Linear algebra

$$\begin{cases} -2(4x + 3y = 700\,g) \\ 2x + 6y = 1000\,g \end{cases} \Rightarrow \begin{cases} -8x - 6y = -1400\,g \\ 2x + 6y = 1000\,g \end{cases}$$

$$-6x = -400\,g$$

$$\boxed{x = 66.7}$$

$$2\times(66.7) + 6y = 1000$$

$$6y = 1000 - 133.5$$

$$\boxed{y = 144.42}$$

$$2\times 66.7 + 6\times 144.42 \approx 1000 \checkmark$$

$$\begin{cases} 5x + 3y + 5z = 6 \\ 7x + 2y + 8z = 17 \\ 4x + 3z = 8 \end{cases} \Rightarrow \underset{(3\times3)}{\begin{bmatrix} 5 & 3 & 5 \\ 7 & 2 & 8 \\ 4 & 0 & 3 \end{bmatrix}} \times \underset{(3\times1)}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{(3\times1)}{\begin{bmatrix} 6 \\ 17 \\ 8 \end{bmatrix}}$$

Determinant:

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 5 \\ 4 & 3 & 8 \end{bmatrix} = 1(3\times8 - 5\times3) - 2(0\times8 - 5\times4) + 1(0\times3 - 3\times4)$$

$$9 + 40 - 12 = \boxed{37}$$

$$\begin{cases} a+b=10 \\ a+2b=12 \end{cases} \rightarrow a=10-b$$

$10-b+2b=12$

$b=2$ (unique solution)

$a=8$ [complete non-redandant]

$$\begin{cases} a+b=10 \\ 2a+2b=20 \end{cases}$$ $\times$

the system is redandant (the same equation twice)

$b=10-a$

(∞ solutions possible)

$2a+2(10-a)=20$

[redandant singular]

$2a+20-2a=20$ $\times$

$20=20$

$$\begin{cases} a+b=10 \\ 2a+2b=24 \end{cases}$$

the system is contradictory (there is no solution.)

$b=10-a$

$2a+2(10-a)=24$

[contradictory singular]

$2a+20-2a=24$

$20\neq 24$

Linear equations

$a+b=10$

$2a+3b=15$

$3.4a+4.8b-2c=122.5$

non-linear

$a^2+b^2=10$

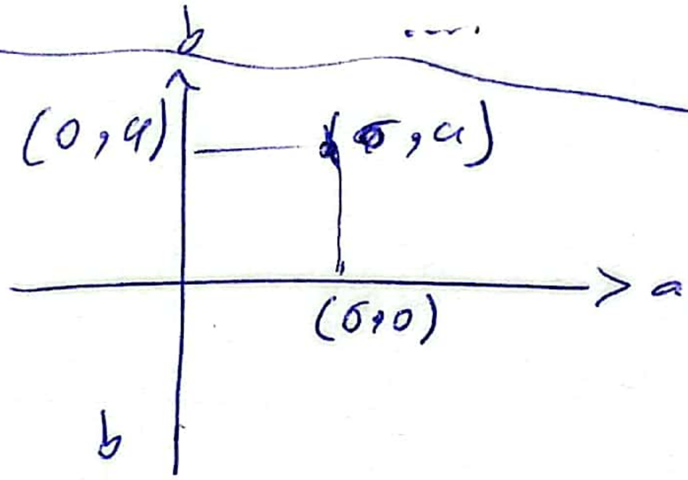
$\sin(a)+b^5=12$

$2^a+3^b=0$

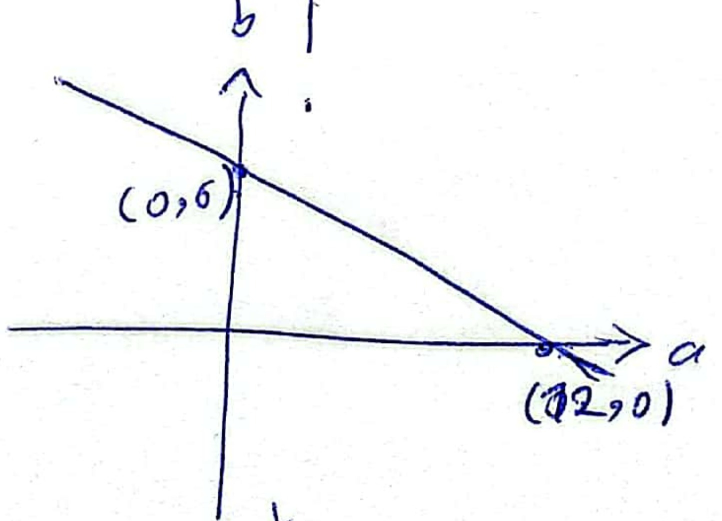
$ab^2+\frac{b}{a}-\frac{3}{b}=4^a$

...

