Numerical Integrals

Using the Trapezoidal Rule and Simpson's Rule

Mathematics is not about numbers, equations, computations, or algorithms: it is about understanding. "Ah, if we could only do the all the integrals . . . But we can't."

— Beta Mathematica

Infinity Tutors

Trapezoidal Rule

Suppose we need to approximate the integral

$$\int_a^b f(x)\,dx$$

using the Trapezoidal Rule with n subdivisions. Then:

$$\int_a^b f(x)\,dx \approx T_n = \frac{h}{2}\Big[f(x_0) + f(x_n) + 2\Big(f(x_1) + f(x_2) + \ldots + f(x_{n-1})\Big)\Big]$$

where:

$$h = \frac{b-a}{n}, \quad x_i = a + ih, \quad \text{for } i = 0, 1, 2, \ldots, n$$

Also,

$$x_0 = a, \quad x_n = b$$

Computation Table:

i	x_i	$f(x_i)$
0		
1		
2		
$\vdots$	$\vdots$	$\vdots$
$n-1$		
n		

Activity 1

Use Trapezoidal Rule to approximate the following integrals with the given n.

(a). $\displaystyle\int_0^1 e^{4x^2}\,dx, \quad n = 7$

(b). $\displaystyle\int_8^{30} \left[2000 \ln\left(\frac{140000}{140000 - 2100t}\right) - 9.8t\right] dt, \quad n = 5$

(c). $\displaystyle\int_0^4 x \sin x\,dx, \quad n = 5$

(d). $\displaystyle\int_0^4 \sqrt{x}e^x\,dx, \quad n = 7$

(e). $\displaystyle\int_1^3 \ln x\,dx, \quad n = 6$

(f). $\displaystyle\int_0^2 xe^{-x}dx, \quad n = 8$

(g). $\displaystyle\int_0^\pi f(x)dx,$ where $f(x) = \begin{cases} \frac{\sin x}{x}, & x > 0 \\ 1, & x = 0 \end{cases} \quad ; \quad n = 6$

Simpson's Rule

Suppose we need to approximate the integral

$$\int_a^b f(x)\,dx$$

using Simpson's Rule with n subdivisions (note: n must be even). Then:

$$\int_a^b f(x)\,dx \approx S_n = \frac{h}{3}\Big[f(x_0)+f(x_n)+4\Big(f(x_1)+f(x_3)+\ldots+f(x_{n-1})\Big)+2\Big(f(x_2)+f(x_4)+\ldots+f(x_{n-2})\Big)\Big]$$

where:

$$h = \frac{b-a}{n}, \quad x_i = a + ih, \quad \text{for } i = 0, 1, 2, \ldots, n$$

Also,

$$x_0 = a, \quad x_n = b$$

Computation Table:

i	x_i	$f(x_i)$
0		
1		
2		
$\vdots$	$\vdots$	$\vdots$
$n-1$		
n		

Activity 2

Use Simpson's Rule to approximate the following integrals with the given n.

(a). $\displaystyle\int_0^1 e^{4x^2}\,dx, \quad n = 6$

(b). $\displaystyle\int_8^{30} \left[2000\ln\left(\frac{140000}{140000-2100t}\right) - 9.8t\right] dt, \quad n = 8$

(c). $\displaystyle\int_0^4 x\sin x\,dx, \quad n = 8$

(d). $\displaystyle\int_0^4 \sqrt{x}e^x\,dx, \quad n = 6$

(e). $\displaystyle\int_1^3 \ln x\,dx, \quad n = 6$

(f). $\displaystyle\int_0^2 xe^{-x}dx, \quad n = 8$

(g). $\displaystyle\int_0^\pi f(x)dx,$ where $f(x) = \begin{cases} \frac{\sin x}{x}, & x > 0 \\ 1, & x = 0 \end{cases} \quad ; \quad n = 6$

