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UNIVERSITÀ DI ROMA
DIPARTIMENTO DI INGEGNERIA INFORMATICA
AUTOMATICA E GESTIONALE ANTONIO RUBERTI

Master in Artificial Intelligence and Robotics

Data-driven identification of a one-link robot with flexible joint

Lagrangian Neural Network applied to elastic robots

Course: Robotics 2
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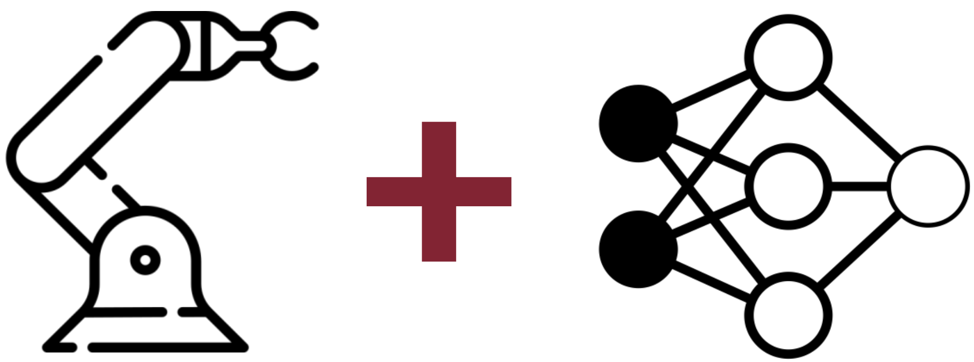
INTRODUCTION

- Robot dynamic model
  - Relationship between applied forces and motion
  - D’Alembert, Hamilton
- Euler-Lagrange equations
  - Modeling and control
  - Symbolic form equations of motions
- Modeling systems → Neural networks → Lagrangian NN



PARAMETRIC LAGRANGIAN

Lagrangian

$$\boxed{L(q,\dot{q}) = T(q,\dot{q}) - U(q)}$$

- Kinetic $T(q,\dot{q})$ and potential $U(q)$ energy
- Generalized coordinates $q \in \mathbb{R}^N$
  - N = robot DOF

Euler-Lagrange equations

$$\boxed{\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = u_i} \qquad i = 1, \dots, N$$

- Non-conservative forces u_i
  - Considered null in our application

PARAMETRIC LAGRANGIAN

Parametric Lagrangian

$$\boxed{\ddot{q} = \left(\nabla_{\dot{q}}\nabla_{\dot{q}}^{T}L\right)^{-1}\left[\nabla_{q}L - \left(\nabla_{q}\nabla_{\dot{q}}^{T}L\right)\dot{q}\right]}$$

- Lagrangian analytical expression not always known
- Alternative approach
  - Learning L + Automatic differentiation

RIGID ROBOT

1R robot with y-axis agreeing with gravity

Dynamic Model: $\boxed{(I + md^2)\ddot{q} + mg_0 d\sin(q) = 0}$

Acceleration: $\boxed{\ddot{q} = -\frac{1}{I + md^2} mg_0 d\sin(q)}$

- Coriolis and Centrifugal term not present
  - One single link
  - Contribute to the joint when it is moving is null

ELASTIC ROBOT

1R robot with y-axis agreeing with gravity

Dynamic model:

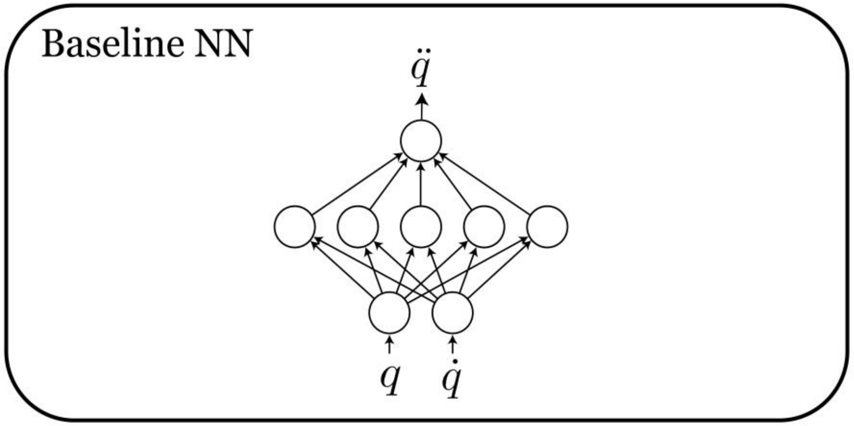
$$\begin{cases} (I + md^2)\ddot{q} + mg_0 d\sin(q) + k(q-\theta) = 0 \\ I_m\ddot{\theta} + k(\theta - q) = 0 \end{cases}$$

Accelerations:

$$\begin{cases} \ddot{q} = -\dfrac{1}{I + md^2}\left[mg_0 d\sin(q) + k(q-\theta)\right] \\ \ddot{\theta} = \dfrac{1}{I_m}k(q-\theta) \end{cases}$$

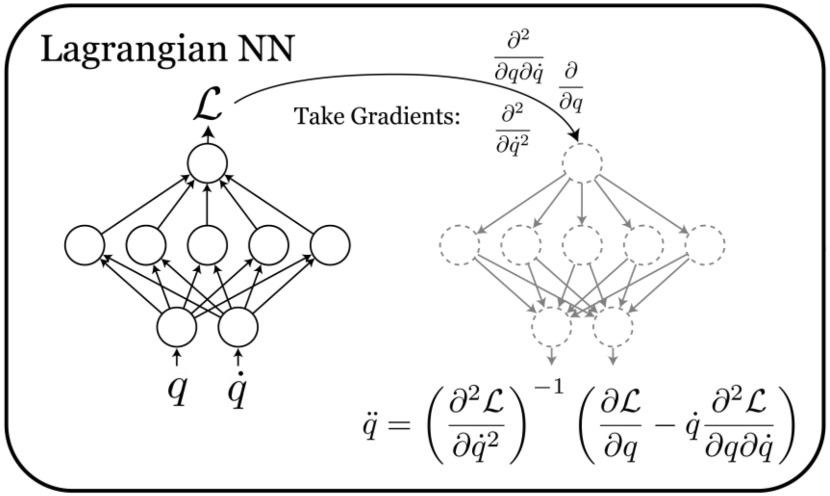
- Again Coriolis and Centrifugal term not present
  - Constant inertia matrix

BASELINE NN



- Classic feedforward neural network
  - Only fully connected layers
- Learn $\ddot{q}$ directly from $\dot{q}$ and q
  - Not considering Lagrangian and relative constraints
- Function of a comparison baseline
  - Same structure of Lagrangian NN to comparison reasons

LAGRANGIAN NN



- Same baseline structure, completely different approach
  - Very close to robot nature
- Learn Lagrangian
  - Functional programming context
  - Predictions benefit of the Lagrangian approach features
- $\ddot{q}$ obtained from parametric Lagrangian

SIMULATIONS

- **Type 1:** Observe behavior of the two networks
  - Trajectory tracking
  - Trajectory error
  - Energy conservation
- **Type 2:** Introduce noise
  - Same tests as in type 1
  - Highlight network robustness in not ideal context

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RIGID CASE

Robot parameters

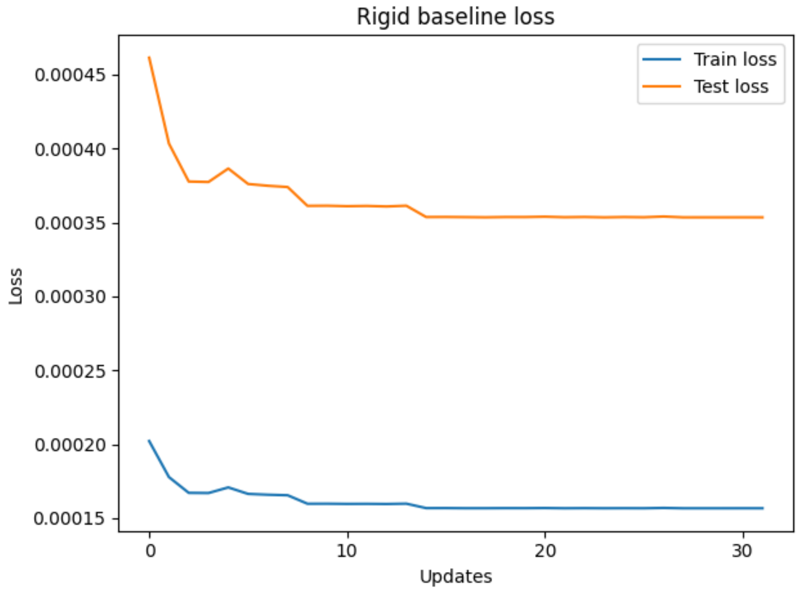
Parameter	Measure
Length	$0.30\ m$
Base radius	$0.01\ m$
CoM distance	$0.15\ m$
Mass	$0.25\ kg$
Inertia	$0.01125\ kg\ m^2$

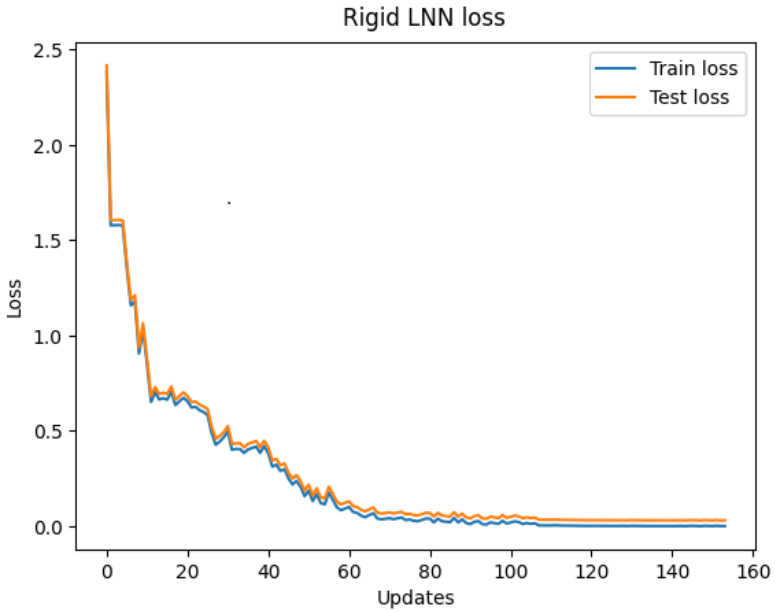
Network setup

Characteristic	Value
Layers	4
Neurons	500
Activation	Softplus
Loss	MSE
Regularization	L2 penalty
Train epochs	10 000