$a+b=10$

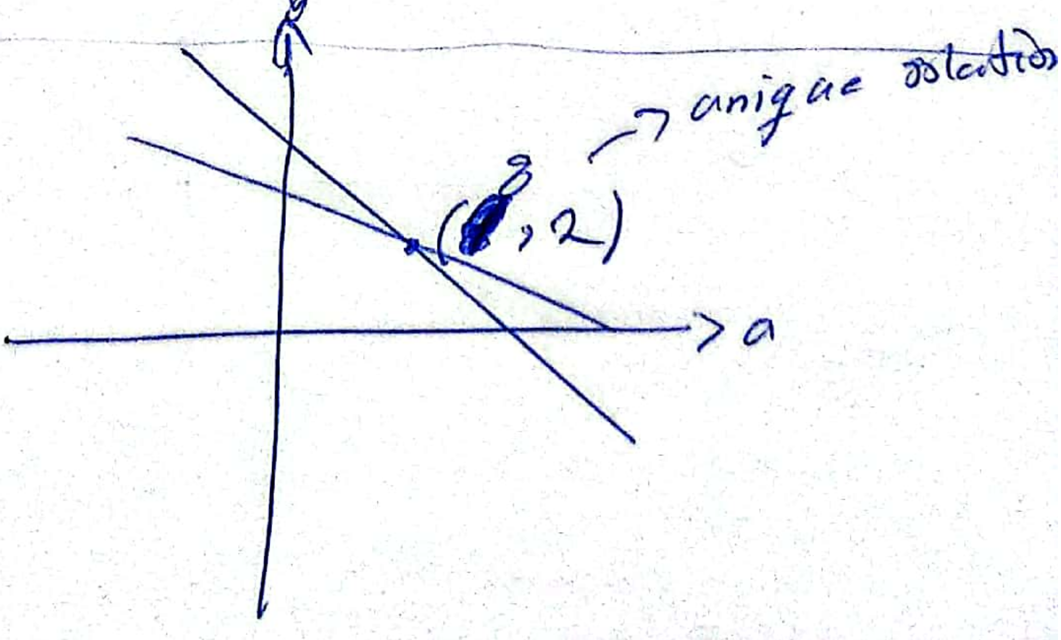


$a+2b=12$



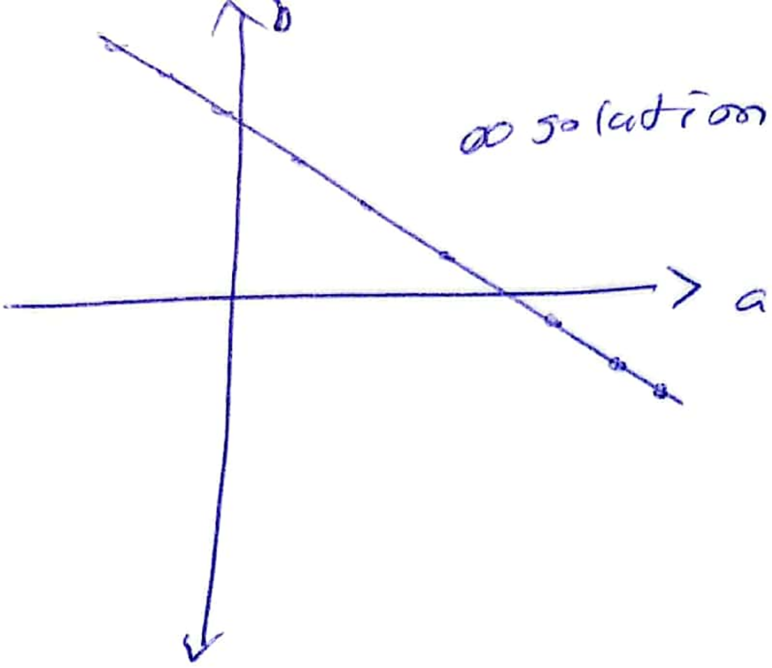
$a+b=10$

$a+2b=12$



$a + b = 10$

$2a + 2b = 20$

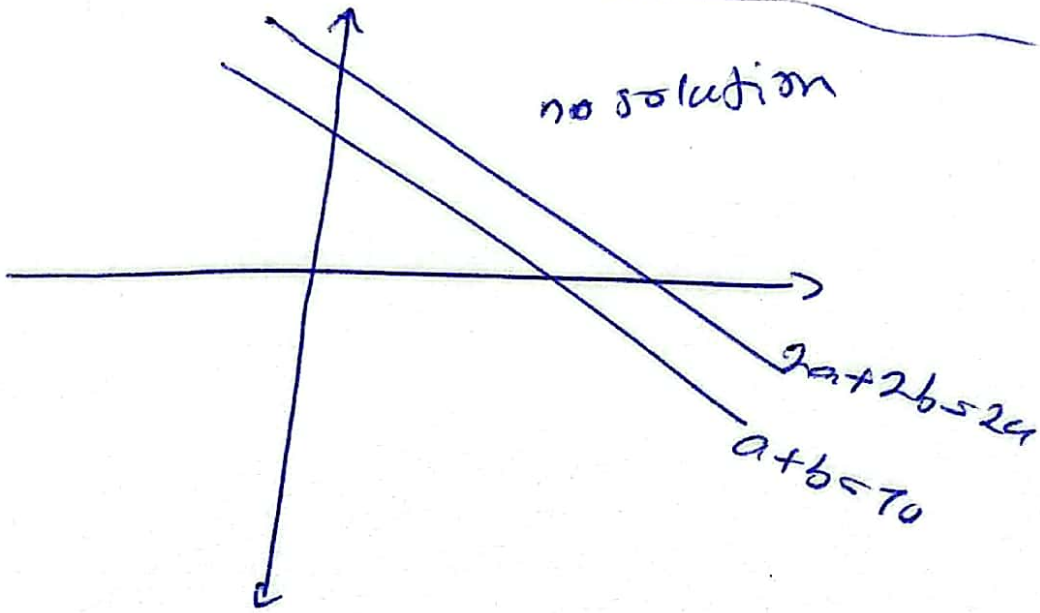


∞ solutions

$a + b = 10$

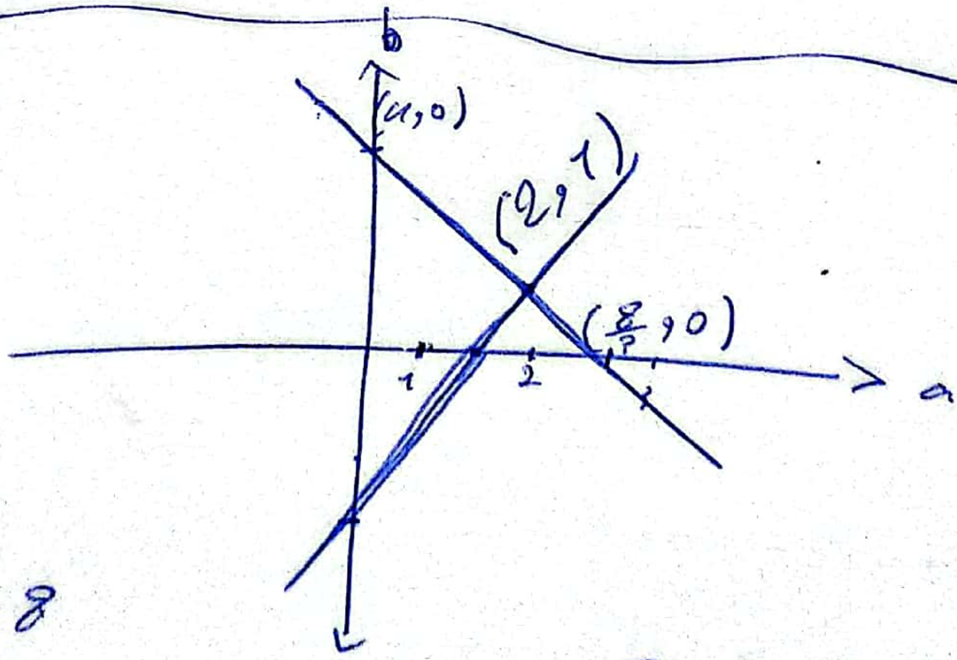
$2a + 2b = 24$

no solution



$3a + 2b = 8$

$2a - b = 3$



$a = \frac{3 + b}{2}$

$3\left(\frac{3+b}{2}\right) + 2b = 8$

$\frac{6 + 3b}{2} + 2b = 8 \Rightarrow 5b = 8 - 3$

$\boxed{b = 1}$ $\boxed{a = 2}$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

matrix is singular if:

$a \; b \times k = c \; d$

$ak = c \quad , \; bk = d$

$\frac{c}{a} = \frac{d}{b} = k \rightarrow$ Determinant $= \boxed{ab - bc}$

$\Rightarrow \boxed{ad - cb = 0}$

$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \Rightarrow$ determinant $= 1 \times 2 - 1 \times 1 = \underline{1}$

non-singalar

$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \Rightarrow$ determinant $= 1 \times 2 - 1 \times 2 = \underline{0}$

singalar.

$\begin{bmatrix} 5 & 1 \\ -1 & 3 \end{bmatrix} \Rightarrow D = (5 \times 3) - (1 \times -1) = 15 + 1 = \underline{16}$

