac{q}{C_1}$

$$\left.\begin{aligned} V_2 &= \frac{q}{C_2} \\ V_3 &= \frac{q}{C_3} \end{aligned}\right\} \text{-------------(ii)}$$

Thus, equation (i) becomes,

$$V = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) \quad -----(iii)$$

If C be the equivalent capacitance of effective capacitance of the combination, then we can write,

$$V = \frac{q}{C} \quad -----(iv)$$

Thus, from equation (iii) and (iv), we get,

$$\frac{q}{C} = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\therefore \quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad -----(v)$$

Thus, reciprocal of equivalent capacitance is the sum of the reciprocal of the individual capacitance of the capacitor grouped in series.

$C < C_1$

$C < C_2$

$C < C_3$

It is to be noted that the potential difference across the capacitor does not depend, upon its position on the combination.

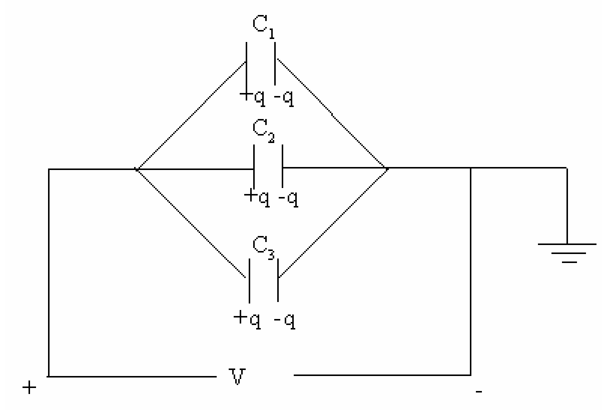
Note: For any two capacitor, we can write

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$or, \quad \frac{1}{C} = \frac{C_1 + C_2}{C_1 C_2} \qquad \left[C = \frac{C_1 C_2}{C_1 + C_2}\right]$$

Capacitor in parallel:

The capacitor are said to be grouped in parallel if the potential difference across each capacitor remain same which is equal to the applied potential across the combination.



Let, C_1, C_2, C_3 are the capacitance of capacitor group in parallel

V = applied potential difference is applied across the combination

q = total charge send by the cell or battery. Since the capacitance of the capacitor is difference. Hence in the persent case on different capacitor is different.

If q_1, q_2, q_3 being the charge on the C_1, C_2 and C_3 respectively then we can write,

$q = q_1 + q_2 + q_3$ ---------- (i)

But,
$$\boxed{\left.\begin{aligned} q_1 &= C_1V \\ q_2 &= C_2V \\ q_3 &= C_3V \end{aligned}\right\} \rightarrow (ii)}$$

Thus, equation (i) becomes,

$$q = V(C_1 + C_2 + C_3) \quad -----(iii)$$

If C be the equivalent capacitor of effective capacitance of the combination of the combination then, we can write,

$$q = CV$$

Thus, from equation (iii) and (ii), we get

$$CV = (C_1 + C_2 + C_3)V$$

$$C = C_1 + C_2 + C_3 \quad -----(v)$$

Thus, the capacitance of the capacitor is the sum of individual capacitance of the capacitor when connected parallel.

Thus,

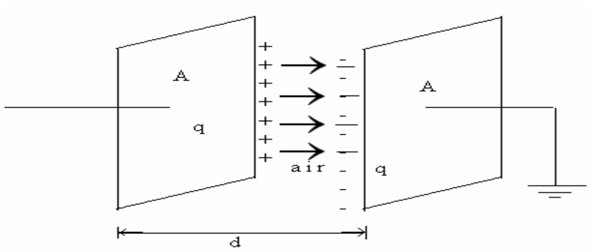
$C > C_1$

$C > C_2$

$C > C_3$

It is noted that the charge on capacitor does not depend upon its position in the combination.

*** Energy stored in the capacitor (or in an electric field):**



Let us consider a parallel plate capacitor one part is given charge another is earthed.

Let, q = Charge given to the capacitor

V = rise in potential of the capacitance of the capacitor

C = capacitance of the capacitor.

Thus,

$$q = CV \quad -----(V)$$

during the process of charging of capacitor worked has to be done against the repultion which is stored in the form of potential energy.

Let, $q^{|}$ = charge on the capacitor at any instant during the process of charging.

$V^{|}$ = potential difference of the capacitor at that instant.

Thus, $V^{|} = \frac{q^{|}}{C} \quad -----(ii)$

If 'dq' be the small charge further supplied the capacitor than work done is given by,

$$dw = V^{|}dq^{|}$$

$$dw = \frac{q^{|}}{C} \cdot dq^{|} \quad -----(iii) \qquad \text{[Using equation (ii)]}$$

Thus total work done on supplying charge +q is given by

$$W = \int_0^q dw$$

$$or, \quad W = \frac{1}{C}\int_0^q a^{|}dq^{|}$$

$$or, \quad W = \frac{1}{C}\left[\frac{q_1^2}{2}\right]_0^q$$

$$U_E = \frac{1}{2}\frac{q^2}{2} \quad -----(iv)$$

when the energy stored in the capacitor or in the electric field using equation (i) n (iv) we obtained

$$U_E = \frac{1}{2}CV^2 \quad -----(v)$$

and, $U_E = \frac{1}{2}qV \quad -----(iv)$

*** Energy density in a electric field:**

It is denoted as the energy stored per unit volume in the electric field. It is denoted by 'U_E'.

Thus, $\frac{U_E}{V} = \frac{\frac{1}{2} \times CV^2}{A \times d}$

$$= \frac{1}{2} \times \frac{\varepsilon_o A}{d} \times \frac{V^2}{Ad}$$

$$= \frac{1}{2}\varepsilon_o \left(\frac{V}{d}\right)^2$$

$$\therefore \quad U_E = \frac{1}{2}\varepsilon_o E^2$$

which is the required expression for energy density.

*** Dielectric constant of Relative permittivity:**

The capacitance of parallel plate capacitor with air between the plates is given by,

$$C = \frac{\varepsilon_o A}{d} \quad -----(i)$$

where, ε_o = permittivity in air

A = Area of each plate

d = separation between the plate

Again, capacitance of same parallel plate capacitor with a dielectric between the plate is given by,

$$C^| = \frac{\varepsilon_o KA}{d} \quad -----(ii)$$

where, K = dielectric constant of medium

Thus, dividing equation (ii) by (i), we get,

$$\frac{C^|}{C} = K \quad -----(iii)$$

Thus, the dielectric constant be defined as the ratio of capacitance of parallel plate capacitor with dielectric to the capacitance of same parallel plate capacitor without dielectric.

*** Parallel plate capacitor partially filled with dielectric**:

Let us consider plate capacitor having capacitance

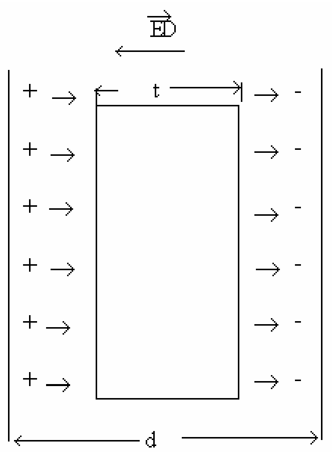
$$C_o = \frac{\varepsilon_o A}{d} \quad -----(i)$$

when an electric field $\overrightarrow{\varepsilon_o}$ is applied between the plates in the absence of dielectric

where, ε_o = permittivity in air or vacuum

A = Area of each plate

d = Separation between the plate



Now, A slab of dielectric medium of thickness 't' is introduced between the plates of parallel plate capacitor. Thus, polarization the molecules inside dielectric take place due to which –ve charge are induced on the surface of dielectric closer to +vely charge plate of the capacitor and +ve charge induced on the surface of dielectric closer to the -ve charge plate of the capacitor.

Hence, an electric field $\overrightarrow{E_P}$ i.e. field due to polarization is setup I the opposite of $\overrightarrow{E_o}$. Hence, resultant field setup inside the slap is $\overrightarrow{E_o} - \overrightarrow{E_P}$.

i.e. $\vec{E} = \overrightarrow{E_o} - \overrightarrow{E_P}$

If V be the potential difference between the plates then we can write,

$$V = E_o(d-t) + Et$$

$$or, \quad V = E_o(d-t) + \frac{E_o}{K}t$$

$$or, \quad V = E_o(d-t) + \frac{t}{K} \quad -----(ii)$$

If C be the capacitance of parallel plate capacitor with dielectric then, we can write

Where, q = charge on the capacitor.

$$C = \frac{q}{V}$$

$$C = \frac{q}{E_o\left(d - t + \frac{t}{K}\right)}$$

$$C = \frac{q}{\frac{q}{E_o A}\left(d - t + \frac{t}{K}\right)}$$

$$\therefore \quad C = \frac{E_o A}{d - t\left(1 - \frac{1}{K}\right)} \quad -----(iii)$$

which is the required expression.

From equation (iii) and (ii) is clear that,

$C > C_o$

Special Case:

(i) In the absence of dielectric

t = 0

$$\therefore C = \frac{E_o A}{d}$$

(ii) From equation (iii) we can write

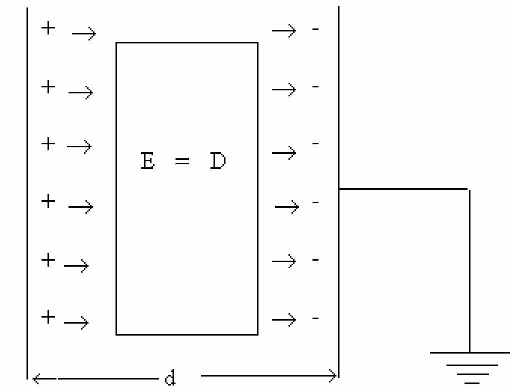
$$C = \frac{E_o A}{d\left\{1 - \frac{t}{d}\left(1 - \frac{1}{K}\right)\right\}}$$

$$C = \frac{C_o}{1 - \frac{t}{d}\left(1 - \frac{1}{K}\right)}$$

*** Parallel plate capacitor field partly with a conductor:**

Let us consider a parallel plate capacitor having capacitance

$$C_o = \frac{E_o A}{d} \quad -----(i)$$



when an electric field $\overrightarrow{E_o}$ is applied between the palate in the absence of dielectric

where, E_o =

A =

d =

Now, A slap of conductor of thickness 't' is introduced between the plate of parallel plate capacitor. Thus, electro-static induction take place, -ve charge are induced at the slap closer to the +vely charge plate of the capacitor and the +ve charge on closer to the –vely charge plate of the capacitor. Inside the slap there is no electric field. Hence, the electric field exist between the plate in air of thickness (d – t) --------- (i)

If V be the potential difference between the plate then, we can write,

$$V = E_o(d - t) \quad -----(ii)$$

If C be the capacitance of capacitor with the induction between the plate then, we can write,

$$C = \frac{q}{V}$$

$$or, \quad C = \frac{q}{E_o(d - t)}$$

$$or, \quad C = \frac{q}{\frac{q(d - t)}{E_o A}}$$

where, q = charge on the capacitance

$$or, \quad E_o = \frac{\sigma}{\varepsilon_o}$$

$$= \frac{q}{\varepsilon_o A}$$

$$\therefore C = \frac{E_o A}{d - t} \quad -----(iii)$$

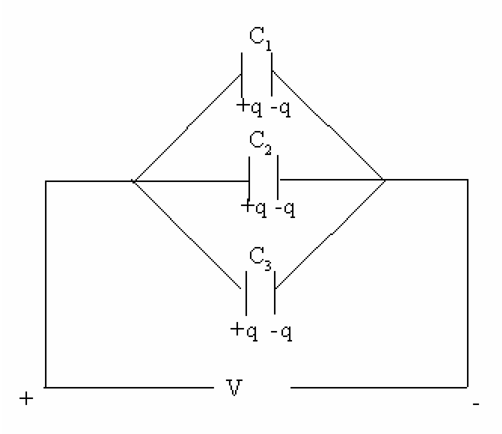
Thus, from equations (i) and (iii), we have, $C > C_o$

$$\text{Again, } C = \frac{\varepsilon_o A}{d\left(1-\frac{t}{d}\right)}$$

$$\therefore \quad C = \frac{C_o}{1-\frac{t}{d}}$$

which is required expression.