Suggested Solutions for Activity 1

(a). We want to approximate the integral:

$$\int_0^1 e^{4x^2}\,dx$$

using the **Trapezoidal Rule** with $n = 7$ subdivisions.
Here we have:

$$\begin{aligned} a &= x_0 = 0, \\ b &= x_7 = 1, \\ n &= 7, \\ f(x) &= e^{4x^2}. \end{aligned}$$

Now,

$$h = \frac{b-a}{n} = \frac{1-0}{7} = \frac{1}{7}$$

and

$$x_i = a + ih = 0 + i\left(\frac{1}{7}\right) = \frac{i}{7}, \quad \text{for } i = 0, 1, 2, \ldots, 7.$$

Thus, we tabulate the data as follows:

i	x_i	$f(x_i) = e^{4(x_i)^2}$
0	0	1.00000000000
1	$\frac{1}{7}$	1.08505714419
2	$\frac{2}{7}$	1.38615068230
3	$\frac{3}{7}$	2.08484367736
4	$\frac{4}{7}$	3.69183066048
5	$\frac{5}{7}$	7.69688981037
6	$\frac{6}{7}$	18.89269822665
7	1	54.59815003314

Therefore,

$$\begin{aligned} \int_0^1 e^{4x^2}\,dx &\approx T_7 \\ &= \frac{h}{2}\left[f(x_0) + f(x_7) + 2\left(f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_4) + f(x_5) + f(x_6)\right)\right] \\ &= \frac{\frac{1}{7}}{2}\left[1 + 54.59815003314 + 2\left(1.08505714419 + 1.38615068230 + 2.08484367736\right.\right. \\ &\quad \left.\left. + 3.69183066048 + 7.69688981037 + 18.89269822665\right)\right] = \boxed{8.94807788827}. \end{aligned}$$

(b). We want to approximate the integral:

$$\int_{8}^{30}\left[2000\ln\left(\frac{140000}{140000-2100t}\right)-9.8t\right]dt$$

using the **Trapezoidal Rule** with $n=5$ subdivisions.

Here we have:

$$\begin{aligned}a &= t_0 = 8,\\ b &= t_5 = 30,\\ n &= 5,\\ f(t) &= 2000\ln\left(\frac{140000}{140000-2100t}\right)-9.8t.\end{aligned}$$

Now,

$$h=\frac{b-a}{n}=\frac{30-8}{5}=\frac{22}{5}$$

and

$$t_i = a+ih = 8+i\left(\frac{22}{5}\right), \quad \text{for } i=0,1,2,3,4,5.$$

Thus, we tabulate the data as follows:

i	t_i	$f(t_i)$
0	8	177.26674301977
1	$8+\frac{22}{5}=\frac{62}{5}$	290.06982595919
2	$8+\frac{44}{5}=\frac{84}{5}$	416.06460201532
3	$8+\frac{66}{5}=\frac{106}{5}$	557.69124227735
4	$8+\frac{88}{5}=\frac{128}{5}$	718.13663089723
5	$8+\frac{110}{5}=\frac{150}{5}=30$	901.67400151124

Therefore,

$$\begin{aligned}\int_{8}^{30} f(t)\,dt &\approx T_5\\ &= \frac{h}{2}\left[f(t_0)+f(t_5)+2\left(f(t_1)+f(t_2)+f(t_3)+f(t_4)\right)\right]\\ &= \frac{\frac{22}{5}}{2}[177.26674301977+901.67400151124+2(290.06982595919+416.06460201532\\ &\quad +557.69124227735+718.13663089723)]\\ &= \frac{11}{5}[1078.94074453101+2(1981.96230014909)]\\ &= \frac{11}{5}[1078.94074453101+3963.92460029818]\\ &= \frac{11}{5}\times 5042.86534482919\\ &= \boxed{11094.30376302425}\end{aligned}$$

(c). We want to approximate the integral:

$$\int_0^4 x \sin x \, dx$$

using the **Trapezoidal Rule** with $n = 5$ subdivisions.

