A5) Applying Laplace to linearised System

① $s\bar{Z}_1(s) = \bar{Z}_2(s)$

② $s\bar{Z}_2(s) = \frac{5m}{7}\left[c\left(a_1\bar{Z}_3(s) + a_2\bar{Z}_1(s)\right) - k\bar{Z}_1(s) - b\bar{Z}_2(s)\right]$

③ $s\bar{Z}_3(s) = a_3\left[\bar{V}(s) - R\bar{Z}_3(s)\right] - a_4\bar{Z}_1(s)$

$\Rightarrow$ ② $s^2\bar{Z}_1(s) = \frac{5m}{7}\left[ca_1\bar{Z}_3(s) + ca_2\bar{Z}_1(s) - k\bar{Z}_1(s) - bs\bar{Z}_1(s)\right]$

$\cancel{s^2\bar{Z}_1 = \frac{5mca_1\bar{Z}_3(s)}{7} + \frac{5mca_2\bar{Z}_1(s)}{7} - \frac{5mk\bar{Z}_1(s)}{7} - \frac{5mbs\bar{Z}_1(s)}{7}}$

$\frac{7}{5m}s^2\bar{Z}_1(s) + bs\bar{Z}_1(s) + k\bar{Z}_1(s) - ca_2\bar{Z}_1(s) = ca_1\bar{Z}_3(s)$

$\bar{Z}_1(s)\left(\frac{7s^2}{5m} + bs + k - ca_2\right) = ca_1\bar{Z}_3(s)$

$\bar{Z}_3(s) = \frac{\bar{Z}_1(s)}{ca_1}\left(\frac{7s^2}{5m} + bs + k - ca_2\right)$

③ $\Rightarrow$ $s\bar{Z}_3(s) = a_3\bar{V}(s) - a_3R\bar{Z}_3(s) - a_4\bar{Z}_1(s)$

$s\bar{Z}_3(s) + a_3R\bar{Z}(s) = a_3\bar{V}(s) - a_4\bar{Z}_1$

$\bar{Z}_3(s)(s + a_3R) = a_3\bar{V}(s) - a_4\bar{Z}_1$

$\frac{\bar{Z}_1(s)}{ca_1}(s+a_3R)\left(\frac{7s^2}{5m} + bs + k - ca_2\right) + a_4\bar{Z}_1(s) = a_3\bar{V}(s)$

$\bar{Z}_1(s)\left(\frac{ca_1a_4 + (s+a_3R)(\frac{7}{5m}s^2 + bs + k - ca_2)}{(ca_1)}\right) = a_3\bar{V}(s)$

$$\frac{\bar{Z}_1(s)}{\bar{V}(s)} = \frac{ca_1a_3}{(s+a_3R)\left(\frac{7s^2}{5m} + bs + k - ca_2\right) + ca_1a_4}$$

A5) This is a 3^{rd}-order transfer function thus has 3 poles. This can be modelled using the product of a first and second order transfer function for analytical purposes. Furthermore, the system's response can be approximated using the sum of the responses from the decomposed functions.

~~Due to the nature of the summation the slower response would become dominant For example;~~
~~An oscillatory impulse response would be generated if the second order function was underdamped~~

Due to the nature of the summation, oscillations present in an impulse response from the second-order system would become apparent in the overall system's impulse response; ~~Hence, Therefore, an underdamped second-order system would produce oscillations within~~ These oscillations would be created due the 2^{nd} order system being underdamped.

A second order transfer function has the form

$$G(s) = \frac{k}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

where $k, \tau, \zeta > 0$ k - static gain ζ - damping factor τ - time constant of sys

A underdamped 2^{nd} order system is catagorized ~~through by having the poles being~~ by a pair of complex conjugate, non-real poles. ~~which is found by~~ The nature of the poles which ~~can be identified by~~ can be identified by analysing the discriminent of the denominator.

$$\Delta\ \tau^2 s^2 + 2\zeta\tau s + 1 = (2\zeta\tau)^2 - 4(\tau^2)(1)$$
$$= 4\zeta^2\tau^2 - 4\tau^2$$
$$= 4\tau^2(\zeta^2 - 1)$$

~~Therefore, if the damping factor, ζ, is~~

A damping factor $0 < \zeta < 1$ would create a negative discriminant hence indicating an underdamped sys due to prescence of a pair of non-real conjugate poles.

Analysing poles of the transfer function

$$0 = (s + a_3 R)\left(\frac{7}{5m}s^2 + bs - h - ca_2\right) + ca_1 a_4$$

$$-ca_1 a_4 = (s + a_3 R)\left(\frac{7}{5m}s^2 + bs - h - ca_2\right)$$

mapping the Quadratic's co-efficients to (eq no of gen form)

$$\tau^2 = \frac{7}{5m} \qquad 2\zeta\tau = b$$

$$\tau = \sqrt{\frac{7}{5m}} \qquad \zeta = \frac{b}{2\tau}$$

$$\zeta = \frac{b}{2}\left(\sqrt{\frac{5m}{7}}\right)$$

$$= \frac{b\sqrt{5m}}{2\sqrt{7}}$$

~~Thus providing the damping coefficient, $\zeta = \frac{b\sqrt{5m}}{2\sqrt{7}}$~~, is

The system would present oscillations within an impulse response providing:

$$0 < \frac{b\sqrt{5m}}{7} < 1$$