*** Total energy stored in the capacitors joined in series:**



If U_1, U_2, U_3 are the energy stored in capacitor C_1, C_2, C_3 respectively then we can write,

$$U_1 + U_2 + U_3 = \frac{1}{2}\frac{q^2}{C_1} + \frac{1}{2}\frac{q^2}{C_2} + \frac{1}{2}\frac{q^2}{C_3}$$

$$= \frac{q^2}{2}\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

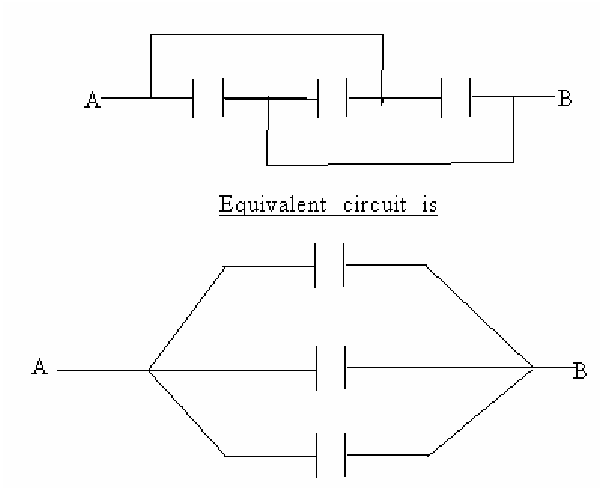
$$= \frac{q^2}{2C} \qquad \left[\therefore \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{C}\right]$$

C is equivalent capacitance

$$\therefore U_1 + U_2 + U_3 = U$$

*** Total energy stored in the capacitor joined in parallel combination**:
]

If U_1, U_2, U_3 are the energy stored in capacitor C_1, C_2, C_3 respectively. We can write,



$$U_1 + U_2 + U_3 = \frac{1}{2}C_1V^2 + \frac{1}{2}C_1V^2 \frac{1}{2}C_3V^2$$

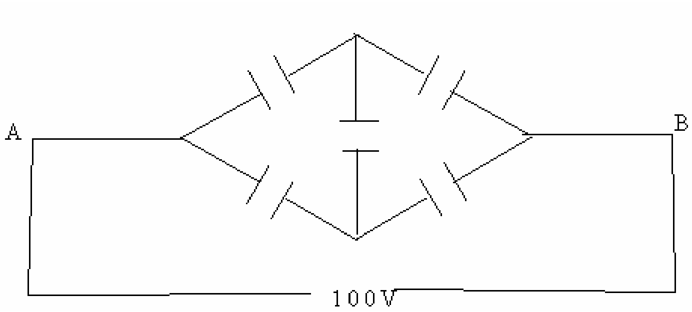
$$= \frac{1}{2}(C_1 + C_2 + C_3)V^2$$

$$= \frac{1}{2}CV^2$$

$$U_1 + U_2 + U_3 = U$$

Problem:

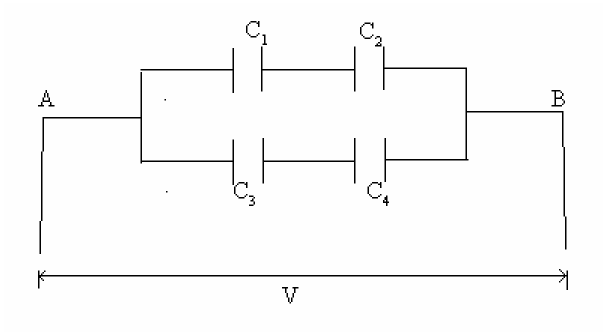
(I) In fig. determined capacitance between points A and B



Three capacitor are grouped in parallel, hence equivalent capacitance is given,

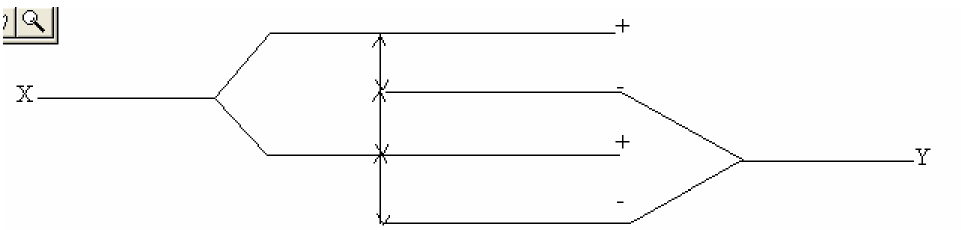
$$\therefore C = C_1 + C_2 + C_3$$

(II) Five capacitor each of capacitance 10µF are connected to a 100V de supply as shown in figure. Determine equivalent capacitor



In this case, there is no Charge on capacitor C_5 thus, from the circuit C_5 is omitted. Hence the equivalent circuit becomes,

(III) For metallic plate,



For metallic plate each having area 'A' are placed as shown in the fig. shown that distance between the consecutive plate 'd' alternate plates are convert to point x and y determine the equivalent capacitance of the system,

Here, three capacitor are in parallel

$$C = C_1 + C_2 + C_3 = 3C = 2\frac{\varepsilon_o A}{d}$$

Group 'D'

L-C Oscillation:

Elements of Electric circuit:

R – Resistor

C – Capacitor

L – Inductor

*** Energy stored in an electric field (or in an capacitor)**

$$U_E = \frac{1}{2}\frac{q^2}{c}$$

where, q = charge on the capacitor

c = capacitance of the capacitor

*** Energy stored in a magnetic field (or in an inductor)**

$$U_B = \frac{1}{2}LI^2$$

where, I = electric field

L = inductance of the inductor

L-C Oscillation (Qualitatively):

In case of L.C. circuit the charge, current and potential difference vary Sinusoid ally with period (T) and angular frequency (w). The resulting oscillations of capacitors electric field and the inductors magnetic field are said to be electro-magnetic oscillations (or L-C Oscillation). Such a circuit is said to oscillate

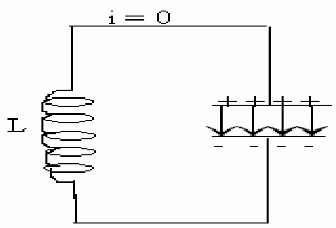


Fig. 1

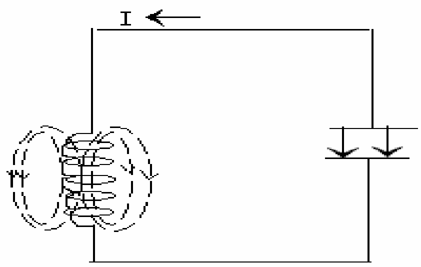


Fig. II

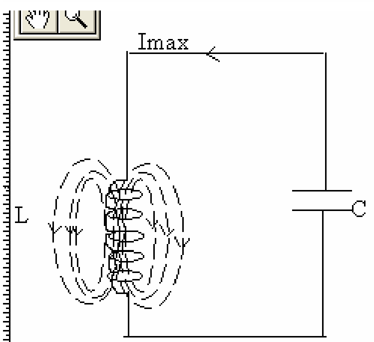


Fig. III

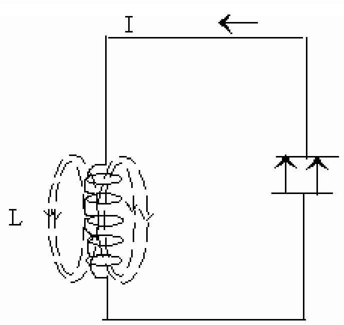


Fig. IV

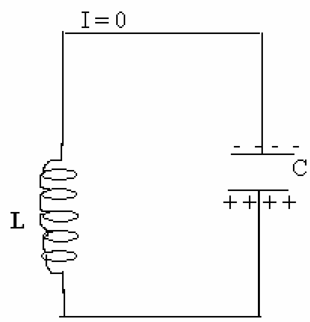
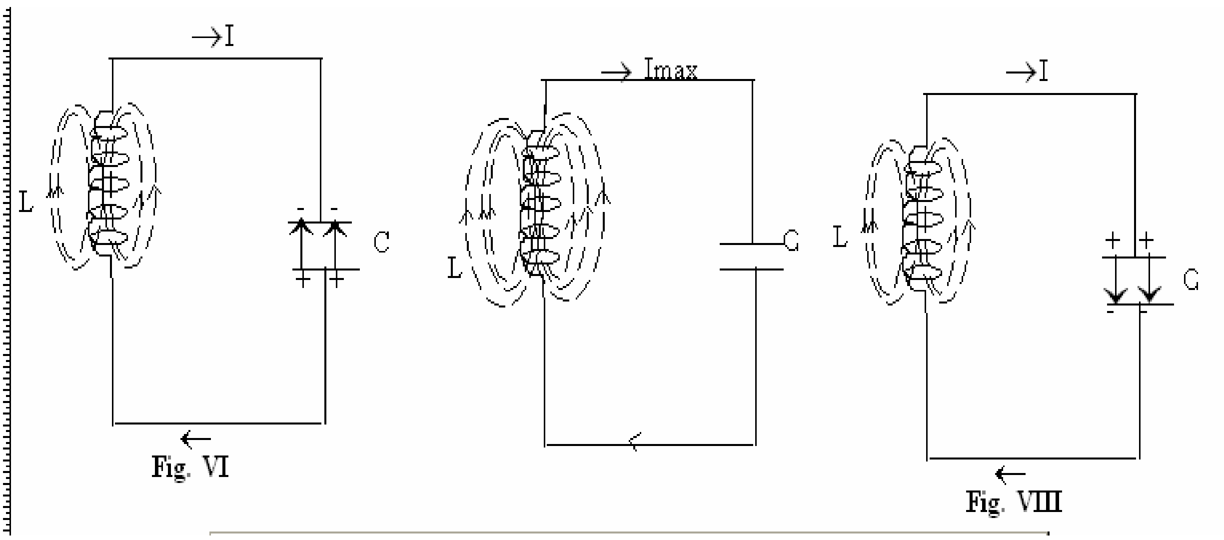


Fig. V



The fig. from (i) to (viii) show the succeeding stages of the oscillation in a simple L-C circuit.

Let us assume that the capacitor C initially carriage a charge q and the current in the inductor is O (i.e. i = 0).

In this case, the energy stored in the electric field of the capacitor is $U_E = \frac{1}{2}\frac{q^2}{c}$ and the energy stored in the magnetic field of the inductor $U_B = 0$ fig(i). The capacitor now begins to discharge through the inductor and a current starts flowing in anticlockwise (fig. ii). As the charge in the capacitor decreases and the current in the inductor increases, hence energy stored in the capacitor decreases and energy stored in the inductor increases.

