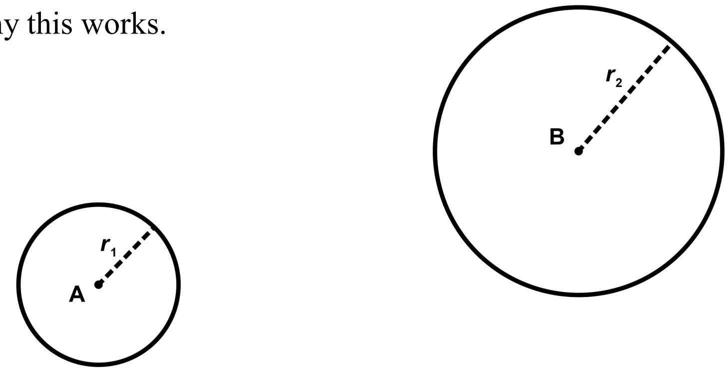
Name: ______________________________ Date: _______________

Radian Angle Measurement
Common Core Geometry

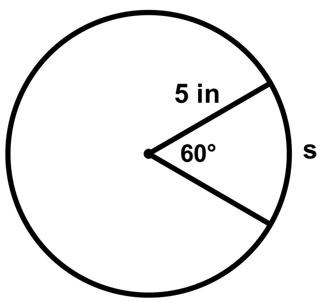
Up until this point, you have only ever **measured angles** in units of **degrees**. This would be like only measuring distance in inches or weight in pounds. There are other ways of measuring angles. We will examine measuring angles in **radians** today. Radians are based on a simple fact: **all circles are similar**.

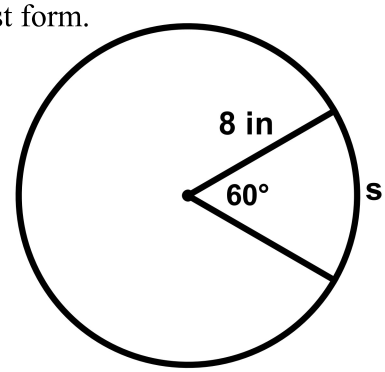
***Exercise* #1:** Below are circle A with radius r_1 and circle B with radius r_2. Give a similarity transformation that shows that the two circles are similar. Fully explain why this works.



Now, since all circles are similar, **ratios of corresponding portions of the circles will stay constant**. Let's see how this works in the next exercise.

***Exercise* #2:** Two circles are shown below with sectors defined by central angles of 60°. One has a radius of 5 inches and one has a radius of 8 inches. Calculate the arc length, s, for both sectors and then the ratio of arc length to radius. Write all your answers in terms of pi and in simplest form.





arc length $= s =$

ratio $= \dfrac{s}{r} =$

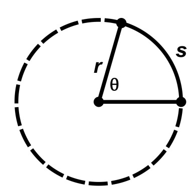
arc length $= s =$

ratio $= \dfrac{s}{r} =$

Notice that in both cases, the ratio of arc length to radius is the same. It only depends on how much we have rotated because the two circles are **similar**. This gives rise to the definition of radian rotation.

Radian Rotation Definition/Formula

A **radian** rotation of arc length s about circle whose radius is r is given by: $\theta = \dfrac{s}{r}$



It appears that the radian measure of a rotation about a circle depends on the size of a circle. But, it doesn't because all circles are similar. Thus, the ratio only relies on the rotation itself. Let's investigate this by determining the number of radians in a full rotation about a circle.

