PHYS422 Assignment 5

Due: March 21, 2019

You **must** show all work - if your solution is not supported by your work, you will not be given points for either. It is not the marker's responsibility to *decode* your work; they will not award marks if they cannot understand your work. **Solutions should be reasonably simplified to assist the marker.** Simplifying is an important aspect of readability.

1. Griffiths skips many steps in deriving the Laplacian of V and the Jefimenko equations. Fill in the missing steps and write out the full derivation. This includes, but is not limited to:
   - proving Eq. 10.28
   - showing the intermediate steps to prove each equation after Eq. 10.29 (you do not need to prove the delta function step)
   - fill in the steps for Eq. 10.36
   - fill in any other steps to get Eq. 10.37 and 10.38
2. Quasi-static approximation. Throughout the text, Griffiths employs the quasi-static approximation without much justification. Now let's examine it in closer detail.

   a) Taylor expand $\rho(\vec{r}', t_r)$, $\dot{\rho}(\vec{r}', t_r)$, $\vec{J}(\vec{r}', t_r)$, and $\dot{\vec{J}}(\vec{r}', t_r)$ about time t. Truncate your result with an ellipsis so that each is expanded up to the third order derivative.
   b) Use these to simplify Jefimenko's Equations.
   c) Based on this, what are the quasi-static approximation conditions necessary to recover Coulomb's Law and the Biot-Savart Law? (Recover them independently, don't assume you need the same conditions for both.)
3. The electric field for a moving point charge is $\vec{E} = \frac{q}{4\pi\epsilon_o}\frac{1-\beta^2}{(1-\beta^2\sin^2\theta)^{3/2}}\frac{\hat{R}}{R^2}$, where $\vec{R}$ is the displacement vector from the current location of the charge to the observation position. The magnetic field for the moving point charge is $\vec{B} = \frac{1}{c^2}(\vec{v} \times \vec{E})$. Letting the charge become differential, such that $q \to dq = \lambda dz'$, solve for the electric field and magnetic field for an infinitely long, neutral wire carrying a current. (Treat the current as a moving charge density $-\lambda$ with stationary charge density $+\lambda$.)
4. A charge q_1 is located at $\vec{r}_1 = -vt_o\hat{x}$ and traveling at speed v in the $+\hat{x}$ direction. Another charge q_2 is located at $\vec{r}_2 = -vt_o\hat{y}$ and traveling at speed v in the $+\hat{y}$ direction. Solve for the force on each charge due to the other one and verify whether Newton's Third Law applies.
5. Two equal charges q are moving towards each other along the z-axis at speed v. When the charges are each a distance a away from the origin, find the force between them by integrating Maxwell's Stress Energy Tensor over an appropriate region of space.
6. COMPUTATION: Expand upon your FDTD code from the previous assignment to include fully reflective boundaries a distance of 10σ from the source location, and a dielectric with $\epsilon_r = 5$ at 7σ from the source location. (Do this symmetrically, so there are reflective boundaries and dielectrics on either side of the origin, from which the wave originates.) Run the simulation until the wave has reflected off of the bounds of the plot at least twice.