RIGID CASE – NETWORK TRAINING

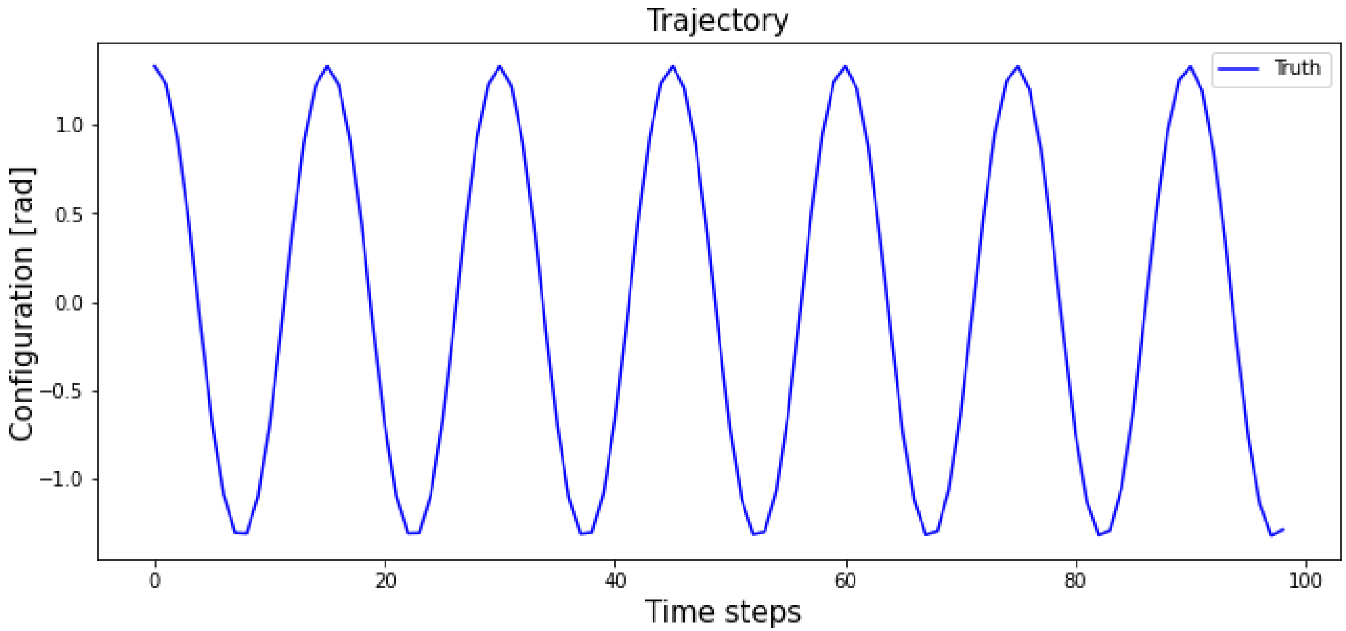
Train and test losses





RIGID CASE – SIMULATION 1

Initial conditions: $q(0) = 1{,}33\, rad \quad \dot{q}(0) = 0{,}2\, \frac{rad}{s}$



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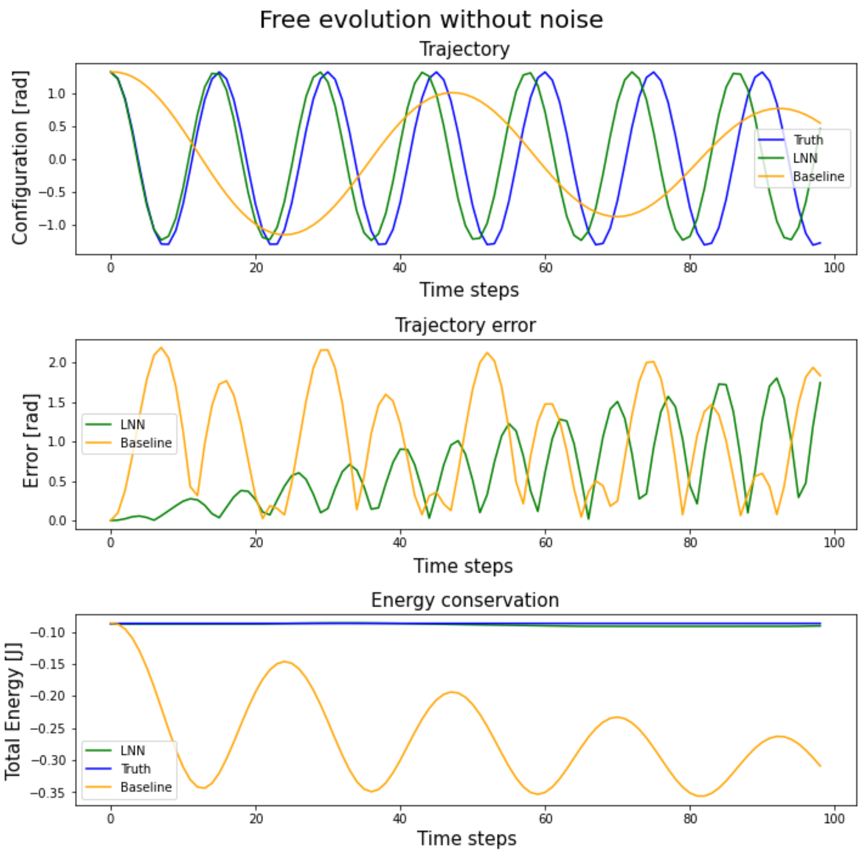
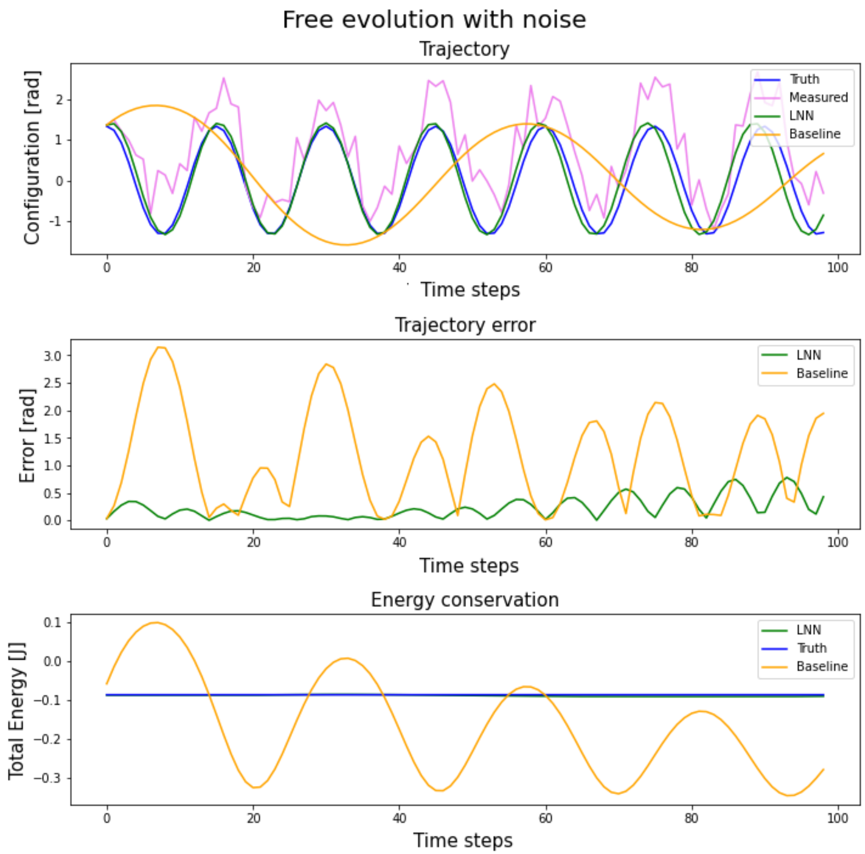
RIGID CASE – SIMULATION 1