***Exercise* #3:** Given a circle whose radius is r answer the following questions.

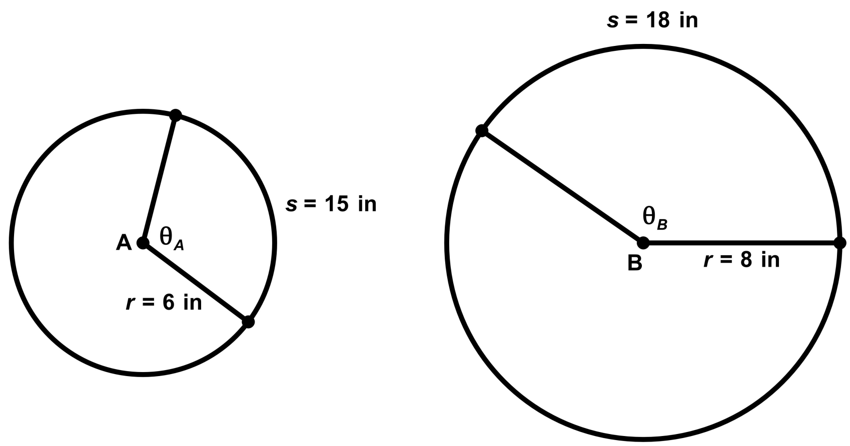
(a) What arc length, s, is traveled in a full 360° rotation? Express your answer in terms of pi and the circle's radius, r.

(b) Given that the radian measure is always $\theta = \frac{s}{r}$, what radian measure represents a full rotation around the circle?

So we see that a full rotation of 360° is equivalent to a rotation of 2π radians (or about 6 radians).

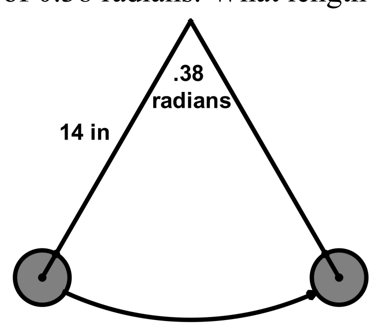
The formula for radians is exceptionally easy to use, although it will likely seem like a very foreign way to measure the size of a rotation.

***Exercise* #4:** In the following diagram, two sectors are shown with central angles labeled. Which has the greater radian rotation? Note: not drawn to scale.



The radian rotation measures the **number of radii** that can be laid out along any curve length in a given rotation. This can be used to help determine arc lengths if the radius and radian angle of rotation are known.

***Exercise* #5:** A pendulum whose length is 14 inches swings through a central angle of 0.38 radians. What length does its tip move through in a single swing?



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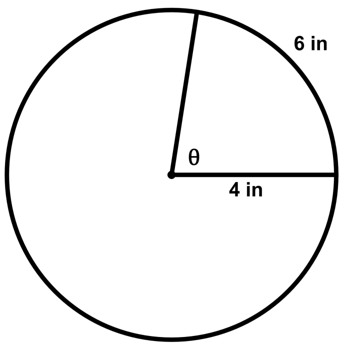
RADIAN ANGLE MEASUREMENT
COMMON CORE GEOMETRY HOMEWORK

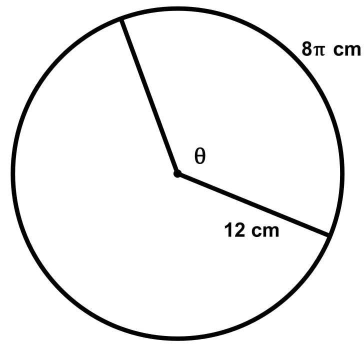
PROBLEM SOLVING

1. In each diagram below, a circle is shown with a particular radius and a particular arc length. Determine the radian measure of the central angle. Express your answer in simplest terms of pi when applicable.

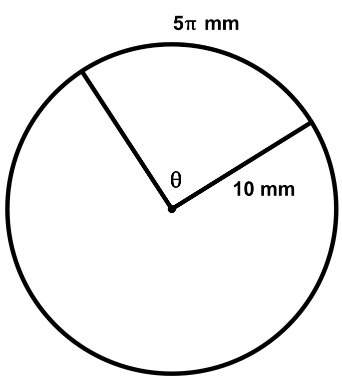
(a)



