Phys 515 Assignment #3 Due: Nov 4, 1 pm

Consider an experiment whose observable is modeled by a random variable, X, following the beta distribution:

$$f(x|\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$$

With repeated measurements of the observable, we estimate the values of the parameters α and β.

1. Use the provided data file (40 measurements) to produce point estimates for the two parameters using maximum likelihood. Estimate the standard deviations and correlation coefficient for the estimators. Use numerical methods (see example posted on course website), not an analytic approach.
2. Plot the data as a histogram, and overlay the scaled pdf with the point estimates.
3. Show the log-likelihood function (using the data file) as a contour plot (as a function of α and β). The range of the plot should be $[1,5]$ in α and $[2,10]$ in β. Show the point of maximum likelihood as a point. Show lines of constant log likelihood 0.5, 1.15, 2.31, 3.00 below maximum likelihood. (See page 288 in notes.)
4. Overlay on the contour plot, lines that indicate the 1-D intervals for the two parameters: $[\hat{\alpha}-\sigma_\alpha, \hat{\alpha}+\sigma_\alpha]$ and $[\hat{\beta}-\sigma_\beta, \hat{\beta}+\sigma_\beta]$.
5. Produce 1000 datasets with sample size 40, with $\alpha = \hat{\alpha}$ and $\beta = \hat{\beta}$. Estimate the parameters for each dataset.
    a. Are the standard deviations from (1.) good estimates of the standard deviations of the 1000 parameter estimates?
    b. Is the correlation coefficient from (1.) a good estimate of the correlation coefficient from the 1000 pairs of parameter estimates?
    c. Are the estimators biased? Estimate the significance of the bias for each parameter.
    d. For what fraction of the datasets is the log-likelihood for the true parameter values within 1.15 of the maximum log-likelihood? (See page 288 in notes).

You will probably find the methods provided by scipy.stats.beta to be helpful in doing this assignment.

Submit your python notebook file (.ipynb) to the website.