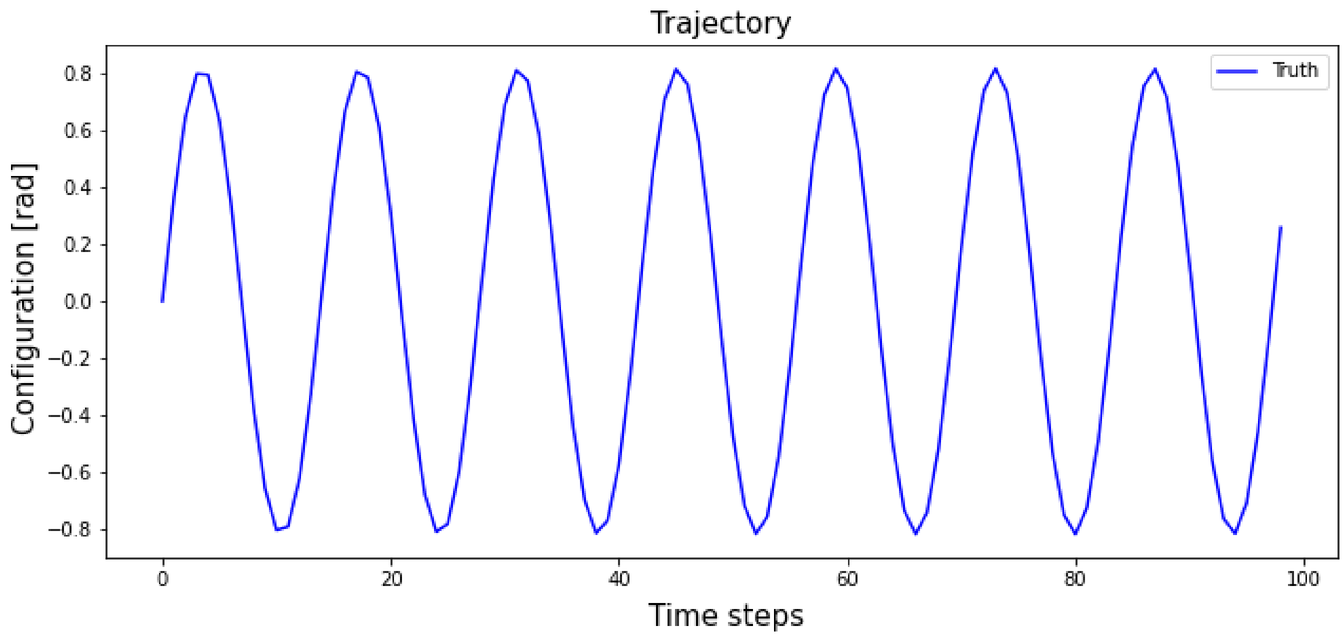




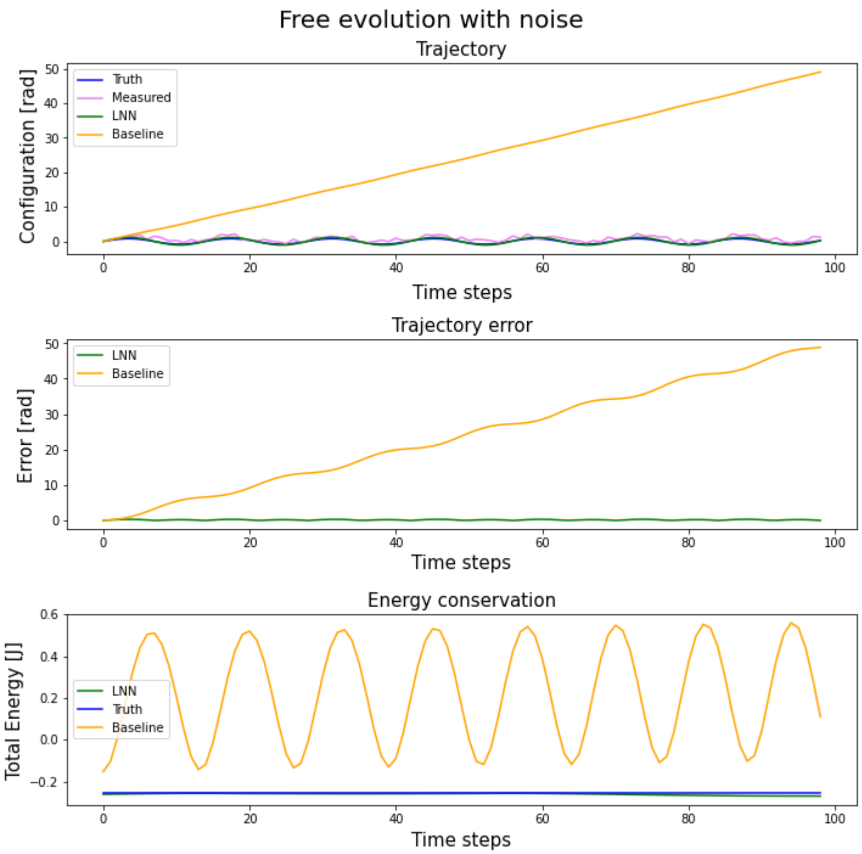
RIGID CASE – SIMULATION 2

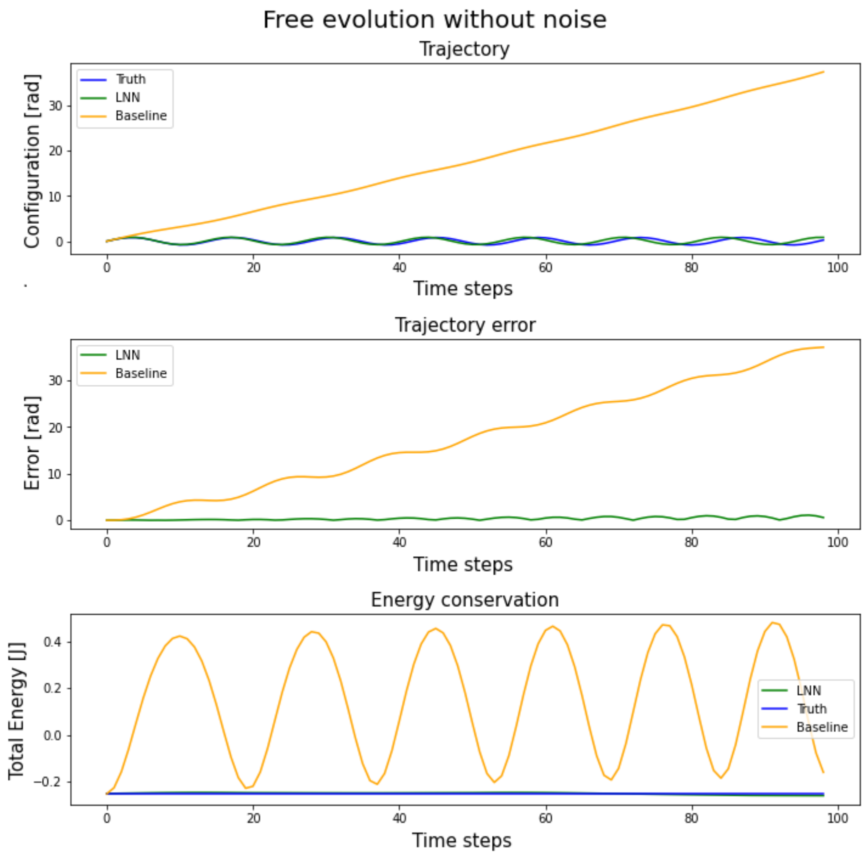
Initial conditions:

$$q(0) = 0\,rad \quad \dot{q}(0) = 4\,\frac{rad}{s}$$



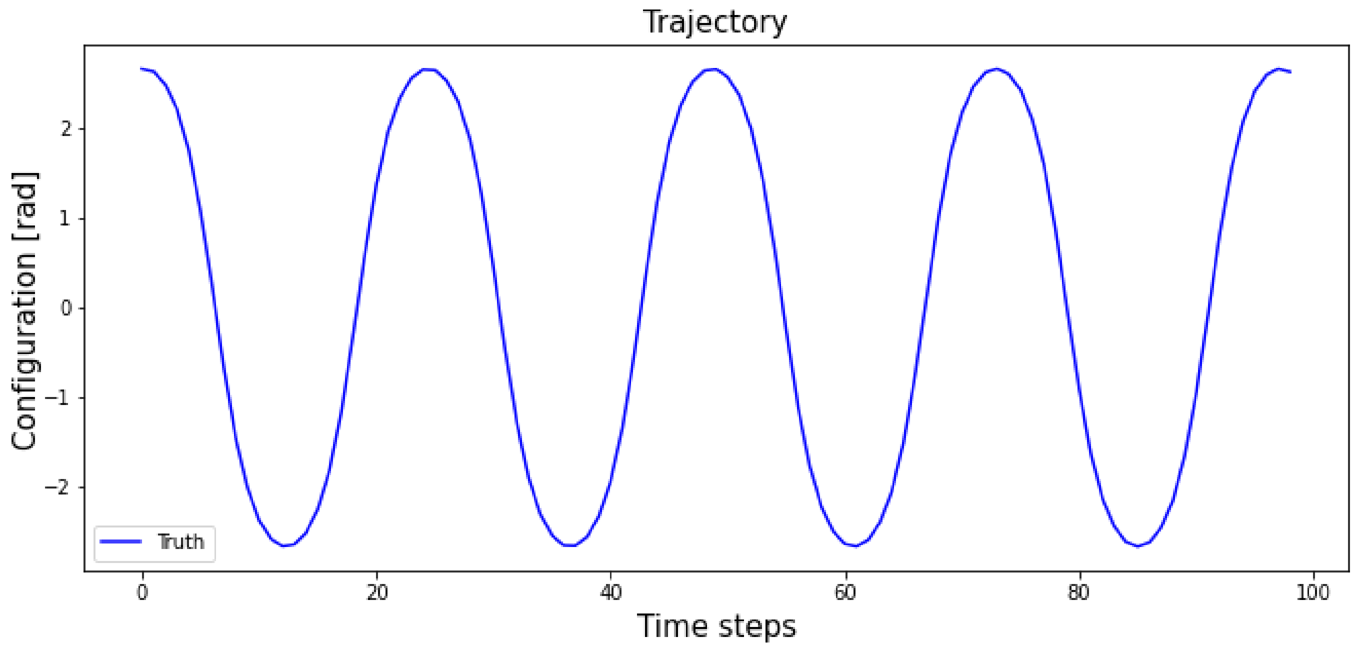
RIGID CASE – SIMULATION 2





RIGID CASE – SIMULATION 3

Initial conditions: $q(0) = 2{,}66\, rad \quad \dot{q}(0) = 0{,}5\, \frac{rad}{s}$