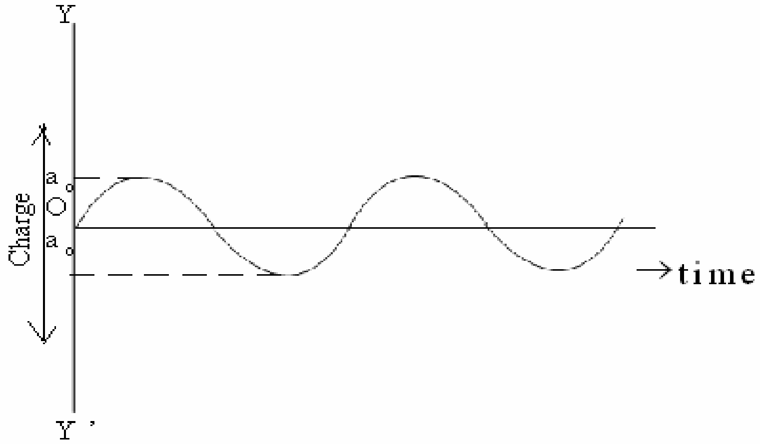
$$\therefore U_B = \frac{1}{2}LI^2$$

The energy transferred from the capacitor to the inductor.

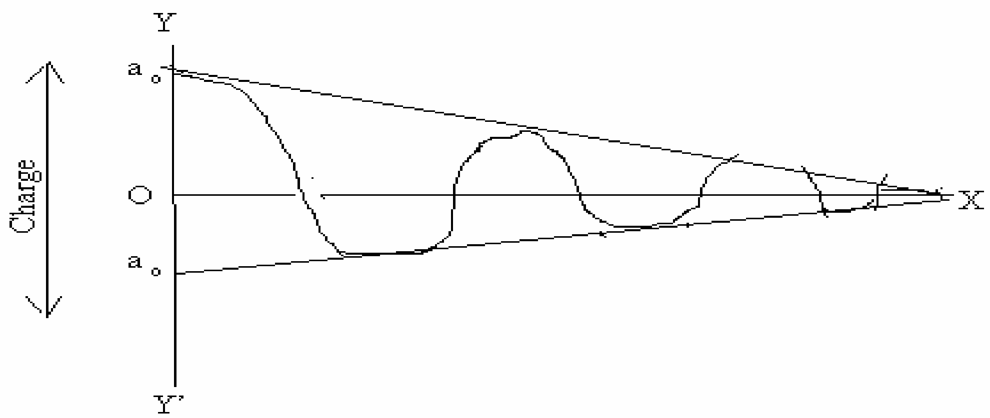
Fig(iii) shows the stage when the total charge on the capacitor disappears. The energy stored in the capacitor is transferred entirely to the magnetic field of the inductor. In this case, current is maximum in the inductor. This current continues to transfer +ve charge from the top plate of the capacitor the bottom plate of the capacitor fig(iv). The energy now flows from inductor back to capacitor as the electric field starts setting up again. Fg(v) shows the complete transfer energy back to the capacitor which is just like the situation of fig(i), except that the capacitor is oppositely charge.

The capacitor will begin to discharge again and the current will now flow clockwise as shown in fig(vi). Proceeding as before we find that the circuit eventually returns to the situation Shown in fig(i). The process continues at a definite frequency f and hence an angular frequency ω=2πf. The energy is continuously transferred between the electric field of the capacitor and the magnetic field of the inductor.

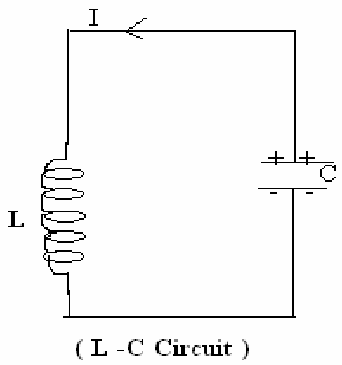
If no resistance is present in L-C circuit then L-C Oscillation will continue in definitely as shown below

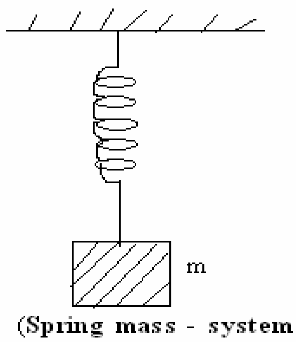


In actual L-C circuit some resistance is always present. This resistance will drain energy from the electric and magnetic field and stores in the form of heat energy. Thus, the L-C oscillation will not continue in definitely.



Electrical mechanical Analogy:





Let us consider an oscillating L-C system and an oscillating spring mass system. The L-C system posses two type of energy i.e. energy stored in electrical field $\left(U_E = \frac{1}{2}\frac{q^2}{c}\right)$ and energy stored in the magnetic field $\left(U_B = \frac{1}{2}LI^2\right)$ while the spring mass system has two energy i.e. the potential energy stored in the spring (= $1/2Kx^2$) and kinetic energy of the moving mass (= $1/2mv^2$). Thus an analogy can be seen between the two pairs of he energy. The equations for the velocity (v) and current (I).

$$v = \frac{dx}{dt}$$

$$I = \frac{dq}{dt}$$

Tell us that q corresponds to x and I correspond to v. These corresponds suggest that in the energy expressions 1/c corresponds to k and L corresponds to m. Thus, it is suggested that in an L-C system, the capacitor is mathematically like the spring in spring mass system and the inductor is like the mass.

The angular frequency of a spring mass system (frictionless) is $\varpi = \sqrt{k/m}$.

The corresponded mention above suggest that to find angular frequency of oscillating L-C circuit (Resistance less), K should be replaced by 1/c and m/L so that

$$\omega = \sqrt{1/Lc}$$

*** For numerical:**

Spring mass system:

Energy in spring = ½ Kx^2 = ½ Fx

F = -Kx

K = force constant or spring constant.

K.E. of mass = ½ mv^2

$$\omega = \sqrt{K/m}$$

ω = 2πf

*** For L-C system:**

$$U_E = \frac{1}{2}\frac{q^2}{c}$$

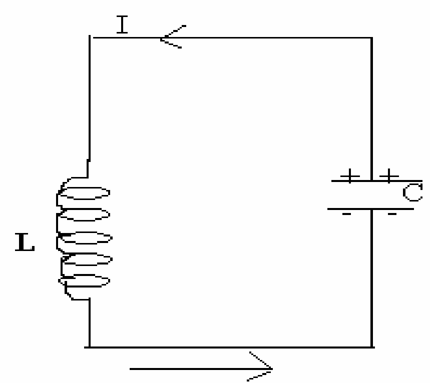
$$V_m = \frac{1}{2}LI^2$$

$$\omega = \sqrt{1/Lc}$$

ω = 2πf

$$\therefore f = \frac{1}{2\pi} \times \frac{1}{\sqrt{Lc}}$$

L-C oscillation (quantitatively):



Let us consider an oscillating L-C circuit (resistance less). The total energy present at any time is given by

$U = U_E + U_B$

[where U_E = Energy stored in the electric field of capacitor]

and, U_B = Energy stored in the magnetic field of Inductor

$$\therefore U = \frac{1}{2}\frac{q^2}{c} + \frac{1}{2}LI^2 \quad -----(i)$$

where, c = capacitance of the capacitor

q = charge present on the capacitor

L = Inductance of the inductor

I = current flowing through inductor

Since, there is no resistance in the circuit hence, no energy is transferred to heat and total energy i.e. U remains constant with respect to time

$$i.e. \frac{du}{dt} = 0$$

Thus, equation (i) becomes,

$$\frac{du}{dt} = \frac{d}{dt}\left(\frac{1}{2}LI^2 + \frac{1}{2}\frac{q^2}{c}\right) = 0$$

$$= \frac{1}{2} \times L \times 2I \times \frac{dI}{dt} + \frac{1}{2} \times \frac{1}{c} \times 2q \times \frac{dq}{dt} = 0$$

$$or,\ LI\frac{dI}{dt} + \frac{q}{c} \times \frac{dq}{dt} = 0 \quad -----(ii)$$

$$but,\ \frac{dq}{dt} = I$$

$$and,\ \frac{dI}{dt} = \frac{d}{dt}\left(\frac{dq}{dt}\right) = \frac{d^2q}{dt^2} \quad -----(iii)$$

Thus, equation (ii) reduces to

$$LI\frac{d^2q}{dt^2}+\frac{q}{c}I=0$$

Dividing both sides by I, we get,

$$L\frac{d^2q}{dt^2}+\frac{q}{c}=0$$

$$\therefore \frac{d^2q}{dt^2}+\frac{1}{Lc}q=0 \;-----(iv)$$

which is the differential equation that describes the oscillations of a resistance **L-C circuit.**

From equation (iv), we can write,

$$\frac{d^2q}{dt^2}+\omega^2 q=0 \;-----(v) \qquad \text{where, } \omega=\frac{1}{\sqrt{Lc}}$$

The general solution of equation (v) is q = A Cos ω t + B Sin ω t ----- (vi)

Where, A and B = constant

Initially, t = 0, q = q_o (the initial charge)

Thus, from equation (vi), we get,

A = q_o ----- (vii)

Now, the instantaneous current is given by,

$$I=\frac{dq}{dt}=\frac{d}{dt}\left[ACos\,\omega t+BSin\,\omega t\right]$$

$$or,\; I=-A\omega Sin\,\omega t+B\omega Cos\,\omega t \;-----(viii)$$

when, t = 0 and I = 0 (initial current)

Thus, above equation provides,

B = 0 ----- (ix)

Putting the values of A and B in equation (vii), we get,

q = q_oCosωt ----- (x)

Thus, the charge on the oscillator is an oscillatory charge. From equation (x) is follows that the maximum value of charge on the capacitor is q_o because the maximum value of Cosωt is 1(one).

Now,

$$I=\frac{dq}{dt}=\frac{d}{dt}\left(q_o Cos\,\omega t\right) \qquad \text{[Using equation (x)]}$$

$$\therefore I=-q_o\omega Cos\,\omega t \;-----(xi) \qquad \left[\therefore \frac{dq_o}{dt}=0\right]$$

Thus, the maximum value of current is $q_o\omega$,

Again,

$$\omega^2=\frac{1}{LC}$$

$$or,\ (2\pi f)^2 = \frac{1}{LC} \qquad \text{[where, } f = \text{frequency]}$$

$$or,\ 2\pi f = \frac{1}{\sqrt{LC}}$$

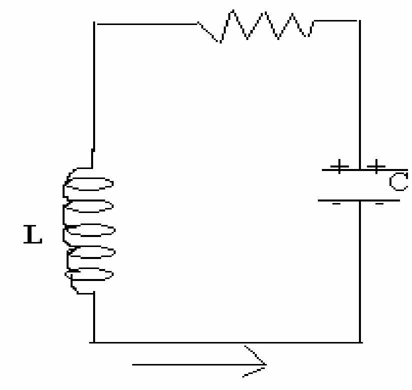
$$or,\ f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \ -----(xii)$$

which determines the frequency of L-C oscillation.

Damped oscillation (L-C-R circuit):

[Free Oscillation: there is no friction or opposition (ideal) {oscillation made in the absence of external resistance

Damped Oscillation: Oscillation made in this case amplitude, decreases gradually]



Oscillations in an L-C circuit are damped when dissipative element i.e. resistance R is also present in the circuit. In this case the total electro-magnetic energy of the circuit does not remain constant but it decreases with time because energy is transferred to thermal energy. So, the L-C oscillation will not continue in definitely and the amplitude of oscillation decreases gradually. These oscillations are said to be damped oscillation.

