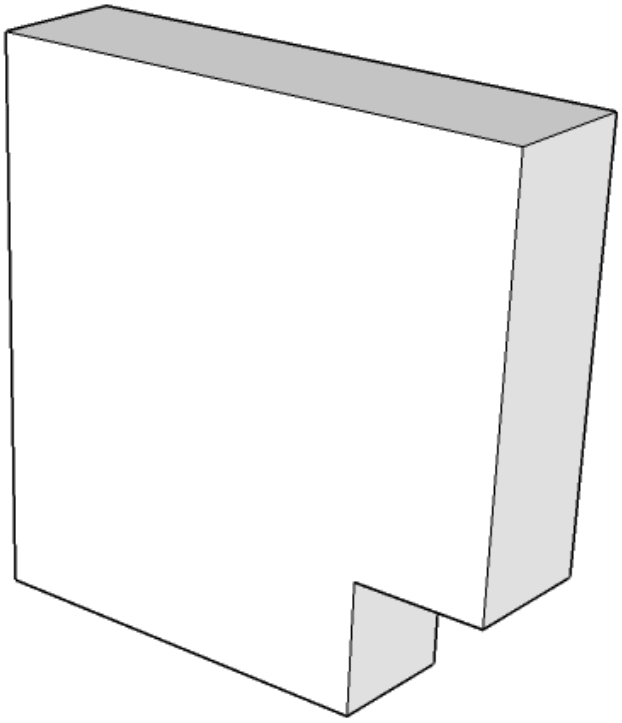
LUOV

Signature Scheme proposal for NIST PQC Project



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Contents

1 Introduction **4**

2 Algorithm specification (part of 2.B.1) **4**

2.1 Overview of the scheme 4

2.2 Relation to the UOV scheme 5

2.3 Parameter space 6

2.4 Key Generation Algorithm 6

2.4.1 Finding the remaining coefficients of $\mathcal{P}$ 6

2.5 Signature Generation Algorithm 7

2.6 Signature Verification Algorithm 9

2.7 Signatures with message recovery 12

2.8 Encoding of objects 13

2.8.1 Encoding of finite field elements 13

2.8.2 Encoding of private key 15

2.8.3 Encoding of public key 15

2.8.4 Encoding of signature 15

2.9 Sampling objects with the SHAKE function 16

2.9.1 Squeezing public seed 16

2.9.2 Squeezing $\mathbf{T}$ 16

2.9.3 Squeezing hash digest and vinegar variables 17

2.9.4 Squeezing most part of the public map 17

3 List of parameter sets (part of 2.B.1) **17**

4 Detailed performance analysis (2.B.2) **18**

4.1 Description of platform 18

4.2 Time 18

4.3 Space 18

4.4 How parameters affect performance 19

4.5 Optimizations 20
4.5.1 Bit slicing 20
4.5.2 Precomputing $\mathcal{P}$ and $\mathcal{F}$ 20

5 Expected strength (2.B.4) 20

6 Analysis of known attacks (2.B.5) 22

6.1 Direct attack 22
6.2 Key recovery attacks. 26
6.2.1 UOV attack 26
6.2.2 Reconciliation attack 26
6.3 Hash collision attack 27

7 Advantages and limitations (2.B.6) 27

7.1 Advantages 27
7.2 Limitations 28

References 29

A Statements 30

A.1 Statement by Each Submitter 31
A.2 Statement by Reference/Optimized Implementations' Owner(s) 33

1 Introduction

One of the major candidates for providing secure cryptographic primitives in a post-quantum world is Multivariate Cryptography. Multivariate Cryptography is based on the hardness of problems related to multivariate polynomials over finite fields, such as solving systems of multivariate polynomial equations. In general, Multivariate Cryptography is very fast and requires only moderate computational resources, which makes it attractive for applications in low-cost devices. In the field of Multivariate Cryptography, the Unbalanced Oil and Vinegar signature scheme (UOV) is one of the oldest and best studied cryptosystems. Since the proposal of the Oil and Vinegar scheme in 1997 by Patarin [15], UOV has successfully withstood almost two decades of cryptanalysis. The UOV scheme is very simple, has small signatures and is fast. The main disadvantage of UOV is arguably that its public keys are quite large. This document presents the Lifted Unbalanced Oil and Vinegar signature scheme (LUOV), which is a simple improvement of the UOV scheme that greatly reduces the size of the public keys.

2 Algorithm specification (part of 2.B.1)

2.1 Overview of the scheme

The LUOV signature scheme uses a one-way function $\mathcal{P} : \mathbb{F}_{2^r}^n \to \mathbb{F}_{2^r}^m$, which is a multivariate quadratic polynomial map in $n = m + v$ variables with coefficients in the binary subfield $\mathbb{F}_2 \subset \mathbb{F}_{2^r}$. The trapdoor is a factorization $\mathcal{P} = \mathcal{F} \circ \mathcal{T}$, where $\mathcal{T} : \mathbb{F}_{2^r}^n \to \mathbb{F}_{2^r}^n$ is an invertible linear map, and $\mathcal{F} : \mathbb{F}_{2^r}^n \to \mathbb{F}_{2^r}^m$ is a quadratic map whose components $f_1, \cdots, f_m$ are of the form

$$f_k(\mathbf{x}) = \sum_{i=1}^{v} \sum_{j=i}^{n} \alpha_{i,j,k} x_i x_j + \sum_{i=1}^{n} \beta_{i,k} x_i + \gamma_k \,,$$

where the $\alpha_{i,j,k}, \beta_{i,k}$ and γ are chosen randomly from $\mathbb{F}_2$ and $v = n - m$. We say that the first v variables $x_1, \cdots, x_v$ are the *vinegar* variables, whereas the remaining m variables are the *oil* variables. Equivalently, the components of $\mathcal{F}$ are quadratic polynomials with random binary coefficients in the variables x_i such that there are no quadratic terms which contain two oil variables. One could say that the vinegar variables and the oil variables are not fully mixed, which is where their names come from.

How does the trapdoor $\mathcal{P} = \mathcal{F} \circ \mathcal{T}$ help to invert the function $\mathcal{P}$? Given a target $\mathbf{x} \in \mathbb{F}_{2^r}^m$ a solution $\mathbf{y}$ for $\mathcal{P}(\mathbf{y}) = \mathbf{x}$ can be found by first solving $\mathcal{F}(\mathbf{y}') = \mathbf{x}$ for $\mathbf{y}'$ and then computing $\mathbf{y} = \mathcal{T}^{-1}(\mathbf{y}')$. The system $\mathcal{F}(\mathbf{y}') = \mathbf{x}$ can be solved efficiently by fixing the vinegar variables to some pseudo-randomly chosen values. If we substitute these values in the equations the remaining system only contains linear equations, because every quadratic term contains at least one vinegar variable and thus turns into a linear or constant term after substitution. The remaining linear system can be solved using linear algebra. In the event that there are no solutions we can simply try again with a different assignment to the vinegar variables.

The trapdoor function is then combined with a collision resistant hash function $\mathcal{H} : \{0,1\}^* \to \mathbb{F}_{2^r}^m$ into a signature scheme using the standard hash-and-sign paradigm. The resulting key generation, signature generation and verification algorithms are described in the next few sections.

A large part of the coefficients of $\mathcal{P}$ is generated from a seed. This seed is included in the public key and replaces all the generated coefficients to make the public key much smaller. In order to reduce the size of the secret key we do not store $\mathcal{F}$ nor $\mathcal{T}$. Instead, we only store a private seed that was used to generate the public seed and $\mathcal{T}$.

The LUOV scheme can be used in two modes. One option is the usual appended signature mode where a message is authenticated by appending a signature. A different option is the message recovery mode, which can be used to reduce the size of a message-signature pair. In message recovery mode (part of) the message is not transmitted but recovered from the signature.

2.2 Relation to the UOV scheme

The LUOV scheme is an adaptation of the Unbalanced Oil and Vinegar signature scheme. It differs from the original UOV scheme in a number of ways. The first modification, due to Petzoldt [16], changes the key generation algorithm to make it possible to choose a large part of the public key. One can then choose this part to correspond with the output of a pseudo-random number generator and replace a large part of the public key by a seed. The modified key generation algorithm generates a distribution of public polynomial maps $\mathcal{P}$ that is indistinguishable from the original signature scheme if we assume the output of the PRNG (we have used the Keccak1600 Sponge construction) is indistinguishable from true randomness.

A second modification is that a public key $\mathcal{P} : \mathbb{F}_2^n \to \mathbb{F}_2^m$ for the UOV scheme over $\mathbb{F}_2$ is used as a public key for the UOV scheme over a large extension field $\mathbb{F}_{2^r}$. The public key is 'lifted' to the extension field by just extending the polynomial map $\mathcal{P}$ to a map from $\mathbb{F}_{2^r}^n$ to $\mathbb{F}_{2^r}^m$. This is were the Lifted UOV scheme gets its name from. The advantage of this approach is that the public key remains small (since the coefficients of the public key are 0 or 1), while solving the system $\mathcal{P}(x) = y$ for some y in $\mathbb{F}_{2^r}^m$ becomes more difficult compared to the case where y is in $\mathbb{F}_2^m$. This adaptation is due to Beullens and Preneel. [5].

