Name: _____   Date: _____

THE CIRCUMFERENCE OF A CIRCLE
COMMON CORE GEOMETRY

If a piece of string was laid out around a circle and then its length was measured, we would call this measurement the **circumference** of the circle. Circumference is truly just a special type of **perimeter** measurement. The circumference of a circle can be calculated by the following formula that you should be familiar with from middle school geometry.

THE CIRCUMFERENCE OF A CIRCLE

For a circle whose diameter is *D* and whose radius is *r*, the circumference, *C* is:

$$C = \pi \cdot D \qquad \text{or} \qquad C = 2\pi \cdot r$$

Exercise #1: For the circle whose equation is $x^2 + y^2 = 25$, which of the following is closest to its circumference? Show how you found your answer.

(1) 15.7   (3) 31.4

(2) 25.9   (4) 78.5

                                                         _____

Since objects that are circular or nearly circular arise everywhere in our world, the circumference formula for a circle is an important one.

Exercise #2: A tree trunk has a roughly circular cross-section. A logger uses a tape measure to find that the circumference of the cross section is six feet long. Which of the following would be the best estimate of the diameter of the cross section?

(1) 8 inches   (3) 34 inches

(2) 23 inches   (4) 46 inches

                                                          _____

We can also combine the perimeter work from the previous lesson with the circumference formula from today.

Exercise #3: A garden is to be enclosed by a combination of a rectangle and a semicircle. Fencing is to be put up around this garden (but not along the dashed line). How many feet of fencing would be needed? Round to the nearest foot.
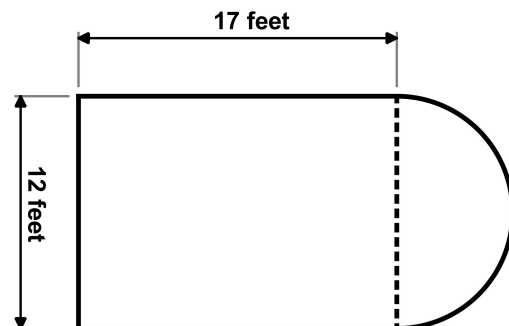
The circumference formula and the number pi, π, are somewhat of a mystery. The next set of exercises is designed to help you understand where the formula comes from and how pi gets involved. It takes calculus to fully derive this formula.

Exercise #4: The picture below shows regular polygons whose length from center to any vertex is staying a constant 1 inch. As the number of sides increases, what should the perimeter of the regular polygons approach and why?

n=4   n=6   n=8   n=10   n=15
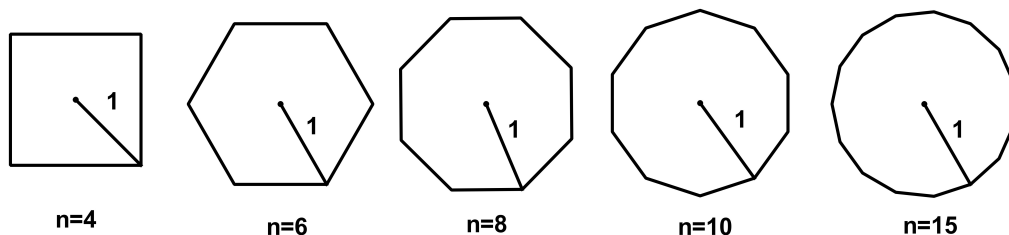
The perimeters of these polygons can be found using right triangle trigonometry as we did in the last lesson.

Exercise #5: Shown below is the regular 10-sided figure from above along with a related isosceles triangle. Determine the perimeter of the decagon. Round your answer to the nearest hundredth.
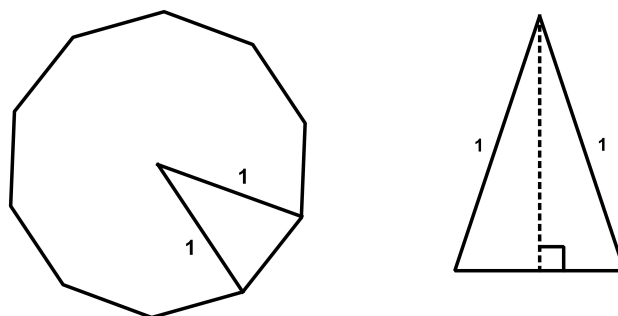
Exercise #6: Show that the perimeter of any of these regular polygons with the distance from the center to a vertex of 1 is given by $P = 2n\sin\left(\dfrac{360}{2n}\right)$, where n is the number of sides.