Here we have:

$$\begin{aligned} a = x_0 &= 0, \\ b = x_5 &= 4, \\ n &= 5, \\ f(x) &= x \sin x. \end{aligned}$$

Now,

$$h = \frac{b-a}{n} = \frac{4-0}{5} = \frac{4}{5}$$

and

$$x_i = a + ih = 0 + i\left(\frac{4}{5}\right) = \frac{4i}{5}, \quad \text{for } i = 0, 1, 2, 3, 4, 5.$$

Thus, we tabulate the data as follows:

i	x_i	$f(x_i) = x_i \sin x_i$
0	0	0.00000000000
1	$\frac{4}{5}$	0.57388487272
2	$\frac{8}{5}$	1.59931776487
3	$\frac{12}{5}$	1.62111163332
4	$\frac{16}{5}$	-0.18679725897
5	4	-3.02720998123

Therefore,

$$\begin{aligned} \int_0^4 x \sin x \, dx &\approx T_5 \\ &= \frac{h}{2}\left[f(x_0) + f(x_5) + 2\left(f(x_1) + f(x_2) + f(x_3) + f(x_4)\right)\right] \\ &= \frac{\frac{4}{5}}{2}[0 + (-3.02720998123) + 2(0.57388487272 + 1.59931776487 \\ &+ 1.62111163332 - 0.18679725897)] \\ &= \frac{2}{5}[-3.02720998123 + 2(3.60751701294)] \\ &= \frac{2}{5}[-3.02720998123 + 7.21503402588] \\ &= \frac{2}{5} \times 4.18782404465 \\ &= \boxed{1.67512961706} \end{aligned}$$

(d). We want to approximate the integral:

$$\int_0^4 \sqrt{x}\, e^x \, dx$$

using the **Trapezoidal Rule** with $n = 7$ subdivisions.

Here we have:

$$\begin{aligned} a = x_0 &= 0, \\ b = x_7 &= 4, \\ n &= 7, \\ f(x) &= \sqrt{x}\, e^x. \end{aligned}$$

Now,

$$h = \frac{b-a}{n} = \frac{4-0}{7} = \frac{4}{7}$$

and

$$x_i = a + ih = 0 + i\left(\frac{4}{7}\right) = \frac{4i}{7}, \quad \text{for } i = 0, 1, 2, \ldots, 7.$$

Thus, we tabulate the data as follows:

i	x_i	$f(x_i) = \sqrt{x_i}e^{x_i}$
0	0	0.00000000000
1	$\frac{4}{7}$	1.33859516201
2	$\frac{8}{7}$	3.35222008798
3	$\frac{12}{7}$	7.27020118627
4	$\frac{16}{7}$	14.86565579667
5	$\frac{20}{7}$	29.43115830362
6	$\frac{24}{7}$	57.09081634954
7	4	109.19630006629

Therefore,

$$
\begin{aligned}
\int_0^4 \sqrt{x}e^x\,dx &\approx T_7 \\
&= \frac{h}{2}\left[f(x_0)+f(x_7)+2(f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)+f(x_6))\right] \\
&= \frac{\frac{4}{7}}{2}[0+109.19630006629+2(1.33859516201+3.35222008798+7.27020118627 \\
&\quad +14.86565579667+29.43115830362+57.09081634954)] \\
&= \frac{2}{7}[109.19630006629+2(113.34864688509)] \\
&= \frac{2}{7}[109.19630006629+226.69729377018] \\
&= \frac{2}{7}\times 335.89359383647 \\
&= \boxed{95.96959823956}
\end{aligned}
$$

(e). We want to approximate the integral:

$$\int_1^3 \ln x\,dx$$

using the **Trapezoidal Rule** with $n = 6$ subdivisions.