Thirdly, the linear map $\mathcal{T}$ is chosen to have a matrix representation of the form

$$\begin{pmatrix} \mathbf{1}_v & \mathbf{T} \\ 0 & \mathbf{1}_m \end{pmatrix},$$

where $\mathbf{T}$ is a v-by-m matrix. This choice makes the key generation algorithm and the signing algorithm much faster, but does not affect the security of the scheme because for a random public key there exists an equivalent private key with $\mathcal{T}$ of this form with high probability [18]. This implies that if there is an attack against the modified signature scheme, the same attack would work on nearly all public keys of the original UOV scheme. This choice of $\mathcal{T}$ was first

proposed by Czypek [7], where it was used to speed up the key generation algorithm. LUOV makes the same choice of $\mathcal{T}$, but uses different key generation and signature generation algorithms that are even faster.

Lastly, in the signing algorithm, instead of choosing the assignments to the vinegar variables truly randomly, the assignments are deterministically generated from the message M and the private key. This ensures that when a message is signed multiple times, the generated signatures will be identical. If the vinegar variables were chosen at random, an attacker could query many different signatures for the same message. We are not aware of an attack that exploits this fact, but it is cautious to block this kind of attack anyway.

2.3 Parameter space

The parameters for the LUOV algorithm are :

- r — The degree of the field extension $\mathbb{F}_2 \subset \mathbb{F}_{2^r}$.
- m — The number of polynomials in the public key, also the number of oil variables.
- v — The number of vinegar variables.
- $n = m + v$ — The total number of variables
- SHAKE — The extendable output function that is used, either SHAKE128 or SHAKE256.

2.4 Key Generation Algorithm

The key generation algorithm (Alg. 4) first uses a private seed to pseudo-randomly generate a seed that will be published, as well as the v-by-m matrix that determines the linear map $\mathcal{T}$. Then, the public seed is used to generate $\mathbf{C} \in \mathbb{F}_2^m$, the constant part of the public map $\mathcal{P}$, $\mathbf{L} \in \mathbb{F}_2^{m \times n}$, the linear part of $\mathcal{P}$ and $\mathbf{Q}_1 \in \mathbb{F}_2^{m \times \frac{v(v+1)}{2} + vm}$, the first $\frac{v(v+1)}{2} + vm$ columns of the Macaulay matrix of the quadratic part of $\mathcal{P}$ in the lexicographic ordering. Then $\mathbf{Q}_2 \in \mathbb{F}_2^{m \times \frac{m(m+1)}{2}}$, the remaining part of the Macaulay matrix of the quadratic part of $\mathcal{P}$ is calculated (see Sect. 2.4.1). The public key consists of the public seed and $\mathbf{Q}_2$. The private key is simply the seed that was used as input for the key generation algorithm. The details of how the different objects are sampled from the SHAKE function are described in Sect. 2.9.

2.4.1 Finding the remaining coefficients of $\mathcal{P}$

For each polynomial p_k in the public map $\mathcal{P}$ there is a uniquely determined upper triangular matrix $\mathbf{P}_k \in \mathbb{F}_2^{n \times n}$, such that $\mathbf{x}^\top \mathbf{P}_k \mathbf{x}$ is equal to the evaluation of the quadratic part of p_k

at $\mathbf{x}$. The matrix corresponding to the polynomial f_k in the secret map $\mathcal{F}$ is then, up to the addition of a skew-symmetric matrix, equal to

$$\begin{pmatrix} \mathbf{1}_v & \mathbf{0} \\ -\mathbf{T}^\top & \mathbf{1}_m \end{pmatrix} \begin{pmatrix} \mathbf{P}_{k,1} & \mathbf{P}_{k,2} \\ \mathbf{0} & \mathbf{P}_{k,3} \end{pmatrix} \begin{pmatrix} \mathbf{1}_v & -\mathbf{T} \\ \mathbf{0} & \mathbf{1}_m \end{pmatrix} = \begin{pmatrix} \mathbf{P}_{k,1} & -\mathbf{P}_{k,1}\mathbf{T} + \mathbf{P}_{k,2} \\ -\mathbf{T}^\top\mathbf{P}_{k,1} & \mathbf{T}^\top\mathbf{P}_{k,1}\mathbf{T} - \mathbf{T}^\top\mathbf{P}_{k,2} + \mathbf{P}_{k,3} \end{pmatrix},$$

where we have split up the matrix $\mathbf{P}_k$, into $\mathbf{P}_{k,1} \in \mathbb{F}_2^{v\times v}$, $\mathbf{P}_{k,2} \in \mathbb{F}_2^{v\times m}$ and $\mathbf{P}_{k,3} \in \mathbb{F}_2^{m\times m}$. The terms of f_k that are quadratic in the vinegar variables have to vanish, so

$$\mathbf{P}_{k,3} = -\mathbf{T}^\top\mathbf{P}_{k,1}\mathbf{T} + \mathbf{T}^\top\mathbf{P}_{k,2}\,,$$

up to the addition of a skew-symmetric matrix. This formula completely determines the upper triangular matrix $\mathbf{P}_{k,3}$. The entries of the $\mathbf{P}_{k,1}$ and $\mathbf{P}_{k,2}$ are generated from the public seed and the matrix $\mathbf{T}$ is known, so the matrices $\mathbf{P}_{k,3}$ can easily be computed. The entries of the matrices $\mathbf{P}_{k,3}$ are then arranged in the Macaulay matrix $\mathbf{Q}_2$. A detailed implementation of this procedure is shown in Alg. 3.

Algorithm findPk1

input: k — An integer between 1 and m.
$\mathbf{Q}_1$ — First part of Macaulay matrix of the quadratic part of $\mathcal{P}$

output: $\mathbf{P}_{k,1}$ — The v-by-v matrix representing the part of p_k that is quadratic in the vinegar variables.

```
1: P_{k,1} ← 0_v
2: column ← 1
3: for i from 1 to v do
4:     for j from i to v do
5:         P_{k,1}[i,j] ← Q_1[k, column]
6:         column ← column + 1                    ▷ move to the next term
7:     end for
8:     column ← column + m                        ▷ Skip the terms x_i x_{v+1} up to x_i x_{v+m}
9: end for
10: return P_{k,1}
```

Alg. 1: Algorithm for reading $\mathbf{P}_{k,1}$ from $\mathbf{Q}_1$.

2.5 Signature Generation Algorithm

The signature generation algorithm first generates $\mathbf{C}, \mathbf{L}, \mathbf{Q}_1$ and $\mathbf{T}$ from the private seed in the same way as the key generation algorithm. Then, it calculates $\mathbf{h}$, the hash digest of the message that will be signed, concatenated with a zero. Concatenating the message with zero is done to make signatures generated in appended signature mode unrelated to signatures generated in message recovery mode (see Sect. 2.7). Then, the algorithm produces

Algorithm findPk2

input: k — An integer between 1 and m.
$\mathbf{Q}_1$ — First part of Macaulay matrix of quadratic part of $\mathcal{P}$

output: $\mathbf{P}_{k,2}$ — The v-by-m matrix representing the part of p_k that is bilinear in the vinegar variables and the oil variables.

$\mathbf{P}_{k,2} \leftarrow \mathbf{0}_{v\times m}$
column $\leftarrow 1$
for i from 1 to v **do**
  column $\leftarrow$ column $+ v - i + 1$ ▷ Skip terms x_i^2 up to $x_i x_v$
  for j from 1 to m **do**
    $\mathbf{P}_{k,2}$[i,j] $\leftarrow \mathbf{Q}_1[k,$ column$]$
    column $\leftarrow$ column $+ 1$ ▷ Move to the next term
  end for
end for
return $\mathbf{P}_{k,2}$

Alg. 2: Algorithm for reading $\mathbf{P}_{k,2}$ from $\mathbf{Q}_1$.

Algorithm findQ2

input: $\mathbf{Q}_1$ — First part of Macaulay matrix of quadratic part of $\mathcal{P}$
$\mathbf{T}$ — A v-by-m matrix

output: $\mathbf{Q}_2$ — The second part of Macaulay matrix for quadratic part of $\mathcal{P}$

$\mathbf{Q}_2 \leftarrow 0_{m\times D_2}$
for k from 1 to m **do**
  $\mathbf{P}_{k,1} \leftarrow$findPk1$(k, \mathbf{Q}_1)$
  $\mathbf{P}_{k,2} \leftarrow$findPk2$(k, \mathbf{Q}_1)$
  $\mathbf{P}_{k,3} \leftarrow -\mathbf{T}^\top \mathbf{P}_{k,1}\mathbf{T} + \mathbf{T}^\top \mathbf{P}_{k,2}$ ▷ Compute $\mathbf{P}_{k,3}$ up to skew-symmetric matrix
  column $\leftarrow 1$
  for i from 1 to m **do** ▷ Read off $\mathbf{Q}_2$
    $\mathbf{Q}_2[k,$ column$] \leftarrow \mathbf{P}_{k,3}[i,i]$
    column $\leftarrow$ column $+ 1$
    for j from $i+1$ to m **do**
      $\mathbf{Q}_2[k,$ column$] \leftarrow \mathbf{P}_{k,3}[i,j] + \mathbf{P}_{k,3}[j,i]$
      column $\leftarrow$ column $+ 1$
    end for
  end for
end for
return $\mathbf{Q}_2$

Alg. 3: Algorithm for determining $\mathbf{Q}_2$ from $\mathbf{Q}_1$ and $\mathbf{T}$.