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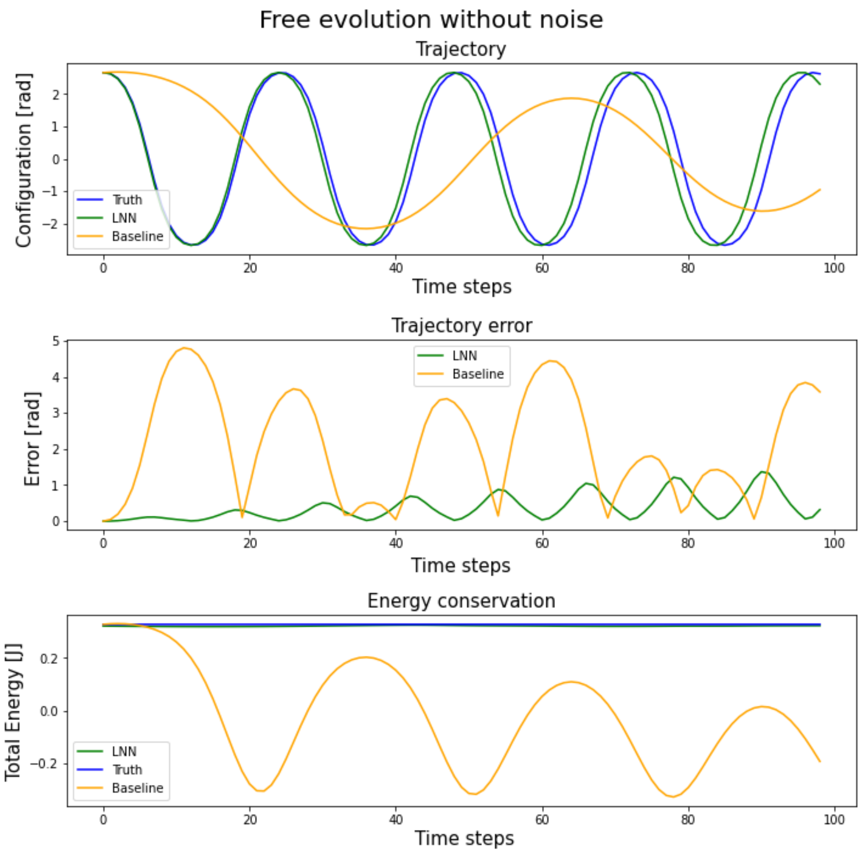
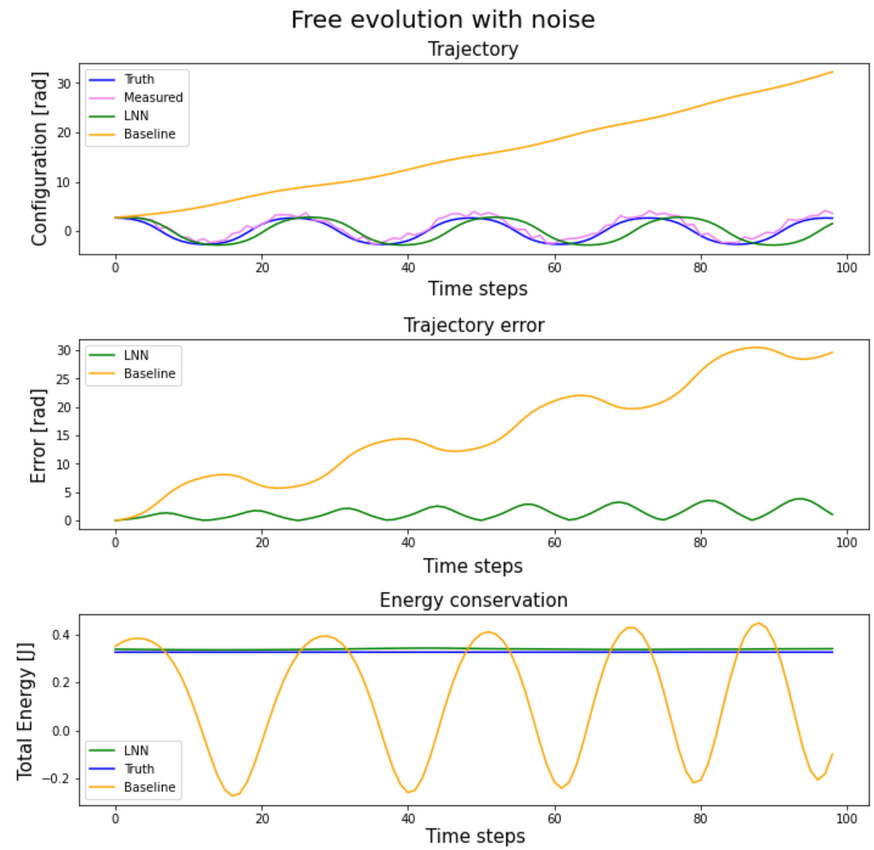
RIGID CASE – SIMULATION 3





ELASTIC CASE

Motor parameters

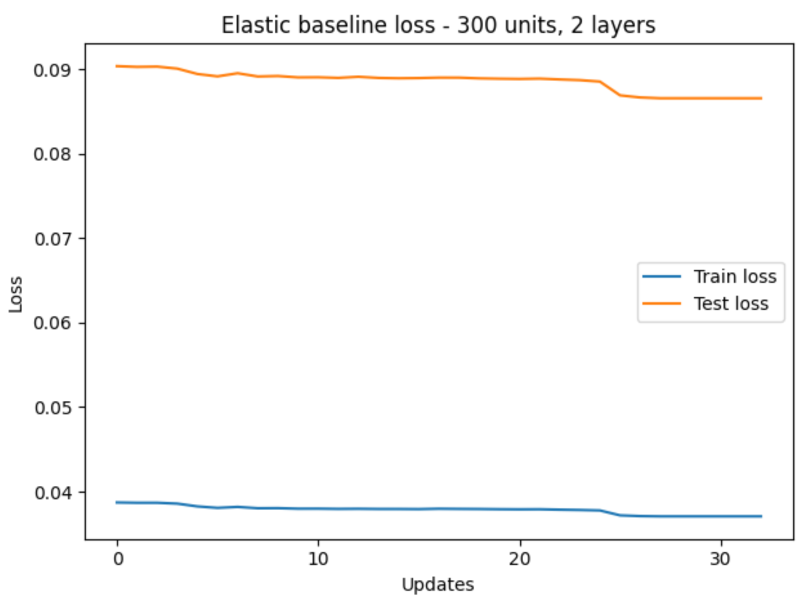
Parameter	Measure
Height	$0.04\ m$
Base radius	$0.02\ m$
Mass	$0.4\ kg$
Reduction ratio	160
Inertia	$0.0128\ kg\ m^2$
Stiffness constant	$50\ Nm/rad$

