

# AADS project N4.

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### Dynamic programming algorithm

In order to define the dynamic algorithm used, we will explain the 4 steps of the algorithm in details:

#### How the problem is divided in subproblems:

The optimal solution for a tree, is obtained considering the optimal solutions of the subtrees rooted in the children and grandchildren of the root.

#### How the optimal solution of the subproblems are combined in order to solve a problem of bigger size

Define  $VC(r)$  as the minimum vertex cover for the tree rooted in  $r$ .

For each node  $i$  of the tree we consider two cases:

1. The node  $i$  is inserted in the vertex cover, in this case the optimal solution of his children must be considered.

$$VC_{in}(i) = 1 + \sum_{k=1}^{children(i)} VC(k)$$

2. The node  $i$  is not inserted in the vertex cover.

In this case, all the children of  $i$  have to be inserted in the vertex cover, and the optimal solutions of his grandchildren must be considered.

$$VC_{out}(i) = children(i) + \sum_{k=1}^{grandchildren(i)} VC(k)$$

At the end the minimum of this two quantities is considered:

$$VC(i) = \min(VC_{in}(i), VC_{out}(i))$$

#### Borderline cases:

1. If the tree is empty, the solution is 0
2. If the node  $i$  has no children, the solution for  $VC(i)$  is 0

#### In which order the subproblems are solved:

Recursion is used in order to visit the tree, so the problems are solved in the same order of an inorder visit of a tree.

$VC(i)$  could be evaluated multiple times, because there is overlapping between subproblems.

When the algorithm computes  $VC(i)$  for a node  $i$  it memorizes it in the node in order to avoid this problem and optimize the complexity.