Algorithm KeyGen

input: private_seed — seed to generate a key-pair

output: (public_seed, $\mathbf{Q}_2$) — A public key
private_seed — A corresponding private key

1: private_sponge ← InitializeAndAbsorb(private_seed)
2: public_seed ← SqueezePublicSeed(private_sponge)
3: $\mathbf{T}$ ← SqueezeT(private_sponge)
4: public_sponge ← InitializeAndAbsorb(public_seed)
5: $\mathbf{C}, \mathbf{L}, \mathbf{Q}_1$ ← SqueezePublicMap (public_sponge)
6: $\mathbf{Q}_2$ ← FindQ2($\mathbf{Q}_1, \mathbf{T}$)
7: **return** (public_seed, $\mathbf{Q}_2$) and private_seed

Alg. 4: The key generation algorithm

a signature in two steps. First, the special structure of $\mathcal{F}$ is exploited to produce a solution $\mathbf{s}'$ to the equation $\mathcal{F}(\mathbf{s}') = \mathbf{h}$. Then, the signature $\mathbf{s}$ is calculated as

$$\mathbf{s} = \begin{pmatrix} \mathbf{1}_v & -\mathbf{T} \\ \mathbf{0} & \mathbf{1}_m \end{pmatrix} \mathbf{s}' .$$

Solving $\mathcal{F}(\mathbf{s}') = \mathbf{h}$ is done by repeatedly substituting pseudo-randomly generated values into the vinegar variables and trying to solve the resulting linear system until a unique solution is found. A unique solution is almost always found on the first try, the probability of failing being roughly 2^{-r}. For a particular assignment to the vinegar variables $\mathbf{v} \in \mathbb{F}_{2^r}^v$, the augmented matrix for the linear system $\mathcal{F}((\mathbf{v}||\mathbf{o})^\top) = \mathbf{h}$ can be derived as in Alg. 5. This algorithm relies on the fact that after fixing the vinegar variables to $\mathbf{v}$, the map $\mathcal{F}$ is a linear map with constant part

$$\mathbf{C} + \mathbf{L} \begin{pmatrix} \mathbf{v} \\ \mathbf{0} \end{pmatrix} + \begin{pmatrix} \mathbf{v}^\top \mathbf{P}_{1,1} \mathbf{v} \\ \cdots \\ \mathbf{v}^\top \mathbf{P}_{m,1} \mathbf{v} \end{pmatrix} ,$$

and a linear part with the matrix representation

$$\mathbf{L} \begin{pmatrix} -\mathbf{T} \\ \mathbf{1}_m \end{pmatrix} + \begin{pmatrix} \mathbf{v}^\top [(\mathbf{P}_{1,1} + \mathbf{P}_{1,1}^\top)\mathbf{T} + \mathbf{P}_{1,2}] \\ \cdots \\ \mathbf{v}^\top [(\mathbf{P}_{m,1} + \mathbf{P}_{m,1}^\top)\mathbf{T} + \mathbf{P}_{m,2}] \end{pmatrix} .$$

Pseudocode for the signature generation algorithm is provided in Alg. 6.

2.6 Signature Verification Algorithm

First, the signature verification algorithm uses the public seed to generate $\mathbf{C}, \mathbf{L}$ and $\mathbf{Q}_1$. Together with $\mathbf{Q}_2$, which is included in the public key, this completely determines the public

Algorithm BuildAugmentedMatrix

input: $\mathbf{C} \in \mathbb{F}_{2^r}^m$ — The constant part of the public map $\mathcal{P}$
$\mathbf{L} \in \mathbb{F}_{2^r}^{m \times n}$ — The linear part of $\mathcal{P}$
$\mathbf{Q}_1 \in \mathbb{F}_{2^r}^{m \times \frac{v(v+1)}{2} + vm}$ — The first part of quadratic part of $\mathcal{P}$
$\mathbf{T} \in \mathbb{F}_2^{v \times m}$ — The matrix that determines the linear transformation $\mathcal{T}$.
$\mathbf{h} \in \mathbb{F}_{2^r}^m$ — The hash digest to target.
$\mathbf{v} \in \mathbb{F}_{2^r}^v$ — An assignment to the vinegar variables.

output: $\mathbf{LHS}||\mathbf{RHS} \in \mathbb{F}_{2^r}^{m \times m+1}$ — The augmented matrix for $\mathcal{F}(\mathbf{v}||\mathbf{o}) = \mathbf{h}$

$\mathbf{RHS} \leftarrow \mathbf{h} - \mathbf{C} - \mathbf{L}_s(\mathbf{v}||0)^\top$ ▷ Right hand side of linear system
$\mathbf{LHS} \leftarrow \mathbf{L}\begin{pmatrix} -\mathbf{T} \\ \mathbf{1}_m \end{pmatrix}$ ▷ Left hand side of linear system
for k from 1 to m **do**
  $\mathbf{P}_{k,1} \leftarrow \text{findPk1}(k, \mathbf{Q}_1)$
  $\mathbf{P}_{k,2} \leftarrow \text{findPk2}(k, \mathbf{Q}_1)$
  $\mathbf{RHS}[k] \leftarrow \mathbf{RHS}[k] - \mathbf{v}^\top \mathbf{P}_{k,1} \mathbf{v}$ ▷ evaluation of terms of f_k that are quadratic in vinegar variables
  $\mathbf{F}_{k,2} \leftarrow -(\mathbf{P}_{k,1} + \mathbf{P}_{k,1}^\top)\mathbf{T} + \mathbf{P}_{k,2}$ ▷ Terms of f_k that are bilinear in the vinegar and the oil variables
  $\mathbf{LHS}[k] \leftarrow \mathbf{LHS}[k] + \mathbf{v}\mathbf{F}_{k,2}$ ▷ Insert row in the left hand side
end for
return LHS||RHS

Alg. 5: Builds the augmented matrix for the linear system $\mathcal{P}(\mathbf{v}||\mathbf{o}) = \mathbf{h}$ after fixing the vinegar variables.

Algorithm Sign

input: $\mathsf{private\_seed}$ — A private key
M — A message to sign

output: $\mathbf{s}$ — A signature for the message M

$\mathsf{sponge} \leftarrow \text{InitializeAndAbsorb}(\mathsf{private\_seed})$
$\mathsf{public\_seed} \leftarrow \text{SqueezePublicSeed}(\mathsf{sponge})$
$\mathbf{T} \leftarrow \text{SqueezeT}(\mathsf{sponge})$
$\mathsf{public\_sponge} \leftarrow \text{InitializeAndAbsorb}(\mathsf{public\_seed})$
$\mathbf{C}, \mathbf{L}, \mathbf{Q}_1 \leftarrow \text{SqueezePublicMap}\ (\mathsf{public\_sponge})$
$\mathsf{hash\_sponge} \leftarrow \text{InitializeAndAbsorb}(M||\mathtt{0x00})$
$\mathbf{h} \leftarrow \text{SqueezeHashDigest}(\mathsf{hash\_sponge})$ ▷ Calculate hash digest
$\mathsf{vinegar\_sponge} \leftarrow \text{InitializeAndAbsorb}(M||\mathtt{0x00}||\mathsf{private\_seed})$ ▷ Sponge for determining vinegar variables
while No solution $\mathbf{s}'$ to the system $\mathcal{F}(\mathbf{s}') = \mathbf{h}$ is found **do**
  $\mathbf{v} \leftarrow \text{SqueezeVinegar}\ (\mathsf{vinegar\_sponge})$
  $\mathbf{A} \leftarrow \text{BuildAugmentedMatrix}\ (\mathbf{C}, \mathbf{L}, \mathbf{Q}_1, \mathbf{T}, \mathbf{h}, \mathbf{v})$
    ▷ Build the augmented matrix for the linear system $\mathcal{F}(\mathbf{v}||\mathbf{o}) = \mathbf{h}$
  $\text{GaussianElimination}(\mathbf{A})$
  if $\mathcal{F}(\mathbf{v}||\mathbf{o}) = \mathbf{h}$ has a unique solution $\mathbf{o}$ **then**
    $\mathbf{s}' \leftarrow (\mathbf{v}||\mathbf{o})^\top$
  end if
end while
$\mathbf{s} \leftarrow \begin{pmatrix} \mathbf{1}_v & -\mathbf{T} \\ \mathbf{0} & \mathbf{1}_m \end{pmatrix} \mathbf{s}'$
return $\mathbf{s}$

Alg. 6: The signature generation algorithm

map $\mathcal{P}$. To verify a signature $\mathbf{s}$ for a message M, the verification algorithm simply checks whether $\mathcal{P}(\mathbf{s})$ is equal to the rm-bit long hash digest of the message M, appended with `0x00`. Pseudocode for this algorithm is provided in Alg. 9.

