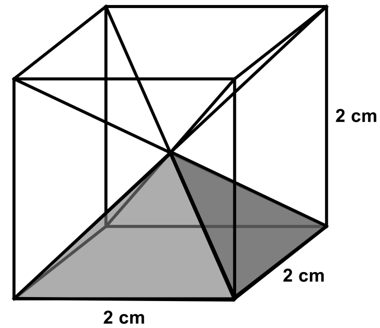
Name: ______________________________ Date: ________________

THE VOLUME OF PYRAMIDS AND CONES
COMMON CORE GEOMETRY

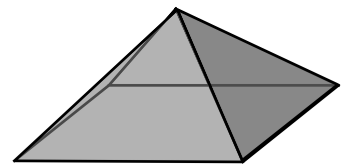
Pyramids and cones have volume formulas that are easy to use but are somewhat of a mystery until you reach calculus. We will explore the volume of one pyramid and see the relationship that ultimately holds for all pyramids and cones.

***Exercise* #1:** Consider a cube that has side lengths of 2 cm each. The center of the cube is located, and segments are drawn to each of the eight vertices of the cube. This creates the pyramid whose volume we are going to explore.

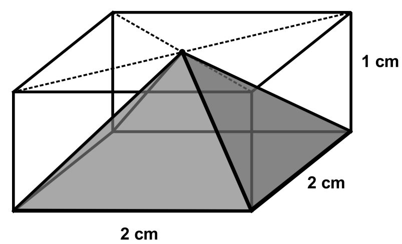


(a) What is the volume of the cube? Include appropriate units.

(b) How many of these pyramids would fit into the original cube? Why? Use this to determine the volume of the pyramid in simplest form.

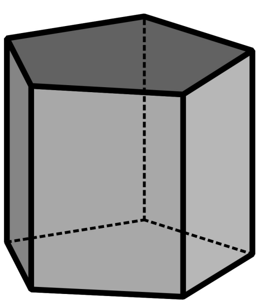


(c) The pyramid fits inside of a **prism** that has the same base as before and a height of 1 cm. What fraction of the prism's volume is filled by the pyramid? Fill in the missing fraction in the equation below.

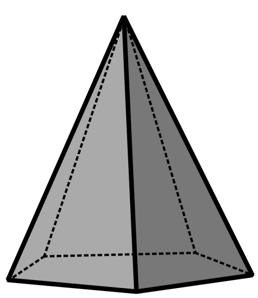


$$V_{\text{pyramid}} = \_\_\_\_\_\_\_\_\_\_ \times V_{\text{prism}}$$

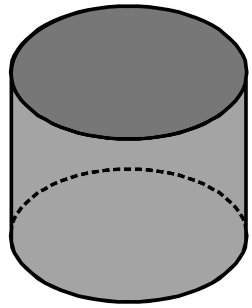
This relationship is remarkable in that it holds for all pyramids and cones as the following formulas will suggest.



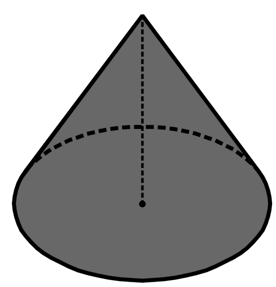
$V_{\text{prism}} = B \cdot h$



$V_{\text{pyramid}} = \dfrac{1}{3} B \cdot h$

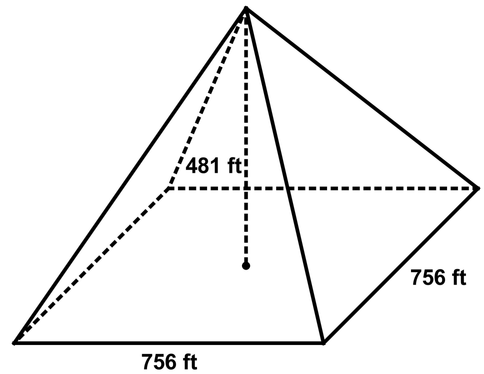


$V_{\text{cylinder}} = \pi r^2 h$

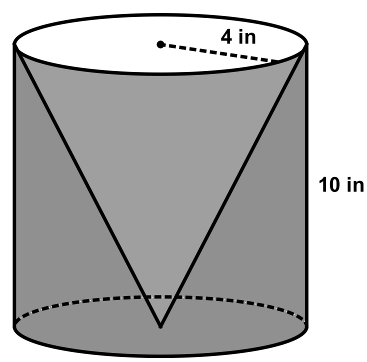


$V_{\text{cone}} = \dfrac{1}{3} \pi r^2 h$

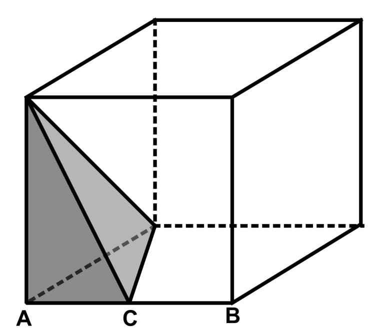
***Exercise* #2:** The Great Pyramid of Giza in Egypt has a square base that measures 756 feet on a side and a height of 481 feet tall. To the nearest million cubic feet, what is the volume of the Great Pyramid?