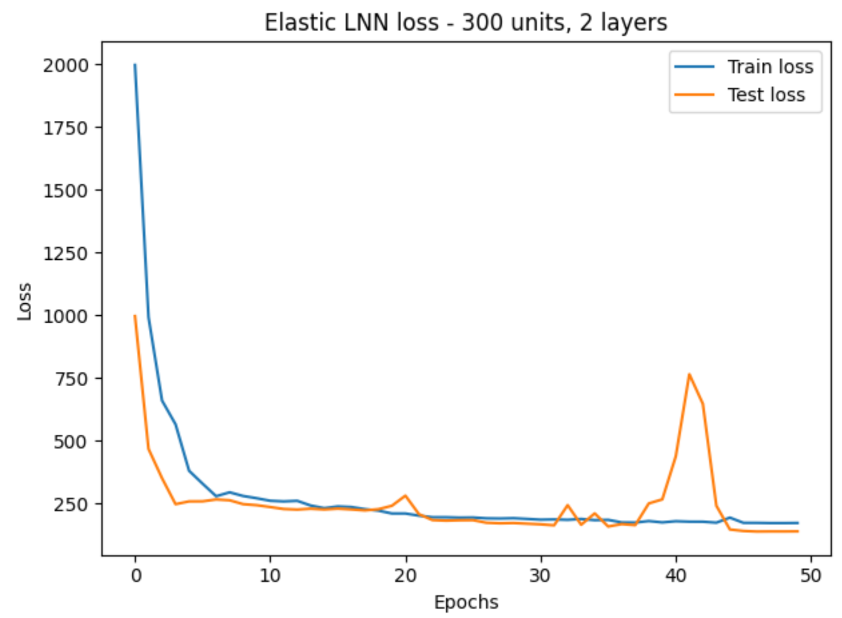
Network setup

Configuration	Units	Layers
A	300	2
B	500	4
C	700	5

ELASTIC CASE – CONFIGURATION A

Train and test losses

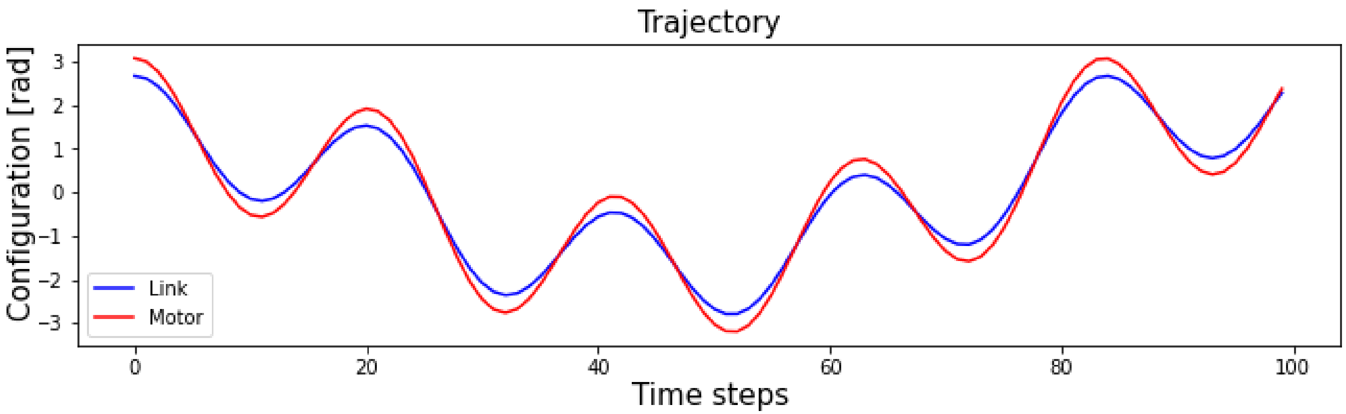




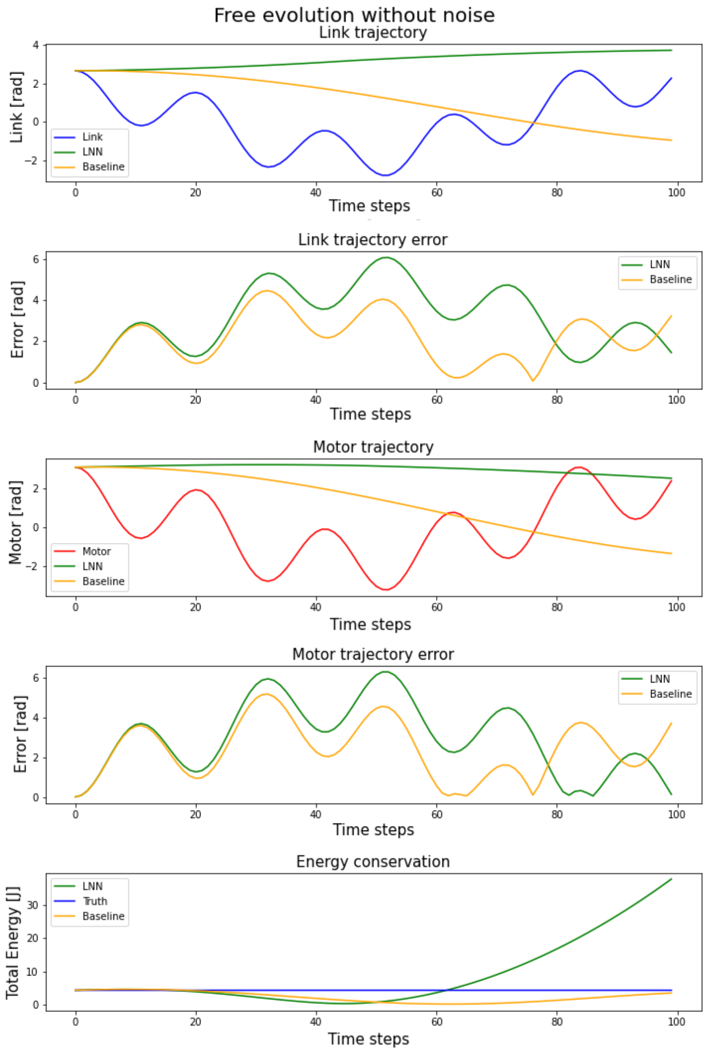
ELASTIC CASE – SIMULATION 1

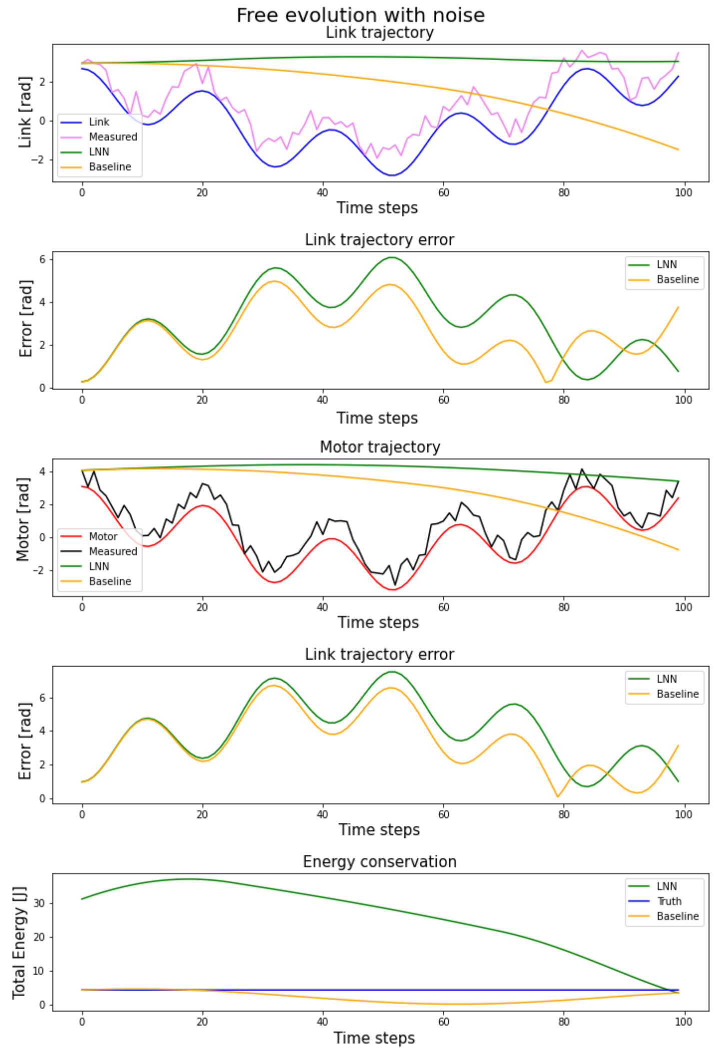
Initial conditions:

$$\begin{cases} q(0) = 2{,}66\,rad \quad \dot{q}(0) = 0{,}5\,\dfrac{rad}{s} \\ \theta(0) = 3{,}06\,rad \quad \dot{\theta}(0) = 1\,\dfrac{rad}{s} \end{cases}$$