Exercise #7: Using the formula from Exercise #6, fill in the table below. Round to three decimal places. Compare to your answer from Exercise #4.

n	5	10	50	100	200
$P(n)$					

THE CIRCUMFERENCE OF A CIRCLE
COMMON CORE GEOMETRY HOMEWORK

PROBLEM SOLVING

1. Which of the following is the circumference of a circle whose equation is $x^2 + y^2 = 100$?

   (1) 5π (3) 20π

   (2) 10π (4) 50π

   _____

2. The distance around a circular dance hall room is 135 feet. Which of the following is closest to the longest distance across the dance hall?

   (1) 19 feet   (3) 36 feet

   (2) 28 feet   (4) 43 feet

   _____

3. In the following shape, two semicircles have been placed at the end of a rectangle whose length is twice its height. Which of the following gives the length around this figure in terms of the semicircle's radius, r?

   (1) $2\pi r + 8r$

   (2) $4\pi r + 4r$

   (3) $8\pi r + 2r$

   (4) $4\pi r + 4r$

   _____

4. The following regular polygon has 15 sides. This distance from its center to any given vertex is 12 inches. Which of the following is the best approximation for its perimeter?

   (1) 68 inches   (3) 84 inches

   (2) 75 inches   (4) 180 inches

   12 in

   _____
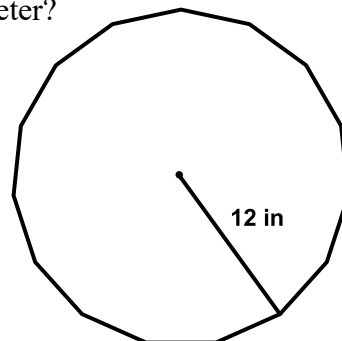
5. A metal ornament is being designed such that its perimeter is created by four identical three-quarter circles as shown whose centers are connected to form a square. The ornament, both circular and square portions, is made of wire that weighs 1.8 grams per inch. The square has sides that are 4 inches long.

(a) Determine the total length of the circular portions in terms of pi. Show the work that leads to your answer.

(b) Determine the total length of wire needed, both circular and square portions, to the nearest tenth of an inch.

(c) Determine the total weight of the ornament to the nearest gram.
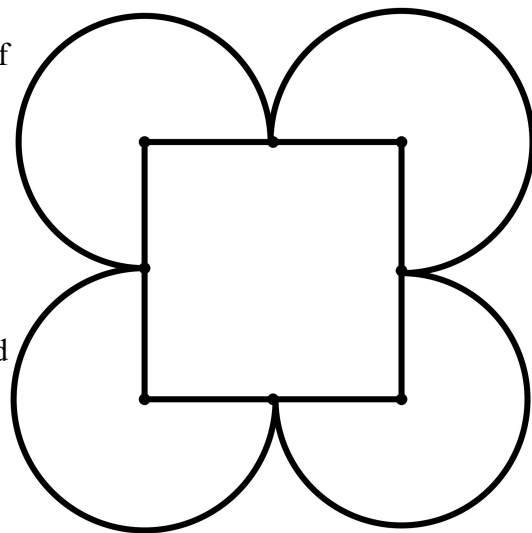
6. Semicircles and quarter circles are types of **arc lengths**. Recall that an **arc** is simply part of a circle. We learned about the **degree measure** of an **arc**, but they also have physical lengths.

(a) Determine the arc length to the nearest tenth of an inch.
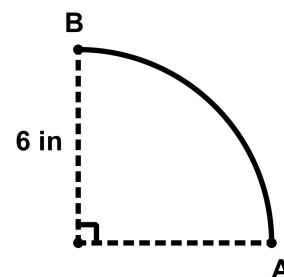
(b) Explain why the following proportion would solve for the length of $\overset{\frown}{AC}$ below.

$$\frac{x}{12\pi} = \frac{130}{360}$$
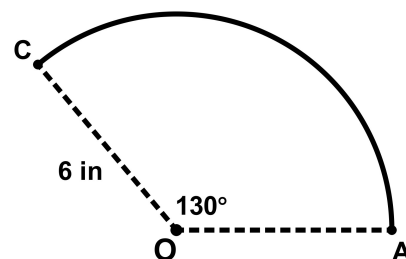
(c) Solve the proportion in (b) to find the length of $\overset{\frown}{AC}$ to the nearest tenth of an inch.