Algorithm EvaluatePublicMap

input: $(\mathsf{public\_seed}, \mathbf{Q}_2)$ — A public key
$\mathbf{s}$ — A candidate–signature

output: The evaluation of $\mathcal{P}$ at $\mathbf{s}$

1: $\mathsf{sponge} \leftarrow$ InitializeAndAbsorb($\mathsf{public\_seed}$)
2: $\mathbf{C}, \mathbf{L}, \mathbf{Q}_1 \leftarrow$ SqueezePublicMap (sponge)
3: $\mathbf{Q} \leftarrow \mathbf{Q}_1 || \mathbf{Q}_2$
4: $\mathbf{e} \leftarrow \mathbf{C} + \mathbf{Ls}$ ▷ Evaluate constant and linear part of $\mathcal{P}$ at $\mathbf{s}$
5: $\mathsf{column} \leftarrow 1$
6: **for** i from 1 to n **do** ▷ Evaluate quadratic parts of $\mathcal{P}$ at $\mathbf{s}$
7: **for** j from i to n **do**
8: **for** k from 1 to m **do**
9: $\mathbf{e}[k] \leftarrow \mathbf{e}[k] + \mathbf{Q}[k, \mathsf{column}]\mathbf{s}[i]\mathbf{s}[j]$ ▷ Evaluate terms in $x_i x_j$
10: **end for**
11: $\mathsf{column} \leftarrow \mathsf{column} + 1$
12: **end for**
13: **end for**
14: **return** $\mathbf{e}$

Alg. 7: The algorithm for evaluating the public map at a point

2.7 Signatures with message recovery

It is possible to use the signature scheme in a message recovery mode. Whether or not message recovery is used does not affect the signature generation algorithm. The same key pair can be used to sign messages in message recovery mode and in appended signature mode, a signature for M in appended signature mode is unrelated to a signature for the same message in message recovery mode, because a different byte is appended to the message in each mode. The signing algorithm in message recovery mode differs from the signing algorithm in appended signature mode (Alg. 6) because the message is padded with `0x01` instead of `0x00` in lines 6 and 8. Furthermore, the procedure to determine the target of the public map is altered to make message recovery possible. In the appended signature mode, the target was determined by interpreting the $\frac{r}{8}m$ byte long output of a SHAKE function as a vector of m elements of $\mathbb{F}_{2^r}$. In message recovery mode, the target is obtained by interpreting

$$\text{SHAKE}(M||\texttt{0x01}, l_1)||\text{SHAKE}(\text{SHAKE}(M||\texttt{0x01}, l_1), l_2) \oplus M'$$

as a vector of m elements in $\mathbb{F}_{2^r}$, where l_1 is equal to 256 if SHAKE128 is used, or equal to 512 if SHAKE265 is used, and l_2 is equal to $\frac{r}{8}m - l_1$, and M' is formed by taking the last

Algorithm Verify

input: $(\mathsf{public\_seed}, \mathbf{Q}_2)$ — A public key
M — A message
$\mathbf{s}$ — A candidate–signature

output: **Accept** if $\mathbf{s}$ is a valid signature for M, **Reject** otherwise

1: $\mathsf{sponge} \leftarrow$InitializeAndAbsorb$(M||\mathtt{0x00})$
2: $\mathbf{h} \leftarrow$SqueezeHashDigest(sponge)
3: $\mathbf{e} \leftarrow$EvaluatePublicMap$((\mathsf{public_seed}, \mathbf{Q}_2), \mathbf{s})$
4: **if** $\mathbf{e} = \mathbf{h}$ **then** ▷ Check if $\mathcal{P}(\mathbf{s}) = \mathbf{h}$
5: | **return Accept**
6: **else**
7: | **return Reject**
8: **end if**

Alg. 8: The signature verification algorithm in appended signature mode

$l_2 - 1$ bytes of the message M , appending the byte `0x01` from the right, and padding with zeros in the case that the message M is shorter that $l_2 - 1$ bytes.

The signature verification algorithm evaluates the public map $\mathcal{P}$ at the signature $\mathbf{s}$, and interprets the output as a sequence $\mathsf{first\_bytes}$ of l_1 bytes, concatenated with a sequence $\mathsf{last\_bytes}$ of l_2 bytes. The signature verification algorithm recovers up to $l_2 - 1$ bytes of the message M, by calculating

$$M' = \mathsf{last\_bytes} \oplus \mathrm{SHAKE}(\mathsf{first\_bytes}, l_2)$$

and removing the padding. If the computed value of M' does not end in a `0x01`, followed by a (possibly empty) sequence of `0x00`s, the signature is rejected. Otherwise, the signature is accepted if t_1 is equal to $\mathrm{SHAKE}(M||\mathtt{0x01}, l_1)$.

2.8 Encoding of objects

2.8.1 Encoding of finite field elements

The finite fields that are used by the various instantiations of the LUOV signature scheme are $\mathbb{F}_{2^8}, \mathbb{F}_{2^{16}}, \mathbb{F}_{2^{48}}, \mathbb{F}_{2^{64}}$ and $\mathbb{F}_{2^{80}}$.

Field of size 2^8. Field elements in the field $\mathbb{F}_{2^8}$ are represented as binary polynomials modulo the irreducible polynomial $f_8 = x^8 + x^4 + x^3 + x + 1$. This choice is arbitrary and does not affect the security of the scheme. An element of $\mathbb{F}_2[x]/(f_8)$ is encoded as the byte obtained by concatenating its coefficients, where the least significant bits correspond to the lowest degree terms.

Algorithm Verify

input: $(\mathsf{public\_seed}, \mathbf{Q}_2)$ — A public key
M — The first part of a message (possibly the empty string)
$\mathbf{s}$ — A candidate–signature

output: The full message M if $\mathbf{s}$ is a valid signature, **Reject** otherwise

$\mathbf{e} \leftarrow$EvaluatePublicMap$((\mathsf{public_seed}, \mathbf{Q}_2), \mathbf{s})$
$\mathsf{first_bytes}, \mathsf{last_bytes} \leftarrow$Enc$(\mathbf{e})$ ▷ Split $\mathbf{e}$ into l_1 and l_2 bytes
$\mathsf{padded_message} \leftarrow \mathsf{last_bytes} \oplus$SHAKE$(\mathsf{first_bytes}, l_2)$
if $\mathsf{padded\_message}$ is **not** properly padded **then**
  return Reject ▷ Reject if $\mathsf{padded\_message}$ doesn't end in $\mathtt{0x01}\,\mathtt{0x00}\cdots\mathtt{0x00}$
end if
$M \leftarrow M||$RemovePadding$(\mathsf{padded_message})$
$\mathsf{hash_digest} \leftarrow$SHAKE$(M||\mathtt{0x01}, l_1)$
if $\mathsf{first\_bytes} = \mathsf{hash\_digest}$ **then**
  return M
else
  return Reject
end if

Alg. 9: The signature verification algorithm in message recovery mode

Example.

$$\begin{aligned}\mathsf{Enc}(1) &= \mathtt{0x01}\\ \mathsf{Enc}(x^6) &= \mathtt{0x40}\\ \mathsf{Enc}(x + x^5 + x^7) &= \mathtt{0xa2}\end{aligned}$$

Field of size 2^{16}. Field elements in the field $\mathbb{F}_{2^{16}}$ are represented as binary polynomials modulo the irreducible polynomial $f_{16} = x^{16} + x^{12} + x^3 + x + 1$. This choice is arbitrary and does not affect the security of the scheme. An element of $\mathbb{F}_2[x]/(f_{16})$ is encoded as the two bytes obtained by concatenating its coefficients. The first byte represents the terms of degree 0 op to 7, the second byte represents the terms of degree 8 up to 15.

Example.

$$\begin{aligned}\mathsf{Enc}(1) &= \mathtt{0x01}\,\mathtt{0x00}\\ \mathsf{Enc}(x^8 + x^9) &= \mathtt{0x00}\,\mathtt{0x03}\\ \mathsf{Enc}(x + x^5 + x^7 + x^{15}) &= \mathtt{0xa2}\,\mathtt{0x80}\end{aligned}$$

Larger fields. The larger fields used by the scheme are seen as simple field extensions of $\mathbb{F}_{2^{16}}$. The irreducible polynomials of these field extensions are given in Table 1. If F is

Table 1: Irreducible polynomials used for representing finite fields.

Finite Field	Irreducible polynomial in $\mathbb{F}_2[X,x]/(f_{16})$
$\mathbb{F}_{2^{48}}$	X^3+X+1
$\mathbb{F}_{2^{64}}$	X^4+X^2+xX+1
$\mathbb{F}_{2^{80}}$	X^5+X^2+1

such an irreducible polynomial of degree d, an element of $\mathbb{F}_2[X,x]/(f_{16},F)$ is encoded by the $2d$ bytes obtained by concatenating the encodings of its coefficients in order of increasing degrees, i.e.

$$\mathsf{Enc}(c_0+c_1X+\cdots+c_{d-1}X^{d-1}) = \mathsf{Enc}(c_0)\cdots\mathsf{Enc}(c_{d-1})$$

2.8.2 Encoding of private key

A private key for the LUOV signature scheme is a sequence of 256 random bits (used to seed a Keccak1600 Sponge) and is simply encoded as a sequence of 32 bytes.