Since electromagnetic energy is not stored in the resistance. Hence, the total electromagnetic energy at any instant is given by,

$$U = U_B + U_E$$

$$U = \frac{1}{2}LI^2 + \frac{1}{2}\frac{q^2}{C} \ -----(i)$$

where, U_B = Energy stored in magnetic field of inductor
U_E = Energy stored in electric field of capacitor.
C = capacitance of the capacitor
q = charge present on the capacitor
L =Inductance of the inductor
I = current flowing through inductor

The rate of transfer of energy to the thermal energy is given by,

$$\frac{du}{dt} = -I^2R \ -----(ii)$$

The –ve sign shows that U decreases with time.
From equations (i) and (ii), we get,

$$\frac{du}{dt} = U_E + U_B = \frac{1}{2}LI^2 + \frac{1}{2}\frac{q^2}{C}$$

$$\frac{du}{dt} = LI\frac{dI}{dt} + \frac{q}{C}\times\frac{dq}{dt} = -I^2R$$

putting $I = \frac{dq}{dt}$ and $\frac{dI}{dt} = \frac{d^2q}{dt^2}$

$$LI\frac{d^2q}{dt^2} + \frac{q}{C}I = -I^2R$$

$$or,\ L\frac{d^2q}{dt^2} + \frac{q}{C} = -IR$$

$$or,\ L\frac{d^2q}{dt^2} + IR + \frac{q}{C} = 0$$

$$or,\ L\frac{d^2q}{dt^2} + R\cdot\frac{dq}{dt} + \frac{q}{C} = 0 \qquad \left[\therefore I = \frac{dq}{dt}\right]$$

$$\therefore \frac{d^2q}{dt^2} + \frac{R}{L}\cdot\frac{dq}{dt} + \frac{q}{LC} = 0 \ -----(iii)$$

which is the differential equation for damped oscillation in an L-C-R circuit.
The solution of above differential equation is given by,

$$q = q_o e^{\frac{-R}{2L}} Cos\left(\omega^| t + \phi\right) \ -----(iv)$$

where, $\omega^| = \sqrt{\omega^2 - \left(R/2L\right)^2} \ -----(v)$

and, $\omega = \frac{1}{\sqrt{LC}} \ -----(vi)$

For small damping R should be very small.

Hence, $\omega \approx \omega^|$

**** Forced oscillation and resonance:**

In undamped (free) L-C circuit (without resistance) and damped L-C-R circuit (with very small R) the charge, current and potential difference oscillate at angular frequency of $\omega = 1/\sqrt{LC}$. Such oscillation are said to be free oscillation and the angular frequency ω is said to be the circuit natural angular frequency.

*** Forced Vibration:**

Vibration of a body in the presence of a periodic force with a frequency other than its natural frequency.

*** Resonant or sympathetic vibration (special case of force vibration):**

If the frequency of periodic force and natural frequency of the body become equal then body vibrates with maximum amplitude. This vibration of the body with maximum amplitude is called resonant or sympathetic vibration.

*** Resonance:**

The phenomenon in which a body vibrates with maximum amplitude under in effect of a periodic force of frequency equals to its natural frequency is called resonance.

R = opposite of current

L = opposite of current by coil = inductive reactance ($X_L = \omega_L = \Omega$)
C = capacitance = opposition offered by the capacitor is called capacitive reactance

$$X_C = \frac{1}{\omega_C}$$

*** Forced oscillation (vibration) and resonance:**

In undamped L-C circuit (without resistance) and damped L-C-R circuit (with very small R), the charge, current and potential difference oscillate at angular frequency of $\omega = 1/\sqrt{LC}$. Such oscillation are said to be the circuit natural angular frequency.

When the external alternating e.m.f. $E = E_o \sin \omega_d t$, where ω_d is the driving angular frequency, is connected to an L-C-R circuit the oscillations of the charge, current and potential difference are said to be force oscillation (or driven oscillation). These oscillation always occur at the driving angular frequency ω_d. However the amplitude of the oscillations very mch depend upon how close ω_d is toω. When ω_d becomes equal to ω the resonance occurs and the current in the circuit becomes maximum. In this case the capacitive reactance (X_C) becomes equal to the inductive reactance and the current and e.m.f. remain in phase i.e. phase difference = 0.

At resonance,

$$X_C = X_L$$

$$or,\ \frac{1}{\omega_C} = \omega L$$

$$or,\ \omega^2 = \frac{1}{LC`}$$

$$or,\ \omega = \frac{1}{\sqrt{LC}}$$

$$or,\ 2\pi f_o = \frac{1}{\sqrt{LC}}$$ [where, f_o = resonant frequency]

$$\therefore f_o = \frac{1}{2\pi\sqrt{LC}}$$

which determines the resonant frequency.

Formula:

(i) $U_E = \frac{1}{2}\frac{q^2}{C}$ [Energy stored in electric circuit]

(ii) $U_B = \frac{1}{2}LI^2$ [Energy stored in magnetic field]

(iii) L-C circuit

$$U = U_E + U_B = \text{Constant}$$

when $U_B = \max imum, \quad U_E = 0$

$$\therefore U = U_B^{\max.}$$

again, when $U_E = \max$, $U_B = 0$

$$\therefore U = U_E^{\max.}$$

(iv) $\omega = \dfrac{1}{\sqrt{LC}}$

$$\omega = 2\pi f$$

For L-C-R:

$$\omega^{|} = \sqrt{\omega^2 - \left(\frac{R}{2L}\right)^2}$$

$$\omega^{|} = 2\pi f^{|}$$

$$\omega = 2\pi f$$

(v) Resonant frequency (f_C)

$$f_C = \frac{1}{2\pi\sqrt{LC}}$$

** REMEMBER:

SI unit of L = H (henry)
SI unit of C = F (Farad)
SI unit of R = Ω (Ohm)
Angular frequency = rev./sec
Frequency = H_Z

Numericals:

1. What is the capacitance of an oscillating L-C circuit if the maximum charge on the capacitor is 1.60μc and the total energy is 140μJ.

So|^n: Given,

q_m = 1.60μc = 1.60×10^{-6}c
Total energy (v) = 140μJ
Capacitance of capacitor (C) = ?

When charge or the capacitor is maximum, we have

$$U = U_E^m$$

Now, $U_E^m = \dfrac{1}{2}\dfrac{q_{\max}^2}{C}$

$$\therefore C = ?$$

2. A 1.50mH inductor in an oscillating L-C circuit stores a maximum energy of 10μJ. What is the maximum current?

So|n: Given,

$L = 1.50mH = 1.50\times10^{-3}H$

$U_B^{max} = 10\mu J = 10\times10^{-6}J$

$I_{max} = ?$

3. In an oscillating L-C circuit L is 1.10mH and C is 4.0μF. The maximum charge on the capacitor is 3.0μC. Find the maximum current.

So|n: Given,

$L = 0.10mH = 0.1\ 0\times10^{-3}H$

$C = 4\mu F = 4\times10^{-6}H$

$q_{max} = 3\mu C = 3\times10^{6}C$

$I_{max} = ?$

For maximum current,

$$U_E^m = U_B^{max}$$

$$or, \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}LI^2 = \frac{1}{2}CV^2$$

Displacement current:-

The displacement current is that current which comes into plate in the region, whenever the electric field and hence the electric flux is changing with time. The displacement current is given by,

$$I_D = \varepsilon_o = \frac{d\phi_E}{dt}$$

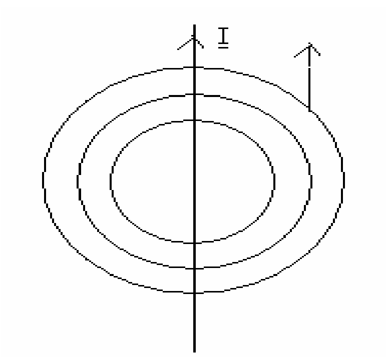
where, ε_0 = permittivity of air (or vacuum)

$\frac{d\phi_E}{dt}$ = rate of change of electric flux

In case of a steady (constant) electric flux linked with the region, the displacement current is O.

Application:

Maxwell's displacement current:



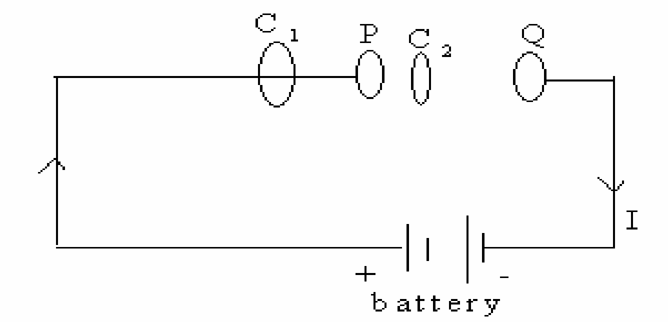
From Ampere's circuital law, we have,

$$\oint_C \vec{B} \cdot \vec{dt} = \mu_o I \;-----(i)$$

where, μ_o = permittivity of free space

I = a steady current threading the surface bounded by a closed loop C.

$\vec{B}$ = magnetic field



Maxwell's showed that the reflection (i) is logically in consistant .

Let us consider a parallel plate capacitor having plates P and Q being charge by a battery. During the charging a current I flows through the connecting wire which changes with time and hence produces a magnetic field around the wire which can be detected on using a magnetic niddle. Let us consider two loops C_1 and C_2 parallel to the plate P and Q of the capacitor. The loop C_1 encloses the connecting wire attach to the plate P and C like in region between the . Thus for loop C_1, the amper circuit law provides,

$$\oint_{C1} \vec{B} \cdot \vec{dt} = \mu_o I \;-----(ii)$$

Similarly for loop C_2, we can write,

$$\oint_{C2} \vec{B} \cdot \vec{dt} = 0 \;-----(iii)$$ [C_2 does not encloses current]

The equations (ii) and (iii), continue to be true even if C_1 and C_2 are very-very close to the plate P. But, it is expected that

$\oint_{C1} \vec{B} \cdot \vec{dt} = \oint_{C2} \vec{B} \cdot \vec{dt} \;-----(iv)$ [$\therefore$ as C_1 and C_2 are very very closed to P and hence close to each other]

The equation (iv) shows contradiction with equations (ii) and (iii). This led Maxwell to point out that amperes circuital law given by equation (i) is logically inconsistent.

Maxwell modified ampere circuital law to avoid the contradiction as follows:

$$\oint_C \vec{B} \cdot \vec{dl} = \mu_o \left(I + \varepsilon_o \frac{d\phi_E}{dt} \right) \;-----(v)$$

where, ϕ_E = Electric flux across a loop.

And, $\varepsilon_o \frac{d\phi_E}{dt}$ = I_D, the displacement current

Thus, in general ampere circuital law is stated as

$$\oint_C \vec{B}\cdot\vec{dl} = \mu_o\left(I + I_D\right) \;-----(vi)$$

which is known as Ampere Maxwell's law. Here, I is the conduction current which arises due to the flow of electrons and I_P is the displacement current, which exist so long as electric field produces magnetic field.

Maxwell's modification of ampere circuital law given by equation (vi) shows that the some of the conduction current and displacement current (i.e. $I+I_D$) has the important property of continuity along a closed path although individually they may not be continuous.