non-singalar

$\begin{bmatrix} 2 & -1 \\ -6 & 3 \end{bmatrix} \Rightarrow D = (2 \times 3) - (-6 \times -1) = 6 - 6 = \underline{0}$

singalar.

6

$$\begin{cases} 2x+3y=15 \\ 2x+4y=16 \end{cases} \rightarrow 2x+3=15$$

$\boxed{x=6}$

$$\begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 15 \\ 16 \end{bmatrix}$$

$2x=15-3y$

$x=\frac{15-3y}{2}$

$$\begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix}$$

$2\left(\frac{15-3y}{2}\right)+4y=16$

$D=(2\times4)-(2\times3)=8-6=2$

non-singular

$\frac{30-6y}{2}+4y=16$

linear independent rows

$15-3y+4y=16$

$y=16-15$

$\boxed{y=1}$

$$\begin{cases} a+b+c=10 \quad)\times2 \\ a+2b+c=15 \\ a+b+2c=12 \end{cases}$$

$\cancel{2a+2b+2c=20}$

$-\cancel{a+2b+c=16}$

$a \cancel{+c} = 5$

$\boxed{\cancel{a=5-c}}$

$\cancel{5-c+b+2c=12}$

$a=3, \; b=5, \; c=2$

$a+b+c=10$

$a+b+2c=15 \Rightarrow c=5$

$a+b+3c=20$

$a+b=5$

$a=5-b$

There are infinitely many solutions.

$a+b+c=10$

$a+b+2c=15$

$a+b+3c=18$

There are no solutions.

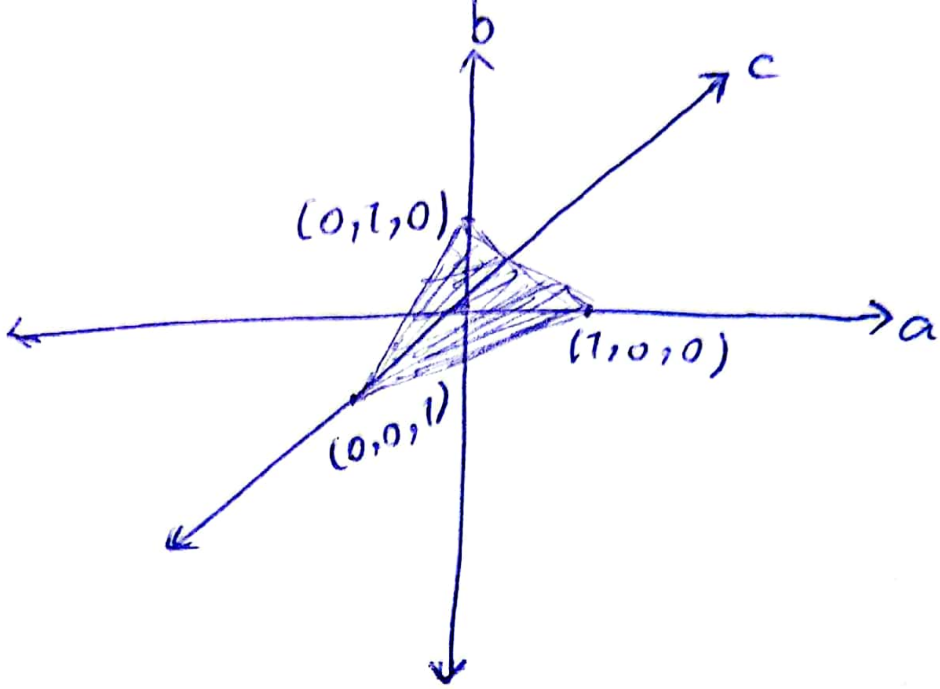
$a+b+c=10$

$2a+2b+2c=20$

$3a+3b+3c=30$

There are infinitely many solutions..