Here we have:

$$
\begin{aligned}
a &= x_0 = 1, \\
b &= x_6 = 3, \\
n &= 6, \\
f(x) &= \ln x.
\end{aligned}
$$

Now,

$$h = \frac{b-a}{n} = \frac{3-1}{6} = \frac{1}{3}$$

and

$$x_i = a + ih = 1 + i\left(\frac{1}{3}\right) = \frac{3+i}{3}, \quad \text{for } i = 0, 1, 2, \ldots, 6.$$

Thus, we tabulate the data as follows:

i	x_i	$f(x_i) = \ln x_i$
0	$\frac{3}{3} = 1$	0.00000000000
1	$\frac{4}{3}$	0.28768207245
2	$\frac{5}{3}$	0.51082562377
3	$\frac{6}{3} = 2$	0.69314718056
4	$\frac{7}{3}$	0.84729786039
5	$\frac{8}{3}$	0.98082925301
6	$\frac{9}{3} = 3$	1.09861228867

Therefore,

$$
\begin{aligned}
\int_1^3 \ln x\, dx &\approx T_6 \\
&= \frac{h}{2}\left[f(x_0) + f(x_6) + 2(f(x_1) + f(x_2) + f(x_3); + f(x_4) + f(x_5))\right] \\
&= \frac{\frac{1}{3}}{2}\left[0 + 1.09861228867 + 2(0.28768207245 + 0.51082562377 + 0.69314718056 \right.\\
&\quad \left. + 0.84729786039 + 0.98082925301)\right] \\
&= \frac{1}{6}\left[1.09861228867 + 2(3.31978199018)\right] \\
&= \frac{1}{6}\left[1.09861228867 + 6.63956398036\right] \\
&= \frac{1}{6} \times 7.73817626903 \\
&= \boxed{1.28969604484}
\end{aligned}
$$

(f). We want to approximate the integral:

$$\int_0^2 xe^{-x}\, dx$$

using the **Trapezoidal Rule** with $n = 8$ subdivisions.

Here we have:

$$
\begin{aligned}
a &= x_0 = 0, \\
b &= x_8 = 2, \\
n &= 8, \\
f(x) &= xe^{-x}.
\end{aligned}
$$

Now,

$$h = \frac{b-a}{n} = \frac{2-0}{8} = \frac{1}{4}$$

and

$$x_i = a + ih = 0 + i\left(\frac{1}{4}\right) = \frac{i}{4}, \quad \text{for } i = 0, 1, 2, \ldots, 8.$$

Thus, we tabulate the data as follows:

i	x_i	$f(x_i) = x_i e^{-x_i}$
0	0	0.00000000000
1	$\frac{1}{4}$	0.19470019577
2	$\frac{1}{2}$	0.30326532986
3	$\frac{3}{4}$	0.35427491456
4	1	0.36787944117
5	$\frac{5}{4}$	0.35813099608
6	$\frac{3}{2}$	0.33469524022
7	$\frac{7}{4}$	0.30410440104
8	2	0.27067056647

Therefore,

$$
\begin{aligned}
\int_0^2 xe^{-x}\,dx &\approx T_8 \\
&= \frac{h}{2}\left[f(x_0) + f(x_8) + 2\left(f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6) + f(x_7)\right)\right] \\
&= \frac{\frac{1}{4}}{2}[0.00000000000 + 0.27067056647 + 2(0.19470019577 + 0.30326532986 + 0.35427491456 \\
&\quad + 0.36787944117 + 0.35813099608 + 0.33469524022 + 0.30410440104)] \\
&= \frac{1}{8}[0.27067056647 + 2(2.21605051870)] \\
&= \frac{1}{8}[0.27067056647 + 4.43210103740] \\
&= \frac{1}{8} \times 4.70277160387 \\
&= \boxed{0.58809645048}
\end{aligned}
$$

(g). We want to approximate the integral:

$$\int_0^{\pi} f(x)\,dx, \quad \text{where } f(x) = \begin{cases} \frac{\sin x}{x}, & x > 0 \\ 1, & x = 0 \end{cases}$$

using the **Trapezoidal Rule** with $n = 6$ subdivisions.