Now for loop C_1:

$$\phi_E = 0$$

$$\therefore \frac{d\phi_E}{dt} = 0$$

Thus, $I + I_D = I + 0 = I \;-----(vii)$

For loop C_2:

I = 0 (because there is no conduction current between the plates of the capacitor)

Thus, $I + I_D = I_D = \varepsilon_o \dfrac{d\phi_E}{dt} \;-----(viii)$

Now, the electric field at any instant between the plates of capacitor having charge q is given by,

$$E = \frac{q}{\varepsilon_o A}$$

where, A = area

Now, electric flux,

$$\phi_E = EA = \frac{q}{\varepsilon_o A}\times A = \frac{q}{\varepsilon_o}$$

Thus, $\dfrac{d\phi_E}{dt} = \dfrac{d}{dt}\left(\dfrac{q}{\varepsilon_o}\right) = \dfrac{1}{\varepsilon_o}\times\dfrac{dq}{dt} = \dfrac{1}{\varepsilon_o}I$

$$\therefore \varepsilon_o \frac{d\phi_E}{dt} = I$$

Thus, equation(viii) becomes,

$$I + I_D = \varepsilon_o \frac{d\phi_E}{dt} = I \;-----(ix)$$

From equations (vii) and (ix), it can be concluded that the sum $I+I_D$ has the same value on the left and right sides of plate P of the capacitor. Thus, $I+I_D$ has the property of continuity although individually they may not be continuous.

*** Relation between displacement current and the capacitance of a capacitor:**

The displacement current is given by

$$I_D = \varepsilon_o \frac{d\phi_E}{dt} \quad -----(i)$$

where ε_o = permittivity of air or vacuum

ϕ_E = Electric flux

But, ϕ_E = EA ----- (ii)

Where, E = Electric field between the two plates of the parallel plate capacitor.

A = area of each plate of the capacitor.

Thus, equation (i) becomes,

$$I_D = \varepsilon_o \frac{d}{dt}(EA)$$

$$\therefore I_D = \varepsilon_o A \frac{dE}{dt} \quad -----(iii)$$

Now, $E = \dfrac{V}{d}$

Where, V = potential difference between the plates of capacitor

d = separation between the plates of capacitor.

Thus, equation (ii) becomes,

$$I_D = \varepsilon_o A \frac{d}{dt}\left(\frac{V}{d}\right)$$

$$or,\ I_D = \frac{\varepsilon_o A}{d}\frac{dV}{dt}$$

$$\therefore I_D = C\frac{dV}{dt} \quad -----(iv)$$

where $C = \dfrac{\varepsilon_o A}{d}$, capacitance of parallel plate of capacitor with air between the plates.

Which is the required equation.

Numerical:

1. A L-C-R circuit has inductance L = 12mH, C = 1.6μF and resistance R = 1.5Ω
   a. At what time t will the amplitude of charge oscillation in the ckt. be 50% of its initial value.
   b. How many oscillations are completed within this time?

So|[n]: Given,

L = 12mH = 12×10^{-3}H

C = 1.6μF = 1.6×10^{-6}F

R = 1.5Ω

(a) $q = \dfrac{q_o}{2}$

t = ?

(b) no. of oscillation in time t = ?

we know that the charge oscillation in LCR circuit is

$$q = q_o e^{-R/2it} \cdot Cos(\omega^{|} t + \phi)$$

when charge is maximum,

$$Cos(\omega^{|} t + \phi) = 1$$

$\therefore$ we can write,

$$or, \frac{q_o}{2} = q_o e^{-R/2it}$$

$$t = ?$$

$$T = \frac{i}{f}$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

T = ?

$$\therefore \frac{t}{T} =$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$T = \frac{2\pi}{\omega}$$

$\therefore$ numbers of oscillations in time t = t/T

*** Induced magnetic field:**

From Faraday's law of induction, we know that a changing magnetic flux induces an electric field.

i.e. $\oint \vec{E} \cdot \vec{dl} = -\frac{d\phi_B}{dt} \quad -----(i)$

where, E is the electric field induces along a closed loop by the changing magnetic field ϕ_B encircled by that loop.

Similarly Maxwell's law of induction states that a changing electric flux induces a magnetic field.

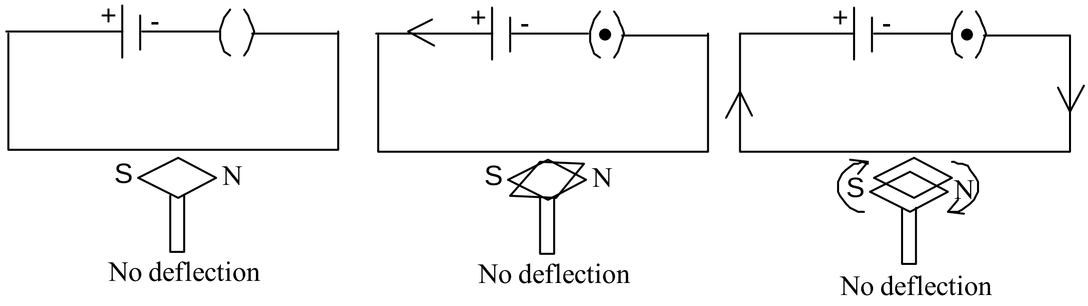
i.e. $\oint \vec{B} \cdot \vec{ds} = \mu_o E \frac{d\phi_E}{dt} \quad -----(ii)$

B is the magnetic field induces along a closed loop by the changing electric flux in the region encircled by that loop.

Electric flux

$$\phi_E = EA \qquad \therefore \frac{d\phi_E}{dt} = A\frac{dE}{dt} = \frac{A}{d} \times \frac{dV}{dt}$$

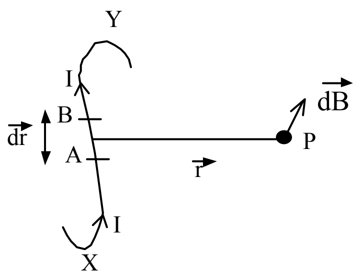
Oersted Discovery:



Oersted found that when a current is allowed to passed through a conductor magnetic field is produced around it. This phenomenon of production of magnetic field around a current carrying conductor is called magnetic effect of current.

Biot-Savart's Law:

This law deals with the magnetic field at a point due to a small current element i.e. a part of any conductor carrying current.



Let us consider a small element of length dl of the conductor XY carrying a current i. Let $\vec{r}$ be the position vector of the point P from the current element $\vec{dl}$ and θ be the agle between $\vec{dl}$ and $\vec{r}$.

According to Biot-Savart's law the magnetic field dB at the point P due to current element depends upon the factors stated below:

(i) $dB \propto I$
(ii) $dB \propto dl$
(iii) $dB \propto Sin\theta$
(iv) $dB \propto \frac{1}{r^2}$

Thus, on combining the above laws, we get,

$$dB \alpha \frac{IdlSin\theta}{r^2}$$

$$\therefore dB = K\frac{IdlSin\theta}{r^2} \quad -----(i)$$

Where, K = a constant of proportionality which depends upon the system of unit in which the various quantities are measured and also on the medium between point P and the current element.

In S.I. Unit:

$$K = \frac{\mu_o}{4\pi}$$

Where, μ_o is permeability.

$\mu_o = 4\pi\times10^{-7}Hm^{-1}$

Thus,

$$dB = \frac{\mu_o}{4\pi}\frac{IdlSin\theta}{r^2} \quad -----(ii)$$

In Vector notation:

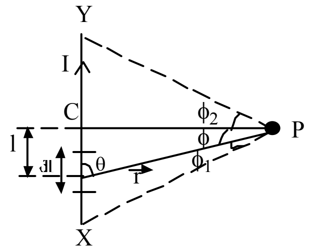
$$\overrightarrow{dB} = \frac{\mu_o}{4\pi}\frac{I\left(\overrightarrow{dl}\times\vec{r}\right)}{\left|\vec{r}\right|^3}$$

Thus, the magnetic field at point P due to whole conductor,

$$B = \int dB = \int \frac{\mu_o}{4\pi}\frac{IdlSin\theta}{r^2}$$

Application of Biot-Savart's law:

Magnetic field at a point due to a straight current carrying conductor:



Let, XY = a long straight conductor lies in the plane of the paper.

I = current flowing through the conductor from X to Y

P = a point at a distance PC = a, from the conductor where the magnitude field is to be determined.

dl = the length of the current element of the conductor having its centre O.

r = The position vector of the point P with respect to current element $\overrightarrow{dl}$

θ = angle between $\overrightarrow{dl}$ and $\vec{r}$

l = OC, the separation of point C from O.

Thus, from Biot-Savart's law the magnitude of the magnetic field at point P due to the current element is given by,

$$dB = \frac{\mu_o}{4\pi}\frac{IdlSin\theta}{r^2} \quad -----(i)$$

Where, μ_o = permeability of free space

From right-angled triangle OPC, we have

$$\theta + \phi = 90^o$$

$$or, \quad \theta = 90^o - \phi$$

$$or, \quad SSi\theta = Sin(90^o - \phi)$$

$$or, \quad SSi\theta = Cos\phi \quad -----(ii)$$

Again, $Cos\phi = \frac{q}{r}$

$$\therefore \quad r = \frac{a}{Cos\phi} \quad -----(iii)$$

Again, $\tan\phi = \dfrac{l}{a}$

$or, \quad l = a\tan\phi$

$\therefore \quad dd = a\,Sec^2\phi\, d\phi \quad -----(iv)$

Thus, using (ii), (iii) and (iv) in equation (i), we get,

$$dB = \frac{\mu_o}{4\pi}\frac{I\left(aSec^2\phi\, d\phi\right)Cos\phi}{\left(\dfrac{a^2}{Cos^2\phi}\right)}$$

$$\therefore dB = \frac{\mu_o}{4\pi}\frac{I}{a}Cos\,\phi\, d\phi \quad -----(v)$$

Thus, total magnetic field at point P due to whole conductor is given by

$$B = \int_{-\phi 1}^{\phi 2} dB$$

$$or, \quad B = \frac{\mu_o}{4\pi}\frac{I}{a}\int_{-\phi 1}^{\phi 2} Cos\,\phi\, d\phi \qquad \text{[Using equation (v)]}$$

$$or, \quad B = \frac{\mu_o}{4\pi}\frac{I}{a}\left[Sin\phi\right]_{-\phi 1}^{\phi 2}$$

$$or, \quad B = \frac{\mu_o}{4\pi}\frac{I}{a}\left[Sin\,\phi_2 - Sin\left(-\phi_1\right)\right]$$

$$\therefore \quad B = \frac{\mu_o}{4\pi}\frac{I}{a}\left[Sin\phi_2 + Sin\phi_1\right] \quad -----(vi)$$

Which is the re3quired expression for magnetic field at a point due to straight current carrying conductor.