$a + b + c = 1$



- Determine if the following matrices have linearly dependent or independent rows:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 3 & 2 & 3 \end{bmatrix}$$

3 Row 1 + 2 row 2 = Row 3

Dependent (Singular)

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix}$$

row 1 - row 2 = row 3

Dependent (Singular)

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

no relations

Independent

(non-singular)

$$\begin{bmatrix} 1 & 2 & 5 \\ 0 & 3 & -2 \\ 2 & 4 & 10 \end{bmatrix}$$

2 row 1 = row 3

Dependent (Singular)

$$\begin{bmatrix} 1 & 2 & 5 \\ 0 & 3 & -2 \\ 2 & 4 & 10 \end{bmatrix} = (30+8) - 2(+4) + 5(-6)$$

$$38 - 8 - 30 = 0$$

singular

$$\begin{bmatrix} 2 & 1 & 5 \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

$$d = 2\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} + 5\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}$$

$$d = 2(6-1) - (3-2) + 5(\overset{-3}{1-4})$$

$$10 - 1 - 15 = -6$$

non singular

$$\begin{bmatrix} 2 & 1 & 5 \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

$$\begin{cases} 2b + m + 5c = 20 \\ b + 2m + c = 10 \\ 2b + m + 3c = 15 \end{cases}$$

$$2c = 5 \Rightarrow \boxed{c = 2.5}$$

$$5 + m + 12.5 = 20$$

$$\cancel{m = 5 +}$$

$$m = 20 - 12.5 - 5 \quad \boxed{m = 2.5}$$

$$2b + m = 7.5$$

$$b + 2m = 7.5$$

$$\cancel{2b + m = 7.5}$$

$$m = 7.5 - 2b$$

$$b + 2(7.5 - 2b) = 7.5$$

$$b + 15 - 4b = 7.5$$

$$-3b = -7.5$$

$$\boxed{b = 7.5/3} \quad \boxed{b = 2.5}$$

(11)

$$\begin{bmatrix} \cancel{2} & \cancel{7} & \cancel{5} \\ \cancel{1} & \cancel{2} & \cancel{1} \\ \cancel{x} & \cancel{y} & \cancel{z} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 1 & 4 & 5 \end{bmatrix}$$

$$d = 1\begin{vmatrix} 2 & 2 \\ 4 & 5 \end{vmatrix} - 2\begin{vmatrix} 0 & 2 \\ 1 & 5 \end{vmatrix} + 3\begin{vmatrix} 0 & 2 \\ 1 & 4 \end{vmatrix}$$

$$d = (10 - 8) - 2(-2) + 3(-2)$$

$$d = 2 + 4 - 6$$

$$\boxed{d = \cancel{0}} \quad \boxed{d = 0}$$

$$\begin{cases} (4x + 3y = 6) \times \frac{5}{3} \\ x - 5y = 8 \end{cases} \Rightarrow \begin{cases} \frac{20}{3}x + 5y = 10 \\ x - 5y = 8 \end{cases}$$

?

$$\cancel{5x = 18}$$

$x =$

$$2.35 - 5y = 8$$

$$y = \frac{8 - 2.35}{-5}$$

$$\boxed{y \simeq 3.76}$$

$$\frac{23}{3}x = 18$$

$$x = \frac{18 \times 3}{23}$$

$$\boxed{x = \frac{54}{23} \simeq 2.35}$$

$$\begin{cases} 4x + 3y + z = 6 \\ -4\,(x - 5y + 7z = 8) \\ 5x - 2y + 8z = 14 \end{cases}$$

$$4x + 3y + z = 6$$
$$-4x + 20y - 28z = -32$$

$$23y - 27z = -26$$

$$23y = -26 + 27z$$

$$y = \frac{-26 + 27z}{23}$$

$$\begin{cases}(5a + b = 17) \times 4 \\ (4a - 3b = 6) \times 5\end{cases} \Rightarrow \begin{cases}20a + 4b = 68 \\ 20a - 15b = 30\end{cases}$$

$\cancel{19b = 28}$ $19b = 38$

$\cancel{b = \frac{28}{19}}$ $\boxed{b = 2}$

$5a + 2 = 17$

$\boxed{a = 3}$

$$\begin{cases}2a + 5b = 46 \\ (8a + b = 32) \times \frac{1}{4}\end{cases} \Rightarrow \begin{cases}2a + 5b = 46 \\ 2a + \frac{1}{4}b = 8\end{cases}$$

$0 + \frac{19}{4}b = 38$

$\boxed{b = \frac{\overset{2}{\cancel{38}} \times 4}{\underset{1}{\cancel{19}}} = 8}$

$2a + (5 \times 8) = 46$

$2a = 6$

$\boxed{a = 3}$

$$\begin{cases}(5a + b = 11) \times 2 \\ 10a + 2b = 22\end{cases} \Rightarrow \begin{cases}10a + 2b = 22 \\ 10 + 2b = 22\end{cases} \text{ redundant}$$

$0 = 0$

the system is singular and has infinite solutions

$$\begin{cases} a + b + 2c = 12 \\ 3a - 3b - c = 3 \;\; | \times \frac{1}{3} \\ 2a - b + 6c = 24 \;\; | \times \frac{1}{2} \end{cases} \Rightarrow \begin{cases} a + b + 2c = 12 \\ a - b - \frac{1}{3}c = 1 \\ a - \frac{1}{2}b + 3c = 12 \end{cases}$$

$$\Rightarrow \begin{cases} a + b + 2c = 12 \\ 0 + 2b - \frac{7}{3}c = -11 \\ 0 - \frac{3}{2}b + c = 0 \end{cases} \Rightarrow \boxed{a = 4}$$

$$\Rightarrow \begin{cases} \left(+2b - \frac{7}{3}c = -11\right) \times \frac{1}{2} \\ \left(-\frac{3}{2}b + c = 0\right) \times -\frac{2}{3} \end{cases}$$

$$\Rightarrow \begin{cases} b - \frac{7}{6}c = -\frac{11}{2} \\ b - \frac{2}{3}c = 0 \end{cases} \Rightarrow \boxed{b = 2}$$

$$0 - \frac{11}{6}c = \cancel{0} - \frac{11}{2}$$

$$\boxed{\cancel{c = 11}} \quad \boxed{c = 3}$$

"Row echelon form of a matrix"

$$\begin{bmatrix} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 1 & * \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

→ any number ~~or default~~ possible.