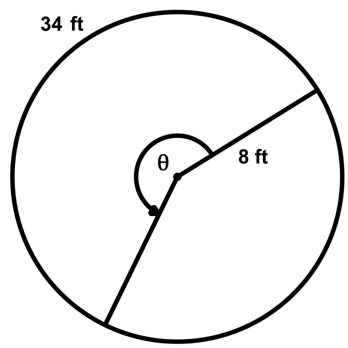
(b)



(c)



(d)

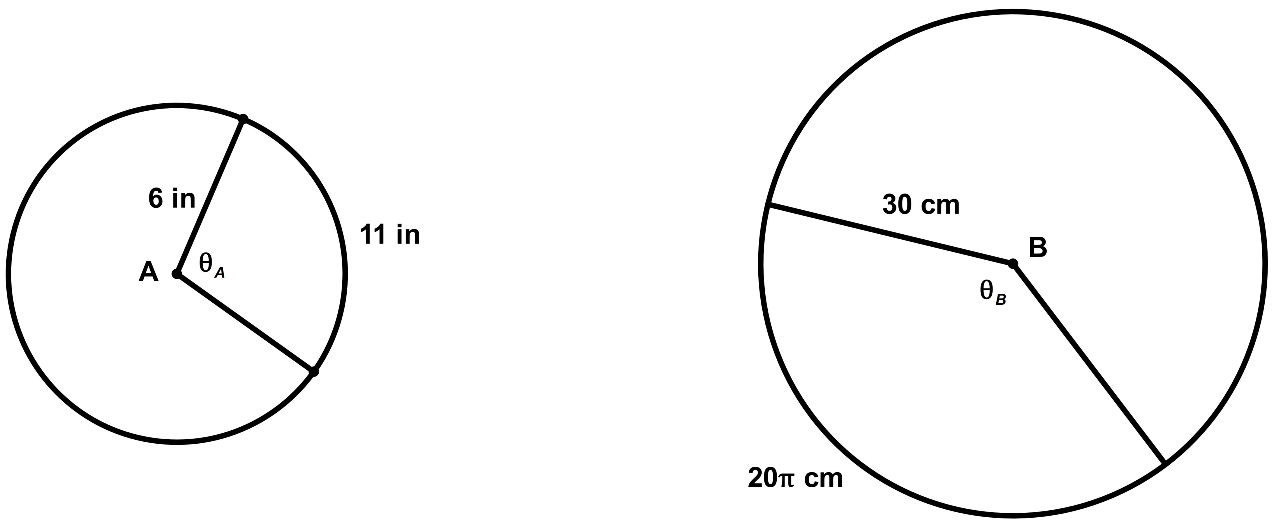


2. Janine walks on a circular track whose radius is 60 meters. If she walks a total of 100 meters, which of the following represents the radian angle she rotates about the center of the track?

(1) $\frac{5}{3}$ (3) $\frac{5\pi}{3}$

(2) $\frac{3}{5}$ (4) $\frac{3\pi}{5}$ __________

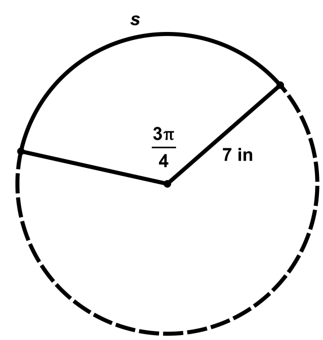
3. Which circle has the greater radian rotation shown below? Justify your answer.



4. An arc is intercepted by two radii that form an angle of $\frac{3\pi}{4}$ radians. If the radius of the circle is 7 inches, then which of the following is closest to the arc length, s, also measured in inches?

(1) 12.8 (3) 18.7

(2) 16.5 (4) 21.8



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REASONING - An important skill to begin mastering is the conversion between angles measured in degrees and angles measured in radians. See if you can fill out the following table. The first two lines are done for you.

5.

Angle in Degrees	Fraction of Circle	Angle in Radians
360°	1	2π
180°	$\frac{1}{2}$	$\frac{1}{2}\cdot(2\pi)=\pi$
90°		
60°		
30°		
45°		