2.8.3 Encoding of public key

A public key of the LUOV signature scheme consists of a sequence of 32 bytes (which are used to seed a Keccak Sponge) and an m-by-$m(m+1)/2$ matrix with binary entries. The matrix is encoded by concatenating the columns and padding the result with zero bits to get a sequence of bits of length divisible by 8. Then, the sequence is interpreted as a sequence of bytes, where the first bits have the least significant values. The encoding of a public keys is $32+\lceil\frac{m^2(m+1)}{2}\frac{1}{8}\rceil$ bytes large.

Example. *For a parameter set with $m=3$, the public key could contain the matrix*

$$\mathbf{Q}_2 = \begin{pmatrix} 010111 \\ 111001 \\ 000101 \end{pmatrix} .$$

Concatenating its columns gives 010110010101100111, which results in the 3 bytes (01011001) (01011001) (11000000), so

$$\mathsf{Enc}(\mathtt{0x36}\cdots\mathtt{0x5d},\mathbf{Q}_2) = \underbrace{\mathtt{0x36}\cdots\mathtt{0x5d}}_{\textit{32-byte Public seed}}\ \underbrace{\mathtt{0x9a}\,\mathtt{0x9a}\,\mathtt{0x03}}_{T} .$$

2.8.4 Encoding of signature

A signature of the UOV signature scheme consists of a vector $\mathbf{s}\in\mathbb{F}_{2^r}^n$ of $n=v+m$ field elements. The encoding of the signature consists of the concatenation of the encodings of

these n field elements. The encoding of a signature is $\frac{nr}{8}$ bytes large. (r is always divisible by 8)

$$\mathsf{Enc}(\mathbf{s}) = \mathsf{Enc}(\mathbf{s}[0])\mathsf{Enc}(\mathbf{s}[1]) \cdots \mathsf{Enc}(\mathbf{s}[n-1])$$

2.9 Sampling objects with the SHAKE function

The LUOV signature scheme uses the SHAKE extendable-output functions to provide cryptographically secure pseudorandom bit-streams. First, a seed is fed into the Keccak1600 sponge construction. Then output bytes are squeezed from the sponge and interpreted as some mathematical object. This approach is used to generate the following objects:

- $\mathsf{public\_seed}$ — The public seed used to generate a large part of the public map $\mathcal{P}$.
- $\mathbf{T}$ — The matrix that determines the linear transformation that hides the UOV structure of the secret map $\mathcal{F}$.
- $\mathbf{h}$ — The hash digest of a message.
- $\mathbf{v}$ — An assignment to the vinegar variables.
- $\mathbf{C}, \mathbf{L}, \mathbf{Q}_1$ — A large part of the public map $\mathcal{P}$.

Before sampling objects from a Keccak sponge, the sponge has to be initialized to the all-zero state and used to absorb a seed. In our pseudocode description of the LUOV algorithm we refer to this operation as InitializeAndAbsorb, which receives a sequence of bytes as input, and outputs a Keccak sponge object that was initialized and has absorbed the input sequence. The sponge can then provide an arbitrarily long sequence of pseudorandom bytes with the Squeeze operation, which takes a sponge object and an integer b as input, outputs a sequence of b bytes and updates the state of the sponge, such that it can be used to squeeze more bytes if needed.

2.9.1 Squeezing public seed

A public seed, represented by 32 bytes, is simply obtained from a sponge by squeezing out the 32 bytes. This operation is called SqueezePublicSeed.

2.9.2 Squeezing T

The matrix $\mathbf{T} \in \mathbb{F}_2^{v \times m}$ is squeezed out of a sponge by squeezing $\lceil \frac{m}{8} \rceil v$ bytes from the sponge, and interpreting the bytes $(i-1)\lceil \frac{m}{8} \rceil + 1$ up to $i\lceil \frac{m}{8} \rceil$ as the i-th row of $\mathbf{T}$. If m is not divisible by 8, the most significant bits of the last byte (i.e. $i\lceil \frac{m}{8} \rceil$-th byte in the sequence) are ignored. This operation is referred to as SqueezeT.

Example. *Suppose $m = 3$, $v = 4$ and the following 4 bytes are squeezed from the Keccak sponge :*

```
0x49 0xa2 0x86 0x4d .
```

Then, the matrix $\mathbf{T} \in \mathbf{F}_2^{v \times m}$ is equal to

$$\begin{pmatrix} 001 \\ 010 \\ 110 \\ 101 \end{pmatrix} .$$

2.9.3 Squeezing hash digest and vinegar variables

The hash digest and the assignment to the vinegar variables are vectors over $\mathbb{F}_{2^r}$ of length $n = m + v$ and length v respectively. They are obtained by squeezing $n\frac{r}{8}$ and $v\frac{r}{8}$ bytes from the sponge and interpreting these as the encoding of n, respectively v elements of $\mathbb{F}_{2^r}$. These operations are referred to as SqueezeHashDigest and SqueezeVinegar.

2.9.4 Squeezing most part of the public map

The matrices $\mathbf{C} \in \mathbb{F}_2^{m \times 1}, \mathbf{L} \in \mathbb{F}_2^{m \times n}$ and $\mathbf{Q}_1 \in \mathbb{F}_2^{m \times (\frac{o(o+1)}{2} + mo)}$ are squeezed column by column from the Keccak sponge. Each column is obtained by squeezing $\lceil \frac{m}{8} \rceil$ bytes from the sponge, and interpreting these as m-bit long columns, ignoring the most significant bits of the last byte in the case that m is not divisible by 8. The process of sampling columns of coefficients of $\mathcal{P}$ is identical to the process of sampling rows of $\mathbf{T}$.

In total, $1+n+\frac{o(o+1)}{2}+mo$ columns are sampled from the sponge. The first column represents $\mathbf{C}$, the next n columns represent $\mathbf{L}$, and the remaining $\frac{o(o+1)}{2}+mo$ columns represent $\mathbf{Q}_1$. The entire operation is called SqueezePublicMap, it takes a sponge object as input and returns the matrices $\mathbf{C}, \mathbf{L}$ and $\mathbf{Q}_1$.

3 List of parameter sets (part of 2.B.1)

We define two sets of parameter choices. The first set aims to provide small signatures, which is suitable for applications where many signatures are communicated. The second set of parameter choices aims to minimize the combined cost of a signature and a public key and is more suitable when the signatures and the public key are both communicated, such as a chain of signatures anchored to a root certificate authority.

Table 2: Different parameter choices for the LUOV signature scheme. The first 3 choices provide small signatures, the last three choices give small public keys at the cost of larger signatures.

	claimed security level	r	m	v	SHAKE	|sig|	|pk|	|sk|	message recovery (optional)
LUOV-8-63-256	lvl 2	8	63	256	128	319 B	15.5 KB	32B	30 B
LUOV-8-90-351	lvl 4	8	90	351	256	441 B	45.0 KB	32B	25 B
LUOV-8-117-404	lvl 5	8	117	404	256	521 B	98.6 KB	32B	52 B
LUOV-48-49-242	lvl 2	48	49	242	128	1.7 KB	7.3 KB	32B	261 B
LUOV-64-68-330	lvl 4	64	68	330	256	3.1 KB	19.5 KB	32B	479 B
LUOV-80-86-399	lvl 5	80	86	399	256	4.7 KB	39.3 KB	32B	795 B

4 Detailed performance analysis (2.B.2)

4.1 Description of platform

The following measurements were collected using `supercop-20171020` running on a computer named `bas`. The CPU on `bas` is an Intel® Core™ i5-7500T running at 3.3 GHz. `bas` has 7.5GB of RAM and runs CentOS Linux release 7.4.1708. Benchmarks used `crypto_sign`, which ran on one core of the CPU. The gcc version 4.8.5 20150623 (Red Hat 4.8.5-16) was used.

4.2 Time

The median number of cycles consumed by the different algorithms are reported in Table 3. The measurements are made in appended signature mode, but there is no noticeable difference between the cycle count in appended signature mode and in message recovery mode. A more optimized implementation that uses vectorization instructions is likely to reduce the cycle counts significantly.

4.3 Space

For all parameter choices, the secret key consists of a **32-byte** seed.

The public key consists of a 4 byte seed, and the remaining $\frac{m^2(m+1)}{2}$ coefficients of the public map $\mathcal{P}$. This makes a total of $\mathbf{4} + \lceil \frac{\mathbf{m^2(m+1)}}{\mathbf{16}} \rceil$ **bytes**. If message recovery is used, the messages can be shortened by roughly 15% of the signature size.

Table 3: Median cycle counts of optimized implementation. Measured with `supercop20171020`. The SUPERCOP output files with the compiler flags that were used and the exact cycle counts for various message sizes are included in the Supporting_Documentation folder.

	claimed security level	Key generation (million cycles)	Signing (million cycles)	Verification (million cycles)
LUOV-8-63-256	lvl 2	21.0	5.87	4.93
LUOV-8-90-351	lvl 4	81.8	21.6	17.3
LUOV-8-117-404	lvl 5	146	36.5	29.7
LUOV-48-49-242	lvl 2	14.8	34.1	23.6
LUOV-64-68-330	lvl 4	50.8	111	66.1
LUOV-80-86-399	lvl 5	96.8	216	124

A signature consists of $v + m$ elements of the field $\mathbb{F}_{2^r}$, good for a total of $\frac{\mathbf{r(v+m)}}{\mathbf{8}}$ **bytes**.