→ one or zero in the diagonal

all must be zero

$$\begin{cases} x + y = 4 \\ (-6x + 2y = 16) \times -\frac{1}{6} \end{cases} \Rightarrow \begin{cases} x + y = 4 \\ x - \frac{2}{6}y = -\frac{16}{6} \end{cases}$$

$$\Rightarrow \frac{4}{3}y = \frac{40}{6}$$

$$\Rightarrow \boxed{y = 5}$$

$$\Rightarrow \boxed{x = -1}$$

$$\begin{bmatrix} 4 & -3 \\ 7 & -8 \end{bmatrix}$$

$$d = [4 \times (-8)] - [(-3) \times 7]$$

$$d = -32 + 21 \Rightarrow d = -11$$ non-singular

$$\begin{bmatrix} -3 & 8 & 1 \\ 2 & 2 & -1 \\ -5 & 6 & 2 \end{bmatrix}$$

$$d = -3\begin{vmatrix} 2 & -1 \\ 6 & 2 \end{vmatrix} - 8\begin{vmatrix} 2 & -1 \\ -5 & 2 \end{vmatrix} + 1\begin{vmatrix} 2 & 2 \\ -5 & 6 \end{vmatrix}$$

$$d = -3[4+6] - 8[4-5] + 1[12+10]$$

$$d = -30 + 8 + 22 = \underline{0}$$ singular

Rank = 2 - (Dimension of solution space)

- if the rank of a matrix is equal to the number of the rows of the matrix (full rank) this is a non-singular matrix.

* Rank ?

$$\begin{bmatrix} 5 & 1 \\ -1 & 3 \end{bmatrix} \Rightarrow R = 2 = \text{number of the rows}$$

so: non-singular

solution space has dimension $\underline{\underline{2}}$

$$\begin{bmatrix} 2 & -1 \\ -6 & 3 \end{bmatrix} \times(-3) \quad R = 1 \quad \text{so: singular.}$$

→ solution space had dimension 1

Original matrix

$$\begin{bmatrix} 5 & 1 \\ 4 & -3 \end{bmatrix} \begin{matrix} \div 5 \\ \div 4 \end{matrix} \longrightarrow \begin{bmatrix} 1 & 0.2 \\ 1 & -0.75 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0.2 \\ 0 & -0.95 \end{bmatrix} \div -0.95$$

$$\longrightarrow \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix} \rightarrow \underline{\text{Rank } 2}$$

Row echelon form.

$$\begin{bmatrix} 5 & 1 \\ 10 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0.2 \\ 1 & 0.2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0.2 \\ 0 & 0 \end{bmatrix} \rightarrow \underline{\text{Rank } 1}$$

Row echelon form

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \rightarrow \underline{\text{Rank} = 0}$$

→ Row echelon form

Row echelon form in General

$$\begin{bmatrix} \# & \# & \# & \# \\ 0 & \# & \# & \# \\ 0 & 0 & \# & \# \\ 0 & 0 & 0 & \# \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\#$ = any number

→ Pivots (every pivot is to the right of the pivots on the rows above.)

the rank of a matrix is the same as the number of pivot.

---

original matrix		Row echelon form		reduced row echelon form.
$\begin{bmatrix} 5 & 1 \\ 4 & -3 \end{bmatrix}$	$\longrightarrow$	$\begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}$	$\longrightarrow$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(Sum and difference of vectores)

$\vec{a} = (3,6)$

$\vec{v} = (5,2)$

$\vec{a} + \vec{v} = (3,6) + (5,2) = (8,8)$

$\vec{a} - \vec{v} = (3,6) - (5,2) = (-2,4)$

Vector properties:

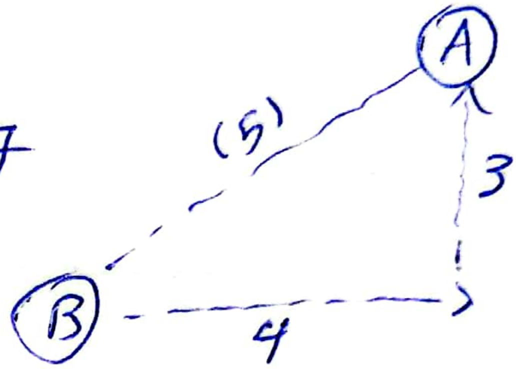
- Two very components of a vector are:

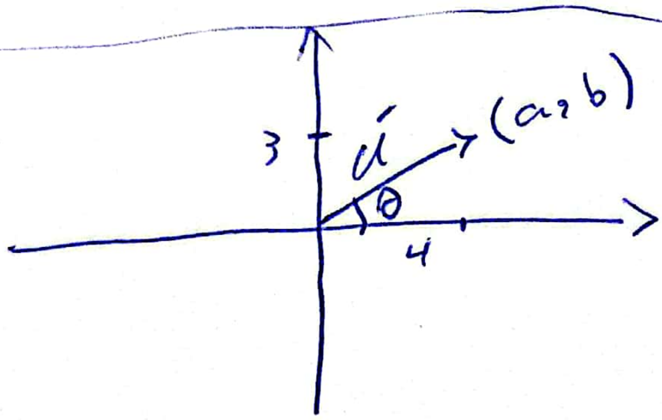
1) magnitude 2) direction

size

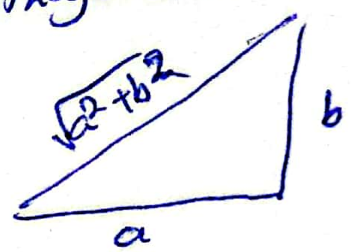
- Taxicab distance = $4+3=7$

- Helicopter distance $\sqrt{3^2+4^2}=5$





Pythagorean theorem



L1-norm $= \|(a,b)\|_1 = |a|+|b|$

L2-norm $= \|(a,b)\|_2 = \sqrt{a^2+b^2}$

norm of a vector $= \sqrt{a^2+b^2} = 5$

direction of a vector: $\tan(\theta) = \frac{3}{4}$

$\theta = \arctan(3/4) = 0.64 = 36.87^\circ$

what is the distance between $\vec{u}$ and $\vec{v}$? 20

$\vec{u} = (3, 5)$

$\vec{v} = (5, 2)$

$L_1\text{-distance} = |3-5| + |5-2| = 5$

$L_2\text{-distance} = \sqrt{(3-5)^2 + (5-2)^2}$

cosine-distance = $\cos(\theta)$

$\vec{u} = (3, 5)$

$6\vec{u} = 6(3, 5) = (18, 30)$

(dot ~~product~~)

$\vec{u} = (1, 2, 3)$ $\quad (1, 2, 3) \times \begin{pmatrix} 0 \\ 3 \\ 5 \end{pmatrix} = \underline{27}$

$\vec{v} = (0, 3, 5)$

dot-product = $\boxed{\vec{u} \cdot \vec{v} = \sum_{i=1}^{n} a_i b_i}$

$\vec{u} \cdot \vec{v} = (1 \times 0) + (2 \times 3) + (3 \times 5) = \underline{27}$

$$\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

$$a + b + c = 10$$

$$(1, 1, 1) \times \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 10$$

$$M = \begin{bmatrix} 4 & 5 & 9 \\ 7 & 1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

$$M.\vec{v} = \underset{(3\times3)}{\begin{bmatrix} 4 & 5 & 9 \\ 7 & 1 & 0 \\ 4 & 2 & 1 \end{bmatrix}} \times \underset{(3\times1)}{\begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}}$$

$$\Rightarrow \begin{bmatrix} \overset{4}{4\times1} + \overset{10}{5\times2} + \overset{45}{9\times5} \\ \overset{7}{7\times1} + \overset{2}{1\times2} + 0\times5 \\ \underset{4}{4\times1} + \underset{4}{2\times2} + \underset{5}{1\times5} \end{bmatrix} = \begin{bmatrix} 59 \\ 9 \\ 13 \end{bmatrix}$$

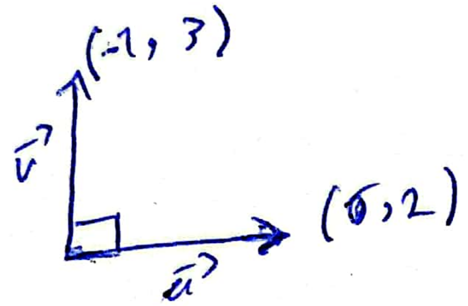
$$\begin{aligned} a + b + c &= 10 \\ a + 2b + c &= 15 \\ a + b + 2c &= 12 \end{aligned} \Rightarrow \begin{matrix} (1, 1, 1) \\ (1, 2, 1) \\ (1, 1, 2) \end{matrix} \times \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 10 \\ 15 \\ 12 \end{pmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \\ 12 \end{bmatrix}$$

linear transformation

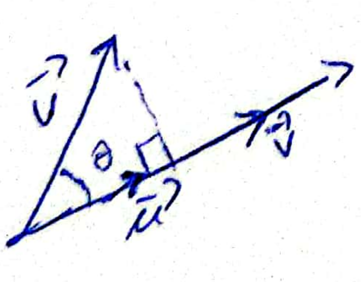
if: $T(0,1) = (1,2)$ and $T(1,0) = (4,1)$
find the matrix which represents this transformation

- orthogonal vectors have dot product "0" zero



$$\underbrace{(6,2)\begin{pmatrix} -1 \\ 3 \end{pmatrix} = -6+6 = 0}_{\langle \vec{u}, \vec{v} \rangle = 0}$$

$\langle u, u \rangle = |u|^2$



$$\langle \vec{u}, \vec{v} \rangle = |\vec{u}| \cdot |\vec{v}|$$

$$= |u| \cdot |V| \cdot \cos(\theta)$$

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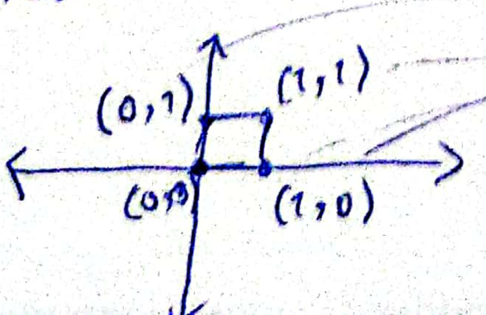
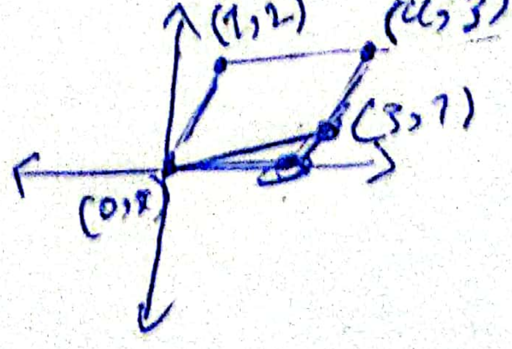
matrices as linear transformation

Linear transformation:

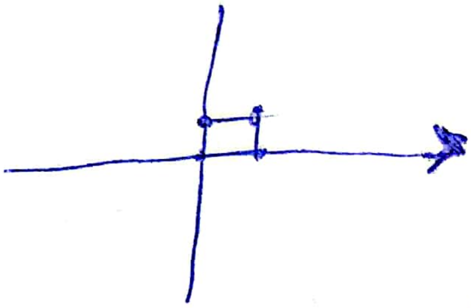


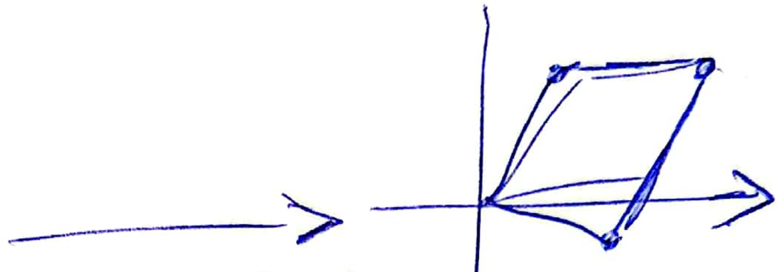
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$



"Linear transformation as matrices"

$M = ?$





$(0,0) \rightarrow (0,0)$

$\boxed{\begin{matrix}(1,0) \rightarrow (3,-1) \\ (0,1) \rightarrow (2,3)\end{matrix}}$

$(1,1) \rightarrow (5,2)$

$$(\cancel{x},\cancel{x})\begin{bmatrix}3 & 2\\ -1 & 3\end{bmatrix} = \begin{bmatrix}\cancel{x}\\ -\cancel{x}\end{bmatrix}$$

$T(0,1) = (1,2)$

$T(1,0) = (4,1)$

$M = ? \Rightarrow \begin{bmatrix}4 & 1\\ 1 & 2\end{bmatrix}$

$$M = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$$

$$N = \begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix}$$

$$M \cdot N = ? \quad \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 4+2 & 0+4 \\ 12+5 & 0+10 \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ 17 & 10 \end{bmatrix}$$

$$N^{-1} = \frac{1}{8} \begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ \frac{1}{8} & \frac{1}{4} \end{bmatrix}$$

$$d_n = (4 \times 2) - (0 \times 1) = \underline{8}$$

(determinant)

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 6-1 & 2-2 \\ 0+2 & 0+4 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

Idendity matrix: is the matrix that when multiplied by any other matrix it gives the same matrix. (ones in the diagnoal and zeros everywhere else).

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \underbrace{\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}}_{\text{same}}$$

"Inverse matrix"

$M = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

$M^{-1} = ?$

$$\underset{(A)}{\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}} \times \underset{\text{Inverse of } (A)}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} = \underset{\text{Identity}}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{cases} 3a + c = 1 \\ 3b + d = 0 \\ a + 2c = 0 \\ b + 2d = 1 \end{cases}$$

$a = \frac{2}{5}$

$b = -\frac{1}{5}$

$c = -\frac{1}{5}$

$d = \frac{5}{3}$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{cases} 5a + 2c = 1 \\ 5b + 2d = 0 \\ a + 2c = 0 \\ b + 2d = 1 \end{cases}$$

$$\begin{array}{r} 5a + 2c = 1 \\ a + 2c = 0 \\ \hline 4a = 1 \end{array}$$

$\boxed{a = \frac{1}{4}}$

$d = \frac{5}{8}$

$$\begin{array}{r} 5b + 2d = 0 \\ - \quad b + 2d = 1 \\ \hline 4b = -1 \end{array}$$

$\boxed{b = \frac{-1}{4}}$

$\frac{5}{4} + 2c = 1$

$2c = \frac{1}{1} - \frac{5}{4} = \frac{4-5}{4} = \frac{-1}{4} / 2$

$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

$\boxed{c = -\frac{1}{8}}$

$A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

$A^{-1} = ?$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{cases} a + 2c = 1 \\ b + 2d = 0 \\ 2a + 2c = 0 \\ 2b + 2d = 1 \end{cases}$$

$a + 2c = 1$

$2a + 2c = 0$

$\boxed{a = -1}$

$\boxed{c = 1}$

$$\begin{cases} 2b + 2d = 1 \\ b + 2d = 0 \end{cases}$$

$\boxed{b = 1}$

$-1 + 2c = 2$

$c = \frac{2}{2}$

$= 1$

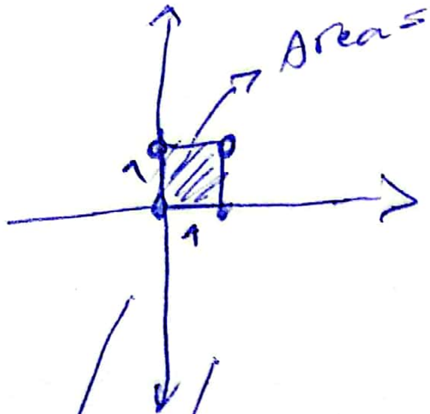
!

?

$d = \frac{-1}{2}$

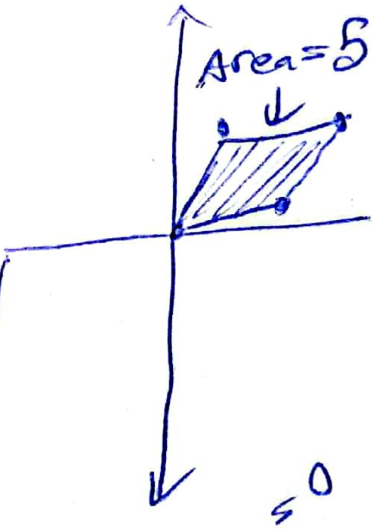
I am reaching a dead end!

-The determinate of amatrix is equal to the area of the Image of the fundamental basis.



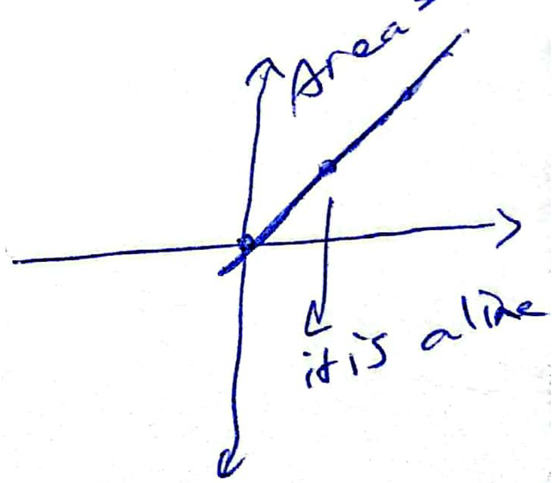
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

$$Det = (3\times 2) - (1\times 1) = \boxed{5}$$

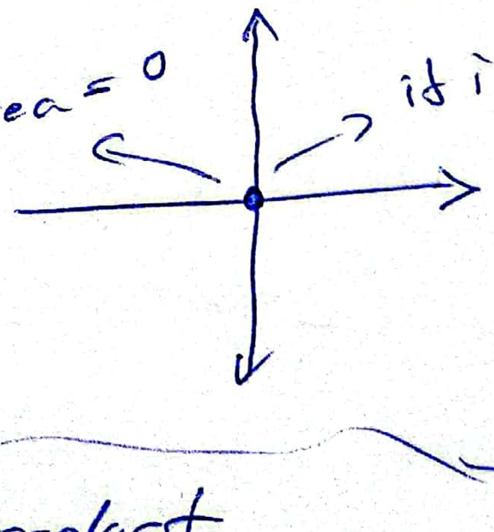


$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

$$Det = (1\times 2) - (1\times 2) = 0$$



$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$



$$Det = 0$$

Determinant of a product

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 3-2 & 3+1 \\ 1-4 & 1+2 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ -3 & 3 \end{bmatrix}$$

$Det_1 = 6-1 = 5$ $\quad det_2 = 1+2 = 3$ $\quad det_3 = 3+12 = 15$

50, Determinant of the product of matrices is equal to the product of the determinant. 29

$$\det(AB) = \det(A) \times \det(B)$$

- The product of a singular and a non-singular matrix (in any order) is singular.

$$\begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix} \rightarrow \det = (0.4 \times 0.6) - (0.2 \times 0.2)$$

$$0.24 - 0.04 = \underline{0.2}$$

$$\begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix} \rightarrow \det = (0.25 \times 0.625) - (0.125 \times 0.25)$$

$$= \underline{0.125}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

$\downarrow$ $\det = 5$ $\quad$ $\downarrow$ $\det = 0.2$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

$\downarrow$ $\det = 8$ $\quad$ $\downarrow$ $\det = 0.125$

$$8^{-1} = 0.125$$

$$5^{-1} = \frac{1}{5}$$

- If det $M = 20$ and det $N = 10$ and M, N have the same size. what is the velae of ~~det~~ det $M \times N$ and det (N^{-1})? (37)

$det(M) = 20$

$det(N) = 10$

$det(M \cdot N) = 20 \times 10 = 200$

$det(N^{-1}) = \frac{1}{10}$

$$W = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$det(W) = 1\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} + -1\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= \underbrace{1(-1)}_{-1} \underbrace{-2(0)}_{0} \underbrace{-1(1)}_{-1} = \boxed{-2}$$

$$W^{-1} = \frac{1}{-2}\begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & -1 & \frac{1}{2} \\ -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & -\frac{1}{2} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0.5 & 0.5 & -1 \\ 0 & 0 & 1 \\ -0.5 & 0.5 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

det = 0 det = ?

$$0^{-1} = ???$$

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

$$\det(AB) = \det(A).\det(B)$$

if $B = A^{-1}$

$$\det(AA^{-1}) = \det(A) \cdot \det(A^{-1})$$

$\downarrow$

$$\det(I) = \det(A) \det(A^{-1})$$

$\downarrow$ $\downarrow$

1 $\frac{1}{\det(A)}$

Identity matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1 \times 1 - 0 \times 0 = \underline{1}$$

$$\begin{bmatrix} 5 & -2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 5-2 & 10 & -5-2 \end{bmatrix}$$

(1x3) (3x3)

$$\begin{bmatrix} 3 & 10 & -7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 5-4 \\ 5 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix}$$

$$\begin{pmatrix} 3 & 1 & 7 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = 6$$

3x1 3x1

"Bases in linear algebra"

-anything that comprises two vectors that go in the same direction and they could be opposites or they could go in the same direction, but as long as they belong to the same line, the two vectors do not form a basis.

- number of elements in the basis is the dimension.

"Eigenvalue"

- if λ is an eigenvalue: $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

$$\begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\det\begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} = 0$$

$$(2-\lambda)(3-\lambda)-1-0=0 \quad \begin{cases} \lambda=2 \\ \lambda=3 \end{cases}$$

characteristic polynomial

eigenvalues

$\cancel{\begin{bmatrix}9 & 4\\ 4 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}}$

$\begin{bmatrix}9 & 4\\ 4 & 3\end{bmatrix}$ = eigenvectors? eigenvalues?

characteristic polynomial: $\det\begin{bmatrix}9-\lambda & 4\\ 4 & 3-\lambda\end{bmatrix} =$

$$= (9-\lambda)(3-\lambda) - 16 = 0$$

$$\lambda^2 - 12\lambda + 11 = (\lambda - 11)(\lambda - 1)$$

$\lambda = 11$ → eigenvalues

$\lambda = 1$

finish :)