Here we have:

$$
\begin{aligned}
a &= x_0 = 0, \\
b &= x_6 = \pi, \\
n &= 6, \\
f(x) &= \frac{\sin x}{x}, \text{ with } f(0) = 1.
\end{aligned}
$$

Now,

$$h = \frac{b-a}{n} = \frac{\pi - 0}{6} = \frac{\pi}{6}$$

and

$$x_i = a + ih = i \cdot \frac{\pi}{6}, \quad \text{for } i = 0, 1, 2, \ldots, 6.$$

Thus, we tabulate the data as follows:

i	x_i	$f(x_i) = \dfrac{\sin x_i}{x_i}$
0	0	1.00000000000
1	$\frac{\pi}{6}$	0.95492965855
2	$\frac{\pi}{3}$	0.82699334313
3	$\frac{\pi}{2}$	0.63661977237
4	$\frac{2\pi}{3}$	0.41349667157
5	$\frac{5\pi}{6}$	0.19098593171
6	π	0.00000000000

Therefore,

$$\begin{aligned}
\int_0^\pi f(x)\,dx &\approx T_6 \\
&= \frac{h}{2}\left[f(x_0) + f(x_6) + 2\left(f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)\right)\right] \\
&= \frac{\frac{\pi}{6}}{2}[1.00000000000 + 0.00000000000 + 2(0.95492965855 + 0.82699334313 + 0.63661977237 \\
&\quad + 0.41349667157 + 0.19098593171)] \\
&= \frac{\pi}{12}[1.00000000000 + 2(3.02202537733)] \\
&= \frac{\pi}{12}[1.00000000000 + 6.04405075466] \\
&= \frac{\pi}{12} \cdot 7.04405075466 \\
&= \boxed{1.84431223665}
\end{aligned}$$

Suggested Solutions for Activity 2

(a). We want to approximate the integral:

$$\int_0^1 e^{4x^2}\,dx$$

using the **Simpson's Rule** with $n = 6$ subdivisions.

Here we have:

$$\begin{aligned} a &= x_0 = 0, \\ b &= x_6 = 1, \\ n &= 6, \\ f(x) &= e^{4x^2}. \end{aligned}$$

Now,

$$h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$$

and

$$x_i = a + ih = i \cdot \frac{1}{6} = \frac{i}{6}, \quad \text{for } i = 0, 1, 2, \ldots, 6.$$

Thus, we tabulate the data as follows:

i	x_i	$f(x_i) = e^{4x_i^2}$
0	0	1.00000000000
1	$\frac{1}{6}$	1.11751906874
2	$\frac{1}{3}$	1.55962349761
3	$\frac{1}{2}$	2.71828182846
4	$\frac{2}{3}$	5.91669359066
5	$\frac{5}{6}$	16.08324067206
6	1	54.59815003314

Therefore,

$$
\begin{aligned}
\int_0^1 e^{4x^2}\,dx &\approx S_6 \\
&= \frac{h}{3}\left[f(x_0)+f(x_6)+4(f(x_1)+f(x_3)+f(x_5))+2(f(x_2)+f(x_4))\right] \\
&= \frac{\frac{1}{6}}{3}[1.00000000000+54.59815003314+4(1.11751906874+2.71828182846+16.08324067206 \\
&\quad +2(1.55962349761+5.91669359066)] \\
&= \frac{1}{18}[55.59815003314+4(19.91904156926)+2(7.47631708827)] \\
&= \frac{1}{18}[55.59815003314+79.67616627704+14.95263417654] \\
&= \frac{1}{18}\cdot 150.22695048672 \\
&= \boxed{8.34594169371}
\end{aligned}
$$

(b). We want to approximate the integral:

$$\int_8^{30}\left[2000\ln\left(\frac{140000}{140000-2100t}\right)-9.8t\right]dt$$

using the **Simpson's Rule** with $n=8$ subdivisions.