The concrete sizes for the proposed parameter choices are displayed in Table 2.

When implemented properly, the signing and verification algorithms require very little RAM memory. The RAM usage of the signing algorithm is dominated by storing the augmented matrix for the linear system after fixing the vinegar variables. This requires storing $m(m+1)$ elements of $\mathbb{F}_{2^r}$. For the LUOV-8-63-256 parameter set this is 4032 bytes. Besides storing the public key and a signature, the memory requirements of the verification algorithm is dominated by the state of the Keccak sponge (i.e. 200 bytes), or storing the evaluation of the public map $\mathcal{P}$, (i.e. $rm/8$ bytes).

4.4 How parameters affect performance

Table 3 shows that key generation is faster for the parameter sets with large extension fields. This is so because key generation benefits from the smaller polynomial systems, without paying the price of more complex field arithmetic, since key generation works in $\mathbb{F}_2$.

In contrast, in our implementation of the signing and verification algorithms, the smaller size of the polynomial systems does not make up for the increased complexity of the field arithmetic. Therefore, signing and verification is faster for the parameter sets with smaller field extensions.

The size of the public key is only impacted by the parameter m, and scales as $O(m^3)$, therefore to keep the public key small m should not be too large. By increasing r, the degree of the field extension $\mathbb{F}_2 \subset \mathbb{F}_{2^r}$, the required value of m to achieve a fixed security level decreases. However, increasing r also increases the size of the signatures. Therefore, it is possible to make a trade-off between small public keys (i.e. large r) or small signatures (i.e. small r). We propose two sets of parameter choices, one aiming at small signatures, the other aiming at small public keys. By varying the parameter r it is possible to interpolate

between these parameter sets.

Example. *One might want a signature scheme that attains security level 2 with signatures as small as possible, subject to the condition that the public key is smaller than* $10KB$*. The best option from the proposed parameter sets would be LUOV-48-49-242, having signatures of* $1.7KB$ *and public keys of* $7.3KB$*. We can do better by adjusting the parameter* r*. For the choice* $r = 28$*, the python script that is included in the submission proposes the parameters* $m = 54, v = 247$*, resulting in signatures of* $1.0KB$ *and public keys of just under* $10KB$*.*

4.5 Optimizations

4.5.1 Bit slicing

The i-th row of $\mathbf{Q}_2$ is calculated using only the data $\mathbf{T}$ and the i-th row of $\mathbf{Q}_1$ and this calculation is exacly the same for each row. This is an ideal situation for using bit slicing. The bits in the columns of $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are packed into words and the computation is performed for all rows simultaneously. This greatly speeds up the key generation algorithm. This optimization is included in the reference implementation, because it does not affect the legibility of the code.

4.5.2 Precomputing $\mathcal{P}$ and $\mathcal{F}$

With each verification of a signature a lot of coefficients of the public map $\mathcal{P}$ have to be generated with the SHAKE function. According to the gprof profiler, this computation is responsible for roughly 75% of the cycle usage of the verification algorithm in our optimized implementation of the first parameter set. If enough memory is available (e.g. roughly 380 KB for the first parameter set) the coefficients of $\mathcal{P}$ can be precomputed and stored to speed up the verification of signatures. Similarly, the coefficients of the secret map $\mathcal{F}$ can be precomputed to speed up the signing algorithm. This optimization was not used in the reference or optimized implementation.

5 Expected strength (2.B.4)

The LUOV signature system is designed for EUF-CMA security. The parameters of the LUOV scheme are chosen such that lower bounds to the bit complexity of all the known attacks exceed the required complexity level by a margin of 10 percent to account for possible future improvements in the attacks. The process of choosing the parameters is implemented in a python script which is included in the submission package. The designer specifies the desired security level and chooses the size of the field extension, then the script determines the parameters m and v to reach the required security level. Larger field extensions lead to smaller public keys at the cost of larger signatures. Table 4 summarizes the lower bounds

to the complexity of the various attacks. An overview of the known attacks and what the lower bounds to their complexities are is given in section 6.

To reach security level 2 i.e. "Any attack that breaks the relevant security definition must require computational resources comparable to or greater than those required for collision search on a 256-bit hash function (e.g. SHA256/ SHA3-256)" we assure that all known attacks (except hash collision attacks) require at least 2^{160} operations. This number was determined by considering the estimated number of gates required to find a hash collision in SHA3-256 (i.e. 2^{146}), and increasing the exponent by a margin of 10 percent to allow for future improvements of the attacks. Similarly, to reach security level 4, we require that all known attacks require at least $2^{231} = 2^{210 \times 1.1}$ operations.

To reach security level 5, i.e. "any attack that breaks the relevant security definition must require computational resources comparable to or greater than those required for key search on a block cipher with a 256-bit key (e.g. AES 256)" we require that all classical attacks require at least 2^{299} operations, and all quantum attacks require at least 2^{257} operations. These numbers are obtained by considering the estimated number of classical gates (i.e. 2^{272}) or quantum gates (i.e. 2^{234}) and increasing their exponent by 10 percent to allow for future improvements of the attacks. In all attack scenarios the depth of a quantum computation is assumed to be bounded by 2^{64} quantum gates.

Table 4: Summary of attacks against our parameters. The table reports $\log_2$ of a lower bound to the number of operations required for each attack. Quantum computations are bounded to a depth of 2^{64} field operations.

		Direct forgery		UOV attack		Reconciliation attack	
(r, m, v)	security	optimal k	complexity	classical	quantum	classical	quantum
$(8, 63, 256)$	lvl 2	2	161	225	161	192	192
$(8, 90, 351)$	lvl 4	3	231	295	231	263	287
$(8, 117, 404)$	lvl 5	4	300	322	258	303	340
$(48, 49, 242)$	lvl 2	1	165	224	160	181	178
$(64, 68, 330)$	lvl 4	1	235	295	231	247	266
$(80, 86, 399)$	lvl 5	1	300	347	283	299	335

6 Analysis of known attacks (2.B.5)

The signature scheme is an adaptation of Oil and Vinegar [15] scheme that was proposed by Patarin in 1997. The Oil and Vinegar scheme is one of the best studied multivariate signature schemes which has, with the right parameter choices, withstood all cryptanalysis since 1997.

All the adaptations that LOUV makes to the Unbalanced Oil and Vinegar scheme (see Sect. 2.2) can be shown not to impact the security of the scheme (assuming the output of the Keccak1600 sponge construction is indistinguishable from random bits), an exception being the adaptation of lifting a public key of UOV over $\mathbb{F}_2$ to a large extension field. It requires some argument to show that a direct signature forgery against the modified scheme is as difficult as a direct signature forgery against UOV over the extension field. However, since the key generation algorithm is not changed by this adaptation, it is clear that a key recovery attack against LUOV is equivalent to a key recovery attack against UOV over $\mathbb{F}_2$.

We now give an overview of known attacks. The overview is based on the overview given in [5]; We have adapted the example to match one of the proposed parameter sets.

6.1 Direct attack

This attack tries to forge a signature for a certain message M by trying to find a solution $\mathbf{s} \in \mathbb{F}_{2^r}^n$ for the system $\mathcal{F}(\mathbf{s}) = \mathcal{H}(M)$. This is an instance of the MQ (Multivariate Quadratic) problem.

MQ Problem. Given a quadratic polynomial map $\mathcal{P} : \mathbb{F}_q^n \to \mathbb{F}_q^m$ over a finite field $\mathbb{F}_q$, find $\mathbf{x} \in \mathbb{F}_q^n$ that satisfies $\mathcal{P}(\mathbf{x}) = \mathbf{0}$.

Thomae and Wolf showed that finding a solution for an underdetermined system with $n = \alpha m$ can be reduced to finding a solution of a determined system with only $m + 1 - \lfloor \alpha \rfloor$ equations [17]. This means that as a system becomes more underdetermined it becomes easier to solve.

For all but very small values of q, (e.g. $q = 2, q = 3$), the best known classical algorithms to solve the MQ-problem for generic determined systems over finite fields use a hybrid approach [3, 4] that combines exhaustive search with Gröbner basis computations. In this approach k variables are fixed to random values and the remaining $n-k$ variables are found with a Gröbner basis algorithm such as F_4, F_5 or XL. If no assignment to the remaining $n-k$ variables exists that solves the system, the procedure starts again with a different guess for the first k variables. We require on average roughly q^k Gröbner basis computations until a solution is found. As a result, the optimal value of k decreases as q increases. The complexity of computing a Gröbner basis for a system of polynomials depends critically on the degree of regularity (d_{reg}) of that system. We refer to Bardet [1] for a precise definition of the degree of regularity.

