PHYS422 Assignment 2

Due: January 31, 2019

You **must** show all work - if your solution is not supported by your work, you will not be given points for either. It is not the marker's responsibility to *decode* your work; they will not award marks if they cannot understand your work. **Solutions should be reasonably simplified to assist the marker.** Simplifying is an important aspect of readability.

1. Two concentric, conducting, spherical shells of radii a and $a + \delta$ are separated by a weakly conducting material of conductivity σ.
   a) Given that the potential difference between them is ΔV, solve for the charge on the inner sphere in terms of ΔV. Why is the charge on the outer sphere irrelevant?
   b) Solve for the current flowing between the two shells in terms of the potential difference. For what value of δ is the effective resistance between the two shells maximized? What is that maximum value?
   c) Two identical spherical shells of radius a are placed a large distance $L \gg a$ apart under the Earth. What is the effective resistance between them, given the above results? Look up Single-Wire Earth Return systems and give a brief overview of why this question is useful to understand.

2. Consider two lines of current, I, separated by a distance $2a$, travelling in the same direction.
   a) Use Maxwell's stress-energy tensor to determine the force per unit length between the wires. (Refer to problem 8.4 in Griffiths for some help understanding this.)
   b) Repeat this for two lines of current travelling in opposite directions.

3. The energy density for a complex valued electric and magnetic field is given by $u = \frac{1}{2}(\epsilon_o|\vec{E}|^2 + \frac{1}{\mu_o}|\vec{B}|^2)$. Use this expression to solve for an expression of the Poynting Vector for complex valued electric and magnetic fields. (Note: It should work out to be the same as the Poynting Vector for real valued fields in the limit the the imaginary component is zero.)

4. The following time varying electric and magnetic fields are observed in a region of space: $\vec{E} = A\frac{\sin\theta}{r}\cos(kr - \omega t)\hat{\theta}$ and $\vec{B} = \frac{1}{c}A\frac{\sin\theta}{r}\cos(kr - \omega t)\hat{\phi}$. Find an expression for the energy density of that region of space, as a function of time.

5. COMPUTATION: A circular loop of wire of radius $R = 10$cm with central axis along $\hat{z}$ carries a current I. A second circular loop of wire with radius $r = 5$cm is centred at a point along the $\hat{z}$ axis, 12cm away from the larger loop, and initially parallel to the larger loop.
   a) Produce a vector/stream plot of the magnetic field around the larger circular wire in the $\hat{s} - \hat{z}$ plane.
   b) Assuming the smaller loop of wire is rotating about about its centroid in the $\hat{x}$ axis, solve for the induced current as a function of time in the smaller loop.