Here we have:

$$
\begin{aligned}
a &= t_0 = 8, \\
b &= t_8 = 30, \\
n &= 8, \\
f(t) &= 2000\ln\left(\frac{140000}{140000-2100t}\right)-9.8t.
\end{aligned}
$$

Now,

$$h=\frac{b-a}{n}=\frac{22}{8}=\frac{11}{4},\quad t_i=a+i\cdot h=8+i\cdot\frac{11}{4}=\frac{32+11i}{4}$$

Thus, we tabulate the data as follows:

i	t_i	$f(t_i)$
0	8	177.26674301977
1	$\frac{43}{4}$	246.33518139249
2	$\frac{27}{2}$	320.24688864628
3	$\frac{65}{4}$	399.51653927417
4	19	484.74547257626
5	$\frac{87}{4}$	576.64231351786
6	$\frac{49}{2}$	676.05011676092
7	$\frac{109}{4}$	783.98267835199
8	30	901.67400151124

Therefore,

$$\begin{aligned}
\int_8^{30} f(t)\,dt &\approx S_8 \\
&= \frac{h}{3}\left[f(t_0)+f(t_8)+4\big(f(t_1)+f(t_3)+f(t_5)+f(t_7)\big)+2\big(f(t_2)+f(t_4)+f(t_6)\big)\right] \\
&= \frac{\frac{11}{4}}{3}[177.26674301977+901.67400151124 \\
&\quad +4(246.33518139249+399.51653927417+576.64231351786+783.98267835199) \\
&\quad +2(320.24688864628+484.74547257626+676.05011676092)] \\
&= \frac{11}{12}[1078.94074453101+4\times 2006.47671253651+2\times 1481.04247898346] \\
&= \frac{11}{12}[1078.94074453101+8025.90685014604+2962.08495796692] \\
&= \frac{11}{12}\times 12066.93255264397 \\
&= \boxed{11061.35483809031}
\end{aligned}$$

(c). We want to approximate the integral:

$$\int_0^4 x\sin x\,dx$$

using the **Simpson's Rule** with $n=8$ subdivisions.

Here we have:

$$\begin{aligned}
a &= x_0 = 0, \\
b &= x_8 = 4, \\
n &= 8, \\
f(x) &= x\sin x.
\end{aligned}$$

Now,

$$h=\frac{b-a}{n}=\frac{4-0}{8}=\frac{1}{2},\quad x_i=a+ih=0+i\cdot\frac{1}{2}=\frac{i}{2}$$

Thus, we tabulate the data as follows:

i	x_i	$f(x_i)$
0	0	0.00000000000
1	$\frac{1}{2}$	0.23971276930
2	1	0.84147098481
3	$\frac{3}{2}$	1.49624247991
4	2	1.81859485365
5	$\frac{5}{2}$	1.49618036026
6	3	0.42336002418
7	$\frac{7}{2}$	-1.22774129691
8	4	-3.02720998123

Therefore,

$$\begin{aligned}
\int_0^4 x \sin x \, dx &\approx S_8 \\
&= \frac{h}{3}\left[f(x_0) + f(x_8) + 4\left(f(x_1) + f(x_3) + f(x_5) + f(x_7)\right) + 2\left(f(x_2) + f(x_4) + f(x_6)\right)\right] \\
&= \frac{\frac{1}{2}}{3}\Big[0 + (-3.02720998123) + 4(0.23971276930 \\
&\quad + 1.49624247991 + 1.49618036026 - 1.22774129691) \\
&\quad + 2(0.84147098481 + 1.81859485365 + 0.42336002418)\Big] \\
&= \frac{1}{6}\Big[-3.02720998123 + 4(2.00439431256) + 2(3.08342586264)\Big] \\
&= \frac{1}{6}\Big[-3.02720998123 + 8.01757725024 + 6.16685172528\Big] \\
&= \frac{1}{6} \times 11.15721899429 \\
&= \boxed{1.85953649904}
\end{aligned}$$

(d). We want to approximate the integral:

$$\int_0^4 \sqrt{x} e^x \, dx$$

using the **Simpson's Rule** with $n = 6$ subdivisions.

Here we have:

$$\begin{aligned}
a &= x_0 = 0, \\
b &= x_6 = 4, \\
n &= 6, \\
f(x) &= \sqrt{x}\, e^x.
\end{aligned}$$

Now,

$$h = \frac{b-a}{n} = \frac{4}{6} = \frac{2}{3}, \quad x_i = a + ih = i \cdot \frac{2}{3}$$

Thus, we tabulate the data as follows:

i	x_i	$f(x_i)$
0	0	0.00000000000
1	$\frac{2}{3}$	1.59031818508
2	$\frac{4}{3}$	4.38055036042
3	2	10.44970334824
4	$\frac{8}{3}$	23.50190056938
5	$\frac{10}{3}$	51.17851092752
6	4	109.19630006629

Therefore,

$$\int_0^4 \sqrt{x}e^x\,dx \approx S_6$$

$$= \frac{h}{3}\left[f(x_0) + f(x_6) + 4\left(f(x_1) + f(x_3) + f(x_5)\right) + 2\left(f(x_2) + f(x_4)\right)\right]$$

$$= \frac{\frac{2}{3}}{3}\Big[0 + 109.19630006629 + 4(1.59031818508 + 10.44970334824 + 51.17851092752)$$

$$+ 2(4.38055036042 + 23.50190056938)\Big]$$

$$= \frac{2}{9}\left[109.19630006629 + 4(63.21853246084) + 2(27.88245092980)\right]$$

$$= \frac{2}{9}\left[109.19630006629 + 252.87412984336 + 55.76490185960\right]$$

$$= \frac{2}{9} \times 417.83533176925$$

$$= \boxed{92.85229594872}$$

(e). We want to approximate the integral:

$$\int_1^3 \ln x\,dx$$

using the **Simpson's Rule** with $n = 6$ subdivisions.

Here we have:

$$a = x_0 = 1,$$
$$b = x_6 = 3,$$
$$n = 6,$$
$$f(x) = \ln x.$$

Now,

$$h = \frac{b-a}{n} = \frac{3-1}{6} = \frac{2}{6} = \frac{1}{3}, \quad x_i = a + ih = 1 + \frac{i}{3} = \frac{3+i}{3}$$

Thus, we tabulate the data as follows:

i	x_i	$f(x_i)$
0	$\frac{3}{3}$	0.00000000000
1	$\frac{4}{3}$	0.28768207245
2	$\frac{5}{3}$	0.51082562377
3	2	0.69314718056
4	$\frac{7}{3}$	0.84729786039
5	$\frac{8}{3}$	0.98082925301
6	3	1.09861228867

Therefore,

$$\begin{aligned}
\int_1^3 \ln x\, dx &\approx S_6 \\
&= \frac{h}{3}\left[f(x_0) + f(x_6) + 4\left(f(x_1) + f(x_3) + f(x_5)\right) + 2\left(f(x_2) + f(x_4)\right)\right] \\
&= \frac{\frac{1}{3}}{3}\,[0 + 1.09861228867 + 4(0.28768207245 + 0.69314718056 + 0.98082925301) + 2(0.51082562 \\
&= \frac{1}{9}\,[1.09861228867 + 4(1.96165850602) + 2(1.35812348416)] \\
&= \frac{1}{9}\,[1.09861228867 + 7.84663402408 + 2.71624696832] \\
&= \frac{1}{9} \times 11.66149328107 \\
&= \boxed{1.29572147567}
\end{aligned}$$

(f). We want to approximate the integral:

$$\int_0^2 xe^{-x}\, dx$$

using the **Simpson's Rule** with $n = 8$ subdivisions.

Here we have:

$$\begin{aligned}
a = x_0 &= 0, \\
b = x_8 &= 2, \\
n &= 8, \\
f(x) &= xe^{-x}.
\end{aligned}$$

Now,

$$h = \frac{b-a}{n} = \frac{2-0}{8} = \frac{1}{4}, \quad x_i = a + ih = i \cdot \frac{1}{4} = \frac{i}{4}$$

Thus, we tabulate the data as follows:

i	x_i	$f(x_i)$
0	0	0.00000000000
1	$\frac{1}{4}$	0.19470019577
2	$\frac{1}{2}$	0.30326532986
3	$\frac{3}{4}$	0.35427491456
4	1	0.36787944117
5	$\frac{5}{4}$	0.35813099608
6	$\frac{3}{2}$	0.33469524022
7	$\frac{7}{4}$	0.30410440104
8	2	0.27067056647

Therefore,

$$\begin{aligned}
\int_0^2 xe^{-x}\,dx &\approx S_8 \\
&= \frac{h}{3}\left[f(x_0) + f(x_8) + 4\left(f(x_1) + f(x_3) + f(x_5) + f(x_7)\right) + 2\left(f(x_2) + f(x_4) + f(x_6)\right)\right] \\
&= \frac{\frac{1}{4}}{3}[0 + 0.27067056647 + 4(0.19470019577 + 0.35427491456 + 0.35813099608 + 0.304104401 \\
&\quad +2(0.30326532986 + 0.36787944117 + 0.33469524022)] \\
&= \frac{1}{12}[0.27067056647 + 4(1.21121050745) + 2(1.00584001125)] \\
&= \frac{1}{12}[0.27067056647 + 4.84484202980 + 2.01168002250] \\
&= \frac{1}{12} \times 7.12719361877 \\
&= \boxed{0.59393271823}
\end{aligned}$$

(g). We want to approximate the integral:

$$\int_0^\pi f(x)\,dx, \quad \text{where } f(x) = \begin{cases} \frac{\sin x}{x}, & x > 0 \\ 1, & x = 0 \end{cases}$$

using the **Simpson's Rule** with $n = 6$ subdivisions.

Here we have:

$$\begin{aligned} a &= x_0 = 0, \\ b &= x_6 = \pi, \\ n &= 6, \\ f(x) &= \begin{cases} \frac{\sin x}{x}, & x > 0 \\ 1, & x = 0 \end{cases}. \end{aligned}$$

Now,

$$h = \frac{b-a}{n} = \frac{\pi}{6}, \quad x_i = a + ih = \frac{i\pi}{6}$$

Thus, we tabulate the data as follows:

i	x_i	$f(x_i)$
0	0	1.00000000000
1	$\frac{\pi}{6}$	0.95492965855
2	$\frac{\pi}{3}$	0.82699334313
3	$\frac{\pi}{2}$	0.63661977237
4	$\frac{2\pi}{3}$	0.41349667157
5	$\frac{5\pi}{6}$	0.19098593171
6	π	0.00000000000

Therefore,

$$\begin{aligned} &\int_0^\pi f(x)\,dx \approx S_6 \\ &\quad= \frac{h}{3}\left[f(x_0) + f(x_6) + 4\left(f(x_1) + f(x_3) + f(x_5)\right) + 2\left(f(x_2) + f(x_4)\right)\right] \\ &\quad= \frac{\frac{\pi}{6}}{3}[1.00000000000 + 0.00000000000 + 4\left(0.95492965855 + 0.63661977237 + 0.19098593171\right) \\ &\quad+ 2\left(0.82699334313 + 0.41349667157\right)] \\ &\quad= \frac{\pi}{18}\left[1 + 4(1.78253536263) + 2(1.24049001470)\right] \\ &\quad= \frac{\pi}{18}\left[1 + 7.13014145052 + 2.48098002940\right] \\ &\quad= \frac{\pi}{18} \times 10.61112147992 \\ &\quad= \boxed{1.85199007154} \end{aligned}$$