For initially long conductor, we have,

$\phi_1 = \phi_2 = 90^o$

Thus, equation (vi) becomes,

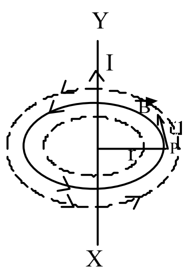
$$B = \frac{\mu_o}{4\pi}\frac{2I}{a} \quad -----(vii)$$

*** Ampere's circuital law:**

It states that the line integral of magnetic field $\vec{B}$ around any closed path in vacuum is equal to μ_o times the total current threading (flowing) the close path.

i.e. $\oint \vec{B}\cdot\vec{dl} = \mu_o I$

Proof:



Let, I = a current flowing through a long st. conductor XY lying in the plane of paper

B = magnitude of the magnetic field produced at a point P at a distance r (i.e. OP = r) from the conductor

Thus, we can write,

$$B = \frac{\mu_o}{4\pi}\frac{2I}{r} \quad -----(i)$$

Since, the magnetic lines of force are concentric circles perpendicular to the plane of paper. Hence, direction of $\vec{B}$ at every point is along the tangent to the circle. Let us consider a small element $\vec{dl}$ of the circle of radius r at P having direction same as $\vec{B}$. Thus, the line integral of $\vec{B}$ around the complete circle of radius r is given by,

$$\oint \vec{B}\cdot\vec{dl} = \oint B dl\, Cos\, 0^o$$

$$or, \quad \oint \vec{B}\cdot\vec{dl} = \oint \frac{\mu_o}{4\pi}\frac{2I}{r} dl \qquad \text{[using equation (i)]}$$

$$or, \quad \oint \vec{B}\cdot\vec{dl} = \frac{\mu_o}{4\pi}\frac{2I}{r}\oint dl$$

$$or, \quad \oint \vec{B}\cdot\vec{dl} = \frac{\mu_o I}{2\pi r}\times 2\pi r$$

$$\therefore \quad \oint \vec{B}\cdot\vec{dl} = \mu_o I \quad -----(ii)$$

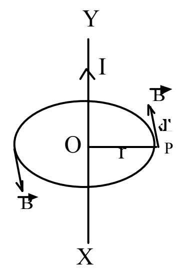
Which is proved.

From equation (ii), it is clear that the line integral of $\vec{B}$ around any close path is independent of the size of the path i.e. r.

Application of Ampere's circuital law:

(i) Magnetic field due to a long st. conductor carrying current:

(ii)



Let us consider a st. conductor (i.e. wire) XY carrying current I. Let us draw a circular loop of radius r (i.e. OP = r) around XY. The lines of force are circles hence at every point of a close line $\vec{B}$ is directed along the tangent to the circle at that point P. Thus,

$$\oint \vec{B}\cdot\vec{dl} = \oint B dl\, Cos\, 0^o$$

Where, $\vec{dl}$ = small element of the close path having the direction same as $\vec{B}$.

$$or, \quad \oint \vec{B}\cdot\vec{dl} = B\oint dl$$

$$\therefore \quad \oint \vec{B}\cdot\vec{dl} = B\times 2\pi r \qquad \left[\therefore \quad \oint dl = 2\pi r\right]$$

But from Ampere's circuital law, we have

$$\oint \vec{B}\cdot\vec{dl} = \mu_o I \quad -----(ii)$$

Where, μ_o =

Thus, from equation (i) and (ii), we get,

$$B\times 2\pi r = \mu_o I$$

$$\therefore \quad B = \frac{\mu_o I}{2\pi r} \quad -----(iii)$$

(ii) Magnetic field due to a solenoid carrying current:

A solenoid consists of an insulated long wire closely in the form of helix, having its length very large as compared to its diameter.

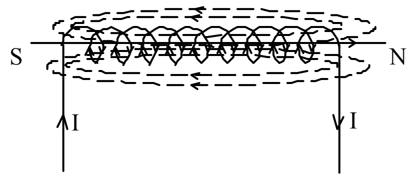


Fig: I

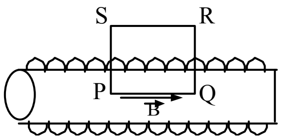


Fig: II

Let, n = no. of turns per unit length of a long solenoid
I = current flowing through the solenoid

When current flows through the solenoid the magnetic field set up in it is uniform and parallel to the length of the solenoid at points inside it. At point outside the solenoid the magnetic field is almost zero.

Let us consider a rectangle PQRS, so that PQ = L

Now, the line integral of magnetic field $\vec{B}$ over the closed path PQRS is given by,

$$\oint \vec{B}\cdot\vec{dl} = \int_P^Q \vec{B}\cdot\vec{dl} + \int_Q^R \vec{B}\cdot\vec{dl} + \int_R^S \vec{B}\cdot\vec{dl} + \int_S^P \vec{B}\cdot\vec{dl}$$

$$= \int_P^Q Bdl\,Cos\,0^o + \int_Q^R Bdl\,Cos\,90^o + \int_R^S \vec{B}\cdot\vec{dl} + \int_S^P Bdl\,Cos\,90^o$$

$$= BL + 0 + 0 + 0 \qquad \left[\therefore \int_R^Q dl = L \;\; and \;\; B = 0, \quad \therefore \int_R^S \vec{B}\cdot\vec{dl} = 0\right]$$

$$\therefore \oint \vec{B}\cdot\vec{dl} = BL \quad -----(i)$$

From, Ampere's circuital law, we have,

$$\therefore \oint \vec{B}\cdot\vec{dl} = BL \quad -----(i)$$

$$\oint \vec{B}\cdot\vec{dl} = \mu_o \times \text{no. of turns in the rectangle} \times I$$

$$or, \quad \vec{B}\cdot\vec{dl} = \mu_o \times nL \times I \qquad \text{[Using equation (i)]}$$

$$\therefore B = \mu_o nI \quad -----(ii) \qquad \text{[for air core]}$$

Which is the required expression

*** For iron-core**

$B = \mu nI$

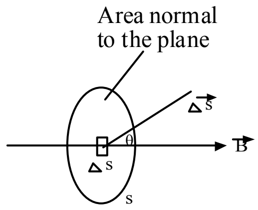
Where, μ = permittivity of iron

$\mu = \mu_o \mu_r$

μ_r = relative permittivity of iron

$$\boxed{\therefore \quad B = \mu_o \mu_r nI} \qquad \text{B increases} \quad [\therefore \mu_r > 1]$$

*** Magnetic flux:**



The magnetic flux through any surface held on a magnetic field is defined as the total no. of lines of force crossing the surface.

If $\overrightarrow{\Delta S}$ be the small element of area on the surface S held in the uniform magnetic field $\vec{B}$ then magnetic flux through it is given by $\Delta\phi$.

$$\Delta\phi = \vec{B}\cdot\overrightarrow{\Delta S} = B\Delta S\,Cos\,\theta$$

Where, θ = angle made by $\vec{B}$ with the normal to the surface.

Thus, the total magnetic flux over entire surface is given by,

$$\phi = \sum \vec{B}\cdot\overrightarrow{\Delta S} \qquad [\because \Delta S = 0]$$

Hence, we can write,

$$\phi = \int_S \vec{B}\cdot\overrightarrow{\Delta S}$$

If the surface is plane and has area A, then we can write,

$$\phi = \vec{B}\cdot\vec{A} = BA\,Cos\,\theta$$

For a coil of N turns is having area A,

The flux is given by,

$$\phi = BA\,Cos\,\theta$$

Special cases:

(i) If θ = 0º i.e. $\vec{B}$ is perpendicular to the surface, then

$\phi = BA$ (max.) → for 1 turn

$\phi = BAN$ (max.) → for nth term

(ii) If θ = 90º i.e. $\vec{B}$ is parallel to the plane of surface, then,

$\phi = 0$ (minimum)

*** Magnetic flux density(β):**

The quantity $\beta = \phi / A$ (i.e. magnetic flux per unit area normal to the surface is called magnetic flux density or magnetic induction)

Its S.I. unit is Wb (weber)

Since, $\phi = BA$

$$\therefore Wb = Tesla \times m^2$$

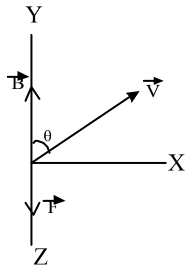
Thus, $1Wb = 1T \times 1m^2$

Thus, 1 weber is the amount of mag. flux passing normally through area 1m^2 held normal to a uniform mag. field of 1T.

*** Tesla:** the tesla is the S.I. unit of magnetic flux or magnetic field intensity.

The force acting on a charge moving on a uniform mag. field is given by,

$$F = BqVSin\,\theta$$

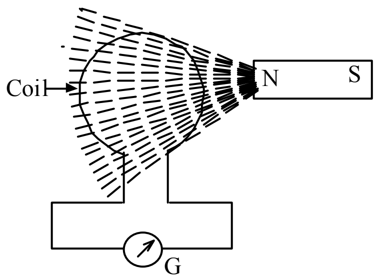


Where, B = magnitude of uniform magnetic field.
q = charge on the particle
θ = angle between $\vec{B}$ and $\vec{V}$

If q = 1C,
V = 1m/sec.
$\theta = 90^o$
and, F = 1N
then, B = 1T

Thus, the mag. field strength or mag flux density is said to be 1 tesla in which a charge of 1C moving with a velocity of 1m/s in the direction at right angles to the direction of magnetic field experiences a force of 1N.

 # Elector-magnetic Induction:
Faradays experiment:



Faraday experimentally found that induced emf. Appears in a circuit when ever the magnetic flux linked with a coil charges.

Experimental arrangement:
G = A galvanometer connected across a circular coil having one or more turns.
NS = A bar magnet which can be moved with respect to the coil.

Experimental Observations:

(i) Whenever there is a relative motion between the coil and magnetic the galvanometer shows a deflection indicating that current is induced in the coil.
(ii) Deflection is temporary and lasts so long as the relative motion between the coil and mag. continues.
(iii) The deflection increases when the magnet moved faster and decreases when moved slowly.
(iv) The direction of deflection is reversed when the same pole (N or S) moved away from the coil instead of moving it towards the coil or N pole is moved towards the coil instead of S pole.

The motion of the magnet implies that the no. of mag. lines of force crossing the coil is changing due to which induced emf. appears to the coil. Thus, the caused of induced emf. Is changed in the magnetic flux.

*** Faraday's laws of electro magnetic Induction:**

The phenomenon of generation of induced emf. in a coil due to charge in magnetic flux linked with it is called electro-magnetic induction.

There are two laws of electro magnetic induction given by Faradays.

(i) First law: Whenever the amount of magnetic flux linked with a coil changes, an induced emf. is developed in the circuit. The induced emf. last os long as the charge in magnetic flux continues.

(ii) The magnitude of induced emf. in a circuit is directly proportional to the rate of charge of magnetic flux linked with the circuit.

Explanation:

(i) First law:

When there is no relative motion between the magnet and the coil, the magnetic flux linked to the coil remains constant due to which the galvanometer does not give deflection. When the magnet is moved towards the coil (or coil is moved towards the magnet) the no. of magnetic lines of force linked with the coil increases. When the mag. is moved away from the magnet) the mag. lines of force linked with the coil decrease and hence magnetic flux also decreases. Ion the both cases he galvanometer shows deflection indicating that emf. is induced in the coil. Ths, the first law isexplained.

(ii) 2^{nd} law:

When magnet is moved faster the magnetic flux linked with the coil charges at a faster rate due to which the galvanometer deflection more. However deflection in the galvanometer becomes slower when the magnet is moved at the slower rate because the rate of change of magnetic flux is smaller. Hence, the magnitude of induced emf. varies of directly as the rate of charge of mag. flux linked with the coil. This is second law.

