PHYS422 Assignment 4

Due: March 7, 2019

You **must** show all work - if your solution is not supported by your work, you will not be given points for either. It is not the marker's responsibility to *decode* your work; they will not award marks if they cannot understand your work. **Solutions should be reasonably simplified to assist the marker.** Simplifying is an important aspect of readability.

1. Conducting media.
   a) Simplify the skin depth formula in the two limits of bad conductors ($\frac{\omega\epsilon}{\sigma} \gg 1$) and good conductors ($\frac{\omega\epsilon}{\sigma} \ll 1$) to illustrate the dependence of the skin depth on wavelength in both cases.
   b) Simplify the magnetic field phase shift formula to approximate its value in each of these two limits. (Simplify as much as possible!)
   c) Simplify the ratio of the magnetic and electric field amplitudes in each of these two limits.
   d) Find an expression for the time averaged energy density in a conducting medium (the energy density will not be uniform in space). Which term is the dominant term in each of the two limits - electric or magnetic contributions?

2. For a material with a single natural frequency, ω_1, find the frequencies at which the absorption coefficient, α, is half its maximum value. Find the frequencies at which the index of refraction takes on its minimum and maximum values.

3. Transverse Electric Waves
   a) What is the electric field of a TE wave?
   b) Find the Poynting vector and energy density for TE waves in a rectangular waveguide.
   c) What does the integral of the Poynting vector and the energy density over the cross section of the rectangular waveguide represent? Use this information to find the speed at which energy is transmitted along the TE waveguide.

4. COMPUTATION: Use the Finite Difference Time Domain method to solve for E_z and H_y for a Gaussian source located at $x = 0$ in free space. The source should take the form $E_z(0,t) = E_o e^{-(t-t_o)^2/\sigma^2}$. You should choose appropriate values of t_o and σ (hint: try different values and see what you get! choose one that makes for nice looking plots). You can either produce a river plot or an animated GIF. You should run the simulation until the wave passes the edges of the plot.
   Note: You should ensure that your waveform does not reflect at the boundaries of your calculation!