***Exercise* #3:** A cylinder of solid material with a radius of 4 inches and a height of 10 inches has a cone with the same radius and height removed from it. What is the volume of the remaining material? Round to the nearest cubic inch.



***Exercise* #4:** A pyramid is constructed from a cube as shown below. Point C is the midpoint of edge $\overline{AB}$. If the cube has a volume of 216 cm^3, then what is the volume of the pyramid?



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THE VOLUME OF PYRAMIDS AND CONES
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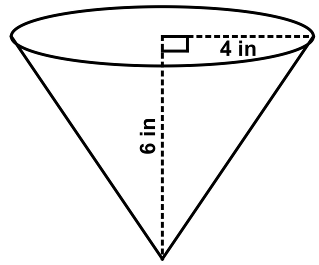
PROBLEM SOLVING

1. Which of the following is the volume of the cone pictured in cubic inches?



(1) 32π

(2) 48π

(3) 64π

(4) 96π

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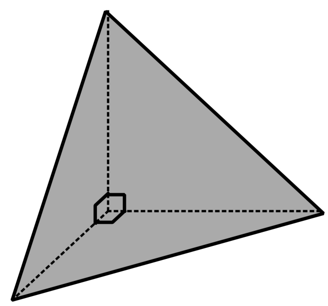
2. A pyramid is built such that three of its faces are congruent, isosceles right triangles whose legs are 10 inches long. The fourth face is an equilateral triangle. Which of the following is closest to the volume of the pyramid in cubic inches?



(1) 87 (3) 167

(2) 136 (4) 208

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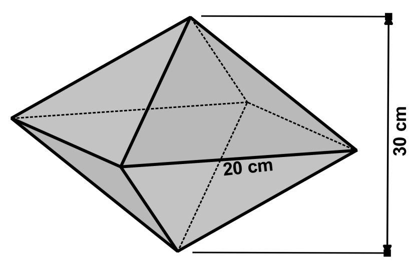
3. An octahedron (an 8 faced solid) is created by connecting two pyramids by their congruent square bases as shown. The square bases measure 20 cm on each side and the overall height of the octahedron is 30 centimeters as shown. What is the volume of the octahedron, in cubic centimeters?



(1) 1,500 (3) 4,000

(2) 2,575 (4) 6,400

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4. Which of the following is the volume of the largest cone that can fit inside of a cube whose volume is 8 cubic inches?

(1) $\frac{2}{3}\pi \text{ in}^3$ (3) $7\pi \text{ in}^3$

(2) $\frac{8}{3}\pi \text{ in}^3$ (4) $12\pi \text{ in}^3$

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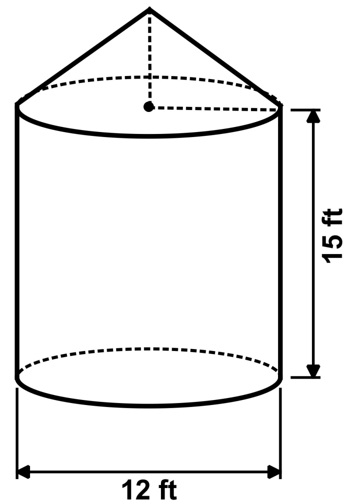
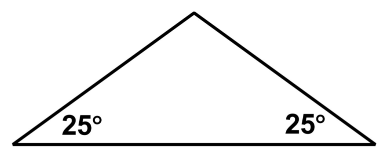
5. A water tower in New York City has the shape of a cylinder with a cone on top. The cylinder has a diameter of 12 feet and a height of 15 feet. The roof has an inclination angle of 25°.



(a) Determine the height of the cone to the nearest tenth of a foot.

(b) Determine the overall volume of the tower to the nearest cubic foot. Show the work that leads to your answer.

(c) There are 7.48 gallons in a cubic foot. If residents of the apartment building are using the water from the tower at an average rate of 56 gallons per minute, determine how long it will take to drain the entire tower.



Roof Cross-Section

REASONING

6. The cone pictured has the same diameter as the cylinder but only half of its height. If the cone is used to fill the cylinder with water, how many of the full cones will be needed to fill the cylinder completely? Explain how you arrived at your answer.