The most costly part of the F_5 algorithm is doing Gaussian elimination on a large matrix with roughly $\binom{n+d_{reg}}{d_{reg}}$ rows and columns. The complexity of the F_5 algorithm is thus given by

$$C_{F_5}(n, d_{reg}) = O\left(\binom{n + d_{reg}}{d_{reg}}^{\omega}\right),$$

where $2 \leq \omega < 3$ is the constant in the complexity of doing Gaussian reduction on the matrices constructed in the Gröbner basis computation. These matrices are structured and sparse, which can be exploited to make Gaussian elimination more efficient [9]. The complexity of the hybrid approach is

$$C_{\text{Hybrid}F_5(n,d_{reg},k)} = O\left(q^k \binom{n - k + d_{reg}(k)}{d_{reg}(k)}^{\omega}\right), \tag{1}$$

where $d_{reg}(k)$ stand for the degree of regularity of the system after fixing the values of k variables.

Determining the degree of regularity for a specific polynomial system is difficult, but for a certain class of systems, called semi-regular systems, it is known that the degree of regularity can be deduced from the number m of equations and the number n of variables [1, 8]. In particular, for quadratic semi-regular systems the degree of regularity is the degree of the first term in the power series of

$$S_{m,n}(x) = \frac{(1 - x^2)^m}{(1 - x)^n}$$

that has a non-positive coefficient. This gives a practical method to calculate the degree of regularity of any semi-regular system. Empirically, polynomial systems that are randomly chosen have a very large probability of being semi-regular and it is conjectured that most systems are semi-regular systems. For the definition and the theory of semi-regular systems we refer to chapter 3 of the PhD thesis of Bardet [1].

In a direct attack against the LUOV scheme all the coefficients of the system that needs to be solved lie in $\mathbb{F}_2$, except those of the constant terms, because those coefficients come from the message digest. We claim that this property does not significantly reduce the hardness of finding solutions relative to the case where the coefficients are generic elements of $\mathbb{F}_{2^r}$. By definition [1], the degree of regularity of a polynomial system does only depend on its quadratic part, and it is apparent that lifting a polynomial system to a field extension does not affect its degree of regularity. Therefore, the degree of regularity of a LUOV public key follows the same distribution of a UOV public key over the field $\mathbb{F}_2$, even after fixing a number of variables. It has been observed by Faugère and Perret [10] that polynomial systems that result from fixing $\approx v$ variables in a UOV system behave like semi-regular systems, whose degree of regularity does not depend on q. Therefore, the degree of regularity of a LUOV public polynomial system is distributed identically to that of a UOV public polynomial system, independently of the size q of the finite field that is used.

Since the degree of regularity, in combination with the number of variables, determines the complexity of a Gröbner basis computation (measured in number of field operations), a Gröbner basis computation on the LUOV polynomial system is not significantly more efficient than a Gröbner basis computation against regular UOV with the same parameters. This argument is confirmed by the experimental data in Table 5. There we see that a direct attack is slightly faster against the modified scheme than against the original UOV scheme, but only by a small constant factor. Even though the Gröbner basis is computed over $\mathbb{F}_{2^r}$, the largest part of the arithmetic only involves the field elements 0 and 1, so the arithmetic is faster than with generic elements of $\mathbb{F}_{2^r}$. This is where the difference observed in Table 5 comes from. If we do the same experiment with a smaller extension field such as $\mathbb{F}_{2^8}$ there is no observed difference between the running time of a direct attack against a regular UOV scheme and our modified scheme.

Remark. *In a direct attack one fixes $\approx v$ variables randomly to make the system a slightly overdetermined system. In our experiments we have fixed these variables to values in $\mathbb{F}_2$ to make sure that we do not introduce linear terms with coefficients in $\mathbb{F}_{2^r}$ instead of $\mathbb{F}_2$ in the case of the modified UOV scheme.*

Table 5: Running time of a direct attack against the regular UOV scheme over $\mathbb{F}_{2^{64}}$ and the modified UOV scheme, with the MAGMA v2.22-10 implementation of the F4 algorithm. We did not implement the method of Thomae and Wolf [17].

(m, v)	Regular UOV (s)	Lifted UOV (s)	difference
(7,35)	0.43	0.21	-52%
(8,40)	1.56	0.76	-51%
(9,45)	7.00	3.21	-54%
(10,50)	33.50	17.44	-48%
(11,55)	132.88	76.60	-42%
(12,60)	828.31	588.33	-29%

To obtain a lower bound to the complexity of a Gröbner basis computation we assume that the parameter ω in the complexity of Gaussian elimination on the matrices constructed

in the Gröbner basis algorithm is equal to 2 and that the constant factor hidden by the big O notation is equal to 1. That is, in Eqn. (1) we put $\omega = 2$ and we drop the big O notation to get a concrete lower bound to the number of bit operations of a hybrid attack. Even though this is a generous lower bound, we require that this lower bound exceeds the required bit complexity by 10 percent when choosing parameters. This is done to allow for future improvements in algorithms that find solutions to polynomial equations.

Example. *We will estimate the complexity of a direct attack against LUOV with the parameter set* $(r = 8, m = 63, v = 256)$*; this set is proposed as a set that achieves security level 2. Using the method of Thomae and Wolf. we can reduce finding a solution to this underdetermined system to finding a solution of a determined system with* $63+1-\lfloor(63+256)/63\rfloor = 59$ *equations. We assume this system, and the systems that are derived by fixing a number of variables, to be semi-regular. If we fix* k *extra variables the degree of regularity is equal to the degree of the first term in the power series of*

$$S_{59,59-k}(x) = \frac{(1-x^2)^{59}}{(1-x)^{59-k}}$$

which has a non-positive coefficient. For $k = 0$ *we have* $S_{59,59}(x) = (1+x)^{59}$*, so the degree of regularity is* 60*. For* $k = 1$ *we have*

$$S_{59,58}(x) = 1 + 58x + 1652x^3 + \cdots + 3814986502092304x^{29} + 0x^{30} + O(x^{31}),$$

where all the omitted terms have positive coefficients, so the degree of regularity is 30*. We can now use (1) to obtain a lower bound to the complexity of the hybrid approach. For* k *equal to* 0 *and* 1 *this is equal to*

$$\binom{59+60}{60}^2 \approx 2^{230.4} \qquad \text{and} \qquad 2^8\binom{59-1+30}{30}^2 \approx 2^{164.0}$$

respectively. Repeating this calculation for higher values of k *we eventually see that the optimal value of* k *is 2, the corresponding degree of regularity is* 27 *and the complexity of the direct attack is estimated as* $2^{161.3}$*. Thus, this lower bound exceeds* $2^{146\times 1.1}$*, as required.*

In theory, a quantum attacker could use Grover search instead of the brute force part of the hybrid approach to speed up a direct attack. The complexity of this attack would be

$$C_{\text{Hybrid}F_5(n,d_{reg},k)} = O\left(q^{k/2}\binom{n-k+d_{reg}(k)}{d_{reg}(k)}^{\omega}\right), \tag{2}$$

where the only difference with (1) is that the factor q^k is replaced by $q^{k/2}$. However, this attack is not possible if the depth of a quantum computation is limited to, say, 2^{64} operations. For all our parameter choices and all practical values of k, the complexity of even a single Gröbner basis computation is beyond 2^{64}, and the Grover algorithm should do a large number of these computations sequentially in order to enjoy a noticeable speedup over the classical brute force search.

6.2 Key recovery attacks.

Since the key pair generation algorithm used by the LUOV scheme is identical to that of the original UOV scheme over the field $\mathbb{F}_2$ it is clear that a key recovery attack against the Lifted UOV scheme is equivalent to a key recovery attack against a regular UOV scheme over $\mathbb{F}_2$. Key recovery attacks against UOV have been investigated ever since the invention of the Oil and Vinegar scheme in 1997 [15], so it is well understood which attacks are possible and what the complexities of these attacks are. It is also clear that we can make key recovery attacks harder by increasing the number of vinegar variables.

6.2.1 UOV attack

Patarin [15] suggested in the original version of the Oil and Vinegar scheme to choose the same number of vinegar and oil variables, or $v = m$. This choice was cryptanalyzed by Kipnis and Shamir [14]: they showed that an attacker can find the inverse image of the oil variables under the map $\mathcal{T}$. This is enough information to find an equivalent secret key, so this breaks the scheme. This approach generalizes for the case $v > m$; the complexity then increases to $O(q^{v-m}n^4)$ [13] and is thus exponential in $v - m$. Since a UOV attack on the Lifted UOV scheme is equivalent to a UOV attack over $\mathbb{F}_2$, we have that the complexity of a UOV attack against the Lifted UOV scheme is approximately $2^{v-m-1} \cdot n^4$ binary operations.

The generalized UOV attack chooses a random linear combination of the matrices that represent the quadratic parts of the polynomials in the public system and computes the minimal eigenspaces of the matrix. With probability 2^{m-v+1} this computation yields a vector in the oil subspace. This means that a quantum attacker can use the Grover search algorithm [11] to look for a random linear combination that will yield a vector in the oil subspace. Ignoring issues of 'Groverizing' the algorithm such as making the computation reversible and the probabilistic nature of the eigenspace computation, the complexity of a quantum attack becomes $2^{\frac{v-m-1}{2}}n^4$. If we limit the depth of a quantum computation to 2^{depth}, and we ignore the depth of the eigenspace-finding subroutine, the complexity of an attack is at least $\max(2^{\frac{v-m-1}{2}}n^4, 2^{v-m-1}n^4/2^{depth})$.