If ϕ_1 and ϕ_2 be the initial and final (i.e. after time t) magnetic flux linked with the coil then from Faraday's 2^{nd} law the induced emf.

$$e \alpha \frac{(\phi_2 - \phi_1)}{t}$$

$$e = K\frac{(\phi_2 - \phi_1)}{t}$$

Where, K = constant of proportionality

K= 1 (for all systm of unit)

If $d\phi$ be the small change of the mag. flux in small

$$e = \frac{-d\phi}{dt}$$

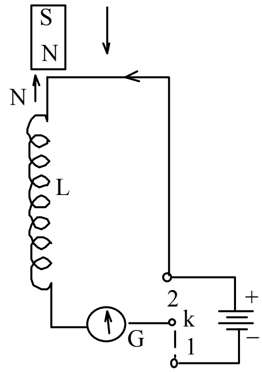
The –ve sign shows that induced emf. always opposes any charge in magnetic flux.

Lenz-law:

Lenz-law gives the direction of induced emf. or current in a circuit. According to this law, "the direction of induced emf. in a circuit is always such as to oppose the charge in mag. flux responsible for its production."

Thus, when mag. flux through a circuit is increase or decreased the induced emf. changed to oppose this increase or decrease.

 # Experimental verification:



Let us consider a coil L a few turn is connected to a battery B and a galvanometer G through a two way key K.

When the plug is inserted in the gap 1 a current flows in the circuit. The current at the upper end of the coil is anticlockwise which would produce N pole on this end the galvanometer gives deflection to the right. It is clear that the galvanometer deflection were to the left, current would be clockwise at the upper end, which would behave as S pole.

Now, the plug is inserted into the gap 2 on removing from from gap 1 of the key. A bar magnet is moved towards the coil so that N pole remains closer to it. The galvanometer shows a certain deflection to the right, whichindicates that current induced in the coil is anticlockwise and upper end of the coil behaves as North pole. Thus, inward motion of N-pole of the magnet is opposed due to which the increase in mag. flux, which is responsible for induced emf. is opposed. Similarly, when N-pole of magnet moved away from the coil the galvanometer shows a certain deflection to the left indicating that the induced current is clockwise and upper ends of coil behaves as South Pole. Thus, outward motion of N-pole of the magnet is oppose due to which decrease in magnetic flux is opposed.

If S-pole of magnet is moved instead of N-pole then exactly similar result are found thus induced emf. always opposes charge in mag. flux which produces it, which verifies Lenz law.

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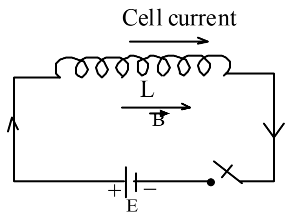
Lenz law and energy conservation:

The Lenz law is in accordance with the law of conservation of energy. For example, in the experimental verification of Lenz law when N pole of the magnet is moves towards the coil, the upper end of the coil acquires North polarity. Thus, to bring the N pole of the magnet closer to the coil, a work has to be done against the repulsion. Similarly, when N pole of the magnet is moved away from the coil, South polarity is developed on the upper end of the coil and hence a work has to be done against attraction. Thus, the mechanical work done in moving the magnet wit respect to the coil, changes into electrical energy to produce induced emf. in the coil. Thus, the energy is being transformed only.

When the magnet has no relative motion with respect to the coil, workdone is zero and hence nduced emf. is also not produced.

Hence, the Lenz law abeys the law of conservation of energy and can be treated as a consequence of the principle of energy conservation.

- **Self Induction:**



The self induction is the property of a coil by virtue of which the coil opposes any change in the straight of current flowing through it by inducing an emf. in itself.

Let us consider a circuit containing a coil, a cell E and a tapping key K.

When K is pressed current through the coil grows from zero to a max. value in a certain interval of time and hence magnetic flux linked with the coil also increases causing an induced current in the coil which opposes the growth of current in the coil by flowing in the opposite direction to the direction of flow of cell current, according to Lenz law.

When K is released, the current through the coil deceases from max. to zero value in certain interval of time and hence magnetic flux also decreases to cause an induced current in the coil by flowing in opposite direction of self current.

*** Coefficient of self induction:**

The magnitude of magnetic flux linked with a coil at any time is directly proportional to the current flowing through it at that time

i.e. $\phi \alpha I$

where, I = current flowing through a coil at any time when the flux linked with it is ϕ

$\therefore \phi \alpha LI$

Where, L = coefficient of proportionality known as the coefficient of self induction of given coil

If I = 1 then $\phi = L$

Thus, the coefficient of self induction of a coil is numerically equal to the amount of magnetic flux linked with it when a unit current flows through the coil. Again, from Faradays law of electromagnetic induction

$$e = -\frac{d\phi}{dt}$$

Where, e = induced emf. in the coil

$$or, \quad e = -\frac{d}{dt}(LI)$$

$$\therefore \quad e = -L\frac{dI}{dt}$$

When $\frac{dI}{dt} = 1, \qquad e = -L$

Thus, the coefficient of self induction of a coil is numerically equal to the induced emf. in the coil when the rate of change of current through the coil is unity.

Units of L:

The S.I. unit of L is Henry.

$\because \quad e = L\frac{dI}{dt}$ (neg. (-)ve sign)

$or, \quad L = \frac{e}{dI/dt}$

$\therefore \quad 1H = \frac{1V}{1A/\sec}$

Thus, the coefficient of self induction of a coil is said to be 1 Henry when a current changes at the rate of 1A/sec through the coil induces an emf. of 1V in it.

Again, $L = \frac{\phi}{I} = \frac{weber}{Ampere}$

$\therefore \quad 1H = 1wbA^{-1}$

*** Expression for the coefficient of self induction of a long solenoid:**

Let us consider a solenoid of cross-sectional area A and the number of turns per unit length n.

The magnetic field at any point of the solenoid is given by,

$B = \mu_o nI$

Where, μ_o = permeability of free space

I = current through the solenoid.

Hence, the magnetic flux linked with each term of the solenoid is given by

$\phi = BA$

$\therefore \quad \phi = \mu_o nIA \quad -----(iii)$ [Using equation (i)]

If ϕ_B be the total flux linked with the solenoid then, we can write,

$\phi_B = \mu_o nIA \times nl$

Where, nl = total no. of turns in length l of the solenoid.

If L be the coefficient of self induction of the long solenoid then, we can write,

$L = \frac{\phi_B}{I}$

$\therefore \quad L = \mu_o n^2 Al \quad -----(iv)$

Special case: For solenoid with iron core:

$L = \mu n^2 Al$

Where, μ =permeability of iron = $\mu_o \mu_r$

Where, μ_o = relative permeability of iron

$$\mu_o = \frac{\text{permeability of iron } (\mu)}{\text{permeability of free space } (\mu_r)}$$

Thus, $L = \mu_o \mu_r n^2 Al$

L α μ_r

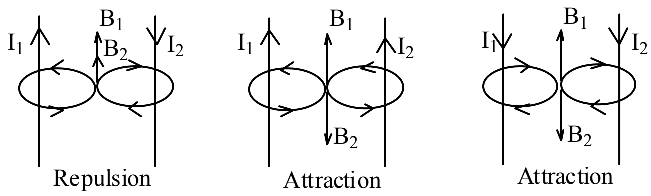
L α n^2

L α A

L α l

Numerical point of view:

* Forces acting between two parallel current carrying conductors:

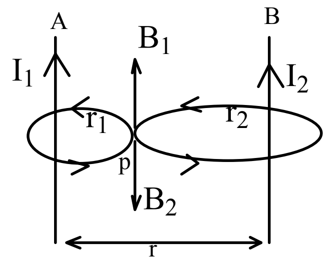


** when direction of current is same the force acting between the two parallel conductor is attractive.
** If the directions of the currents are opposite to each other then, the parallel conductors repel each other.

1. *Two long straight current carrying conductors are placed parallel to each other at 12cm apart. If the direction of the current in the conductor is same then determine the separation of a point where the net magnetic field is zero, when the current through the A and B are respectively 4A and 6A.*

So|^n: Given,

$I_1 = 4A$
$I_2 = 6A$
$r = 12cm = 0.12m$
$B_1 = B_2$ (at point P)
r_1 (or r_2) = ?



From question, at point P

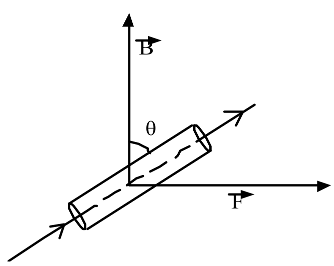
$B_1 = B_2$

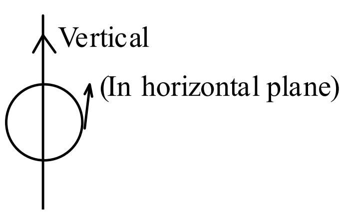
$$\frac{\mu_o}{4\pi}\frac{2I_1}{r_1} = \frac{\mu_o}{4\pi}\frac{2I_2}{r_2}$$

$$or, \quad \frac{4}{r_1} = \frac{6}{r - r_1} \qquad (\because \quad r = r_1 + r_2)$$

$r_1 = ?$ (m)

* Expression for the force acting between two parallel current carrying conductor:
Special cases:
(i) If θ = 0º or 180º,
Then, F=0 (minimum)
(ii) If θ = 90º i.e. $\vec{B}$ perpendicular to the conductor
F = BIl (maximum)





Therefore, **B** perpendicular **F** perpendicular **B**.

Force acting on a current carrying conductor in a uniform magnetic filed.

$F = BIL \sin\theta$

θ = angle made by with conductor

Let I_1 and I_2 = currents flowing through parallel conductors which are long A & B resp.
r = separation between the conductors

Now, the force acting per unit length of B due to a current in A is given by,
F = amg.field due to A ×current in B

$$\therefore \quad F = \frac{\mu_o}{4\pi}\frac{2I_1 I_2}{r} \quad -----(i)$$

Similarly, the force acting per unit length on A due to current n B may be obtain are

$$F^{|} = \frac{\mu_o}{4\pi}\frac{2I_2}{r} \times I_1 \quad -----(ii)$$

Thus, from equations (ii) and (iii), we have,

$$F = F^{|} = \frac{\mu_o I_1 I_2}{2\pi r} \quad -----(iii)$$

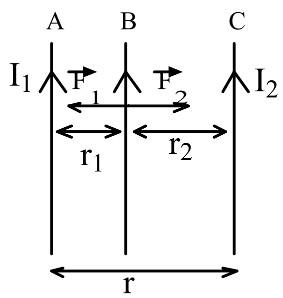
Numericals:

Two long parallel conductors are placed 10cm apart, the current flowing through them are 8A and 12A respectively, in the same direction. Determine the position of a third current carrying st. conductor placed between them. So that, net force acting on it is zero.

So$|^n$: Given,

$I_1 = 8A$
$I_2 = 12A$
r = 10cm = 0.10m
$F_1 = F_2$
r_1 (or r_2) = ?

From question, net force acting on third conductor see is zero
i.e. force $F_1 = F_2$
F_1 = force per unit length on C due to A
F_2 = force per unit length on C due to B



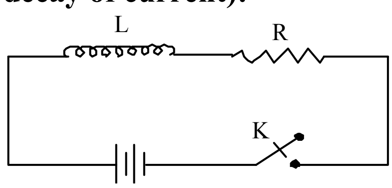
$$\frac{\mu_o I_1 I}{2\pi r_1} = \frac{\mu_o I_2 I}{2\pi r_2}$$

Where, I = current flowing through

$$\frac{I_1}{r_1} = \frac{I_2}{r - r_1} \qquad (\because \quad r = r_1 + r_2)$$

$r_1 = ?$

- **L – R circuit (growth and decay of current):**



Let us consider an electric circuit in which an inductor and a resistor, a battery E and a tapping key K are connected in the series. When 'K' is open, there is no current in the circuit.