ELASTIC CASE – SIMULATION 1

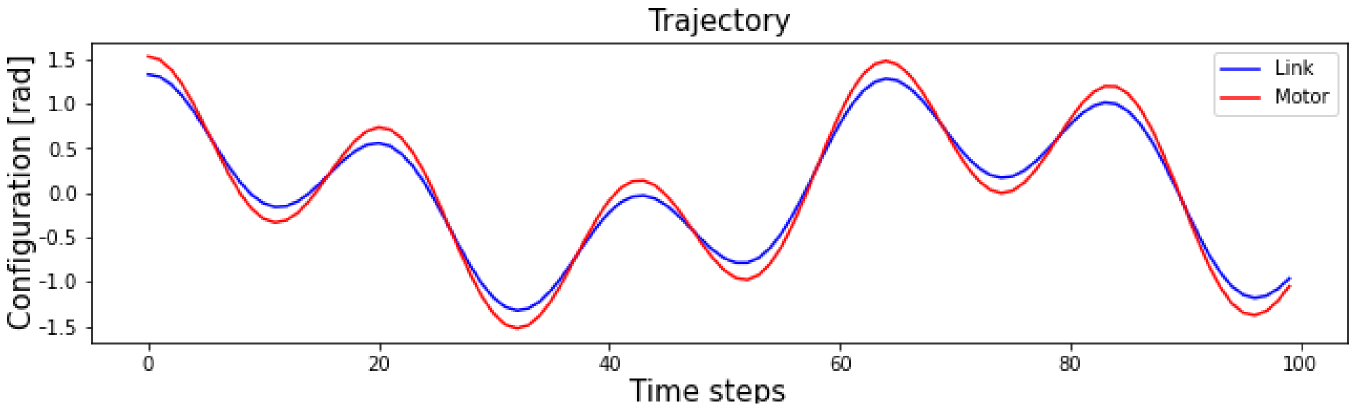




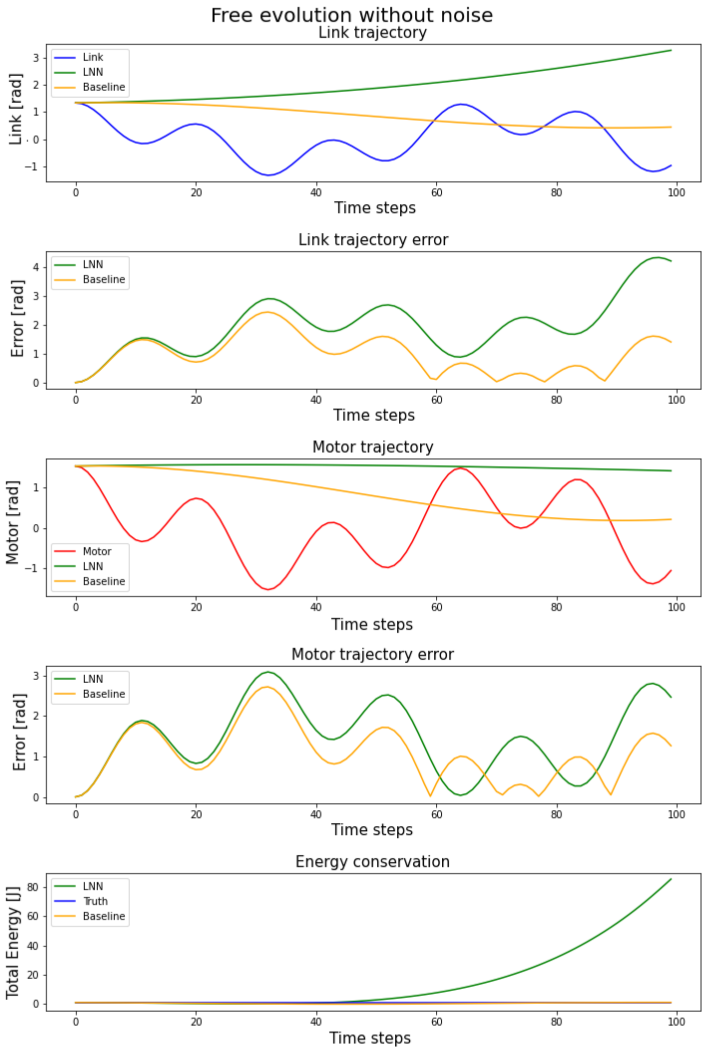
ELASTIC CASE – SIMULATION 2

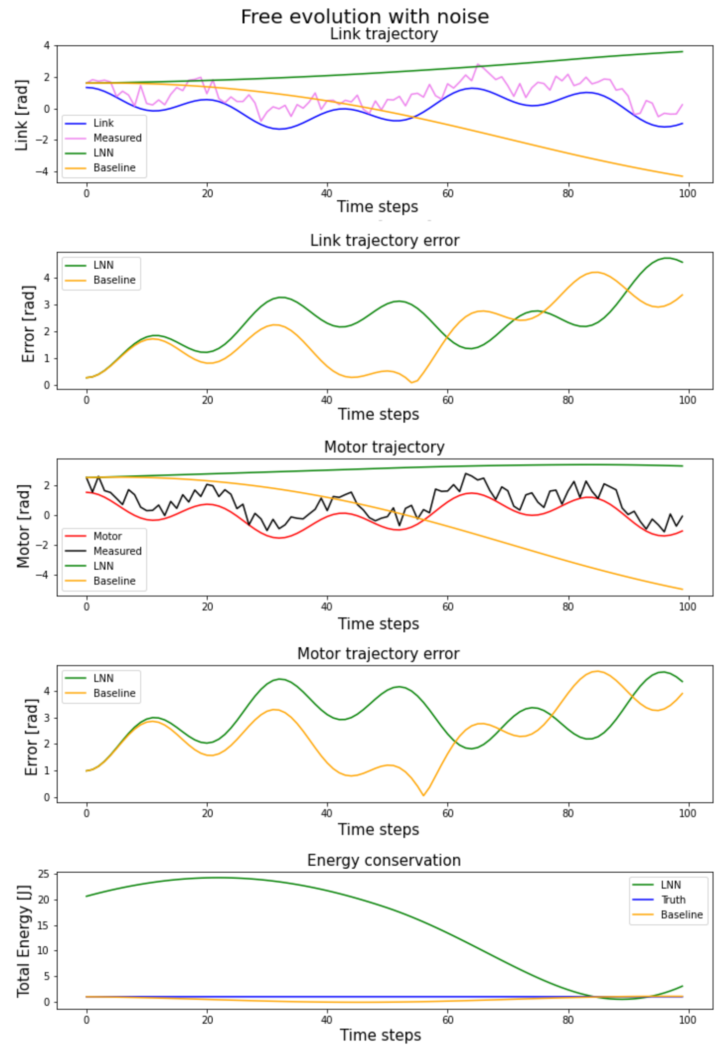
Initial conditions:

$$\begin{cases} q(0) = 1{,}33\, rad \quad \dot{q}(0) = 1\,\dfrac{rad}{s} \\ \theta(0) = 1{,}51\, rad \quad \dot{\theta}(0) = 0{,}5\,\dfrac{rad}{s} \end{cases}$$



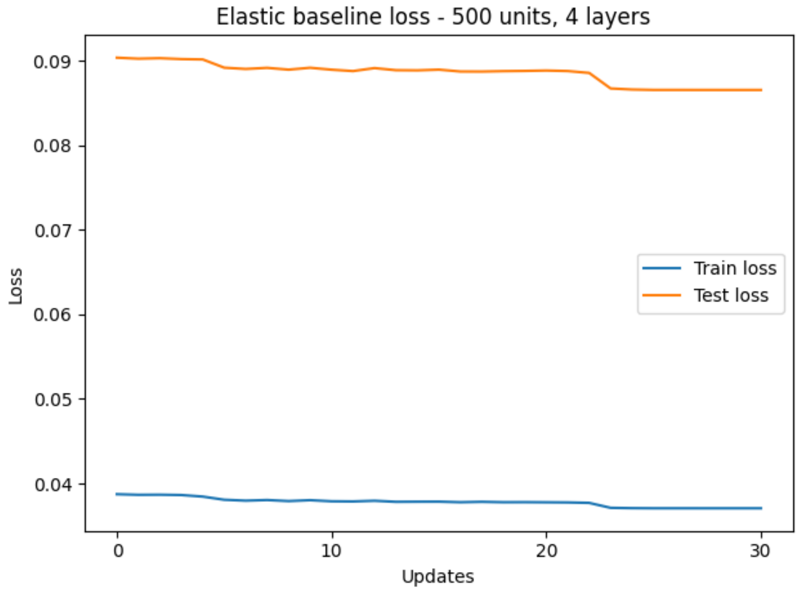
ELASTIC CASE – SIMULATION 2

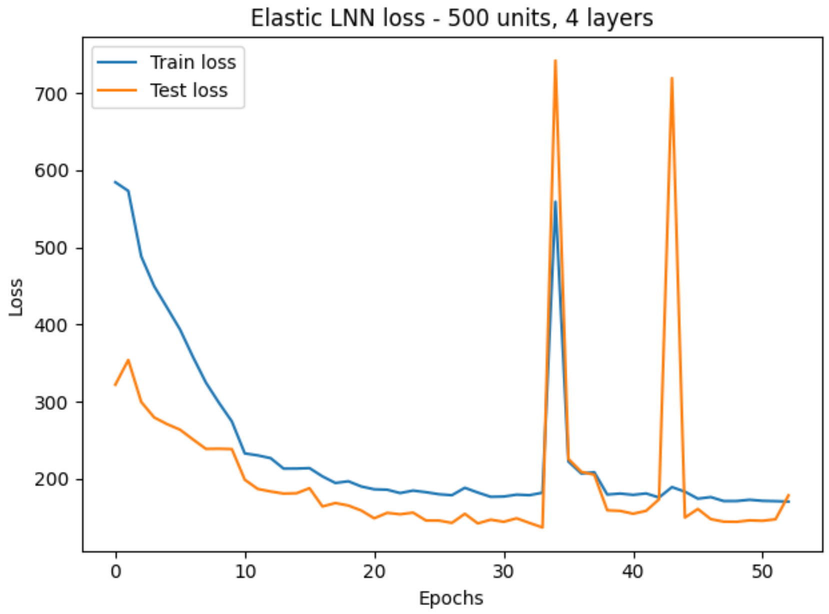




ELASTIC CASE – CONFIGURATION B

Train and test losses





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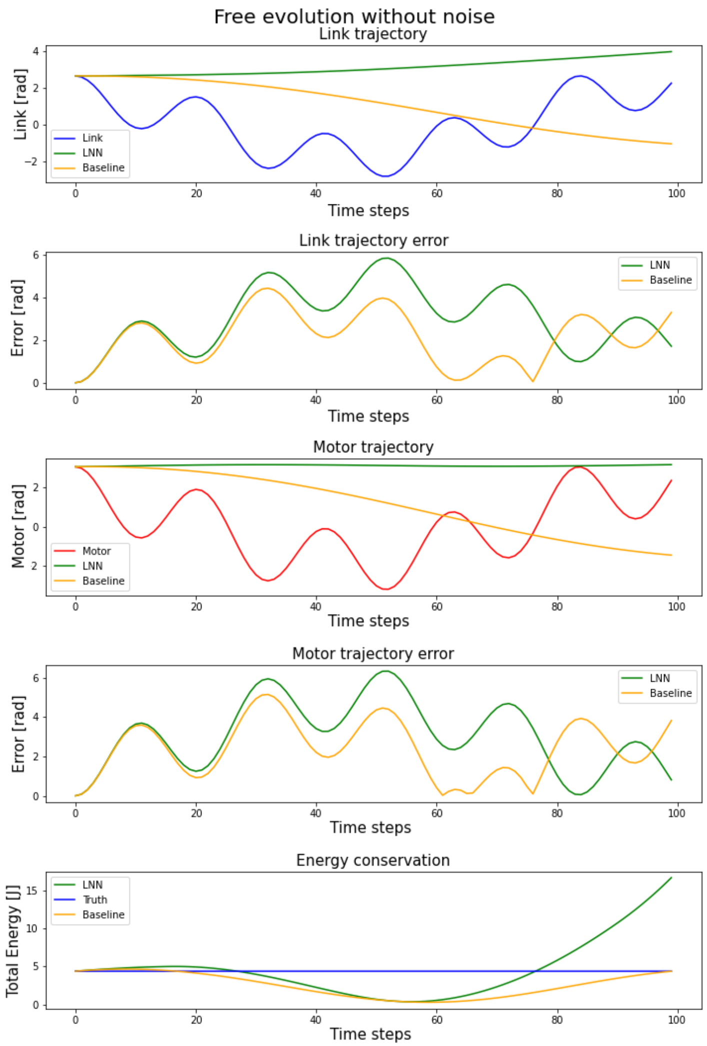
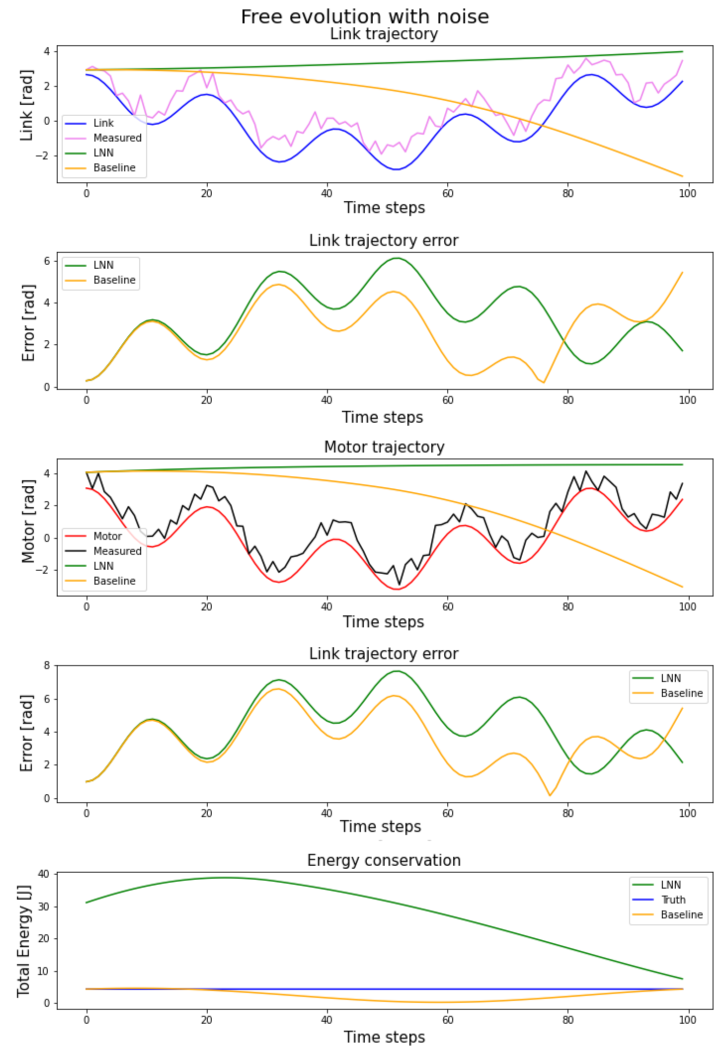
ELASTIC CASE – SIMULATION 3





