PHYS422 Assignment 1

Due: January 17, 2019

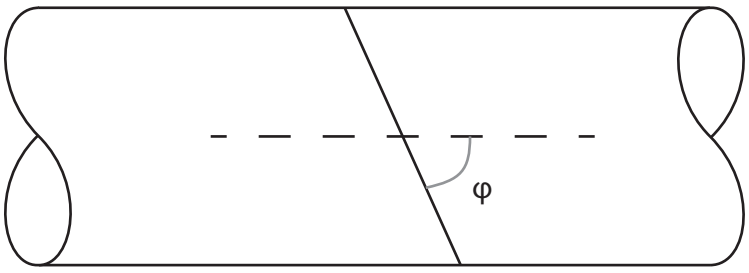
You **must** show all work - if your solution is not supported by your work, you will not be given points for either. It is not the marker's responsibility to *decode* your work; they will not award marks if they cannot understand your work. **Solutions should be reasonably simplified to assist the marker.** Simplifying is an important aspect of readability.

1. The multipole expansion can be written as a spherical harmonic expansion, assuming $r \gg r'$. Using this, solve for and write expressions for the generic a) monopole, b) dipole and c) quadrupole terms of the electric field for an arbitrary charge distribution.

2. The hydrogen wavefunctions can be used to solve for the charge distribution of the electron surrounding the nucleus. Taking the nucleus to be a positively charged point particle, solve for the monopole and dipole terms of the electric potential and electric field of the following states:
   a) $\psi_{2,1,0} = \dfrac{1}{4\sqrt{2\pi}a^{3/2}} \dfrac{r}{a} e^{-r/2a} \cos\theta$
   b) $(\psi_{2,1,+1} + \psi_{2,1,-1})/\sqrt{2} = \frac{1}{8\sqrt{\pi}a^{3/2}} \frac{r}{a} e^{-r/2a} \sin\theta\cos\phi$

3. Solve for the polarizability of hydrogen in the $1s^1$ and $2s^1$ orbitals.

4. Two cylinders with magnetic susceptibilities χ_a and χ_b have their ends sliced off at an angle ϕ so that they fit together to make one long cylinder, as in the figure. The material with susceptibility χ_a (left of the interface in the figure) has a constant magnetization directed along the central axis of the cylinder ($\vec{M}_a = \vec{M}_0\hat{z}$). What is the angle of the induced magnetic field in the material with χ_b right at the boundary layer, in terms of the angle ϕ and susceptibilities χ_a and χ_b?



5. COMPUTATION: Produce plots of the electric field of four point charges: two positive ($+q_o$) located at $\vec{r}_{1,2} = \pm a\hat{z}$ and two negative ($-q_o$) located at $\vec{r}_{3,4} = \pm a\hat{x}$. You can use either a stream plot or a vector plot to indicate the direction of the electric field, but the magnitude of the electric field should be plotted as a density plot (or a 2D histogram). Note: The analytical expression for the field should be determined on paper, and is trivially easy. Your grade will be based on how well you convey information through the plotting.