Now, K is pressed to complete the circuit due to which the current grows from O to I in time t=0 to t=t. When current through L changes an induced emf is produced in it which opposes the growth of current if E be the potential difference applied in the circuit, the we can write,

$$E+\left(-L\frac{dI}{dt}\right)=IR$$

$$or,\quad E-IR=L\frac{dI}{dt}$$

$$or,\quad \frac{dI}{E-IR}=\frac{1}{L}dt$$

$$or,\quad \int_0^I\frac{dI}{E-IR}=\frac{1}{L}\int_0^t dt$$

$$or,\quad -\frac{1}{R}\log\left(\frac{E-IR}{E}\right)=\frac{1}{L}t$$

$$or,\quad \log\left(\frac{E-IR}{E}\right)=-\frac{R}{L}t$$

$$or,\quad \frac{E-IR}{E}=e^{-\frac{R}{L}t}$$

$$or,\quad 1-\frac{I}{E}R=e^{-\frac{R}{L}t}$$

$$or,\quad \frac{IR}{E}=1-e^{-\frac{R}{L}t}$$

$$or,\quad I=\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$$

$$\boxed{\therefore\quad I=I_o\left(1-e^{-\frac{R}{L}t}\right)}$$

Where, $I_o=E/R$ max. current in the circuit.

It is clear from above equation that, the current gradually grows from O (i.e. at t=0) to a max. value after a long time.

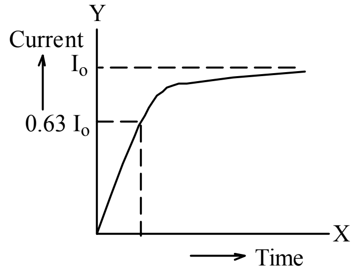
At $t=\tau$ $(where,\quad \tau=L/R)$

We get,

$$I=I_o\left(1-\frac{1}{e}\right)$$

$$or,\quad I=I_o\left(1-\frac{1}{2.718}\right)$$

$$\therefore\quad I=0.63I_o$$



Thus, in one time constant current remains 63% of the max. value. The constant L/R=τ, is known as the time constant of the L – R circuit which tells us how faster the current grows. If the time constant is small the growth is steep (sharp). It is clear that $I = I_o$ when $t = \infty$. In practice a small no. of time constants may be sufficient for the current to rach almost the max. value.

$$I = I_o\left(1 - e^{-\frac{R}{L}t}\right)$$

$$also, \quad I = I_o\left(1 - r^{-\frac{t}{\tau}}\right)$$

for numerical problems

*** Decay of current:**

When the key is left (i.e. the battery is disconnected) the current decreases in the circuit due to which the induced emf. $-L\,{dI}/{dt}$ is produced in the inductor. Thus, we can write,

$$-L\frac{dI}{dt} = IR$$

$$or, \quad \frac{dI}{I} = -\frac{R}{L}dt$$

At, t = 0, I = I_o

And at t = t, I = I

Thus,

$$\int_{Io}^{I}\frac{dI}{I} = -\frac{R}{L}\int^{t} dt$$

$$or, \quad [\ln I]_{Io}^{I} = -\frac{R}{L}[t]_o^t$$

$$or, \quad \ln\frac{I}{I_o} = -\frac{R}{L}t$$

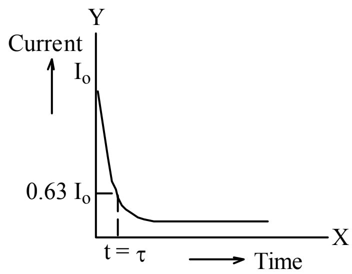
$$or, \quad \frac{I}{I_o} = e^{-\frac{R}{L}t}$$

$$\therefore \quad I = I_o e^{-\frac{R}{L}t} = I_o e^{-\frac{t}{\tau}} \qquad \text{[where, } \tau = L/R = \text{time constant]}$$

Thus, it is clear that, the current gradually decreases with respect to time, when t = τ, we have,

$$\boxed{I = \frac{I_o}{e} = 0.37 I_o}$$

i.e. the current decreases to 37% of the initial value in one time constant. Decay is steep (sharp).



Numericals:

1. An inductor (L=20mH), a resistor (R=100Ω) and a battery (E=10V) are connected in series. Find
   a. The time constant
   b. The maximum current
   c. The time elapsed before the current reaches 99% of the maximum value

So|ⁿ: Given,

L = 20mH = 20×10^{-3}H
R = 100Ω
E = 10volts

(a) time constant, τ = ?

$$\tau = \frac{L}{R} = \frac{20\times10^{-3}}{100} = 2\times10^{-4}\text{ sec}$$

(b) maximum current, I_o = ?

$$I_o = \frac{E}{R} = \frac{10}{100} = 0.1A$$

(c) I = 99% of I_o
t = ?

$$\therefore \quad I = I_o\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$or, \quad \frac{99}{100} I_o = I_o\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$or, \quad 0.99 = 1 - e^{\frac{-t}{\tau}}$$

$$or, \quad e^{\frac{-t}{\tau}} = 1 - 0.99$$

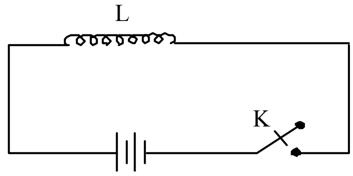
$$or, \quad -\frac{t}{\tau} = \ln(0.01)$$

$$or, \quad \frac{t}{\tau} = \ln(100)$$

$$\therefore \quad t = \tau\times\ln(100) = 0.92m\text{sec} = 0.92\times10^{-3}\text{ sec}$$

2. An inductor (L=20mH), a resistor (R=100Ω) and a battery (E=10V) are connected in series. After a long time, the circuit is short circuited and then battery is disconnected. Find the current, n the circuit 1msec after short circuiting.
    After long time ($I=I_o=E/R=$………..A)
   $I^{|} = I_o e^{-t/\tau}$
   $t = ?$
   $\tau = ?$
ans(6.7×10^{-4}A)

- **Energy stored in a magnetic field (energy stored in an inductor):**



Let us consider an electric circuit having an inductor of seet inductance L when a current builds up in the inductor, the magnetic flux linked with the circuit changes and hence a back emf. is induced in the circuit which is given by,

$$e = -L\frac{dI}{dt} \quad -----(i)$$

Where, I = instantaneous current in the circuit

The work done in moving the charge dQ against this emf. is given by,

$$dw = -edq = +L\frac{dI}{dt}\cdot dq \qquad \text{[Using equation (i)]}$$

$$or, \quad \left[\therefore \frac{dq}{dt} = I\right]$$

$$\therefore \quad dd = LIdt \quad -----(ii)$$

Hence, total work done in building up the current from O to I in the circuit, is given by

$$W = \int_0^I dw = \int_0^I LIdI$$

$$\therefore \quad U_B = \frac{1}{2}LI^2 \quad -----(iii)$$

Which is required expression for the energy stored in the magnetic field of the inductor.

*** Energy density of the magnetic field:**

The energy stored per unit volume of an inductoris called energy density in magnetic field.

Let, l = length near the middle of a long solenoid
A = cross-sectional area of the solenoid
n = no. of turns per unit length of solenoid

Thus, the volume associated with length l of the solenoid,

$$V = Al$$

The energy stored must be uniformly distributed within the solenoid, because the magnetic field is uniform everywhere, inside the solenoid, hence the energy stored per unit volume in the magnetic field of the solenoid is given by,

$$U_B = \frac{U}{Al}$$

$$or, \quad U_B = \frac{1}{2} LI^2 \times \frac{1}{Al}$$

$$or, \quad U_B = \frac{1}{2} \frac{L}{al} \times I^2$$

But, $L = \mu_o n^2 Al$ where, μ_o = permittivity of free space

$$\therefore \frac{L}{Al} = \mu_o n^2$$

Thus, $U_B = \frac{1}{2} \mu_o n^2 I^2$

$$or, \quad U_B = \frac{1}{2} \frac{(\mu_o nI)^2}{\mu_o} \qquad [B = \mu_o nI]$$

$$\therefore \quad U_B = \frac{1}{2} \frac{B^2}{\mu_o}$$ where, B = mag. field inside the long solenoid

Which is the required expression for energy density

Remember:

$$U_E = \frac{1}{2} \frac{q^2}{C}$$

$$U_E = \frac{1}{2} \varepsilon_o E^2$$

$$U_B = \frac{1}{2} LI^2$$

$$U_B = \frac{1}{2} \frac{B^2}{\mu_o}$$

*** Diamagnetism, paramagnetism and ferromagnetism:**

Diamagnetism:

The diamagnetism are the substances which are feebly (weakly) repel by a magnet. For example: Copper, Zinc, Gold, Silver, Water, H_e, etc.

Diamagnetism is due to the fact that the atoms of a diamagnetic substance do not posses any magnetic moment in the absence of external magnetic field. When an external magnetic field is applied the atoms acquire small magnetic moment in the direction opposite to the direction of external magnetic field and hence get feebly repeled.

Paramagnetism:

The substances which are feebly (weakly) attracted by a magnet are called paramagnetic substances. For example: Platinum, Chromium, Aluminum, etc.

The paramagnetism is due to the fact that the atoms of a paramagnetic substance posses small, magnetic moment even in the absence of external magnetic field. When an external magnetic field is applied the atoms acquire small magnetic moment in the direction of applied magnetic field and hence get feebly attracted by the magnet.

Ferro-magnetism:

The substances which are strongly attracted by a magnet are called ferro-magnetic substances. For example: Iron, Cobalt, Nickel, etc.

The ferro-magnetism is due to the fact that the atoms of a ferromagnetic substance posses appreciable magnetic moments due to vacancies in the inner electronic shells in which electrons are not pair with equal and opposite orbital magnetic moment and anti-parallel spin. Also, the magnetic moment of unpaired electron gets aligned in the same direction due to exchange interaction (i.e. the interaction between the unpaired electrons of the neighbouring atoms)

Properties of dia, para and ferromagnetic substances:

	Diamagnetic	Para-magnetic	Ferromagnetism
1.	They are feebly repelled by a magnet.	They are feebly attracted by a magnet.	They are strongly attracted.
2.	They tend to move stronger region to the weaker region of the field.	They tend to move from weaker to stronger region o the field.	They tend to move from weaker to stronger region of the field.
3.	When a rod of diamagnetic substance is suspended freely it sets slowly iself in the direction perpendicular o the direction of applied magnetic field.	When a rod of para-magnetic substances is suspended freely it set up slowly in the direction of applied mag. field.	When a rod of ferromagnetism substances is suspended freely it set up quickly in the direction of applied ield.
4.	When a piece of diamagnetic substance is placed in a magnetiing field, it gets freely magnetized in the opposite direction of te applied field.	When a piece of para-magnetic substance is placed in a magnetizing field, it gets feebly magnetized in the direction of the applied field.	When a piece of ferromagnetism substance is placed in a magnetizing field, it gets strongly magnetized in the direction of the applied field.
5.	The intensity of magnetization (I) is small and (-)ve.	I=small and (+)ve	I=large and (+)ve
6.	$B=H+4\pi I$ (in CGS system) $B=\mu_o(H+I)$ (in SI unit) Where, H = magnetic field B=magnetic induction I = (-)ve Thus, $B<H$	$B>H$	$B>>H$