Phys 515 Assignment #2 Due: Oct 21, 1 pm

Consider an experiment that measures times y_i between successive light flashes. Model these times as outcomes of a random variable $Y = X + T$, in which X which follows the uniform distribution $[0, d]$ and T follows the exponential distribution, with mean lifetime $\tau = 1$. The null hypothesis has $d = 5$ and there is a single alternative hypothesis with $d = 4$. From the Bayesian viewpoint, the two hypotheses are deemed equally likely prior to the measurement.

Use the sample mean as a test statistic to test the Null Hypothesis for the dataset posted for the assignment (n = 40).

1. Make a histogram that shows the data in 40 bins from 0. to 10. Overlay the pdfs (derived by an analytic convolution) for the two hypotheses appropriately scaled to make a direct comparison.
2. Evaluate the data test statistic value for the dataset.
3. Find sampling distributions for the test statistic by generating 10,000 toy samples for each hypothesis (produce each toy sample with 40 events). Plot normalized histograms of the test statistic for these toy samples, with bin size 0.1. These represent the sampling distributions and an example is shown below. Using the central limit theorem, derive approximate pdfs for the sampling distributions, and overlay these on the plot. Are these good approximations?
4. Frequentist Test: Calculate the p-value to test the null hypothesis. Is the null hypothesis rejected at the 95% confidence level?
5. Bayesian Test: Calculate the posterior probability for the null hypothesis. To estimate the probability densities for the sampling distributions at the value of the data test statistic, count the number of toy samples that have a test statistic within 0.1 of the data test statistic value.
6. Repeat 5, using the approximate pdfs derived using the central limit theorem.

In your electronic submission, summarize your approach and your results and include the plots in the same file. Create a single pdf (portable document file) and submit it, along with your software code to the course web site before the due date. Alternatively, you can submit a python notebook file (.ipynb) that contains the summary in “markdown cells” and python code.