① Solve the equation systems using Elimination

$$(x_1 - 2x_2 + x_3 = 0)\,4$$

$$3\,(2x_2 - 8x_3 = 8)$$

$$-4x_1 + 5x_2 + 9x_3 = -9$$

$$\cancel{4x_1} - 8x_2 + 4x_3 = 0$$

$$\cancel{-4x_1} + 5x_2 + 9x_3 = -9$$

$$(-3x_2 + 13x_3 = -9)\,2$$

$$\cancel{6x_2} - 24x_3 = 24$$

$$\cancel{-6x_2} + 26x_3 = -18$$

$$2x_3 = 6$$

$$\boxed{x_3 = 3}$$

$$2x_2 - 8x_3 = 8$$

$$2x_2 - 8(3) = 8$$

$$\boxed{x_2 = 16}$$

$$x_1 - 2(16) + 3 = 0$$

$$\boxed{x_1 = 29}$$

Existence and uniqueness

1) determine if the system is consistent (does a solution exist?)

2) if so, determine if the solution is unique (just one solution?)

$$x_2 - 4x_3 = 8$$

$$2x_1 - 3x_2 + 2x_3 = 1$$

$$4x_1 - 8x_2 + 12x_3 = 1$$

$$\longrightarrow \left[\begin{array}{ccc|c} 0 & 1 & -4 & 8 \\ 2 & -3 & 2 & 1 \\ 4 & -8 & 12 & 1 \end{array}\right] \xrightarrow{R_1 \Leftrightarrow R_2} \left[\begin{array}{ccc|c} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 4 & -8 & 12 & 1 \end{array}\right]$$

2

$$\xrightarrow{\frac{1}{2}R_1 \to R_2} \left[\begin{array}{ccc|c} 1 & -\frac{3}{2} & 1 & \frac{1}{2} \\ 0 & 1 & -4 & 8 \\ 4 & -8 & 12 & 1 \end{array}\right] \xrightarrow{-4R_1 + R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & -\frac{3}{2} & 1 & \frac{1}{2} \\ 0 & 1 & -4 & 8 \\ 0 & -2 & 8 & -1 \end{array}\right]$$

$$\xrightarrow{2R_2 + R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & -\frac{3}{2} & 1 & \frac{1}{2} \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & 75 \end{array}\right] \to \text{Inconsistent}$$

does not a solution exist.

consistent system with ~~infinity~~ infinited many solutions

$$\left[\begin{array}{ccc|c} 1 & 0 & -5 & 1 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \begin{array}{l} x_1 - 5x_3 = 1 \\ x_2 + x_3 = 4 \\ 0 = 0 \end{array} \to \begin{cases} x_1 = 5x_3 + 1 \\ x_2 = 4 - x_3 \\ x_3 \text{ is free} \end{cases}$$

free ~~values~~ variables can take on any values.

Let $x_3 = 2$

than $\begin{cases} x_1 = 11 \\ x_2 = 2 \end{cases}$

let $x_3 = -6$

$\begin{cases} x_1 = -29 \\ x_2 = 10 \end{cases}$

there are many solutions.

Algebraic Properties of $\mathbb{R}^n$ (vectors) ③

$\vec{u} + \vec{v} = \vec{v} + \vec{u}$ $\quad$ $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$

$\vec{u} + 0 = 0 + \vec{u} = \vec{u}$ $\quad$ $\vec{u} + (-\vec{u}) = (-\vec{u}) + \vec{u} = 0$

$c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$ $\quad$ $(c+d)\vec{u} = c\vec{u} + d\vec{u}$

$c(d\vec{u}) = (cd)\vec{u}$ $\quad$ $1\vec{u} = \vec{u}$

$$\begin{aligned} 2x_1 + 3x_2 - x_3 &= 3 \\ 2x_2 + 3x_3 &= 4 \end{aligned} \Rightarrow \left[\begin{array}{ccc|c} 2 & 3 & -1 & 3 \\ 0 & 2 & 3 & 4 \end{array}\right]$$

$$\Rightarrow x_1\begin{bmatrix} 2 \\ 0 \end{bmatrix} + x_2\begin{bmatrix} 3 \\ 2 \end{bmatrix} + x_3\begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 3 & -1 \\ 0 & 2 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

Address $= a_{ij} = a_{23} = 7$ $\quad$ $\begin{bmatrix} 0 & 1 & 4 & 5 \\ 2 & -3 & 7 & 6 \\ 4 & -1 & -5 & 2 \end{bmatrix}$

Diagonal matrix

Diagonal entries $= a_{11}, a_{22}, a_{33}, \ldots$

Square matrix $= n \times n \ (n = m) = 3\times3$, 4×4

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Zero matrix $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad$ Identity matrix. (I_4)

In general $AB \neq BA$

if $AB = AC$ it is not required that $B = C$

if $AB = 0$ you cannot conclude $A = 0$ or $B = 0$

Properties of transpose matrices

$(A^T)^T = A \longrightarrow a_{ij} \longrightarrow a_{ji} \longrightarrow a_{ij}$

$(A+B)^T = A^T + B^T$ $\begin{cases} \begin{bmatrix}1\\2\end{bmatrix} + \begin{bmatrix}3\\4\end{bmatrix} = \begin{bmatrix}4\\6\end{bmatrix} \\ \begin{bmatrix}1 & 2\end{bmatrix} + \begin{bmatrix}3 & 4\end{bmatrix} = \begin{bmatrix}4 & 6\end{bmatrix} \end{cases}$

$(rA)^T = r(A^T) \longrightarrow$ $\begin{cases} \left(\begin{bmatrix}1\\2\end{bmatrix} 3\right)^T = \begin{bmatrix}3 & 6\end{bmatrix} \\ 3\begin{bmatrix}1 & 2\end{bmatrix} = \begin{bmatrix}3 & 6\end{bmatrix} \end{cases}$

$(AB)^T = B^T A^T$

Inverses and Identities

In matrices $\begin{cases} AI_n = A \\ I_n A = A \end{cases}$ $\quad AA^{-1} = I_n \quad A^{-1}A = I_n$

determinate of matrix A

~~let if~~

let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, if $\boxed{ad - bc \neq 0}$, A is invertible.

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Solve the system using the inverse verify with row operations:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \begin{aligned} 3x_1 + 4x_2 &= 3 \\ 5x_1 + 6x_2 &= 7 \end{aligned} \Rightarrow \begin{array}{cc} A & b \\ \left[\begin{matrix} 3 & 4 \\ 5 & 6 \end{matrix}\right. & \left|\begin{matrix} 3 \\ 7 \end{matrix}\right] \end{array}$$

$$A^{-1} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\Rightarrow \underbrace{\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 3 \\ 7 \end{bmatrix}}_{b}$$

$$A^{-1} = \frac{1}{18-20} \begin{bmatrix} 6 & -4 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ \frac{5}{2} & \frac{-3}{2} \end{bmatrix}$$

$$\text{Solution: } X = \begin{bmatrix} -3 & 2 \\ \frac{5}{2} & \frac{-3}{2} \end{bmatrix} \begin{bmatrix} 3 \\ 7 \end{bmatrix} = \begin{bmatrix} -9+14 \\ \frac{15}{2} + \frac{-21}{2} \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

---

second method

$$\Rightarrow \left[\begin{matrix} 3 & 4 \\ 5 & 6 \end{matrix} \middle| \begin{matrix} 3 \\ 7 \end{matrix}\right] \sim \left[\begin{matrix} 1 & \frac{4}{3} \\ 5 & 6 \end{matrix} \middle| \begin{matrix} 1 \\ 7 \end{matrix}\right] \sim \left[\begin{matrix} 1 & \frac{4}{3} \\ 0 & \frac{-2}{3} \end{matrix} \middle| \begin{matrix} 1 \\ 2 \end{matrix}\right]$$

$$\sim \left[\begin{matrix} 1 & \frac{4}{3} \\ 0 & 1 \end{matrix} \middle| \begin{matrix} 1 \\ -3 \end{matrix}\right] \sim \left[\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \middle| \begin{matrix} 5 \\ -3 \end{matrix}\right]$$

Matrix determinante

find det A for $A = \begin{bmatrix} 1 & 5 & 0 \\ 2 & 4 & -1 \\ 0 & -2 & 0 \end{bmatrix} \Rightarrow \pm a_{ij} \det A_{ij}$

$$\det A = a_{11} \det A_{11} - a_{12} \det A_{12} + a_{13} \det A_{13}$$

$$\det A = (1) \det \begin{bmatrix} 4 & -1 \\ -2 & 0 \end{bmatrix} - (5) \det \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix} + (0) \det \begin{bmatrix} 2 & 4 \\ 0 & -2 \end{bmatrix}$$

$$\det A = 1 \begin{vmatrix} 4 & -1 \\ -2 & 0 \end{vmatrix} - 5 \begin{vmatrix} 2 & -1 \\ 0 & 0 \end{vmatrix} + 0 \begin{vmatrix} 2 & 4 \\ 0 & -2 \end{vmatrix}$$

$$\det A = 1(0-2) - 5(0-0) + 0(-4-0)$$

$$\det A = -2 - 0 + 0 = \boxed{-2}$$

Find The Least Squares solution to the inconsistent system $AX = b$ for $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}$ $b = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$

Step ① ensure the system is Inconsistent.

$$\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 2 & 2 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{array}\right]$$

impossible, so inconsistent

$\boxed{0x_1 + 0x_2 = 1}$

Step② calculate $A^T A$ and $A^T b$ $(A^T A \hat{x} = A^T b)$

$$A^T A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

Step③ solve for $\hat{x}$ Here $A^T A \hat{x} = A^T b$

$$\left[\begin{array}{cc|c} 2 & 3 & 4 \\ 3 & 6 & 7 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & \frac{3}{2} & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & \frac{2}{3} \end{array}\right]$$

$$\hat{x} = \begin{bmatrix} 1 \\ \frac{2}{3} \end{bmatrix}$$

Step④ solve $A\hat{x} = \hat{b}$ Here $\hat{x} = \begin{bmatrix} 1 \\ 2/3 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ \frac{2}{3} \end{bmatrix} = \begin{bmatrix} 5/3 \\ 2/3 \\ 7/3 \end{bmatrix} \longrightarrow \hat{b}$$

$\to$ an estimation for b

Linear Algebra

$$v = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad \|v\| = \sqrt{\sum_{i=1}^{n} v_i^2}$$

or

$$\|v\| = \sqrt{v_1^2 + v_2^2 + v_3^2 + \dots + v_n^2}$$

Scalar multiplication

$$k \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} kx \\ ky \\ kz \end{bmatrix}$$

Vector Addition and subtraction

vectors can be added or subtracted from each other when they are of the same dimension (Same number of components)

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + 2 \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} - 3 \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} x_1 + 2x_2 - 3x_3 \\ y_1 + 2y_2 - 3y_3 \\ z_1 + 2z_2 - 3z_3 \end{bmatrix}$$

vector addition is commutative (the order of the terms does not matter). $(a+b = b+a)$

vector addition is also associative $(a+(b+c) = (a+b)+c)$

vector dot products

$$a \cdot b = \sum_{i=1}^{n} a_i b_i$$

the dot product operation is
1) commutative
2) distributive

1) commutative : $(a \cdot b = b \cdot a)$

2) distributive : $(a \cdot (b+c) = a \cdot b + a \cdot c)$

$$a = \begin{bmatrix} 3 \\ 2 \\ -3 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ -3 \\ -6 \end{bmatrix}$$

$$a \cdot b = 3 \times 0 + 2 \times (-3) + (-3) \times (-6) = \boxed{12}$$

to find the magnitude of a vector

$$\|a\| = \sqrt{a \cdot a}$$

to find the angle between two vectors

$$\theta = \cos^{-1}\left(\frac{a \cdot b}{\|a\| \, \|b\|}\right)$$

an example:

$$a=\begin{bmatrix}3\\2\\-3\end{bmatrix},\quad \begin{bmatrix}0\\-3\\-6\end{bmatrix}$$

$$\theta=\cos^{-1}\left(\frac{0-6+18}{\sqrt{3^2+2^2+(-3)^2}\times\sqrt{0^2+(-3)^2+(-6)^2}}\right)$$

$$\theta=\cos^{-1}\left(\frac{12}{3\sqrt{110}}\right)\approx\boxed{67.58^\circ}$$

matrices

A matrix is a quantity with m rows and n columns of data. for example we can combine multiple vectors into a matrix where each column is one of the vectors.

scalar	vector or a single-column matrix	a matrix
1	$\begin{bmatrix}1\\2\end{bmatrix}$	$\underset{\underset{m\ \ n}{3\times 2}}{A}=\begin{bmatrix}1&2&3\\2&9&6\\7&1&5\end{bmatrix}$

$\boxed{\text{matrix operations}}$

$$2\begin{bmatrix}a_1&b_1\\a_2&b_2\end{bmatrix}+3\begin{bmatrix}c_1&d_1\\c_2&d_2\end{bmatrix}=\begin{bmatrix}2a_1+3c_1&2b_1+3d_1\\2a_2+3c_2&2b_2+3d_2\end{bmatrix}$$

matrix multiplication

it works by computing the dot product between each Row of the first matrix and each column of the second one in multiplication ~~must~~ the number of columns in the first matrix must be equal to the number of Rows in the second one.

$$A_{m_1 \times n_1} \times B_{m_2 \times n_2} = C_{m_1 \times n_2}$$

$\boxed{AB \neq BA}$ not commutative

$\boxed{A(BC) = (AB)C}$ associative

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ 139 & 154 \end{bmatrix}$$

First step:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ 139 & 154 \end{bmatrix}$$

$(1,2,3) \cdot (7,8,9) = 1 \times 7 + 2 \times \cancel{8} + 3 \times \cancel{11}$
$= 58$

second step:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ 139 & 154 \end{bmatrix} = (1,2,3) \cdot (7,10,12) =$$

$1 \times 7 + 2 \times 10 + 3 \times 12 = 64$

fourth step

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ 139 & 154 \end{bmatrix}$$

$(4,5,6)\cdot(7,9,11) =$

$4\times7+5\times9+6\times11 = 139$

fifth step

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ 139 & 154 \end{bmatrix}$$

$(4,5,6)\times(8 \text{ to } 12) =$

$4\times8+5\times10+6\times12 =$
$154.$

(Identity matrix)

it is a square matrix of element equal to 0 (except for the elements along the diagonal). any matrix multiplied by this matrix is equal to itself.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times A_{m\times n} = A_{m\times n}$$

(transpose matrix)

transpose of amatrix

it is amatrix ~~which~~ wich computed by swapping the rows and columns.

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

Permutation matrix

it is a square matrix. it allows us to flip rows and columns of a separate matrix.

$$PA = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} d & e & f \\ g & h & i \\ a & b & c \end{bmatrix}$$

← Row Swap

$$AP = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} c & a & b \\ f & d & e \\ i & g & h \end{bmatrix}$$

← column swap

"Gauss-Jordan Elimination"

coefficent matrix

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \longrightarrow \left[\begin{array}{ccc|c} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{array}\right]$$

Augmented matrix

$$\boxed{\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}}$$

it is an algorithm that can be used to solve systems of linear equations and to find the invers of any inversible matrix.

an Example:

a system of equations:

$$x + y - 2z = 1$$
$$-x - y + z = 2$$
$$-2x + 2y + z = 0$$

first step →

augmented matrix

matrix form

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ -1 & -1 & 1 & 2 \\ -2 & 2 & 1 & 0 \end{array}\right]$$

second step

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 0 & -1 & 3 \\ -2 & 2 & 1 & 0 \end{array}\right] \rightarrow R_2 + R_1$$

third step ~~second~~

$$\frac{1}{2} R_3 + R_1 \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 0 & -1 & 3 \\ 0 & 2 & -\frac{3}{2} & 1 \end{array}\right]$$

fourth step

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 2 & -\frac{3}{2} & 1 \\ 0 & 0 & -1 & 3 \end{array}\right]$$

R_2 R_3

step five

$$R_2 \times \frac{1}{2}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 1 & -\frac{3}{4} & \frac{1}{2} \\ 0 & 0 & 1 & -3 \end{array}\right]$$

normalize the diagonal

last step

$$x + y - 2z = 1$$
$$y - \frac{3}{4} z = \frac{1}{2}$$

$$\boxed{z = -3}$$

Inverse matices (A^{-1})

$$AA^{-1} = A^{-1}A = I$$

I = an Identity matrix is a sqaare matrix in which all the elements of principal diagonals are one and all other elements are zeros

one example:

$$I_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

---

solving equations with Inverse matrix

$$XA = BC$$

$$XAA^{-1} = BCA^{-1} \longrightarrow X = BCA^{-1}$$

Singular matrix = a matrix which have no Inverse.

an example: $[A|I] \longrightarrow [I|A^{-1}]$

$$A = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -2 & 0 \\ -1 & 1 & 2 \end{bmatrix}$$

one

$$\left[\begin{array}{ccc|ccc} 0 & 2 & 1 & 1 & 0 & 0 \\ -1 & -2 & 0 & 0 & 1 & 0 \\ -1 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 0 & 0 & 1 \\ -1 & -2 & 0 & 0 & 1 & 0 \\ 0 & 2 & 1 & 1 & 0 & 0 \end{array}\right]$$

zero

2

thre

$$R_2 - R_1 \rightarrow \left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 0 & 0 & 1 \\ 0 & -3 & -2 & 0 & 1 & -1 \\ 0 & 2 & 1 & 1 & 0 & 0 \end{array}\right]$$

four

$$\tfrac{3}{2}R_3 + R_2 \rightarrow \left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 0 & 0 & 1 \\ 0 & -3 & -2 & 0 & 1 & -1 \\ 0 & 0 & -\frac{1}{2} & \frac{3}{2} & 1 & -1 \end{array}\right]$$

five

$$\left[\begin{array}{ccc|ccc} -3 & 0 & 4 & 0 & 1 & 2 \\ 0 & -3 & -2 & 0 & 1 & -1 \\ 0 & 0 & -\frac{1}{2} & \frac{3}{2} & 1 & -1 \end{array}\right] \leftarrow 3R_1 + R_2$$

six

$$-\tfrac{1}{4}R_2 + R_3 \rightarrow \left[\begin{array}{ccc|ccc} -3 & 0 & 4 & 0 & 1 & 2 \\ 0 & \frac{3}{4} & 0 & \frac{3}{2} & \frac{3}{4} & -\frac{3}{4} \\ 0 & 0 & -\frac{1}{2} & \frac{3}{2} & 1 & -1 \end{array}\right] \rightarrow$$

seven

$$\left[\begin{array}{ccc|ccc} -\frac{3}{8} & 0 & 0 & \frac{3}{2} & \frac{9}{8} & -\frac{3}{4} \\ \frac{3}{2} & \frac{3}{4} & -\frac{3}{4} & \frac{3}{2} & \frac{3}{4} & -\frac{7}{4} \\ 0 & 0 & -\frac{1}{2} & \frac{3}{2} & 1 & -1 \end{array}\right] \leftarrow -\tfrac{1}{8}R_1 + R_3$$

eight

$$\begin{array}{cc} I & A^{-1} \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -4 & -3 & 2 \\ 0 & 1 & 0 & 2 & 1 & -1 \\ 0 & 0 & 1 & -3 & -2 & 2 \end{array}\right] \begin{array}{l} -\frac{8}{3}R_1 \\ \frac{4}{3}R_2 \\ -2R_3 \end{array}$$

normalize the diagonals

Mit CamScanner gescannt