6.2.2 Reconciliation attack

The reconciliation attack against the lifted UOV scheme is equivalent to the UOV reconciliation attack against UOV over the field $\mathbb{F}_2$. A lower bound on the complexity of this attack is given by the complexity of solving a quadratic system of v variables and v equations over $\mathbb{F}_2$, but the problem is expected to be harder [5]. There exists specialized algorithms for solving polynomial systems over $\mathbb{F}_2$ that are more efficient than the generic hybrid approach. One method is a smart exhaustive search, which requires approximately $log_2(n)2^{n+2}$ bit operations [6]. The BooleanSolve algorithm [2] combines an exhaustive search with sparse linear algebra to achieve a complexity of $O(2^{0.792n})$. However the method only becomes faster than

the exhaustive search method when $n > 200$. Recently, Joux and Vitse proposed a new algorithm that was able to solve a Boolean system of 146 quadratic equations in 73 variables in one day [12]. The algorithm beats the exhaustive search algorithm, even for small systems. The complexity of this algorithm is still under investigation, but a rough estimate based on the reported experiments suggests that the number of operations scales like $2^{\alpha n}$ with α between 0.8 and 0.85 and with a constant factor between 2^7 and 2^{10}. For choosing the parameters of the LUOV signature scheme, we have assumed that finding a solution to a determined system of n quadratic Boolean equations requires $2^{0.75n}$ operations in $\mathbb{F}_2$, even though this is likely to seriously overestimate the capabilities of the state of the art algorithms.

Due to the limit on the circuit depth of quantum computations, the Gröbner based methods of solving a Boolean system cannot be 'Groverised'. In contrast, quantum attackers can still use a brute force Grover search to solve systems over $\mathbb{F}_2$ with $2^{n/2}$ sequential evaluations of the polynomials in the system. However, if the depth of a quantum computation is restricted to 2^{depth} evaluations of the polynomials, the required number of polynomial evaluations in a Grover search is at least $\max(2^{n-depth}, 2^{n/2})$. Asymptotically this is worse than the classical Gröbner basis based methods, which is why the reported hardness of a quantum reconciliation attack in Table 6 is higher than the hardness of the classical reconciliation attack. One would expect quantum attacks to be at least as efficient as classical attacks, because a quantum computer can simulate a classical computer. In our analysis this is not the case, because the depth of a quantum computation is assumed to be limited, which is not the case for a classical computation.

6.3 Hash collision attack

As is the case for all hash-and-sign digital signature algorithms, a hash collision can be exploited to break the EUF-CMA security definition. The SHAKE extendable output functions are used to generate a hash digest of the required length. The parameter sets claiming a security level 2 use SHAKE-128, those claiming security level 4 or 5 use SHAKE-256. In each proposed parameter set the output length (i.e. rm bits) is large enough to reach the required hardness of finding collisions. Therefore, a hash collision attack does not threaten the claimed security levels.

7 Advantages and limitations (2.B.6)

7.1 Advantages

- **Small signatures.** Like many other MQ signature schemes, the signatures of the LUOV scheme are very small. For security level 2 the signatures are only 319 bytes long.

- **A wide security margin** Instead of trying to estimate the complexity of existing attacks and choosing the parameters such that these estimates match the required security level we have formulated conservative lower bounds to plausible attacks. For example, we have assumed that a classical attacker can solve a determined system of n Boolean quadratic polynomials with only $2^{0.75n}$ bit operations, whereas the best known algorithms seem to require $2^{0.80n+7}$ operations at best. On top of our conservative lower bounds, we require the $\log_2$ of this lower bound to exceed the $\log_2$ of the required number of operations by 10% (see Sect. 5).

- **Simple arithmetic.** The scheme only uses SHA-3 and simple arithmetic operations over $\mathbb{F}_2$ or over an extension field. Arithmetic over $\mathbb{F}_2$ translates to the operations AND and XOR, while the arithmetic over an extension field can be implemented with XOR, additions and table lookups in small tables. This makes the algorithm very suitable for hardware implementations.

- **Message recovery.** It is possible to use the LUOV scheme in a message recovery mode. In this mode, a part of the message can be recovered from the signature and does not need to be communicated. This can reduce the size of a message-signature pair by up to 15 percent of the signature size.

- **Deterministic signatures.** The generation of a signature does not require any external source of randomness. This makes a secure implementation easier and excludes any attack that might exploit the usage of a poor source of randomness.

- **Stateless.** The signing algorithm does not need to maintain a state between signing sessions and can sign an unbounded number of messages. This makes a secure implementation of the algorithm easier.

- **Flexible.** The parameters of the signature are easily adjustable to reach a specific security level. It is also possible to choose parameters to make a trade-off between small signatures and small public keys.

- **Diversity.** Multivariate cryptography relies on a different hard problem than other branches such as lattice cryptography or hash-based cryptography. It is prudent to have cryptographic algorithms that rely on a diverse set of hard problems such that if one hard problem is broken and wipes out a branch of cryptography, there are alternative algorithms available.

7.2 Limitations

- **Public key size.** Even though the public key size of the LUOV scheme is much smaller than the public key size of other MQ signature schemes, it remains larger than the public key size of some other post quantum signature schemes. It is possible to mitigate this problem by making a trade-off for a smaller public key at the cost of larger signatures.

- **No encryption or KEM.** The LUOV scheme is a digital signature scheme. This submission does not include an encryption scheme or a key encapsulation mechanism.

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A Statements

These statements "must be mailed to Dustin Moody, Information Technology Laboratory, Attention: Post-Quantum Cryptographic Algorithm Submissions, 100 Bureau Drive – Stop 8930, National Institute of Standards and Technology, Gaithersburg, MD 20899-8930, or can be given to NIST at the first PQC Standardization Conference (see Section 5.C)."

First blank in submitter statement: full name. Second blank: full postal address. Third, fourth, and fifth blanks: name of cryptosystem. Sixth and seventh blanks: describe and enumerate or state "none" if applicable.

First blank in patent statement: full name. Second blank: full postal address. Third blank: enumerate. Fourth blank: name of cryptosystem.

First blank in implementor statement: full name. Second blank: full postal address. Third blank: full name of the owner.

A.1 Statement by Each Submitter

I, Ward Beullens, of Afdeling ESAT - COSIC, Kasteelpark Arenberg 10 - bus 2452, 3001 Heverlee, Belgium, do hereby declare that the cryptosystem, reference implementation, or optimized implementations that I have submitted, known as LUOV, is my own original work, or if submitted jointly with others, is the original work of the joint submitters. I further declare that (check one):

☑ *I do not hold and do not intend to hold any patent or patent application with a claim which may cover the cryptosystem, reference implementation, or optimized implementations that I have submitted, known as LUOV OR (check one or both of the following):*

☐ *to the best of my knowledge, the practice of the cryptosystem, reference implementation, or optimized implementations that I have submitted, known as LUOV may be covered by the following U.S. and/or foreign patents: None*

☐ *I do hereby declare that, to the best of my knowledge, the following pending U.S. and/or foreign patent applications may cover the practice of my submitted cryptosystem, reference implementation or optimized implementations: None*

I do hereby acknowledge and agree that my submitted cryptosystem will be provided to the public for review and will be evaluated by NIST, and that it might not be selected for standardization by NIST. I further acknowledge that I will not receive financial or other compensation from the U.S. Government for my submission. I certify that, to the best of my knowledge, I have fully disclosed all patents and patent applications which may cover my cryptosystem, reference implementation or optimized implementations. I also acknowledge and agree that the U.S. Government may, during the public review and the evaluation process, and, if my submitted cryptosystem is selected for standardization, during the lifetime of the standard, modify my submitted cryptosystem's specifications (e.g., to protect against a newly discovered vulnerability).

I acknowledge that NIST will announce any selected cryptosystem(s) and proceed to publish the draft standards for public comment.

I do hereby agree to provide the statements required by Sections 2.D.2 and 2.D.3, below, for any patent or patent application identified to cover the practice of my cryptosystem, reference implementation or optimized implementations and the right to use such implementations for the purposes of the public review and evaluation process.

I acknowledge that, during the post-quantum algorithm evaluation process, NIST may remove my cryptosystem from consideration for standardization. If my cryptosystem (or the derived cryptosystem) is removed from consideration for standardization or withdrawn from consideration by all submitter(s) and owner(s), I understand that rights granted and assurances made under Sections 2.D.1, 2.D.2 and 2.D.3, including use rights of the reference and optimized implementations, may be withdrawn by the submitter(s) and owner(s), as appropriate.

Signed: Ward Beullens

Title:
Date:
Place:

A.2 Statement by Reference/Optimized Implementations' Owner(s)

I, Ward Beullens, Afdeling ESAT - COSIC, Kasteelpark Arenberg 10 - bus 2452, 3001 Heverlee, Belgium, am the owner or authorized representative of the owner Ward Beullens of the submitted reference implementation and optimized implementations and hereby grant the U.S. Government and any interested party the right to reproduce, prepare derivative works based upon, distribute copies of, and display such implementations for the purposes of the post-quantum algorithm public review and evaluation process, and implementation if the corresponding cryptosystem is selected for standardization and as a standard, notwithstanding that the implementations may be copyrighted or copyrightable.

Signed: Ward Beullens
Title:
Date:
Place: