Calculus Study Guide

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For this study guide we will be going over how to solve the types of problems you will be seeing on the test. This includes related rates and optimization problems.

1 Related Rates Step-by-Step Solution Guide

To solve for a related rates problem following these step by step instructions will help you find a solution:

1. Determine the desired value
    - What value are we looking for? Is that value a rate?
2. Identify what information is given to you
    - What do we already know about the problem?
    - What can we infer about the problem?
3. Determine the equation that relates the variables
    - What equation can we use or find that will relate the variables?
    - Is the equation well known?
    - Or will you have to look in the problem?
4. Take the derivative of the equation that relates the variables
    - Be careful here and make sure that you are taking the derivative with respect to the independent variable.
5. Plug in the numbers initially given
    - Be sure to double check that you are plugging in the correct numbers.
6. Find any other unknown variables
    - Make sure that if there are any other unknown variables that you identify them and solve for them.

7. Solve the equation
    - Make sure with word problems that your solutions sounds reasonable.
    - Always double check your work if you are unsure about your answer.

2 Optimization Step-by-Step Solution Guide

To solve for an optimization problem following these step by step instructions will help you find a solution:

1. Set up an equation for the desired
    - What equation can we use or find that will represent the problem?
    - Is the equation well known?
    - Or will you have to look in the problem?
2. Set up an equation for the constraint
    - What equation can we use or find that will represent the problem?
    - Is the equation well known?
    - Or will you have to look in the problem?
3. Use the constraint equation to get the desired function in terms of one variable
    - rewrite the constraint equation in terms of one variable and then plug it in to the desired equation
4. Determine the domain of the desired equation
    - What values can the independent variable take on?
    - make sure that your domain represents the constraints of the question
5. Take the derivative of the desired equation and set it to zero

6. Solve for the other unknown variable
   - Make sure with word problems that your solutions sounds reasonable.
   - Always double check your work if you are unsure about your answer.

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3 Related Rates Example Problem

Captain America is fighting Iron man during the marvel civil war. Captain America trades blows with Iron Man and then Iron Man then does an Repulsor Beam Blast that Captain America blocks with his circular shield. This causes his shield to heat up and increase the radius at a rate of $\frac{3}{1000}\frac{meters}{sec}$. What is the rate of the area of Captain America's shield increasing when the radius is 0.7 $meters$?

1. **Determine the Desired Value**
   - We are trying to find the rate at which the area of Captain America's shield is increasing. So the desired value that we are trying to solve is:

$$\frac{da}{dt}. \qquad \text{(Desired Rate)}$$

2. **Identify what information is given to you**
   - We are given that we are given the rate the radius is increasing, thus:

$$\frac{dr}{dt} = \frac{3}{1000}. \qquad \text{(Rate of Radius Change)}$$

   - We are also told that the radius of Captain America's shield, Thus:

$$r = 0.7meters. \qquad \text{(Radius of Shield)}$$

   - In the question we are also told that Captain America's shield is circular

3. **Determine the equation that relates the variables**

   - To form this equation we know that we are trying to find a change in the area (Desired Rate). We are given the change in the radius (Rate of Radius Change), the radius of the shield (Radius of Shield), and that the shield is circular. From this information we can determine that the best equation to use is the equation of a circle given by:
   $$A = \pi r^2. \tag{Related Rate Equation}$$

4. **Take the derivative of the equation that relates the variables**

   - With our related rate equation (Related Rate Equation) we then can take the derivative of that equation resulting in:
   $$\begin{aligned} A &= \pi r^2 \\ \frac{da}{dt} &= 2\pi r \frac{dr}{dt} \end{aligned} \tag{Derivative of Equation}$$

5. **Plug in the numbers initially given**

   - Knowing the derivative of the equation we then plug in our variables that we do know. We know the radius (Radius of Shield) and the change in the radius (Rate of Radius Change). Plugging those into the derivative of our equation (Derivative of Equation) we get:
   $$\begin{aligned} \frac{da}{dt} &= 2\pi r \frac{dr}{dt} \\ \frac{da}{dt} &= (2)(\pi)(0.7)(\frac{3}{1000}) \end{aligned} \tag{Values plugged in}$$

6. **Find any other unknown variables**

   - we have no other unknown variables so we then skip onto the next step

7. **Solve the equation**

- we have our equation set up,(Values plugged in) all we have to do is then solve for our desired rate (Desired Rate):

$$\begin{aligned} \frac{da}{dt} &= (2)(\pi)(0.7)(\frac{3}{1000}) \\ \frac{da}{dt} &= 0.0131946891 \end{aligned} \qquad \text{(Final Answer)}$$

- So then the rate at which the area of Captain America's Shield is expanding is $0.0131946891\frac{meters}{sec}$.

4 Optimization Example Problem

A king is planning on getting a new castle built. He wants to make sure that the castle is easily defendable and thus wants to have the smallest perimeter possible. The problem is that the king needs space to fit all of his belongings. To fit all of his stuff he needs the castle to be about $127acres^2$. What is the smallest possible perimeter of the castle if you represent the castle as a rectangle?

1. **Set up an equation for the desired**

   - Since we are trying to solve for the minimal parameter of a rectangle, or in this case a castle. Let us denote the width and the length of the castle by w and l respectively. Our equation is then:

   $$P = 2w + 2l. \qquad \text{(Parimeter of Castle)}$$

2. **Set up an equation for the constraint**

   - We are trying to find the width and the length of the castle whose area is $127acres^2$. Our equation is then:

   $$\begin{aligned} A &= wl \\ 127 &= wl \end{aligned} \qquad \text{(Area of Castle)}$$

3. **Use the constraint equation to get the desired function in terms of one variable**

- for this step we will be rewriting the constraint equation (Area of Castle) in terms of l to solve for our desired function (Parimeter of Castle):

$$\begin{aligned} 127 &= wl & P &= 2w + 2l \\ l &= 127/w & P &= 2w + 2(\frac{127}{w}) \qquad \text{(Desired in terms of L)} \\ & & P &= 2w + \frac{254}{w} \end{aligned}$$

4. **Determine the domain of the desired equation**

   - In the domain of the desired equation (Desired in terms of L), we see that our width w is in the denominator of the equation. As such we see that w cannot equal zero otherwise we would be dividing by zero. This also makes sense because we cannot have a non existent width. We are also dealing with units measuring length and thus cannot be zero so our domain is:

$$w > 0 \qquad \text{(Domain)}$$

5. **Take the derivative of the desired equation and set it to zero**

   - We then take the perimeter of our desired equation (Desired in terms of L), and then set it to zero:

$$\begin{aligned} P &= 2w + \frac{254}{w} \\ \frac{dp}{dx} &= 2 - \frac{127}{w^2} \\ 0 &= 2 - \frac{254}{w^2} \qquad \text{(Width Value)} \\ w^2 &= 127 \\ w &= \pm 11.269 \end{aligned}$$

   - Since our domain (Domain) is $x > 0$ our width value is then positive.

6. **Solve for the other unknown variable**

- with our width solved for (Width Value) we then need to then solve for our length l. We have an equation that can give us a numeric value for l (Area of Castle).

$$\begin{aligned} A &= wl \\ 127 &= (11.269)l \\ l &= 11.269 \end{aligned} \tag{Length}$$

- Now that we know the length we must then find the the perimeter:

$$\begin{aligned} P &= 2w + 2l \\ P &= 2(11.269) + 2(22.269) \\ P &= 45.07 \end{aligned} \tag{Perimeter}$$

- We have now solved for the smallest perimeter (Perimeter) while still maintaining the desired area.