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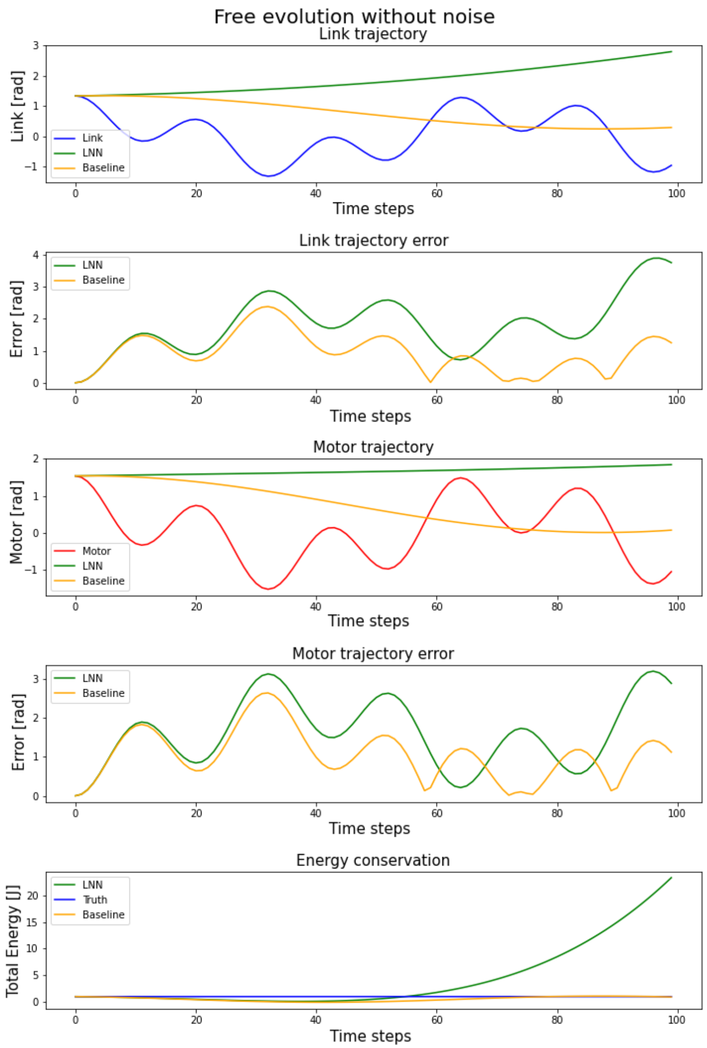
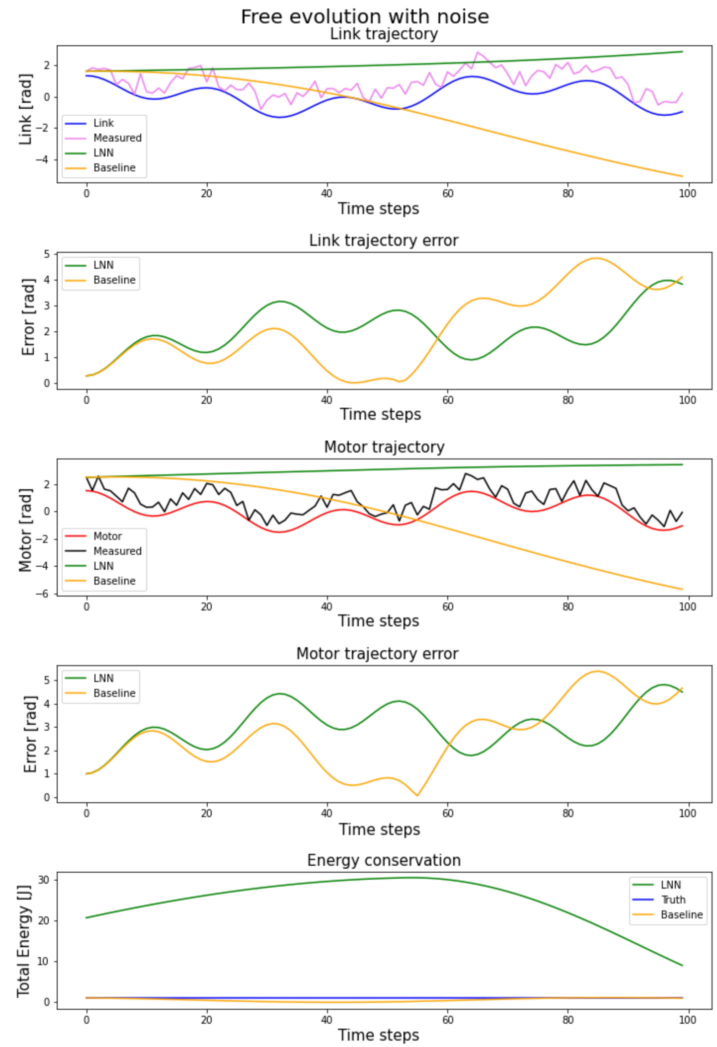
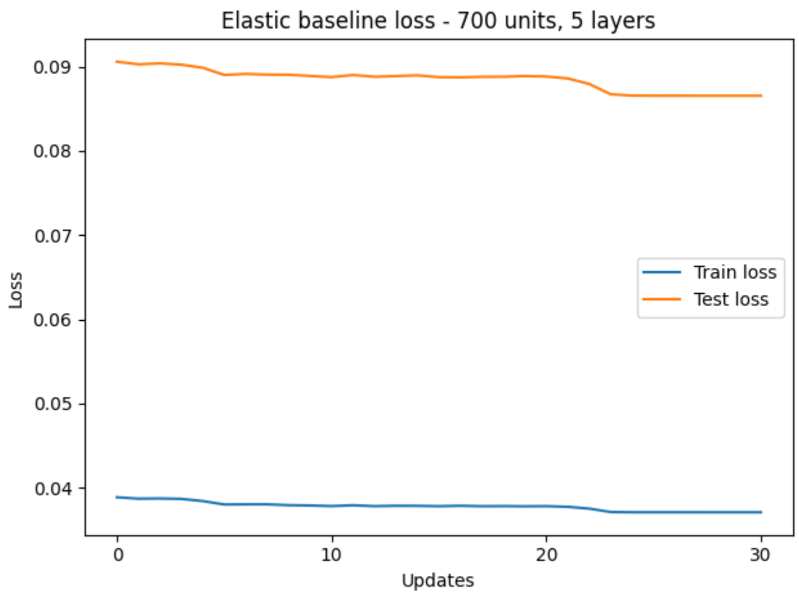
ELASTIC CASE – SIMULATION 4

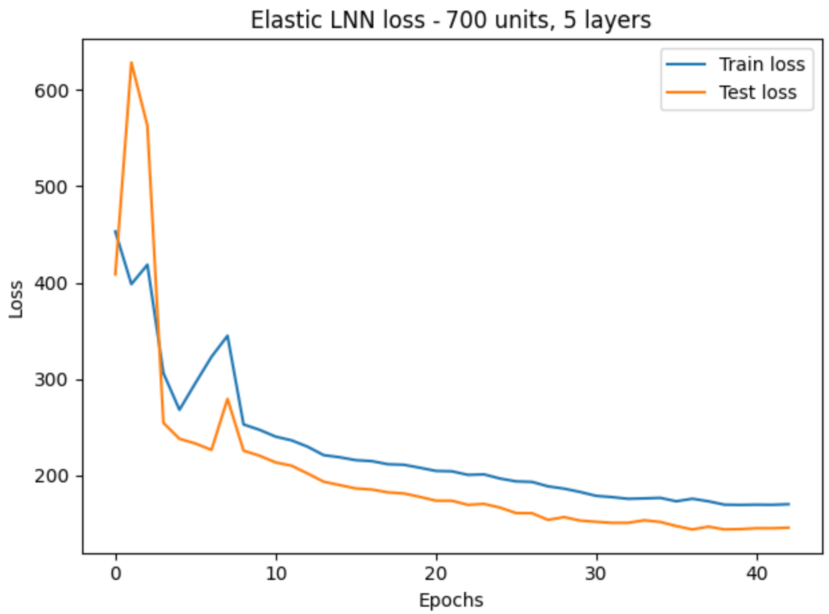




ELASTIC CASE – CONFIGURATION C

Train and test losses





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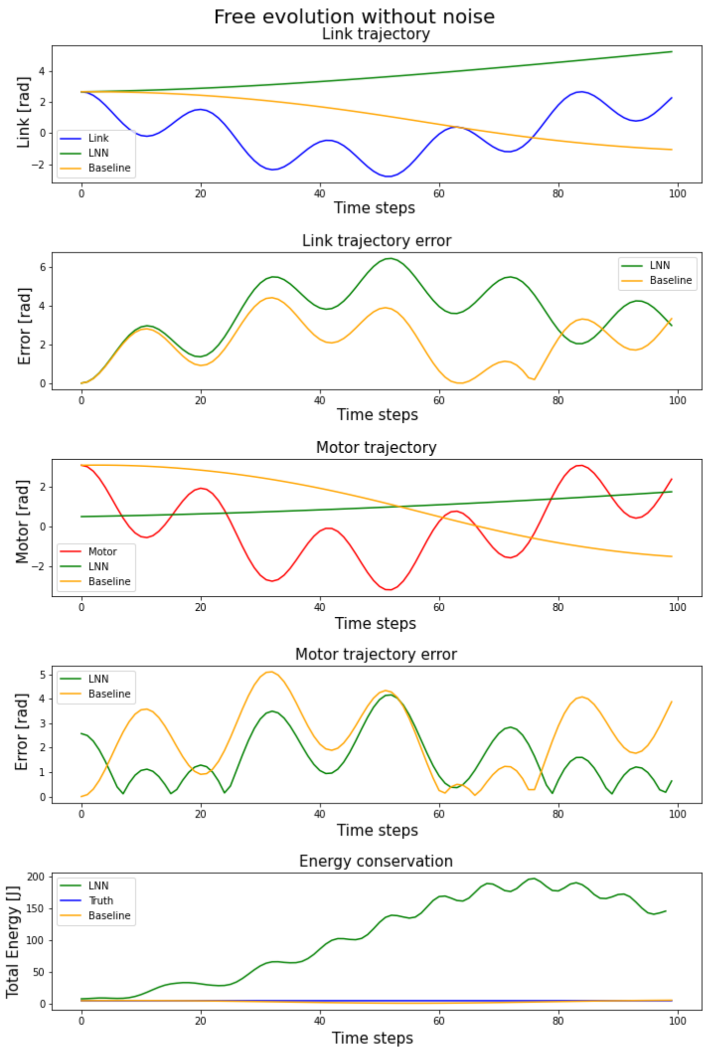
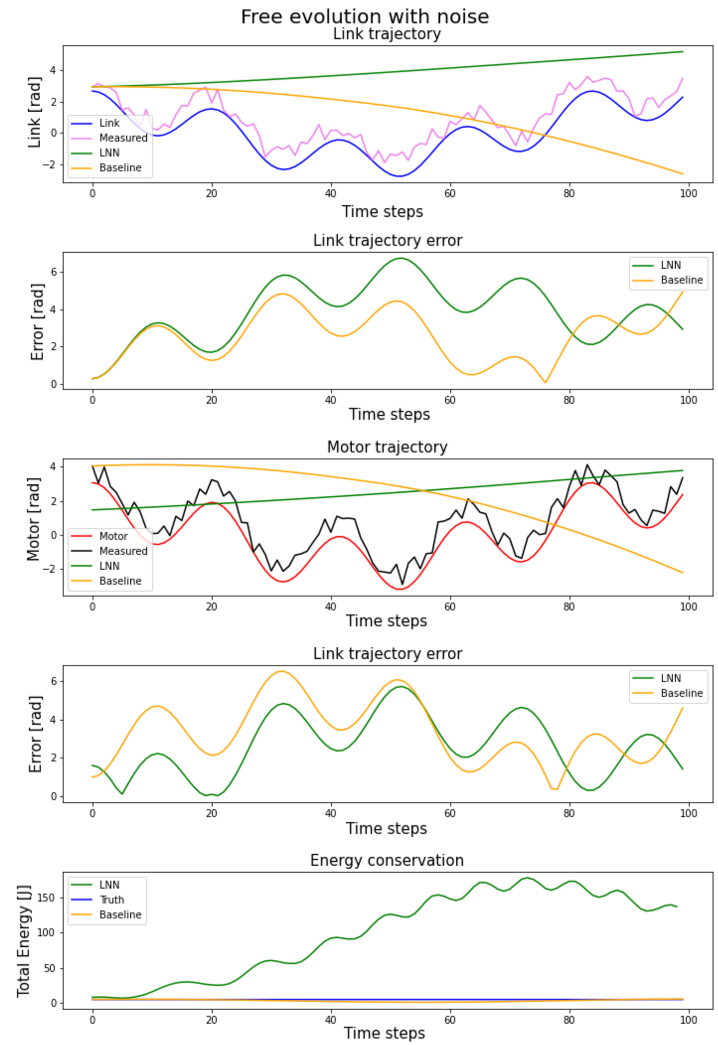
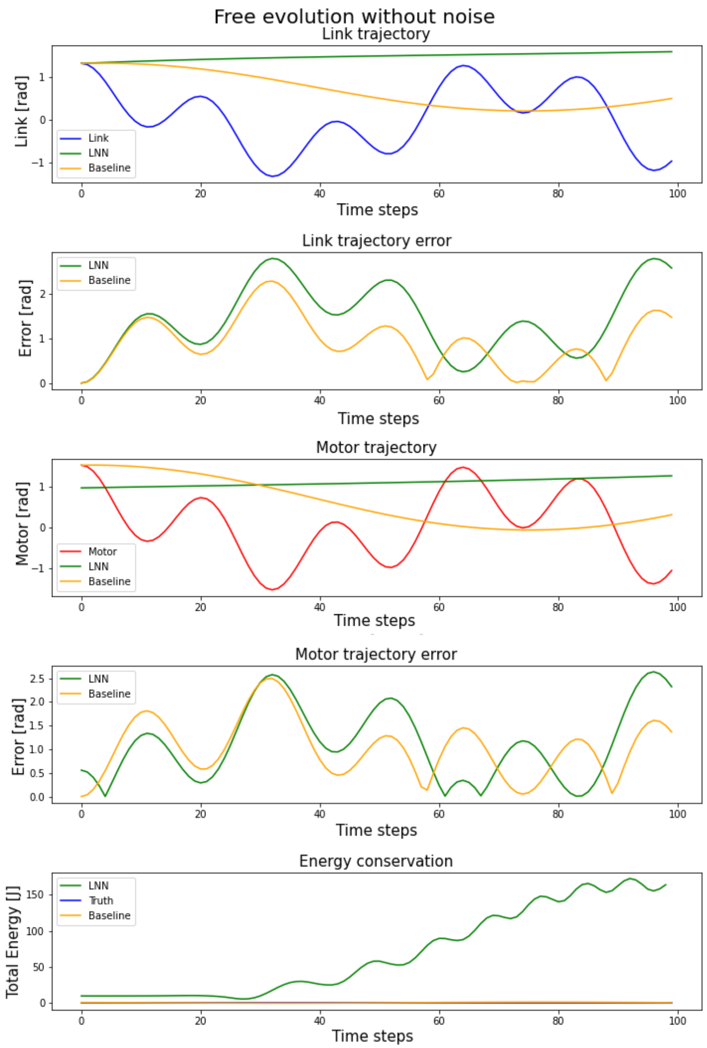
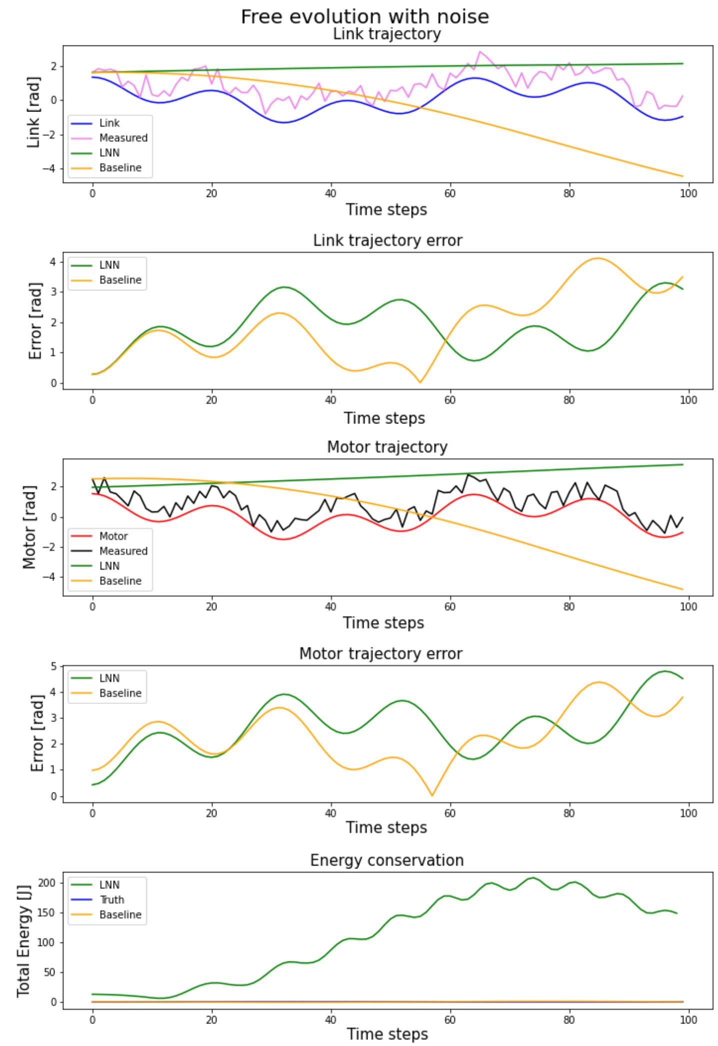
ELASTIC CASE – SIMULATION 5





ELASTIC CASE – SIMULATION 6





CONCLUSIONS

- At least in the rigid case, LNN performances are very good
  - Both in trajectory error and energy conservation
- Unfortunately, bad results in elastic case
- Elastic LNN seems to improve as network complexity increase
  - More wide/deep network and more initial conditions
  - High computational resources needed
- Paper LNN approach works on unitary configurations
  - We reject this in preference to real robot parameters

THANK YOU FOR THE ATTENTION